

**CS 880: Quantum Algorithm**  
**Homework NUM: 1**

## Exercise 1

Consider a system with  $m = 1$ . Determine the output distributions of each of the following processes:

1. Start in state  $|0\rangle$  and apply  $H$ , apply again, and measure.
2. Start in state  $|0\rangle$  and apply  $H$ , measure, apply  $H$ , and measure.

## Solution

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (1)$$

1.

$$S_0 = HH|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2)$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (3)$$

$$(4)$$

2.

$$S_0 = H|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (5)$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (6)$$

$$(7)$$

If the state collapse, it will be a probability distributions of  $|0\rangle$  and  $|1\rangle$ .

$$P(|0\rangle) = \frac{1}{2} \quad (8)$$

$$P(|1\rangle) = \frac{1}{2} \quad (9)$$

$$(10)$$

Then if we apply the  $H$  again, we will get a similar equal probability distributions for each state we observes.