



# Compilers

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## Type Environments

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 $\vdash \text{false} : \text{Bool}$ 

[False]

$s$  is a string literal

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 $\vdash s : \text{String}$ 

[String]

new T produces an object of type T

– Ignore SELF\_TYPE for now . . .

$$\frac{}{\vdash \text{new } T : T} \quad [\text{New}]$$

$$\frac{\vdash \underline{e} : \text{Bool}}{\vdash \underline{!e} : \text{Bool}} \quad [\text{Not}]$$

$$\left[ \frac{\frac{\vdash e_1 : \text{Bool}}{\vdash e_2 \text{ T}}}{\vdash \text{while } e_1 \text{ loop } e_2 \text{ pool} : \text{Object}} \right] \quad [\text{Loop}]$$

*Handwritten annotations in red:*

- A large red bracket on the left and right of the [Loop] rule.
- A red circle around the  $\text{T}$  in the premise  $\vdash e_2 \text{ T}$ .
- A red underline under  $e_1$  in the conclusion.
- A red underline under  $e_2$  in the conclusion.
- A red underline under  $\text{pool} : \text{Object}$  in the conclusion.
- A red arrow pointing from the  $\text{T}$  in the premise to the  $\text{pool} : \text{Object}$  in the conclusion.

- What is the type of a variable reference?

$$\frac{x \text{ is a variable}}{\vdash \underline{x}: ?} \quad [\text{Var}]$$

- The local, structural rule does not carry enough information to give  $x$  a type.

- Put more information in the rules!
- A type environment gives types for free variables
  - A type environment is a function from ObjectIdentifiers to Types
  - A variable is free in an expression if it is not defined within the expression

x  
x+y  
let y ← ... in x+y  
bound  
x is free  
x, y are free  
x is free

Let  $O$  be a function from ObjectIdentifiers to Types

The sentence  $O \vdash e : T$

is read: Under the assumption that <sup>free</sup>variables have the types given by  $O$ , it is provable that the expression  $e$  has the type  $T$

The type environment is added to the earlier rules:

$$\frac{i \text{ is an integer literal}}{\underline{O} \vdash \underline{i} : \underline{\text{Int}}} \quad [\text{Int}]$$

$$\frac{\underline{O} \vdash \underline{e_1} : \underline{\text{Int}} \quad \underline{O} \vdash \underline{e_2} : \underline{\text{Int}}}{\underline{O} \vdash \underline{e_1 + e_2} : \underline{\text{Int}}} \quad [\text{Add}]$$



And we can write new rules:

$$\frac{\underline{Q(x)} = \underline{T}}{\underline{Q} \vdash \underline{x} : \underline{T}} \quad [\text{Var}]$$

$$\underline{O[T/x]}(x) = T$$

$$O[T/x](y) = O(y) \\ x \neq y$$

$$\frac{\underline{O[T_0/x]} \vdash \underline{e_1 : T_1}}{\underline{O} \vdash \underline{\text{let } x:T_0 \text{ in } e_1 : T_1}}$$

[Let-No-Init]

↙  
symbol table

Fill in the correct type environments in the following type rule

$$\frac{\begin{array}{c} O_1 \vdash e_1 : T_1 \\ O_2 \vdash e_2 : T_2 \end{array}}{O \vdash \text{let } x : T_1 \leftarrow e_1 \text{ in } e_2 : T_2}$$

[Let-Init]

- ☐  $O_1 = O[T_1/x]; O_2 = O[T_1/x]$
- ☐  $O_1 = O[T_1/x]; O_2 = O[T_2/x]$
- ☐  $O_1 = O; O_2 = O[T_1/x]$
- ☐  $O_1 = O; O_2 = O[T_2/x]$

- The type environment gives types to the free identifiers in the current scope
- The type environment is passed down the AST from the root towards the leaves
- Types are computed up the AST from the leaves towards the root