CT561: Systems Modelling & Simulation

1. Introduction to System Dynamics

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https://github.com/JimDuggan/SDMR

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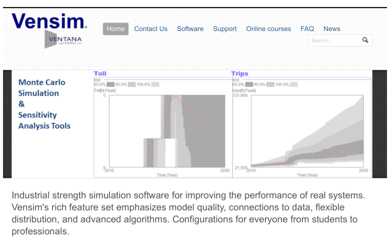
Overview

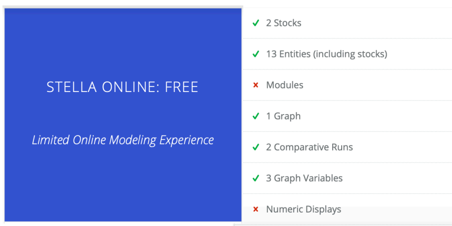
Structure

- Lectures, and Workshops/Tutorials
- 30% Continuous Assessment
  - MCQs
  - Assignment
  - Lab Exam



Software

- https://vensim.com/free-download/#ple
- https://www.iseesystems.com/store/products/stella-online.aspx



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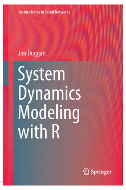
Lecturer – Jim Duggan



- Lectures in
  - Programming (R, MATLAB),
  - Modelling & Simulation
- Research interests:
  - System Dynamics
  - Computational Epidemiology
  - Data Science & Artificial Intelligence

System Dynamics

System dynamics is a modeling methodology used to build simulation models of social systems, and these computerized models can support policy analysis and decision making. This simulation method is based calculus, and models of real-world dynamic processes are constructed using integral equations. The models presented here illustrate the breadth of application of system dynamics, and include:

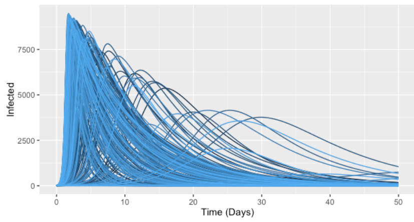
- **Epidemiology**, with a focus on the contagious disease SIR model, and an interesting extension of this to a disaggregate form, based on a vectorized R implementation.
- **Health Systems Design**, which provides a system-wide model comprising population demographics, a supply chain of general practitioners, and a demand-capacity model of general practitioner services to overall population.
- **Economics and Business**, ranging from simple customer model, and onto models of limits to growth, capital investment, and the impact of non-renewable resources on growth.

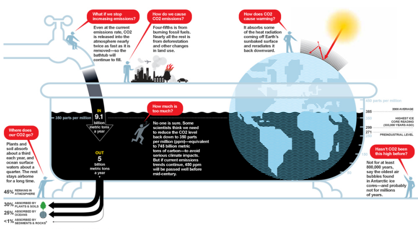
Models are implemented using R and Vensim. The model catalog is as follows:

- Chapter 1 contains the Vensim model of customer growth, with a single stock (Customers), and two flows, an inflow (Recruits) and an outflow (Losses)
- Chapter 2 has an implemention of the customer model in R
- Chapter 3 presents models of (1) S-Shaped growth, (2) Solow's Economic model and (3) a model of overshoot and collapse.
- Chapter 4 introduces a Vensim model for a health systems example, where the model is divided into three distinct sectors
- Chapter 5 contains the SIR model and a vectorised diffusion model.
- Chapter 6 shows how RUnit can be used to test system dynamics models
- Chapter 7 illustrates how statistical screening can be used to analyse system dynamics models

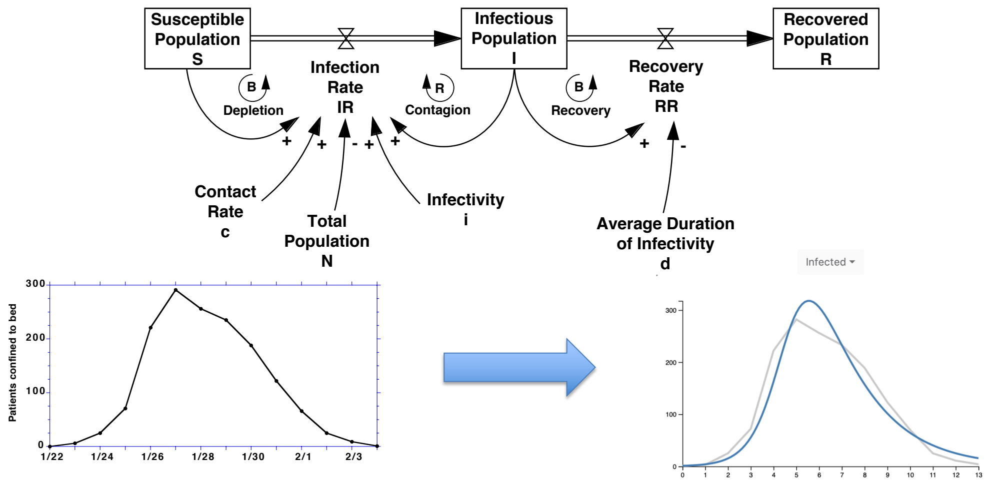
(1) What is a model?

- "An external and explicit representation of part of reality as seen by the people who wish to use that model to understand, to change, to manage and to control that part of reality." Pidd 1996.
- System Dynamics models are *dynamic models* (behaviour over time)





A Dynamic Model



Dynamics of epidemic disease

Sources: British Medical Journal, 4 March 1978, p. 587;.

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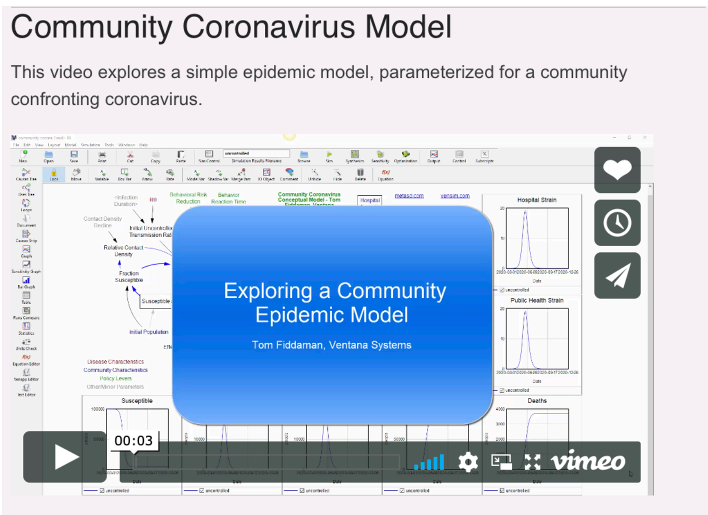
https://vensim.com/coronavirus/



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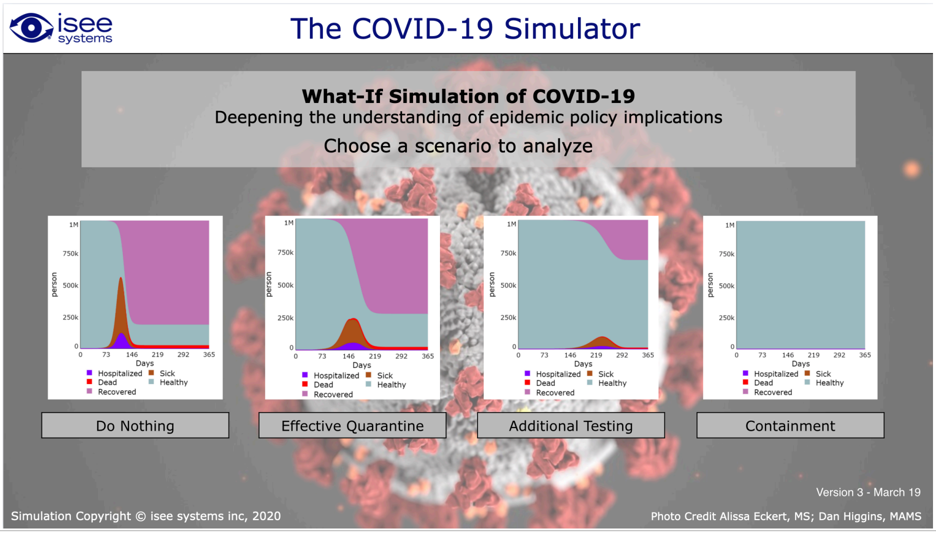
https://exchange.iseesystems.com/public/isee/covid-19-simulator/index.html#page1



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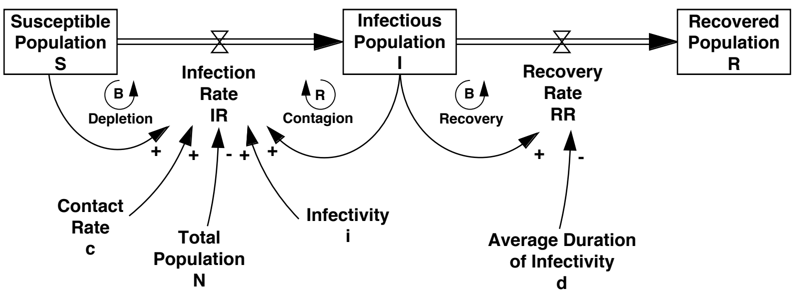
Stocks and Flows

- **Stock** and **flow** variables modeled with stock-flow diagrams
- The stock **accumulates** its inflows to it, less the outflows from it.



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(2) Stocks and Flows

- A **stock** is the foundation of any system.
- **Stocks** are the elements of the system that you can see, feel, count, or measure at any given time.
- A **system stock** is, an accumulation of material or information that has built up over time
- Dimensions are units (litres, people, lines of code)

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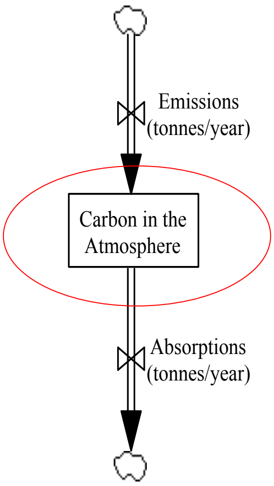
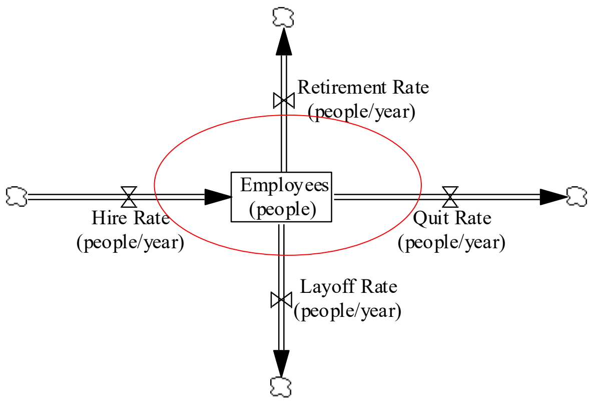
Models: **Stocks** and Flows





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Flows

- Stocks change over time through the actions of a **flow**.
- Flows are:
  - filling and draining,
  - births and deaths,
  - purchases and sales,
  - deposits and withdrawals
  - enrolments and graduations
- Dimensions are units/time period (litres/day, people/year)

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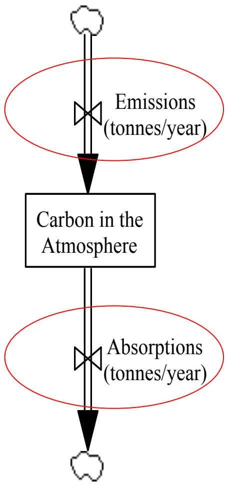
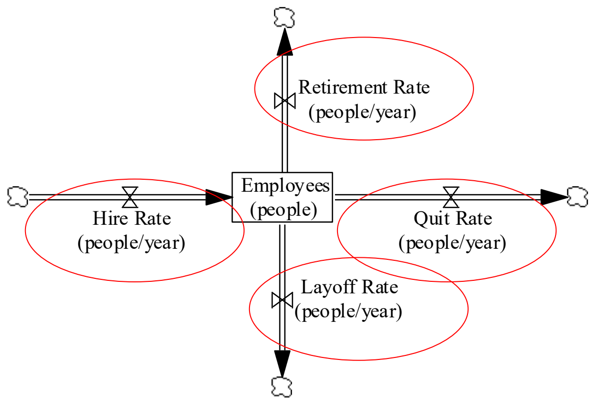
Models: Stocks and **Flows**





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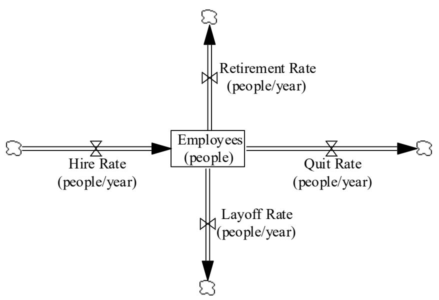
General Principle of Stock/Flow Systems

- From this simple bathtub model you can deduce **several important principles** that extend to more complicated systems:
- As long as the sum of all inflows exceeds the sum of all outflows, the level of the stock will **rise**.
- As long as the sum of all outflows exceeds the sum of all inflows, the level of the stock will **fall**.
- If the sum of all outflows equals the sum of all inflows, the stock level **will not change**; it will be held in dynamic equilibrium at whatever level it happened to be when the two sets of flows became equal.

Meadows, Donella H. *Thinking in systems: A primer*. Chelsea Green Publishing, 2008.



Year	Employees (Jan)	Hires (Jan-Dec)	Retires (Jan-Dec)	Layoff (Jan-Dec)	Quits (Jan-Dec)	Net Flow Jan-Dec
2018	1000	100	20	0	50	30
2019	1030	50	5	0	15	30
2020	1060	30	0	0	0	30
2021	1090					

The stock (Employees) ***accumulates*** *its inflows, less the outflows.*

Summary

- **Systems thinkers see the world as a collection of stocks along with the mechanisms for regulating the levels in the stocks by manipulating flows.**

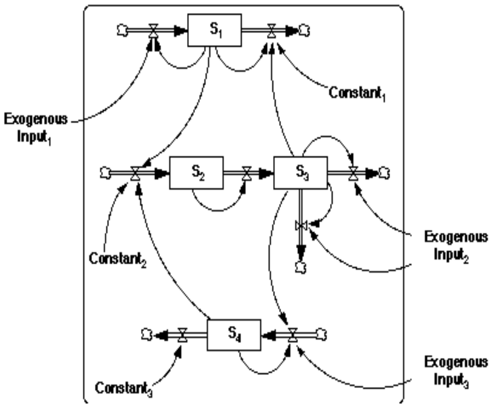
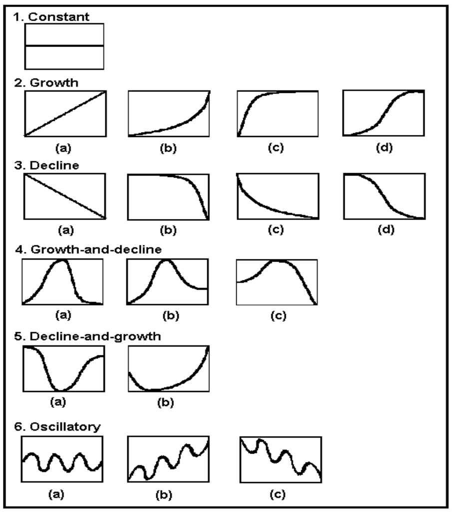


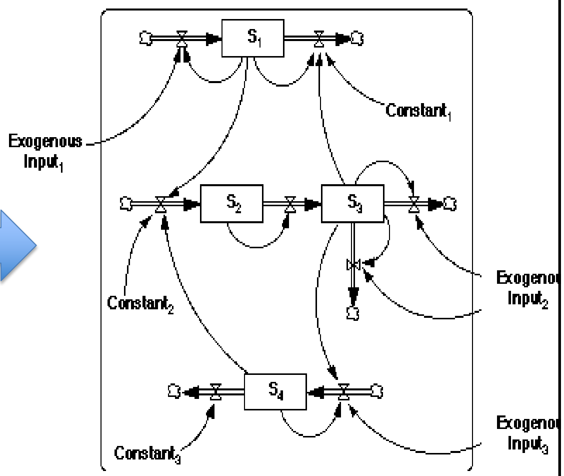
Diagram source: J.D. Sterman,Business Dynamics: Copyright © 2001 by the McGraw-Hill Companies

Challenge 1.1

- For each of the following variables, identify the stock, the flows (distinguish between inflows and outflows), the units for each variable, and the most likely time horizon for a simulation model representation.
  - Enrollments, Students, Graduations
  - Account Balance, Credits, Debits
  - Retirements, Staff, Recruitments, Total Staff Retired
  - Absorptions, Emissions, Carbon in the Atmosphere
  - People Entering, People Leaving, People in the Store
  - Water in the Lake, Rainfall, Evaporations
  - Customers Joining, Customers, Customers Leaving
  - People Infected, New Infections, People Recovering, People Recovered
- Build a simple model in Excel.

Categories of Dynamic Behaviour Patterns (Barlas 2019)





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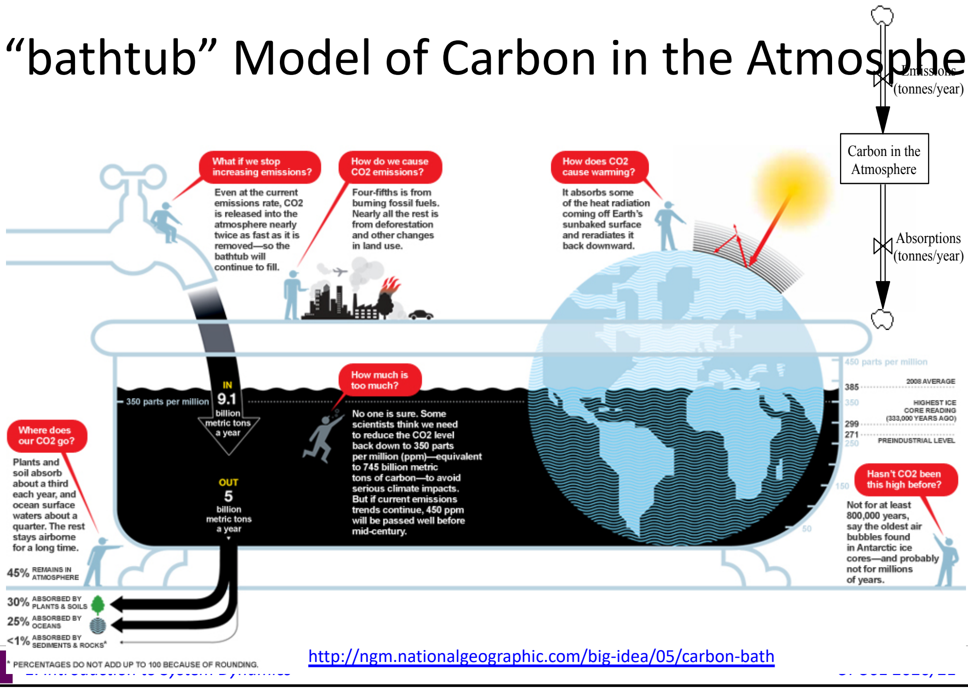
A "bathtub" Model of Carbon in the Atmosphere



http://ngm.nationalgeographic.com/big-idea/05/carbon-bath

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CT561: Systems Modelling & Simulation

2: Integration – Graphical and Numerical

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https://github.com/JimDuggan/SDMR

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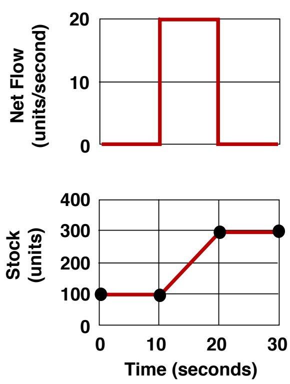
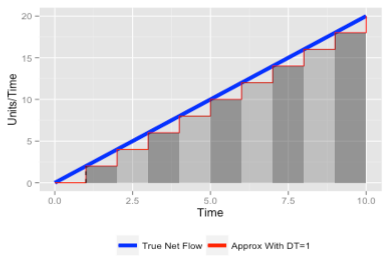
Lecture Topics

- Stocks and Flows Recap
- Integration
  - Graphical
  - Numerical (Euler's Equation)
- Useful flow formulations
  - Fractional increase
  - Fractional decrease





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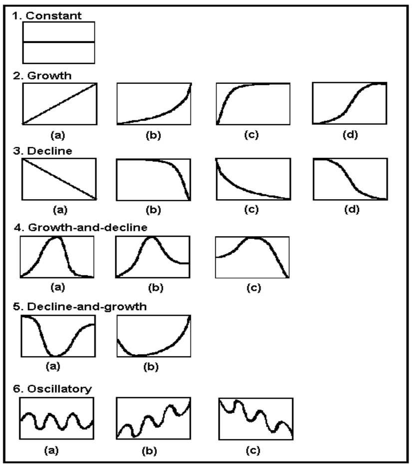
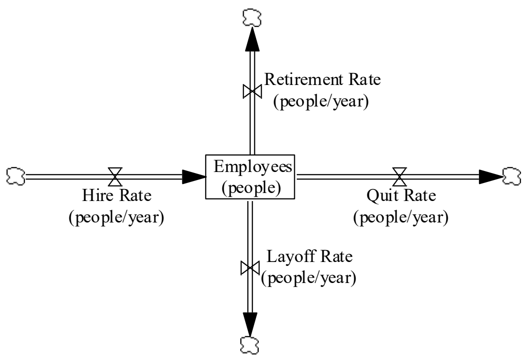
Stocks and Flows

- Enrollments, **Students**, Graduations
- **Account Balance**, Credits, Debits
- Retirements, **Staff**, Recruitments, **Total Staff Retired**
- Absorptions, Emissions, **Carbon in the Atmosphere**
- People Entering, People Leaving, **People in the Store**
- **Water in the Lake**, Rainfall, Evaportions
- Customers Joining, **Customers**, Customers Leaving
- **People Infected**, New Infections, People Recovering, **People Recovered**



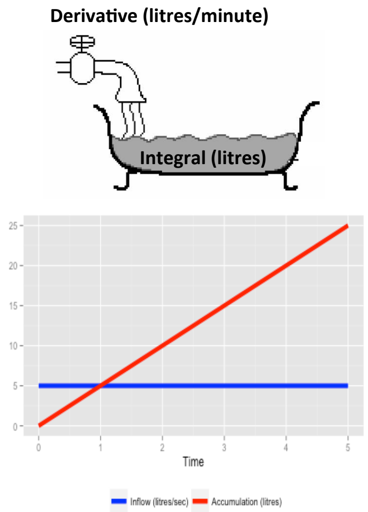
Stock and Flow Systems

- All stock-flow systems share the same underlying structure.
- The stock **accumulates** its inflows to it, less the outflows from it.
- This is a fundamental concept of calculus (integrals and derivatives)



Calculus – Integration

- Calculus is the study of how things **change over time**, and is described by Strogratz (2009) as "*perhaps the greatest idea that humanity has ever had.*"
- **Given the dynamics of the flows, what is the behaviour of the stock?**
- **Integration is the mathematical process of calculating the area under the net flow curve, between initial and final times.**



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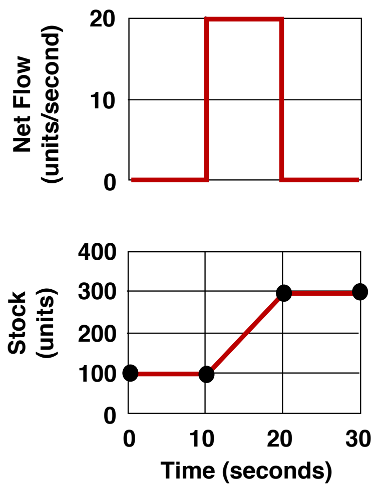
Graphical Integration

- Stocks accumulate their net flow
- The quantity added to a stock over any interval is the area bounded by the graph of the net rate between the start and end of the interval.
- Net Flow = Inflows - Outflows



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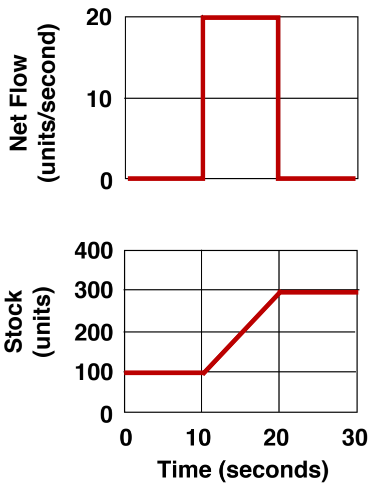
Graphical Integration (1/8)

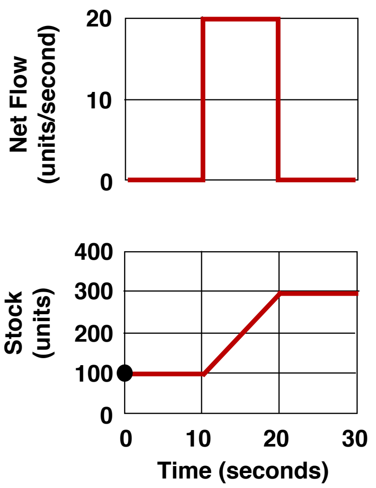
Make a set of axes to graph the stock. Stocks (units) and flows (units per time period) have different units of measure, and must be graphed on different scales.

Make a separate graph for the stock under the graph for the flows, with the time axes lined up.



Graphical Integration (2/8)

Plot the initial value of the stock on the stock graph. The initial value MUST be specified.

