hw9

Problem 1

We know $\sin(\theta_0)^2 = p \implies \sin(\theta_0) = \sqrt{p}$

Every iteration we add the angle by θ_0^2, which means the success probability is a sequence

$$\sin(\theta_0)^2, \sin(3\theta_0)^2, \sin(5\theta_0)^2 \cdots$$

From wolframalpha, the result

$$(1 - \sin(\theta)^2)(1 - \sin(3\theta)^2) \cdots (1 - \sin((2k-1)\theta)^2$$

is

$$4^{-k-1} e^{-2i\theta(k+1)^2} (e^{-2i\theta})^{2k} (e^{4i\theta})^k ((-e^{2i\theta}; e^{4i\theta})_{k+1})^2$$

where $\theta = \sin^{-1}(\sqrt{p})$ and $(a; q)_n$ is the Pochhammer symbol.

Approximate the $\sin(\theta) \approx \theta$

we can get

$$(1-p)(1-9p)(1-81p) \cdots (1-(2k-1)^2 p) = (4^k p^k \Gamma(\frac{k+1}{2})^2)/\pi$$

which grows in p.

Problem 2

With a similar approximation, we can get

$$\sum_{n=1}^{\infty} n \left(\prod_{i=1}^{n-1} (1 - (2i-1)p) \right) (2n-1)p$$

which should be approximately a geometric grow in order p, so the expected number of iterations is approximately $O(1/p)$.