Well Loss Estimation: Variable Pumping Replacing Step Drawdown Test

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Abstract: An optimization method is presented for simultaneous estimation of aquifer parameters and well loss parameters utilizing all the drawdowns observed during a variable rate pumping or multiple step pumping test. The proposed method does not require any graphical analysis. It is shown that a variable rate pumping test is a better substitute for the conventional step drawdown test to estimate well loss parameters. It suggests that the pumping rate may be changed frequently without waiting for a near steady state to be reached (or a selected duration, say 60 min) in each step of a conventional step drawdown test. This can result in a substantial saving of time and money involved in conducting a step drawdown test with a view to estimate well loss parameters. This gives a greater number of distinct discharges, which improves the estimates of the well loss parameters. Application of the method is demonstrated on published data sets, the results of which show that the parameters estimated using the new method are more reliable as compared to those obtained using prior methods.

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Introduction

The drawdown in a pumping well consists of two parts. The first part is formation (aquifer) loss and the second part is well loss (Jacob 1947). Well loss occurs due to the resistance to the flow of water through the well screen and filter pack. It also includes the loss due to the flow of water through the well bore. The term ''loss'' denotes the piezometric head loss, and shall be used hereafter. The formation or aquifer loss occurs because of resistance to the flow offered by the aquifer medium. Aquifer parameters, i.e., transmissivity and storage coefficient, govern the formation loss. For constant aquifer parameters, the formation loss varies with time measured since start of the pump and is directly proportional to the rate of pumping in a constant rate pumping test. The well loss consists of laminar or turbulent loss or a combination of the two. The laminar loss component is directly proportional to the rate of pumping, while the turbulent loss component varies roughly with the second power of pumping rate. The laminar loss component may be due to screen blockage, partial penetration of the well, and screen location in the aquifer, while the turbulent loss component may be due to the resistance to the flow through the well screen and inside the well bore (Sheahan 1971). The well loss excludes the formation loss. Therefore, the term ''well loss'' denotes the loss in piezometric head due to the flow through and inside the well.

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The parameters defining well loss are presently obtained only by using the drawdown observed during a step drawdown test. A step drawdown test consists of pumping at several successively higher rates and recording the drawdowns during each pumping stage, which generally continues until nearly steady state is reached. Generally, three and preferably four or more steps of pumping rate are required for estimation of well loss parameters (Jacob 1947; Rorabaugh 1953; Lennox 1966).

There have been several attempts to identify the parameters of well loss using step drawdown data. Graphical methods were given by Jacob (1947), Rorabaugh (1953), Lennox (1966), Sheahan (1971), and Birsoy and Summers (1980). Numerical or optimization methods were proposed by Labadie and Helweg (1975), Sheahan (1975), Gupta (1989), and Avci (1992). None of these methods is fully objective as each involves some graphical analysis with the numerical or optimization procedure. Methods published so far either use few drawdown points with a single drawdown selected from each step of the step drawdown test or estimate the parameters in two steps. This is done because of the popularity of the Jacob (1947) method. However, it is always advisable to utilize all of the measured data. In the first step, the parameters of well loss are estimated and in the second step, the well loss is subtracted from the measured drawdowns to get the corrected drawdowns. These corrected drawdowns are then analyzed for the aquifer parameters. To the author's knowledge, a method for simultaneous estimation of both aquifer and well loss parameters and having the capability to utilize all data observed during a step drawdown test or a variable rate pumping test is not available.

In this paper, an optimization method is presented for simultaneous estimation of aquifer parameters and well loss parameters utilizing all the drawdowns observed during a variable rate pumping test or a step drawdown test. It will be shown that a variable rate pumping test can outperform the traditional step drawdown test for the estimation of well loss parameters. In a variable rate pumping test, the pumping rate may be changed without waiting for near steady state in each step. This results in manyfold saving

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of time and money involved in conducting a multiple step drawdown test.

