CT561: Systems Modelling & Simulation

Lecture 13: Dimensional Analysis

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https://github.com/JimDuggan/SDMR

Dimensional Analysis

- In the physical sciences and engineering, any equation representing a real-world process needs to have the units (i.e. dimensions) balanced on each side of the '='
- This checking – also known as **dimensional analysis** – is also an important activity in system dynamics, as it provides a validation mechanism for the model being designed.
- As a starting point, the units for system stocks are identified, and examples from a range of modeling are now shown

Stock Units - Example

Application Area	Stock	Units
Business	Inventory	Stock Keeping Unit (SKU)
Financial Planning	Cash	€, $
Education Planning	Students	People
Epidemiology	Infected	People
Demographics	Population	People
Climate Change	Carbon in the Atmosphere	Metric Tons

Table 1.2 Sample stock variables along with indicative values for units

Flow Units

- Stocks change over time through their flows, and therefore, in order to maintain dimensional consistency, a flow must have units of the stock it feeds, *divided by the units in which time is measured* (Coyle 1996).
- The selection of time unit depends on the problem being explored, for example, planning in a higher education context has annual student intakes, therefore the most suitable time unit would be *year*.

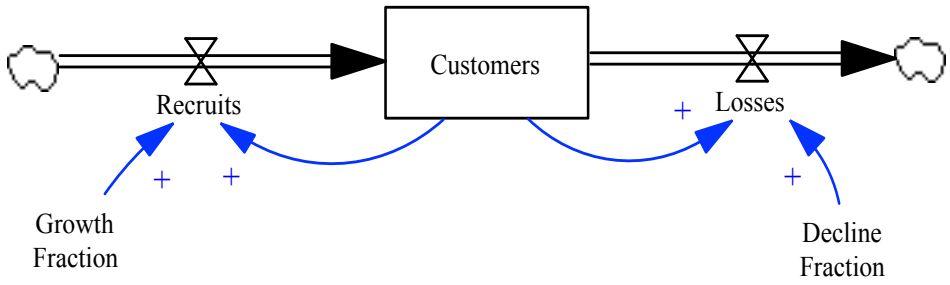
Flow Units - Example

Stock	Inflow	Outflow	Flow Units
Inventory	Arrivals	Shipments	SKU/week
Cash	Deposits	Withdrawals	€/day, $/day
Student	Registrations	Graduations	People/year
Infected	Incidence	Recovery	People/day
Population	Births	Deaths	People/year
Carbon in the Atmosphere	Emissions	Absorptions	Metric Tons/year

Table 1.3 Sample flow variables along with indicative values for units

Dimensional Analysis

- Once the units for stocks and flows are identified, dimensional analysis can be performed, where both sides of an equation are simplified to their basic units.
- If the two sides of the dimensional equation are equal, then the equation is dimensionally consistent



Eulers Equation (the Stock)

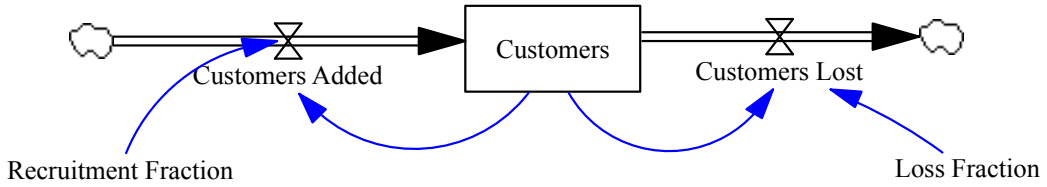
$$Customers_t = Customers_{t-dt} + (Recruits - Losses) * DT$$

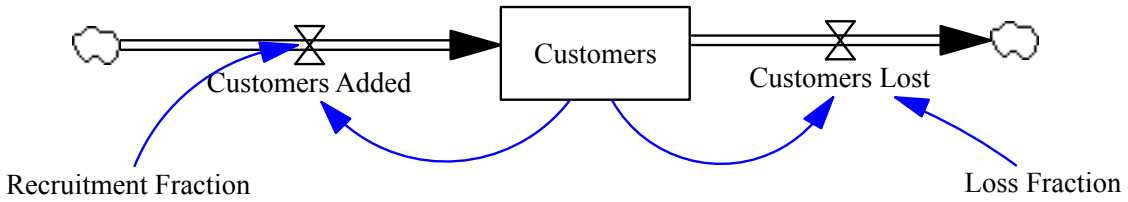
$$\text{people} = \text{people} + (\text{people/year} - \text{people/year}) * \text{year}$$

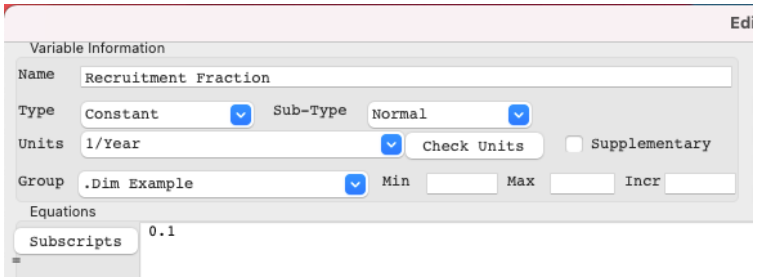
- This equation is dimensionally consistent, as the inflow and outflow denominator (year) cancels with the dimensions of DT (year) to arrive at the dimension (people).

Fraction Increase and Decrease Rates

- The units for these are 1/time







Customers = INTEG (Customers Added-Customers Lost, 1000)
Units: People

Customers Added = Customers*Recruitment Fraction
Units: People/Year

Customers Lost = Customers*Loss Fraction
Units: People/Year

Loss Fraction = 0.03
Units: 1/Year

Recruitment Fraction = 0.1
Units: 1/Year