A Virtual Actuator and Sensor Approach for Event-Triggered Fault-Tolerant Control of Multi-Agent Systems ⋆

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Abstract: The increasing emphasis on system safety and reliability has heightened the demand for fault-tolerant ability of control systems. Fault-tolerant cooperative control (FTCC) is one of the safety issues in multi-agent systems (MASs). This paper studies the leader-following consensus control of MASs in the presence of actuator and sensor faults. A unified framework of active FTCC is proposed under a dynamic event-triggered mechanism (DETM) by employing the virtual actuator and virtual sensor approach. The proposed DETM serves to reduce the communication cost among agents. The FTCC issue is formulated by linear matrix inequalities (LMIs) and can be implemented in a distributed architecture. Without the need of re-tuning the pre-designed observer-based controller, the FTCC exhibits a notable efficiency for fault compensation and plant reconfiguration. Numerical simulations demonstrate the validity of the proposed methods.

Keywords: Multi-Agent Systems, Fault-Tolerant Control, Event-Triggered Control, Virtual Actuator, Virtual Sensor, LMI approach, Networked System.

1. INTRODUCTION

Technological advances have led to an increasing amount of research and applications on multi-agent systems (MASs), the goal being the collaboration of multiple autonomous systems to accomplish specific tasks. One intriguing subject is the leader-following (L-F) consensus problem, which aims to ensure that follower agents follow a leader and achieve consensus. This topic has been extensively explored and can be easily extended to formation control and other further applications. A more detailed survey of recent advances in MASs' consensus issues can be found in (Amirkhani and Barshooi (2021)).

There is a growing demand for enhancing systems' security and reliability. In the context of MASs, the fault of one agent can be propagated through network communication to other agents. Consequently, fault-tolerant cooperative control (FTCC) of MASs has gained greater importance. This paper focuses on actuator faults and sensor faults of MASs. It is worth noting that the majority of current studies primarily examine actuator faults, as seen in (Wang et al. (2021); Li et al. (2023)). In (Chadli et al. (2017); Yadegar and Meskin (2021)), more general non-linear MASs FTCC problems are addressed. In (Ju et al. (2022)), discrete FTCC based on event-triggered and observer-based control is investigated. The FTCC for compound fault is more difficult to handle since it requires a unified framework to deal with actuator faults and sensor faults simultaneously (Gao et al. (2022)). It is noticeable that there is still some research on this topic, such as in (Chen et al. (2019); Yan et al. (2019); Wu et al. (2023)), to list a few. Recently, in (Yang et al. (2023)), additive compound faults are considered and DETM are also employed based on H_∞/H_- fault detection. A more comprehensive review of FTCC could be found in (Yang et al. (2020)).

The utilization of virtual actuators (VA) and virtual sensors (VS) for control reconfiguration is one approach of FTCC (Blanke et al. (2006)). VA and VS can compensate for fault signals without re-tuning the nominal controller. This method is considered as a generalized form of the pseudo-inverse technique, the latter of which exhibits significant limitations when dealing with a complete loss of effectiveness. Widespread application in single-agent systems has shown the capability of this VA/VS approach, which is also extended to more general linear parameter-varying (LPV) systems, as presented in (Tabatabaeipour et al. (2012); Rotondo et al. (2014)). As regards to MASs, (Yadegar and Meskin (2021)) employs the VA in FTCC for LPV systems. (Trejo et al. (2022)) explores FTCC based on VA and static event-triggered strategies, but the final result is not distributed. It is important to note that there is currently a dearth of research that simultaneously considers the utilization of both VA and VS for compound faults in conjunction with the observer-based event-triggered framework.

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As indicated in (Yang et al. (2020)), reducing communication costs becomes a significant issue for FTCC, especially for large-scale networked systems. The event-triggered mechanisms (ETMs) represent a promising method for this problem. There have been numerous investigations of ETMs for MASs, including ETMs to address communication delays (Antonio et al. (2021)), ETMs for packet loss scenarios (Viel et al. (2022)), and dynamic event-triggered mechanisms (DETM) with adjustable inter-event times (Wang and Chadli (2023b,a)). Notably, the DETM incorporates auxiliary dynamic variables and thus can extend the inter-event time (IET) and reduce communication times. A more comprehensive survey of ETMs for MASs can be found in (Nowzari et al. (2019)).

Based on the studies mentioned above, this work aims to employ the VA and VS approach for FTCC of MASs under an observer-based DETM. The main contribution of this work is as follows: 1) Development of a unified framework of dynamic event-triggered FTCC incorporating multiplicative compound faults by using the VA/VS method, where a DETM is also employed to reduce the utilization of communication resources. 2) The proposed approach can manage not only minor faults but also critical failures, including complete loss of effectiveness. 3) A synthesis of the proposed method is developed by linear matrix inequalities (LMIs). In addition, the proposed controller, observer, and FTCC approach are distributed, which can facilitate the practical implementation.

The rest of the paper is organized as follows: section 2 provides notations and basic graph theory. Section 3 presents the problem formulation and nominal controller/observer design. The distributed FTCC strategy and the DETM are detailed in section 4. The numerical example is demonstrated in section 5. A conclusion with a summary is given in section 6.

2. PRELIMINARIES

For clarity of notation, matrices or vectors are written in bold characters, while scalars are in non-bold characters. Let $\mathbb{R}^{m\times n}$ denote the set of matrices of dimension $m \times n$. $\mathbb{N}^+$ denotes the set of positive natural numbers, respectively. Given a matrix $\boldsymbol{P}$, $\boldsymbol{P}^T$ denotes its transpose, and $\boldsymbol{P}^\dagger$ represents its Moore-Penrose inverse. $\boldsymbol{P} < 0$ denotes that $\boldsymbol{P}$ is negative definite, and $\boldsymbol{P} > 0$ means $-\boldsymbol{P} < 0$. $\boldsymbol{I}_n$ denotes an identity matrix of dimension n. $\boldsymbol{0}_n$ denotes a zero matrix of dimension n. $\otimes$ denotes the Kronecker product. Define $\mathrm{Sym}(\boldsymbol{P}) = \boldsymbol{P} + \boldsymbol{P}^T$. Let $*$ denote the symmetric entries in a matrix. $\mathrm{Im}(\boldsymbol{P})$ and $\mathrm{Ker}(\boldsymbol{P})$ denote the image space and the null space of matrix $\boldsymbol{P}$, respectively. $\mathrm{diag}(\boldsymbol{P}_1, \ldots, \boldsymbol{P}_N)$ denotes a block diagonal matrix with its diagonal blocks $\boldsymbol{P}_1, \ldots, \boldsymbol{P}_N$. Let $\lambda_{\min}(\boldsymbol{P})$ denote the minimum eigenvalue of $\boldsymbol{P}$.

