Additional Results

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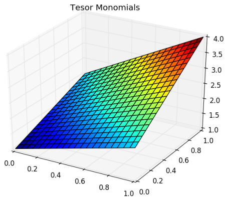
SCgPC Results

Tensor Monomials

Tensor Monomials

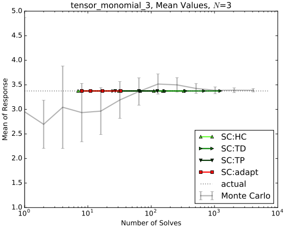
$$u(Y) = \prod_{n=1}^{N} (y_n + 1)$$

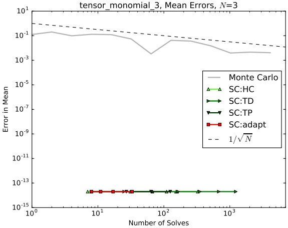


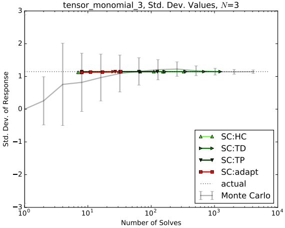
- Linear response
- All polynomial combinations

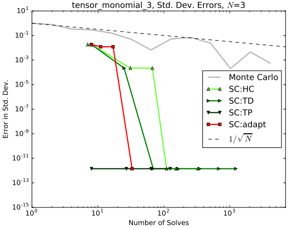
SCgPC Results

Tensor Monomials, $N = 3$



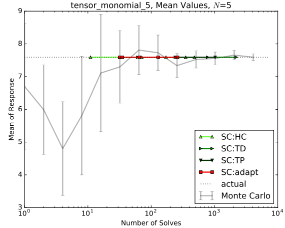


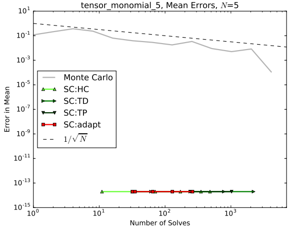


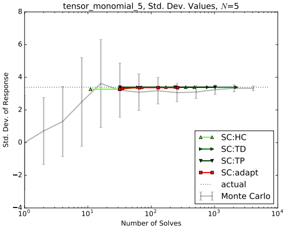


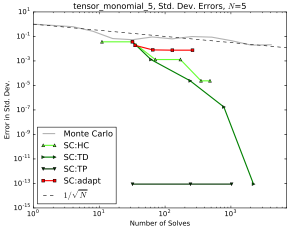
SCgPC Results

Tensor Monomials, $N = 5$



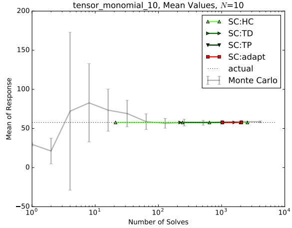


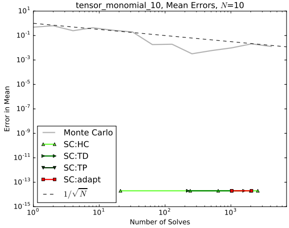


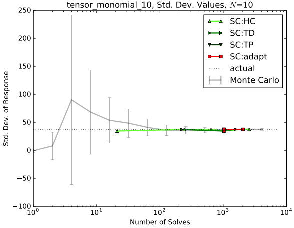


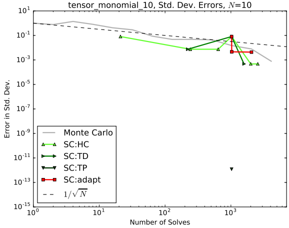
SCgPC Results

Tensor Monomials, $N = 10$







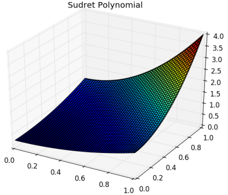


SCgPC Results

Sudret Polynomials

Sudret Polynomials

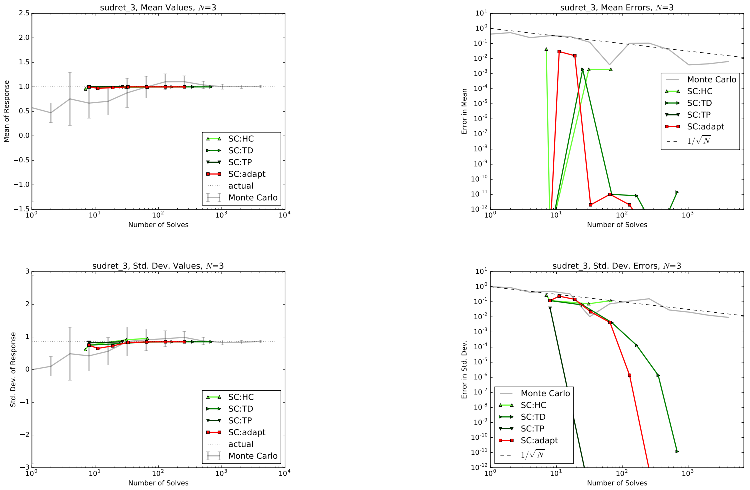
$$u(Y) = \frac{1}{2^N} \prod_{n=1}^{N} (3y_n^2 + 1)$$



- Exclusively second-order interactions
- All second-order polynomial combinations

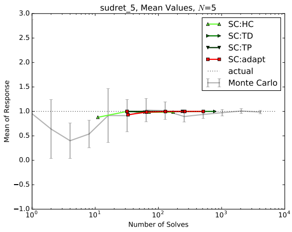
SCgPC Results

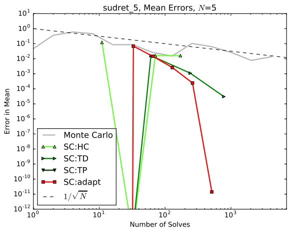
Sudret Polynomials, $N = 3$

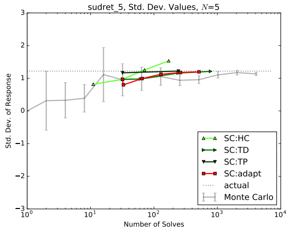


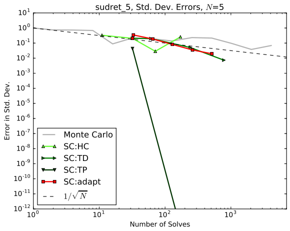
SCgPC Results

Sudret Polynomials, $N = 5$







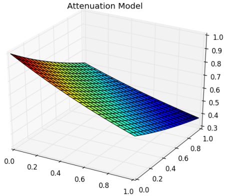


SCgPC Results

Attenuation

Attenuation

$$u(Y) = \prod_{n=1}^{N} \exp(-y_n/N)$$



- Tensor of decreasing-importance polynomials
- Combination terms over single-variable

SCgPC Results

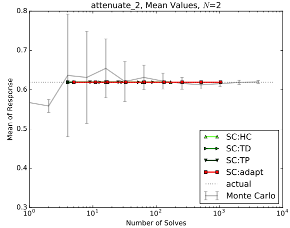
Attenuation, Taylor Expansion

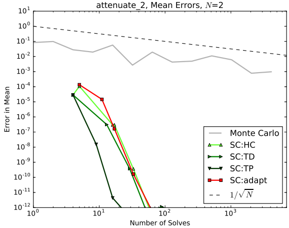
$$e^{-ay} = 1 - ay + \frac{(ay)^2}{2} - \frac{(ay)^3}{6} + \frac{(ay)^4}{24} - \frac{(ay)^5}{120} + \mathcal{O}(y^6)$$