Method

The drawdown in a pumped well experiencing well loss is given by

$$s(t)=s_a(t)+s_w(t) \tag{1}$$

where $s(t)$=drawdown or the well, $s_a(t)$=drawdown in the piezometric head at the well face due to resistance to the flow offered by the aquifer medium, and $s_w(t)$=well loss. The expression for s_a for constant rate pumping may be written (Theis 1935) as

$$s_a(t)=Q\alpha_1 W\left(\frac{\alpha_2}{t}\right) \tag{2}$$

$$\alpha_1=\frac{1}{4\pi T}$$

and

$$\alpha_2=\frac{r^2 S}{4T} \tag{3}$$

where Q=constant rate of pumping, T=transmissivity of aquifer, S=storage coefficient of the aquifer, r=effective radius of the well, and t=time measured since start of the pumping, and, well function $W(.)$ is given by

$$W(u)=\int_u^\infty \frac{e^{-x}}{x}dx \tag{4}$$

The well loss for variable rate pumping may be expressed as

$$s_w(t)=\alpha_3[Q(t)]^n \tag{5}$$

where $Q(t)$=pumping rate at time t, $\alpha_3=C$=coefficient of well loss, and n=well loss exponent. Jacob (1947) assumed $n=2$. Rorabaugh (1953) found n to vary between 2.4 and 2.8 for high discharges. Rorabaugh (1953) differentiated between laminar and turbulent loss components and suggested laminar loss ($n=1$) for low discharges and turbulent loss ($n>2$) for high discharges. Lennox (1966) reported n as high as 3.5. Taking n as a parameter to be varied, the estimation of the aquifer and well loss parameters can be improved in situations where the value of n other than 2 is anticipated. It is worth mentioning here that the concepts of hydraulics suggest $n=2$ for turbulent head loss. Available methods do not utilize all the measured drawdowns for simultaneous estimation of aquifer and well loss parameters. A minimum of two distinct discharges is required and three are sufficient (with a degree of freedom of one). More data from successively stepped pumping rates give more degrees of freedom. The more degrees of freedom, the better the estimates of well loss parameters. Therefore, more distinct discharges are needed for better estimation of well loss parameters. In the conventional step drawdown test, it costs more and consumes more time for a larger number of stepped discharges. Rather, a variable rate pumping test can be adopted to serve the intended purpose; this would save substantial time and money.

Many investigators (e.g., Butler and McElwee 1990; Edward and Jones 1993; Singh 1998) have estimated the aquifer parameters from drawdown at an observation well due to variable rate pumping. However, these methods do not address or estimate the parameters of well loss and as such are not applicable to the data on a pumped well. A method to estimate simultaneously the aquifer parameters and well loss parameters utilizing the drawdowns due to the variable rate pumping test is outlined below.

For variable rate pumping, $s_a(t)$ may be expressed by the following convolution equation in discretized form (Singh 1998):

$$s_a^i=\sum_{\gamma=1}^{i} Q_\gamma \delta_{i-\gamma+1} \tag{6}$$

where i and γ=indices denoting time steps. The kernel δ_m is the discrete pulse kernel for drawdown at the mth time step. Equations similar to Eq. (6) have been proposed by Morel-Seytoux (1975) and Bear (1979). The discrete kernel method is for accounting temporal variation in excitation or input (see Morel-Seytoux 1975). The kernel δ_m is given by $\delta_m=F(m)-F(m-1)$, where m=index for time steps. The function $F(m)$ is expressed as

$$F(m)=\alpha_1 W\left(\frac{\alpha_2}{m\Delta t}\right) \tag{7}$$

where Δt=the size of the uniform time steps. The kernel proposed here renders a dimensionally homogeneous equation, which does not require special care for the units of the parameters for different values of Δt. Using Eqs. (1), (5), and (6), drawdown in the pumped well due to variable rate pumping is given by

$$s^i=s_a^i+\alpha_3 Q_i^n \tag{8}$$

where i=index for time steps, and s^i=drawdown in the well at the end of ith time step. Measured drawdowns may contain errors due to imprecise measurements and lack of fit of the model [Eq. (8)] in representing the reality. Therefore, the measured drawdown may be expressed as

$$s_0^i=s^i+\varepsilon_i \tag{9}$$

where ε_i=white noise associated with the ith drawdown measurement. The drawdown in the well at the end of the ith time step is expressed from Eq. (8) in a functional form as

$$s^i=f(Q,\alpha_j,i\Delta t); \quad j=1,2,...,4 \tag{10}$$

where j=index for parameters and α_j=the parameters of the model ($\alpha_4=n$). These parameters can be estimated by minimizing the integral squared errors (ISEs) given by