The communication topology of N ($N \in \mathbb{N}^+$) follower agents is represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consisting of a vertex set $\mathcal{V} = \{v_1, ..., v_N\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Follower agent i and the leader are represented as vertices v_i and v_0, respectively. Denote $\mathcal{N}_i$ the set of neighbor of agent i. The adjacency matrix $\boldsymbol{E} = (E_{ij}) \in \mathbb{R}^{N\times N}$ of $\mathcal{G}$ is defined such that $E_{ii} = 0$, $E_{ij} = 1$ if v_i is connected to v_j and $E_{ij} = 0$ otherwise. The Laplacian matrix $\boldsymbol{\mathcal{L}} = (L_{ij}) \in \mathbb{R}^{N\times N}$ is defined as $L_{ii} = \sum_{i\neq j} E_{ij}$ and $L_{ij} = -E_{ij}, i \neq j$. Define the augmented graph of $\mathcal{G}$ as $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$, with $\bar{\mathcal{V}} = \mathcal{V} \cup \{v_0\}$ and $(v_i, v_0) \in \bar{\mathcal{E}}$ if agent i is connected to the leader. Define leader adjacency matrix $\boldsymbol{\mathcal{D}} = \mathrm{diag}(d_1, ..., d_N)$ where $d_i = 1$ if $(v_i, v_0) \in \bar{\mathcal{E}}$ otherwise $d_i = 0$. Define $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{L}} + \boldsymbol{\mathcal{D}}$.

3. PROBLEM STATEMENT

Consider a linear MAS with N followers and one leader represented by the following equations:

$$\begin{cases} \dot{\boldsymbol{x}}_i^c(t) = \boldsymbol{A}\boldsymbol{x}_i^c(t) + \boldsymbol{B}\boldsymbol{u}_i^c(t), \quad i = 1, \ldots, N \\ \boldsymbol{y}_i^c(t) = \boldsymbol{C}\boldsymbol{x}_i^c(t) \\ \dot{\boldsymbol{x}}_0(t) = \boldsymbol{A}\boldsymbol{x}_0(t) \end{cases} \tag{1}$$

where $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, $\boldsymbol{B} \in \mathbb{R}^{n\times m}$, $\boldsymbol{C} \in \mathbb{R}^{r\times n}$. $\boldsymbol{x}_i(t)$ and $\boldsymbol{x}_0(t)$ denote the states of follower agents and the leader, respectively. $\boldsymbol{y}_i(t)$ is the output of agent. The objective is to achieve the leader-following (L-F) consensus, i.e.,

$$\lim_{t\to+\infty} \|\boldsymbol{x}_i(t) - \boldsymbol{x}_0(t)\| = 0, \quad \forall i \in \{1, \ldots, N\} \tag{2}$$

In the presence of multiplicative actuator faults and sensor faults on follower agents, the faulty plant is presented as

$$\begin{cases} \dot{\boldsymbol{x}}_i^f(t) = \boldsymbol{A}\boldsymbol{x}_i^f(t) + \boldsymbol{B}_i^f\boldsymbol{u}_i^f(t), \quad i = 1, \ldots, N \\ \boldsymbol{y}_i^f(t) = \boldsymbol{C}_i^f\boldsymbol{x}_i^f(t) \end{cases} \tag{3}$$

where $\boldsymbol{B}_i^f \in \mathbb{R}^{n\times m}$ is the input matrix with actuator faults of agent i, $\boldsymbol{C}_i^f \in \mathbb{R}^{r\times n}$ is the output matrix with sensor faults of agent i. Furthermore, $\boldsymbol{B}_i^f$ is represented by the actuator effectiveness matrix $\boldsymbol{\Theta}_i$: $\boldsymbol{B}_i^f = \boldsymbol{B}\boldsymbol{\Theta}_i$, $\quad \boldsymbol{\Theta}_i = \mathrm{diag}(\theta_1^i, \ldots, \theta_m^i)$, where $\theta_k^i \in [0, 1]$ represents the effectiveness of k-th input of agent i. $\theta_k^i = 1$ if the actuator works in a normal state. Oppositely, a faulty actuator with complete (resp. partial) loss of effectiveness is represented by $\theta_k = 0$ (resp. $\theta_k = a$, $a \in (0, 1)$). By applying the similar representation of $\boldsymbol{B}_i^f$, $\boldsymbol{C}_i^f$ can be written as $\boldsymbol{C}_i^f = \boldsymbol{\Phi}_i\boldsymbol{C}$, $\quad \boldsymbol{\Phi}_i = \mathrm{diag}(\phi_1^i, \ldots, \phi_r^i)$, where $\phi_k^i = 1$ means a healthy sensor, while $\phi_i^i \in (0, 1)$ or 0 means partial or complete loss of effectiveness.

The following assumptions hold in this paper:

Assumption 1. The nominal process presented by the matrix pair $(\boldsymbol{A}, \boldsymbol{B})$ and $(\boldsymbol{A}, \boldsymbol{C})$ is stabilizable and observable. The fault process presented by the matrix pair $(\boldsymbol{A}, \boldsymbol{B}_i^f)$ and $(\boldsymbol{A}, \boldsymbol{C}_i^f)$ is still stabilizable and observable.

Remark 2. Assumption 1 guarantees that even if one actuator is in complete failure, there will still be redundant actuators, or actuators that can serve as indirect control, to take over (the same case for the sensor).

Assumption 3. The communication topology is undirected and fixed, and the augmented graph $\bar{\mathcal{G}}$ contains a spanning tree with the leader agent being its root.

Assumption 4. The values of θ_k^i and ϕ_k^i are assumed available.

