Advanced Uncertainty Quantification with RAVEN

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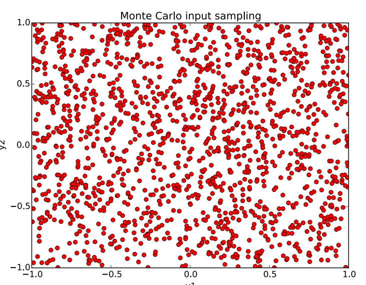
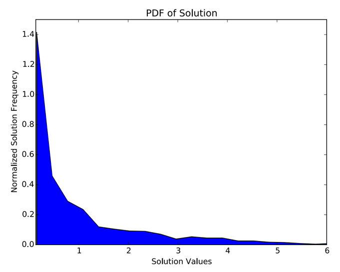
Uncertainty Quantification





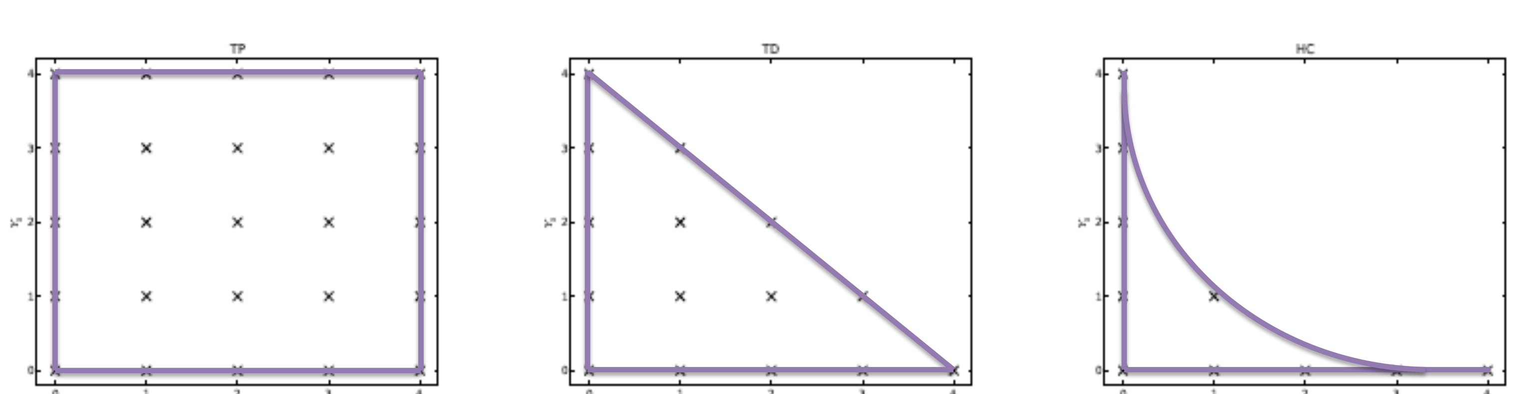
Uncertainty Quantification (UQ) is the science of understanding the output response of a model as a function of the uncertainty in the inputs. Traditional UQ uses Monte Carlo method to explore the model's output space by randomly sampling from the input space. While this method works well regardless of the number of uncertain model inputs, one of its largest failings is relatively slow convergence. There are several other advanced methods that seek to speed up convergence. We consider here the deterministic method Sparse Grid Collocation on generalized Polynomial Chaos.