$$\text{ISE}=\sum_{i=1}^{N} \varepsilon_i^2 \tag{11}$$

The Marquardt (1963) algorithm, a technique to estimate nonlinear parameters, may be used to identify the parameters α_j. The algorithm suggested may be stated as

$$(\mathbf{A}^T\mathbf{A}+\lambda^{(\nu)}\mathbf{I})\mathbf{d}^{(\nu)}=\mathbf{A}^T\delta\mathbf{s} \tag{12}$$

where

$$\mathbf{A}=\left[\frac{\partial f_i}{\partial \alpha_j}\right] \tag{13}$$

and

$$\delta\mathbf{s}=[s_0^i-f_i] \tag{14}$$

where $\mathbf{I}$=identity matrix, $\mathbf{d}$=vector containing the increment over parameter values, f_i is given by Eq. (10), and ν=index denoting iteration number. In Eq. (12), $\mathbf{A}$ may be used in normalized form. The initial value of λ is taken to be large and is reduced by a factor at each successive iteration giving ISE(ν+1)<ISE(ν), oth-

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erwise, it is increased by the same factor. Eq. (12) is solved for **d** at each iteration and α_j for the next iteration is calculated using the following relation:

$$\alpha_j^{(\nu+1)} = \alpha_j^{(\nu)} + d_j^{(\nu)} \quad (15)$$

As the convergence is approached, λ, ISE, and d approach zero. Analytically calculated derivatives in application of the Marquardt algorithm substantially reduce the number of function evaluations and ensure proper convergence making the process robust. The expressions for the derivatives obtained by differentiating Eq. (8) are

$$\frac{\partial s^i}{\partial \alpha_1} = \frac{1}{\alpha_1} \sum_{\gamma=1}^{i} Q_\gamma \delta_{i-\gamma+1} \quad (16)$$

$$\frac{\partial s^i}{\partial \alpha_2} = \frac{\alpha_1}{\alpha_2} \sum_{\gamma=1}^{i} Q_\gamma \delta'_{i-\gamma+1} \quad (17)$$

$$\frac{\partial s^i}{\partial \alpha_3} = Q_i^n \quad (18)$$

$$\frac{\partial s^i}{\partial \alpha_4} = Q_i^n \ln(Q_i) \quad (19)$$

The discrete pulse kernel δ'_m for the derivative of drawdown with respect to α_2 at mth time step is given by $\delta'_m = F'(m) - F'(m-1)$. The function $F'(m)$ is expressed as

$$F'(m) = \exp\left[-\left(\frac{\alpha_2}{m\Delta t}\right)^2\right] \quad (20)$$

Once α_1 and α_2 are identified, estimates of transmissivity and the product of storage coefficient and square of the effective radius of the well are obtained using the following equations:

$$T = \frac{1}{4\pi\alpha_1}, \qquad r^2 S = \frac{\alpha_2}{\pi\alpha_1} \quad (21)$$

The estimate of storage coefficient can be obtained if drawdowns at an observation well are also available. This is because observation well drawdowns are not affected by the well loss; thus S and T can be estimated separately. With the use of analytical derivatives, the algorithm returns the reliable estimates of the parameters even when the initial guess for the parameters differ by 3 orders of magnitude from the respective true values. Therefore, the method gives unique estimates of the parameters irrespective of the initial guesses of the parameters. The effect of the number of data points on the estimates of the parameters is discussed later in the "Application" section. The method can also be used to estimate the aquifer parameters from drawdowns at an observation well during a variable rate pumping test. In this case, r is to be substituted by the distance of the observation well from the pumped well and well loss parameters are assigned values for zero well loss.

The method discussed above can utilize all the data collected during a step drawdown test to yield the estimate of aquifer transmissivity and the parameters of well loss. It can utilize initial drawdowns much before the steady state in each pumping step, to yield the estimate of the parameters. The pumping rate may be varied even before a steady state. Thus, it suggests a much short duration for a pumping test at variable rate pumping, which is a better substitute for the step drawdown test in order to estimate the well loss parameters. The reliability and performance of the model can be assessed by the standard error of estimate (SEE) given by the following expressions:

$$\mathrm{SEE} = \left(\frac{1}{N-p} \sum_{i=1}^{N} (s_0^i - s^i)^2\right)^{0.5} \quad (22)$$

where N=number of observed drawdowns and p=number of optimized parameters. The lower the value of SEE, the more reliable the estimated parameters. Sometimes outliers in the data affect the estimates of the parameters. These outliers need to be identified and removed before getting the final estimates of the parameters. An outlier may be identified with the error associated with it, which is substantially greater than SEE. A criterion for the error [say, >2(SEE)] for identifying the outliers is fixed and qualifying outliers are removed successively until SEE improves (i.e., becomes lower in a practical sense).

