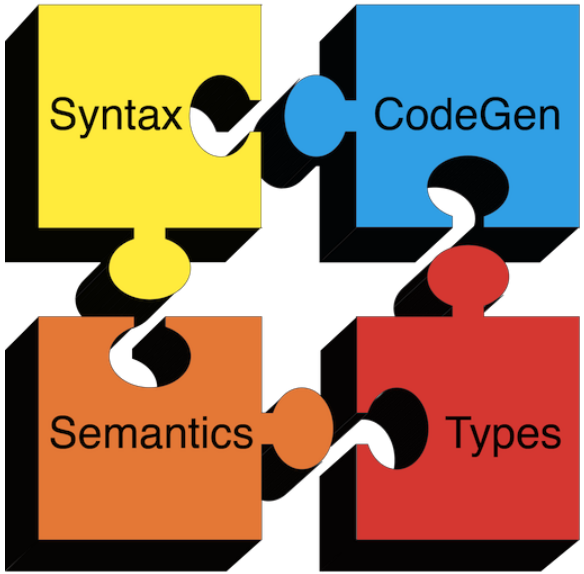


Compilers

Type Checking

Type Checking

- We have seen two examples of formal notation specifying parts of a compiler
  - Regular expressions
  - Context-free grammars

- The appropriate formalism for type checking is logical rules of inference

Type Checking

- Inference rules have the form

  If Hypothesis is true, then Conclusion is true

- Type checking computes via reasoning

  If E_1 and E_2 have certain types, then E_3 has a certain type

- Rules of inference are a compact notation for "If-Then" statements

Type Checking

- The notation is easy to read with practice
- Start with a simplified system and gradually add features
- Building blocks
  - Symbol $\wedge$ is "and"
  - Symbol $\Rightarrow$ is "if-then"
  - x:T is "x has type T"

Type Checking

If e_1 has type Int and e_2 has type Int, then $e_1 + e_2$ has type Int

(e_1 has type Int $\wedge$ e_2 has type Int) $\Rightarrow$ $e_1 + e_2$ has type Int

$(e_1 : \text{Int} \wedge e_2 : \text{Int}) \Rightarrow e_1 + e_2 : \text{Int}$

Type Checking

The statement

$$(e_1: \text{Int} \wedge e_2: \text{Int}) \Rightarrow e_1 + e_2: \text{Int}$$

is a special case of

$$\text{Hypothesis}_1 \wedge \ldots \wedge \text{Hypothesis}_n \Rightarrow \text{Conclusion}$$

This is an inference rule

Type Checking

- By tradition inference rules are written

$$\frac{\vdash \text{Hypothesis} \ldots \vdash \text{Hypothesis}}{\vdash \text{Conclusion}}$$

- Cool type rules have hypotheses and conclusions

$$\vdash e:T$$

- $\vdash$ means "it is provable that . . ."

Type Checking

$$\frac{i \text{ is an integer literal}}{\vdash i : \text{Int}} \quad [\text{Int}]$$

$$\frac{\vdash e_1 : \text{Int} \quad \vdash e_2 : \text{Int}}{\vdash e_1 + e_2 : \text{Int}} \quad [\text{Add}]$$

Type Checking

- These rules give templates describing how to type integers and + expressions

i $\quad e_1 \quad e_2$

- By filling in the templates, we can produce complete typings for expressions

Type Checking

$$\frac{\dfrac{1 \text{ is an int literal}}{\vdash 1 : \text{Int}} \qquad \dfrac{2 \text{ is an int literal}}{\vdash 2 : \text{Int}}}{\vdash 1 + 2 : \text{Int}}$$

