

# Nonreciprocal coherent coupling of nanomagnets by exchange spin waves

Hanchen Wang<sup>1,§</sup>, Jilei Chen<sup>1,§</sup>, Tao Yu<sup>2,§</sup>, Chuanpu Liu<sup>1,§</sup>, Chenyang Guo<sup>3,§</sup>, Song Liu<sup>4,§</sup>, Ka Shen<sup>5,§</sup>, Hao Jia<sup>4,§</sup>, Tao Liu<sup>6</sup>, Jianyu Zhang<sup>1</sup>, Marco A. Cabero Z<sup>4,1</sup>, Qiuming Song<sup>4</sup>, Sa Tu<sup>1</sup>, Mingzhong Wu<sup>6</sup>, Xiufeng Han<sup>3</sup>, Ke Xia<sup>4</sup>, Dapeng Yu<sup>4</sup>, Gerrit E. W. Bauer<sup>7,8,9</sup>, and Haiming Yu<sup>1</sup> (✉)

<sup>1</sup> Fert Beijing Institute, School of Integrated Circuit Science and Engineering, Beijing Advanced Innovation Center for Big Data and Brain Computing, Beihang University, Beijing 100191, China

<sup>2</sup> Max Planck Institute for the Structure and Dynamics of Matter, 22761 Hamburg, Germany

<sup>3</sup> Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, University of Chinese Academy of Sciences, Chinese Academy of Sciences, Beijing 100190, China

<sup>4</sup> Shenzhen Institute for Quantum Science and Engineering (SIQSE), and Department of Physics, Southern University of Science and Technology (SUSTech), Shenzhen 518055, China

<sup>5</sup> Department of Physics, Beijing Normal University, Beijing 100875, China

<sup>6</sup> Department of Physics, Colorado State University, Fort Collins, Colorado 80523, USA

<sup>7</sup> Kavli Institute of Nanoscience, Delft University of Technology, 2628 CJ Delft, The Netherlands

<sup>8</sup> Institute for Materials Research, WPI-AIMR and CSNR, Tohoku University, Sendai 980-8577, Japan

<sup>9</sup> Zernike Institute for Advanced Materials, University of Groningen, Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands

<sup>§</sup> Hanchen Wang, Jilei Chen, Tao Yu, Chuanpu Liu, Chenyang Guo, Song Liu, Ka Shen and Hao Jia contributed equally to this work.

© Tsinghua University Press and Springer-Verlag GmbH Germany, part of Springer Nature 2020

Received: 1 August 2020 / Revised: 10 November 2020 / Accepted: 16 November 2020

## ABSTRACT

Nanomagnets are widely used to store information in non-volatile spintronic devices. Spin waves can transfer information with low-power consumption as their propagations are independent of charge transport. However, to dynamically couple two distant nanomagnets via spin waves remains a major challenge for magnonics. Here we experimentally demonstrate coherent coupling of two distant Co nanowires by fast propagating spin waves in an yttrium iron garnet thin film with sub-50 nm wavelengths. Magnons in two nanomagnets are unidirectionally phase-locked with phase shifts controlled by magnon spin torque and spin-wave propagation. The coupled system is finally formulated by an analytical theory in terms of an effective non-Hermitian Hamiltonian. Our results are attractive for analog neuromorphic computing that requires unidirectional information transmission.

## KEYWORDS

spintronics, nanomagnets, spin waves, coherent coupling, nonreciprocity

## 1 Introduction

Coupled nanomagnets offer a variety of functionalities towards non-volatile memories and logic gates and form essential building blocks for spintronic devices [1–4]. The interlayer exchange coupling between magnetic layers leads to the discovery of the giant magnetoresistance [5–7] and stimulates the broad applications of magnetic tunnel junctions [8–10]. Very recently, the nanomagnets are laterally coupled via the Dzyaloshinskii-Moriya interaction, where two static magnetic domains are connected through a chiral domain wall [11]. In addition to static magnetic coupling, GHz magnetization dynamics of nanomagnets can be coupled via e.g., dynamic exchange coupling [12, 13] and dynamic dipolar coupling [14, 15]. However, these magnetic couplings are quickly weakened when the nanomagnets are separated apart. The dynamics of distant nanomagnets can be synchronized using an electrical current due to the spin-transfer torque effect [16, 17], which has important technological implications e.g., microwave generators [18–21].

Yet, a high current density is required to activate the synchronization, which would inevitably produce significant energy dissipation due to the Joule heating. Spin waves or its quanta, the magnons [22–31], are collective excitations of the magnetic order and can convey information free of charge transport. Therefore, spin waves are regarded as a promising candidate for the next-generation information carriers with low-power consumption. Magnons and photons are both bosons [32, 33], but have different dispersion relations: at the same frequencies magnons have much shorter wavelengths than photons, which offers spin waves an innate advantage for integrated nanodevices [34–38]. Incoherent-magnon-mediated coupling between magnets are found in the magnon valves with both vertical [39, 40] and lateral [41] structures. However, the long-distance coherent coupling between two nanomagnets via propagating spin waves has not been achieved yet [42–46].

Here, we experimentally demonstrate a nonreciprocal coupling of two distant ultra-thin film Co nanowires (> 50 nm wide) that is mediated by fast exchange spin waves in 20 nm-thin

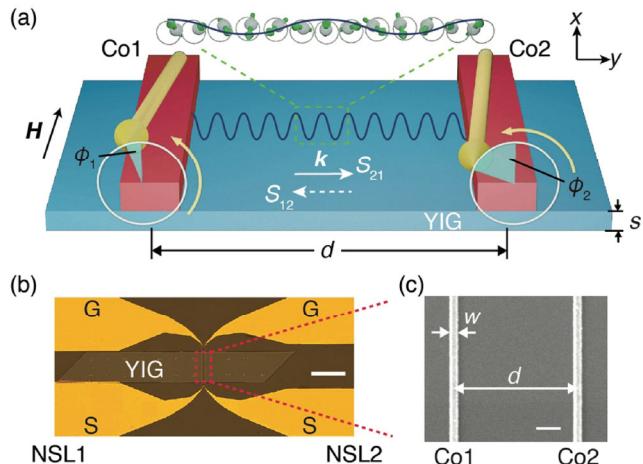
Address correspondence to [haiming.yu@buaa.edu.cn](mailto:haiming.yu@buaa.edu.cn)



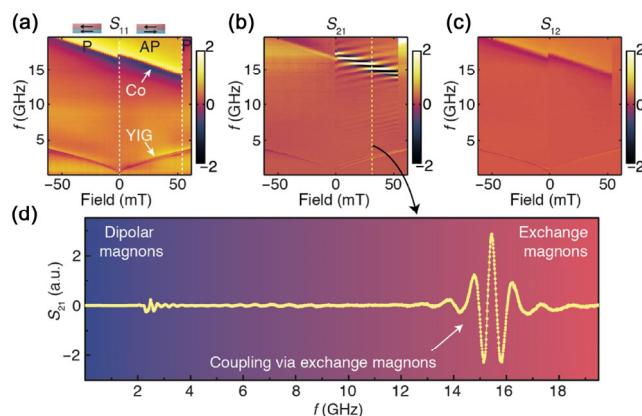
yttrium iron garnet (YIG) films. Nonreciprocal spin waves are of great interest in magnonic applications due to the controllable wave flow for the transfer of information and logic functions [47–50]. The excitation and detection of coherent exchange spin waves is non-trivial by itself [46–49]. In this work, coherent exchange spin waves are excited with wavelengths down to  $\lambda = 48$  nm, which are the fastest reported so far [50–52] (see Electronic Supplementary Material (ESM)). More importantly, the Co wires are coherently and unidirectionally coupled by propagating spin waves with a phase shift  $\Delta\phi$  that is freely tunable from 0 to  $2\pi$  by propagation-induced phase delay. The coupling of spin waves in Co and YIG can be interpreted as a dynamic interlayer magnon spin torque [53–56] and the strong nonreciprocity is mainly caused by the chiral spin pumping [57, 58], while the microwave antennas and propagating magnetostatic surface waves might also induce spin-wave nonreciprocity [59–61]. Theoretically, the coupled system of two Co wires and magnetic substrate can be formulated in terms of an effective non-Hermitian Hamiltonian [62–64]. The coupling strongly enhances the spin-wave mediated transmission signal [65], quite different from the level attraction of magnon and photon levels reported in a microwave cavity [66, 67].

## 2 Nanomagnonic device information

Figure 1(a) illustrates two identical Co nanowires in direct contact with a 20 nm-thin YIG film. The Co wires are 30 nm thin, 100 nm wide and separated by 1.5  $\mu\text{m}$ . A scanning electron microscope (SEM) image is shown in Fig. 1(c). Magnons in two Co wires are coupled through spin waves in the YIG film with low magnetic damping [68, 69]. On each Co wires, gold nano-stripline (NSL) antennas (200 nm wide) [70] are integrated to coherently excite and detect spin waves (Fig. 1(b)). The microwave reflection spectrum  $S_{11}$  (Fig. 2(a)) is sensitive to the ferromagnetic resonance (FMR) of the material close to NSL1. The Co wire Kittel mode is observed at frequencies (14.5–19.5 GHz) much higher than that of the YIG film (0.5–3.5 GHz) due to the large form anisotropy of Co nanowires [71, 72]. An in-plane magnetic field  $H$  is applied parallel to Co



**Figure 1** (a) A schematic diagram of two Co nanowires coupled by spin waves in the YIG film. Parameters are the film thickness  $s$ , the distance between the nanowires  $d$ , the spin-wave wavevector  $k$  and the in-plane applied field  $H$ . The coupling between Co1 (2) and Co2 (1) is evident in the microwave transmission spectra  $S_{21}$  ( $S_{12}$ ) measured by nano-stripline antennas. The shift  $\Delta\phi = \phi_2 - \phi_1$  of the precession phases of the Kittel modes in two wires can be controlled. (b) An optical microscope image of the device with two ground-signal antennas, NSL1 and NSL2. Scale bar, 100  $\mu\text{m}$ . (c) An SEM image of two Co wires with width  $w = 100$  nm and center-to-center distance  $d = 1.5$   $\mu\text{m}$ . Scale bar, 300 nm.



**Figure 2** (a) Reflection spectra  $S_{11}$  measure the microwave absorption by NSL1. The ferromagnetic resonances of the Co wires and YIG film are indicated by white arrows. By sweeping the magnetic field  $H$  from negative to positive values, the magnetization of the Co/YIG bilayer switches from the parallel (P) to antiparallel (AP) magnetic configuration from 0 to 54 mT (white dashed lines). Transmission spectra  $S_{21}$  plotted in (b) exhibits strong spin-wave oscillations in the AP state close to the Co resonance. In the reversed transmission  $S_{12}$  (c) such oscillations are not detected. (d) is a line plot of the spectrum at 30 mT, tracing the yellow dashed line in (b), in which the blue and red areas represent the dipolar and exchange magnon regimes, respectively. The enhanced transmission signal indicated by the white arrow is caused by the coupling of the two Co wires through exchange magnons in YIG.

wires. By sweeping  $H$  from negative to positive values, the magnetic configuration of Co/YIG device switches from parallel (P) to antiparallel (AP) because of the high coercivity of Co nanowires [71, 72] compared to the YIG film, returning to a parallel configuration only at  $H > 50$  mT.