Table 1. (a) Synthetic Drawdowns after Avci (1992) and (b) Estimated Parameters (Avci 1992 Data)

(a) Time	Drawdown
25	1.914
50	2.123
100	2.305
175	7.456
250	7.915
300	8.114
360	11.126
400	11.363
450	11.582
500	13.358
525	13.388
575	13.589

(b) Parameters and SEE	Actual	Avci (1992) Set 1	Avci (1992) Set 2	Avci (1992) Set 3	Present method
n	2.46	2.44	2.44	2.44	2.458
α_3 or C	0.11	0.115	0.115	0.115	0.112
T	0.21	0.21	0.21	0.21	0.21
$r^2 S \times 10^3$	8.8	8.8	9.0	8.8	8.77
SEE (based on all data)	—	0.0091	0.0156	0.0091	0.0004

Note: Unit of length is in meters and unit of time is in minutes.

Application

The application of the proposed method is demonstrated in this section on two sets of published data. The first set is synthetic data and the second set is field data.

Synthetic Data

The synthetic data were taken from Avci (1992) (Table 1). The aquifer parameters and well loss parameters were estimated using the present method. In the application of the present method all drawdowns were utilized while Avci (1992) selected one drawdown from each step of four steps of synthetic data (three such sets were selected). The drawdowns are given in Table 1(a). The pumping steps were of 100, 200, 150, and 125 min durations, respectively, with corresponding pumping rates of 0.6944,

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Table 2. Effect of Number of Drawdowns on Estimated Parameters (Avci 1992 Data)

m[a]	n	T	$r^2S\times10^3$	C	SEE
1[b]	2.46	0.21	8.7	0.11	0.0002
2	2.46	0.21	8.8	0.11	0.0004

Note: Unit of length is in meters and unit of time is in minutes.

[a]Number of initial drawdown from each step.

[b]One additional drawdown corresponding to 50 min is also considered.

2.0833, 2.7778, and 3.1250 m^3/min, respectively. The estimated values of the parameters are compared to those obtained by Avci (1992) in Table 1(b), which shows that the present method gives low SEE. It is also observed that the actual parameter values are returned using the present method, while the estimates obtained by Avci (1992) are different from the actual values for each set. Thus, the present method gives better estimates of the parameter than that of Avci (1992).

In order to test the method for early data, only the first and second drawdowns were selected from each step and the parameters were optimized. Similarly, selecting the first drawdown from each step, one gets only four drawdowns for estimating four parameters. In this case, optimization cannot be performed; hence the second drawdown from the first step was also selected to have a total of five drawdowns. The estimated parameters are given in Table 2, which show that the values of the optimized parameters are the same as the true values.

Field Data

The method was applied on published data taken from Clark (1977). It consists of 119 drawdown observations during a step drawdown test with six steps of pumping rate. The effect of variation in the value of n (assuming it to be fixed) on the estimated parameters is given in Table 3. Thus, only three parameters, i.e., α_1, α_2, and α_3, were optimized. It is observed that the value of n affects the estimates of the parameters. Therefore, a better choice is to take n as the parameter and optimize it as suggested in the proposed method. The optimized value of n, thus obtained, may be subjected to further analysis, especially in comparison to the case for which $n=2$ is assumed. This is in view of the many investigators reporting higher values (>2) for n.