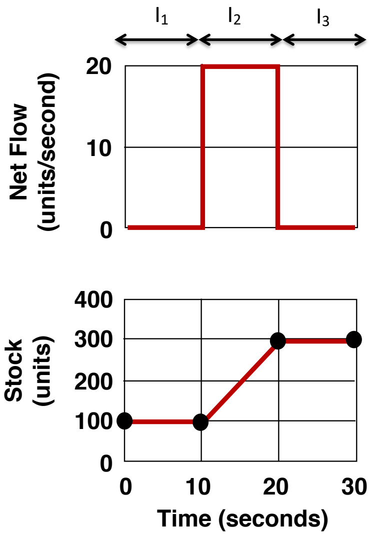


Graphical Integration (3/8)

Break the net flow into intervals with the same behaviour and calculate the amount added to the stock during the interval.

The amount added or subtracted to the stock during an interval is the area under the net rate curve for that same interval.

The total area is then added to the original value of the stock, and this point is then plotted on the stock graph.



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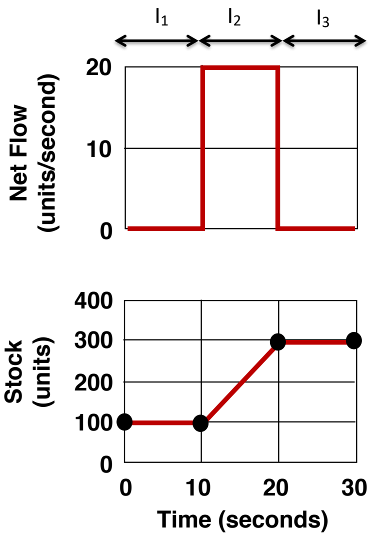
Graphical Integration (4/8)

(6) Sketch the trajectory of the stock between the start and end of each interval.

Find the value of the net rate at the start of the interval.

If the net rate is positive, the stock will be increasing at that time, if it is negative, the stock will be decreasing.



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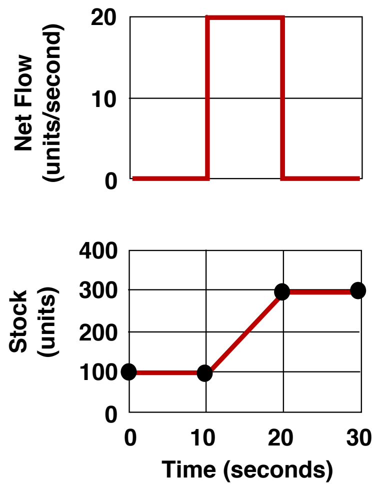
Graphical Integration (5/8)

The behaviour of the stock can be inferred from the net flow according to the following rules:

- If the net rate is positive and increasing, the stock increases at an increasing rate (the stock accelerates upwards)
- If the net rate is positive and decreasing, the stock increases at a decreasing rate (the stock is decelerating but still moving upwards)



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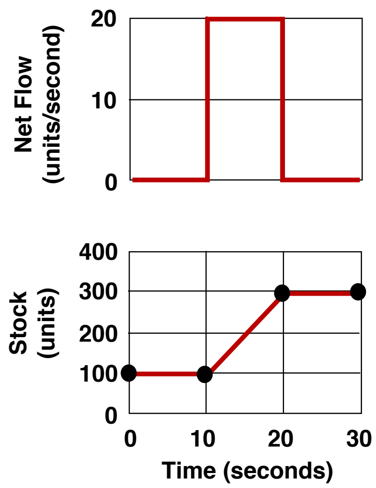
Graphical Integration (6/8)

The behaviour of the stock can be inferred from the net flow according to the following rules:

- If the net rate is negative and its magnitude is increasing (the net rate is becoming more negative), the stock decreases at an increasing rate.
- If the net rate is negative and its magnitude is decreasing (becoming less negative), then the stock decreases at a decreasing rate.



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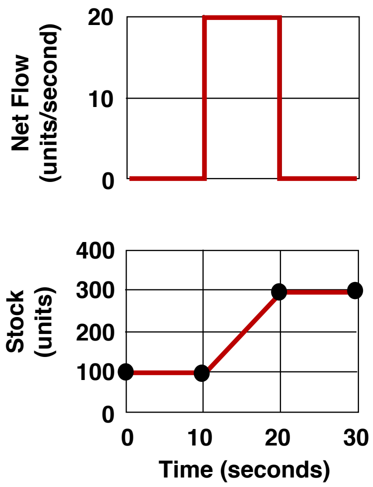
Graphical Integration (7/8)

The behaviour of the stock can be inferred from the net flow according to the following rules:

- If the net rate is positive and its magnitude is constant, the stock increases at an constant rate.
- If the net rate is negative and its magnitude is constant, the stock decreases at an constant rate.



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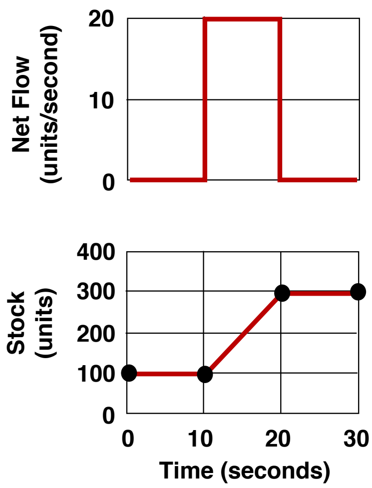
Graphical Integration (8/8)

8. Whenever the net rate is zero, the stock is unchanging. Make sure that your graph of the stock shows no change in the stock everywhere the net rate is zero. At points where the net rate changes from positive to negative, the stock reaches a maximum as it ceases to rise and starts to fall. At points where the net rate changes from negative to positive, the stock reaches a minimum as it ceases to fall and starts to rise.
9. Repeat steps 5 through 8 until completion.



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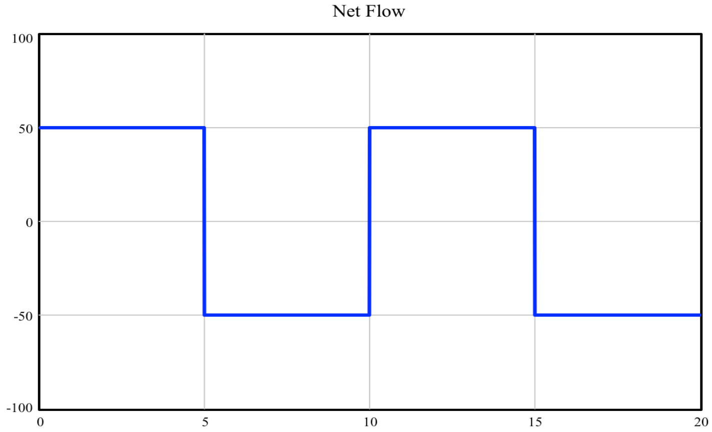
Challenge 2.1

Graphically integrate this net flow.

Assume the starting value of the stock is 100.



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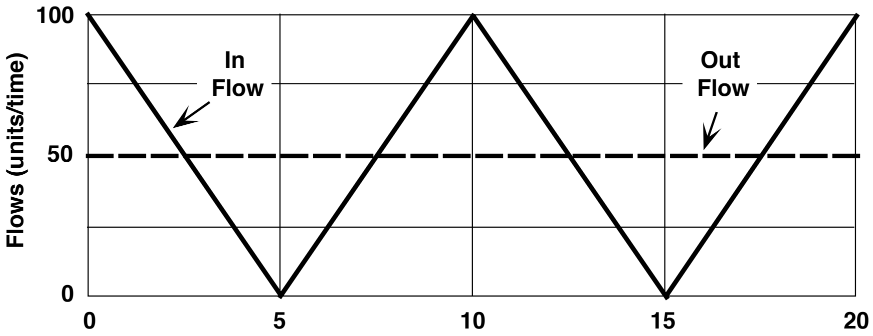
Challenge 2.2

Graphically integrate the net flow.

Assume the starting value of the stock is 100.



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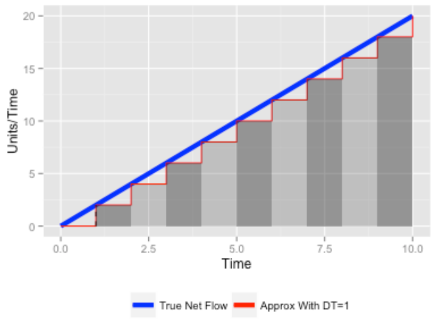
Numerical Integration

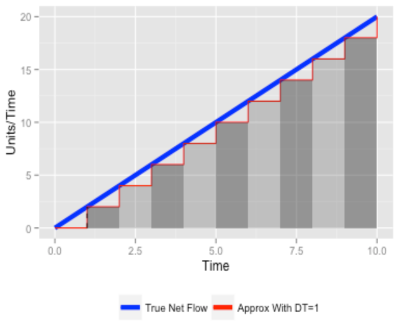
- Euler's Method
- Approximate area under the net flow curve as a summation of rectangles, of width DT
- The smaller DT, the more accurate the result



$$S_t = S_{t-dt} + NF_{t-dt} \times DT$$

Solution, DT=1

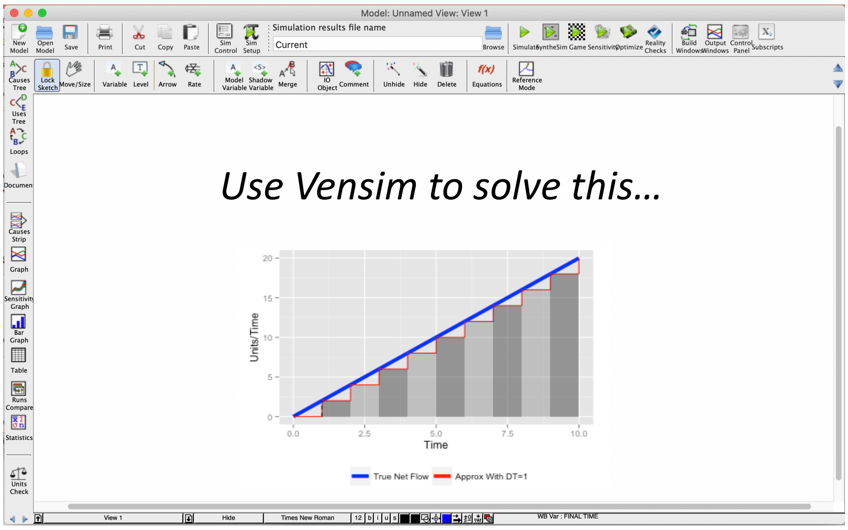
Time	$Stock_t$	Net Flow
0	0	0
1	0+0=0	2
2	0+2=2	4
3	2+4=6	6
4	6+6=12	8
5	12+8=20	10
6	20+10=30	12
7	30+12=42	14
8	42+14=56	16
9	56+16=72	18
10	72+18=90	20



$$S_t = S_{t-dt} + NF_{t-dt} \times DT$$

Note: Stock only depends on previous stock and net flows

Challenge 2.3

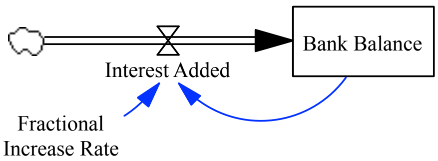


Formulating Flows

- Stocks change over time through the actions of a **flow**.
- Basic flow types:
  – Fractional increase
  – Fractional decrease
- Flow depends on the stock and a constant value (increase or decrease fraction)

Fractional Increase Rate

- Consider a stock *S* with inflow rate R_I
- The inflow is proportional to the size of *S*
- The fractional increase rate is a constant *g*



Bank Balance = INTEG(Interest Added , 1000)

Fractional Increase Rate = 0.01

Interest Added = Bank Balance * Fractional Increase Rate

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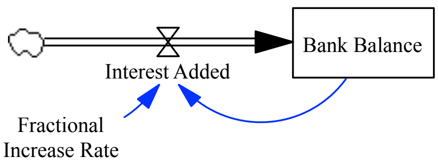
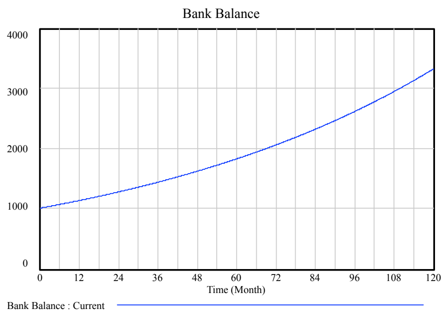
System behaviour
Generates Exponential growth





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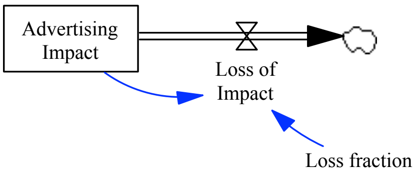
Fractional Decrease Rate

- Consider a stock S with outflow rate R_O
- The outflow is proportional to the size of S
- The fractional decrease rate is a constant d

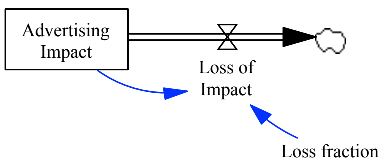


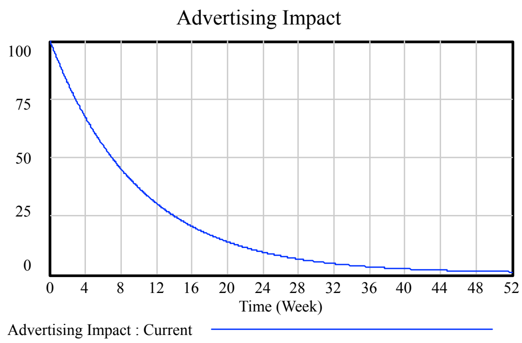
Advertising Impact = INTEG(- Loss of Impact , 100)

Loss fraction = 0.1

Loss of Impact = Loss fraction * Advertising Impact

System behaviour
Generates Exponential decay

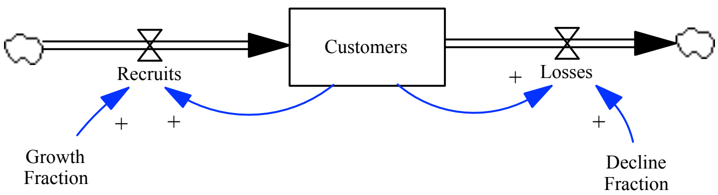




A simple model of Customers

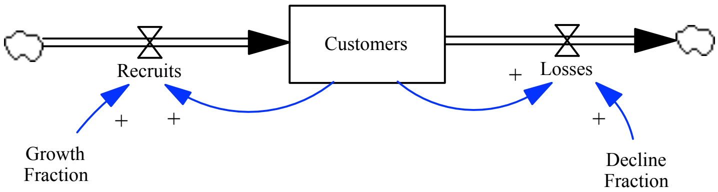
- Given that the customer base is an accumulation, it can be modeled as a stock (assume = 10,000)
- The inflow is recruits, and the outflow are losses, also known as the churn rate.
- The goal of organizations is to limit the losses and maximize the recruits, in order to maintain increasing customers levels, and therefore support company growth.

Stock and Flow Model



Customers= INTEG (Recruits-Losses, 10000)

Flow equations



Recruits = Customers*Growth Fraction
Losses = Customers*Decline Fraction

Decline Fraction=0.03
Growth Fraction=0.07-step(0.04,2020)-step(0.01,2025)

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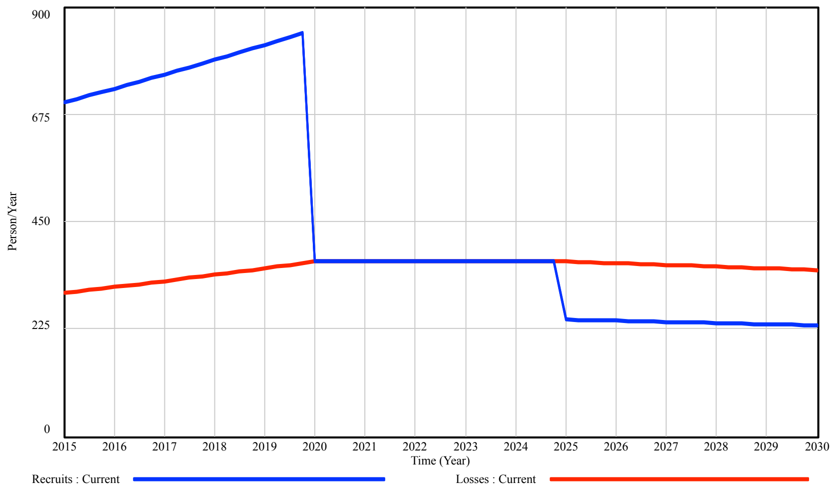
Simulation – Flows



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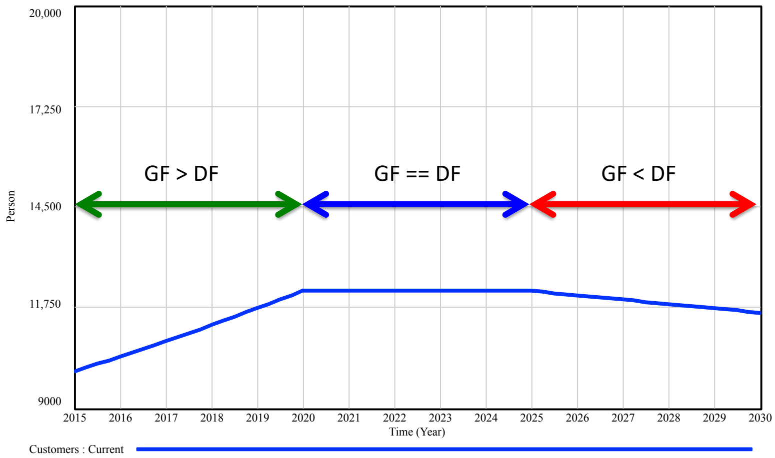
Stock: 3 Phases of behaviour



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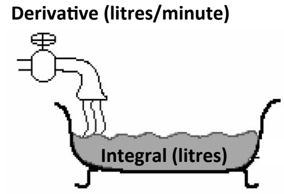
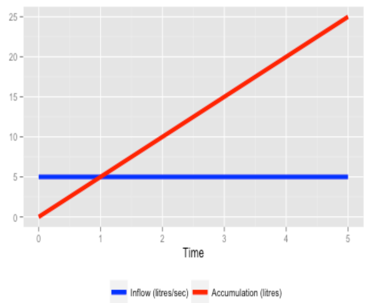
Calculus – Integration

- Calculus is the study of how things **change over time**, and is described by Strogratz (2009) as *"perhaps the greatest idea that humanity has ever had."*
- **Given the dynamics of the flows, what is the behaviour of the stock?**
- **Integration is the mathematical process of calculating the area under the net flow curve, between initial and final times.**





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Challenge 2.4

- A University attracts 30% of its total students as new students each year, and has an initial population of 1000
- It graduates 25% of all students
- For this:
  - Draw a stock and flow model
  - Add the net flow to the model
  - Formulate the equations
  - Build a model in Vensim with DT=0.25
  - Start the model in 2020, and complete in 2030

CT561: Systems Modelling & Simulation

3. Introduction to Feedback

Prof. Jim Duggan,
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https://github.com/JimDuggan/SDMR

1

Challenge 2.4

- A University attracts 30% of its total students as new students each year, and has an initial population of 1000
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  - Formulate the equations
  - Build a model in Vensim with DT=0.25
  - Start the model in 2020, and complete in 2030

2

Recap

- **Systems thinkers see the world as a collection of stocks along with the mechanisms for regulating the levels in the stocks by manipulating flows.**

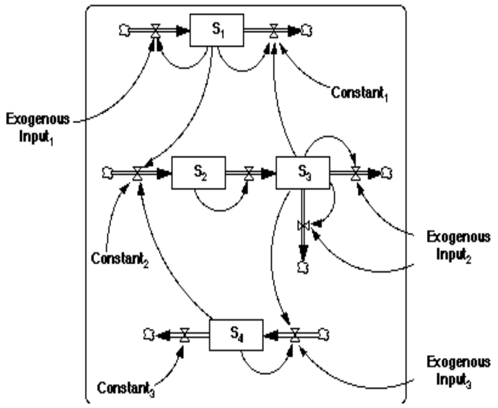
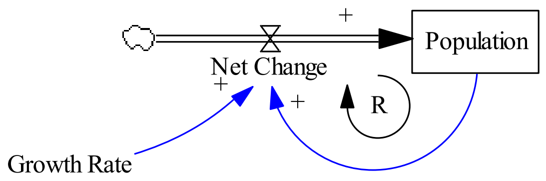


Diagram source: J.D. Sterman,Business Dynamics: Copyright © 2001 by the McGraw-Hill Companies

Causality and Feedback

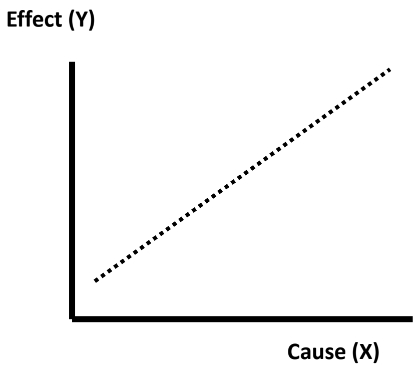
- Exploring causal relationships
  - Link polarity
  - Loop polarity
- Feedback loops
  - Positive
  - Negative



Link polarity – Positive Link

A positive link means that:

- if the cause **increases**, the effect **increases** *above what it otherwise would have been,* and
- if the cause **decreases**, the effect **decreases** below what it would otherwise have been.



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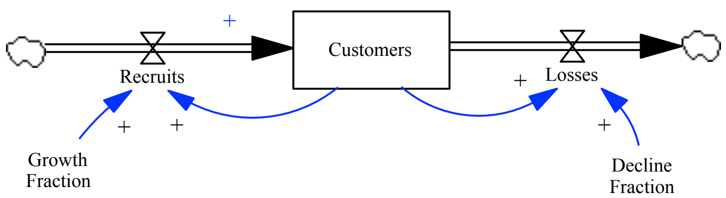
Examples



Recruits	↑	Customers	↑

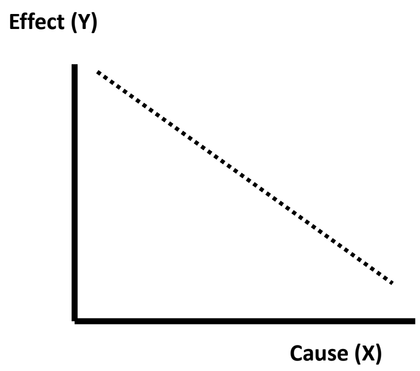
The variables move in the same direction…

6

Link polarity – Negative Link

A negative link means that:

- if the cause **increases**, the effect **decreases** *below what it would otherwise have been*, and
- if the cause **decreases**, the effect **increases** *above what it might otherwise have been*.



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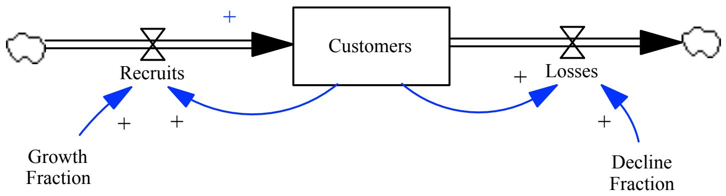
Example



Losses	↑	Customers	↓

The variables move in opposite directions…

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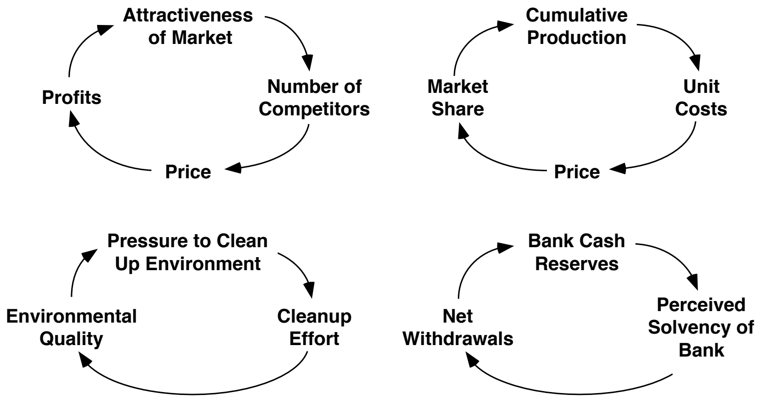
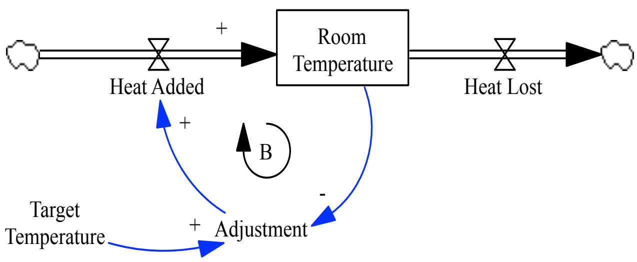
Challenge 3.1

- Add links to the following diagrams



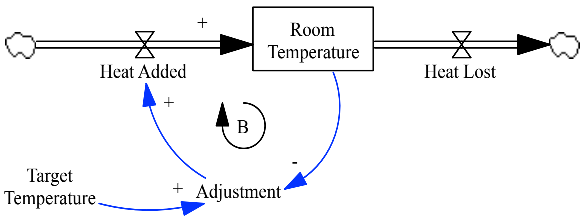
Feedback - Definition

A closed chain of causal connections from a stock, through a set of decisions or rules or physical laws or actions that are dependent on the level of the stock, and back again through a flow to change the stock.



Loop Polarity

The loop is broken down into a set of the causal links, and the impact of a change in one variable is traced through the causal chain, and back to the original variable.



Room Temperature	↓	Adjustment	↑
Adjustment	↑	Heat Added	↑
Heat Added	↑	**Room Temperature**	↑

Calculating Loop Polarity?