Microwaves are transmitted from Co1 to Co2 via propagating spin waves [25, 28, 69, 73] as measured by the transmission spectra  $S_{21}$  (its imaginary part is shown in Fig. 2(b)). We observe remarkable interference fringes (ripples) around the Co resonance in the AP state. The features in  $S_{21}$  (Fig. 2(b)) vanish in the reverse transmission spectra  $S_{12}$  (Fig. 2(c)), revealing a nearly perfect chiral excitation of exchange spin waves by the magnetodipolar fields from the Co nanowires [15, 57, 58]. This can be understood by the stray fields generated by the right (left) moving spin waves that vanish below (above) the film and precess in the opposite direction of the magnetization. Figure 2(d) is a cross section from  $S_{21}$  extracted at a field of 30 mT. The Co FMR related signal near 15 GHz is more than 10 times stronger than the signal at lower frequencies assigned to dipolar magnons close the FMR of YIG [74, 75]. Therefore, the short-wavelength, exchange-dominated magnons with high group velocities [47–52] transmit signals between two Co wires at high frequencies (14–17 GHz) and in one direction only.

## 3 Results and discussion

The mechanism of nonreciprocal coupling is conceptually sketched in Fig. 3(a). Due to the chiral spin pumping [58], the FMR of Co wire 1 can only couple to spin waves in YIG with wavevector  $k$  in one direction. After propagating over a finite distance  $d$ , the spin waves in YIG then drive the magnetization in Co wire 2 to precess. We find a fixed phase relation  $\Delta\phi = \phi_2 - \phi_1$  between the spin precessions of two Co wires that is caused by: (1) Twice a shift of  $\phi_i$  by the dynamical coupling [53–55] between the Co wires and YIG film, i.e., from Co1 to YIG and from YIG to Co2, which depends on the frequency. At resonance this process is purely dissipative, i.e.,  $\phi_i = \pi/2$ ; (2) the phase delay  $\phi_k$  by the propagation of exchange spin



$$\frac{\hat{\mathcal{H}}_{\text{Co|YIG}}}{\hbar} = \omega^{\text{Co}} \hat{m} \hat{m} + \sum_k \omega_k^{\text{YIG}} \hat{p}_k \hat{p}_k + \sum_k (g_k \hat{m} \hat{p}_k^\dagger + g_k^* \hat{p}_k \hat{m}) \quad (2)$$

where  $\hat{m}$  and  $\hat{p}_k$  are bosonic operators associated, respectively, with the Kittel mode in Co and spin waves in YIG with wave-number  $k$ .  $\omega^{\text{Co}}$  is the Co FMR frequency,  $\omega_k^{\text{YIG}}$  the YIG spin-wave frequency dispersion [74] (see the ESM). The interlayer coupling matrix elements  $g_k$  can be caused by the interface exchange or magnetodipolar interaction. The indirect Co-Co coupling strength as sketched in Fig. 2(a) scales with  $|g_k|^2 \propto$

$$\left| \int_0^w e^{iky} dy \right|^2 = \left| \frac{\sin(kw/2)}{k/2} \right|^2, \text{ which is plotted as a function of}$$

wavelength  $\lambda = 2\pi/k$  in Fig. 4(f) (see the ESM). The blue shaded interval  $\lambda = 54.0\text{--}66.8$  nm corresponds to the spectrum plotted in Fig. 4(e) (see the ESM). In this range, the relative coupling strength shows a minimum around 60 nm (Fig. 4(e)), which explains the suppressed amplitude around 19 GHz (Figs. 4(a)–4(c)). In the  $w = 100$  nm sample (Fig. 3(d)), we also observe some amplitude modulation (see the ESM). The experimental results shown in Fig. 4(b) may be directly compared with the calculated transmission based on the theoretical analysis [12, 78–80] (see the ESM).

The coherent excitation of Co2 from Co1 is driven by a magnon spin torque [53–56] mediated by exchange spin waves in the YIG film. The torque transferred at Co1|YIG and YIG|Co2 may be induced by either interfacial exchange or magnetodipolar coupling. In view of an interfacial exchange coupling strength measured to be 0.2 GHz in planar Co|YIG films [53], the exchange coupling strength at the interface between a single Co nanowire and the YIG film is merely 20 kHz, which is negligible in comparison with the magnetodipolar coupling calculated and shown in Fig. 4(f). It has been theoretically studied that the interlayer exchange (magnetodipolar) coupling is larger at the parallel (antiparallel) configuration [58]. The experimental observation that strong transmission between Co1 and Co2 occurs at the antiparallel configuration (Fig. 3(b)) indicates that the magnetodipolar interaction is responsible for the coupling at Co1|YIG and YIG|Co2. The dominance of magnetodipolar coupling and the absence of interlayer exchange give rise to the spin-wave chirality and eventually lead to the nonreciprocal coupling of two Co wires. A chiral spin pumping can also be achieved by microwaves emitted by normal metal antennas that can couple to the dipolar spin waves in thick films [81]. However, this process is very inefficient in ultrathin films, while the exchange spin waves as excited by magnetic nanowires here cannot be accessed at all. Theoretically, the coupling between two Co wires can be described by an effective Hamiltonian as

$$\begin{aligned} \frac{\hat{\mathcal{H}}_{\text{Co|Co}}}{\hbar} &= \sum_{l=1}^2 (\omega_l^{\text{Co}} - i\Gamma_l) \hat{m}_l \hat{m}_l - i\Gamma_{12}(\omega) \hat{m}_1^\dagger \hat{m}_2 - i\Gamma_{21}(\omega) \hat{m}_2^\dagger \hat{m}_1, \\ \Gamma_{l=1,2}(\omega) &\equiv - \int \frac{dk}{2\pi} \frac{|g_{k,l}|^2}{i(\omega - \omega_k) - \frac{\kappa_{\text{YIG}}}{2}}, \\ \Gamma_{12}(\omega) &\equiv - \int \frac{dk}{2\pi} \frac{g_{+,k} g_{+,k}}{i(\omega - \omega_k) - \frac{\kappa_{\text{YIG}}}{2}}, \\ \Gamma_{21}(\omega) &\equiv - \int \frac{dk}{2\pi} \frac{g_{-,k}^* g_{-,k}}{i(\omega - \omega_k) - \frac{\kappa_{\text{YIG}}}{2}}. \end{aligned} \quad (3)$$

where  $\Gamma_{12}$  ( $\Gamma_{21}$ ) is the coupling from Co1 (2) to Co2 (1),  $\kappa_{\text{YIG}}$  stands for the FMR linewidth of the YIG film,  $g_{+,l}$  ( $g_{+,k,2}$ ) the coupling between Co1 (2) and propagating spin waves in YIG with positive wavenumber  $+k$ ,  $g_{-,l}$  ( $g_{-,k,2}$ ) the coupling strength between Co1 (2) and spin waves with  $-k$ . Here the spin-wave coupling is chiral,  $g_{-,l} = 0$  and  $g_{-,k,2} = 0$ , as illustrated in Fig. 3(a) [57, 58] and  $\Gamma_{12} \approx 0$  and hence Eq. (3) is non-Hermitian.

## 4 Conclusions

In summary, we have demonstrated on-chip nonreciprocal coupling of two distant nanomagnets at room temperature. The two Co wires are found to be phase-related by propagating exchange spin waves in the YIG film. Any desired phase shift  $\Delta\phi$  from 0 to  $2\pi$  can be realized by tuning frequencies and applied magnetic fields in the vicinity of the Co FMR. The magnon transmission varies sensitively with propagation distance and the nanowire width, providing additional evidence for the phase-coherent coupling. We theoretically describe the coupling between two Co wires in terms of a Hamiltonian that includes the chiral spin pumping and therefore becomes non-Hermitian [58]. This formulation helps to predict new phenomena, such as the magnon trap [82] and chiral magnetic noise [83]. The chiral coupling of nanomagnets demonstrated in this work is attractive for neuromorphic computing [84, 85], since it mimics the innately unidirectional information transmission between neurons via synapses [86].

## Acknowledgements

The authors thank L. Flacke, M. Althammer, M. Weiler, K. Schultheiss, H. Schultheiss, M. Madami, G. Gubbiotti and C. M. Hu for their helpful discussions. Funding: We wish to acknowledge the support by the National Key Research and Development Program of China (Nos. 2016YFA0300802 and 2017YFA0206200), the National Natural Science Foundation of China (NSFC) (Nos. 11674020, 12074026 and U1801661), and the 111 talent program B16001. G. B. was supported by the Netherlands Organization for Scientific Research (NWO) and Japan Society for the Promotion of Science Kakenhi Grants-in-Aid for Scientific Research (No. 19H006450). T. Y. was funded through the Emmy Noether Program of Deutsche Forschungsgemeinschaft (SE 2558/2-1). K. X. thanks the National Key Research and Development Program of China (Nos. 2017YFA0303304 and 2018YFB0407601) and the National Natural Science Foundation of China (Nos. 61774017 and 11734004). K. S. was supported by the Fundamental Research Funds for the Central Universities (No. 2018EYT02). M. Z. W. were supported by the US National Science Foundation (No. EFMA-1641989).

**Electronic Supplementary Material:** Supplementary material (material characterization, device fabrication, measurement techniques, spin-wave dispersion at different fields, SEM images of the nanowires with different widths, coupling strength with different nanowire widths and wavenumbers, details of micromagnetic simulations of chiral spin-wave emission, theory of spin-wave chiral dynamics including stripline magnetic field, magnon coupling by dipolar interaction, magnon damping by chiral spin pumping, phase relation between nanowires and non-Hermitian coupling and the shortest wavelength observed in the transmission spectra) is available in the online version of this article at <https://doi.org/10.1007/s12274-020-3251-5>.





