Multistep Input Reduction for High Dimensional Uncertainty Quantification

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Outline

Discussion Points

Introduction

Uncertainty Quantification

Introduction

Uncertainty Quantification

How well do we know what we know?

- Quantity of Interest Distribution
- Failure Probabilities
- Accurate Margins

Introduction

Uncertainty Quantification Methods

Monte Carlo, Latin Hypercube

- (Mostly) Agnostic of Dimensionality
- Very slow in converging $\left(\frac{c}{\sqrt{N}}\right)$

Grid-based Polynomial Expansions

- Fast convergence for low (<50) dimensions
- Very slow convergence for high (>1000) dimensions

Introduction

Uncertainty Quantification in Reactor Physics

Specific to Reactor Physics

- Large input spaces (tens of thousands)
- Computationally-intensive models
- Long solve times

Want few samples to characterize high-dimensional input space

Introduction

Uncertainty Quantification in Reactor Physics

Nature of input space

- Mostly cross sections
- Significant correlation between tabulation points, energies...
- Many cross sections have relatively low impact

We can leverage these properties

Methods

Principle Component Analysis

Methods

Principle Component Analysis

Correlated input variables orthogonalized

- Start with many correlated “manifest” input dimensions
- Use linear PCA to pick characteristic “latent” dimensions
- Eliminate dimensions with sufficiently small impact

$$M \approx QL$$

- M is the manifest set of input variables ($Nx1$),
- Q is the PCA reduction matrix (NxM),
- L is the reduced latent variables($Mx1$)
- $M \leq N$

Results

RAVEN: PCA Reduction

Decomposition Eigenvalues

```
<ImportanceRank>
  <pcaindex>
    <ans>
      <variable>y1
        <index>0.434532487017</index>
        <dim>1</dim>
      </variable>
      <variable>y2
        <index>0.289611617218</index>
        <dim>2</dim>
      </variable>
      <variable>y3
        <index>0.188352017943</index>
        <dim>3</dim>
      </variable>
      <variable>y4
        <index>0.0778091082631</index>
        <dim>4</dim>
      </variable>
```

Methods

Principle Component Analysis

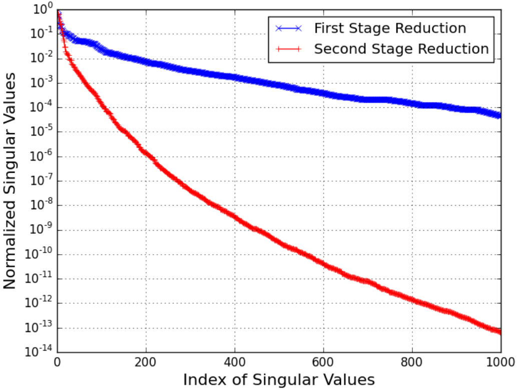


Figure: PCA Reduction

Methods

Sensitivity-Based Reduction

Methods

Sensitivity-Based Reduction

Eliminating low-impact inputs

- Calculate global sensitivity indices
- Remove inputs of low impact

Calculation is often costly (Linear regression)

$$L \approx PR,$$

- L is the set of (once-reduced) input variables ($Mx1$),
- P is the sensitivity reduction matrix (MxK),
- R is the twice-reduced variables($Kx1$)
- $K \leq M$

Results

RAVEN: Sensitivity Reduction

Sensitivity Coefficients


```
<ReducedOrderModel>
  <ans>
    <mean>0.781756851724</mean>
    <variance>0.275604991951</variance>
    <numRuns>9</numRuns>
    <indices>
      <tot_variance>0.275604991951</tot_variance>
      <variables>y1
        <partial_variance>0.264042832714</partial_variance>
        <Sobol_index>0.958048077595</Sobol_index>
      </variables>
      <variables>y2
        <partial_variance>0.00984470571011</partial_variance>
        <Sobol_index>0.035720346139</Sobol_index>
      </variables>
      <variables>y4
        <partial_variance>0.00157893548534</partial_variance>
        <Sobol_index>0.00572897999474</Sobol_index>
      </variables>
```

