Distributions and Quadratures

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December 2, 2014

# 1 General Syntax

- Probability Measure: $f(y)$
- Normalization Factor: $A$
- Probability Distribution: $Af(y)$
- Generic function: $h(y)$

# 2 Distributions

## 2.1 Uniform

$$\mu = \frac{R+L}{2}, \tag{1}$$

$$\sigma = \frac{R-L}{2} \tag{2}$$

$$f(y) = \frac{1}{2\sigma}, \tag{3}$$

$$A = 1, \tag{4}$$

$$1 = \frac{1}{2\sigma}\int_L^R dy, \tag{5}$$

## 2.2 Normal

$$f(y) = \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right), \tag{6}$$

$$A = \frac{1}{\sigma\sqrt{2\pi}} \tag{7}$$

$$1 = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right)dy, \tag{8}$$

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### 2.3 Gamma

$$f(y) = y^{\alpha-1}e^{-\beta y}, \tag{9}$$

$$A = \frac{\beta^\alpha}{\Gamma(\alpha)}, \tag{10}$$

$$1 = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_L^\infty (y-L)^{\alpha-1}e^{-\beta(y-L)}dy, \tag{11}$$

### 2.4 Beta

$$f(y) = y^{\alpha-1}(1-y)^{\beta-1}, \tag{12}$$

$$A = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}, \tag{13}$$

$$1 = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_L^R y^{\alpha-1}(1-y)^{\beta-1}dy. \tag{14}$$

## 3 Quadrature

### 3.1 Legendre

$$\int_{-1}^1 h(x)d(x) = \sum_{\ell=1}^\infty w_\ell h(x_\ell) \tag{15}$$

### 3.2 Hermite

$$\int_{-\infty}^\infty h(x)\exp\left(\frac{-x^2}{2}\right)dy = \sum_{h=1}^\infty w_h h(x_h) \tag{16}$$

### 3.3 Laguerre

$$\int_0^\infty h(x)x^\alpha e^{-x}dx = \sum_{\mathcal{L}=1}^\infty w_\mathcal{L} h(x_\mathcal{L}) \tag{17}$$

### 3.4 Jacobi

$$\int_{-1}^1 h(x)(1-x)^\alpha(1+x)^\beta dx = \sum_{j=1}^\infty w_j h(x_j) \tag{18}$$

### 3.5 Clenshaw-Curtis

$$\int_{-1}^1 h(x)dx = \sum w_{cc}h(x_{cc}) \tag{19}$$

# 4 Conversions

## 4.1 Uniform and Legendre

$$y = \sigma x + \mu, \tag{20}$$

$$x = \frac{y - \mu}{\sigma}, \tag{21}$$

$$\int_a^b h(y) f_\ell(y) dy = \frac{1}{2} \sum_{\ell=1}^{\infty} w_\ell h(\sigma x_\ell + \mu) \tag{22}$$

## 4.2 Normal and Hermite

$$y = \sigma x + \mu, \tag{23}$$

$$x = \frac{y - \mu}{\sigma}, \tag{24}$$

$$\int_{-\infty}^{\infty} h(y) f_h(y) dy = \frac{1}{\sqrt{2\pi}} \sum_{h=1}^{\infty} w_h h(\sigma x_h + \mu) \tag{25}$$

## 4.3 Gamma and Laguerre

$$y = \frac{x}{\beta} + L, \tag{26}$$

$$x = (y - L)\beta, \tag{27}$$

$$\int_L^{\infty} h(y) f_g(y) dy = \frac{1}{(\alpha - 1)!} \sum_{g=1}^{\infty} w_g h\left(\frac{x_g}{\beta} + L\right) \tag{28}$$

## 4.4 Beta and Jacobi