All parameters (α_1, α_2, α_3, and α_4) estimated using the proposed method are compared to those obtained by various investigators in Table 4. Unique values of the parameters were obtained for different initial guesses. This shows that the method returns unique estimates of the parameters and is stable. A substantially low value of SEE for the present method suggests that the parameter estimates are more reliable using the new method as compared to those using the method by Avci (1992) and Gupta (1989). The errors, i.e., the difference between the observed and calculated drawdowns obtained using this method are compared to those obtained using prior methods in Fig. 1. These errors may be

Table 3. Effect of Variation in n on Estimated Parameters

n	T	$r^2S\times10^4$	α_3 or C	SEE
1.5	0.192	43.7	0.982	0.091
2.0	0.180	14.2	0.313	0.070
2.5	0.171	13.6	0.129	0.071
3.0	0.164	15.9	0.059	0.087

Note: Unit of length is in meters and unit of time is in minutes.

Table 4. Comparison of Estimated Parameters

Parameters and SEE	Present method	Avci (1992)	Gupta (1989)	Labadie and Helweg (1975)
n	2.191	2.005	2.709	2.10
α_3 or C	0.2190	0.3071	0.0979	0.3493
T	0.1753	0.1949	0.1742	—
$r^2S\times10^4$	13.9	6.0	9.91	—
SEE (based on all data)	0.068	0.091	0.095	—

Note: Unit of length is in meters and unit of time is in minutes.

due to error in measuring drawdowns, a lack of fit of the model in representing the real situation, and unreliable estimates of the parameters. Fig. 1 shows that the present method gives the lowest error as compared to the methods by Avci (1992) and Gupta (1989).

A comparison of Tables 3 and 4 shows that the SEEs are almost the same in the two cases, i.e., (1) when n is variable and considered as a parameter and (2) when n is fixed and assumed equal to 2. Therefore; optimized values with fewer parameters, i.e., case(2), should be adopted to avoid overparameterization. Also, for turbulent head loss, $n=2$. The aquifer and well loss parameters obtained from these estimates are: $T=0.180$, $r^2S=1.42\times10^{-3}$, $C=0.313$, and $n=2$. However, if for a data set, SEE in case(1) becomes distinctly and practically less than that in case(2), the optimized values of the parameters with n different than 2 [case(1)] should be adopted for further analysis. For the example considered, it is observed that $n=2$. A value of n as high as 2.7 for the example is reported in the literature.

The error, i.e., the difference between the observed and calculated drawdown, is obtained for each drawdown. The drawdown for which the absolute error is found to be more than twice the SEE, is considered an outlier. For the system having been identified to have $n=2$, only three parameters, i.e., α_1, α_2, and α_3, will be optimized in further analysis. The number of identified outliers are 8, 6, 4, and 2 during respectively, the first, second, third, and fourth stage of outlier removal. The aquifer and well loss parameters obtained in each stage are given in Table 5. The residual errors for the data without outliers are shown in Fig. 2 and those for the outliers are shown in Fig. 3. In both cases, the residual errors were calculated using the parameters estimated from the data without outliers.

This method was used to estimate the parameters by taking different number of observations. A successively increasing num-

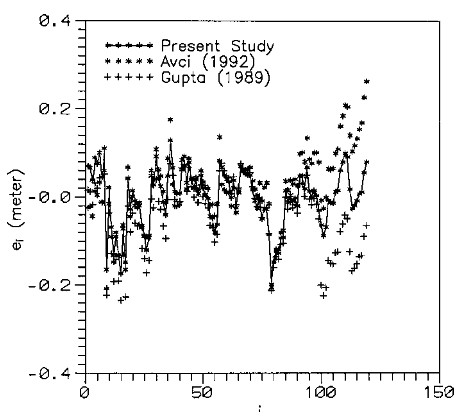

Fig. 1. Residual errors using different methods (Clark 1977 data)

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ber of drawdowns were selected from the early part of each step. Data without outliers are used for this exercise. The optimized values of the parameters obtained in each case are given in Table 6. The estimates of the parameters do not vary substantially with the number of drawdowns considered. Beyond a sufficient number of data points, the estimates would only be affected by noise. It is mentioned that a greater number of data points and hence, more degrees of freedom, gives more reliable estimates. Even only three initial drawdowns from each step yield estimates as good as obtained from all data without outliers. Based upon a very low value of SEE (0.024 m), these estimates may be even better ($T=0.189$, $r^2S=8.5\times10^{-4}$, $C=0.310$). Considering the four initial drawdowns from each step, $T=0.191$ and $r^2S=7.5\times10^{-4}$ is obtained with almost the same value of SEE (i.e., 0.023). Therefore, for this example, the parameter r^2S is assumed to be less sensitive. The values of the parameters in Table 6 show that the method can yield reliable estimates, even when only drawdowns before the steady state in each step are used.