The Fast Way

- Count the number of negative links in the loop
- If this number is even (including zero)
  - Positive Feedback
- If this number is odd
  - Negative Feedback

The Correct Way

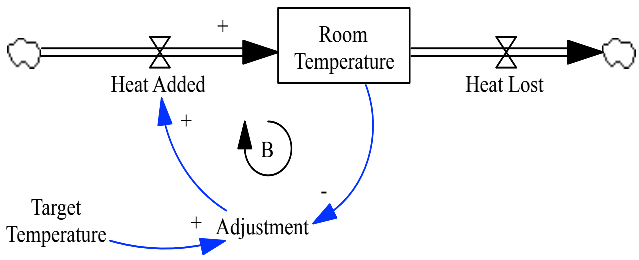
- Trace the effect of a small change in one of the variables as it propagates around the loop
- If the loop reinforces the original change, it's a positive loop
- If it opposes the original change, it's a negative loop

Balancing Loop

Balancing feedback loops are goal-seeking structures in systems and are:

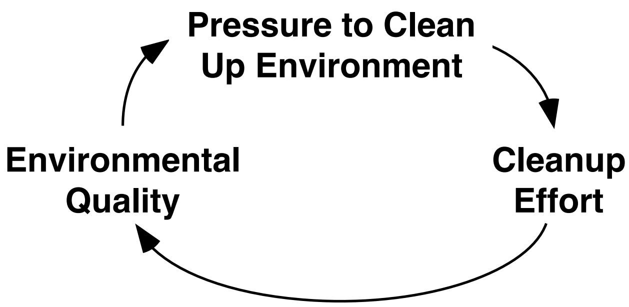
– sources of stability and

– sources of resistance to change.

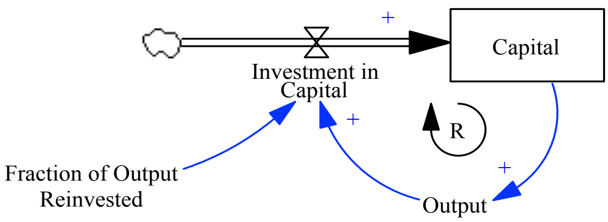


Challenge 3.2

Calculate Link and Loop Polarity



Another type of feedback…



Capital	↑	Output	↑
Output	↑	Investment in Capital	↑
Investment in Capital	↑	**Capital**	↑

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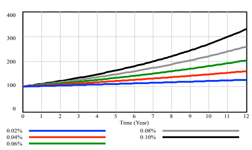
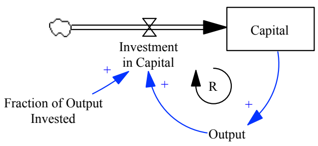
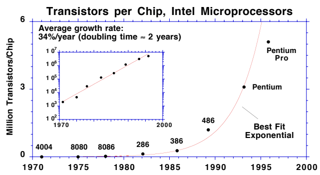
Reinforcing Loops

- **"Reinforcing feedback loops are self-enhancing, leading to exponential growth or to runaway collapses over time.**
- They are found whenever a stock has the capacity to reinforce or reproduce itself."







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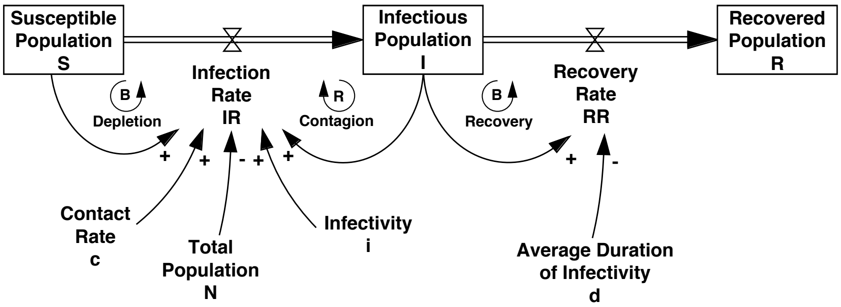
Infectious Disease Example



Cause	Direction	Effect	Direction
Infectious Population	↑	Infection Rate	↑
Infection Rate	↑	Infectious Population	↑

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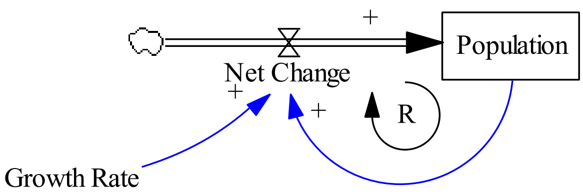
Population Growth Example

Cause	Direction	Effect	Direction
Population	↑	Net Change	↑
Net Change	↑	Population	↑



"The second kind of feedback loop is amplifying, reinforcing, self-multiplying, snowballing—a vicious or virtuous circle that can cause healthy growth or runaway destruction." Meadows (2008)

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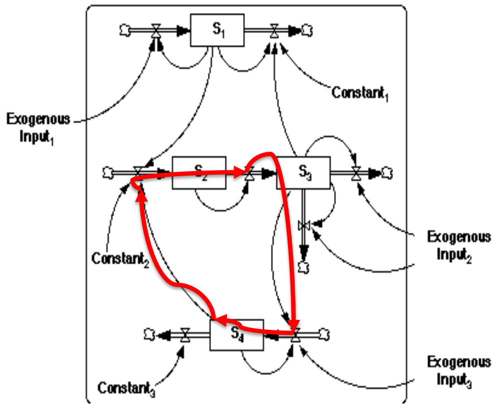
Positive Feedback (Sterman 2000)

Bill Gates quotes…

- "The biggest advantage we have is that good developers like to work with good developers."
- "The growth [Windows NT] continues to amaze us and it's a positive feedback loop. As we got more applications, NT Servers get more popular. As it's gotten more popular, we've got more applications."



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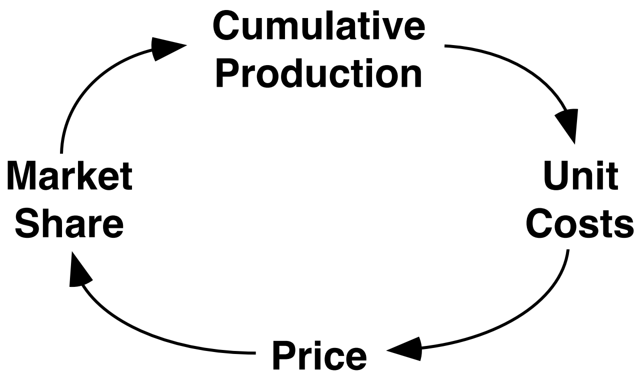
Challenge 3.3

Calculate Link and Loop Polarity



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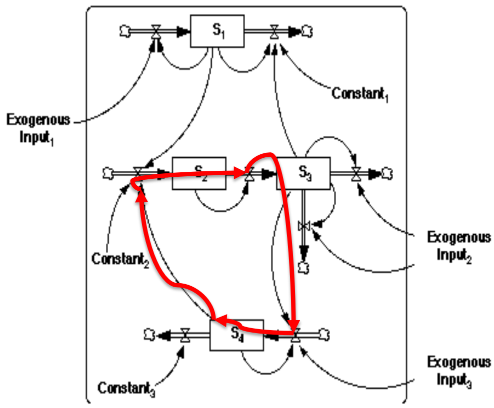
Feedback Summary

- A complex system is an interlocking structure of feedback loops, and this loop structure is found many real-world processes (Forrester 1969).
- A feedback loop is a closed chain of causal links from a stock, through a flow, and back to the original stock again.
- There are two classes of feedback loops. **Negative feedback** counteracts the direction of change, whereas **positive feedback** amplifies change and drives exponential growth.
- Loop polarity is calculated by examining the individual link polarities in a circular causal chain. If there are an odd number of negative links, the loop polarity in negative, otherwise the loop polarity is positive.



Challenge 3.3 – Part (a)

Construct a stock and flow model from the following description of student workload.

- The *assignment backlog* is increased by *new assignments* and reduced by *completions*.
- *Additional rework* also increases the assignment backlog.
- As the *assignment backlog* increases, so too does the *work pressure*
- There are two student responses to increasing work pressure:
  - The *time per assignment* is reduced
  - The *workweek* is increased
- As the time per assignment increases:
  - The *completions* reduce
  - The *Additional rework* reduces
- As the workweek increases:
  - *Completions* increase
  - *Fatigue* increases
- As *fatigue* increases, so too does *time per assignment*.

Part (b)

- Show the feedback loops and calculate their polarity, by tracing an increase in a variable all the way around a feedback loop, and observing its direction of change.

- Discuss how the feedback loops can help identify the implications of leaving assignments to the last possible minute, rather than working on them in a consistent way throughout a semester.

Challenge 3.4

Construct a stock and flow model from the following description of an insurance claims work process:

- *Claims* (the stock) are increased by the *Arrival Rate* and reduced by the *Completion Rate*.
- As *Claims* increase, so to does the *Schedule Pressure*
- In response to increasing *Schedule Pressure*, *Overtime* is increased
- As *Overtime* increases, so too does the *Completion Rate*.
- An increase in *Overtime* leads (after a delay) to increased *Fatigue*.
- Increased *Fatigue* reduces the *Completion Rate.*

Part(b)

Based on the stock and flow model in part(a):

- Show the feedback loops and calculate their polarity, by tracing an increase in a variable all the way around a feedback loop, and observing its direction of change.
- Discuss how the feedback loops can help identify the consequences of having a higher *Arrival Rate* than *Completion Rate.*

CT561: Systems Modelling & Simulation

4. Additional Flows and Higher Order Stock Systems

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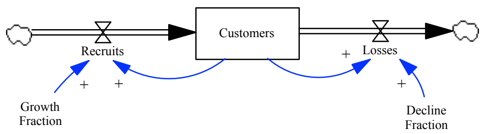
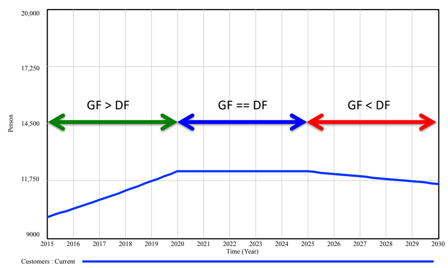
https://github.com/JimDuggan/SDMR

1

Summary to date

- Stocks and Flows
- Feedback
- Integration: Graphical and Numerical Euler's equation (spreadsheet and Vensim)
- One stock examples
  - Customer growth
  - Bank Balance
  - World Population





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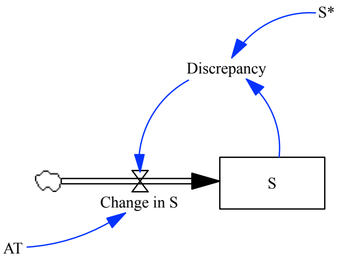
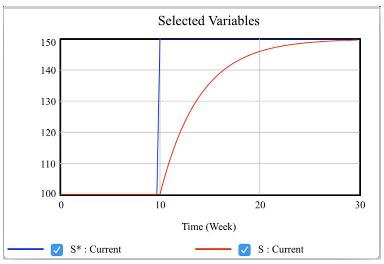
(3) Formulating flows: *adjustment to a goal*

- Managers often seek to adjust the state of the system until it equals a goal or desired state.
- The simplest form of this negative feedback is
  - R_I = Discrepancy/AT = $(S^* - S)$/AT





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Observations on Goal Adjustment

- "**Desired** minus **actual** over **adjustment time**" is the classic linear negative feedback system. (Sterman 2000).
- Examples:
  - **Change in Price** = (Competitor Price – Price)/Price Adjustment Time
  - **Heat Loss from Building** = (Outside Temperature – Inside Temperature)/Temperature Adjustment Time
  - **Net Hiring Rate** = (Desired Labour – Labour)/Hiring Delay

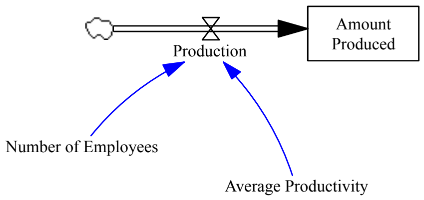
4

Challenge 4.1

- For each of the following, build a one-stock model and show the impact three different adjustment times have on the stock variable.
  - **Net Hiring Rate** = (Desired Labour – Labour)/Hiring Delay
  - **Change in Price** = (Competitor Price – Price)/Price Adjustment Time
  - **Heat Loss from Building** = (Outside Temperature – Inside Temperature)/Temperature Adjustment Time

(4) Flow = Resource * Productivity

- The flows affecting a stock frequently depend on resources other than the stock itself
- The rate is determined by a resource and the productivity of that resource
- Can be applied to an inflow or outflow
- Rate = Resource * Productivity
- Production = Labour Force * Average Productivity



Amount Produced = INTEG(Production , 0)
Units: Units

Average Productivity = 10
*Units: Units/(People*Day)*

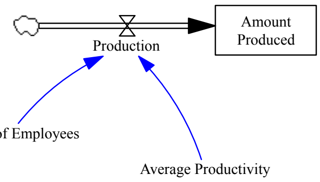
Number of Employees = 100
Units: People

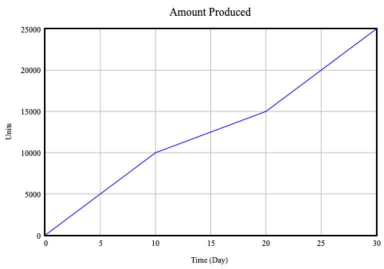
Production = Average Productivity * Number of Employees
Units: Units/Day

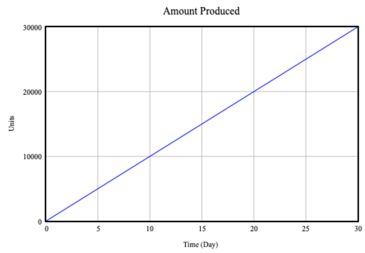
Possible Scenarios

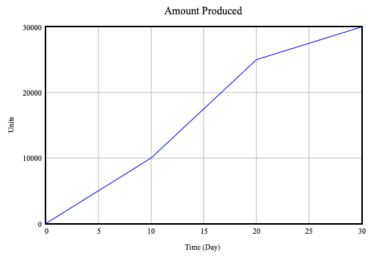
Average Productivity = 10
*Units: Units/(People*Day)*

Number of Employees = 100
Units: People









Number of Employees=100-step(50,10)+step(50,20)

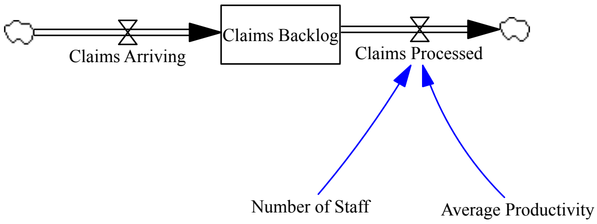
Average Productivity=10+step(5,10)-step(10,20)

Challenge 4.2

- Build a resource/productivity flow equation for *Vaccines Dispensed*
- Variables include *Health Care Worker* and *Health Care Worker Productivity*
- Extend the model to include Vaccines as a resource that gets depleted every time a vaccine is dispensed. Ensure that vaccines cannot be dispensed unless there is a vaccine in stock.

Flow = Resource * Productivity

- Rate = Resource * Productivity
- Can be applied to an outflow
- Example:
  - Insurance Claims Backlog
  - Claims Arriving
  - Claims Completed
    - Number of Staff
    - Productivity (claims/person/day)



Average Productivity = 2

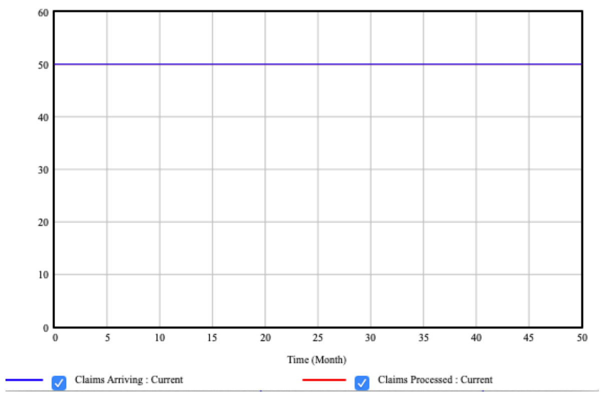
Claims Arriving = 50

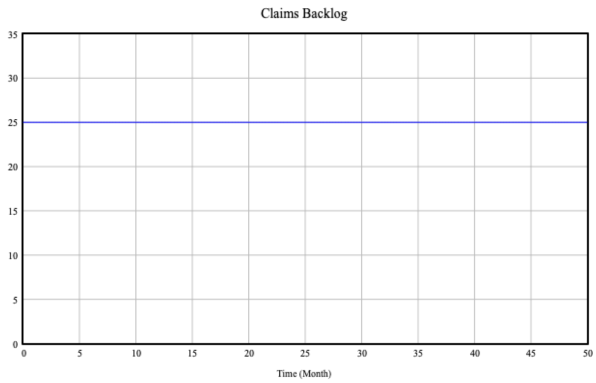
Claims Backlog = INTEG(Claims Arriving - Claims Processed , 25)

Claims Processed = Average Productivity * Number of Staff

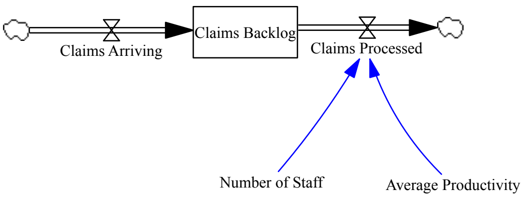
Number of Staff = 25

Model Behaviour – Dynamic Equilibrium





New Scenario: What will happen?



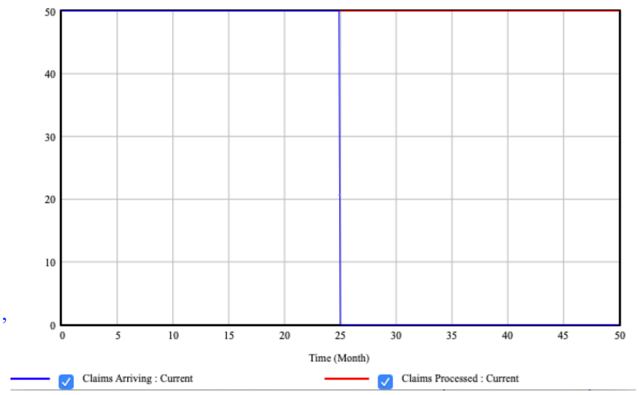
Average Productivity = 2

Claims Arriving = 50 – step(50,25)

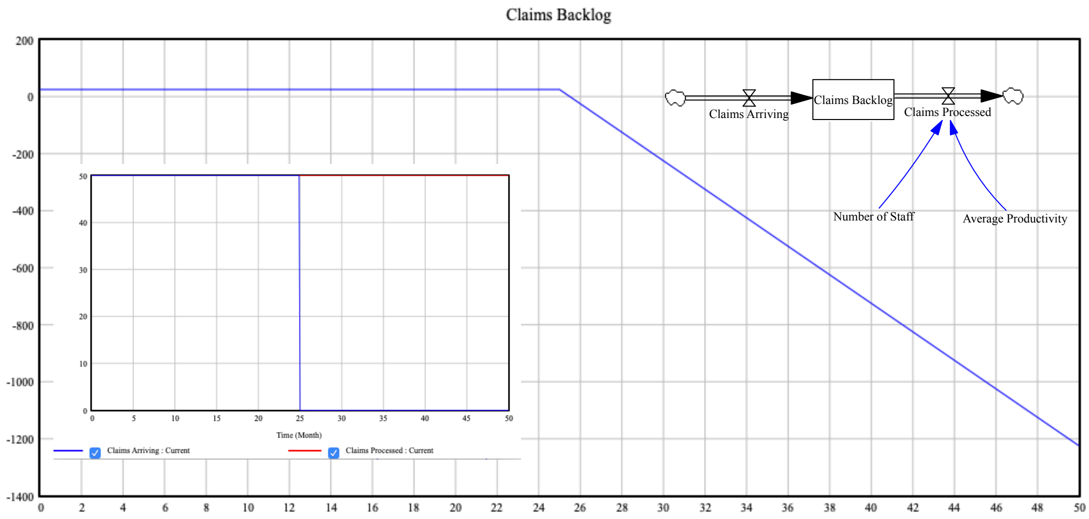
Claims Backlog = INTEG(Claims Arriving - Claims Processed ,

Claims Processed = Average Productivity * Number of Staff

Number of Staff = 25

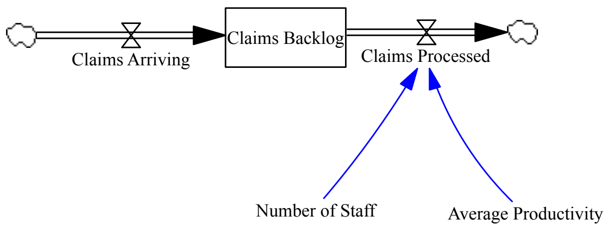


Stock behaviour?



Need to ensure stock stays positive

- Rules:
  - If there is no claims backlog, then the claims processed should be zero
  - If the claims backlog is less than the claims processing capacity, then only the backlog should be processed
  - If the claims backlog is greater than the claims processing capacity, then only the claims processing capacity should be processed.



Claims Backlog	Number of Staff	Average Productivity	Capacity	Claims Processed
0	25	2	50	0
49	25	2	50	49
2000	25	2	50	50

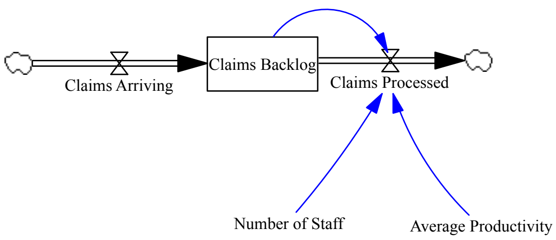
IF-ELSE RULE – MIN Function

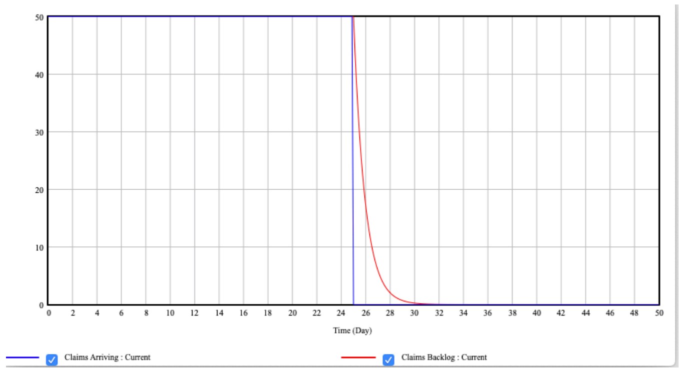
Claims Backlog	Number of Staff	Average Productivity	Capacity	Claims Processed
0	25	2	50	0
49	25	2	50	49
2000	25	2	50	50

Claims Processed = MIN(Claims Backlog, Number Staff * Average Productivity)

Updated Model

Claims Processed = MIN(Claims Backlog, Number Staff * Average Productivity)



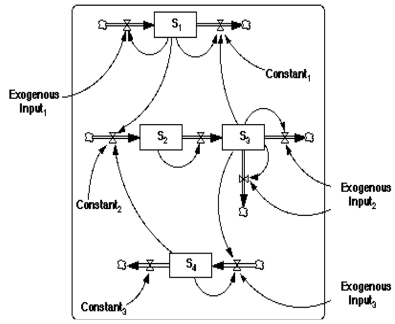


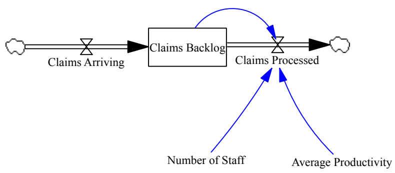
Challenge 4.3

- Extend the Vaccine Model to include
  - Those Yet to be vaccinated
  - Those Vaccinated
  - Those with non-effective vaccines (assume efficacy of 73%)
- Make the vaccination flow subject to two constraints
  - Vaccine availability
  - Health Care workers
  - HCW Productivity
- Identify all possible outcomes (similar to earlier table)

Higher Order Systems (Multiple Stocks)

- We can extend our models to build models with more than one stock (interacting stocks).
- Basic formulations include
  - Fractional increase
  - Fractional decrease
  - Adjustment to a goal
  - Resources/Productivity





Challenge 4.4
2-Stock Model of University

- Create a 2-stock University Model
- Students are recruited and graduate using fractional increase & decrease (0.25/year)
- Start with 10000 students
- Staff (initial value 500) are recruited using a goal adjustment structure
- Assume the desired student/staff ratio is 20:1
- Sketch the stock and flow structure, with equations
- Identify the feedback in the model
- Speculate on how the stocks will react to the following separate scenarios:
  - The desired student staff ratio drops to 15
  - There is a new influx of 1000 students per year from 2025
- How might the stock of staff influence the stock of students?