We design first the controller and the observer when the system is in the normal state, i.e., (1). The controller is designed by

$$\begin{cases} \boldsymbol{u}_i^c(t) = \boldsymbol{K}\boldsymbol{z}_i(t) \\ \boldsymbol{z}_i(t) = \displaystyle\sum_{j\in\mathcal{N}_i} a_{ij}(\tilde{\boldsymbol{x}}_j^c(t) - \tilde{\boldsymbol{x}}_i^c(t)) + d_i(\boldsymbol{x}_0(t) - \tilde{\boldsymbol{x}}_i^c(t)) \end{cases} \tag{4}$$

where $\boldsymbol{K} \in R^{m\times n}$ is the controller gain. $\tilde{\boldsymbol{x}}_i^c(t)$ is the event-generated state, which is defined by a model-based estimation: $\tilde{\boldsymbol{x}}_i^c(t = t_k^i) = \hat{\boldsymbol{x}}_i^c(t_k^i)$, $\frac{d}{dt}\tilde{\boldsymbol{x}}_i^c(t) = A\tilde{\boldsymbol{x}}_i^c(t)$, $t \in (t_k^i, t_{k+1}^i)$. The event moment $t_k^i, k \in \mathbb{N}^+$ is triggered by the DETM in the following. $\hat{\boldsymbol{x}}_i^c(t)$ is the estimated nominal state generated by the nominal observer, presented in the following equations:

$$\begin{cases} \dot{\hat{\boldsymbol{x}}}_i^c(t) = \boldsymbol{A}\hat{\boldsymbol{x}}_i^c(t) + \boldsymbol{B}\boldsymbol{u}_i^c(t) + \boldsymbol{L}_o\left(\boldsymbol{y}_i^c(t) - \hat{\boldsymbol{y}}_i^c(t)\right) \\ \hat{\boldsymbol{y}}_i^c(t) = \boldsymbol{C}\hat{\boldsymbol{x}}_i^c(t) \end{cases} \tag{5}$$

where $\boldsymbol{L}_o \in \mathbb{R}^{n\times r}$ is the observer gain.

The DETM is proposed in a distributed manner by the following rule:

$$t_{k+1}^i = \inf\left\{t | t > t_k^i,\ g_i(t) \geq 0\right\} \tag{6}$$

where $g_i(t) = -\eta_i(t) + \omega(\boldsymbol{\varepsilon}_i(t)^T\boldsymbol{U}\boldsymbol{\varepsilon}_i(t) - \boldsymbol{z}_i(t)^T\boldsymbol{T}\boldsymbol{z}_i(t))$. $\boldsymbol{\varepsilon}_i(t) = \tilde{\boldsymbol{x}}_i^c - \hat{\boldsymbol{x}}_i^c$ is the event error. $\eta_i(t)$ is the auxiliary dynamic variable satisfying $\eta_i(0) = \eta_{i0} > 0$ and $\dot{\eta}_i(t) = -\lambda\eta_i(t) - \boldsymbol{\varepsilon}_i(t)^T\boldsymbol{U}\boldsymbol{\varepsilon}_i(t) + \boldsymbol{z}_i(t)^T\boldsymbol{T}\boldsymbol{z}_i(t)$. It is shown that this type of DETM can significantly prolong the IET (Girard (2015); Yi et al. (2019)). ω and λ are two positive scalars. $\boldsymbol{U}, \boldsymbol{T}$ are two positive definite matrices.

In the following Lemma, we propose a synthesis of the controller, the observer, and the DETM for L-F consensus.

Lemma 5. The MAS (1) under no-fault condition can achieve leader-following consensus with the nominal controller (4), nominal observer (5), and the DETM (6), if the following conditions are satisfied subject to $\boldsymbol{Q}_1 > 0$, $\boldsymbol{Q}_2 > 0$, $\boldsymbol{K}$, $\boldsymbol{L}_o$, $\boldsymbol{T} > 0$ and $\boldsymbol{U} > 0$:

$$\boldsymbol{S} = \begin{pmatrix} \boldsymbol{S}_1 & \boldsymbol{S}_{12} & \boldsymbol{S}_{13} \\ * & \boldsymbol{S}_2 & \overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}} \\ * & * & \overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}} - \overline{\boldsymbol{U}} \end{pmatrix} < 0 \tag{7}$$

with

$$\begin{aligned} &\boldsymbol{S}_1 = \mathrm{Sym}\{\overline{\boldsymbol{Q}}_1(\overline{\boldsymbol{A}} - \overline{\boldsymbol{H}}\overline{\boldsymbol{B}}\overline{\boldsymbol{K}})\} + \overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}} \\ &\boldsymbol{S}_2 = \mathrm{Sym}\{\overline{\boldsymbol{Q}_2}(\overline{\boldsymbol{A}} - \overline{\boldsymbol{L}_o}\overline{\boldsymbol{C}})\} + \overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}} \\ &\boldsymbol{S}_{12} = -\overline{\boldsymbol{Q}}_1\overline{\boldsymbol{H}}\overline{\boldsymbol{B}}\overline{\boldsymbol{K}} + \overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}}, \quad \boldsymbol{S}_{13} = \boldsymbol{S}_{12} \end{aligned} \tag{8}$$

and $\overline{\boldsymbol{Q}}_1 = \boldsymbol{I}_N \otimes \boldsymbol{Q}_1$, $\overline{\boldsymbol{Q}}_2 = \boldsymbol{I}_N \otimes \boldsymbol{Q}_2$, $\overline{\boldsymbol{A}} = \boldsymbol{I}_N \otimes \boldsymbol{A}$, $\overline{\boldsymbol{B}} = \boldsymbol{I}_N \otimes \boldsymbol{B}$, $\overline{\boldsymbol{C}} = \boldsymbol{I}_N \otimes \boldsymbol{C}$, $\overline{\boldsymbol{K}} = \boldsymbol{I}_N \otimes \boldsymbol{K}$, $\overline{\boldsymbol{L}}_o = \boldsymbol{I}_N \otimes \boldsymbol{L}_o$, $\overline{\boldsymbol{H}} = \boldsymbol{\mathcal{H}} \otimes \boldsymbol{I}_n$, $\overline{\boldsymbol{T}} = \boldsymbol{I}_N \otimes \boldsymbol{T}$, $\overline{\boldsymbol{U}} = \boldsymbol{I}_N \otimes \boldsymbol{U}$.

Proof. Referring to Remark 8, Lemma 5 is a special case of Theorem 7 by taking $\boldsymbol{B}_i^f = \boldsymbol{B}$, $\boldsymbol{C}_i^f = \boldsymbol{C}$, and $\boldsymbol{x}_i^{\Delta}(0) = \boldsymbol{x}_i^{vs}(0) = 0$.

Remark 6. There are several methods to solve the bilinear matrix inequality (BMI) $\boldsymbol{S} < 0$ in (7). A possible way is to first choose $\boldsymbol{K} = \boldsymbol{B}^T\boldsymbol{P}$ and solve the Ricatti inequality $\boldsymbol{Q}_1\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{Q}_1 - 2\lambda_{\min}(\boldsymbol{\mathcal{H}})\boldsymbol{Q}_1\boldsymbol{B}\boldsymbol{B}^T\boldsymbol{Q}_1 < 0$ subject to $\boldsymbol{Q}_1 > 0$, then denote $\boldsymbol{F} = \boldsymbol{Q}_2\boldsymbol{L}_o$ and solve the LMI $\boldsymbol{S} < 0$ subject to $\boldsymbol{F}, \boldsymbol{T} > 0, \boldsymbol{U} > 0, \boldsymbol{Q}_2 > 0$. Finally $\boldsymbol{L}_o = \boldsymbol{Q}_2^{-1}\boldsymbol{F}$.