Methods

Sensitivity-Based Reduction

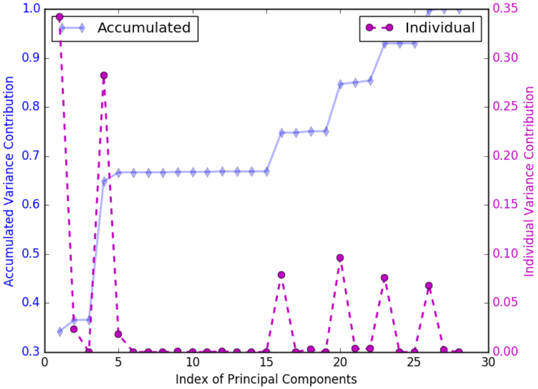


Figure: Sensitivity Reduction

Methods

Combined Reduction

Combine PCA and Sensitivity Reduction

$$M \approx PQR$$

$$|R| < |L| < |M|$$

Reduction can be several orders of magnitude

Results

Demonstration Case

Results

Demonstration Case

Demonstration Case

- 308 correlated uncertain input variables
- Originally cross sections from SCALE 44-group library
- Simulation is simple polynomial of input variables

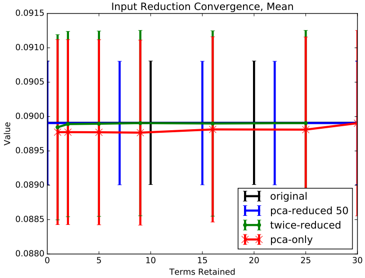
Results

Demonstration Procedure

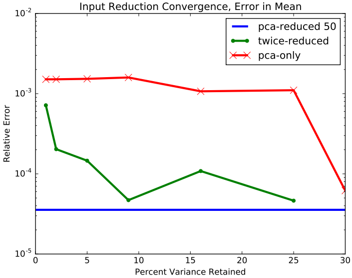
1. Use Monte Carlo to establish benchmark (308)
2. Perform PCA reduction
3. Use Monte Carlo to sample PCA-reduced space (50)
4. Perform Sensitivity Analysis
5. Use Monte Carlo to sample various sensitivity reductions

Results

Demonstration Twice-Reduced Mean



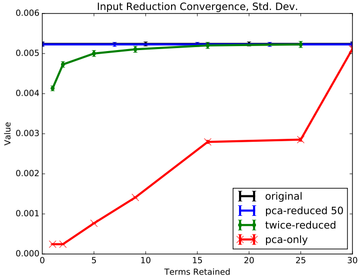
(a) Mean Values



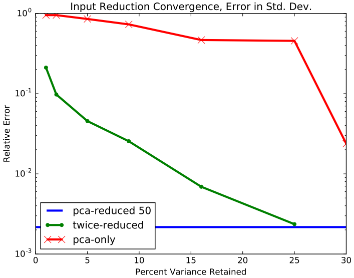
(b) Mean Errors

Results

Demonstration Twice-Reduced Variance



(a) Variance Values



(b) Variance Errors

Results

Automated Sensitivity

What if we could automate reduction?

Results

Automated Sensitivity: Adaptive Sobol

Sobol decomposition:

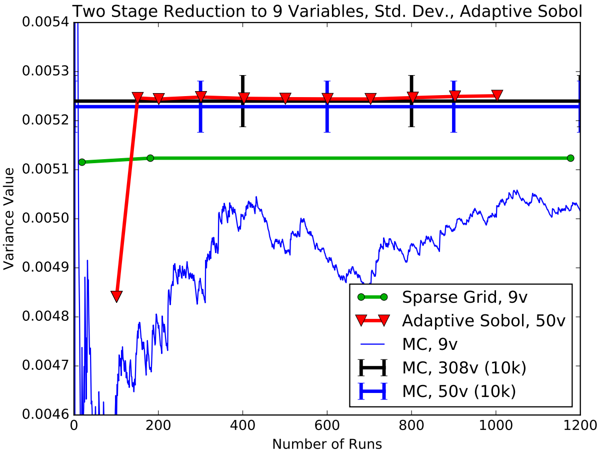
$$\begin{aligned} f(x,y,z) = & f_0 \\ & + f_1(x) + f_2(y) + f_3(z) \\ & + f_{1,2}(x,y) + f_{1,3}(x,z) + f_{2,3}(y,z) \\ & + f_{1,2,3}(x,y,z), \end{aligned}$$

where $f_1(x) = \int \int f(x,y,z)\, dy\, dz - f_0$, etc.

Adaptive Sobol: construction based on sensitivities

Results

Demonstration: Adaptive Sobol



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Questions?

Thank you for attending!

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