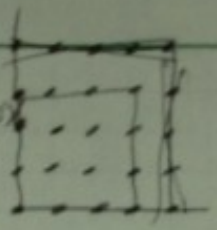
Tensor Product | Coefficients + Tensor Grids

$L=3 \quad N=2$

Λ: (0,0)(0,1)(0,2)(0,3)(1,0)(1,1)(1,2)(1,3)(2,0)(2,1)(2,2)(2,3)(3,0)(3,1)(3,2)(3,3)



$|\Lambda| = \cancel{25}\ 16$

$C(\vec{i}) = \sum_{\vec{j} \text{ valid}} (-1)^{|\vec{i}-\vec{j}|}$

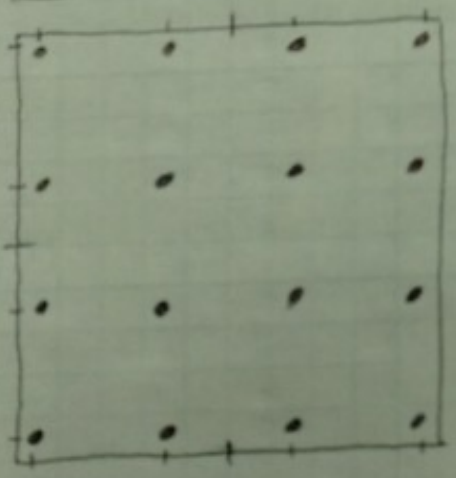
possible $\vec{j}$: (0,0)(0,1)(1,0)(1,1)

$\vec{j} + \vec{i} \in \Lambda$

Λ	Acceptable $\vec{j}$	$\sum (-1)^{\lvert\vec{i}-\vec{j}\rvert}$	$C(\vec{i})$	Pictorial Grid (max i+1)
(0,0)	(0,0)(0,1)(1,0)(1,1)	1-1-1+1	0	–
(0,1)	-all-	1-1-1+1	0	–
(0,2)	-all-	1-1-1+1	0	–
(0,3)	(0,0)(1,0)	1-1	0	–
(1,0)	-all-	1-1-1+1	0	–
(1,1)	-all-	1-1-1+1	0	–
(1,2)	-all-	1-1-1+1	0	–
(1,3)	(0,0)(1,0)	1-1	0	–
(2,0)	-all-	1-1-1+1	0	–
(2,1)	-all-	1-1-1+1	0	–
(2,2)	-all-	1-1-1+1	0	–
(2,3)	(0,0)(1,0)	1-1	0	–
(3,0)	(0,0)(0,1)	1-1	0	–
(3,1)	(0,0)(0,1)	1-1	0	–
(3,2)	(0,0)(0,1)	1-1	0	–
(3,3)	(0,0)	1	1	

total points: 16

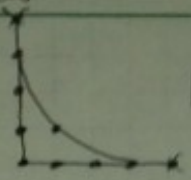
COMBINED (Gauss-Legendre)



Hyperbolic Cross | Coefficients + Tensor Fields

$L=3$ $N=2$

Λ: (0,0)(0,1)(0,2)(0,3)/ 1,0)(1,1)(2,0)(3,0)