Sparse Grid Quadrature

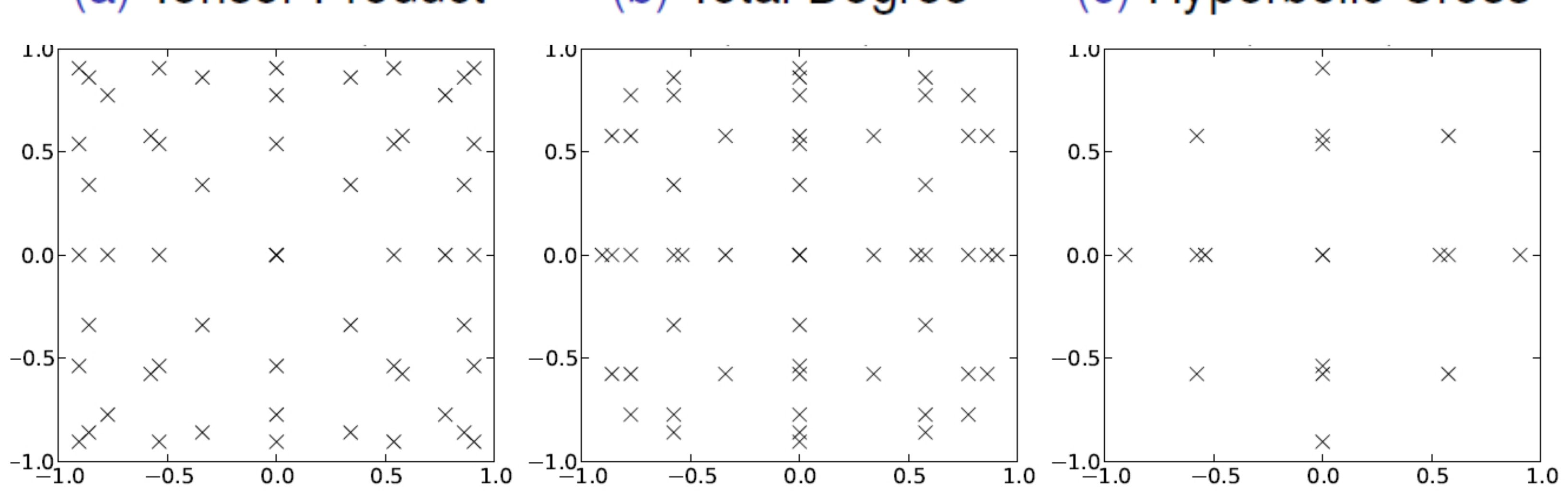
S. A. Smoljak developed a method joining tensor products of small quadratures to replicate large multidimensional quadrature sets using far fewer evaluations. These sparse grid quadratures do not completely remove the curse of dimensionality, but they mitigate it significantly. There are two kinds of sparse quadrature strategies: Total Degree, which seeks to match overall polynomial order and is most effective for analytic response surfaces; and Hyperbolic Cross, which emphasizes monomials and is most effective for discontinuous response surfaces.

$$\Lambda^{iso}_{TD}(L) = \left\{\bar{p} = [p_1, p_2, \dots, p_N] \colon \sum_{n=1}^{N} p_n \le L\right\}$$

$$\Lambda_{TD}(L) = \left\{\bar{p} = [p_1, p_2, \dots, p_N] \colon \sum_{n=1}^{N} \alpha_n p_n \le |\alpha|_1 L\right\}$$