# Electronic Supplementary Material

## Nonreciprocal coherent coupling of nanomagnets by exchange spin waves

Hanchen Wang<sup>1,§</sup>, Jilei Chen<sup>1,§</sup>, Tao Yu<sup>2,§</sup>, Chuanpu Liu<sup>1,§</sup>, Chenyang Guo<sup>3,§</sup>, Song Liu<sup>4,§</sup>, Ka Shen<sup>5,§</sup>, Hao Jia<sup>4,§</sup>, Tao Liu<sup>6</sup>, Jianyu Zhang<sup>1</sup>, Marco A. Cabero Z<sup>4,1</sup>, Qiuming Song<sup>4</sup>, Sa Tu<sup>1</sup>, Mingzhong Wu<sup>6</sup>, Xiufeng Han<sup>3</sup>, Ke Xia<sup>4</sup>, Dapeng Yu<sup>4</sup>, Gerrit E. W. Bauer<sup>7,8,9</sup>, and Haiming Yu<sup>1</sup> (✉)

<sup>1</sup> Fert Beijing Institute, School of Integrated Circuit Science and Engineering, Beijing Advanced Innovation Center for Big Data and Brain Computing, Beihang University, Beijing 100191, China

<sup>2</sup> Max Planck Institute for the Structure and Dynamics of Matter, 22761 Hamburg, Germany

<sup>3</sup> Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, University of Chinese Academy of Sciences, Chinese Academy of Sciences, Beijing 100190, China

<sup>4</sup> Shenzhen Institute for Quantum Science and Engineering (SIQSE), and Department of Physics, Southern University of Science and Technology (SUSTech), Shenzhen 518055, China

<sup>5</sup> Department of Physics, Beijing Normal University, Beijing 100875, China

<sup>6</sup> Department of Physics, Colorado State University, Fort Collins, Colorado 80523, USA

<sup>7</sup> Kavli Institute of Nanoscience, Delft University of Technology, 2628 CJ Delft, The Netherlands

<sup>8</sup> Institute for Materials Research, WPI-AIMR and CSNR, Tohoku University, Sendai 980-8577, Japan

<sup>9</sup> Zernike Institute for Advanced Materials, University of Groningen, Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands

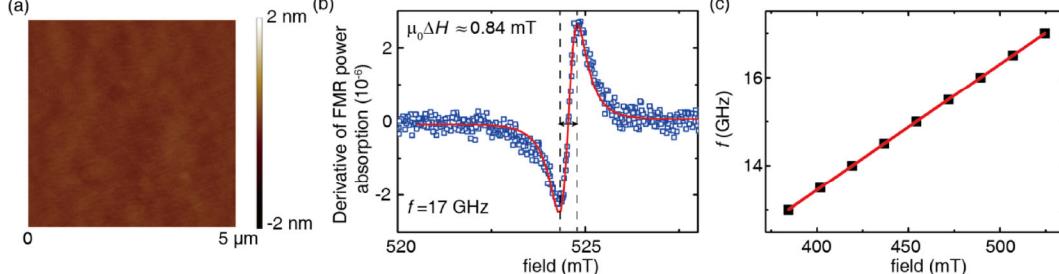
<sup>§</sup> Hanchen Wang, Jilei Chen, Tao Yu, Chuanpu Liu, Chenyang Guo, Song Liu, Ka Shen and Hao Jia contributed equally to this work.

Supporting information to <https://doi.org/10.1007/s12274-020-3251-5>

### Material characterization

The 20 nm-thick yttrium iron garnet film (YIG) on GGG substrate is grown by magnetron sputtering at room temperature. Before growing, the system has a base pressure of  $1.0 \times 10^{-8}$  Torr. During the sputtering, the argon pressure is 10 mTorr, the argon flow rate is 4 sccm and the power is fixed at 75 W. Immediately, after the sputtering, the films are annealed at 750 °C or 800 °C for 4 h in 1.12 Torr oxygen. The surface roughness is measured by the atomic force microscope (AFM) as shown in Fig. S1(a), the size of the presented region is 5 μm × 5 μm. The ferromagnetic resonance (FMR) power absorption spectra provide the high-frequency performance and the linewidth of the YIG films. As present in Fig. S1(b), the frequency is fixed at 17 GHz, one can extract the linewidth at this frequency is about  $\mu_0\Delta H = 0.84$  mT through the Lorentz fitting, and for the frequency at 13 GHz, the extracted linewidth is about  $\mu_0\Delta H = 0.82$  mT. The Gilbert damping  $\alpha$  fitted by  $\mu_0\Delta H = \frac{2}{\sqrt{3}\gamma}\alpha f + \mu_0H_0$  is about  $8 \times 10^{-5}$ , and

$\mu_0H_0 = 0.73$  mT is the film inhomogeneity broadening. After changing the measurement frequency, the FMR dispersion is extracted from the measurement results and showing in Fig. S1(c), from which the saturation magnetization  $\mu_0M_s$  is extracted as 0.18 T.

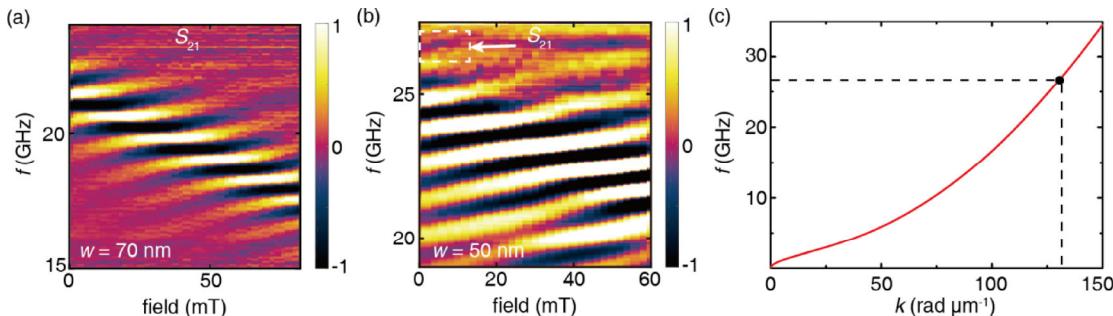


**Figure S1** Sample information of the 20 nm-thick YIG films. (a) The AFM surface image of 20 nm-thick YIG with the roughness of 0.07 nm. (b) By using the Lorentz fitting of the FMR power absorption spectrum at 17 GHz, the linewidth  $\mu_0\Delta H$  is about 0.84 mT. (c) The FMR frequency as a function of field, from which we extract the saturation magnetization  $\mu_0M_s = 0.18$  T.

Address correspondence to [haiming.yu@buaa.edu.cn](mailto:haiming.yu@buaa.edu.cn)

## Exchange spin waves with wavelength down to 48 nm and transmission spectra on other devices

Here, we present the transmission measurement results on other widths Co nanowire samples, namely, 50 nm (Fig. S2(b)) and 70 nm wide (Fig. S2(a)). In our measurement, the shortest wavelength spin waves is observed in the measurement results of the narrowest (50 nm-wide) sample corresponding to the highest frequency of Co FMR mode and shortest spin waves wavelength. For clear observation of the fastest exchange magnons mode, we present the spectra with the field range from 0 mT to 60 mT. As shown in Fig. S2(b), the transmission spectra  $S_{21}$  of the 50 nm wide Co nanowire device, from which one can observe the signal about 26.85 GHz pointed by the white arrow when the magnetic field is zero. The dispersion of dipolar-exchange magnons at zero field is present in (Fig. S2(c)), the black point on the red curve is the observed spin-wave resonance frequency extracted from spectra. After calculation, one can calculate that the wavevector of spin waves is about 130 rad  $\mu\text{m}^{-1}$  corresponding to 48 nm wavelength.

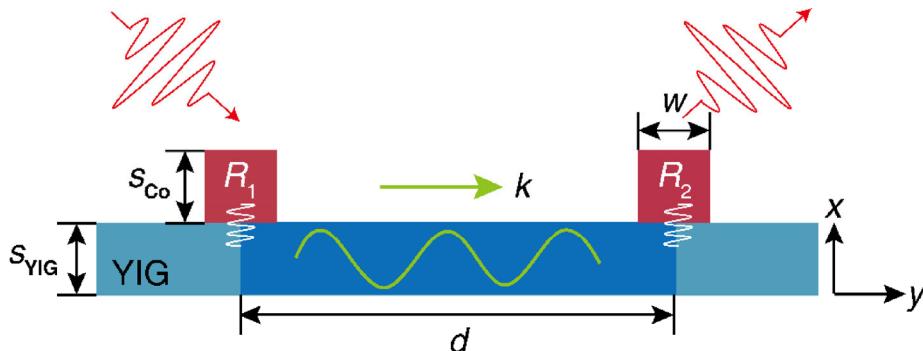


**Figure S2** The color-coded spectra and dispersion of the observation of exchange spin waves with wavelength down to 48 nm. (a), (b), Transmission spectra  $S_{21}$  of the 70 nm and 50 nm wide Co nanowire device, the shortest wavelength exchange magnons mode is pointed by the white arrow. (c) The dispersion when the magnetic field is zero for calculating the wavelength from the results in Fig. S2(b).

## Theory model of spin-wave chiral dynamics

Here, we formulate the theory of chiral spin pumping and (long-range) chiral interaction between ferromagnetic nanowires mediated by unidirectionally excited spin waves in the ferromagnetic film. We focus on the experimental observables, such as the additional damping of Co magnon and the microwave transmission spectra.

We consider the effectively one-dimensional model in Fig. S3 with two sufficiently long magnetic nanowires (thickness  $s_{\text{Co}}$  and width  $w$ ) on the top of a thin YIG film. The distance between the nanowires  $d \gg w$ . The nanowires are excited and detected by local metal stripline antennas with small rectangular cross section of widths  $w_0$  and thicknesses  $s_0$  and uniform AC electric current distributions.



**Figure S3** Two Co nanowires on a thin YIG film. Local stripline antennas (not shown) excite and detect the magnetization dynamics in the nanowires. The green arrow indicates the unidirectionally excited magnon current in the film.

## Theory of stripline magnetic field distribution

A stripline carrying a harmonic current source  $\mathbf{J}(t) = \mathbf{J}(\omega)e^{-i\omega t}$  generates the vector potential

$$\mathbf{A}(\mathbf{r}, \omega) = \frac{\mu_0}{4\pi} \int d\mathbf{r}' \mathbf{J}(\mathbf{r}', \omega) \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}, \quad (\text{S1})$$

where  $k = \omega / c$  and the magnetic field

$$\mathbf{H}(\mathbf{r}) = \nabla \times \frac{\mathbf{A}(\mathbf{r})}{\mu_0} = \frac{(\partial_y A_z, -\partial_x A_z, 0)}{\mu_0}. \quad (\text{S2})$$

The current is non-zero in the antenna for  $0 < x < s_0$  and  $-\frac{w_0}{2} < y < \frac{w_0}{2}$ , which leads to the Fourier components of the magnetic field

$$\begin{aligned} H_x(x, k_y, \omega) &= i \frac{\mathbf{J}(\omega)}{4\pi} \mathcal{F}(s_0, w_0) \frac{k_y}{\sqrt{k_y^2 - k^2}} e^{\sqrt{k_y^2 - k^2}x}, \\ H_y(x, k_y, \omega) &= -\frac{\mathbf{J}(\omega)}{4\pi} \mathcal{F}(s_0, w_0) e^{\sqrt{k_y^2 - k^2}x}, \end{aligned} \quad (\text{S3})$$

with form factor

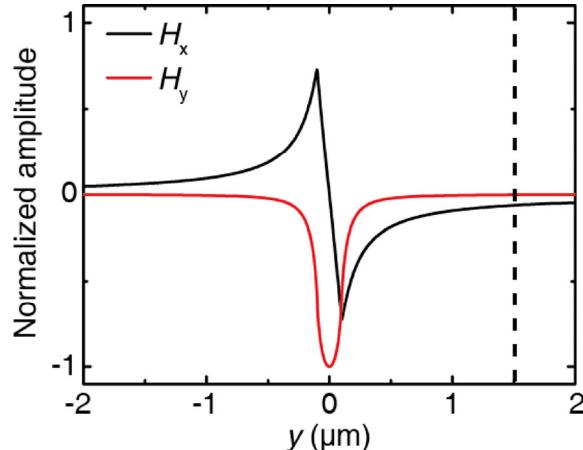
$$\mathcal{F}(s_0, w_0) = \frac{4}{k_x k_y} e^{ik_x \frac{s_0}{2}} \sin\left(k_x \frac{s_0}{2}\right) \sin\left(k_y \frac{w_0}{2}\right). \quad (\text{S4})$$

At microwave frequencies  $\omega / 2\pi \sim 10\text{GHz}$ ,  $k = \omega / c \sim 200\text{radm}^{-1}$  and wavelength  $\lambda = 2\pi / k \sim 3\text{ cm}$ . Since the wavelengths of the excited spin waves at this frequency are much smaller (see below), we are in the near-field limit. The Fourier component of the evanescent magnetic field  $H_x \rightarrow i\text{sgn}(k_y)H_y$  is circularly polarized in the momentum space with a sign locked to its linear momentum.