CT561: Systems Modelling & Simulation

Lecture 5: Limits to Growth

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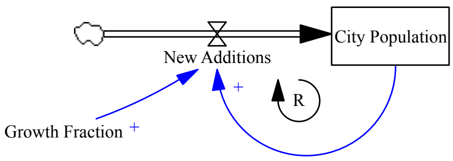
https://github.com/JimDuggan/SDMR

1

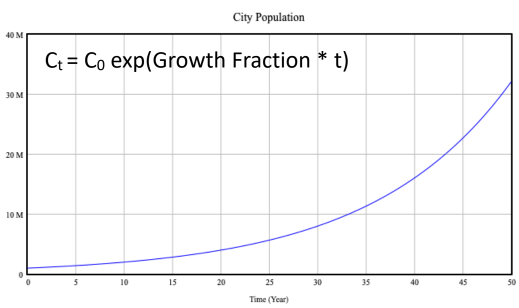
Recap: Exponential Growth

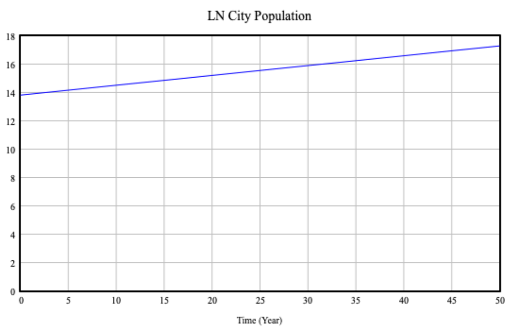


City Population = INTEG(New Additions , 1e+06)

Growth Fraction = 0.07

New Additions = City Population * Growth Fraction





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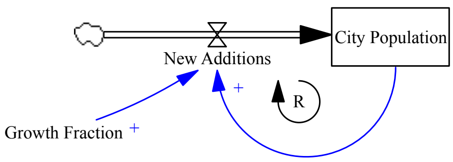
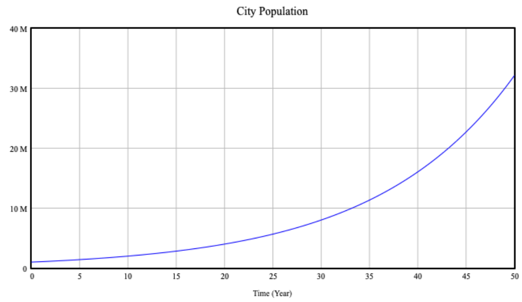
Feedback Analysis

Cause	Direction	Effect	Direction
City Population	↑	New Additions	↑
New Additions	↑	City Population	↑





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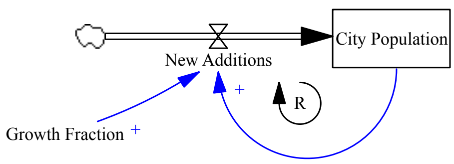
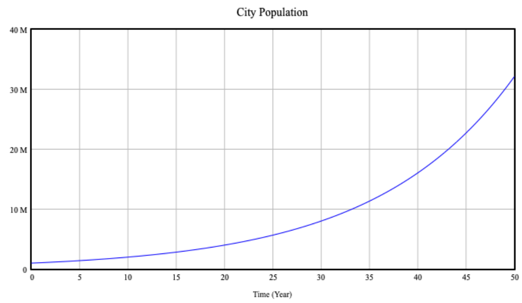
Exponential Growth

- Quantities that grow by a fixed percentage (e.g. 0.018) per time period exhibit exponential growth
- Exponential growth behaves according to a “doubling time”
- “Treacherous and misleading” Forrester (1971)
- Within one doubling time, the quantity goes from half its limit to its limit





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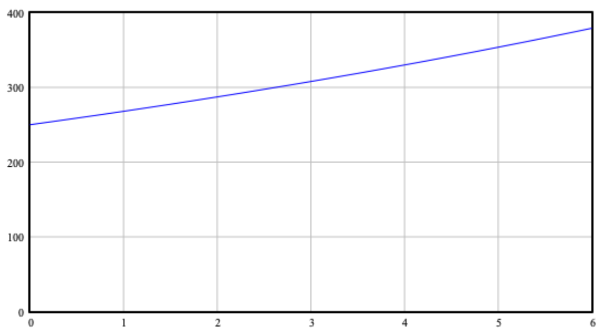
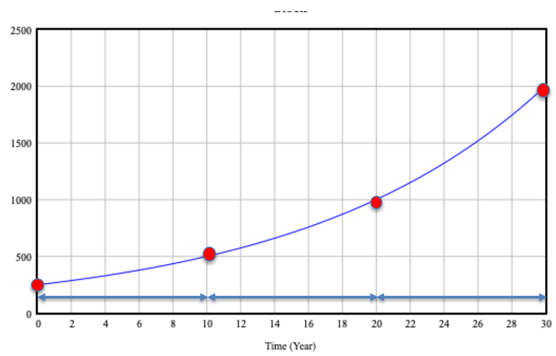
Misperceptions of Exponential Growth

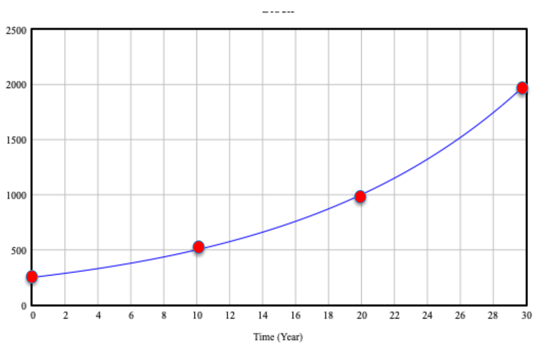
- Studies have shown that people grossly underestimate the rate of growth, by extrapolating *linearly* instead of *exponentially*.
- *Doubling time is* a valuable way to understand exponential growth





Doubling Time Calculation

- S(t) = S(0) exp(growth rate * t)
  - 2 * S(0) = S(0) exp (growth rate * tD)
  - 2 = exp(growth rate * tD)
  - Ln (2) = growth rate * tD
  - tD = Ln(2)/growth rate = (0.6931)/growth Rate
- "The Rule of 70"
  - Doubling time independent of stock size
  - tD = 70 /(100*growth rate)
  - An investment earning 7%/year doubles after 10 years



Growth Fraction = 0.07

New Additions = Stock * Growth Fraction

Stock = INTEG(New Additions , 250)

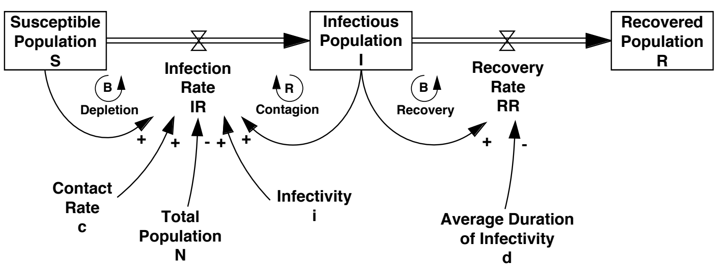
Challenge 5.1

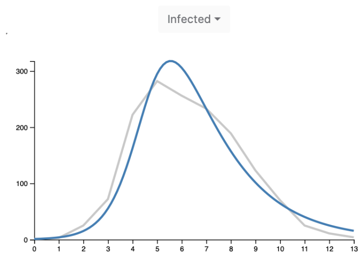
- The growth rate of an epidemic in its early stage is estimated at 15% per day. From that, estimate the doubling time.
- If there are 100 people infected on day 1, estimate (using the growth rate and the integral equation solution for exponential growth), how many have been infected after 30 days.
- Implement a simple model in Vensim and compare the results

Limits to Growth Model

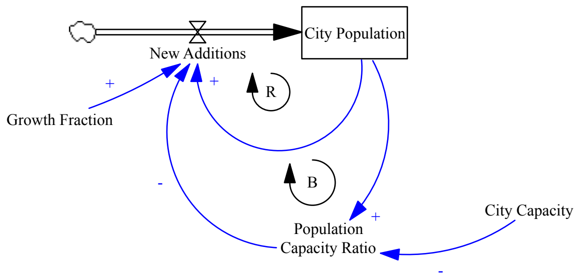
There will always be limits to growth. They can be self-imposed. If they aren't, they will be system-imposed.

Donella H. Meadows, Thinking in Systems: A Primer (2008), p.103





Limit to Growth – *note extension to net flow equation*



City Capacity = 5e+06

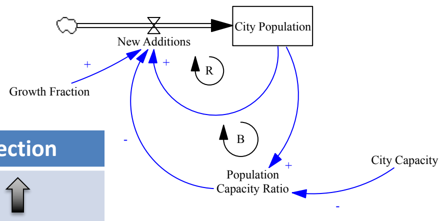
City Population = INTEG(New Additions , 50000)

Growth Fraction = 0.15

New Additions = City Population * Growth Fraction * **(1 - Population Capacity Ratio)**

Population Capacity Ratio = City Population / City Capacity

Feedback Analysis – 2 Loops



Cause	Direction	Effect	Direction
City Population	⇧	New Additions	⇧
New Additions	⇧	City Population	⇧

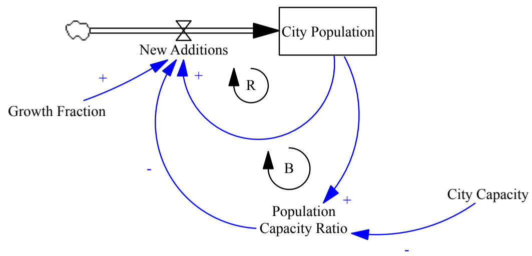
Cause	Direction	Effect	Direction
City Population	⇧	Population Capacity Ratio	⇧
Population Capacity Ratio	⇧	New Additions	⇩
New Additions	⇩	City Population	⇩

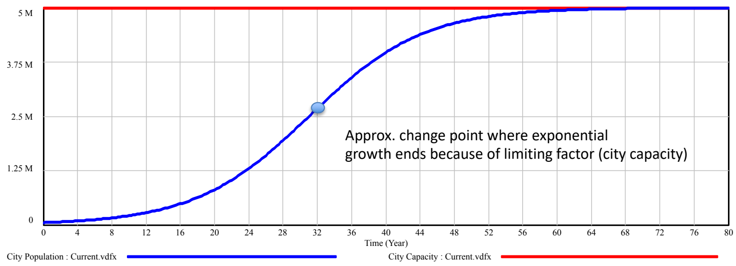
New Additions = City Population * Growth Fraction * **(1 - Population Capacity Ratio)**

Model Outputs

Cause	Direction	Effect	Direction
City Population	⇧	New Additions	⇧
New Additions	⇧	City Population	⇧

Cause	Direction	Effect	Direction
City Population	⇧	Population Capacity Ratio	⇧
Population Capacity Ratio	⇧	New Additions	⇩
New Additions	⇩	City Population	⇩





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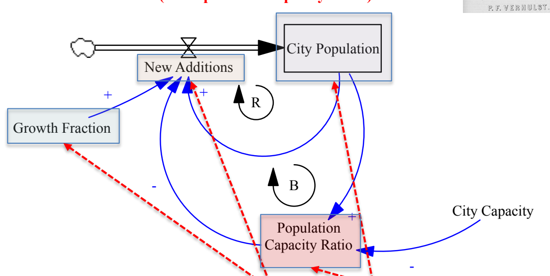
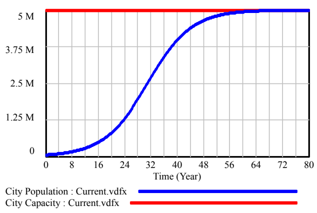
Limits to Growth - Verhulst Model

New Additions = City Population * Growth Fraction *
(1 - Population Capacity Ratio)

- A model of population growth, where the rate of increase is limited by the carrying capacity (K)
- When P is small, it approximates exponential growth
- Model compared to population growth in:
  - France (1817-1831)
  - Belgium (1815-1833)
  - Essex (1811-1831)





$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

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Challenge 5.2

- A fixed amount of land is designated for a new trading estate in order to encourage business development in a town.
- Initially, the trading estate grows rapidly as more businesses attract more business developments
- However, as growth continues, land availability falls, and business construction is reduced
- Draw a Stock and Flow Model and formulate the equations
- Variables include: **Business Structures (S)**, Construction Rate, **Business Construction (F)**, Land Availability, Land Area, Land Per Business Structure

Overshoot and Collapse (Sterman p 123)

- The Verhulst model assumes that the carrying capacity is fixed
- Often, however, the ability of an environment to support a growing population is eroded or consumed by the population itself
- Example: Population of deer rises, leading to overbrowsing, which consumes the vegetation, leading to a decline in the deer population
- Real world examples include overfishing of St George's Bank, population of Easter Island

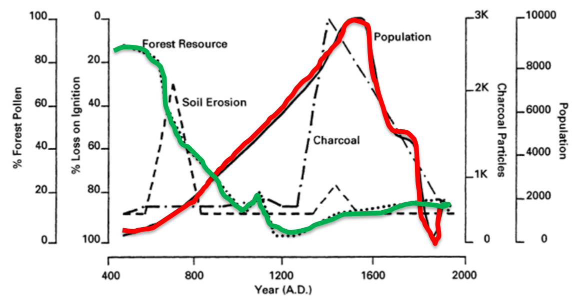
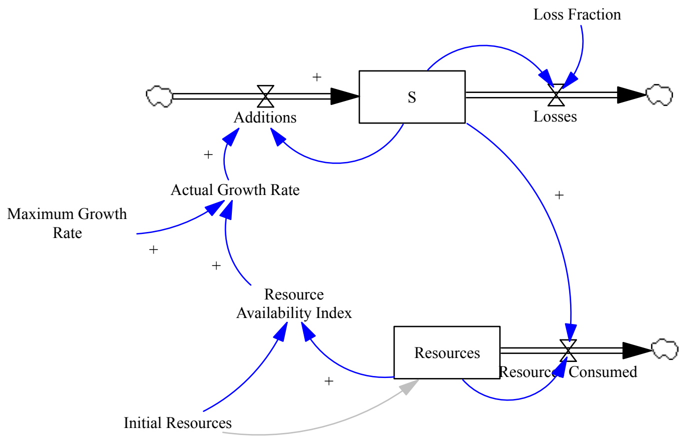
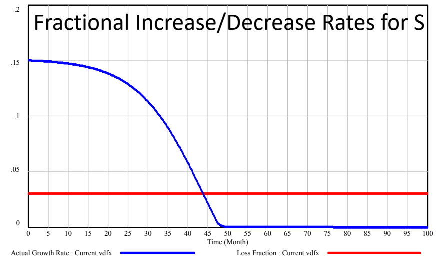


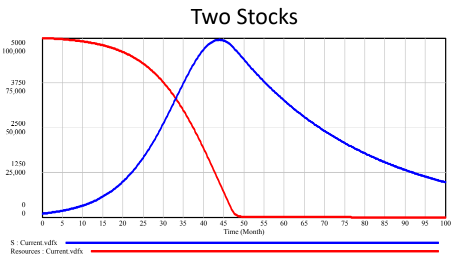
Figure 1. Easter Island dynamics from archaeological study by Bahn and Flenley (1992)

Published in 2012
System Dynamics Implementation of a Model of Population and Resource Dynamics with Adaptation
T. Uehara, Yoko Nagase, W. Wakeland

Exploratory Model







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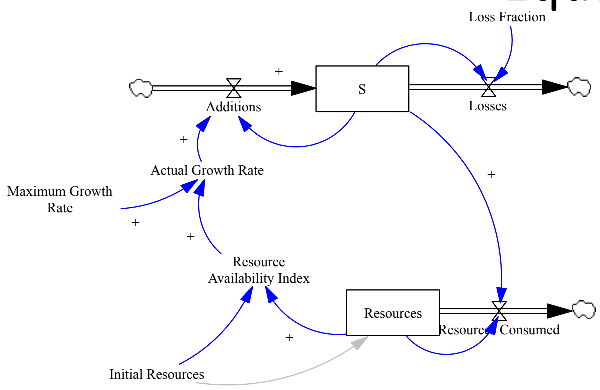
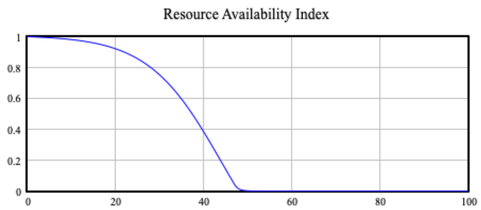
Equations





S = INTEG(Additions - Losses , 100)

Additions = Actual Growth Rate * S

Losses = Loss Fraction * S

Loss Fraction = 0.03

Actual Growth Rate = Maximum Growth Rate * Resource Availability Index

Maximum Growth Rate = 0.15

Resources = INTEG(- Resources Consumed , Initial Resources)

Initial Resources = 100000

Resource Availability Index = Resources / Initial Resources

Resources Consumed = min (S , Resources)

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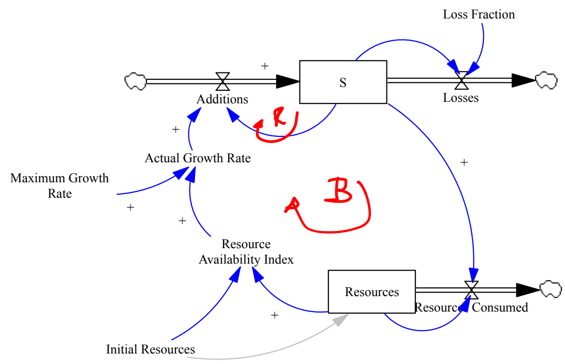
Balancing Loop

Cause	Direction	Effect	Direction
S	⇧	Resources Consumed	⇧
Resources Consumed	⇧	Resources	⇩
Resources	⇩	Resource Availability Index	⇩
Resource Availability Index	⇩	Actual Growth Rate	⇩
Actual Growth Rate	⇩	Additions	⇩
Additions	⇩	S	⇩



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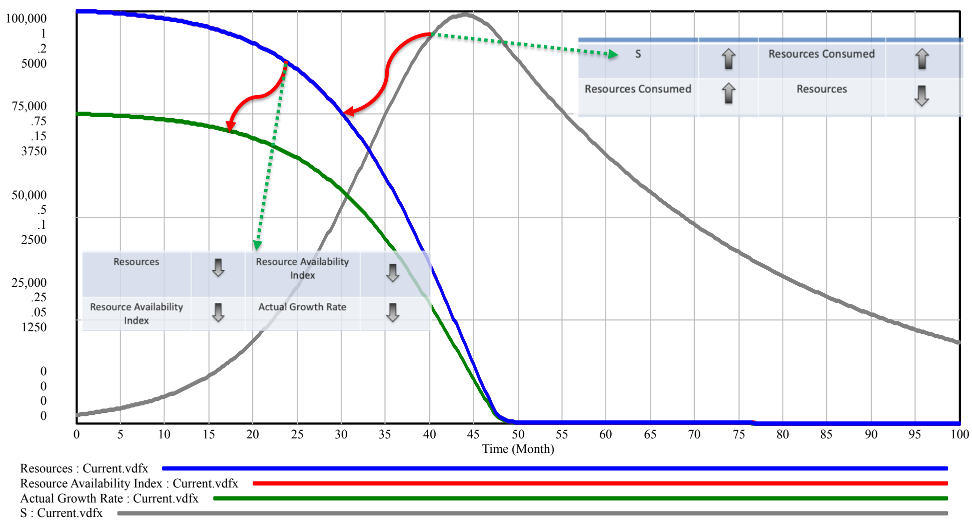
Key Dynamics



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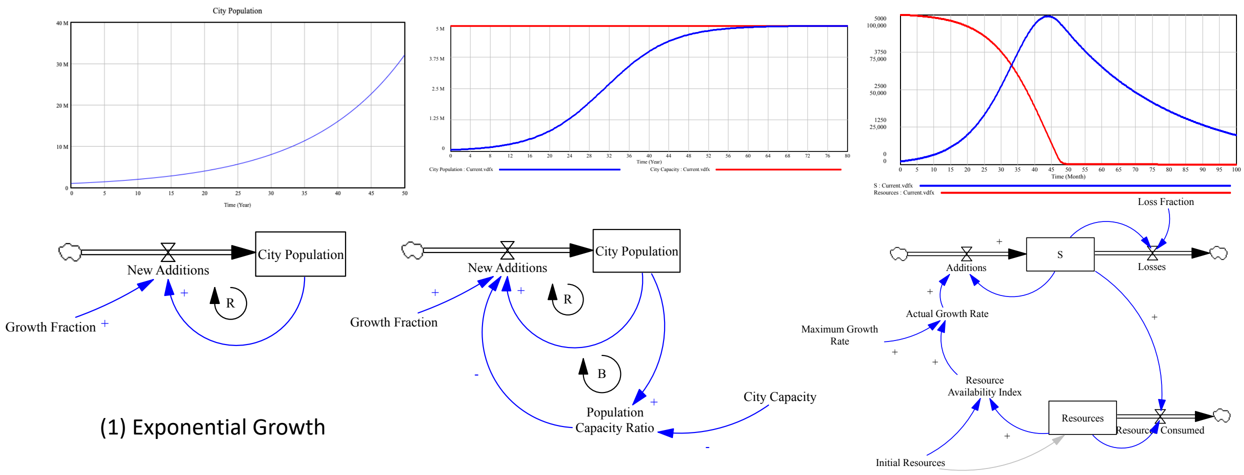

(1) Exponential Growth

(2) Limits to Growth

(3) Overshoot and Collapse

CT561: Systems Modelling & Simulation

Lecture 6: Formulating Effects and the Rework Cycle

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https://github.com/JimDuggan/SDMR

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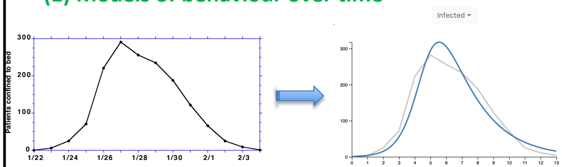
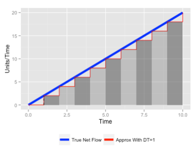
Course Recap

(1) Models of behaviour over time

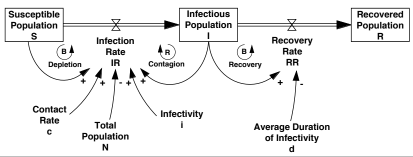


(2) Integration (Calculus)

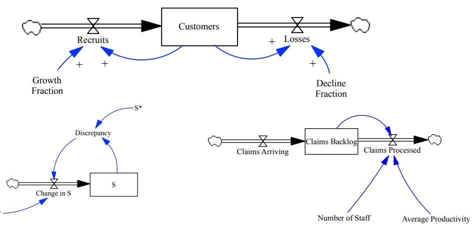


$$S_t = S_{t-dt} + NF_{t-dt} \times DT$$

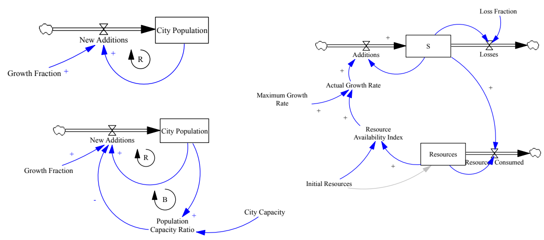
(3) Feedback (Positive & Negative)



(4) Formulating Flows (4)



(5) Limits to Growth



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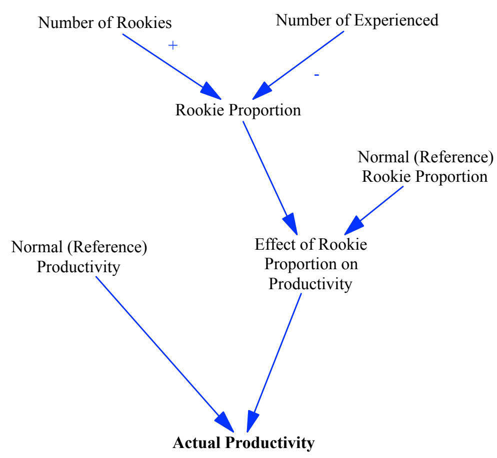
Formulating Effects

- An important building block for models is to capture how variables influence one another over time.
- System dynamics offers a convenient structure for modeling effect variables (Sterman 2000).
- These can be used to help simulate more complex feedback structures



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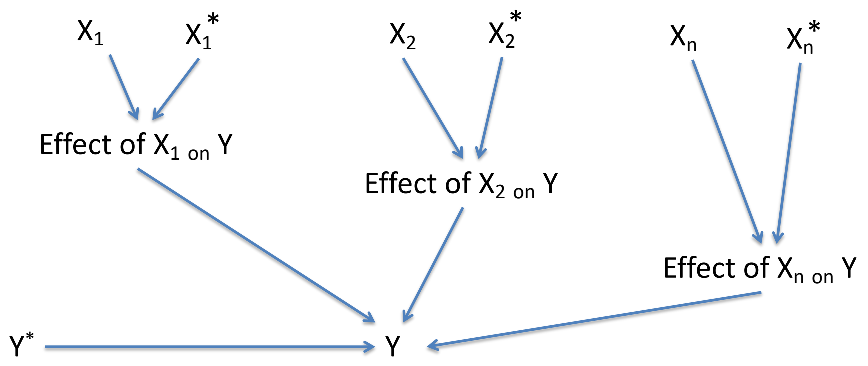

$$Y = Y^{*} \times Effect\,(X_1 on\, Y) \times \ldots \times Effect(X_n on\, Y)$$

$$Effect\,(\,X_i\; on\; Y) = f\left({X_i}/{X_i^{*}}\right)$$

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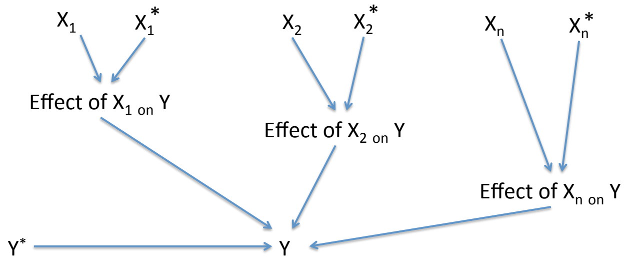
Effects structure (1)

- There is a variable *Y* that is the dependent variable of a causal relationship, and this depends on a set of *n* independent variables ($X_1, X_2, \ldots, X_n$)
- The variable *Y* has a reference value Y^*, and this is multiplied by a sequence of *effect functions* that are calculated based on the normalized ratio of (X_i/X_i^*), where X_i^* is the reference value, and X_i is the actual value.