4. FTCC STRATEGY DESIGN

This section focuses on the FTCC strategy, which helps to fault compensation without altering the nominal controller/observer designed in the previous section.

4.1 The architecture of FTCC

For the sake of brevity, the time notation "(t)" is omitted in the following sections.

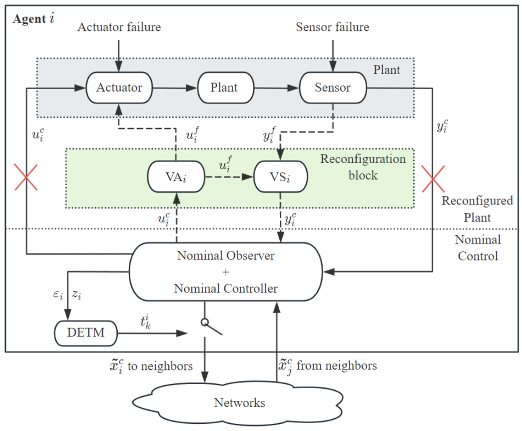

Fig. 1. The architecture of FTCC

We introduce the FTCC strategy in Fig. 1. The proposed approach is to build a reconfiguration block employing a virtual actuator (VA) and a virtual sensor (VS). The reconfigured plant is designed to conceal faults, namely, $\boldsymbol{x}_i^f$ and $\boldsymbol{y}_i^f$, and to generate a reconfigured output $\boldsymbol{y}_i^c$ and the compensated input $\boldsymbol{u}_i^f$. As a result, the controller/observer consistently perceives the plant as before and responds as before, i.e., (4) and (5). For any agent i, if a fault is detected, then the VA and VS will be activated. On the contrary, if the fault disappears, the corresponding VA and VS could be removed, and the system will behave like a healthy agent. The main benefit of this approach is that the reconfiguration block within the active FTCC has the capability to manage not only minor faults but also critical failures, including the complete loss of effectiveness.

4.2 Reconfiguration block

The reconfiguration block is the most critical part in the FTCC, illustrated in Fig. 1, composed of a VA and a VS. The VA of agent i has the following model:

$$\begin{cases} \boldsymbol{u}_i^f = \boldsymbol{N}_i^{va}\boldsymbol{u}_i^c + \boldsymbol{M}_i^{va}\boldsymbol{x}_i^{\Delta}, \quad \boldsymbol{N}_i^{va} = (\boldsymbol{B}_i^f)^{\dagger}\boldsymbol{B} \\ \dot{\boldsymbol{x}}_i^{\Delta} = \left(\boldsymbol{A} - \boldsymbol{B}_i^f\boldsymbol{M}_i^{va}\right)\boldsymbol{x}_i^{\Delta} + \left(\boldsymbol{B} - \boldsymbol{B}_i^f\boldsymbol{N}_i^{va}\right)\boldsymbol{u}_i^c \end{cases} \tag{9}$$

where $\boldsymbol{M}_i^{va}$ is the gain to be designed. Particularly, if $\mathrm{Im}(\boldsymbol{B}) = \mathrm{Im}(\boldsymbol{B}_i^f)$, $\boldsymbol{B} - \boldsymbol{B}_i^f\boldsymbol{N}_i^{va} = \boldsymbol{0}$. $\boldsymbol{x}_i^{\Delta}$ is the internal state of the VA which can be regarded as a difference state between the nominal system and the faulty system.

The VS of agent i has the following model:

$$\begin{cases} \boldsymbol{y}_i^c = \boldsymbol{P}_i^{vs}\boldsymbol{y}_i^f + \boldsymbol{y}_i^{vs}, \quad \boldsymbol{P}_i^{vs} = \boldsymbol{C}(\boldsymbol{C}_i^f)^{\dagger} \\ \boldsymbol{y}_i^{vs} = (\boldsymbol{C} - \boldsymbol{P}_i^{vs}\boldsymbol{C}_i^f)\boldsymbol{x}_i^{vs} \\ \dot{\boldsymbol{x}}_i^{vs} = \left(\boldsymbol{A} - \boldsymbol{L}_i^{vs}\boldsymbol{C}_i^f\right)\boldsymbol{x}_i^{vs} + \boldsymbol{B}\boldsymbol{u}_i^f + \boldsymbol{L}_i^{vs}\boldsymbol{y}_i^f \end{cases} \tag{10}$$

where $\boldsymbol{L}_i^{vs}$ is the gain of the VS to be designed, Particularly, if $\mathrm{Ker}(\boldsymbol{C}) = \mathrm{Ker}(\boldsymbol{C}_i^f)$, $\boldsymbol{C} - \boldsymbol{P}_i^{vs}\boldsymbol{C}_i^f = \boldsymbol{0}$. $\boldsymbol{x}_i^{vs}$ is the internal state of the VS, serving as an estimation of the faulty state $\boldsymbol{x}_i^f$.

4.3 System analysis

Let us define several new variables for further analysis.

$$
\begin{array}{lll}
\text{Consensus error :} & \boldsymbol{\delta}_i^f = \boldsymbol{x}_i^\Delta + \boldsymbol{x}_i^f - \boldsymbol{x}_0 & \\
\text{Observer error :} & \boldsymbol{e}_i = \hat{\boldsymbol{x}}_i^c - (\boldsymbol{x}_i^\Delta + \boldsymbol{x}_i^f) & \\
\text{Sensor error :} & \boldsymbol{s}_i = \boldsymbol{x}_i^{vs} - \boldsymbol{x}_i^f & \\
\text{Event error :} & \boldsymbol{\varepsilon}_i = \widetilde{\boldsymbol{x}}_i^c - \hat{\boldsymbol{x}}_i^c &
\end{array} \tag{11}
$$

and denote $\boldsymbol{\delta}^f = \Big((\boldsymbol{\delta}_1^f)^T \dots (\boldsymbol{\delta}_N^f)^T\Big)^T$, $\boldsymbol{e} = \Big((\boldsymbol{e}_1)^T \dots (\boldsymbol{e}_N)^T\Big)^T$, $\boldsymbol{s} = \Big((\boldsymbol{s}_1)^T \dots (\boldsymbol{s}_N)^T\Big)^T$, $\boldsymbol{\varepsilon} = \Big((\boldsymbol{\varepsilon}_1)^T \dots (\boldsymbol{\varepsilon}_N)^T\Big)^T$, $\boldsymbol{x}^\Delta = \Big((\boldsymbol{x}_1^\Delta)^T \dots (\boldsymbol{x}_N^\Delta)^T\Big)^T$.