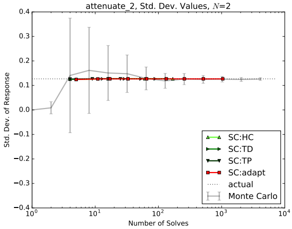
		Polynomial Order (y_1)				
		0	1	2	3	4
Polynomial Order (y_2)	0	1	a	$a^2/2$	$a^3/6$	$a^4/24$
	1	a	a^2	$a^3/2$	$a^4/6$	$a^5/24$
	2	$a^2/2$	$a^3/2$	$a^4/4$	$a^5/12$	$a^6/48$
	3	$a^3/6$	$a^4/6$	$a^5/12$	$a^6/36$	$a^7/144$
	4	$a^4/24$	$a^5/24$	$a^6/48$	$a^7/144$	$a^8/576$

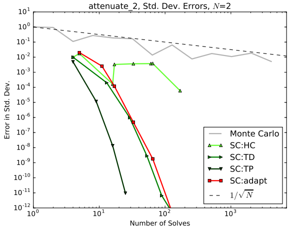
SCgPC Results

Attenuation, $N = 2$



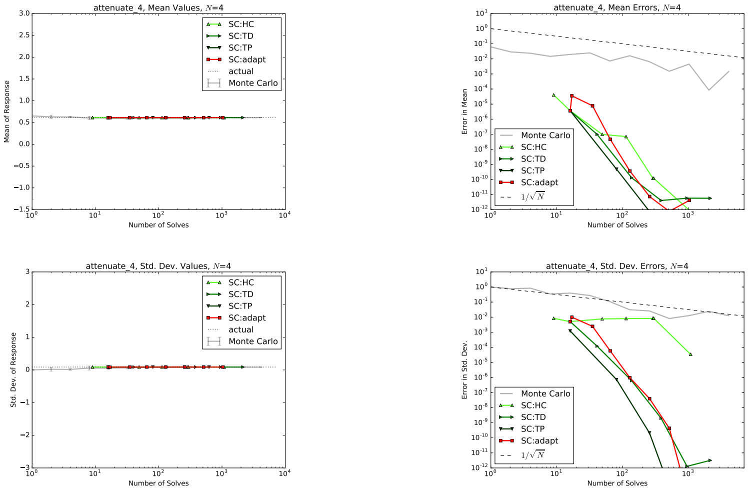






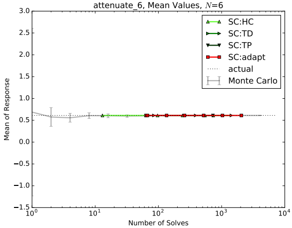
SCgPC Results

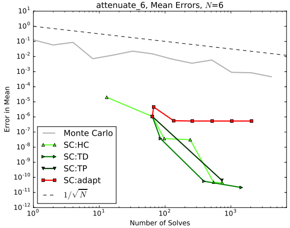
Attenuation, $N = 4$

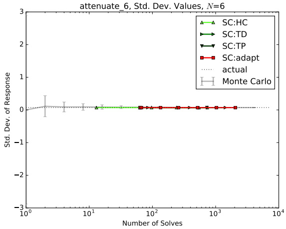


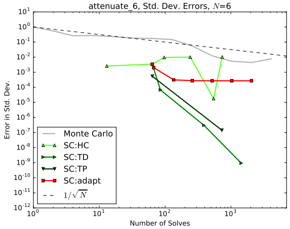
SCgPC Results

Attenuation, $N = 6$







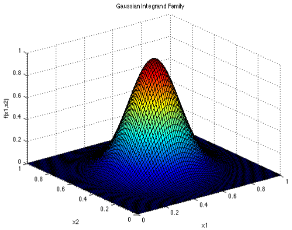


SCgPC Results

Gauss Peak

Gauss Peak

$$u(Y) = \prod_{n=1}^{N} \exp\left(-3^2 (y_n - 0.5)^2\right)$$



- Tensor of polynomials
- Slow, inconsistent decay

SCgPC Results

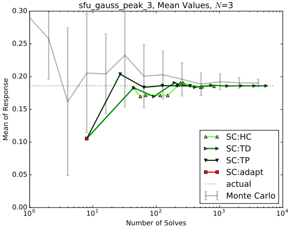
Gauss Peak, Taylor Expansion

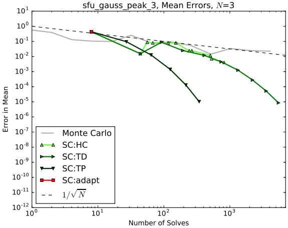
$$e^{-a^2y^2} = 1 - a^2y^2 + \frac{a^4}{2}y^4 - \frac{a^6}{6}y^6 + \frac{a^8}{24}y^8 + \mathcal{O}(y^{10})$$

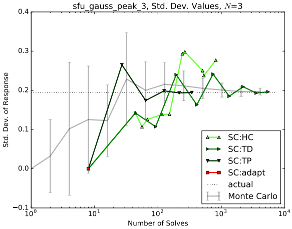
		Polynomial Order (y_1)				
		0	1	2	3	4
Polynomial Order (y_2)	0	1	0	a^2	0	$a^4/2$
	1	0	0	0	0	0
	2	a^2	0	a^4	0	$a^6/2$
	3	0	0	0	0	0
	4	$a^4/2$	0	$a^6/2$	0	$a^8/4$

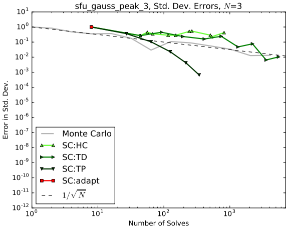
SCgPC Results

Gauss Peak, $N = 3$



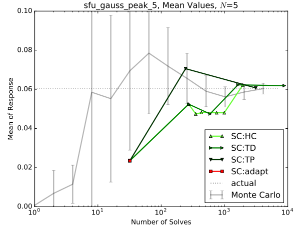


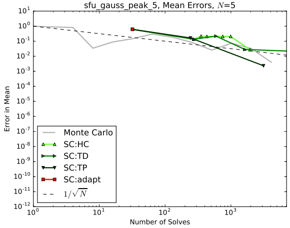


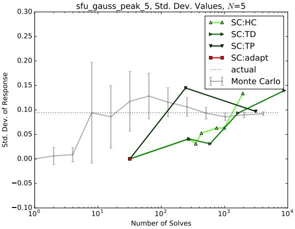


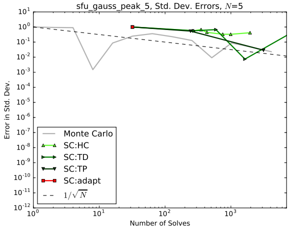
SCgPC Results

Gauss Peak, $N = 5$







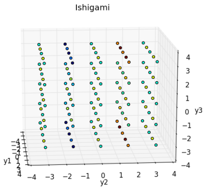


SCgPC Results

Ishigami Function

Ishigami Function

$$u(Y) = \sin y_1 + a \sin^2 y_2 + b\ y_3^4 \sin y_1$$



- Not a tensor combination
- Strange interplay between y_1, y_3

SCgPC Results

Ishigami Function, Taylor Expansion

Ishigami Function

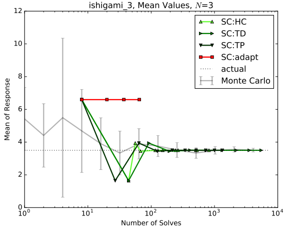
$$u(Y) = \sin y_1 + a \sin^2 y_2 + b\ y_3^4 \sin y_1$$

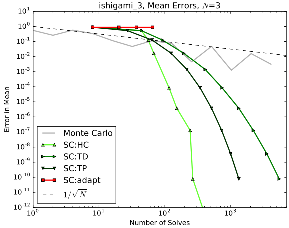
$$\sin y = x - \frac{x^3}{6} + \frac{x^5}{120} + \mathcal{O}(x^7)$$

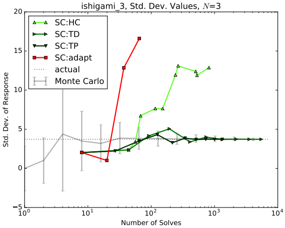
$$\sin^2 y = x^2 - \frac{x^4}{3} + \frac{2x^6}{45} + \mathcal{O}(x^8)$$

