Adaptive gPC Convergence Criteria Norms

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1 Notation

- ξ - The vector of input variables $\xi = (\xi_1, \xi_2, \cdots, \xi_N)$
- Ω - The input space spanned by the input variable distributions
- ϵ - The error between an iteration Λ_2 and Λ_1
- k - A combination of polynomial orders, given as a tuple
- Λ_1 - The set of all desired polynomial combinations k for the current iteration
- Λ_2 - The set of all desired polynomial combinations k for the next iteration
- $c_{i,k}$ - The scalar coefficient for polynomial combination with orders k in index set Λ_i
- Φ_k - The multidimensional orthonormal polynomial basis with degrees k.

2 L2 of Difference Space

$$\epsilon = \sqrt{\int_\Omega d\xi \left[\sum_{k\in\Lambda_2} c_{2,k}\Phi_k(\xi) - \sum_{k\in\Lambda_1} c_{1,k}\Phi_k(\xi)\right]^2}. \tag{1}$$

Let $c_{1,k} = 0 \forall k \notin \Lambda_1$, and noting

$$\int_\Omega \Phi_j(\xi)\Phi_k(\xi)P(\xi)d\xi = \delta_{jk}, \tag{2}$$

$$\epsilon = \sqrt{\int_\Omega d\xi \left[\sum_{m=1}^{2}\sum_{n=1}^{2}\sum_{j\in\Lambda_2}\sum_{k\in\Lambda_2} c_{m,k}c_{n,j}\Phi_k(\xi)\Phi_j(\xi)\right]}, \tag{3}$$

$$= \sqrt{\sum_{k\in\Lambda_2} c_{2,k}^2 - 2\sum_{k\in\Lambda_2} c_{2,k}c_{1,k} + \sum_{k\in\Lambda_2} c_{1,k}^2}, \tag{4}$$

$$= \sqrt{\sum_{k\in\Lambda_2} (c_{2,k} - c_{1,k})^2}. \tag{5}$$

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