Simple problem testing

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Equation to solve:

$$-D\frac{d^2\phi}{d^2x} + \Sigma_a\phi = S,\tag{1}$$

$$\phi(x) = \frac{S}{\Sigma_a} \left(1 - e^{-x/L} \right), \tag{2}$$

$$L^2 \equiv \frac{D}{\Sigma_a}. (3)$$

1 Uniform

We allow Σ_a to vary uniformly on $\Sigma_a \in (0.5, 1)$ and quantify the uncertainty using stochastic collocation for generalized polynomial expansion as well as Monte Carlo sampling.

For increasing orders of expansion, the mean and variance obtained are shown along with the run time. The other parameters for ϕ are taken as follows:

$$S = 1.0 \text{ n/cm}^2/\text{s},$$
 (4)

$$D = 0.5 \text{ /cm}, \tag{5}$$

$$x = 2.0 \text{ cm}.$$
 (6)

The PDFs were obtained by Monte Carlo sampling of the polynomial expansion for the SC cases, and obtained directly for the Monte Carlo case, shown in Fig. ??. The x-axis is the value of the scalar flux, and the y-axis is the probability of obtaining a particular flux.

| $_{\mathrm{type}}$ | runs/order | mean | variance | run time (sec) |
|---------------------|------------|---------------|-----------------|----------------|
| MC | 23400 | 1.25554670335 | 0.0492854851117 | 742.441 |
| SC | 2 | 1.25472215220 | 0 | 2.738 |
| SC | 4 | 1.25569029702 | 0.049198975952 | 2.692 |
| SC | 8 | 1.25569096924 | 0.0492316191443 | 3.854 |
| SC | 16 | 1.25569096924 | 0.0492316191611 | 6.107 |

Table 1: Uniform Uncertainty Means, Variances

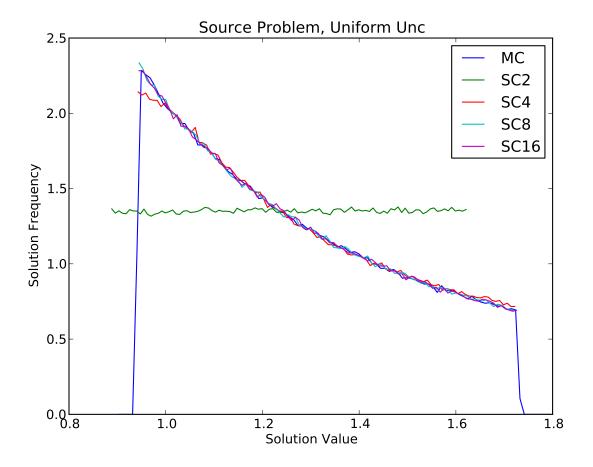


Figure 1: Uniform PDFs

2 Normal

We allow Σ_a to vary uniformly on $\Sigma_a \in \mathcal{N}(0.75, 0.15)$ and quantify the uncertainty using stochastic collocation for generalized polynomial expansion as well as Monte Carlo sampling.

For increasing orders of expansion, the mean and variance obtained are shown along with the run time. The other parameters for ϕ are taken as follows:

$$S = 1.0 \text{ n/cm}^2/\text{s},$$
 (7)

$$D = 0.5 \text{ /cm},$$
 (8)

$$x = 2.0 \text{ cm}.$$
 (9)

| type | runs/order | mean | variance | run time (sec) |
|------|------------|---------------|-----------------|----------------|
| MC | 23400 | 1.24922240195 | 0.0488719424418 | 366.31 |
| SC | 2 | 1.2547221522 | 0 | 2.08 |
| SC | 4 | 1.25569029702 | 0.049198975952 | 3.11 |
| SC | 8 | 1.25569096924 | 0.0492316191443 | 4.74 |
| SC | 16 | 1.25569096924 | 0.0492316191611 | 6.88 |

Table 2: Normal Uncertainty Means, Variances

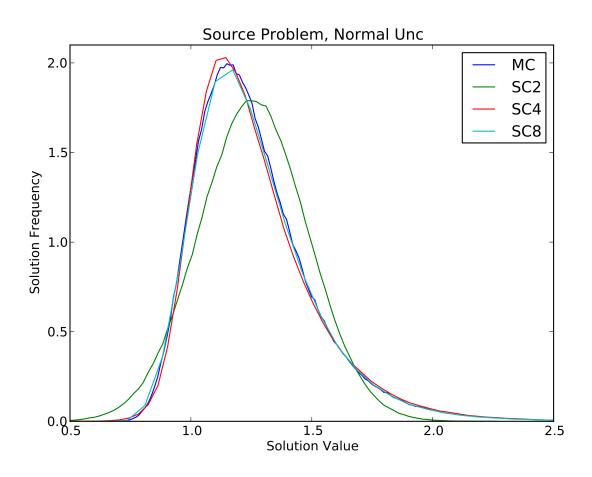


Figure 2: Normal PDFs