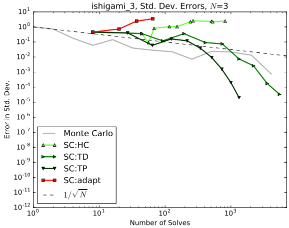
SCgPC Results

Ishigami Function, $N = 3$







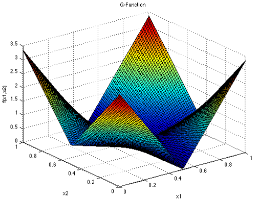


SCgPC Results

Sobol G-Function

Sobol G-Function

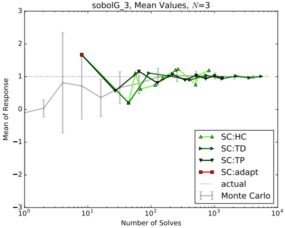
$$u(Y) = \prod_{n=1}^{N} \frac{|4y_n - 2| - a_n}{1 + a_n}, \quad a_n = \frac{n-2}{2}$$

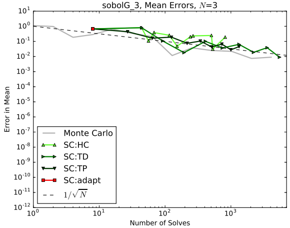


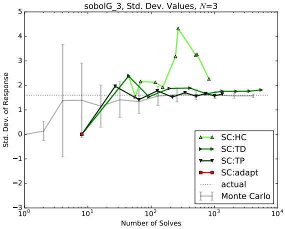
- Tensor combination of terms
- Only zeroth-order continuity

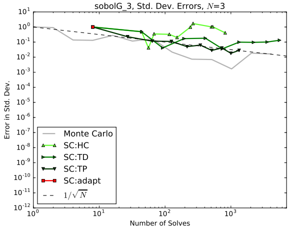
SCgPC Results

Sobol G-Function, $N = 3$



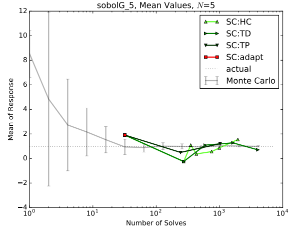


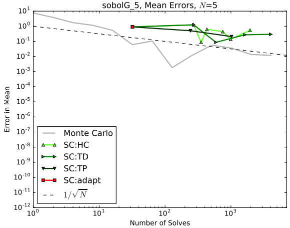


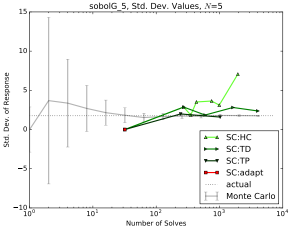


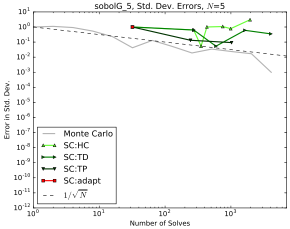
SCgPC Results

Sobol G-Function, $N = 5$









SCgPC Results

Conclusions

Regarding static SCgPC:

- Great in low input space dimensionality
- Better with regular responses
- Total Degree often great choice

Regarding adaptive SCgPC:

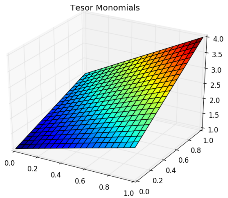
- Optimal for small input dimensionality
- Monotonically-decreasing variance moments
- Very poor if oscillating moments

HDMR Results

Tensor Monomials

Tensor Monomials

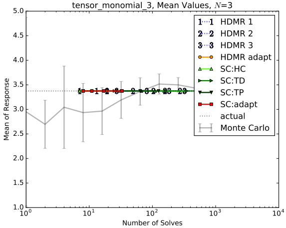
$$u(Y) = \prod_{n=1}^{N} (y_n + 1)$$

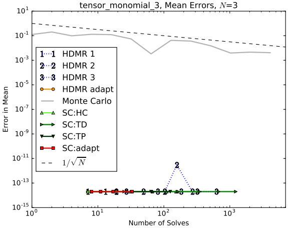


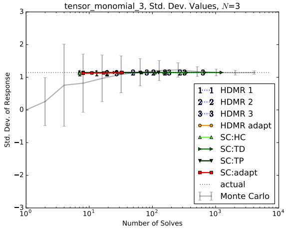
- Linear response
- All polynomial combinations

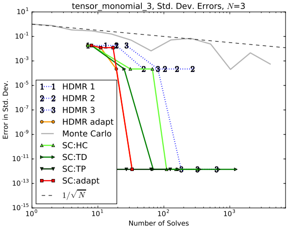
HDMR Results

Tensor Monomials, $N = 3$



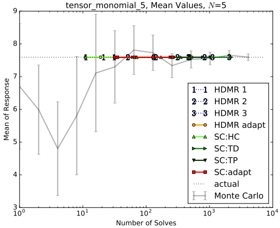


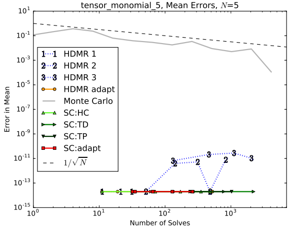


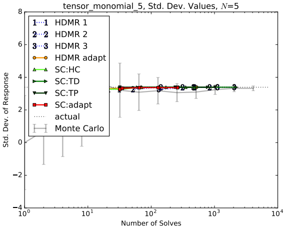


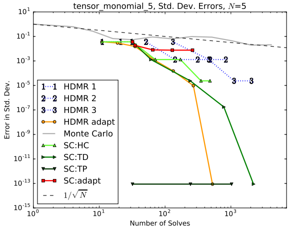
HDMR Results

Tensor Monomials, $N = 5$



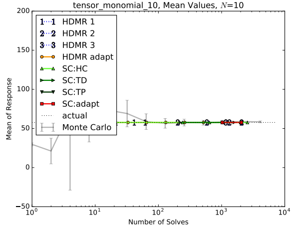


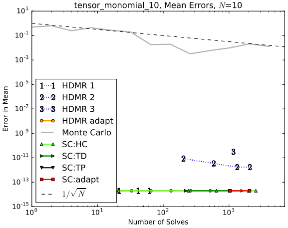


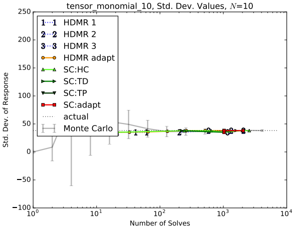


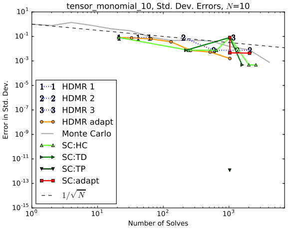
HDMR Results

Tensor Monomials, $N = 10$







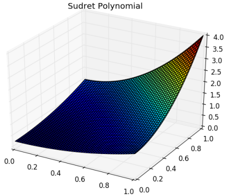


HDMR Results

Sudret Polynomials

Sudret Polynomials

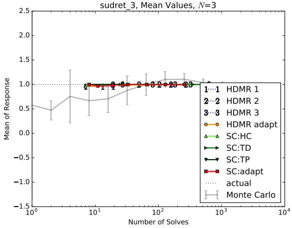
$$u(Y) = \frac{1}{2^N} \prod_{n=1}^{N} (3y_n^2 + 1)$$

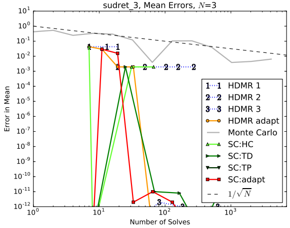


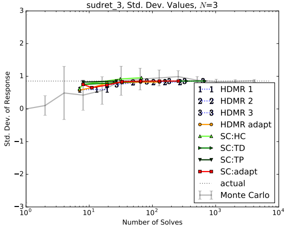
- Exclusively second-order interactions
- All second-order polynomial combinations