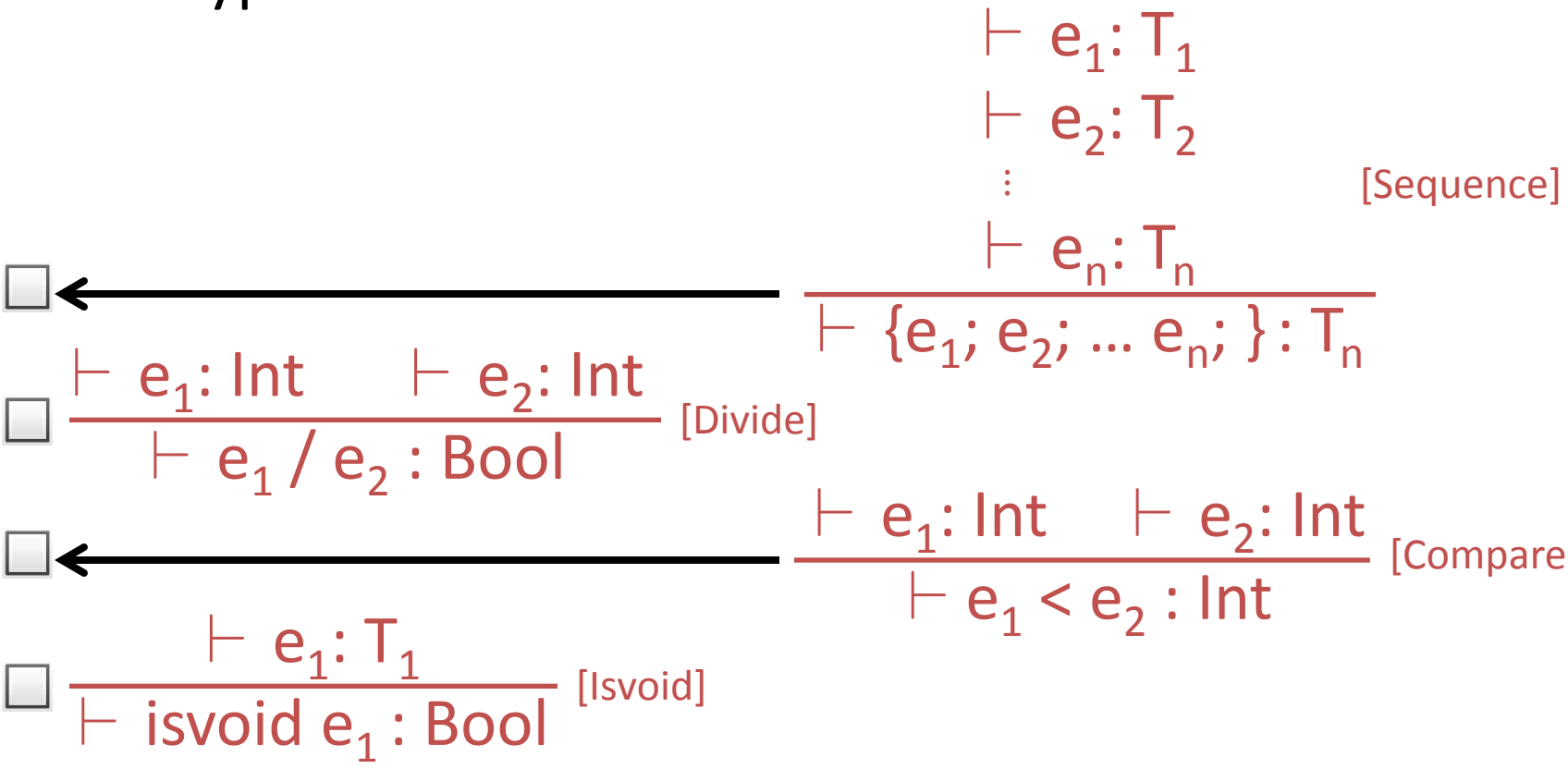
Type Checking

- A type system is *sound* if
  - Whenever $\vdash e : T$
  - Then e evaluates to a value of type T

- We only want sound rules
  - But some sound rules are better than others!

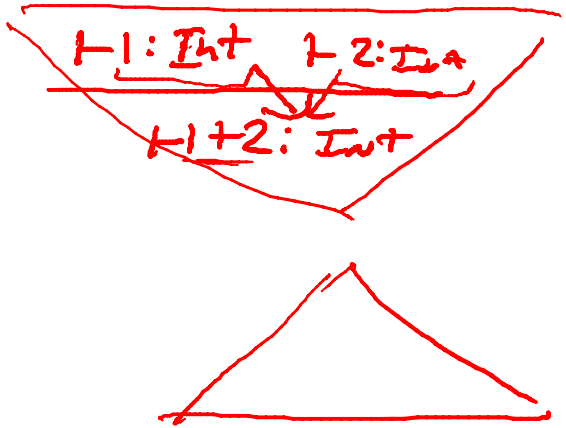
$$\frac{i \text{ is int}}{\vdash i : \text{Int}}$$

Choose the type rules that are sound



Type Checking

- Type checking proves facts e: T
  - Proof is on the structure of the AST
  - Proof has the shape of the AST
  - One type rule is used for each AST node



- In the type rule used for a node e:
  - Hypotheses are the proofs of types of e's subexpressions
  - Conclusion is the type of e

- Types are computed in a bottom-up pass over the AST