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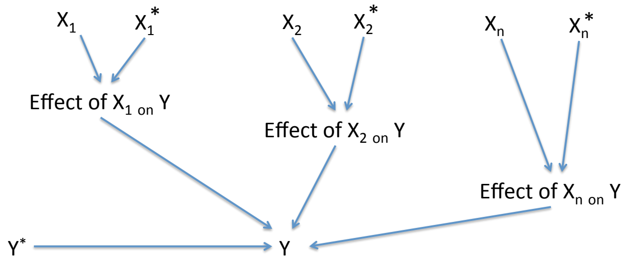
Effects structure (2)

- The effect function (y-axis) has the normalized ratio (X/X^*) on its x-axis, and always contains the point (1,1) although the function itself can be either linear or non-linear around this point.
- This point (1,1) is important for the following reason: if *X* equals its reference value X^*, then the effect function will be 1, and therefore *Y* will then equal its reference value Y^*.



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Challenge 6.1

- Work through the logic of the effect structure, starting with the following structures.

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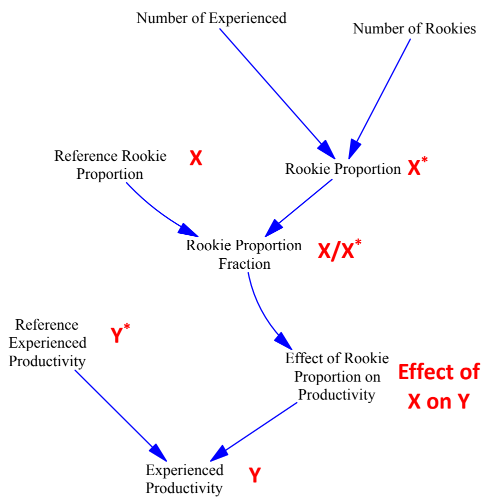
Software Engineering Example

- Reference productivity is 100 loc/person/day
- This assumes a reference rookie proportion in the team (say 20%)
- If we have exactly 20% Rookies
  - Actual Productivity = Reference Productivity
- If we have > 20% Rookies
  - Experienced Productivity < Reference Productivity
- If we have < 20% Rookies
  - Experienced Productivity > Reference Productivity



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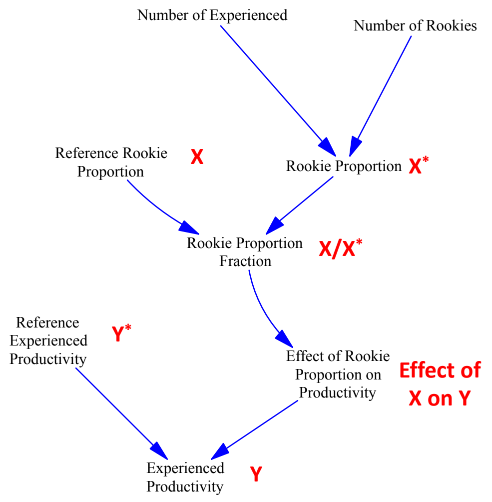
The equation (for experienced productivity)

Experienced Productivity = Reference Experienced Productivity * Effect of Rookie Proportion on Productivity

Reference Experienced Productivity (Y^*)	Reference Rookie Proportion (X^*)	Actual Rookie Proportion (X)	X/X^*	Effect Multiplier	Actual Experienced Productivity (Y)	Comments
100	20%	20%	1	1	100	No effect on experienced productivity, as the benchmark value of 100 is measured when we have 20% rookies in the team.
100	20%	40%	> 1	< 1	< 100	Effect < 1 and experienced productivity goes down, as we have more rookies which will require increased feedback from experienced productivity
100	20%	10%	< 1	> 1	> 100	Effect > 1 and experienced productivity rises, as experienced programmers have more time to focus on coding efforts

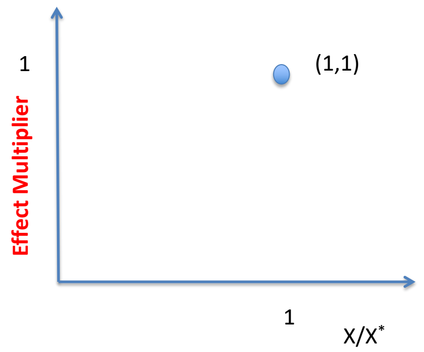
The Effect Equation

- Actual Productivity = Reference Productivity * Effect of Rookie Proportion on Productivity
- Effect of X on Y = $F(X/X^*)$
- Normalised Value
- *When X = X*, F(X) = 1*
- *X^* and Y^* are reference values*



Example

- X = Rookie Proportion
- X^* = Reference Rookie Percentage
- Impact on productivity?
- (1,1) is always on the line



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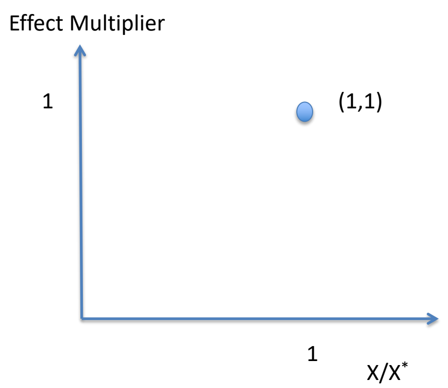
Thinking about the effects…

- X = Actual Rookie Proportion
- X^* = Reference Rookie Proportion (i.e. the number at which our experienced productivity is at its reference value)
- Scenarios:
  – If $X > X^*$, Effect?
  – If $X < X^*$, Effect?



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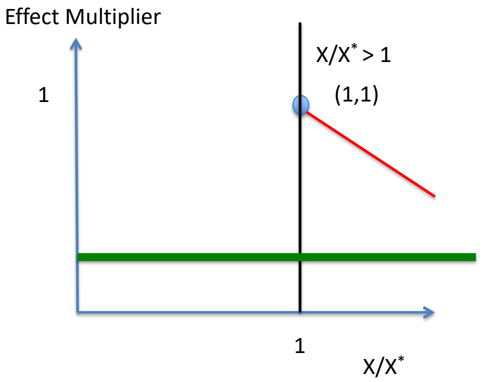
Sketching the relationship, More rookies than reference value

- $X > X^*$
  - We have more Rookies than our target level
  - This will reduce our experienced productivity
  - More work to train rookies
  - Effect will be lower than 1
  - Decide on minimum value (0.25)



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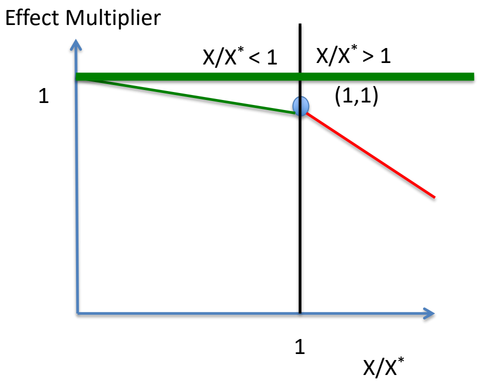
Sketching the relationship, Less rookies than reference value

- $X < X^*$
  - We have less Rookies than our target level
  - This will increase our experienced productivity
  - Less work to train rookies
  - Effect will be greater than 1
  - Decide on a maximum value (1.8)



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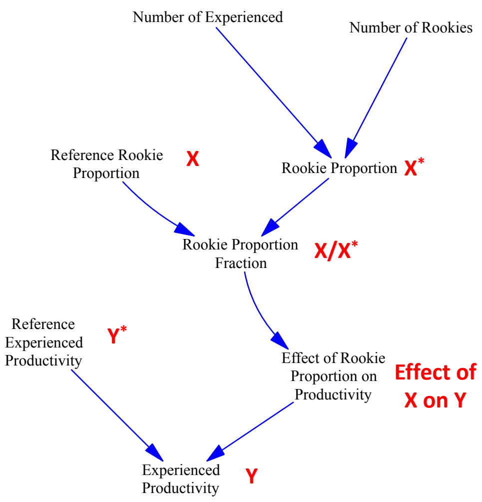
Equations



Reference Rookie Proportion = 0.2

Reference Experienced Productivity = 100

Number of Experienced = 100

Number of Rookies = 20

Rookie Proportion = Number of Rookies / (Number of Experienced + Number of Rookies)

Rookie Proportion Fraction = Rookie Proportion / Reference Rookie Proportion

Effect of Rookie Proportion on Productivity = WITH LOOKUP(Rookie Proportion Fraction
, ([(0,0)-(5,2)],(0,1.8),(1,1),(1.5,0.8),(2,0.7),(3,0.5),(4,0.45)
))

Experienced Productivity = Reference Experienced Productivity * Effect of Rookie Proportion on Productivity

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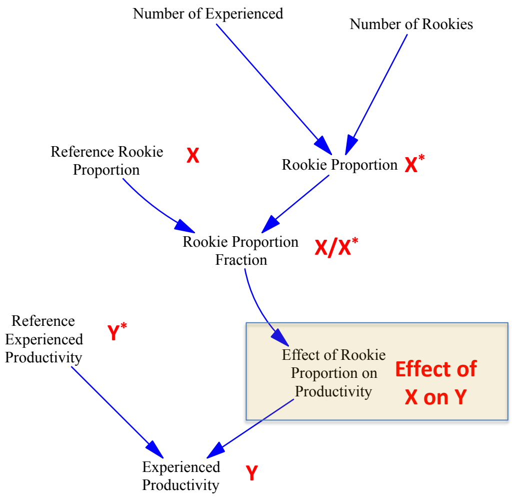
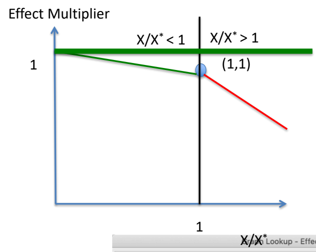
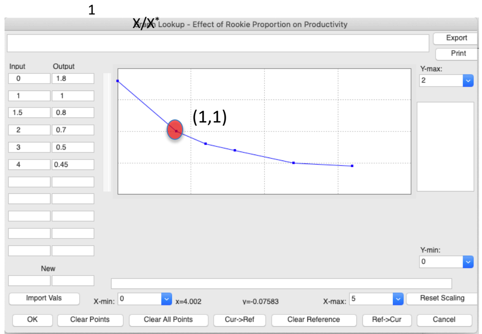
Sample Effect Function (interpolated)





Effect of Rookie Proportion on Productivity = **WITH LOOKUP**(Rookie Proportion Fraction, ([(0,0)(5,2)], (0,1.8), (1,1), (1.5,0.8), (2,0.7), (3,0.5), (4,0.45)))



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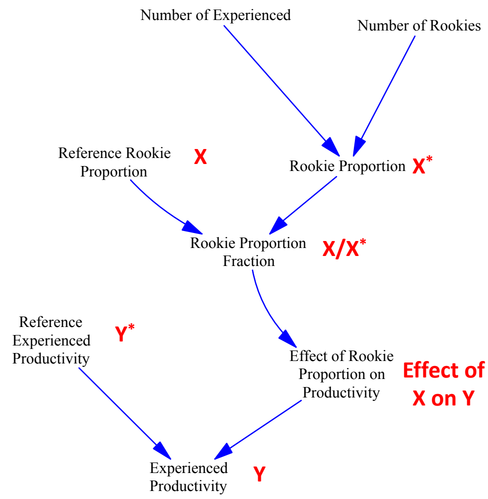
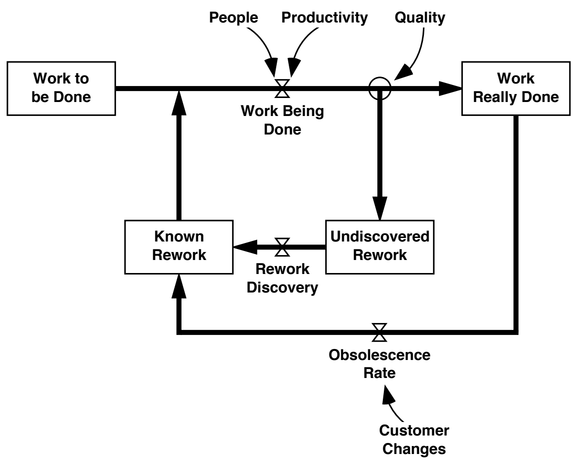
Challenge 6.2

- Explore the model in Vensim and observe the impact of changing X for the three scenarios
- Extend the model to include the following effect variables on experienced productivity:
  - Average time to promotion
  - Average length of working week



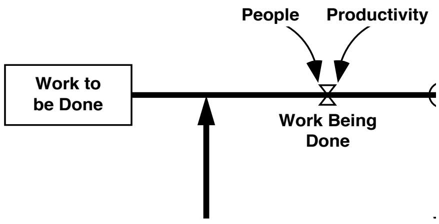
The Rework Cycle (Sterman 2000)

- Productivity of workers
- Capacity of team
- Key Variables
  - Workflow
    - Work to do
    - Work completed
    - Rework: Undiscovered and Known
  - Resources
    - Experience
    - Speed
    - Quality

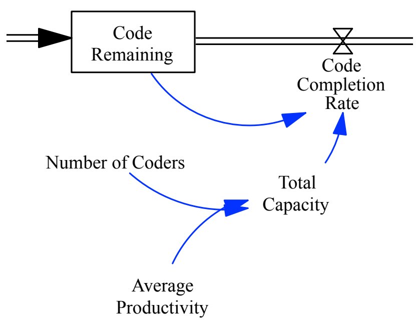


Work Being Done… (Flow)

- WBD = Resource * Productivity
- Production = Labour Force * Average Productivity
- (Units/Period) = (People) * ((Units/Period)/Person)
- Labour Force * Average Productivity also gives the system's ***Capacity***



Software Construction



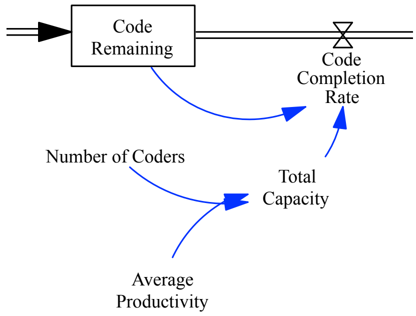
Code Remaining= INTEG (
New Code Added-
Code Completion Rate,
0)

Average Productivity= 50

Number of Coders = 10

Total Capacity= Number of Coders*Average Productivity

Formulating the Outflow

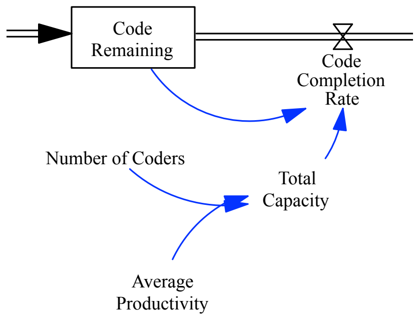


- Stock cannot go negative!
- We cannot produce more than our capacity

Code Remaining	Capacity	Outflow
1,000	200	200
50	200	50

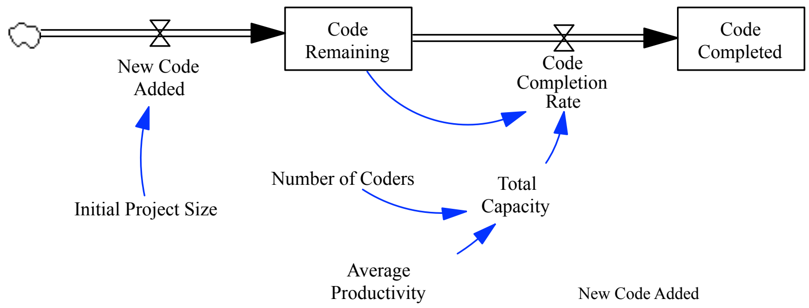
Outflow = f(Code Remaining, Capacity)

Finalising Outflow (simpler version of "first order control")



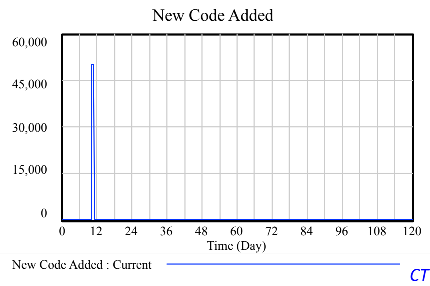
Completion Rate= MIN(
Code Remaining,
Total Capacity)

Extended Model

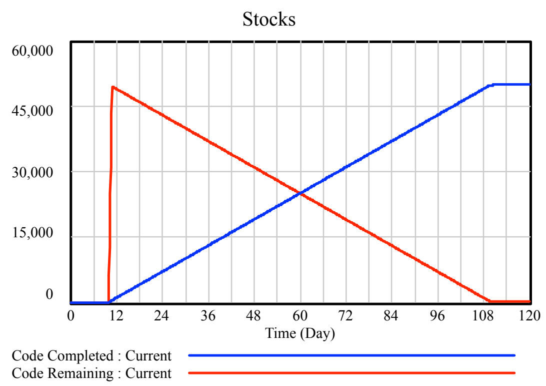


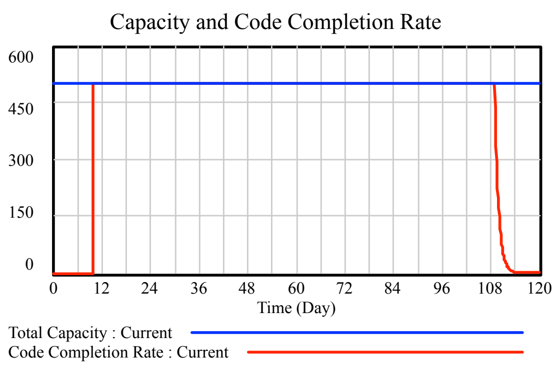
Initial Project Size = 50000

New Code Added = PULSE(10, 1) * Initial Project Size



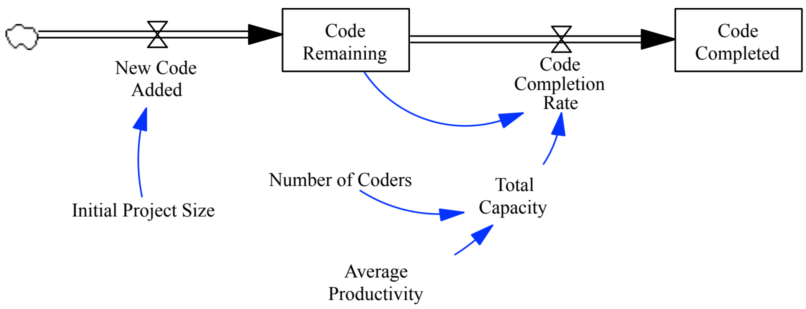
Simulation Output





Challenge 6.3

- Implement the model in Vensim, and explore its behaviour



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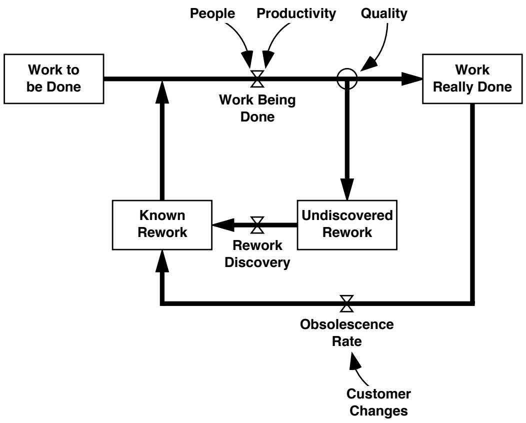
Extension… the Rework Cycle

- Introduce the notion of quality
- All processes have a concept of Yield
  - Output/Input

Input	Output	Yield
1,000	879	87.9%

Input	Output	Defects
1,000	879	12.1%



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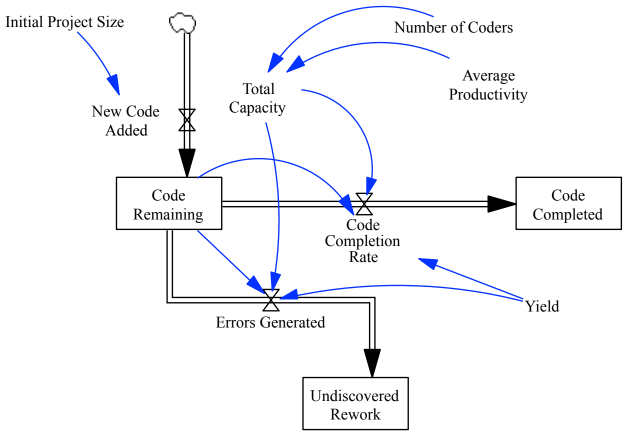
An initial model

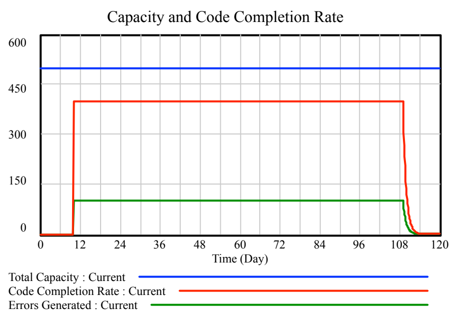


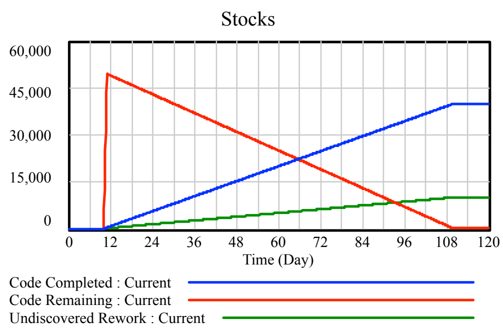
Code Completion Rate=
min(Code Remaining,
Total Capacity)*Yield

Errors Generated=
min(Code Remaining,
Total Capacity) * (1-Yield)

27

Model Behaviour





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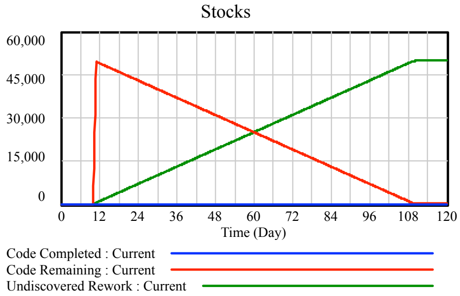
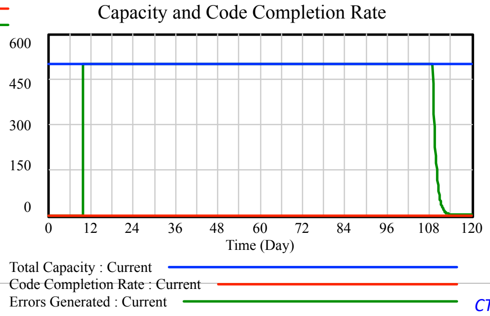
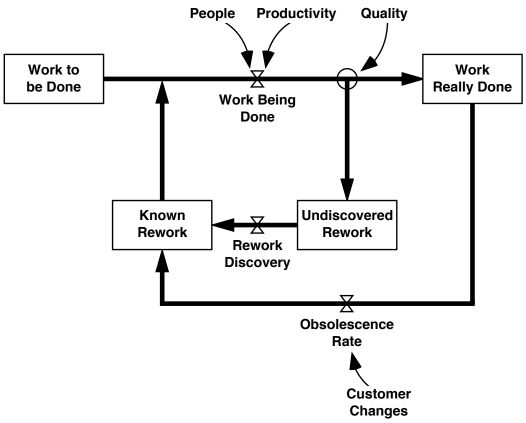
Model Test: Set Yield = 0





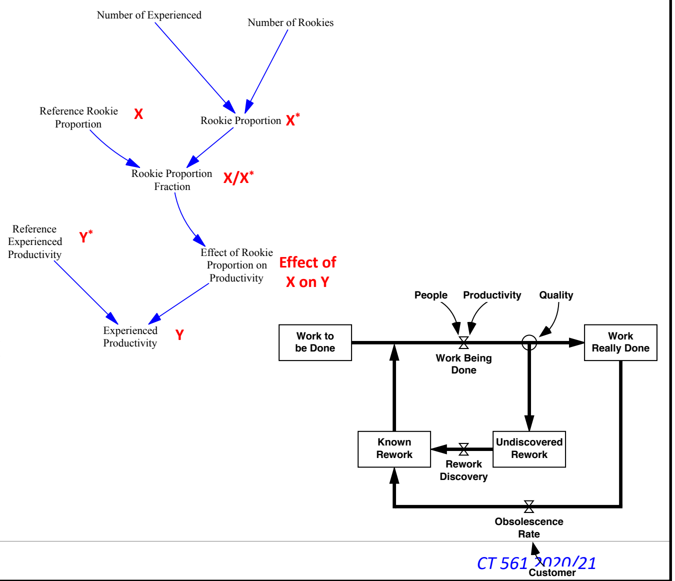
Challenge 6.4

- Complete a model for the software rework cycle.
- Use fractional decrease rate for rework discovery and customer changes
- Assume 100,000 LOC to be done at the start (no need for inflow into *Work to be Done*).



Summary: Effects and the Rework Cycle

- Effects are an important building block for models is to capture how variables influence one another over time.
- Rework cycle and important structure for modelling projects
- Productivity and quality amongst different groups can also be applied to the rework cycle.



