UQ on Two-Group, 2D Criticality Diffusion

A simple problem for testing and learning

Paul Talbot

November 20, 2013

1 Introduction

While the eventual goal of my doctoral work is implementation of stochastic collocation and generalized polynomial chaos as a method of uncertainty quantification within the RAVEN framework in the greater MOOSE environment, it is useful to develop a "toy" problem of smaller dimension and simpler focus on which to develop algorithms. We intend to develop uncertainty quantification in a step-by-step process:

1. Develop a benchmarkable 2-group 2-dimensional criticality diffusion problem with nonlinearity in k-effective.
2. Increase nonlinearity by introducing delayed neutron precursors and material-temperature feedback.
3. Build a framework for non-intrusive uncertainty quantification on the diffusion criticality code.
4. Develop a Monte Carlo sampling algorithm for uncertainty propogation.
5. Develop a Stochastic Collocation sampling algorithm for uncertainty propogation with only uniform uncertainties.
6. Expand the Stochastic Collocation sampling algorithm to include Gaussian-normal uncertainties.
7. Develop a sparse grid method for sampling large numbers of uncertain parameters.

2 Diffusion Criticality Code

2.1 Equations

$$-\nabla \cdot D\nabla\phi + \sigma_a\phi = \frac{1}{k}\nu\sigma_f\phi, \tag{1}$$

2.2 1D-like Problem

The system in question is a two-dimensinoal reactor with a 25 cm reflector on either end, and two 50 cm materials between them. The top and bottom boundaries are reflectors, effectively creating a one-dimensional problem. The basic problem parameters are as follows:

Material	D (cm)	σ_a(cm^{-1})	$\nu\sigma_f$(cm^{-1})
Core 1	0.65	0.12	0.10
Core 2	0.75	0.10	0.11
Reflector	1.15	0.01	0.00

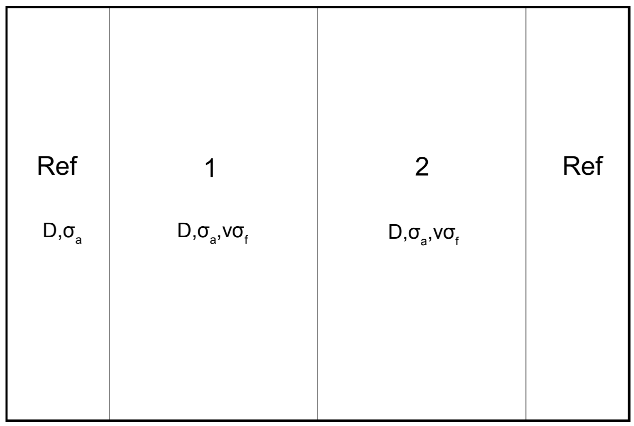

Figure 1: Problem Geometry

2.3 2D Quarter Core problem

The two-dimensional core is shown in Fig. 2.

3 Nonlinear Temperature Feedback

4 Monte Carlo Sampling

4.1 Results

An example of sampling is introducing 30% uniform uncertainty in the absorption and fission cross sections. Using 3000 histories, we arrive at a response surface as shown in Fig. 3. Each black dot is a sampling point, and the black lines indicate the contours of k-eff as a function of changing cross sections. As expected, the highest k is found when the fission cross section is large and the absorption cross section low. Interestingly, the fission cross section has a more dominant effect as the absorption cross section increases, and vice versa.

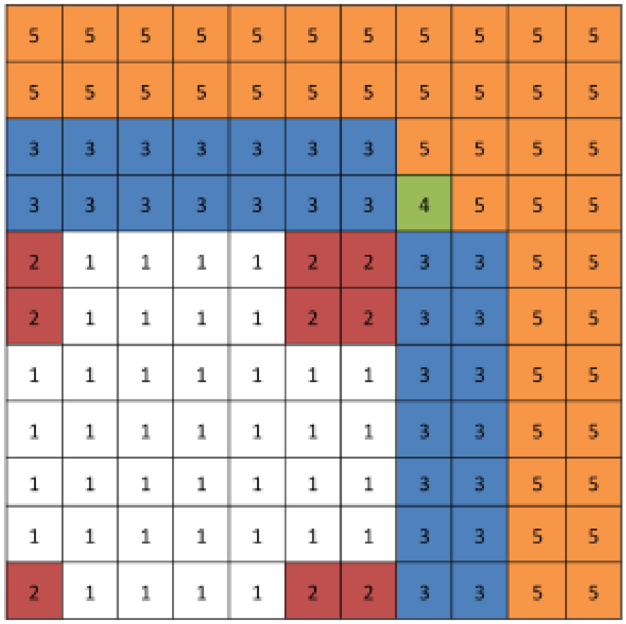

Figure 2: core map

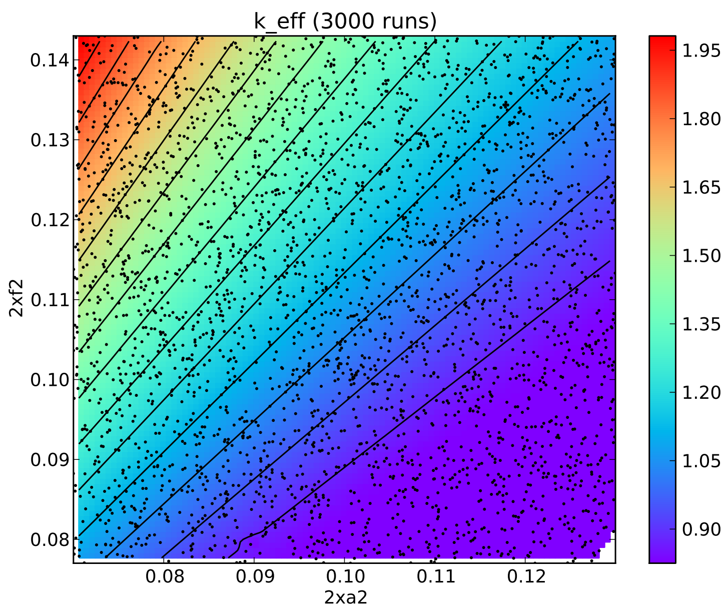

Figure 3: MC Sampling Results

5 Sampling Framework

6 Monte Carlo sampling

7 Simple Stochastic Collocation sampling

8 Mixed Stochastic Collocation[3]sampling

9 Sparse Grid Development