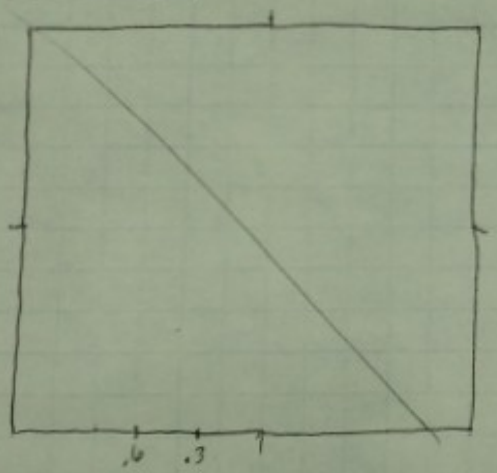


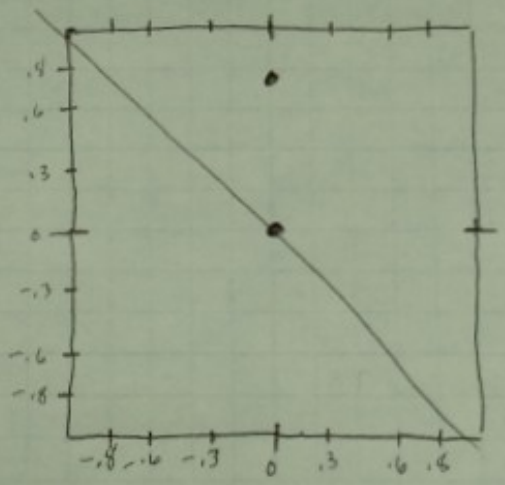
$|\Lambda| = 8$

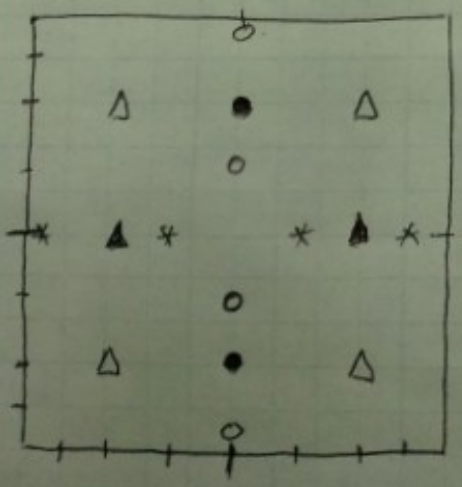
$C(\vec{i}) = \sum_{\vec{j} \in \{0,1\}^m,\ \vec{j}+\vec{i} \in \Lambda} (-1)^{|\vec{j}|}$ possible $\vec{j}$: (0,0)(0,1)(1,0)(1,1)

Λ	Acceptable $\vec{j}$	$\sum (-1)^{\lvert\vec{j}\rvert}$	$C(\vec{i})$	Pictoral ($m=\{1\}$)
(0,0)	(0,0)(0,1)(1,0)(1,1)	1−1−1+1	0	—
(0,1)	(0,0)(0,1)(1,0)	1−1−1	−1	
(0,2)	(0,0)(0,1)	1−1	0	—
(0,3)	(0,0)	1	1	
(1,0)	(0,0)(0,1)(1,0)	1−1−1	−1	
(1,1)	(0,0)	1	1	
(2,0)	(0,0)(1,0)	1−1	0	—
(3,0)	(0,0)	1	1	

COMBINED (Gauss-Legendre)

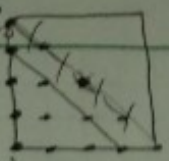






points: 16

Total Degree | Coefficients + Term Structure

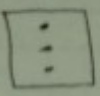
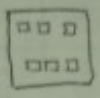
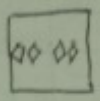


$L = 3 \quad N = 2$

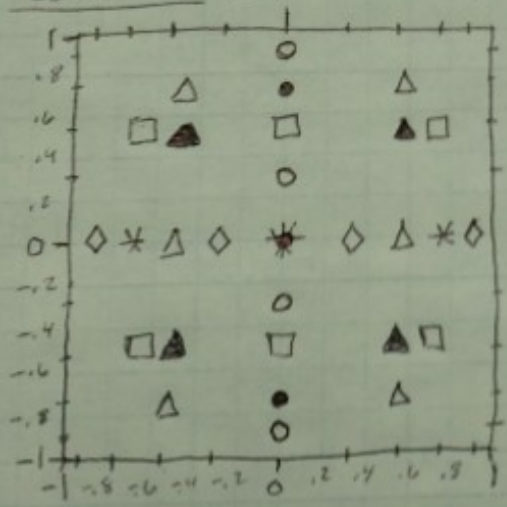
Λ: (0,0) (0,1) (0,2) (0,3) (1,0) (1,1) (1,2) (2,0) (2,1) (3,0)

$|\Lambda| = 10$

$C(i \in \Lambda) = \sum_{j \text{ s.t. } i+j \in \Lambda} (-1)^{|j|}$ possible j: (0,0) (0,1) (1,0) (1,1)

Λ	Accessible j	$\sum (-1)^{\lvert j \rvert}$	$C(i)$	Pictoral Grid ($m = i+1$)
(0,0)	(0,0)(0,1)(1,0)(1,1)	$1-1-1+1$	0	—
(0,1)	(0,0)(0,1)(1,0)(1,1)	$1-1-1+1$	0	—
(0,2)	(0,0)(0,1)(1,0)	$1-1-1$	-1	
(0,3)	(0,0)	1	1	
(1,0)	(0,0)(0,1)(1,0)(1,1)	$1-1-1+1$	0	—
(1,1)	(0,0)(0,1)(1,0)	$1-1-1$	-1	
(1,2)	(0,0)	1	1	
(2,0)	(0,0)(0,1)(1,0)	$1-1-1$	-1	
(2,1)	(0,0)	1	1	
(3,0)	(0,0)	1	1	

COMBINED (Gauss - Legendre)



total points = 29

# points	values
1	0
2	$\pm \frac{1}{\sqrt{3}}$
3	$0, \pm\sqrt{\frac{3}{5}}$
4	$\pm\sqrt{\frac{3}{7} - \frac{2}{7}\sqrt{\frac{6}{5}}}, \pm\sqrt{\frac{3}{7} + \frac{2}{7}\sqrt{\frac{6}{5}}}$

decimal

1	0
2	± 0.5774
3	$0, \pm 0.7746$
4	$\pm 0.3400, \pm 0.8611$