CT561: Systems Modelling & Simulation

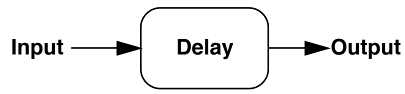
Lecture 7: Formulating Delays

Prof. Jim Duggan,
School of Engineering & Informatics
National University of Ireland Galway.
https://github.com/JimDuggan/SDMR

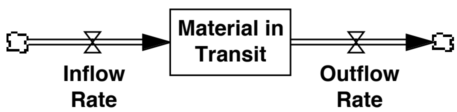
(1) Delays

- "Delays are pervasive.
  - It takes time to measure and report information.
  - It takes time to make decisions.
  - It takes time for decisions to affect the state of the system" (Sterman 2000)
- We need to use delays in many of our models

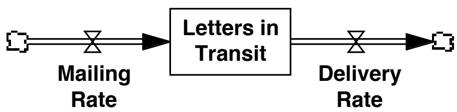
The output of a delay lags behind the input:



General structure of a material delay:



The post office as a delay:



Example of a delay… incubation period

Open access | Original research

BMJ Open **Incubation period of COVID-19: a rapid systematic review and meta-analysis of observational research**

Conor McAloon,[1] Áine Collins,[2] Kevin Hunt,[3] Ann Barber,[2] Andrew W Byrne,[4] Francis Butler,[3] Miriam Casey,[2] John Griffin,[5] Elizabeth Lane,[6] David McEvoy,[7] Patrick Wall,[7] Martin Green,[8] Luke O'Grady,[1,8] Simon J More[2]

Table 2 Percentiles of the pooled lognormal distribution after simulating all possible combinations of mu and sigma within the 95% CIs of the pooled estimates of both parameters

Percentile	Median (days)	Min	Max	Difference (max – min)
2.5th	1.92	1.54	2.38	0.84
5th	2.24	1.83	2.75	0.92
10th	2.69	2.24	3.23	0.99
25th	3.64	3.12	4.25	1.13
50th	5.10	4.53	5.75	1.22
75th	7.15	6.13	8.34	2.21
90th	9.69	8.06	11.60	3.54
95th	11.60	9.49	14.20	4.71
97.5th	13.60	10.9	16.90	6.00

The median days for each percentile are shown along with the minimum and maximum values for that percentile.

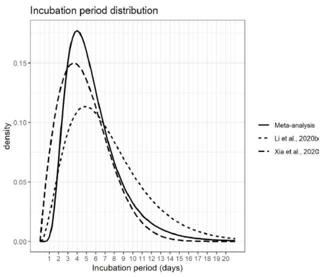


Figure 6 Probability density function of pooled lognormal distribution for incubation period and studies (n=2) not included in the meta-analysis because of the distribution used.

3

Delay Distributions

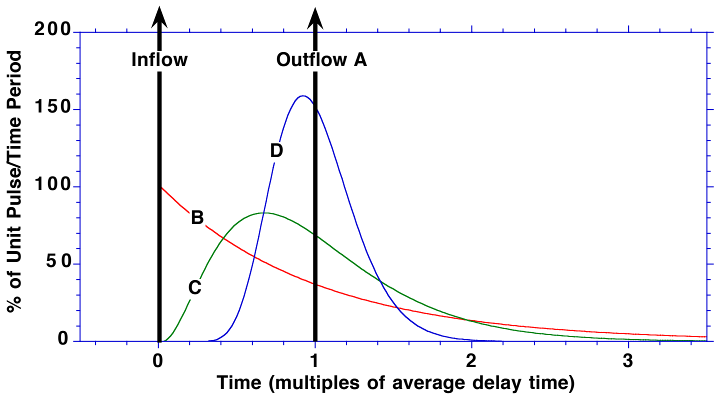


Figure 11-2 Some distributions of the outflow from a delay

The input I all cases is a unit pulse at time zero. Outflow A is a pipeline delay in which all items arrive together exactly 1 delay time after they enter. Outflow distributions B-D exhibit different degrees of variation in processing times for individual items so some arrive before and some after the average delay time. In all cases the average delay time is the same and the areas under each distribution are equal.

4

(1) First Order Material Delay

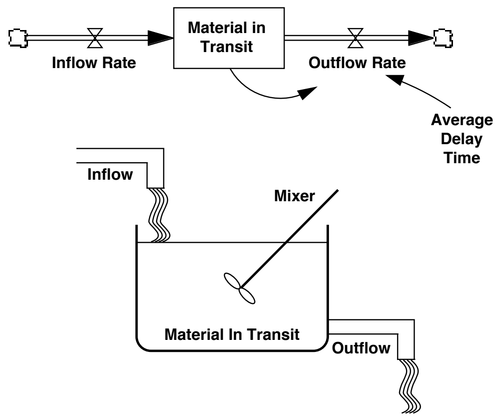
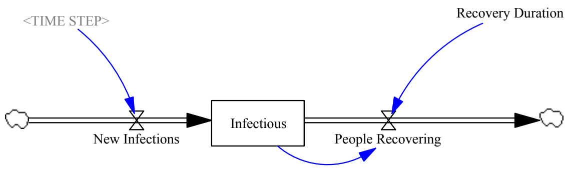


Figure 11-4 First-order material delay: structure

The outflow is proportional to the stock of material in transit. The contents of the stock are perfectly mixed at all times, so all items in the stock have the same probability of exit, independent of their arrival time.

Example 1: Infectious Disease Recovery Delay



The outflow is proportional to the stock of material in transit. The contents of the stock are perfectly mixed at all times, so all items in the stock have the same probability of exit, independent of their arrival time.

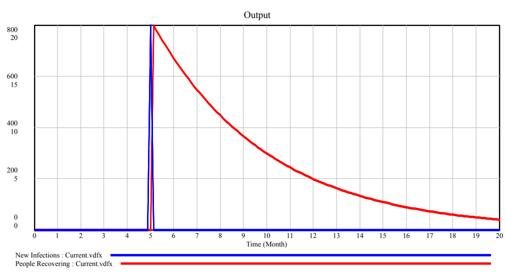
Infectious = INTEG(New Infections - People Recovering , 0)

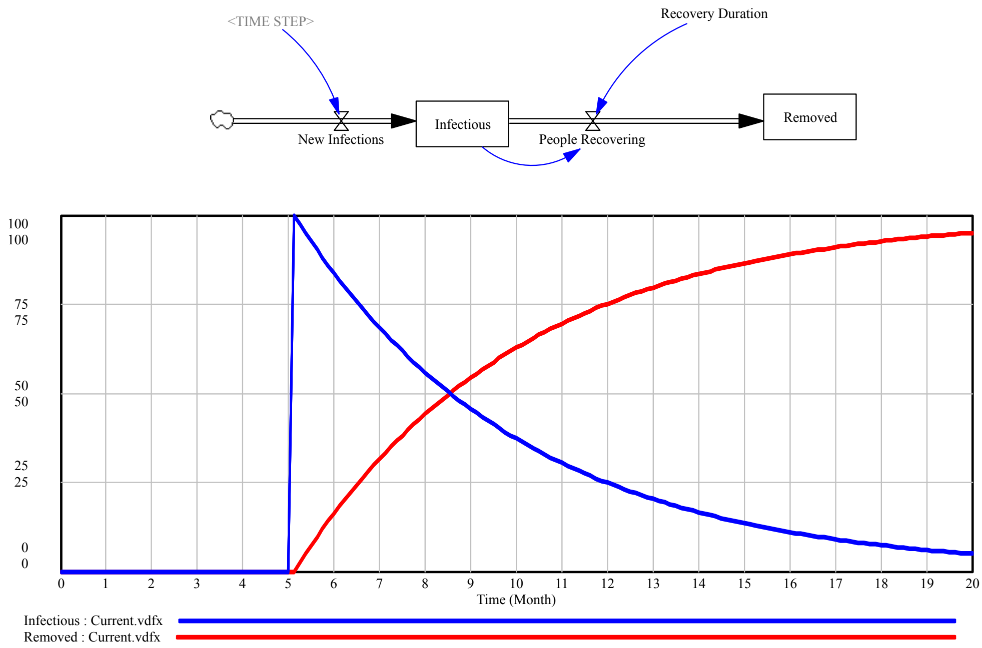
New Infections = PULSE (5, TIME STEP) * 100 / TIME STEP

People Recovering = Infectious / Recovery Duration

Recovery Duration = 5

TIME STEP = 0.125
The time step for the simulation.





Challenge 7.1 – First Order Delay

- Draw a stock and flow model of a software engineering team
- Assume there are 100 rookies to start and 0 experienced
- Assume the average delay for progression is 12 months
- Simulate for 36 months, and show the two stocks together
- Discuss the advanages/limitations of the model

(2) Second Order Material Delay (Sterman 2000)

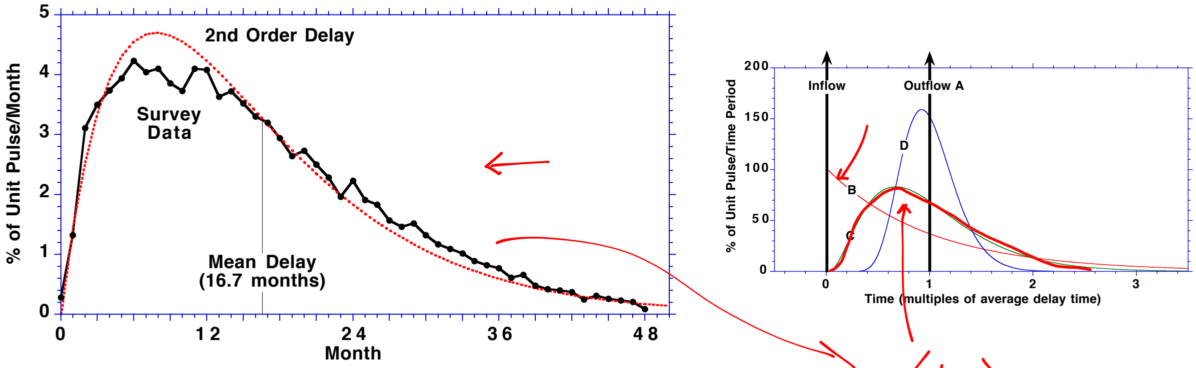


Figure 11-17 The construction lag for capital plant: data vs. model

Data: Distribution of construction completion times for US private nonresidential structures, 1961-1991, as estimated by Montgomery (1995) from US Dept. of Commerce survey data. The mean lag is 16.7 months. *Model:* Second-order material delay with average delay time of 16.7 months.

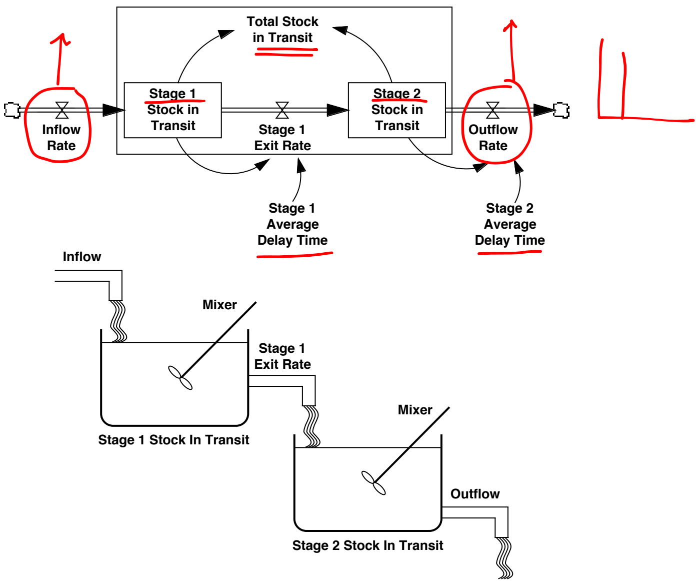
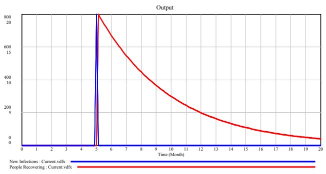


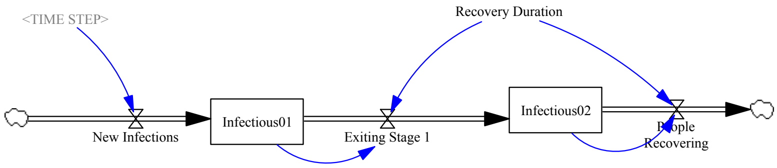
Figure 11-6 Higher-order delays are formed by cascading first-order delays together.

Delay time "shared" by outflows

Stage 1 Exit Rate = Stage 1 Stock in Transit/Stage 1 Average Delay Time
Outflow Rate = Stage 2 Stock in Transit/Stage 2 Average Delay Time

Example 2: Recovery Delay (2nd Order)





Exiting Stage 1 = Infectious01 / (Recovery Duration / 2)

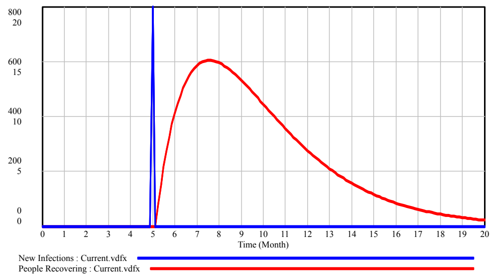
Infectious01 = INTEG(New Infections - Exiting Stage 1 , 0)

Infectious02 = INTEG(Exiting Stage 1 - People Recovering , 0)

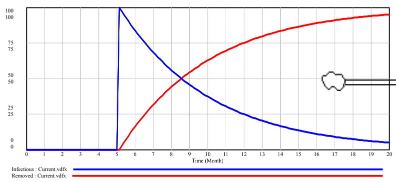
New Infections = PULSE (5, TIME STEP) * 100 / TIME STEP

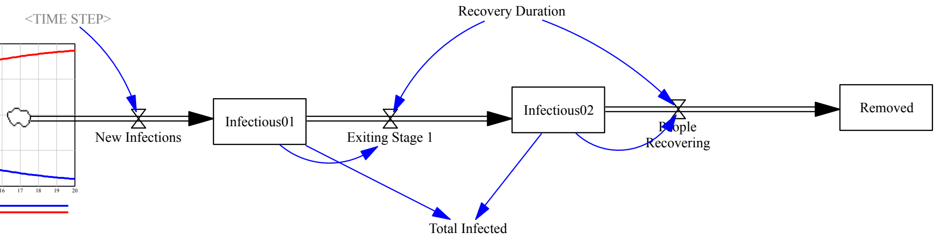
People Recovering = Infectious02 / (Recovery Duration / 2)

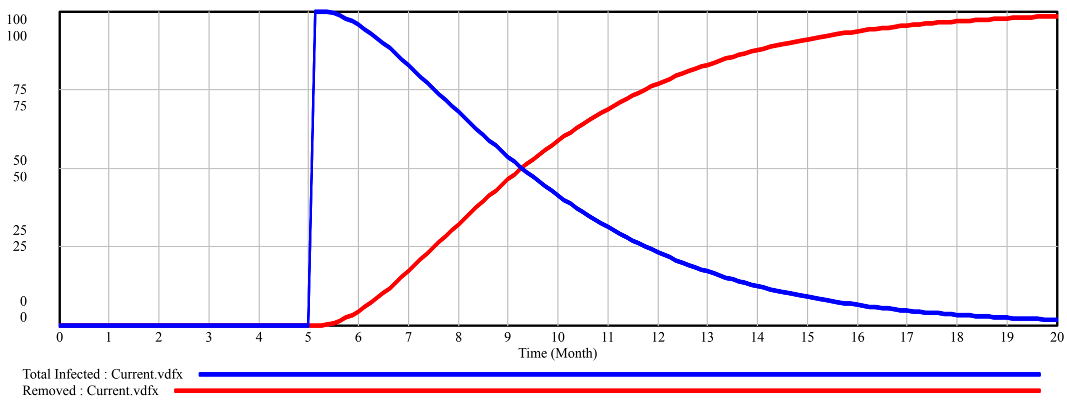
Recovery Duration = 5



11







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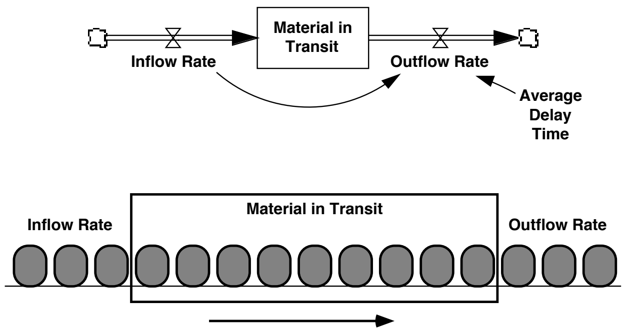
(3) Pipeline Delay

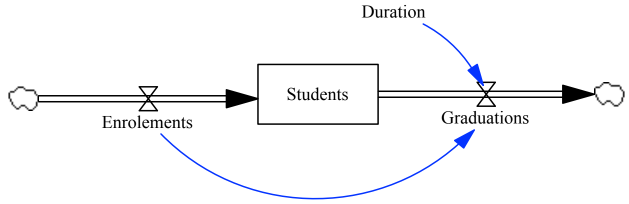
Figure 11-3 Pipeline delay: structure

In a pipeline delay individual items exit the delay in the same order and after exactly the same time, like widgets moving down an assembly line at a constant speed.



Material in Transit(t) = INTEGTRAL(Inflow(t) – Outflow(t), Material in Transit(0))
Outflow(t) = Inflow(t – Average Delay Time)

Student Example – Step Input



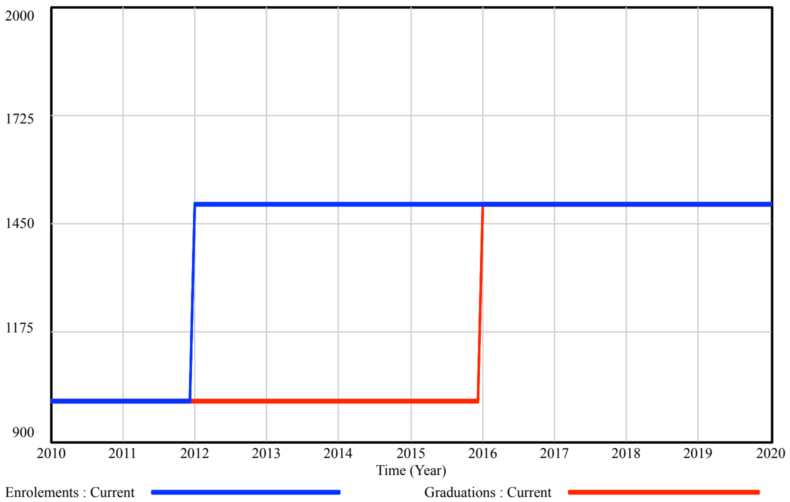
Duration = 4

Enrolements = 1000 + step (500, 2012)

Graduations = DELAY FIXED (Enrolements ,Duration , 1000)

Students = INTEG(Enrolements - Graduations , 4000)

Pipeline Delay Response

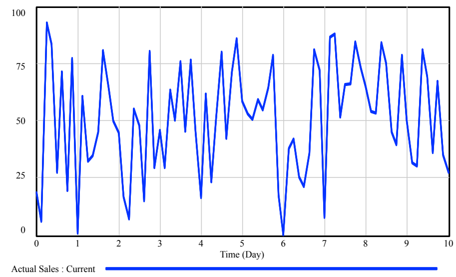


Challenge 7.2 – Second Order Delay

- Draw a stock and flow model of a software engineering team
- Assume there are 100 rookies to start and 0 experienced
- Assume the average delay for progression is 12 months
- Simulate for 36 months, and show the two stocks together
- Discuss the advantages/limitations of the model

Information Delay (Smoothing)

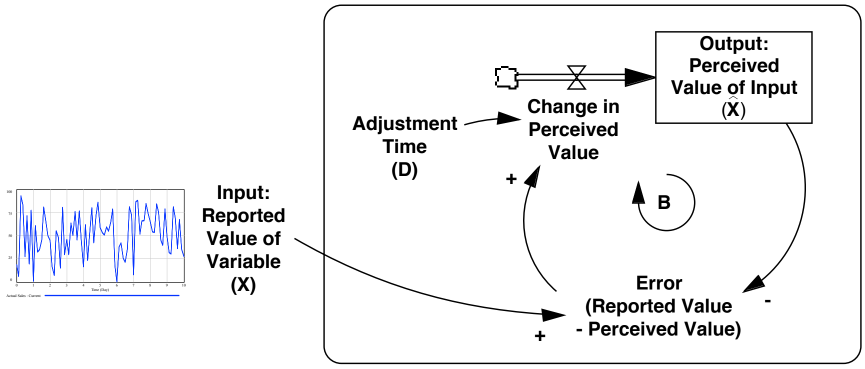
- Separates the signal from the noise
- Used mainly to model expectations (usually of flows)
  - Model of decision maker's expectation (what value might a variable take on?)
  - Similar to a forecast (exponential smoothing)



Actual Sales= RANDOM NORMAL(0, 100 , 50, 30 , 1)

17

Information Delay - Structure



$\hat{X}$ = INTEGRAL(Change in Perceived Value, $\hat{X}(0)$)

Change in Perceived Value = Error/D = $(X - \hat{X})/D$

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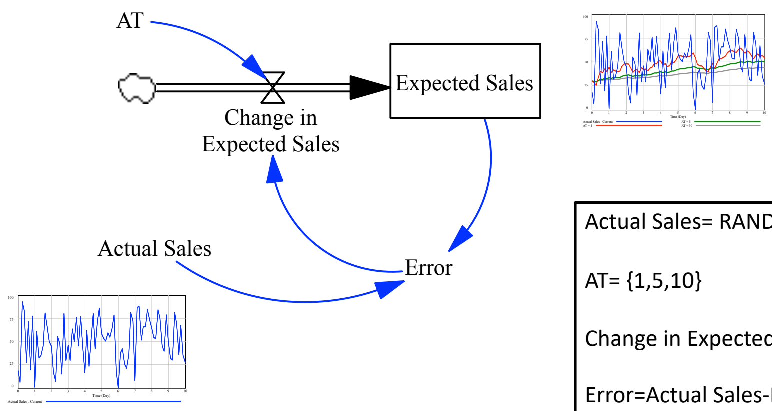
The Stock and Flow Model



Actual Sales= RANDOM NORMAL(0, 100 , 50, 30 , 1)

AT= {1,5,10}

Change in Expected Sales = Error/AT

Error=Actual Sales-Expected Sales

Expected Sales= INTEG (Change in Expected Sales, 30)

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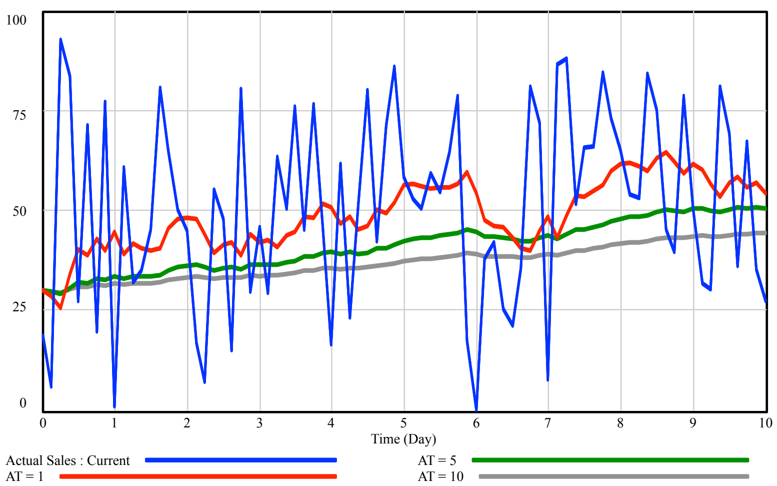
Impact of Adjustment Times



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Challenge 7.3

- For the Student model, add a new variable:
  - Expected Enrollments
- See how this behaves for a step increase in enrollments

CT561: Systems Modelling & Simulation

Lecture 8: Stock Management

Prof. Jim Duggan,
School of Engineering & Informatics
National University of Ireland Galway.
https://github.com/JimDuggan/SDMR

1

Recap

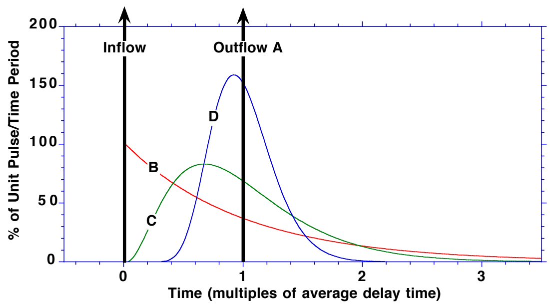
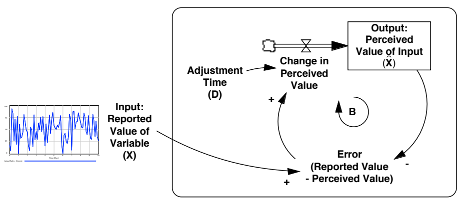


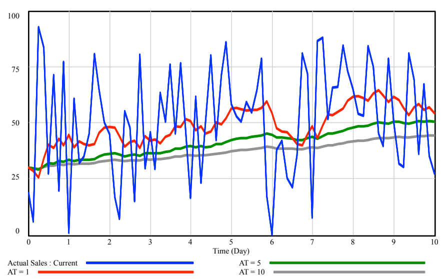
Figure 11-2 Some distributions of the outflow from a delay

The input I all cases is a unit pulse at time zero. Outflow A is a pipeline delay in which all items arrive together exactly 1 delay time after they enter.

Outflow distributions B-D exhibit different degrees of variation in processing times for individual items so some arrive before and some after the average delay time. In all cases the average delay time is the same and the areas under each distribution are equal.



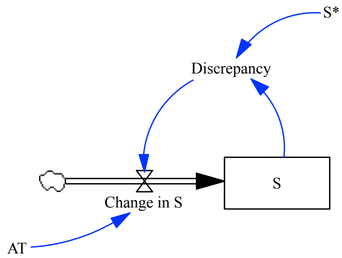
$\hat{X}$ = INTEGRAL(Change in Perceived Value, $\hat{X}$(0))
Change in Perceived Value = Error/D = (X – $\hat{X}$)/D

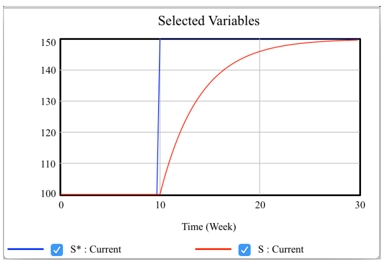


2

A familiar structure…

- Managers often seek to adjust the state of the system until it equals a goal or desired state.
- The simplest form of this negative feedback is
  - R_I = Discrepancy/AT = $(S^* - S)/AT$





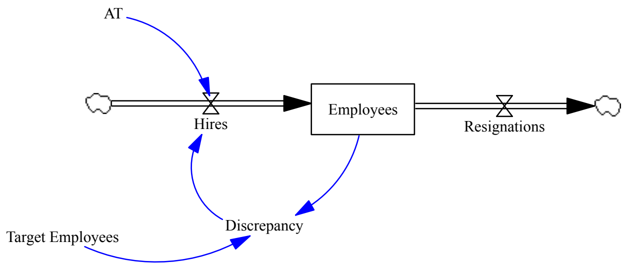
3

Observations on Goal Adjustment

- "**Desired** minus **actual** over **adjustment time**" is the classic linear negative feedback system. (Sterman 2000).
- Examples:
  - **Change in Price** = (Competitor Price – Price)/Price Adjustment Time
  - **Heat Loss from Building** = (Outside Temperature – Inside Temperature)/Temperature Adjustment Time
  - **Net Hiring Rate** = (Desired Labour – Labour)/Hiring Delay
- However, what happens if there is an outflow to the stock?