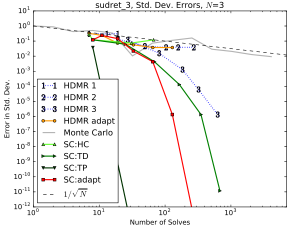
HDMR Results

Sudret Polynomials, $N = 3$



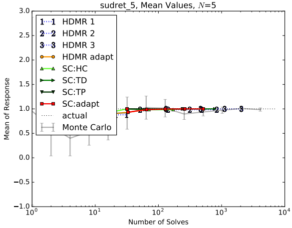


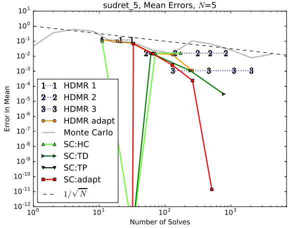


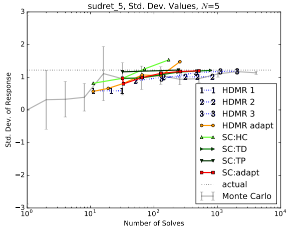


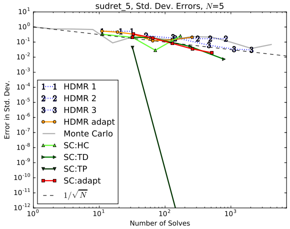
HDMR Results

Sudret Polynomials, $N = 5$







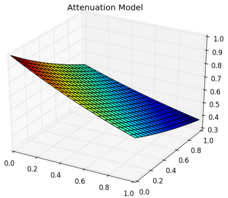


HDMR Results

Attenuation

Attenuation

$$u(Y) = \prod_{n=1}^{N} \exp(-y_n/N)$$



- Tensor of decreasing-importance polynomials
- Combination terms over single-variable

HDMR Results

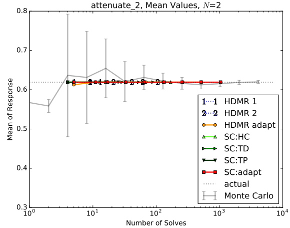
Attenuation, Taylor Expansion

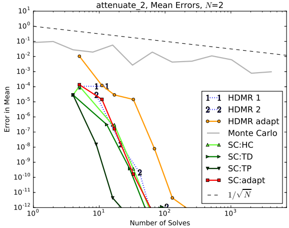
$$e^{-ay} = 1 - ay + \frac{(ay)^2}{2} - \frac{(ay)^3}{6} + \frac{(ay)^4}{24} - \frac{(ay)^5}{120} + \mathcal{O}(y^6)$$

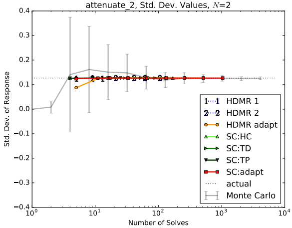
		Polynomial Order (y_1)				
		0	1	2	3	4
Polynomial Order (y_2)	0	1	a	$a^2/2$	$a^3/6$	$a^4/24$
	1	a	a^2	$a^3/2$	$a^4/6$	$a^5/24$
	2	$a^2/2$	$a^3/2$	$a^4/4$	$a^5/12$	$a^6/48$
	3	$a^3/6$	$a^4/6$	$a^5/12$	$a^6/36$	$a^7/144$
	4	$a^4/24$	$a^5/24$	$a^6/48$	$a^7/144$	$a^8/576$

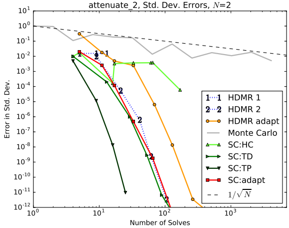
HDMR Results

Attenuation, $N = 2$



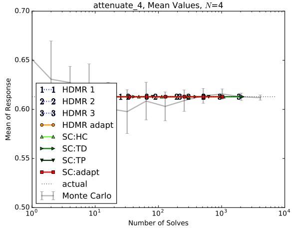


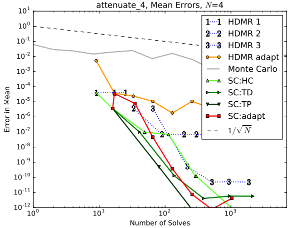


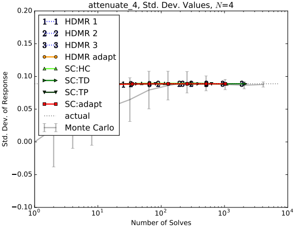


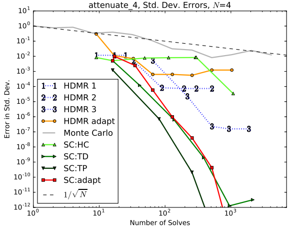
HDMR Results

Attenuation, $N = 4$



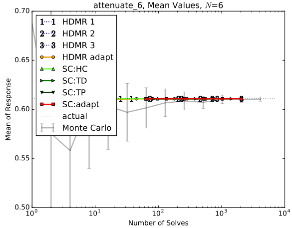


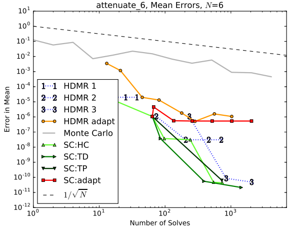


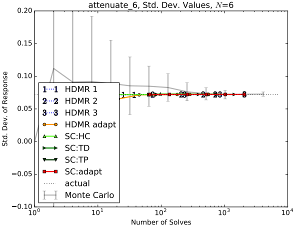


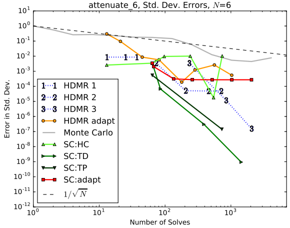
HDMR Results

Attenuation, $N = 6$







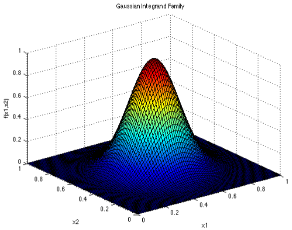


HDMR Results

Gauss Peak

Gauss Peak

$$u(Y) = \prod_{n=1}^{N} \exp\left(-3^2 (y_n - 0.5)^2\right)$$



- Tensor of polynomials
- Slow, inconsistent decay

HDMR Results

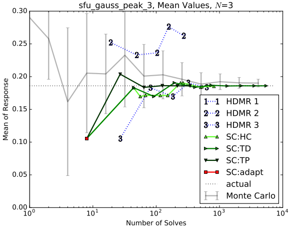
Gauss Peak, Taylor Expansion

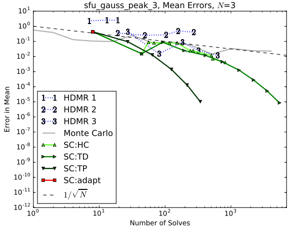
$$e^{-a^2y^2} = 1 - a^2y^2 + \frac{a^4}{2}y^4 - \frac{a^6}{6}y^6 + \frac{a^8}{24}y^8 + \mathcal{O}(y^{10})$$

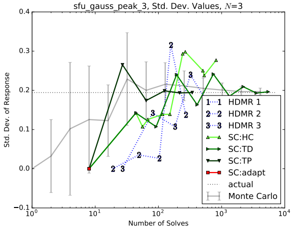
		Polynomial Order (y_1) 0	1	2	3	4
Polynomial Order (y_2)	0	1	0	a^2	0	$a^4/2$
	1	0	0	0	0	0
	2	a^2	0	a^4	0	$a^6/2$
	3	0	0	0	0	0
	4	$a^4/2$	0	$a^6/2$	0	$a^8/4$