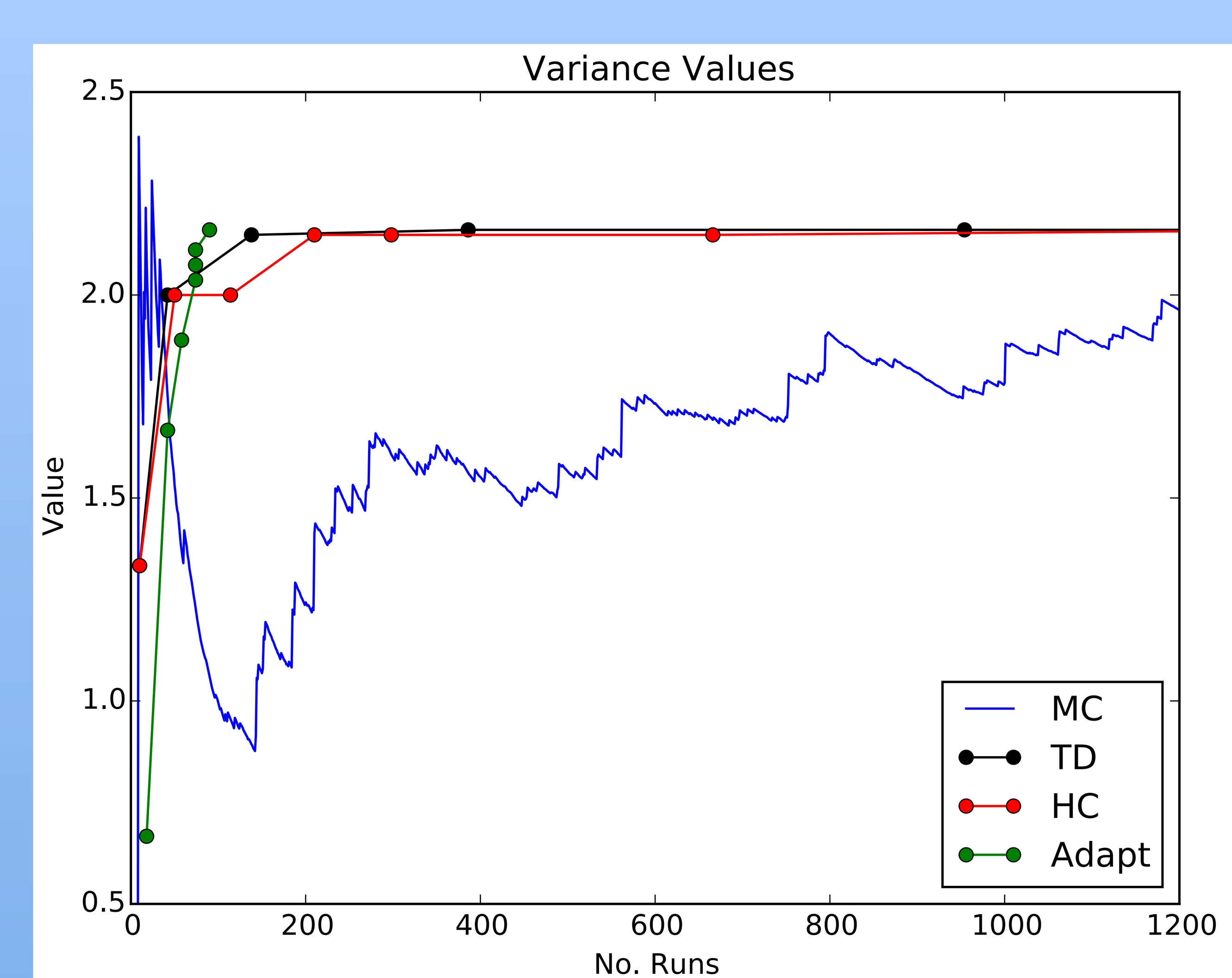


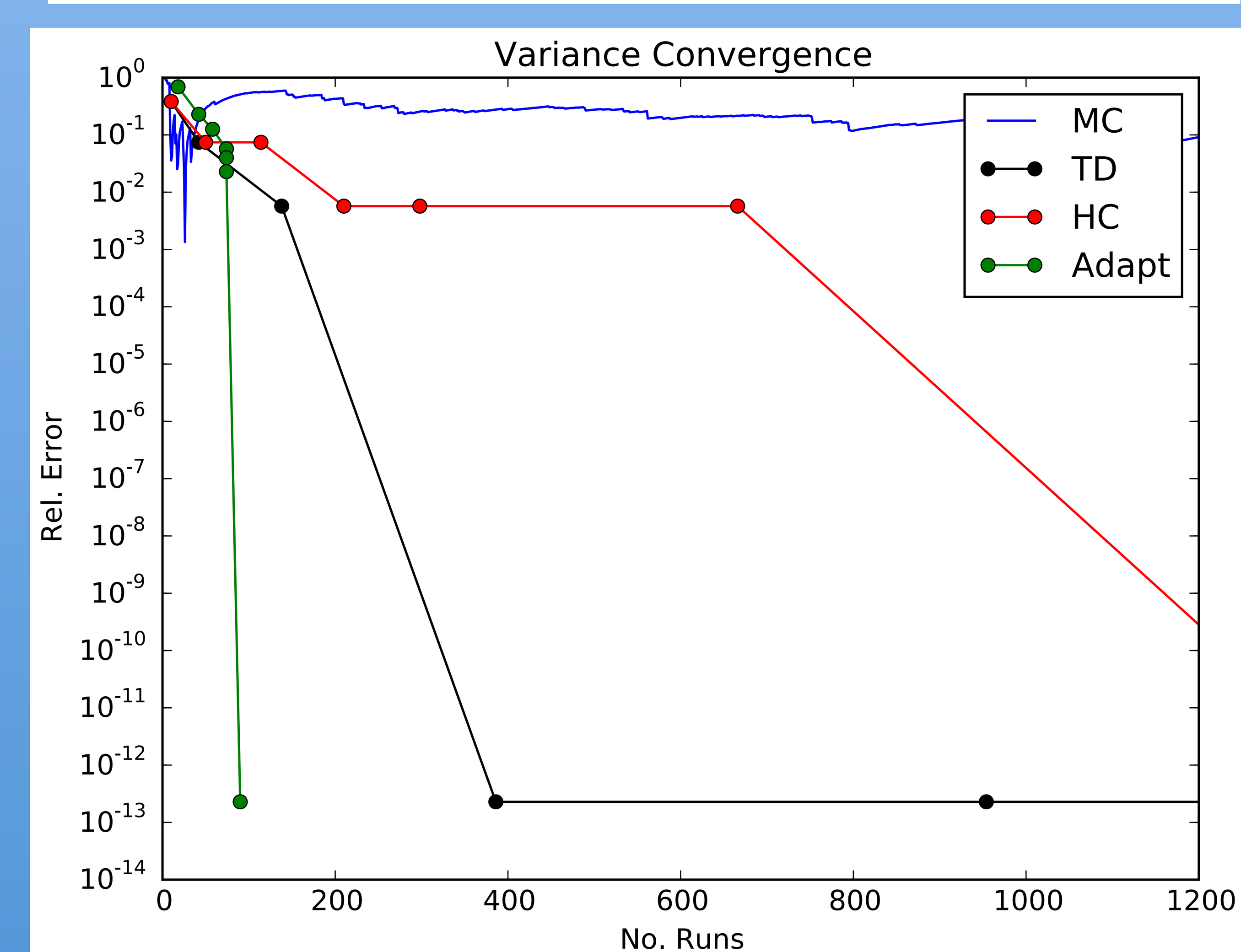
(a) Tensor Product (b) Total Degree (c) Hyperbolic Cross



Generalized Polynomial Chaos

One tool to represent a complex model is to approximate it using Generalized Polynomial Chaos (gPC). In this method, the model is represented as the sum of multidimensional orthonormal polynomials. The coefficients of these polynomials are calculated based on inner products of the polynomials and the original model. This representation of the original model lends itself to an effective UQ technique: Stochastic Collocation on Sparse Grids. This method combines stochastic collocation and sparse grid quadrature to compete favorably with Monte Carlo methods in many cases.





Example: $u(Y) = \prod_{m=1}^{4}(1 + y_m) \Rightarrow u(Y) \approx \sum_{k=0}^{N} u_k\, \Phi_k(Y)$

Stochastic Collocation

$$u_k = \int_{\Omega} u(Y)\, \Phi_k(Y) \rho(Y) dY \approx \sum_{\ell=0}^{L} w_\ell u(Y_\ell) \Phi_k(Y_\ell)$$

Stochastic Collocation (SC) is a mathematical tool that uses multidimensional quadrature to approximate integrals over a probability-weighted domain. The most significant problem with SC is the curse of dimensionality: the more uncertain inputs, the exponentially larger number of samples need to be taken. To alleviate this problem, we use Smoljak sparse quadrature.

Adaptive Collocation

To improve collocation further, the grid of multidimensional polynomials used can be chosen adaptively by checking possible polynomials and adding the one that helps convergence the most. In general, adaptive collocation is the fastest-converging collocation method.