Math 542 HW4

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1 Adjugates and Laplace

1.1

Solution 1.1.1

$$\Lambda^n A(e) = v_1 \wedge \ldots \wedge v_n = \det(A)e$$
$$A_{ij} = v_1 \wedge \ldots \wedge e_j \wedge \ldots \wedge v_n$$

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2 Cayley-Hamilton

3 Free Module

4 Commutator subgroups of matrix group

4.1

This is a direct result of smith normal form.

4.2

Because all elementary row/column operations matrix can be written as a $E_{ij}(a)$. Since every matrix in $\mathrm{SL}(n,k)$ can be written as identity times some $E_{ij}(a)$, we can conclude that $\mathrm{SL}(n,k)$ is product of $E_{ij}(a)$.

4.3

Solution 4.3.1

Consider the commutator of $\left(\begin{smallmatrix} a & 0 \\ 0 & a^{-1} \end{smallmatrix}\right)$ and $\left(\begin{smallmatrix} 1 & b \\ 0 & 1 \end{smallmatrix}\right)$.

$$\begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix}\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a^{-1} & 0 \\ 0 & a \end{pmatrix}\begin{pmatrix} 1 & -b \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & ab \\ 0 & a^{-1} \end{pmatrix}\begin{pmatrix} a^{-1} & -a^{-1}b \\ 0 & a \end{pmatrix} = \begin{pmatrix} 1 & -b+a^2b \\ 0 & 1 \end{pmatrix}$$

Then for any elementary matrix we just need to have $-b + a^2 b = a'$ so $E_{ij}(a') = ABA^{-1}B^{-1}$ for some A, B.

4.4

This can be done by induction on n with the previous question.

5 Determinats of exterior and tensor powers