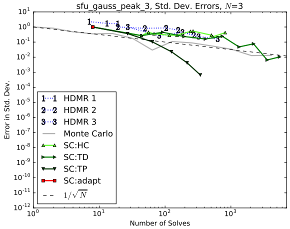
HDMR Results

Gauss Peak, $N = 3$



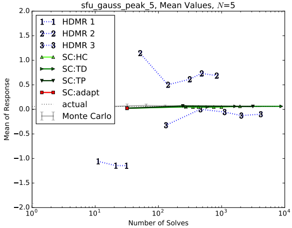


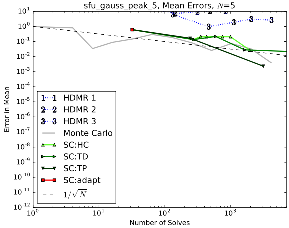


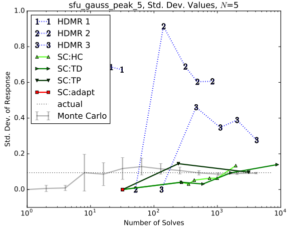


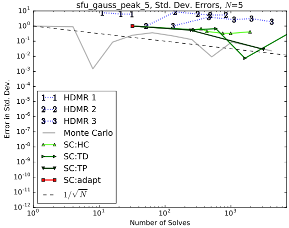
HDMR Results

Gauss Peak, $N = 5$







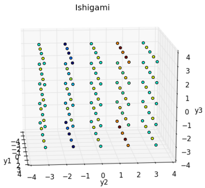


HDMR Results

Ishigami Function

Ishigami Function

$$u(Y) = \sin y_1 + a \sin^2 y_2 + b\ y_3^4 \sin y_1$$



- Not a tensor combination
- Strange interplay between y_1, y_3

HDMR Results

Ishigami Function, Taylor Expansion

Ishigami Function

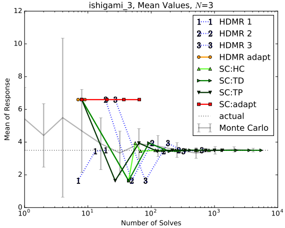
$$u(Y) = \sin y_1 + a \sin^2 y_2 + b\ y_3^4 \sin y_1$$

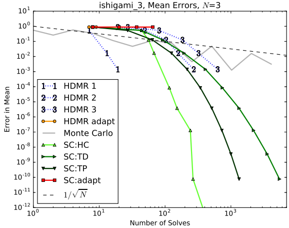
$$\sin y = x - \frac{x^3}{6} + \frac{x^5}{120} + \mathcal{O}(x^7)$$

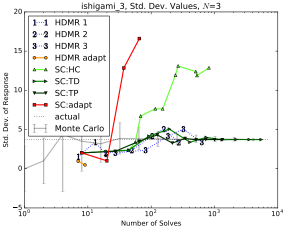
$$\sin^2 y = x^2 - \frac{x^4}{3} + \frac{2x^6}{45} + \mathcal{O}(x^8)$$

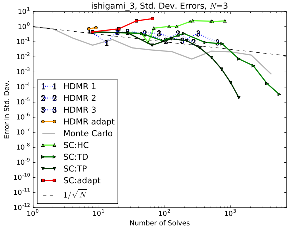
HDMR Results

Ishigami Function, $N = 3$







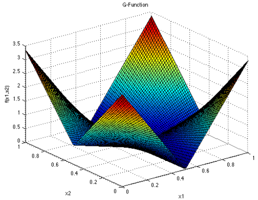


HDMR Results

Sobol G-Function

Sobol G-Function

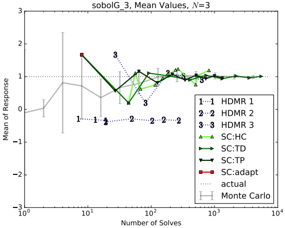
$$u(Y) = \prod_{n=1}^{N} \frac{|4y_n - 2| - a_n}{1 + a_n}, \quad a_n = \frac{n-2}{2}$$

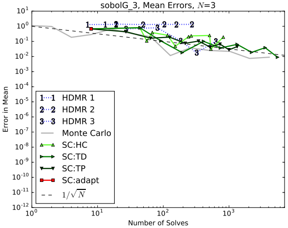


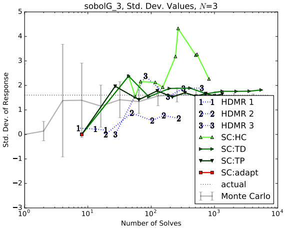
- Tensor combination of terms
- Only zeroth-order continuity

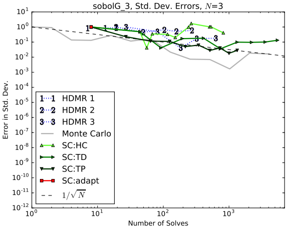
HDMR Results

Sobol G-Function, $N = 3$



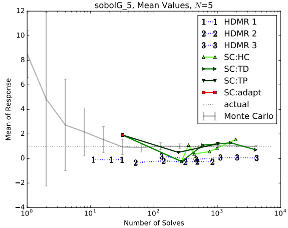


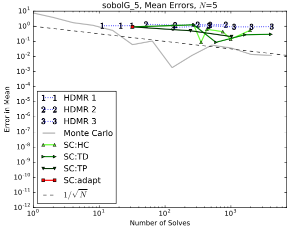


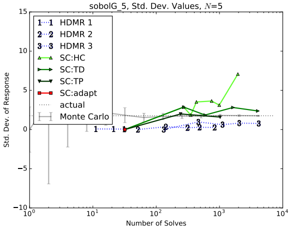


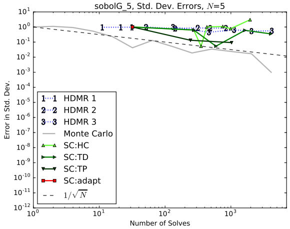
HDMR Results

Sobol G-Function, $N = 5$









HDMR Results

Conclusions

Regarding static HDMR:

- Never outperforms associated SCgPC
- Does produce results with less evaluations

Regarding adaptive HDMR:

- Sometimes outperforms adaptive SCgPC
- Yields results with fewer evaluations

HDMR is most useful when very few evaluations possible

Outline

Neutronics Example

Introduction

More complicated than an analytic case

$$-D_g(\mathbf{r})\nabla^2\phi_g(\mathbf{r}) + \Sigma_{a,g}(\mathbf{r}) = \sum_{g'=1}^{G} \Sigma_{g'\to g}\phi_{g'}(\mathbf{r}) + \frac{\chi_{p,g}}{k}\sum_{g'=1}^{G} \nu\Sigma_{f,g'}(\mathbf{r})\phi_{g'}(\mathbf{r})$$

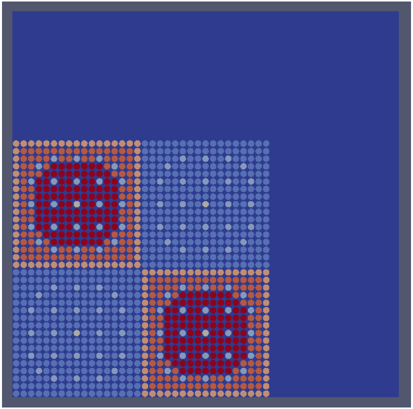
Quantities of interest

- $\phi_g(\mathbf{r})$: Group neutron flux
- k eigenvalue: Neutron multiplication factor

Neutronics Example

Geometry

Quarter-symmetric 4-assembly reactor core



Neutronics Example

Energy Groups

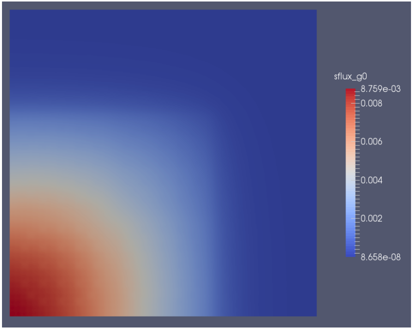
7 energy groups, 7 materials, 32 mesh elements per pin