Table 5. Estimated Parameters with Removal of Outliers

Stage of removal	Number of outliers removed	T	$r^2S\times10^4$	α_3 or C	SEE
1	8	0.184	11.4	0.316	0.055
2	6	0.186	9.8	0.316	0.048
3	4	0.187	9.6	0.316	0.044
4	2	0.189	8.5	0.318	0.042

Note: Unit of length is in meters and unit of time is in minutes.

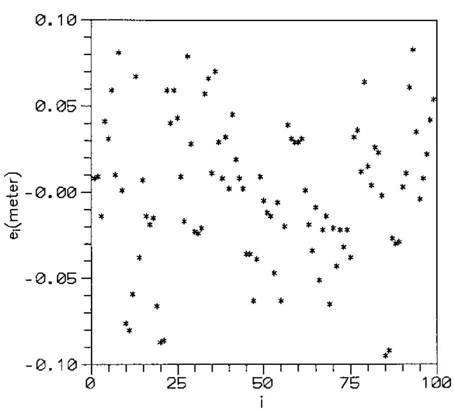

Fig. 2. Residual errors for data without outliers (Clark 1977 data)

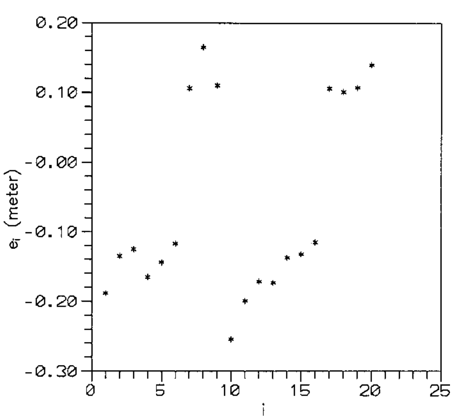

Fig. 3. Residual errors for outliers (Clark 1977 data)

Table 6. Effect of Number of Drawdowns on Estimated Parameters (Clark 1977 Data)

m[a]	T	$r^2S\times10^3$	α_3 or C	SEE
1	0.188	8.8	0.317	0.089
2	0.189	8.2	0.304	0.024
3	0.189	8.5	0.310	0.024
4	0.191	7.5	0.314	0.023
5	0.192	7.3	0.314	0.026
6	0.193	6.8	0.317	0.031
7	0.194	6.8	0.320	0.035
8	0.193	6.9	0.321	0.042
9	0.190	8.1	0.316	0.044
10	0.189	8.6	0.316	0.045

Note: Unit of length is in meters and unit of time is in minutes.
[a]Number of initial drawdowns from each step.

The use of the method suggests that it is really not required or necessary to conduct a step drawdown test in order to estimate the well loss parameters. It is also not necessary to wait until a near steady state is reached in each step before stepping up the pumping rate as is the current practice. The application of the present method requires drawdowns observed in the pumped well due to a variable rate pumping test in order to estimate the well loss parameters. Therefore, well before a near steady state, the pumping rate can be stepped up to have more distinct discharges. This would save considerable time and money. Also, more distinct discharges give more degrees of freedom, which helps in obtaining an unbiased and better estimate of well loss parameters. Thus, with the use of this method, a variable rate pumping test can replace the step drawdown test currently used for estimation of well loss parameters.

Conclusions

A robust optimization method is presented for simultaneous estimation of confined aquifer parameters and parameters of well loss using all the drawdowns observed at the pumped well during a variable rate pumping test or conventional step drawdown test. The method is fully objective and does not require any graphical analysis. In order to identify storage coefficient, drawdowns at an observation well are required. The method returns unique estimates of the parameter for different initial guesses and is stable. A sensitivity with the number of drawdowns considered shows that the method yields reliable estimates of the parameters with even fewer drawdowns. The application of the method on the published data sets shows that the parameters estimated using the new method are more reliable (based upon standard error of estimate) as compared to those using prior methods. It suggests a well loss exponent of 2 for Clark's data. The following conclusions are drawn from the study.