By using the relation $\boldsymbol{z} = -(\boldsymbol{\mathcal{H}} \otimes \boldsymbol{I})(\boldsymbol{\delta}^f + \boldsymbol{e} + \boldsymbol{\varepsilon})$, the closed-loop error dynamic can be obtained as

$$
\begin{cases}
\dot{\boldsymbol{\delta}}^f &= \boldsymbol{F}_1\boldsymbol{\delta}^f - \overline{\boldsymbol{H}}\,\overline{\boldsymbol{B}}\,\overline{\boldsymbol{K}}(\boldsymbol{e} + \boldsymbol{\varepsilon}) \\
\dot{\boldsymbol{e}} &= \boldsymbol{F}_4\boldsymbol{e} + \boldsymbol{F}_5\boldsymbol{s} - \overline{\boldsymbol{L}}_o\overline{\boldsymbol{C}}\boldsymbol{x}^\Delta \\
\dot{\boldsymbol{s}} &= -\boldsymbol{F}_3(\boldsymbol{\delta}^f + \boldsymbol{e} + \boldsymbol{\varepsilon}) + \boldsymbol{G}_3\boldsymbol{s} + \boldsymbol{G}_2\boldsymbol{x}^\Delta \\
\dot{\boldsymbol{x}}^\Delta &= -\boldsymbol{F}_2(\boldsymbol{\delta}^f + \boldsymbol{e} + \boldsymbol{\varepsilon}) + \boldsymbol{G}_1\boldsymbol{x}^\Delta \\
\dot{\boldsymbol{\varepsilon}} &= \overline{\boldsymbol{A}}\boldsymbol{\varepsilon} + \overline{\boldsymbol{H}}\,\overline{\boldsymbol{B}}\,\overline{\boldsymbol{K}}(\boldsymbol{\delta}^f + \boldsymbol{e} + \boldsymbol{\varepsilon}) + \overline{\boldsymbol{L}}_o\overline{\boldsymbol{C}}(\boldsymbol{e} + \boldsymbol{x}^\Delta) - \boldsymbol{F}_5\boldsymbol{s}
\end{cases} \tag{12}
$$

where $\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}}, \overline{\boldsymbol{C}}, \overline{\boldsymbol{K}}, \overline{\boldsymbol{L}}_o, \overline{\boldsymbol{H}}$ are defined in Lemma 5, $\overline{\boldsymbol{B}}^f = \operatorname{diag}(\boldsymbol{B}_1^f, \dots, \boldsymbol{B}_N^f)$, $\overline{\boldsymbol{C}}^f = \operatorname{diag}(\boldsymbol{C}_1^f, \dots, \boldsymbol{C}_N^f)$, $\overline{\boldsymbol{P}}^{vs} = \operatorname{diag}(\boldsymbol{P}_1^{vs}, \dots, \boldsymbol{P}_N^{vs})$, $\overline{\boldsymbol{L}}^{vs} = \operatorname{diag}(\boldsymbol{L}_1^{vs}, \dots, \boldsymbol{L}_N^{vs})$, $\overline{\boldsymbol{M}}^{va} = \operatorname{diag}(\boldsymbol{M}_1^{va}, \dots, \boldsymbol{M}_N^{va})$, $\overline{\boldsymbol{N}}^{va} = \operatorname{diag}(\boldsymbol{N}_1^{va}, \dots, \boldsymbol{N}_N^{va})$, $\boldsymbol{F}_1 = \overline{\boldsymbol{A}} - \overline{\boldsymbol{H}}\,\overline{\boldsymbol{B}}\,\overline{\boldsymbol{K}}$, $\boldsymbol{F}_2 = (\overline{\boldsymbol{B}} - \overline{\boldsymbol{B}}^f\overline{\boldsymbol{N}}^{va})\overline{\boldsymbol{K}}\,\overline{\boldsymbol{H}}$, $\boldsymbol{F}_3 = (\overline{\boldsymbol{B}} - \overline{\boldsymbol{B}}^f)\overline{\boldsymbol{N}}^{va}\overline{\boldsymbol{K}}\,\overline{\boldsymbol{H}}$, $\boldsymbol{F}_4 = \overline{\boldsymbol{A}} - \overline{\boldsymbol{L}}_o\overline{\boldsymbol{C}}$, $\boldsymbol{F}_5 = \overline{\boldsymbol{L}}_o(\overline{\boldsymbol{C}} - \overline{\boldsymbol{P}}^{vs}\overline{\boldsymbol{C}}^f)$, $\boldsymbol{G}_1 = \overline{\boldsymbol{A}} - \overline{\boldsymbol{B}}^f\overline{\boldsymbol{M}}^{va}$, $\boldsymbol{G}_2 = (\overline{\boldsymbol{B}} - \overline{\boldsymbol{B}}^f)\overline{\boldsymbol{M}}^{va}$, $\boldsymbol{G}_3 = \overline{\boldsymbol{A}} - \overline{\boldsymbol{L}}^{vs}\overline{\boldsymbol{C}}^f$.

The design parameter of VA and VS, namely $\boldsymbol{M}_i^{va}$ and $\boldsymbol{L}_i^{vs}$, are given by the following Theorem:

Theorem 7. The MAS (3) in the presence of actuator and sensor faults can achieve L-F consensus by using the FTCC (illustrated in the Fig. 1), with the nominal controller (4), the nominal observer (5), the VA (9), the VS (10) and the DETM (6), if the following matrix inequalities are satisfied subject to $\boldsymbol{Q}_1 > 0$, $\boldsymbol{Q}_2 > 0$, $\boldsymbol{Q}_3 > 0$, $\boldsymbol{Q}_4 > 0$, $\boldsymbol{Q}_{14}$, $\boldsymbol{Q}_{24}$, $\boldsymbol{Q}_{34}$, $\boldsymbol{M}_i^{va}$, $\boldsymbol{L}_i^{vs}$ $(i = 1, \dots, N)$, $\boldsymbol{T} > 0$ and $\boldsymbol{U} > 0$:

$$
\boldsymbol{S} = \begin{pmatrix}
\boldsymbol{S}_1 & \boldsymbol{S}_{12} & \boldsymbol{S}_{13} & \boldsymbol{S}_{14} & \boldsymbol{S}_{15} \\
* & \boldsymbol{S}_2 & \boldsymbol{S}_{23} & \boldsymbol{S}_{24} & \boldsymbol{S}_{25} \\
* & * & \boldsymbol{S}_3 & \boldsymbol{S}_{34} & \boldsymbol{S}_{35} \\
* & * & * & \boldsymbol{S}_4 & \boldsymbol{S}_{45} \\
* & * & * & * & \overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}} - \overline{\boldsymbol{U}}
\end{pmatrix} < 0 \tag{13}
$$