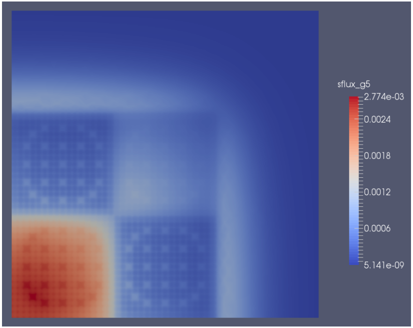
Group	Upper Energy Bound
7	0.02 eV
6	0.1 eV
5	0.625 eV
4	3 eV
3	500 keV
2	1 MeV
1	20 MeV

Solved using `RATTLESNAKE`'s linear CFEM

Neutronics Example

Flux Profiles





Outline

Neutronics Example

Uncertainty

Specific Responses

- k-eigenvalue
- Group 1 flux at reactor center
- Group 5 flux at reactor center

168 correlated uncertain inputs

- Material macroscopic cross sections
- Assigned 10% correlation
  - Same material and reaction, different energies
  - Same material and energy, different reaction
- Relative variance of 5% for all inputs

Neutronics Example

Uncertainty Correlations

Need to de-correlate input space

`RAVEN` has two-step reduction

- Karhunen-Loeve expansion (PCA)
- Sensitivity reduction
- Combined yields *importance rank*

Neutronics Example

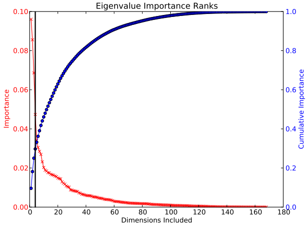
Uncertainty Correlations

	k-eigenvalue		Center Flux, $g = 1$		Center Flux, $g = 5$	
Rank	Dimension	Importance	Dimension	Importance	Dimension	Importance
1	24	0.09606	24	0.07231	24	0.07032
2	9	0.08555	9	0.06472	9	0.06648
3	0	0.06861	0	0.04856	100	0.06474
4	17	0.04737	116	0.03472	13	0.03396
5	23	0.03415	17	0.03470	0	0.03092
6	158	0.03047	10	0.02726	17	0.02716
7	164	0.02852	8	0.02468	10	0.02651
8	50	0.02695	164	0.02174	118	0.02600
9	6	0.02315	20	0.02157	117	0.02420

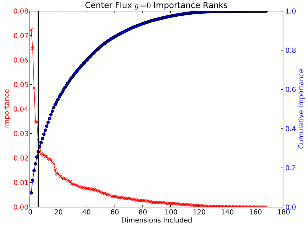
Retained latent dimensions 24, 9, 0, 17, 10, 116, 100, 13

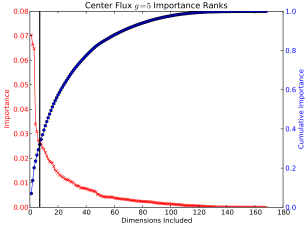
Neutronics Example

Uncertainty Correlations



- Truncated after gradients
- Some importance lost
- Mean preserved well
- Std dev partially preserved