The spatial distribution of the stripline magnetic field is the real part of the inverse Fourier transform ( $k \equiv k_y$ ),

$$\mathbf{H}_a(x, y, t) = \sum_k e^{iky - i\omega t} \mathbf{H}_a(x, k). \quad (\text{S5})$$

We plot a snapshot of the spatial distribution of  $\mathbf{H}(x, 0, t)$  by a stripline antenna with  $s_0 = 120\text{ nm}$  and  $w_0 = 200\text{ nm}$  in Fig. S4, noting that  $H_y$  decays faster than  $H_x$ . The latter is seen to be still significant at a distance of  $1.5\text{ }\mu\text{m}$ .



**Figure S4** Calculated magnetic-field amplitude excited by a metal stripline antenna with  $s_0 = 120\text{ nm}$  and  $w_0 = 200\text{ nm}$ , carrying an AC current with frequency  $\frac{\omega}{2\pi} = 10\text{ GHz}$ . The field components are normalized by the maximum value of  $H_y$ .

## Theory of magnon coupling by dipolar interaction

The magnetization  $\tilde{\mathbf{M}}_l$  of the  $l$ -th magnetic wire couples with the dipolar field  $h$  from the magnetization dynamics of the film  $\mathbf{M}$  by the Zeeman interaction

$$\hat{H}_{int} = -\mu_0 \int_0^{s_{\text{VIG}}} dx d\boldsymbol{\rho} \tilde{\mathbf{M}}_{l,\beta}(x, \boldsymbol{\rho}) h_\beta(x, \boldsymbol{\rho}), \quad (\text{S6})$$

where  $\mu_0$  is the vacuum permeability and we use the summation convention over repeated Cartesian indices  $\beta = \{x, y, z\}$ . The magnetizations  $\tilde{\mathbf{M}}_l \parallel \hat{z}$  are anti-parallel to  $\mathbf{M} \parallel -\hat{z}$ . We focus on the linear regime and disregard magnon interactions in the film. The magnetization operators in the magnetic wires and film are expanded by the magnon mode amplitudes and operators,

$$\begin{aligned} \hat{M}_\alpha(r) &= -\sqrt{2M_s\gamma\hbar} \sum_k (m_\alpha^{(k)}(x) e^{iky} \hat{\alpha}_k + \text{H.c.}), \\ \hat{\tilde{M}}_{l,a}(r) &= -\sqrt{2\tilde{M}_{s,l}\gamma\hbar} (\tilde{m}_{l,a}^K(r) \hat{\beta}_l + \text{H.c.}), \end{aligned} \quad (\text{S7})$$

where  $M_s$  and  $\tilde{M}_{s,l}$  are the saturated magnetizations of film and nanowire,  $-\gamma$  is the electron gyromagnetic ratio,  $m_\alpha^{(k)}(x)$  and  $\tilde{m}_{l,a}^K(r)$ , respectively, represent the amplitude of the spin waves and Kittel modes, and  $\hat{\alpha}_k$  and  $\hat{\beta}_l$  denote the magnon operators in the film and nanowire. The  $l$ -th nanowire centered at  $\mathbf{r}_l = R_l \hat{y}$  feels the dipolar field

$$h_\beta(\mathbf{r}) = -\frac{1}{4\pi} \partial_\beta \partial_a \int d\mathbf{r}' \frac{M_a(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}. \quad (\text{S8})$$

The Hamiltonian then reads:

$$\frac{\hat{\mathcal{H}}}{\hbar} = \sum_l \omega_{0,l} \hat{\beta}_l^\dagger \hat{\beta}_l + \sum_k \omega_k \hat{\alpha}_k^\dagger \hat{\alpha}_k + \sum_l \sum_k \left( g_{k,l} e^{-ikR_l} \hat{\beta}_l \hat{\alpha}_k^\dagger + g_{k,l}^* e^{ikR_l} \hat{\beta}_l^\dagger \hat{\alpha}_k \right), \quad (\text{S9})$$

where the coupling constants

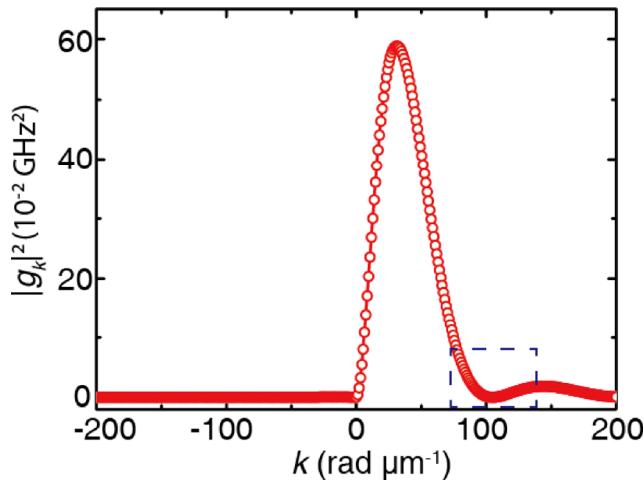
$$g_{k,l} = F_l(k) \left( m_x^{(k)*}, m_y^{(k)*} \right) \begin{pmatrix} |k| & ik \\ ik & -|k| \end{pmatrix} \begin{pmatrix} \tilde{m}_{l,x}^K \\ \tilde{m}_{l,y}^K \end{pmatrix}, \quad (\text{S10})$$

are real  $g_{k,l} = g_{k,l}^*$  and the form factor

$$F_l(k) = -2\mu_0 \gamma \sqrt{M_s \tilde{M}_s / \Lambda} (1 - e^{-|k|s_{\text{Co}}}) (1 - e^{-|k|s_{\text{YIG}}}) \frac{1}{k^3} \sin\left(\frac{k w}{2}\right). \quad (\text{S11})$$

The length  $\Lambda$  of the magnetic nanowire is sufficiently long to warrant the two-dimensional treatment. Perfect chiral coupling corresponds to  $g_{-|k|} = 0$ , which is realized when the spin waves are circularly polarized, i.e. with  $m_y^{(k)} = -im_x^{(k)}$  (for  $\mathbf{M} \parallel -\hat{z}$ ). The chirality of the dipolar coupling can be understood from the distribution of the dipolar field: the dipolar field generated by the right (left) moving spin waves vanishes below (above) the film and precesses oppositely to the magnetization.

With  $\mu_0 M_s = 0.177$  T for YIG and  $\mu_0 \tilde{M}_s = 1.67$  T for Co, we plot  $|g_k|^2$  for spin waves in YIG of thickness 20 nm and a Co nanowire of thickness  $s_{\text{Co}} = 30$  nm and  $w = 60$  nm in Fig. S5. The coupling is indeed chiral and shows minima for spin waves with particular momentum  $k = 2n\pi/w$ , where  $n \geq 0$  is an integer. The coupling is strongest for  $0 < k \lesssim \pi/w$ . In the following, we take unit length of the nanowire such that  $\Lambda = 1$ .



**Figure S5** Momentum dependence of  $|g_k|^2$  between spin waves in a YIG film of thickness 20 nm and Co nanowire with  $s_{\text{Co}} = 30$  nm and  $w = 60$  nm. A zoom-in of this figure (blue box) is shown in Fig. 4(f), in which the shaded area indicated the wavelength interval extracted from the experiments of Fig. 4 in the main text.

## Theory of magnon damping by chiral spin pumping

With one magnetic wire centered at the origin  $R = 0$  and the input of microwave expressed by the operator  $\hat{p}_{in}$ , the magnon equations of motion read

$$\begin{aligned} \frac{d\hat{\beta}(t)}{dt} &= -i\omega_0 \hat{\beta}(t) - i \sum_k g_k \hat{\alpha}_k(t) - \left( \frac{\kappa}{2} + \frac{\kappa_p}{2} \right) \hat{\beta}(t) - \sqrt{\kappa_p} \hat{p}_{in}(t), \\ \frac{d\hat{\alpha}_k(t)}{dt} &= -i\omega_k \hat{\alpha}_k(t) - ig_k \hat{\beta}(t) - \frac{\kappa_k}{2} \hat{\alpha}_k(t). \end{aligned} \quad (\text{S12})$$

Here,  $\alpha_G = \kappa / (2\omega)$  is the intrinsic (Gilbert) damping of the Kittel modes in the nanowire,  $\kappa_p$  is the radiative damping induced by the microwave photons  $\hat{p}_{in}$ , i.e., the coupling of the nanowire to the microwave source, and  $\kappa_k$  denotes the intrinsic (Gilbert) damping of magnons with momentum  $k$  in the films,  $\omega_0$  is the frequency of the Kittel mode of wire. In frequency space, we obtain

$$\begin{aligned} \hat{\alpha}_k(\omega) &= g_k G_k(\omega) \hat{\beta}(\omega), \\ \hat{\beta}(\omega) &= \frac{-\sqrt{\kappa_p}}{-i(\omega - \omega_0) + \frac{\kappa + \kappa_p}{2} + i \sum_k g_k^2 G_k(\omega)} \hat{p}_{in}(\omega), \end{aligned} \quad (\text{S13})$$

where  $G_k(\omega) = \frac{1}{(\omega - \omega_k) + \frac{i\kappa_k}{2}}$  is the magnon Green function. The chiral coupling  $g_{-|k|} = 0$  only excites spin waves with

positive momentum, i.e., a unidirectionally excited magnon current. An imaginary part of the magnon self-energy indicates additional damping. In the on-shell approximation with  $\omega \rightarrow \omega_0$

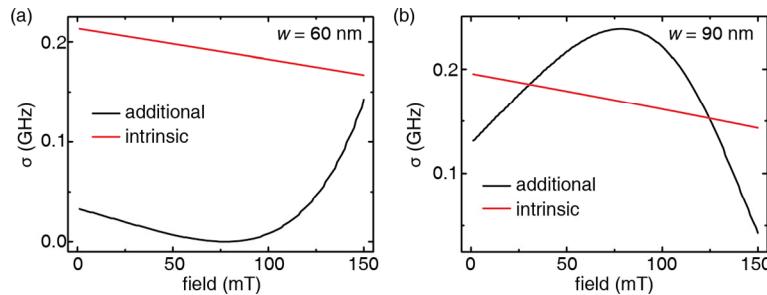
$$\sigma_k(\omega_0) = -\text{Im} \left( \sum_k |g_k|^2 G_k(\omega_k) \right) = \sum_k |g_k|^2 \frac{\frac{\kappa_k}{2}}{(\omega_0 - \omega_k)^2 + \left( \frac{\kappa_k}{2} \right)^2} \rightarrow \frac{1}{2} \frac{|g_k|^2}{v_{k_r}}, \quad (\text{S14})$$

where  $\omega_{k_r} = \omega_0$ ,  $v_{k_r}$  is the magnon group velocity.