$$
\boldsymbol{Q} = \begin{pmatrix}
\boldsymbol{Q}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Q}_{14} \\
* & \boldsymbol{Q}_2 & \boldsymbol{0} & \boldsymbol{Q}_{24} \\
* & * & \boldsymbol{Q}_3 & \boldsymbol{Q}_{34} \\
* & * & * & \boldsymbol{Q}_4
\end{pmatrix} > 0 \tag{14}
$$

where

$$
\begin{aligned}
&\boldsymbol{S}_1 = \operatorname{Sym}\{\overline{\boldsymbol{Q}}_1\boldsymbol{F}_1 - \overline{\boldsymbol{Q}}_{14}\boldsymbol{F}_2\} + \overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}} \\
&\boldsymbol{S}_{12} = -\overline{\boldsymbol{Q}}_1\overline{\boldsymbol{H}}\,\overline{\boldsymbol{B}}\,\overline{\boldsymbol{K}} - \overline{\boldsymbol{Q}}_{14}\boldsymbol{F}_2 - (\overline{\boldsymbol{Q}}_{24}\boldsymbol{F}_2)^T + \overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}} \\
&\boldsymbol{S}_{13} = -(\overline{\boldsymbol{Q}}_3\boldsymbol{F}_3)^T - (\overline{\boldsymbol{Q}}_{34}\boldsymbol{F}_2)^T \\
&\boldsymbol{S}_{14} = -(\overline{\boldsymbol{Q}}_4\boldsymbol{F}_2)^T + \overline{\boldsymbol{Q}}_{14}\boldsymbol{G}_1 + \boldsymbol{F}_1^T\overline{\boldsymbol{Q}}_{14} - \boldsymbol{F}_3^T\overline{\boldsymbol{Q}}_{34} \\
&\boldsymbol{S}_{15} = -\overline{\boldsymbol{Q}}_1\overline{\boldsymbol{H}}\,\overline{\boldsymbol{B}}\,\overline{\boldsymbol{K}} - \overline{\boldsymbol{Q}}_{14}\boldsymbol{F}_2 + \overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}} \\
&\boldsymbol{S}_2 = \operatorname{Sym}\{\overline{\boldsymbol{Q}}_2\boldsymbol{F}_4 - \overline{\boldsymbol{Q}}_{24}\boldsymbol{F}_2\} + \overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}} \\
&\boldsymbol{S}_{23} = \overline{\boldsymbol{Q}}_2\boldsymbol{F}_5 - (\overline{\boldsymbol{Q}}_3\boldsymbol{F}_3)^T - (\overline{\boldsymbol{Q}}_{34}\boldsymbol{F}_2)^T \\
&\boldsymbol{S}_{24} = -\overline{\boldsymbol{Q}}_2\overline{\boldsymbol{L}}_o\overline{\boldsymbol{C}} - (\overline{\boldsymbol{Q}}_4\boldsymbol{F}_2)^T - (\overline{\boldsymbol{H}}\,\overline{\boldsymbol{B}}\,\overline{\boldsymbol{K}})^T\overline{\boldsymbol{Q}}_{14} + \overline{\boldsymbol{Q}}_{24}\boldsymbol{G}_1 \\
&\qquad + \boldsymbol{F}_4^T\overline{\boldsymbol{Q}}_{24} - \boldsymbol{F}_3^T\overline{\boldsymbol{Q}}_{34} \\
&\boldsymbol{S}_{25} = -\overline{\boldsymbol{Q}}_{24}\boldsymbol{F}_2 + \overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}} \\
&\boldsymbol{S}_3 = \operatorname{Sym}\{\overline{\boldsymbol{Q}}_3\boldsymbol{G}_3\} \\
&\boldsymbol{S}_{34} = \overline{\boldsymbol{Q}}_3\boldsymbol{G}_2 + \boldsymbol{F}_5^T\overline{\boldsymbol{Q}}_{24} + \overline{\boldsymbol{Q}}_{34}\boldsymbol{G}_1 + \boldsymbol{G}_3^T\overline{\boldsymbol{Q}}_{34} \\
&\boldsymbol{S}_{35} = -\overline{\boldsymbol{Q}}_3\boldsymbol{F}_3 - \overline{\boldsymbol{Q}}_{34}\boldsymbol{F}_2 \\
&\boldsymbol{S}_4 = \operatorname{Sym}\{\overline{\boldsymbol{Q}}_4\boldsymbol{G}_1 + \overline{\boldsymbol{Q}}_{24}^T\overline{\boldsymbol{L}}_o\overline{\boldsymbol{C}} + \overline{\boldsymbol{Q}}_{34}^T\boldsymbol{G}_2\} \\
&\boldsymbol{S}_{45} = -\overline{\boldsymbol{Q}}_4\boldsymbol{F}_2 - \overline{\boldsymbol{Q}}_{14}^T\overline{\boldsymbol{H}}\,\overline{\boldsymbol{B}}\,\overline{\boldsymbol{K}} - \overline{\boldsymbol{Q}}_{34}^T\boldsymbol{F}_3
\end{aligned}
$$

$\boldsymbol{K}$ and $\boldsymbol{L}_o$ are obtained by Lemma 5, $\boldsymbol{F}_1$, $\boldsymbol{F}_2$, $\boldsymbol{F}_3$, $\boldsymbol{F}_4$, $\boldsymbol{F}_5$, $\boldsymbol{G}_1$, $\boldsymbol{G}_2$, $\boldsymbol{G}_3$ are defined in (12), $\overline{\boldsymbol{T}} = \boldsymbol{I}_N \otimes \boldsymbol{T}$, $\overline{\boldsymbol{U}} = \boldsymbol{I}_N \otimes \boldsymbol{U}$, $\overline{\boldsymbol{Q}}_k = \boldsymbol{I}_N \otimes \boldsymbol{Q}_k$.