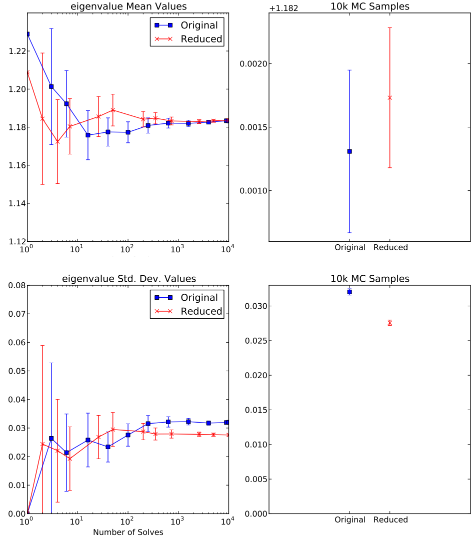




Neutronics Example

Uncertainty Correlations

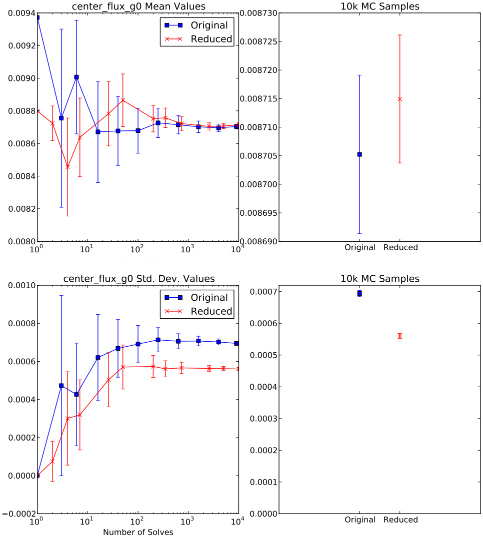
k-Eigenvalue



Neutronics Example

Uncertainty Correlations

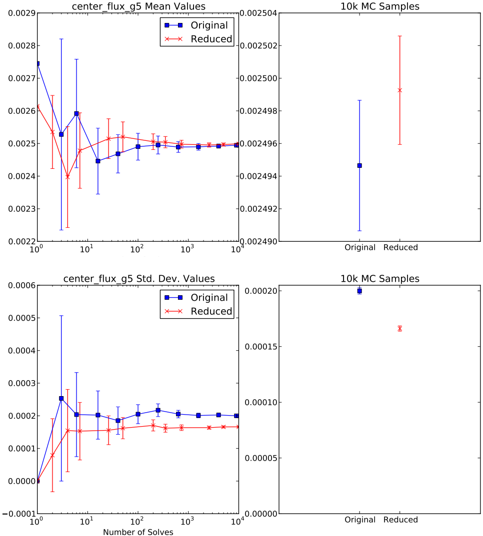
Center Flux, $g = 1$



Neutronics Example

Uncertainty Correlations

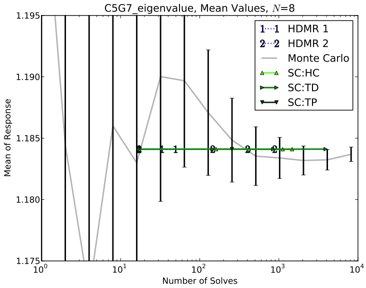
Center Flux, $g = 5$

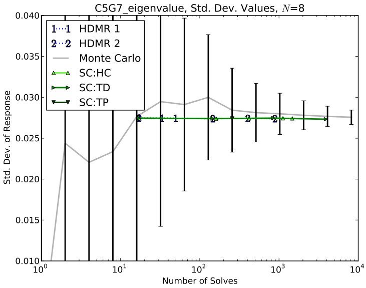


Outline

Neutronics Example

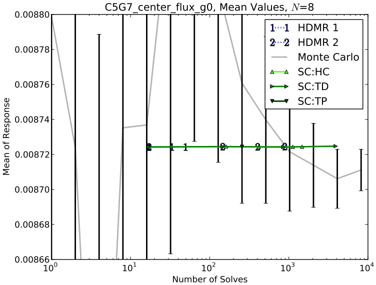
Results: k-eigenvalue

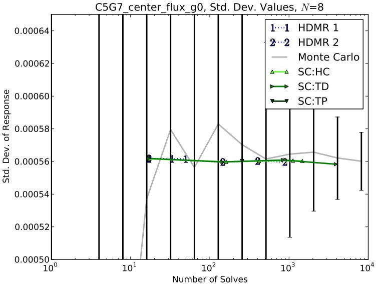




Neutronics Example

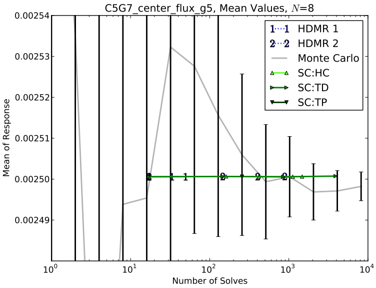
Results: Center Flux, $g = 1$

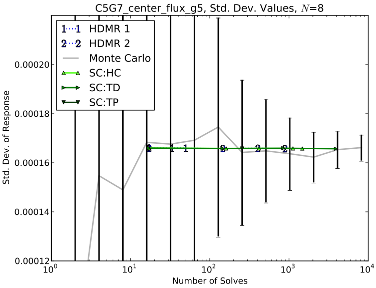




Neutronics Example

Results: Center Flux, $g = 5$





Outline

Multiphysics Example

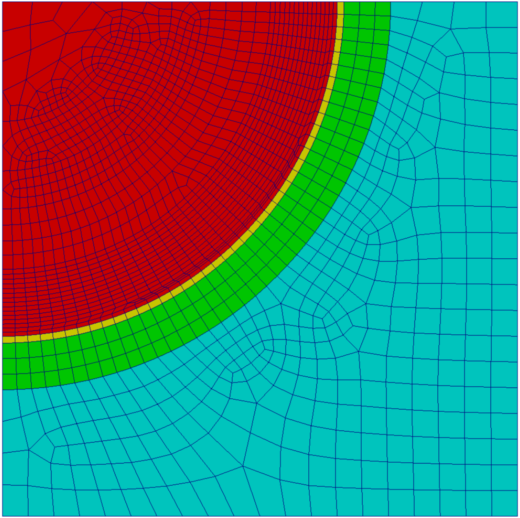
Introduction

Coupled multiphysics problem

- Neutronics
  - neutron transport and interaction
  - Provides flux/power shapes to fuels performance
- Fuels Performance
  - temperature, depletion, fuel oxidation,
  - fission product swelling, densification, fuel fracture,
  - interstitial heat transfer, mechanical contact,
  - cladding creep, thermal expansion, plasticity
  - Provides temperature fields to neutronics

Multiphysics Example

Geometry



Outline

Multiphysics Example

Geometry and Uncertainty

Dimensions

- Domain is 6.3 mm square
- Reflective boundaries
- Fuel pin radius is 4.09575 mm with clad

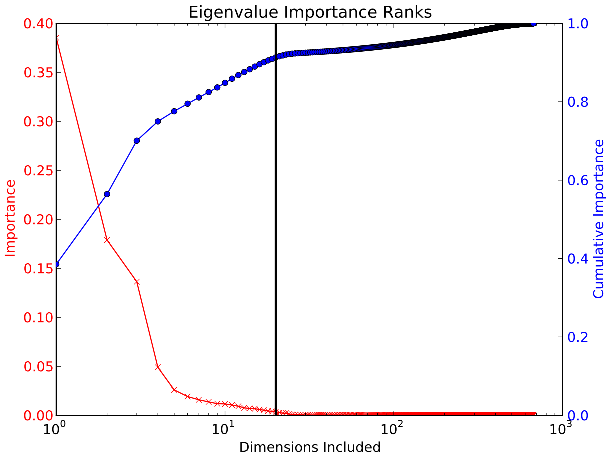
Response is *k*-eigenvalue

Uncertain inputs

- 671 correlated interaction cross sections
- Fuel thermal expansion coefficient
- Clad thermal conductivity
- Fuel thermal conductivity

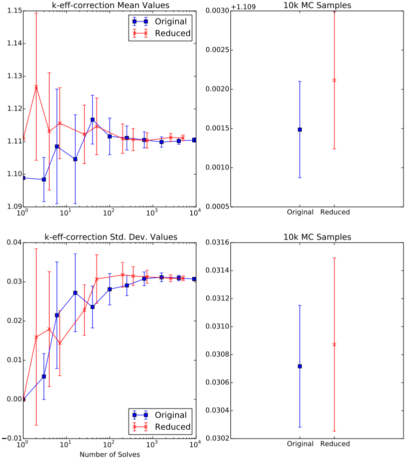
Multiphysics Example

Uncertainty Correlation



Multiphysics Example

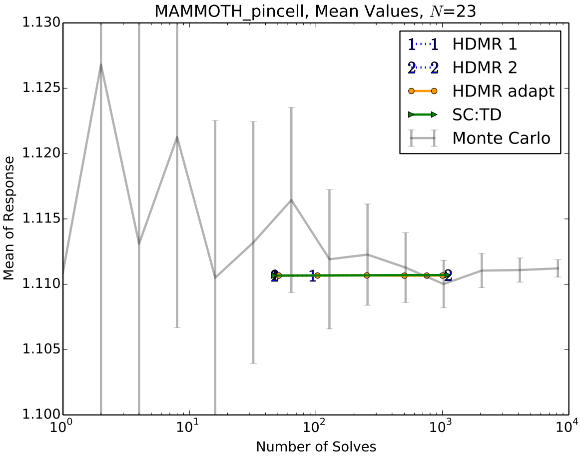
Uncertainty Correlation



Outline

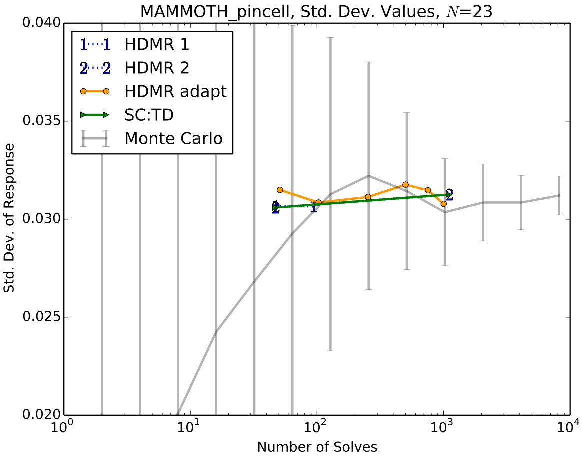
Multiphysics Example

Results



Multiphysics Example

Results



Multiphysics Example

Run Times

Method	Degree	Runs
Total Degree	1	47
Total Degree	2	1105
Total Degree*	3	17389
HDMR (1)	1	47
HDMR (1)	2	47
HDMR (1)	3	93
HDMR (1)	4	93
HDMR (1)	5	139
HDMR (2)	1	47
HDMR (2)	2	1105
HDMR (2)†	3	3221
HDMR (2)†	4	7361
HDMR (2)*	5	13571

Outline

Time-Dependent Analysis

Introduction

Transient Problems

- Response is time-dependent
- Response may evolve in time
- Physics may change in time

"Time" could be any monotonically-increasing parameter

Time-Dependent Analysis

Approach

`RAVEN` approach

- Divide time into snapshots
- Evaluate ROM on each snapshot
- Interpolate between snapshots

We extended SCgPC and HDMR to do this as well

Limitation: Adaptive methods

Outline

Time-Dependent Example

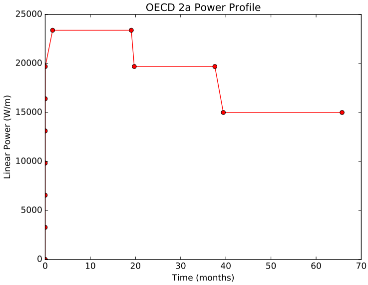
Introduction

Fuels Performance problem

- OECD Benchmark
- PWR Fuel Rod
- "Steady-State"
- Power Changes

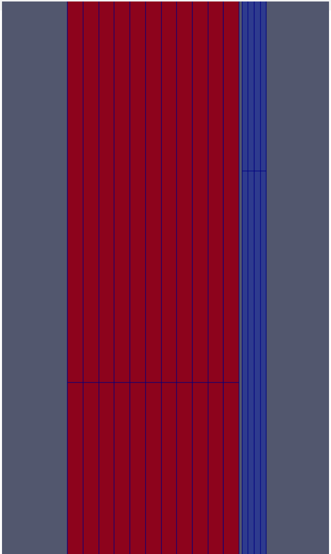
Responses

- max clad temp
- % fission gas released
- clad elongation
- clad creep strain



Time-Dependent Example

Introduction



Geometry

- Fuel and Cladding
- 2D Axisymmetric R-Z
- 4 m by 0.55 cm
- 4290 QUAD8 Elements

Physics

- Displacement/Creep
- Thermal expansion
- Heat conduction
- Heat convection
- Contact stress