We compute the additional damping by spin waves pumping for a Co nanowire of thickness 30 nm and different widths, disregarding any interface exchange. We use the magnetizations  $\mu_0 M_s = 0.177$  T for YIG and 1.67 T for Co. The intrinsic Gilbert damping coefficient of Co wire is taken to be  $\alpha_G = 0.01$ . Figure S6 are the plots of the additional  $\sigma$  and intrinsic broadening  $\kappa_{Co}/2 = \alpha_G \omega_{Co}$  of the wire Kittel magnon as a function of the external magnetic field.

$$\Delta\omega = 2 \left( \sigma_k + \frac{\kappa_{Co}}{2} \right). \quad (\text{S15})$$

$\sigma_k$  depends on the wire width (and thickness) via the form factor (see Fig. S6). By the magnetic field, the additional damping can be tuned to equal the intrinsic one.



**Figure S6** Additional broadening  $\sigma_k$  and intrinsic broadening  $\kappa_{Co}/2$  of the wire magnon as a function of the applied magnetic field. The effective damping can be measured by the line broadening of FMR of the microwave reflection  $S_{11}(\omega)$ .

## Theory of phase relation between nanowires

We now consider two magnetic nanowires located at  $r_1 = R_1 \hat{j}$  and  $r_2 = R_2 \hat{j}$  that act as transducers for microwaves emitted and detected by local striplines.

When the Kittel magnon at  $R_1$  and  $R_2$  are expressed by the operators  $\hat{\beta}_L$  and  $\hat{\beta}_R$ , respectively, the equations of motion of nanowire and film magnetization read

$$\begin{aligned} \frac{d\hat{\beta}_L(t)}{dt} &= -i\omega_{0,L}\hat{\beta}_L(t) - i\sum_k g_{k,L} e^{ikR_1} \hat{\alpha}_k(t) - \left( \frac{\kappa_L}{2} + \frac{\kappa_{p,L}}{2} \right) \hat{\beta}_L(t) - \sqrt{\kappa_{p,L}} \hat{p}_{in}^{(L)}(t), \\ \frac{d\hat{\beta}_R(t)}{dt} &= -i\omega_{0,R}\hat{\beta}_R(t) - i\sum_k g_{k,R} e^{ikR_2} \hat{\alpha}_k(t) - \frac{\kappa_R}{2} \hat{\beta}_R(t), \\ \frac{d\hat{\alpha}_k(t)}{dt} &= -i\omega_k \hat{\alpha}_k(t) - ig_{k,L} e^{-ikR_1} \hat{\beta}_L(t) - ig_{k,R} e^{-ikR_2} \hat{\beta}_R(t) - \frac{\kappa_k}{2} \hat{\alpha}_k(t). \end{aligned} \quad (\text{S16})$$

In frequency space:

$$\begin{aligned} \hat{\alpha}_k(\omega) &= G_k(\omega) [g_{k,L} e^{-ikR_1} \hat{\beta}_L(\omega) + g_{k,R} e^{-ikR_2} \hat{\beta}_R(\omega)], \\ \hat{\beta}_L(\omega) &= \frac{-\sqrt{\kappa_{p,L}}}{-i(\omega - \omega_{0,L}) + \frac{\kappa_L + \kappa_{p,L}}{2} + i\sum_k g_{k,L}^2 G_k(\omega) - f(\omega)} \hat{p}_{in}^{(L)}(\omega), \\ \hat{\beta}_R(\omega) &= \frac{-i\sum_k g_{k,L} g_{k,R} G_k(\omega) e^{ik(R_2 - R_1)}}{-i(\omega - \omega_{0,R}) + \frac{\kappa_R}{2} + i\sum_k g_{k,R}^2 G_k(\omega)} \hat{\beta}_L(\omega), \end{aligned} \quad (\text{S17})$$

where

$$f(\omega) \equiv -\frac{(\sum_k g_{k,L} g_{k,R} G_k(\omega) e^{ik(R_1 - R_2)}) (\sum_k g_{k,L} g_{k,R} G_k(\omega) e^{ik(R_2 - R_1)})}{-i(\omega - \omega_{0,R}) + \frac{\kappa_R}{2} + i\sum_k g_{k,R}^2 G_k(\omega)}. \quad (\text{S18})$$

For perfect chiral coupling  $f(\omega)$  vanishes by the absence of backaction.

The Eq. S19 gives the phase relation between  $\hat{\beta}_R$  and  $\hat{\beta}_L$  when the left nanowire is excited. When the two nanowires have the same parameters  $g_{k,L} = g_{k,R} = g_k$ . Then, at the FMR  $\omega \rightarrow \omega_{0,L} = \omega_{0,R}$ ,

$$\hat{\beta}_R(\omega_0) = -\chi(\omega_0) e^{ik_r(R_2-R_1)} \hat{\beta}_L(\omega_0), \quad (\text{S19})$$

where

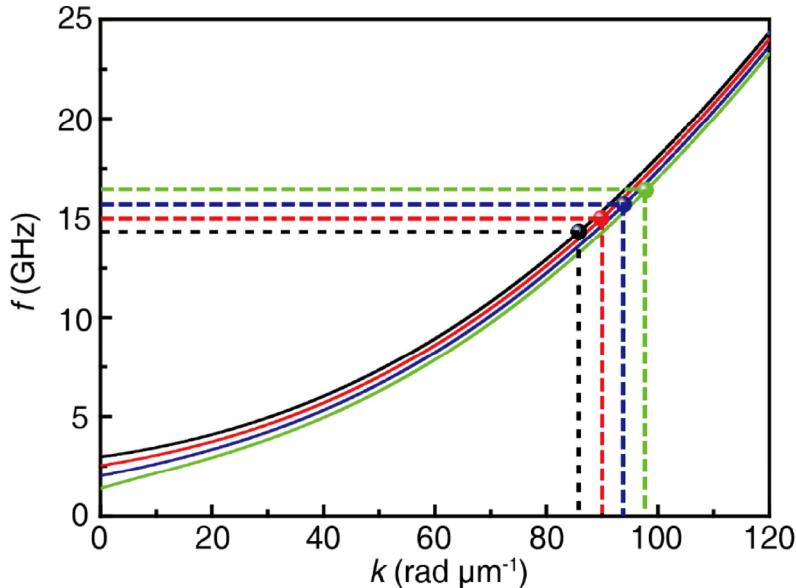
$$\chi(\omega_0) = \frac{2\sigma_k(\omega_0)}{\frac{\kappa_R}{2} + \sigma_k(\omega_0)}, \quad (\text{S20})$$

modulates the excited magnon amplitude. This corresponds to a phase shift

$$\Delta\phi = \pi + k_r(R_2 - R_1), \quad (\text{S21})$$

between the two nanowires.

### Spin-wave dispersion at different fields



**Figure S7** Spin wave dispersion relations of YIG at different magnetic fields. The green, black, red and blue curves correspond to 13 mT, 26 mT, 38.5 mT and 51 mT, respectively. The four data points marked in the figure correspond to the four spin-wave intensity peaks in Fig. 3(d) in the main text.

The dipolar-exchange spin wave dispersion in the Damon-Eshbach configuration reads:

$$f = \frac{\gamma}{2\pi} \left[ (\mu_0 H + A\mu_0 M_s k^2)(\mu_0 H + A\mu_0 M_s k^2 + \mu_0 M_s) + \frac{(\mu_0 M_s)^2}{4} (1 - e^{-2kt}) \right]^{\frac{1}{2}}, \quad (\text{S22})$$

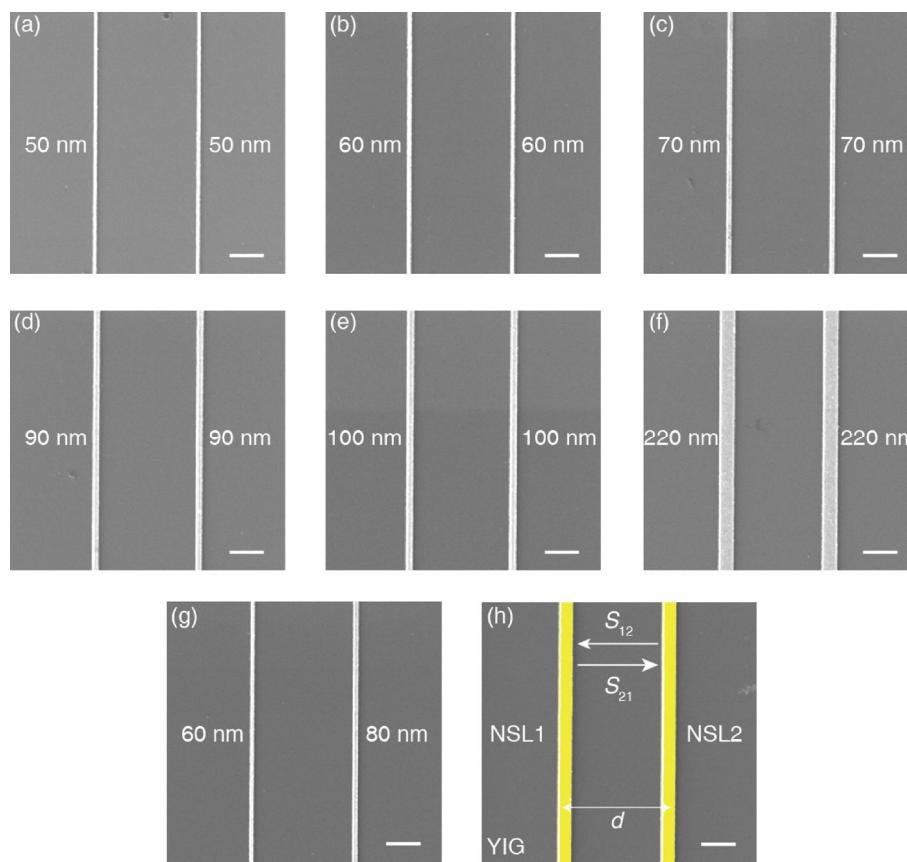
where  $|\gamma/2\pi|$  is an absolute gyromagnetic ratio of YIG with 28 GHz T<sup>-1</sup>,  $\mu_0$  is the vacuum permeability,  $A = 3 \times 10^{-12}$  cm<sup>2</sup> the exchange stiffness constant, and  $\mu_0 M_s = 0.18$  T is the saturation magnetization (see Fig. S1) of the film with 20 nm-thick. Here the wavenumber  $k$  is large, the exchange term  $A\mu_0 M_s k^2$  dominates and the spin-wave dispersion relation is quadratic. Figure S7 shows four dispersion curves, corresponding to the applied field  $\mu_0 H$  of 13 mT, 26 mT, 38.5 mT and 51 mT. Four points are marked in the figure with different wavenumbers, corresponding to the peaks in Fig. 3(d) in the main text. We find a wave vector variation  $\Delta k$  between the neighbouring wavenumbers of 4 rad μm<sup>-1</sup>, which gives a spin wave phase shift of 6.0 rad.