1. With the use of the method, it is really not required or necessary to conduct a step drawdown test in order to estimate the well loss parameters. It is also not necessary to wait until a near steady state is reached (or a selected duration, say 60 min) in each step before stepping up the pumping rate as is the current practice.
2. The pumping rate can be stepped up long before a near steady state, which results in frequently stepped pumping having a variable rate pumping test. Using the present method, a variable rate pumping test can replace the step drawdown test for estimation of well loss parameters. This

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would result in a substantial saving of time and money involved in conducting a multiple step drawdown test for the estimation of well loss parameters. Since a greater number of discharges are available in this way, the well loss parameters can be estimated more reliably.

3. The method can make use of all drawdown data observed during a step drawdown test or a variable rate pumping test. Most methods use only one drawdown selected from each step of a step drawdown test. The method can also estimate the aquifer parameters from the variable rate pumping test data at observation wells by assigning such values to the well loss parameters in order to have zero well loss.

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Notation

The following symbols are used in this paper:

i = index denoting number of time step (dimensionless);
N = number of observed drawdowns (dimensionless);
n = exponent for turbulent loss (dimensionless);
Q = pumping rate (L^3T^{-1});
Q_i = pumping rate during ith time step (L^3T^{-1});
r = effective radius of well (L);
S = storage coefficient of aquifer (dimensionless);
s = drawdown in well (L);
s_a = aquifer loss (L);
s_w = well loss (L);
T = transmissivity of aquifer (L^2T^{-1});
t = time measured since start of pumping (T);
$W(.)$ = well function (dimensionless); and
α_j = parameters of model.

References

Avci, C. B. (1992). ''Parameter estimation for step-drawdown tests.'' *Ground Water,* 30(3), 338–342.

Bear, J. (1979). *Hydraulics of groundwater*, McGraw-Hill, New York, 323–325.

Birsoy, Y. K., and Summers, W. K. (1980). ''Determination of aquifer parameters from step tests and intermittent pumping data.'' *Ground Water,* 18(2), 137–146.

Butler, J. J., Jr., and McElwee, C. D. (1990). ''Variable rate pumping rest for radially symmetric nonuniform aquifers.'' *Water Resour. Res.,* 26(2), 291–306.

Clark, L. (1977). ''The analysis and planning of step drawdown tests.'' *J. Eng. Geol.,* 10, 125–143.

Edwards, K. B., and Jones, L. C. (1993). ''Modeling pumping tests in weathered glacial till.'' *J. Hydrol.,* 150, 41–60.

Gupta, A. D. (1989). ''On analysis of step-drawdown data.'' *Ground Water,* 27(6), 874–881.

Jacob, C. E. (1947). ''Drawdown test to determine effective radius of artesian well.'' *Trans. Am. Soc. Civ. Eng.,* 112, 1047–1070.

Labadie, J. W., and Helweg, O. J. (1975). ''Step-drawdown test analysis by computer.'' *Ground Water,* 13(5), 438–444.

Lennox, D. H. (1966). ''Analysis and application of step-drawdown tests.'' *J. Hydraul. Div., Am. Soc. Civ. Eng.,* 92(HY6), 25–47.

Marquardt, D. W. (1963). ''An algorithm for least squares estimation of nonlinear parameters.'' *J. Soc. Ind. Appl. Math.,* 11, 431–441.

Morel-Seytoux, H. J. (1975). ''Optimal and legal conjunctive operation of surface and ground waters.'' *Proc., 2nd World Congress, Int. Water Resour. Assoc.*, New Delhi, India, 119–129.

Rorabaugh, M. I. (1953). ''Graphical and theoretical analysis of step-drawdown tests of artesian wells.'' *Proc. Am. Soc. Civ. Eng.*, 79, 362.

Sheahan, N. T. (1971). ''Type-curve solution of step-drawdown test.'' *Ground Water,* 9(1), 25–29.

Sheahan, N. T. (1975). ''Discussion of 'Step-drawdown test analysis by computer' by J. W. Labadie and O. J. Helweg.'' *Ground Water,* 13(5), 445–450.

Singh, S. K. (1998). ''Optimization of confined aquifer parameters from variable rate pump test.'' *J. Hydraul. Eng.,* 4(1), 61–70.

Theis, C. V. (1935). ''The relationship between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage.'' *Trans., Am. Geophys. Union,* 16, 519–524.

J. Hydraul. Eng. 2002.128:343-348.