Outline

Time-Dependent Example

Uncertainty

Uncertain Parameter	Mean	Std. Dev.
Clad Thermal Conductivity	16.0	2.5
Cladding Thickness	6.7e-4	8.3e-6
Cladding Roughness	5.0e-7	1.0e-7
Clad Creep Rate	1.0	0.15
Fuel Thermal Conductivity	1.0	0.05
Fuel Density	10299.24	51.4962
Fuel Thermal Expansion	1.0e-5	7.5e-7
Fuel Pellet Radius	4.7e-3	3.335e-6
Fuel Pellet Roughness	2.0e-6	1.6667e-7
Solid Fuel Swelling	5.58e-5	5.77e-6
Gas Conductivity	1.0	0.025
Gap Thickness	9.0e-5	8.33e-6
Mass Flux	3460	57.67
Rod Fill Pressure	1.2e6	40000.0
System Pressure	1.551e7	51648.3
System Power	1.0	0.016667
Uncertain Parameter	**Lower Bound**	**Upper Bound**
Inlet Temperature	558.0	564.0

Time-Dependent Example

Uncertainty

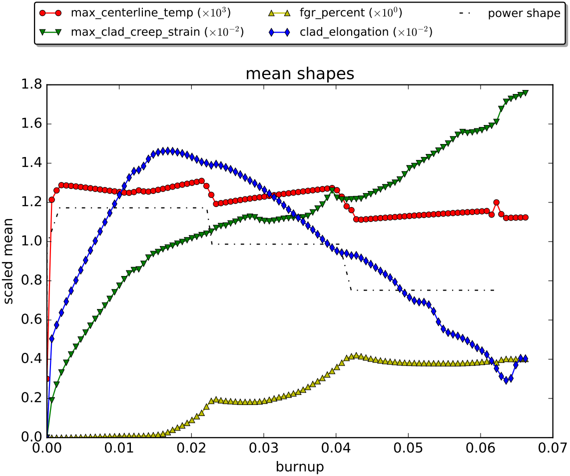
Dependent Parameters

Dependent Parameter	Calculation
Clad inner radius	2*(fuel_rad + gap_width)
Outer Diameter (Hot)	2*(fuel_rad + gap_width + clad_thick)
Outer Diameter (Cool)	2*(fuel_rad + gap_width + clad_thick)
System Pressure (Cool)	sys_press
Thermal Porosity	1 - fuel_dens/10980
Fuel Diameter	2*fuel_rad
Gap Diameter	2*gap_thick
SIFGR Porosity	1 - fuel_dens/10980

Outline

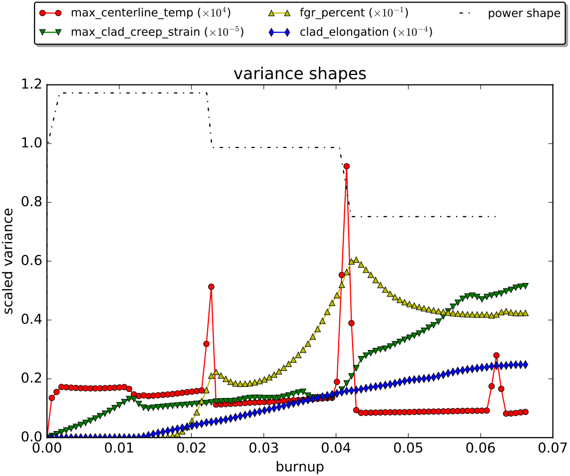
Time-Dependent Example

Results



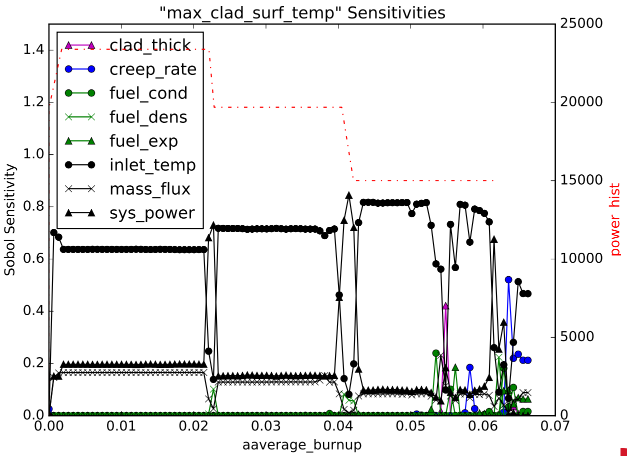
Time-Dependent Example

Results



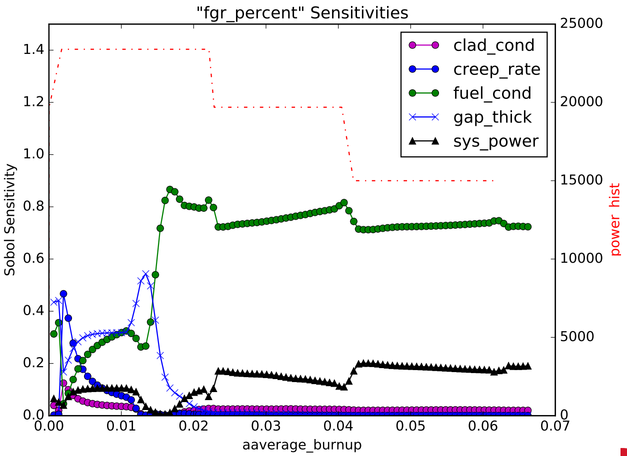
Time-Dependent Example

Results



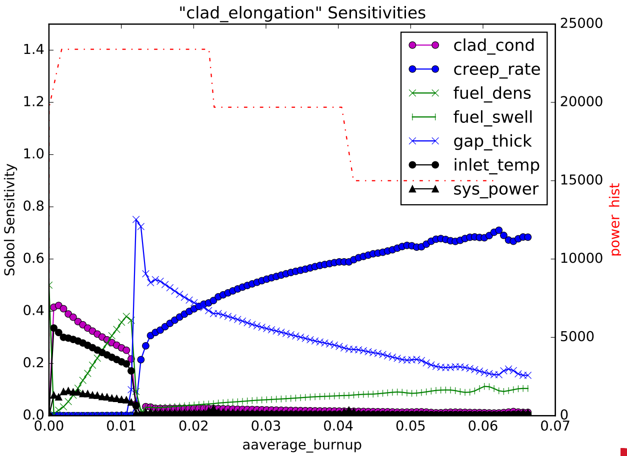
Time-Dependent Example

Results



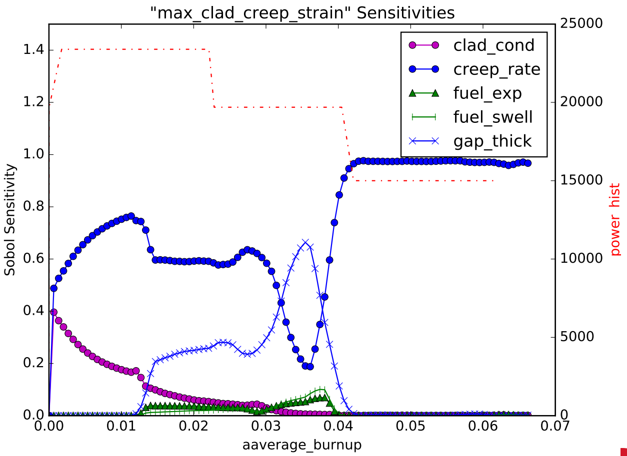
Time-Dependent Example

Results



Time-Dependent Example

Results



Time-Dependent Example

Conclusions

Time-Dependent Analysis with SCgPC and HDMR

- Same benefits as with static analysis
- No additional solves for time-dependent
- Increased understanding of physics
- Informed decision making