### Scanning electron microscope (SEM) images of the devices

The SEM images in Figs. S8(a)-8(f) show Co nanowires on top of YIG that vary in width from 50 to 220 nm. The asymmetric device width of 60 nm and 80 nm shows in Fig. S8(g). The Co nanowires are fabricated on YIG waveguides and the center-to-center distance  $d$  is 1.5 μm. The SEM image of antennas on Co nanowires is shown in Fig. S8(h), with the width of antennas around 200 nm. The scale bars are both 500 nm in Fig. S8.

### Coupling strength modulated by the nanowire width and spin-wave wavenumber

The equation of  $g_k$  in the main text involves the coupling strength between the FMR modes of two Co nanowires on a YIG film that in our model depends on wave number  $k$  and the width of the nanowire  $w$ . In the main text, we discuss the relative coupling strength as a function of the spin-wave wavelength for 60 nm wide Co nanowires for different propagation distances. Here, we show the  $S_{21}$  transmission spectra, the spectrum along the Co nanowire resonance (red dashed lines in  $S_{21}$ ) and the relative coupling strength (or form factors) as a function of the spin-wave wavelength for various samples.



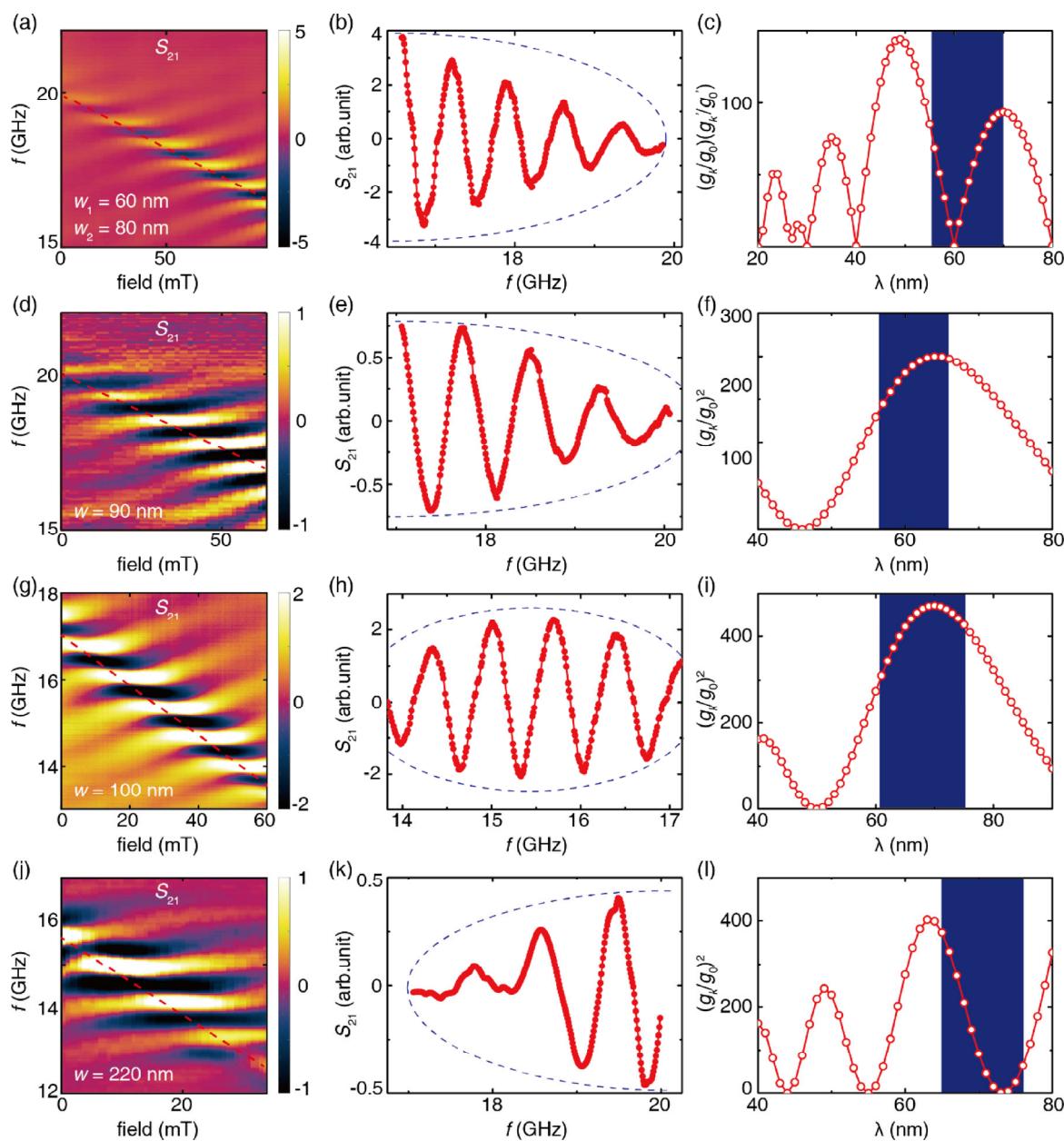
**Figure S8** SEM images of Co nanowire devices. (a)-(f) The SEM images of Co nanowires with the width of 50 nm, 60 nm, 70 nm, 90 nm, 100 nm, and 220 nm. (g) The SEM images of the different widths Co nanowires device with the width of 60 nm and 80 nm. (h) Two antennas (yellow) with width of about 200 nm are fabricated on the Co nanowires. The center-to-center distance  $d$  between the wires and antennas is 1.5  $\mu\text{m}$ . The scale bar in all images is 500 nm.

Fig. S9(a) shows the transmission spectra  $S_{21}$  of the sample with different wire widths  $w \neq w'$ , excited on the  $w = 60$  nm side and detected on the  $w' = 80$  nm side. The transmission decreases with frequencies, which is consistent with the form factor of  $g_k$  in the main text. The relative Co-Co coupling strength of this case reads  $(g_k / g_0) \times (g'_k / g'_0)$  and is plotted in Fig. S9(c) as a function of wavelength  $\lambda$ . The blue background emphasizes the wavelength range from 56.8 nm to 70.1 nm, which is important for the interpretation of the spectrum in Fig. S9(b).

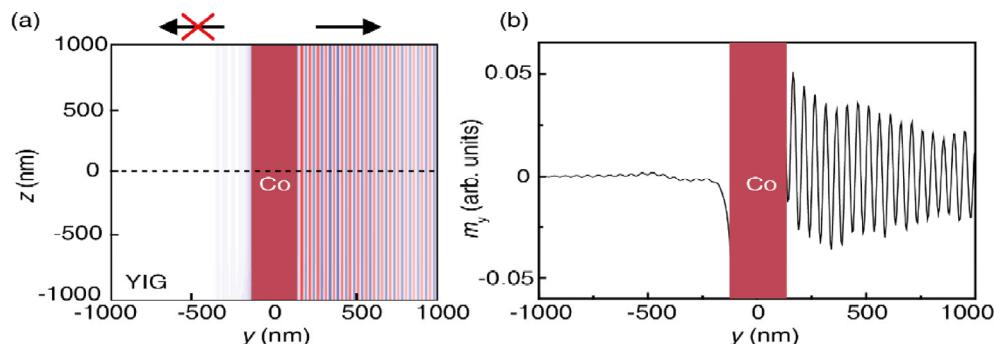
Then, for the same width of two Co nanowires device,  $S_{21}$  spectra of 90 nm wide is presented in Fig. S9(d), in which the red dashed line is along the Co nanowire resonance frequency, 20.1 GHz at 0 mT (spin-wave wavelength is about 56.5 nm) and 17.1 GHz at 64 mT (spin-wave wavelength is about 66.1 nm). Then, as shown in the Fig. S9(e), the lineplot spectrum extracted along the red dashed line in  $S_{21}$ , where the blue dashed line is a guide to the eye. One could find that the tendency of the lineplot along the red dashed line is consistent with the tendency of relative coupling strength from 56.5 nm to 66.1 nm as presented in Fig. S9(f). As shown in Fig. S9(g)-S9(i), we also calculate the relative coupling strength as a function of wavelength from 61.8 nm to 76.0 nm on 100 nm wide Co nanowire device as presented in Fig. 3 in the main text. The lineplot along the red dashed line in Fig. 3(b) from 17.14 GHz at 0 mT and 13.82 GHz at 60 mT is presented in Fig. 3(d) in the main text, from which one can find that the tendency of it is consistent with the relative coupling strength. For the 220 nm wide Co nanowire device, as shown in Fig. S9(j)-S9(l), the Co nanowire resonance frequency is from 12.64 GHz at 0 mT to 15.57 GHz at 34 mT in the  $S_{21}$  spectra (Fig. S9(j)), and relative coupling strength as a function of wavelength from 65.4 nm to 77.8 nm (Fig. S9(l)). These measurements unequivocally present the coupling between the FMR mode of Co wire and the spin-wave mode of YIG film.