Proof. The proof consists of showing the stability of the error system (12) by employing the Lyapunov method. Consider a Lyapunov function $V(t)$:

$$
V(t) = \boldsymbol{\xi}^T(t)\boldsymbol{Q}\boldsymbol{\xi}(t) + \boldsymbol{\eta}(t) \tag{15}
$$

where $\boldsymbol{\xi} = ((\boldsymbol{\delta}^f)^T, \boldsymbol{e}^T, \boldsymbol{s}^T, (\boldsymbol{x}^\Delta)^T)^T$, $\boldsymbol{\eta} = \sum_i \boldsymbol{\eta}_i$, $\boldsymbol{Q} > 0$ defined in Theorem 7. Notice that from the event-triggered rule (6), $g_i(t) < 0$, $\forall t$, that is, $-\boldsymbol{\eta}(t) + \omega(\boldsymbol{\varepsilon}(t)^T\overline{\boldsymbol{U}}\boldsymbol{\varepsilon}(t) - \boldsymbol{z}(t)^T\overline{\boldsymbol{T}}\boldsymbol{z}(t)) < 0$, thus

$$
\boldsymbol{\varepsilon}^T\overline{\boldsymbol{U}}\boldsymbol{\varepsilon} - \boldsymbol{z}^T\overline{\boldsymbol{T}}\boldsymbol{z} \le \frac{\boldsymbol{\eta}}{\omega} \tag{16}
$$

Then by following the dynamic of $\boldsymbol{\eta}$, we have

$$
\dot{\boldsymbol{\eta}} = -\lambda\boldsymbol{\eta} - \boldsymbol{\varepsilon}^T\overline{\boldsymbol{U}}\boldsymbol{\varepsilon} + \boldsymbol{z}^T\overline{\boldsymbol{T}}\boldsymbol{z} \ge -\lambda\boldsymbol{\eta} - \frac{\boldsymbol{\eta}}{\omega} \tag{17}
$$

Since $\boldsymbol{\eta}(0) > 0$, $\lambda > 0$, and $\omega > 0$, we have $\boldsymbol{\eta}(t) > 0$, $\forall t$. Therefore $V(t) > 0$. The next steps is to obtain sufficient conditions for $\dot{V}(t) < 0$. Note that,

$$
\dot{V}(t) = 2\boldsymbol{\xi}^T\boldsymbol{Q}\dot{\boldsymbol{\xi}} + \dot{\boldsymbol{\eta}} \tag{18}
$$

where $\dot{\boldsymbol{\eta}} = -\lambda\boldsymbol{\eta} - \boldsymbol{\varepsilon}^T\overline{\boldsymbol{U}}\boldsymbol{\varepsilon} + \boldsymbol{z}^T\overline{\boldsymbol{T}}\boldsymbol{z} \le -\boldsymbol{\varepsilon}^T\overline{\boldsymbol{U}}\boldsymbol{\varepsilon} + \boldsymbol{z}^T\overline{\boldsymbol{T}}\boldsymbol{z}$. Therefore (18) becomes

$$
\dot{V} \le 2\boldsymbol{\xi}^T Q\dot{\boldsymbol{\xi}} - \boldsymbol{\varepsilon}^T\overline{\boldsymbol{U}}\boldsymbol{\varepsilon} + \boldsymbol{z}^T\overline{\boldsymbol{T}}\boldsymbol{z} \tag{19}
$$

Then, the rest of the calculation just involves using the definition of $\dot{\boldsymbol{\xi}}$ by (12). $\boldsymbol{z}$ is replaced by $\boldsymbol{z} = -(\boldsymbol{\mathcal{H}} \otimes \boldsymbol{I})(\boldsymbol{\delta}^f + \boldsymbol{e} + \boldsymbol{\varepsilon})$. The final expression is obtained as $\dot{V} \le \boldsymbol{\zeta}^T\boldsymbol{S}\boldsymbol{\zeta}$, where $\boldsymbol{S}$ is defined in Theorem 7 and $\boldsymbol{\zeta} = ((\boldsymbol{\delta}^f)^T, \boldsymbol{e}^T, \boldsymbol{s}^T, (\boldsymbol{x}^\Delta)^T, \boldsymbol{\varepsilon}^T)^T$.

Remark 8. If there is no fault, i.e., $\boldsymbol{B}_i^f = \boldsymbol{B}$, $\boldsymbol{C}_i^f = \boldsymbol{C}$, and take the initial value of the VA, VS as $\boldsymbol{x}_i^\Delta(0) = \boldsymbol{x}_i^{vs}(0) = \boldsymbol{0}$, then the condition $\boldsymbol{S} < 0$ in Theorem 7 is simplified to

$$
\left(\begin{array}{c|c|c|c|c}
\begin{matrix}\operatorname{Sym}\{\overline{\boldsymbol{Q}}_1\boldsymbol{F}_1\} \\ +\overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}}\end{matrix} & \begin{matrix}-\overline{\boldsymbol{Q}}_1\overline{\boldsymbol{H}}\,\overline{\boldsymbol{B}}\,\overline{\boldsymbol{K}} \\ +\overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}}\end{matrix} & \boldsymbol{0} & \boldsymbol{0} & \begin{matrix}-\overline{\boldsymbol{Q}}_1\overline{\boldsymbol{H}}\,\overline{\boldsymbol{B}}\,\overline{\boldsymbol{K}} \\ +\overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}}\end{matrix} \\ \hline
* & \begin{matrix}\operatorname{Sym}\{\overline{\boldsymbol{Q}}_2\boldsymbol{F}_4\} \\ +\overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}}\end{matrix} & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}} \\ \hline
\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \hline
\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \hline
* & * & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{H}}^2\overline{\boldsymbol{T}} - \overline{\boldsymbol{U}}
\end{array}\right) < 0 \tag{20}
$$

which is equivalent to Lemma 5 under no-fault condition.

Remark 9. The design condition $\boldsymbol{S} < 0$ is not in LMI term and is difficult to linearize by common methods. Global optimization algorithms are required, which can be found in this tutorial (VanAntwerp and Braatz (2000)). Nevertheless, as reported in (Rotondo et al. (2014)), the dynamic of VA $(\boldsymbol{A} - \boldsymbol{B}_i^f\boldsymbol{M}_i^{va})$ and VS $(\boldsymbol{A} - \boldsymbol{L}_i^{vs}\boldsymbol{C}_i^f)$ for a single agent follows the separation principle, e.g., if $\boldsymbol{A} - \boldsymbol{B}^f\boldsymbol{M}^{va}$, $\boldsymbol{A} - \boldsymbol{L}^{vs}\boldsymbol{C}^f$ and $\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}$ are all Hurwitz then the overall system is stable. Therefore some trials of initial value of M_i^{va} (resp. L_i^{vs}) for a Hurwitz $A - B_i^f M_i^{va}$ (resp. $A - L_i^{vs} C_i^f$) can be used, and then solving the constraint $S < 0$ becoming an LMI problem.

Remark 10. During the implementation phase, the system is discretized, and both the DETM and the controller are realized with a designated sampling period. In this way, Zeno behavior can be avoided.

