Python Code Test Cases

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1 Stochastic Collocation Class

1.1 TEST: Single Uniform Variable

In this test we consider $f(x) = x^2$ with x a random variable uniformly distributed along $\bar{x} \pm \sigma$, where $\bar{x}$ is the average value and σ is the uncertainty in x.. x can be expressed as a function of a random variable ξ distributed from -1 to 1 as

$$x(\xi) = \bar{x} + \sigma\xi, \qquad \xi \in [-1, 1]. \tag{1}$$

This test evalutes the moments n of $f(x)$ as

$$< f(x)^n > \equiv \int P(\xi) f(x)^n d\xi, \tag{2}$$

where $P(\xi)$ is the probability distribution function for xi. In this case, for the uniformly-distributed random variable, $P(\xi)$=1/2 for all values of ξ. Because $\xi \in [-1, 1]$, we integrate

$$\begin{aligned} < f(x)^n > &= \int_{-1}^{1} P(\xi) f(x(\xi))^n d\xi, \\ &= \frac{1}{2} \int_{-1}^{1} \left(x(\xi)^2\right)^n d\xi, \\ &= \frac{1}{2} \int_{-1}^{1} \left(\bar{x}^2 + 2\bar{x}\sigma\xi + \sigma^2\xi^2\right)^n d\xi. \end{aligned} \tag{3}$$

For this test case, we consider x to be uniformly distributed between 1 and 2, so $\sigma = 0.5$ and the first three moments are

$$< f(x) > = 7/3, \tag{4}$$

$$< f(x)^2 > = 31/5, \tag{5}$$

$$< f(x)^3 > = 127/7. \tag{6}$$