4

Model behaviour over time?



AT = 2

Discrepancy = Target Employees - Employees

Employees = INTEG(Hires - Resignations , 100)

Hires = Discrepancy / AT

Resignations = {0,10,50}

Target Employees = 100 + step (50, 6)

Steady State Error (Sterman P524)

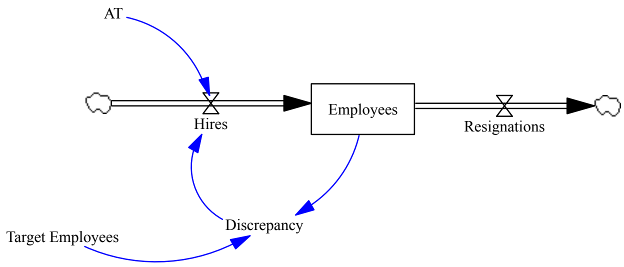
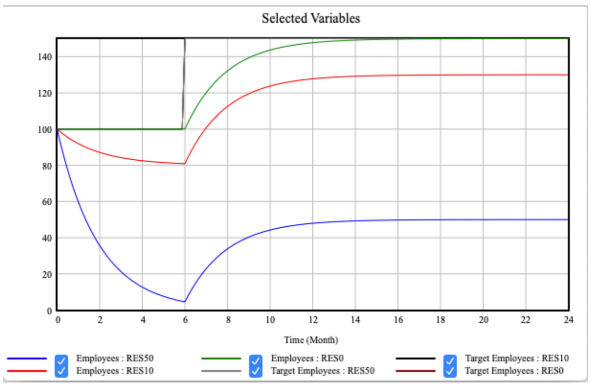


Table | Employees

Time (Month)	23.625	23.75	23.875	24
Employees : RES50	49.9949	49.9952	49.9955	49.9958
Employees : RES10	129.995	129.995	129.995	129.995
Employees : RES0	149.994	149.995	149.995	149.995

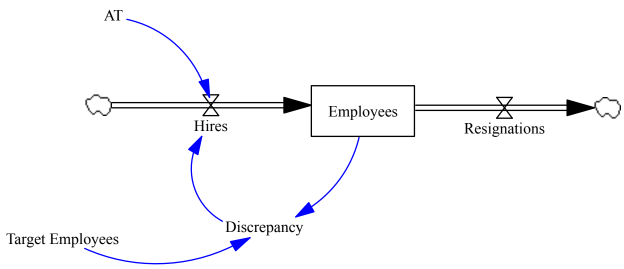


When will the stock be in equilibrium?

When Hires == Resignations

$(E^* - E)/AT = R$
$(E^* - E) = R * AT$
$E = E^* - R * AT$

Steady State Error (Sterman P524)

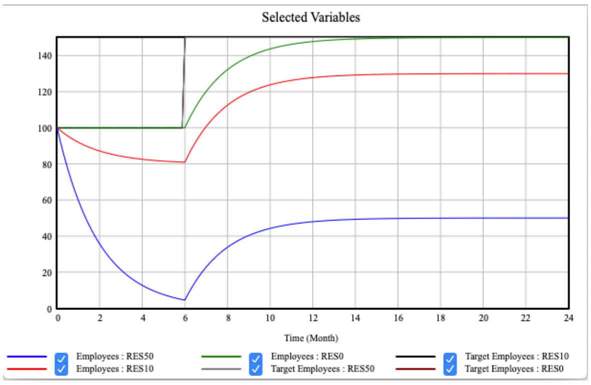


$E = E^* - R * AT$

E^*	R	AT	E
150	0	2	150
150	10	2	130
150	50	2	50

Table | Employees

Time (Month)	23.625	23.75	23.875	24
Employees : RES50	49.9949	49.9952	49.9955	49.9958
Employees : RES10	129.995	129.995	129.995	129.995
Employees : RES0	149.994	149.995	149.995	149.995



7

Challenge 8.1

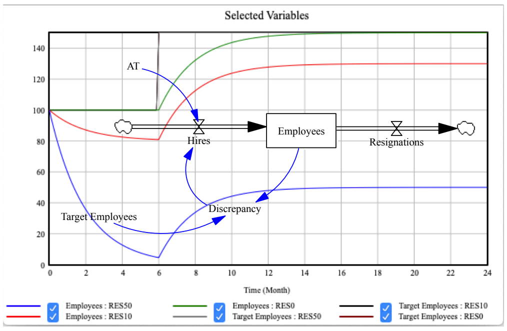
- Explore the steady state error in Vensim
- Confirm the equation $E = E^* - R * AT$

8

How to Manage?

E = E* - R * AT

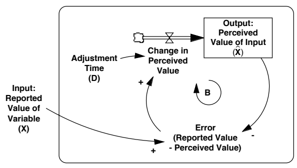
E*	R	AT	E
150	0	2	150
150	10	2	130
150	50	2	50



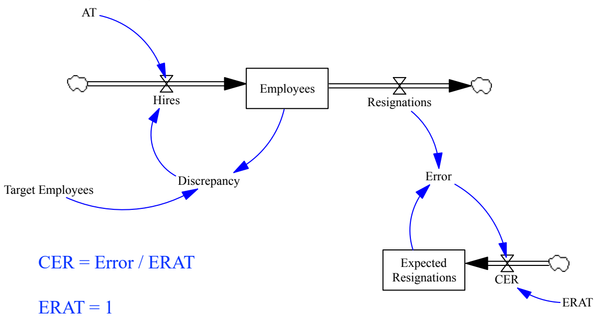
- Where there is an outflow, the equilibrium will be less than the goal
- The larger the outflow, the greater the equilibrium shortfall will be
- To manage this, we need to account for the expected outflow to prevent this steady state error
- This is can be achieved using the stock management structure

9

Add Expected Losses



$\hat{X}$ = INTEGRAL(Change in Perceived Value, $\hat{X}(0)$)
Change in Perceived Value = Error/D = $(X - \hat{X})/D$



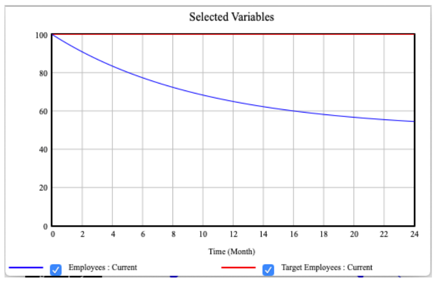
CER = Error / ERAT

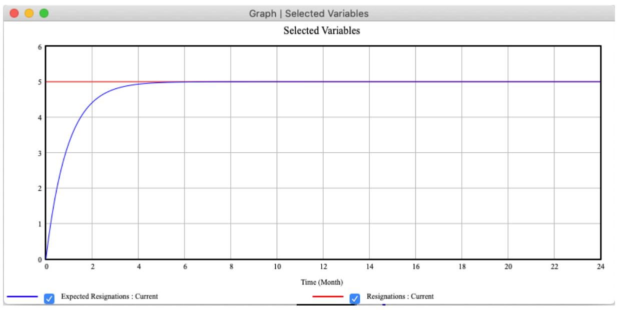
ERAT = 1

Error = Resignations - Expected Resignations

Expected Resignations = INTEG(CER , 0)

Resignations = 5





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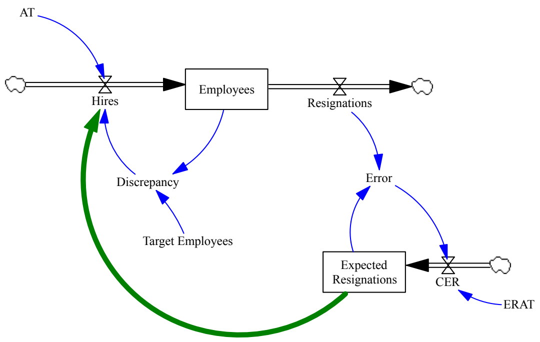
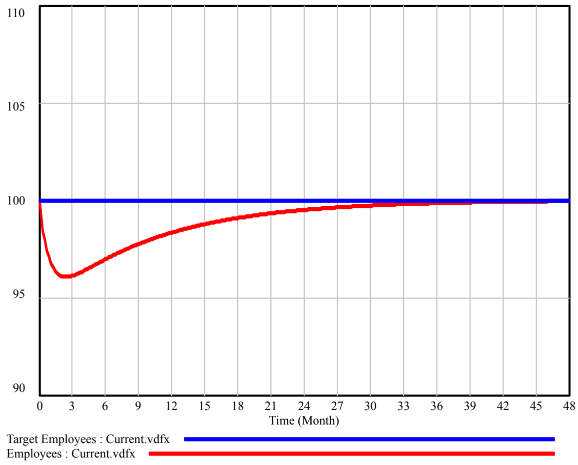
Account for this in the inflow – Stock reaches its goal

Stock Management Structure



Hires=max(0,**Expected Resignations**+Discrepancy/AT)



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Challenge 8.2

- Explore the steady state solution in Vensim
- Show that the state reaches the goal in steady state

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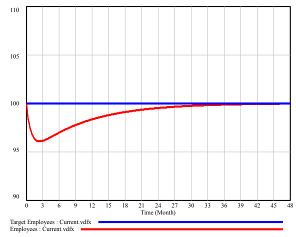
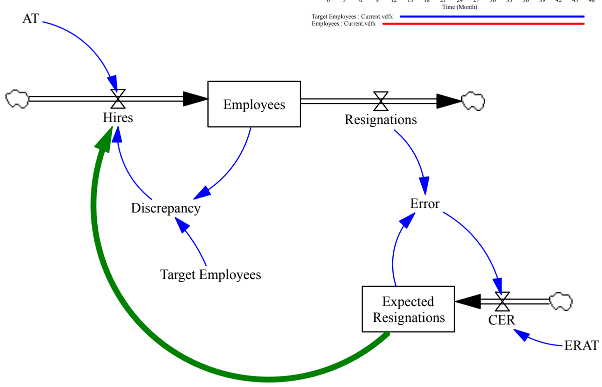
Rules for managing a stock
Anchor and *Adjustment*



- Managers should replace expected losses from the stock **(the anchor)**
- Managers should reduce the discrepancy between the desired and actual stock **(the Adjustment)**. Acquire:
  - more than the expected losses when the stock is less that the desired,
  - less than the expected losses when there is a surplus.



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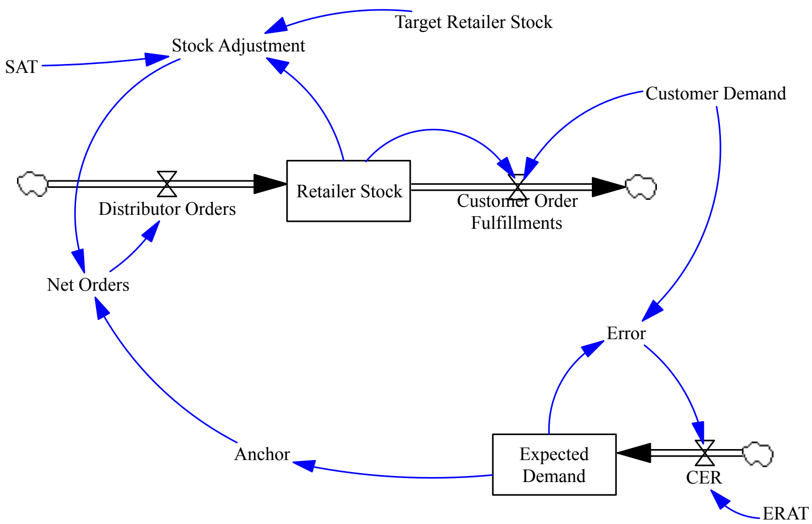
The Stock and Flow Model



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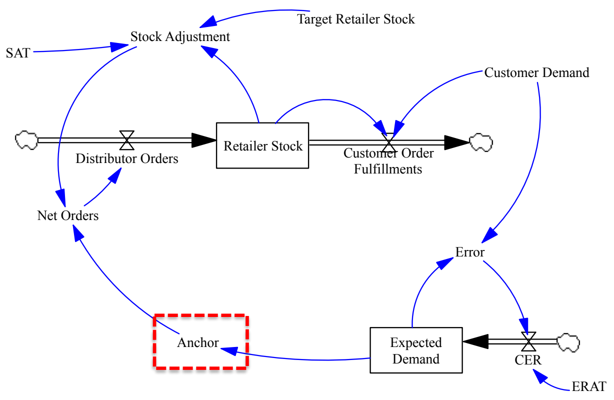
Stock Management: The anchor



Anchor=Expected Demand

CER=Error/ERAT

Customer Demand=100+step(100,30)-step(150,60)

ERAT=5

Error=Customer Demand-Expected Demand

Expected Demand= INTEG (CER,100)

Managers should replace expected losses from the stock

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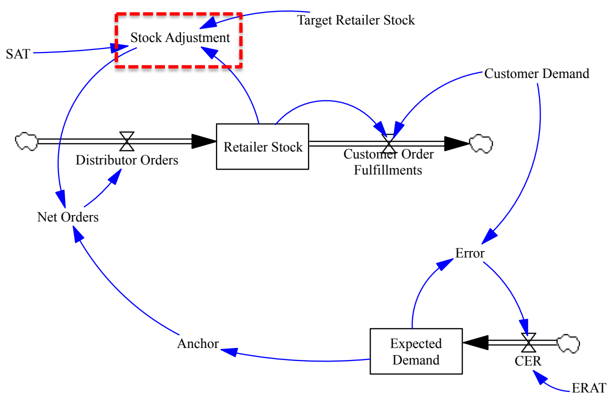
Stock Management: The adjustment



SAT=3

Stock Adjustment= (Target Retailer Stock-Retailer Stock)/SAT

Target Retailer Stock=400

Retailer Stock= INTEG (Distributor Orders-Customer Order Fulfillments, 400)

Managers should reduce the discrepancy between the desired and actual stock

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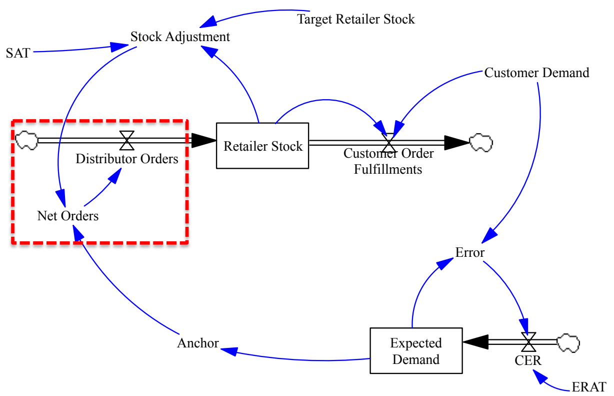
Stock Management: Decision Rule



Distributor Orders= max(0,Net Orders)

Net Orders =Anchor + Stock Adjustment

The ordering amount requested for each time (non-negative)

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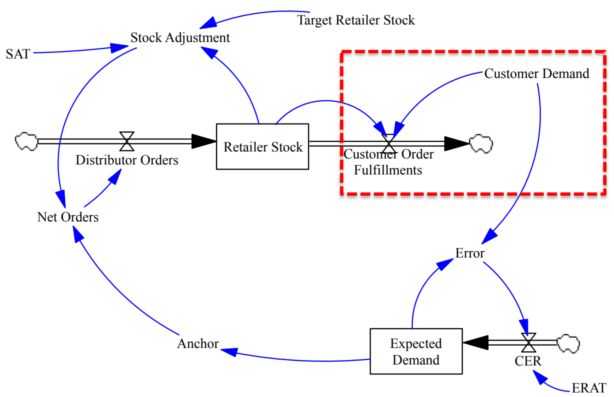
Stock Management: Order Fulfillment



Customer Demand=
100+step(100,5)-step(150,30)-step(30,60)

Customer Order Fulfillments=
min(Customer Demand,Retailer Stock)

Order fulfilment fulfils all demand, constrained by the stock availability

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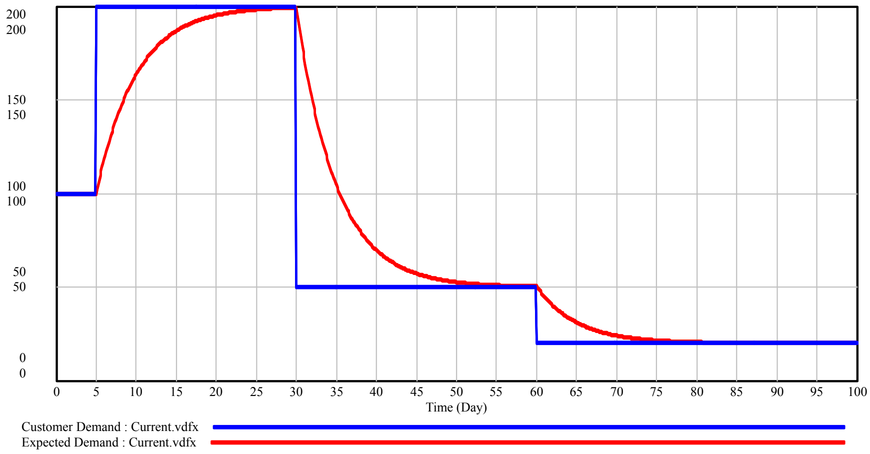
Demand and Expected Demand



Expected demand is driven by actual demand (information smoothing)

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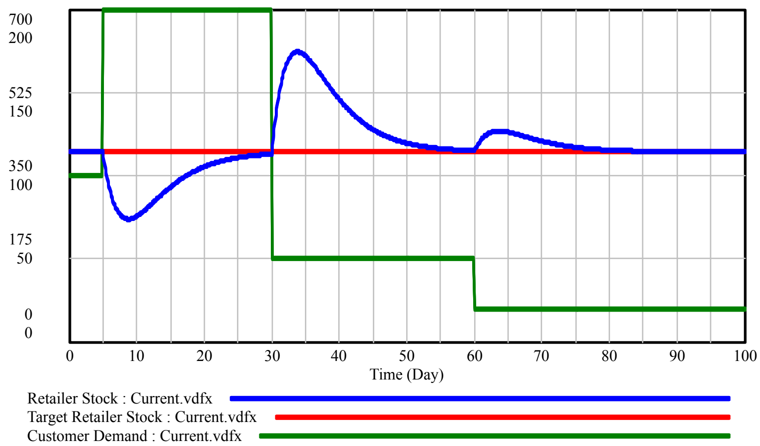
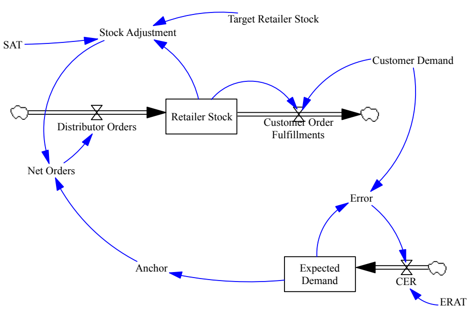
Stock and Target Stock





1. *Step in demand leads to a reduction in stock – Reduction leads to increased adjustment*
2. *Reduction in demand leads to surplus in stock, which then declines to target level*

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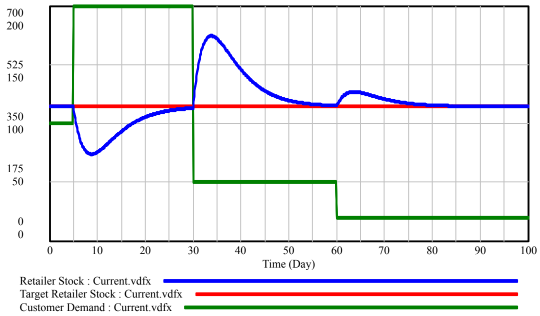
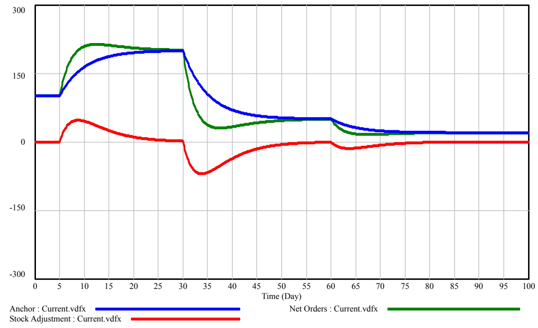
Anchor, Adjustment and Net Orders

1. *Adjustment is positive with a stock deficit compared to target.*
2. *Adjustment is negative with a stock surplus compared to target*
3. *Anchor always approaches true demand (it is expected demand)*
4. *Net orders are the sum of anchor plus adjustment*
5. *Stock reaches its target value*





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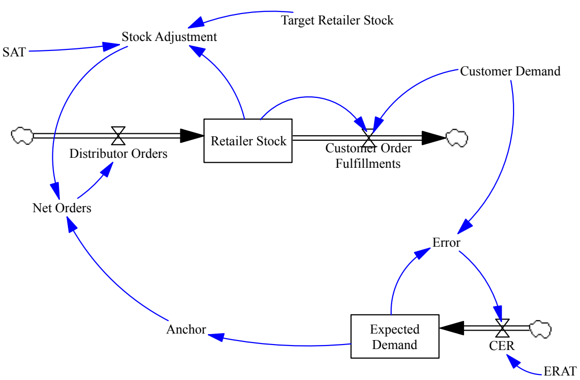
Summary: ***Anchor*** and ***Adjustment***

- Managers should replace expected losses from the stock **(the anchor)**
- Managers should reduce the discrepancy between the desired and actual stock **(the Adjustment).** Acquire:
  - more than the expected losses when the stock is less that the desired,
  - less than the expected losses when there is a surplus.



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Challenge 8.1

Use the stock management structure to build a stock and flow model (with equations) of employee hiring and progression through a software organisation (assume that only Rookies are hired).

There are two kinds of employees: (1) Rookies who are hired and (2) Experienced, who transition from Rookies after a first order time delay of 150. The Rookie quit rate is 10%, and the Experienced quit rate is normally 5%.

Discuss how different values for the hiring adjustment time would impact the number of Rookies in the organisation.

Challenge 8.2

7. Formulate equations for the following description:

   The number of rookies in the organisation impacts the quit rate of experienced employees. If the number of rookies exceeds a reference value of 30%, the quit rate will rise, and if it falls below the reference value, the quit rate will fall.

   The normal quit rate is 5%. The overall quit rate has a maximum value of 10%, and a minimum value of 3%.

CT561: Systems Modelling & Simulation

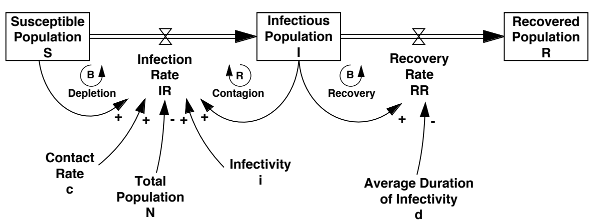
Lecture 9: SIR Model Part 1

Prof. Jim Duggan,
School of Engineering & Informatics
National University of Ireland Galway.
https://github.com/JimDuggan/SDMR

1

Public Health: Modelling Infectious Disease Outbreaks

- Context
- SIR Model
- Related variables



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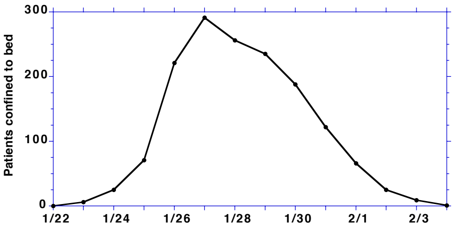
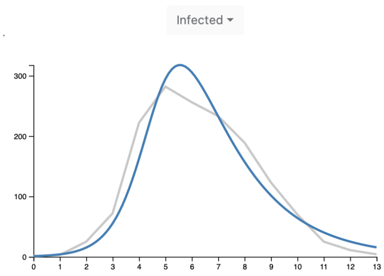
(1) Context – Outbreak Dynamics

Figure 9-3
Dynamics of epidemic disease
Sources: British Medical Journal, 4 March 1978, p. 587;.



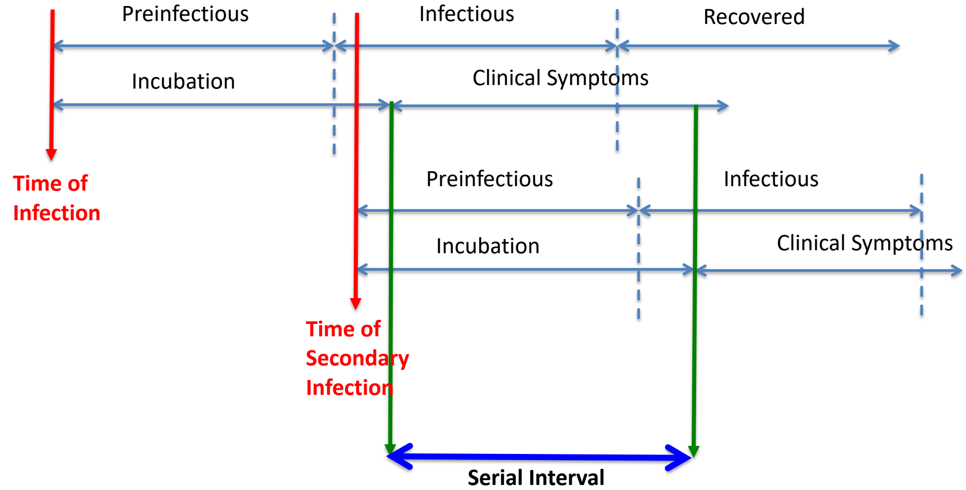


Influenza epidemic at an English boarding school, January 22-February 3, 1978. The data show the number of students Confined to bed for influenza at any time (the stock of symptomatic individuals).