### Micromagnetic simulations of chiral coupling and chiral spin waves

Micromagnetic simulations based on the public object-oriented micromagnetic framework (OOMMF, <http://math.nist.gov/oommf>) confirm and help to understand the chiral coupling between Co nanowire and YIG film. The YIG film has the dimensions 20 nm  $\times$  2  $\mu\text{m}$   $\times$  2  $\mu\text{m}$  ( $xyz$ ) and the Co nanowire with dimensions 20 nm  $\times$  200 nm  $\times$  2  $\mu\text{m}$  ( $xyz$ ) is along the  $z$  axis directly on top of the center of the YIG film. The saturation magnetizations  $\mu_0 M_s = 1.5$  T and 0.175 T and the exchange constant are  $13 \times 10^{-12} \text{ J m}^{-1}$  and  $3.7 \times 10^{-12} \text{ J m}^{-1}$  for the Co nanowire and YIG film, respectively. The interlayer exchange coupling is disregarded. The Gilbert damping of the YIG film  $\alpha = 8 \times 10^{-5}$ . An external field of -200 mT in the  $z$  direction is first applied to align the magnetization of Co and YIG into the same direction. Applying subsequently a positive static magnetic field of 20 mT, we establish a stable AP state. An oscillating external field of  $\mu_0 H_{ex} = 0.2 \text{ mT} \times \sin(2\pi ft)$  is applied on the Co nanowire with the frequency  $f = 15$  GHz. The spatial map of the YIG film magnetization after 1 ns is shown in Fig. S10(a) where the region of Co is marked on the figure. The directionality of the exchange spin waves is evident and caused by the chiral magnetodipolar coupling between the Co nanowire and YIG film. A lineplot extracted from Fig. S10(a) is shown in Fig. S10(b).



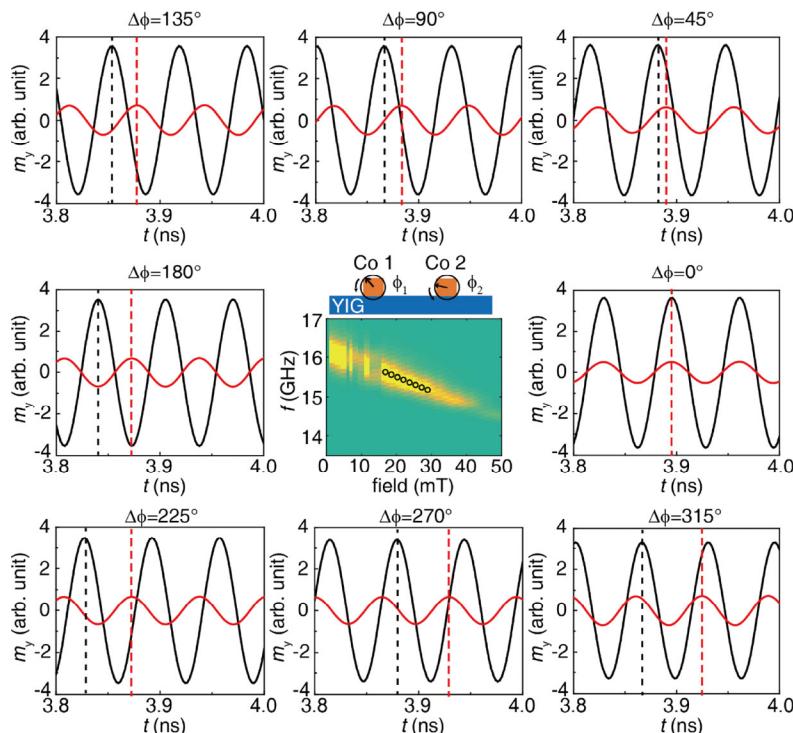
**Figure S9** Coupling between Co Kittel modes via spin waves as a function of the nanowire width  $w$  and wavelength  $\lambda$ . (a) The transmission spectra  $S_{21}$  close to the Co nanowire resonance frequency. (b) A cut of  $S_{21}$  along the red-dashed line of (a), and the relative coupling strength as a function of wavelength when the exciting wire is 60 nm and the detector 80 nm wide. (d), (g), (j), The transmission spectra  $S_{21}$  when both wires are 90 nm (Fig. S9(d)), 100 nm (Fig. S9(g)), and 220 nm (Fig. S9(j)) wide. (e), (h), (k), Along the resonance frequency of Co nanowire (the red dashed line in  $S_{21}$ ), the lineplot spectra are extracted and present in Fig. S9(e) (90 nm wide), Fig. S9(h) (100 nm wide), Fig. S9(k) (220 nm wide). The blue dashed lines in the lineplot spectra indicate a guide to the eye. Base on the form factor of  $g_s$  in the main text, we calculate the relative coupling strength as a function of the spin-wave wavelength. The blue region is the range of spin-wave wavelength extracted from the transmission spectra.



**Figure S10** Exchange spin waves in YIG film excited by the magnetic resonance of a Co wire contact. (a) The spatial map of a snapshot of the  $y$ -component of exchange spin waves 1 ns after switching on the Co FMR with frequency  $f = 15$  GHz. The region covered by the Co nanowire is marked in the figure. (b) Lineplot extracted from (a) at  $z = 0$  nm.

## Micromagnetic simulations of spin-wave coupling between two nanomagnets

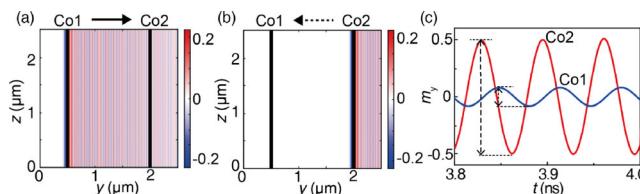
Next, we simulate a sample with two identical Co nanowires with dimensions of  $20\text{ nm} \times 100\text{ nm} \times 90\text{ }\mu\text{m}$  ( $xyz$ ) along the  $z$  axis on top of a YIG thin film with dimensions of  $20\text{ nm} \times 3.5\text{ }\mu\text{m} \times 90\text{ }\mu\text{m}$  ( $xyz$ ). The Co nanowires, Co1 and Co2, have a center-to-center distance of  $1.5\text{ }\mu\text{m}$ . An external field of  $-200\text{ mT}$  is first applied along  $z$  to align the magnetizations of Co and YIG. Next, we establish an equilibrium state by a positive static magnetic field along  $z$  that favors the AP state. An ac magnetic field pulse of the form  $\mu_0 H_{\text{ex}} = 0.2\text{ mT} \times \text{sinc}(2\pi ft)$  is applied in the  $y$  direction on Co1 only. The Fourier transform of the sinc function is rectangular with high-frequency cut-off at  $f$ . The numerical time step is  $20\text{ ps}$  and result are recorded over  $4\text{ ns}$ . The middle image in Fig. S11 is a fast Fourier transform (FFT) of the computed magnetization amplitude  $m_y(t)$  of the Co2 for static applied fields from  $0\text{ mT}$  to  $50\text{ mT}$ . The obtained spectra can be compared with the experimental steady state transmission spectra  $S_{21}$ . The clear resonance mode is the evidence for coupling of nanomagnets via exchange spin waves. In order to obtain the phase information, we apply a periodic AC magnetic field  $\mu_0 H_{\text{ex}} = 0.2\text{ mT} \times \sin(2\pi ft)$  in the  $y$  direction to Co1 with variable  $f$ . For an external field of  $29.4\text{ mT}$  and  $f=15.25\text{ GHz}$ , the phase shift between the Co1 and Co2 dynamics vanishes  $\Delta\Phi=0^\circ$ . By modulating the applied field and frequency, we show in Fig. S11 that the phase difference can be tuned from  $0^\circ$  to  $315^\circ$ .



**Figure S11** Simulated transmission spectra and time-dependent oscillating of two coupled nanomagnets. The centre image shows the simulated transmission spectra of the Co2 from  $0\text{ mT}$  to  $50\text{ mT}$ . The time-dependent oscillations of the Co1 and Co2 with the phase shift  $\Delta\Phi$  from  $0^\circ$  to  $315^\circ$  are shown from  $3.8\text{ ns}$  to  $4.0\text{ ns}$  with the applied field of  $29.4\text{ mT}$  ( $\Delta\Phi=0^\circ$ ),  $27.3\text{ mT}$  ( $\Delta\Phi=45^\circ$ ),  $25.3\text{ mT}$  ( $\Delta\Phi=90^\circ$ ),  $23.3\text{ mT}$  ( $\Delta\Phi=135^\circ$ ),  $21.3\text{ mT}$  ( $\Delta\Phi=180^\circ$ ),  $19.3\text{ mT}$  ( $\Delta\Phi=225^\circ$ ),  $17.3\text{ mT}$  ( $\Delta\Phi=270^\circ$ ) and  $15.3\text{ mT}$  ( $\Delta\Phi=315^\circ$ ), respectively. These eight fields are marked in the transmission spectra by black circles.

## Micromagnetic simulations of crosstalk coupling between two nanomagnets

Next, we discuss the crosstalk coupling between two nanomagnets. A periodic AC magnetic field is applied in the  $y$  direction to Co1 with variable  $f$ . The external field of  $29.4\text{ mT}$  and  $f=15.25\text{ GHz}$  are set and the phase shift between the Co1 and Co2 vanishes. The spatial map is shown in Fig. S12(a) and the Co2 is coupled by exchange spin waves in YIG. In Fig. S12(b), same AC magnetic field is applied to Co2 and the exchange spin waves cannot reach Co1 due to the chiral coupling. The time-dependent oscillations in Co1 (excited by Co2) and Co2 (excited by Co1) is shown in Fig. S12(c) and the coupling via chiral spin waves is about 6.3 times (close to experiments) stronger than that of crosstalk. The origin of the crosstalk may come from the direct dipolar coupling between two nanomagnets.



**Figure S12** Simulated crosstalk coupling. (a) The spatial map of a snapshot of the  $y$ -component of exchange spin waves in YIG  $2\text{ ns}$  after switching on the Co1 FMR with frequency  $f=15.25\text{ GHz}$ . The external field is applied at  $29.4\text{ mT}$ . The region covered by the Co nanowire is marked by black stripes in the figure. (b) The spatial map of a snapshot of the  $y$ -component of exchange spin waves in YIG  $2\text{ ns}$  after switching on the Co2 FMR. (c) The time dependent oscillations of Co1 (excited by Co2) and Co2 (excited by Co1).