4.4 FTCC procedure

The reconfiguration procedure of agent i is described as follows:

(1) A fault is detected in agent i and is estimated by a fault diagnosis module. The estimated fault is represented by the effectiveness matrix Θ_i and Φ_i.
(2) The gain of the VA (M_i^{va}), the gain of the VS (L_i^{vs}) and the parameters of the DETM (T and U) are obtained by Theorem 7.
(3) Agent i activates its FTCC and inserts the reconfiguration block between the faulty plant and the nominal controller/observer as illustrated in Fig. 1.

5. NUMERICAL EXAMPLES

Consider a MAS with 5 follower agents and one leader following the homogeneous dynamic of (1), where A, B, C are defined as follows (Trejo et al. (2022)):

$$A = \begin{pmatrix} -0.05 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & -3 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad (21)$$

The communication topology is represented as

$$\mathcal{L} = \begin{pmatrix} 3 & -1 & -1 & 0 & -1 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 3 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{pmatrix}, \mathcal{D} = \mathrm{diag}(1, 0, 0, 0, 0) \quad (22)$$

The nominal controller and nominal observer are obtained by Theorem 5:

$$\begin{aligned} K &= \begin{pmatrix} 0.7241 & 0.6116 & -0.1107 & -0.6615 \\ -0.0785 & -0.0422 & 0.6331 & -0.6715 \end{pmatrix}, \\ L_o &= \begin{pmatrix} 5.1773 & 5.6029 & -0.6986 & -0.6318 \\ -0.4192 & -1.0930 & 6.2727 & 0.1335 \end{pmatrix}^T \end{aligned} \quad (23)$$

The fault scenario is chosen as follows

(1) $t < 5$: no fault
(2) $5 \leq t < 10$: partial and complete actuator failure of agent 2 $\Theta_2 = \mathrm{diag}(0.5, 0)$.
(3) $10 \leq t < 20$: partial actuator failure of agent 4 $\Theta_4 = \mathrm{diag}(1, 0.2)$, partial sensor failure of agent 1 $\Phi_1 = \mathrm{diag}(0.3, 1)$.
(4) $20 \leq t < 35$: actuators of all agents return to normal state. Agent 1's sensor returns to normal state. Partial sensor failure of agent 4 and agent 5 $\Phi_4 = \mathrm{diag}(0.8, 0.3)$, $\Phi_5 = \mathrm{diag}(0.8, 1)$.
(5) $t \geq 35$: partial and complete actuator failure of agent 1 and agent 5 $\Theta_1 = \mathrm{diag}(0.5, 0)$, $\Theta_5 = \mathrm{diag}(1, 0)$.

$\eta_i(0)$ is set as $\eta_{i0} = 100$, and set $\gamma = \omega = 1$. By employing the proposed FTCC strategy, we illustrate the trajectory of consensus error in Fig. 2. Note that the consensus is successfully achieved in the presence of faults. The perturbation is apparent when new faults arise, for instance, at $t = 5$ s of agent 2 and at $t = 10$ s of agent 4. The reconfiguration block effectively intervenes and contributes to stabilization. Notably, the propagation of faults is effectively restrained, as demonstrated at $t = 35$ s that the fault of agent 1 and agent 5 has only caused minor disturbances on other agents. The VS error and the internal state of VA are presented in Fig. 3, which corresponds to the fault scenario. Note that the VA and the VS of agent 3 are kept deactivated since the agent is free from any fault.

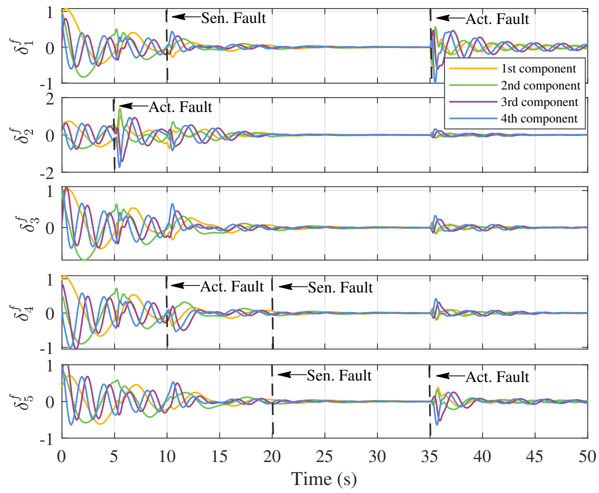

Fig. 2. Consensus error

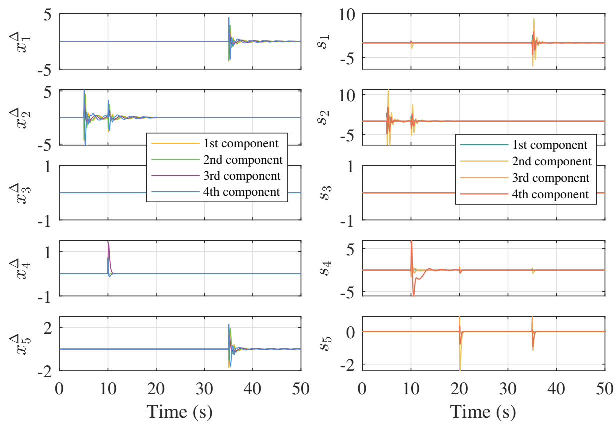

Fig. 3. VA internal state (left) and VS error (right)

The IET of each agent is depicted in Fig.4. It is noticeable that the IET reaches a minimum value when a fault starts to occur. Despite the efficiency of DETM in reducing communication times, the fault event, especially the occurrence of a fault, still needs more attention, and thus, the IET is shorter during these critical moments.

6. CONCLUSION

This paper investigates the FTCC leader-following consensus problem of MASs and proposes a unified framework for handling multiplicative actuator and sensor faults. By employing the virtual actuator and virtual sensor method,

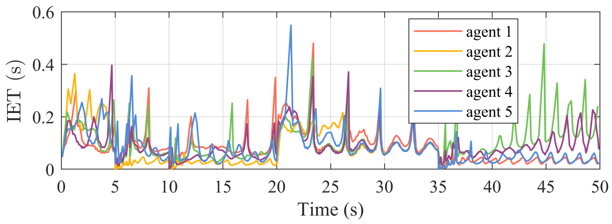

Fig. 4. Inter-event time (IET)

the reconfigured plant can tolerate the partial or complete loss of effectiveness of the actuator and the sensor without re-tuning the designed observer-based controller. The proposed DETM successfully solves the issue of communication cost, and the distributed strategy enables a more practical implementation. It has also been shown that the consensus is asymptotically achieved in the presence of faults, and the reconfiguration block can efficiently compensate for the fault impact. A numerical example corresponding to a MAS is also presented to demonstrate the capabilities of the proposed method.

Further works will aim to design a fault diagnosis strategy and study the robustness of the system under external disturbance.

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