## Theory of scattering matrix

The phase relation Eq. S17 affects the microwaves transmission with excitation (input) at  $R_1$  and the detection (output) at  $R_2$ , which we calculate by the input-output theory. The excitation of the left nanowire propagates to the right one by the spin waves in the film. The observed microwave output of the left and right nanowires inductively detected by the stripline antennas are denoted by  $\hat{p}_{out}^{(L)}(\omega)$  and  $\hat{p}_{out}^{(R)}(\omega)$  with input-output relations

$$\begin{aligned}\hat{p}_{out}^{(L)}(\omega) &= p_m^{(L)} + \sqrt{\kappa_{p,L}}\hat{\beta}_L(\omega), \\ \hat{p}_{out}^{(R)}(\omega) &= \sqrt{\kappa_{p,R}}\hat{\beta}_R(\omega),\end{aligned}\quad (\text{S23})$$

where  $\kappa_{p,R}$  is the additional radiative damping induced by the detector. The microwave reflection ( $S_{11}$ ) and transmission ( $S_{21}$ ) amplitudes become

$$\begin{aligned}S_{11}(\omega) &= \frac{\hat{p}_{out}^{(L)}}{\hat{p}_{in}^{(L)}} = 1 - \frac{\kappa_{p,L}}{-i(\omega - \omega_{0,L}) + \frac{\kappa_L + \kappa_{p,L}}{2} + i \sum_k g_{k,L}^2 G_k(\omega) - f(\omega)}, \\ S_{21}(\omega) &= \frac{\hat{p}_{out}^{(R)}}{\hat{p}_{in}^{(L)}} = [1 - S_{11}(\omega)] \sqrt{\frac{\kappa_{p,R}}{\kappa_{p,L}}} \frac{i \sum_k g_{k,L} g_{k,R} G_k(\omega) e^{ik(R_2 - R_1)}}{-i(\omega - \omega_{0,R}) + \frac{\kappa_R + \kappa_{p,R}}{2} + i \sum_k g_{k,R}^2 G_k(\omega)}.\end{aligned}\quad (\text{S24})$$

Similarly, we obtain  $S_{12}$  and  $S_{22}$  when excite the second wire and detect the response at the first wire (note  $f(\omega) = 0$  by chiral interaction):

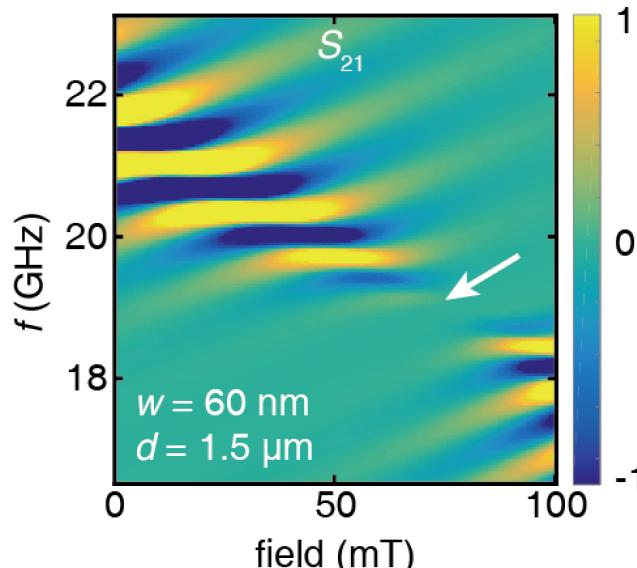
$$\begin{aligned}S_{22}(\omega) &= 1 - \frac{\kappa_{p,R}}{-i(\omega - \omega_{0,R}) + \frac{\kappa_R + \kappa_{p,R}}{2} + i \sum_k g_{k,R}^2 G_k(\omega)}, \\ S_{21}(\omega) &= [1 - S_{22}(\omega)] \sqrt{\frac{\kappa_{p,L}}{\kappa_{p,R}}} \frac{i \sum_k g_{k,R} g_{k,L} G_k(\omega) e^{ik(R_1 - R_2)}}{-i(\omega - \omega_{0,L}) + \frac{\kappa_L + \kappa_{p,L}}{2} + i \sum_k g_{k,L}^2 G_k(\omega)}.\end{aligned}\quad (\text{S25})$$

In particular, then from Eq. S17, we find

$$S_{21}(\omega) = [S_{11}(\omega) - 1] \sqrt{\frac{\kappa_{p,R}}{\kappa_{p,L}}} \frac{\langle \hat{\beta}_R(\omega) \rangle}{\langle \hat{\beta}_L(\omega) \rangle}, \quad (\text{S26})$$

implying that the transmission spectra can measure the phase relation between the Kittel modes in the right and left nanowires.

Based on the information of the 60 nm wide device in Fig. 4(b) (main text) with propagation distance of 1.5  $\mu\text{m}$ , we theoretically calculate the transmission spectra of this device as shown in Fig. S13, which is consistent with the experiment results. For the parameters used in calculation, we adopt the magnetizations  $\mu_0 M_s = 0.177$  T for YIG and 1.67 T for Co. The thickness of YIG film and Co wire is taken to be 20 nm and 30 nm. The exchange stiffness constant is set as  $A = 3 \times 10^{-12} \text{ cm}^2$ .



**Figure S13** Calculated transmission spectra of 60 nm wide device with propagation distance of  $d = 1.5 \mu\text{m}$ . The white arrow shows a valley of amplitude around 19 GHz which is consistent with the observed experiment results in Fig. 4(b) in the main text.

## Theory of non-Hermitian coupling

Starting from the equations of motion Eq. S16, the Markov approximation yields

$$\begin{aligned}\frac{d\hat{\beta}_1}{dt} &= -i\omega_1\hat{\beta}_1 + \sum_k \frac{|g_{k,1}|^2}{i(\omega - \omega_k) - \frac{\kappa_k}{2}} \hat{\beta}_1 + \sum_k \frac{g_{k,1}g_{k,2}^*}{i(\omega - \omega_k) - \frac{\kappa_k}{2}} \hat{\beta}_2 + \hat{P}_1, \\ \frac{d\hat{\beta}_2}{dt} &= -i\omega_2\hat{\beta}_2 + \sum_k \frac{|g_{k,2}|^2}{i(\omega - \omega_k) - \frac{\kappa_k}{2}} \hat{\beta}_2 + \sum_k \frac{g_{k,2}g_{k,1}^*}{i(\omega - \omega_k) - \frac{\kappa_k}{2}} \hat{\beta}_1 + \hat{P}_2,\end{aligned}\quad (\text{S27})$$

which are the coupled equation between  $\hat{\beta}_1$  and  $\hat{\beta}_2$  mediated by the retarded spin waves in the film.  $\hat{P}_1$  and  $\hat{P}_2$  are noise terms from the local antennas felt by the Kittel magnons. By  $g_{k,l} = \tilde{g}_{k,l}e^{ikR_l}$  and with  $R_1 > R_2$ , we obtain the inter-wire coupling

$$\begin{aligned}\Gamma_{12}(\omega) &\equiv -\int \frac{dk}{2\pi} \frac{g_{k,1}g_{k,2}^*}{i(\omega - \omega_k) - \frac{\kappa_k}{2}} = \frac{1}{v(k_\omega)} \tilde{g}_{k_\omega,1}\tilde{g}_{k_\omega,2}^* e^{ik_\omega(R_1-R_2)}, \\ \Gamma_{21}(\omega) &\equiv -\int \frac{dk}{2\pi} \frac{g_{k,2}^*g_{k,1}}{i(\omega - \omega_k) - \frac{\kappa_k}{2}} = \frac{1}{v(k_\omega)} \tilde{g}_{-k_\omega,1}^*\tilde{g}_{-k_\omega,2} e^{ik_\omega(R_1-R_2)},\end{aligned}\quad (\text{S28})$$

and the self-interaction

$$\Gamma_{l=1,2}(\omega) \equiv -\int \frac{dk}{2\pi} \frac{|g_{k,l}|^2}{i(\omega - \omega_k) - \frac{\kappa_k}{2}} \rightarrow \frac{1}{2v(k_\omega)} (|g_{k_\omega,l}|^2 + |g_{-k_\omega,l}|^2). \quad (\text{S29})$$

Here,  $k_\omega$  is the positive root of  $\omega_k = \omega$  and  $v(k_\omega)$  the group velocity. In this equation, a simplified scenario is considered where we disregard the spin-wave attenuation  $e^{-(R_1-R_2)/l_d}$  in terms of the decay  $l_d = v(k_\omega)/(\alpha\omega)$  with  $\alpha = 8 \times 10^{-5}$  being the Gilbert damping of the YIG film as present in Fig. S1. Furthermore, one can estimate that the decay length is about  $180\mu\text{m} \gg |R_1 - R_2| = 1.5\mu\text{m}$  through the experiments of Fig .4 where the group velocity is about  $2\text{ km s}^{-1}$ , and the resonance frequency is about 22 GHz. Generally,  $|\Gamma_{12}(\omega)| \neq |\Gamma_{21}(\omega)|$  as long as  $|g_{k,l}| \neq |g_{-k,l}|$ , i.e. when the coupling is non-reciprocal. For two identical nanowires,  $|\Gamma_{12}(\omega)| = 2\sigma(\omega)$ , i.e., twice of the magnon broadening induced by chiral pumping derived above. In the chiral limit, one of the two couplings vanishes, which means that one magnon affects the other but not the other way around. Such a non-reciprocal interaction cannot be described by a Hermitian Hamiltonian. Even the non-chiral case with  $|\Gamma_{12}(\omega)| = |\Gamma_{21}(\omega)|$  is not trivial since the two magnons can hybridize to form acoustic and optical modes, while the dissipative nature of the coupling causes a level attraction rather than the repulsion associated with a coherent coupling.

The effective non-Hermitian Hamiltonian that via the Heisenberg equation of motion recovers the non-reciprocal dynamics reads

$$\frac{\hat{\mathcal{H}}_{\text{eff}}}{\hbar} = \sum_{i=1}^2 (\omega_i - i\Gamma_i) \hat{\beta}_i^\dagger \hat{\beta}_i - i\Gamma_{12}(\omega) \hat{\beta}_1^\dagger \hat{\beta}_2 - i\Gamma_{21}(\omega) \hat{\beta}_2^\dagger \hat{\beta}_1. \quad (\text{S30})$$

We may divide Eq. S30 into a Hermitian  $\hat{\mathcal{H}}_h$  and non-Hermitian  $\hat{\mathcal{H}}_{nh}$  part:

$$\hat{\mathcal{H}}_{\text{eff}} = \left( \frac{\hat{\mathcal{H}}_{\text{eff}}}{2} + \frac{\hat{\mathcal{H}}_{\text{eff}}^\dagger}{2} \right) + \left( \frac{\hat{\mathcal{H}}_{\text{eff}}}{2} - \frac{\hat{\mathcal{H}}_{\text{eff}}^\dagger}{2} \right), \quad (\text{S31})$$

$$\begin{aligned}\hat{\mathcal{H}}_h &= \sum_l \omega_l \hat{\beta}_l^\dagger \hat{\beta}_l + i(\Gamma_{21}^* - \Gamma_{12}) \hat{\beta}_1 \hat{\beta}_2 + i(\Gamma_{12}^* - \Gamma_{21}) \hat{\beta}_1 \hat{\beta}_2, \\ \hat{\mathcal{H}}_{nh} &= -i \sum_j \Gamma_j \hat{\beta}_j^\dagger \hat{\beta}_j - i \frac{\Gamma_{12} + \Gamma_{21}^*}{2} \hat{\beta}_1^\dagger \hat{\beta}_2 - i \frac{\Gamma_{12}^* + \Gamma_{21}}{2} \hat{\beta}_1 \hat{\beta}_2^\dagger.\end{aligned}\quad (\text{S32})$$

The coupling between magnons has both coherent and dissipative components.