Examples of Infectious Agents – Microparasites (Vynnycky & White)

Type of Agent	Characteristics	Examples
Virus	Small, simple, obligatory parasites of larger cells	Measles, Mumps, Rubella, Ebola, Smallpox, SARS, Influenza
Bacteria	Larger and more complex than viruses- many are able to grow independently but some require a cell host	Bordella pertussis (whooping cough), Mycobacterium tuberculosis (tubercolosis), Salmonella typhi (typhoid fever)
Protozoa	Large single-celled organisms, more complex than bacteria- many are able to grow independently but some require a cell host	Plasmodium falciparum (Malaria)

Typical Life cycle of infection.



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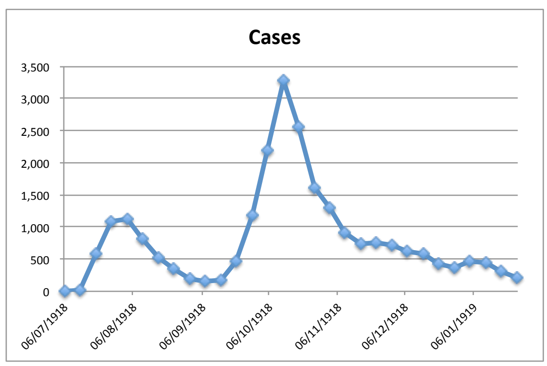
Data from 1918/19

- The second wave of the 1918 pandemic was much deadlier than the first.
- The first wave had resembled typical flu epidemics; those most at risk were the sick and elderly, while younger, healthier people recovered easily.
- But in August, when the second wave began in France, Sierra Leone and the United States, the virus had mutated to a much deadlier form. This has been attributed to the circumstances of the First World War.

Gothenberg Data 1918/19



http://en.wikipedia.org/wiki/1918_flu_pandemic

6

Challenge 9.1

https://github.com/owid/covid-19 data/tree/master/public/data

- Explore the data set our world in data (COVID-19)
- Focus on the epidemic curves (behaviour over time)

README.md

Data on COVID-19 (coronavirus) by *Our World in Data*

Our complete COVID-19 dataset is a collection of the COVID-19 data maintained by *Our World in Data*. It is updated daily and includes data on confirmed cases, deaths, hospitalizations, and testing, as well as other variables of potential interest.

Download our complete COVID-19 dataset : CSV | XLSX | JSON

We will continue to publish up-to-date data on confirmed cases, deaths, hospitalizations, and testing, throughout the duration of the COVID-19 pandemic.

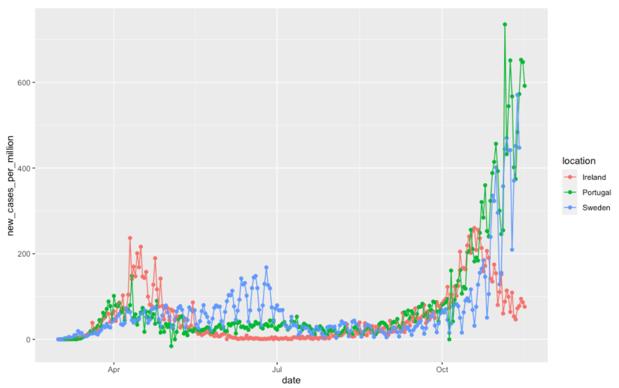
Our data sources

- **Confirmed cases and deaths:** our data comes from the European Centre for Disease Prevention and Control (ECDC). We discuss how and when the ECDC collects and publishes this data here. The cases & deaths dataset is updated daily. *Note: the number of cases or deaths reported by any institution—including the ECDC, the WHO, Johns Hopkins and others—on a given day does not necessarily represent the actual number on that date. This is because of the long reporting chain that exists between a new case/death and its inclusion in statistics.* ***This also means that negative values in cases and deaths can sometimes appear when a country sends a correction to the ECDC, because it had previously overestimated the number of cases/deaths. Alternatively, large changes can sometimes (although rarely) be made to a country's entire time series if the ECDC decides (and has access to the necessary data) to correct values retrospectively.***

A	B	C	D	E	F	G	H	I	J	K	L	M
iso_code	continent	location	date	total_cases	new_cases	cases_smo	otal_death	new_deaths	deaths_smo	cases_per_r	ases_per_n	smoothed_
ABW	North America	Aruba	2020-03-13	2	2			0		18.733	18.733	
ABW	North America	Aruba	2020-03-19			0.286			0			2.676
ABW	North America	Aruba	2020-03-20	4	2	0.286		0	0	37.465	18.733	2.676
ABW	North America	Aruba	2020-03-21			0.286			0			2.676
ABW	North America	Aruba	2020-03-22			0.286			0			2.676
ABW	North America	Aruba	2020-03-23			0.286			0			2.676
ABW	North America	Aruba	2020-03-24	12	8	1.429		0	0	112.395	74.93	13.38
ABW	North America	Aruba	2020-03-25	17	5	2.143		0	0	159.227	46.831	20.071
ABW	North America	Aruba	2020-03-26	19	2	2.429		0	0	177.959	18.733	22.747

(2) Susceptible-Infected-Recovered Model

- The total population of a region/community is divided into three categories:
  - Those susceptible to the disease (S)
  - Those who are infectious (I)
  - Those who have recovered (R)
- Births, deaths & migration ignored
- Population homogenous (no sub-groups)
- Once infected people recover after a time lag



Stocks and Flows

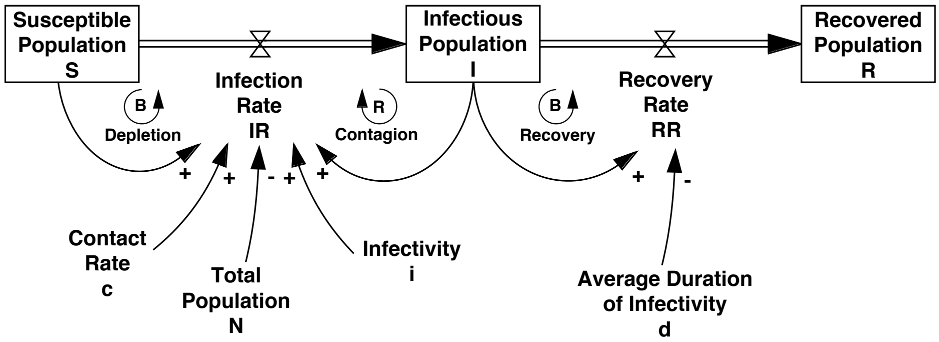


Figure 9-5 People remain infectious (and sick) for a limited time, then recover and develop immunity.

Stock Equations

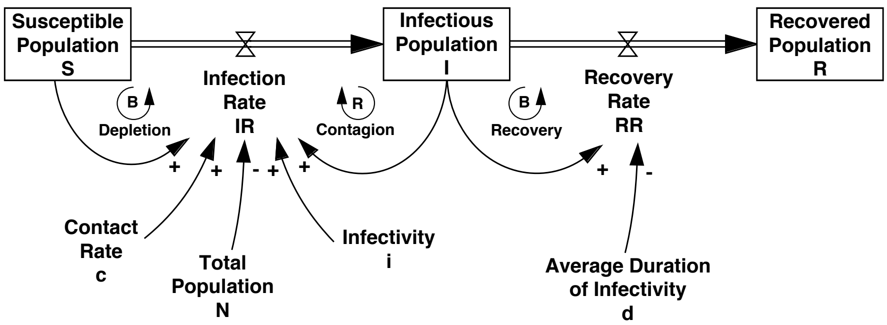


Figure 9-5 People remain infectious (and sick) for a limited time, then recover and develop immunity.

S = INTEG(-IR, 9999)
I = INTEG(IR-RR, 1)
R = INTEG(RR,0)

RR = I / d

Formulate the Infection Rate

- **Contact Rate** (people/person/time period)
- **Number of Infected**

Product is the Number of encounters for Infectious People per time period

- **Number of Susceptible**
- **Total Population**

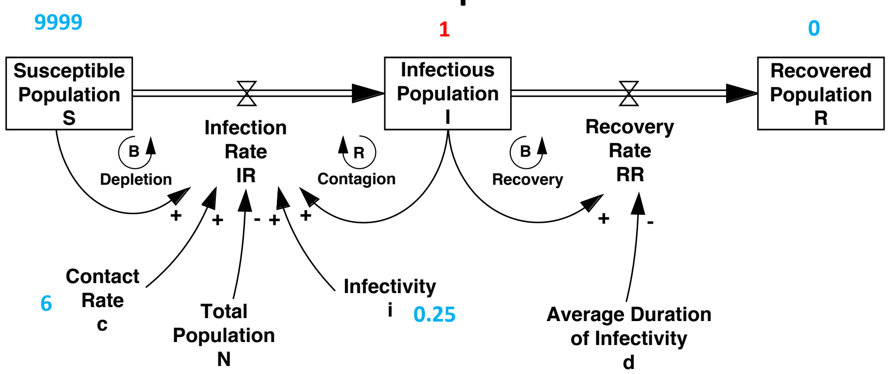
Ratio is the chance of meeting a Susceptible Person

- **Infectivity**

Probability that an encounter with a **Susceptible Person** will lead to an infection

IR = Contact Rate * Infectious * (Susceptible/N) * Infectivity

Example



Number of Infectious Contacts = 1 * 6 = 6 (#Encounters for infectious people)
Chance of Meeting Susceptible = 9999/10000 = 0.999
Probability of Transmission = 0.25

IR = 6 * 0.9999 * 0.25 = 1.4985

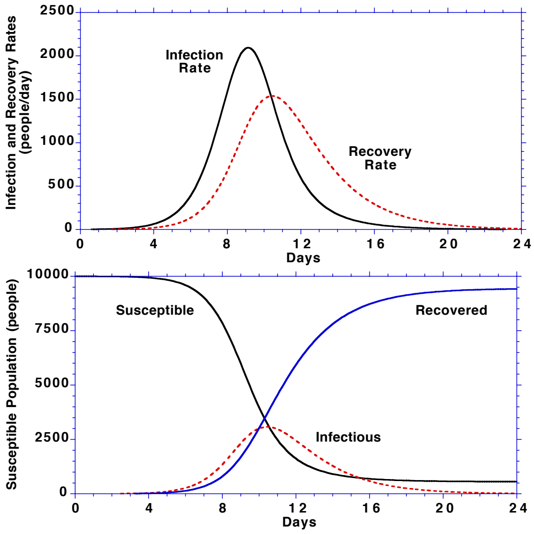
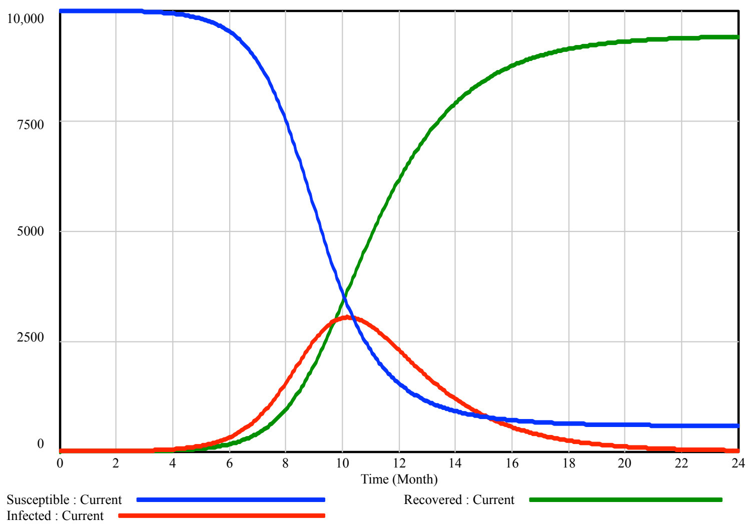


Figure 9-6 Simulation of an epidemic in the SIR model. The total population is 10,000. The contact rate is 6 per person per day, infectivity is 0.25, and average duration of infectivity is 2 days. The initial infective population is 1, and all others are initially susceptible.

Vensim Output



Epidemic dynamics for different contact rates

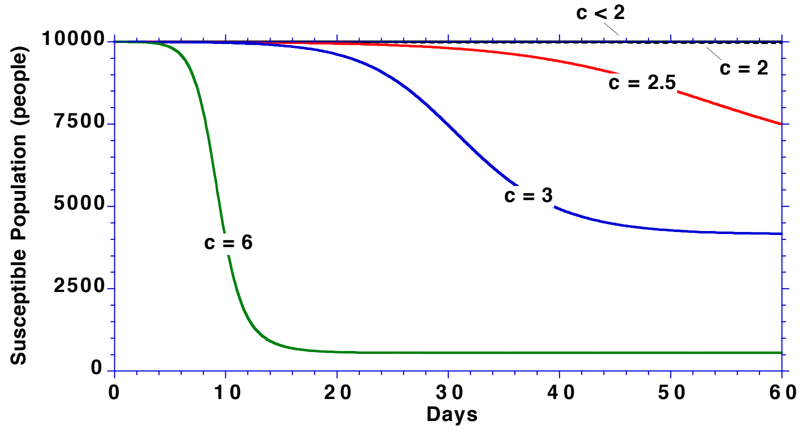


Figure 9-7 The contact rate is noted on each curve; all other parameters are as in Figure 9-6.

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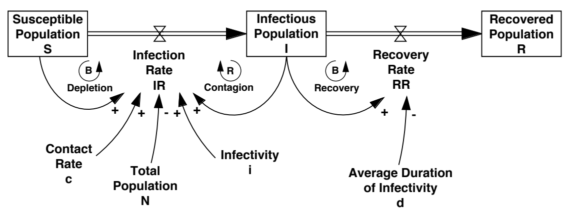
Summary:
Key feature of SIR model

- It captures a fundamental feature of infectious diseases
- The disease spreads through contact between susceptible and infectious
- Nonlinear equation



IR = Contact Rate * Infectious * (Susceptible/N) * Infectivity

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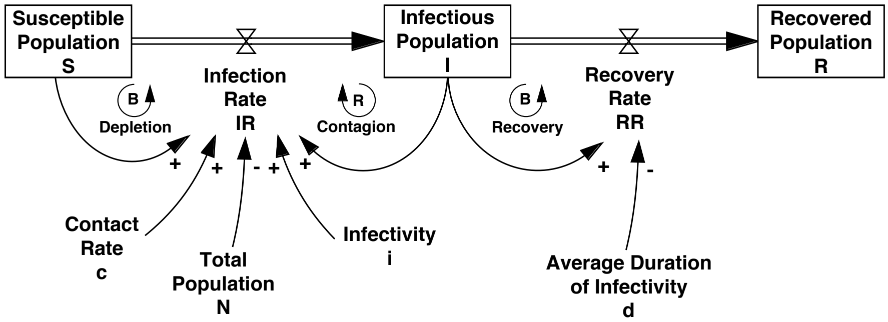
Challenge 9.2



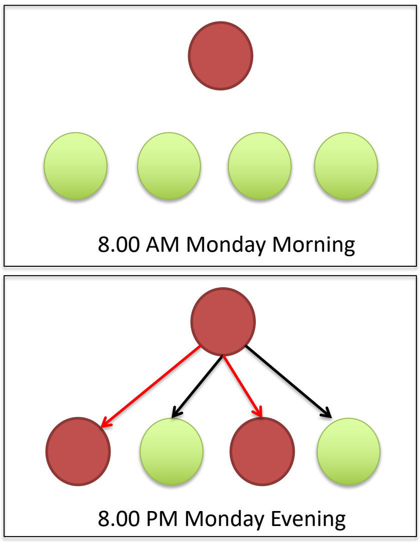
- Explore the SIR Model
- What three model conditions will halt disease spread?

(3) Related variables

- Effective contacts
- Reproduction Number

Effective Contacts (C_e) = c * i

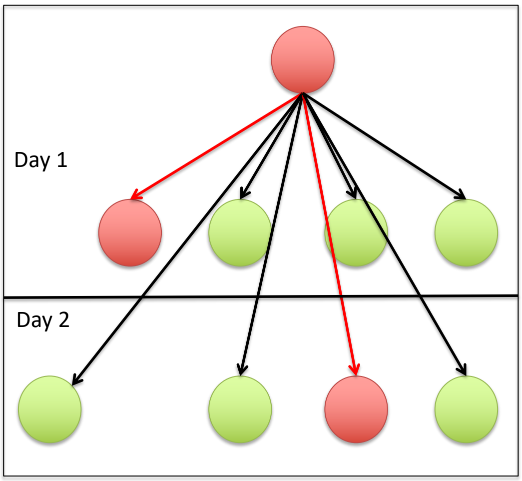
- Defined as one which is sufficient to lead to infection, were it to occur between a **susceptible** and **infectious** individuals
- For example, if $C_e = 2$
  - An infectious person will infect two susceptible people in one day
  - They could meet 4 people, and pass on the virus with probability (0.50)



Reproduction Number – R_0

- Formally defined as the average number of secondary infectious resulting from a typical infectious person being introduced to a totally susceptible population

$$\boxed{R_0 = c_e D}$$



$D = 2 \quad c_e = 1 \quad R_0 = 2$

Approximate data for common potentially vaccine-preventable diseases

Infection	Serial Interval (Range)	R_o	Herd Immunity
Diphtheria	2-30 Days	6-7	85
Influenza	2-4 Days	2-4	50-75
Malaria	20 Days	5-100	80-99
Measles	7-16 Days	12-18	83-94
Pertussis	5-35 Days	12-17	92-94

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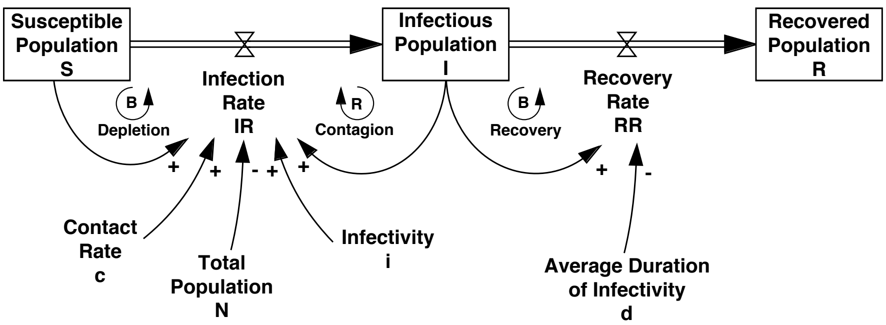
Challenge 9.3



- Add R0 to the SIR Model

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Challenge 9.4 COVID-19 Model

- Build a population model of COVID-19
- Assume an SEIR basic structure
- Assume two streams:
  - Clinical Fraction (Pre-Clinical and Clinical)
  - Sub-clinical Fraction
- Assume subclinical are 50% as infectious as clinical
- Use Population ~ 5M
- 10 People infected at start
- Estimate R0

nature medicine LETTERS
https://doi.org/10.1038/s41591-020-0962-9
Check for updates

Age-dependent effects in the transmission and control of COVID-19 epidemics

Nicholas G. Davies[1], Petra Klepac[1,2], Yang Liu[1,2], Kiesha Prem[1], Mark Jit[1], CMMID COVID-19 working group and Rosalind M. Eggo[1]

Parameter	Description	Value
Latent Time	Time Spent in Exposed Stock	3 Days
Pre-Infectious Clinical Time	Time Spent Pre-Infectious (and spreading)	2.1 Days
Infectious Time (Clinical)	Time spent infectious (and spreading)	2.9 Days
Infectious Time (Sub Clinical)	Time spent infectious	5 Days
Clinical Fraction	Proportion of infected who show symptoms	0.60
Contacts	Contacts/Person/Day	10
Infectivity	Probability of transmission	0.10

CT561: Systems Modelling & Simulation

Lecture 10: SIR Model Part 2

Prof. Jim Duggan,
School of Engineering & Informatics
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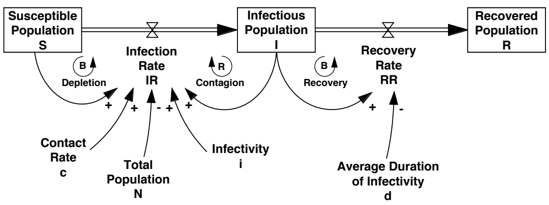
https://github.com/JimDuggan/SDMR

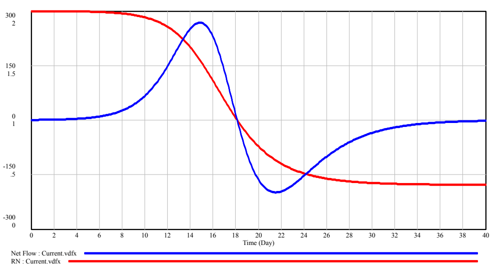
1

Overview

IR = Contact Rate * Infectious * (Susceptible/N) * Infectivity

- Force of Infection (Lambda)
- Threshold Dynamics
- Net Reproduction Number and Herd Immunity
- Exploring downstream effects with the SEIR model (with clinical and sub-clinical streams)





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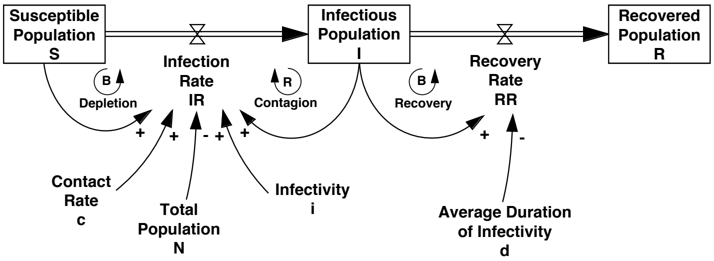
(1) Force of Infection (Lambda)

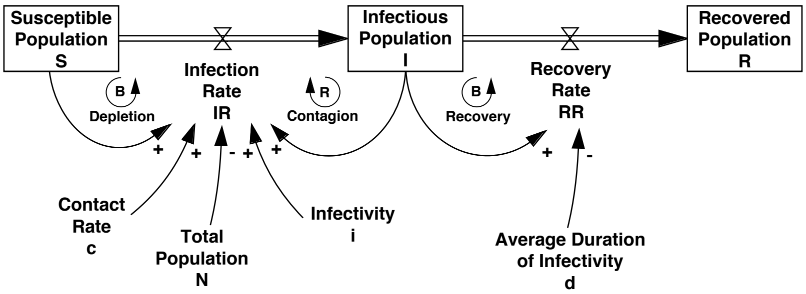
The rate at which susceptible individuals become infected per unit time. It is also known as the incidence rate or the hazard rate.



IR = Susceptible * Contact Rate * Infectivity / N * Infectious
= Susceptible * FOI

Beta

The per capita rate at which two specific individuals come into contact per unit time.



IR = Susceptible * Contact Rate * Infectivity / N * Infectious
= Susceptible * FOI
= Susceptible * Beta * Infectious

Challenge 9.1

- Suppose we have a town with 100,000 (=N) individuals, of which 1% were infectious with a novel pathogen, with R_0 = 14 and recovery delay (D) =7 days. Calculate the:
  - Effective per capital contact rate β
  - Force of infection λ

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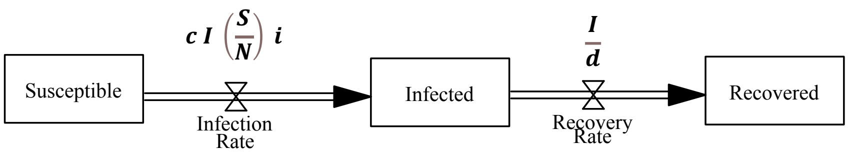
(2) Threshold Dynamics of SIR Model



For the number of infectious people to increase, the inflow must be greater than the outflow.
Assume S==N in a totally susceptible population

$$c\,I\left(\frac{S}{N}\right)i > \frac{I}{d} \longrightarrow c\left(\frac{S}{N}\right)i > \frac{1}{d} \longrightarrow c\;i\,d > 1 \longrightarrow R_0 > 1$$

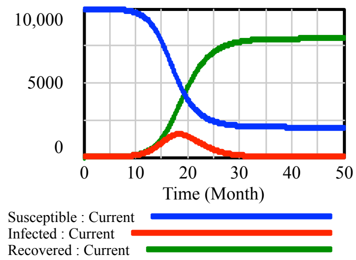
6

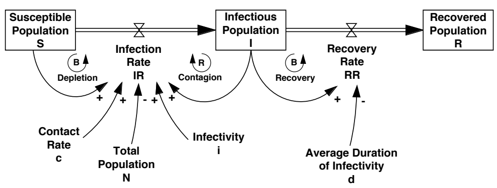
R_0 Threshold Phenomenon (Keeling & Rohani 2008)

- Assuming everyone in the population is initially susceptible, a pathogen can only invade if $R_0 > 1$
- Due to differences in demographic rates, rural-urban gradients, and contact structures, different populations may be associated with different values of R_0 for the same disease.
- *R_0 depends on the disease and the population*

Observation

"The chain of transmission eventually breaks due to a decline in **Infectives**, NOT due to a complete lack of **Suceptibles"**





IR = Contact Rate * Susceptible* (Infectious/N) * Infectivity

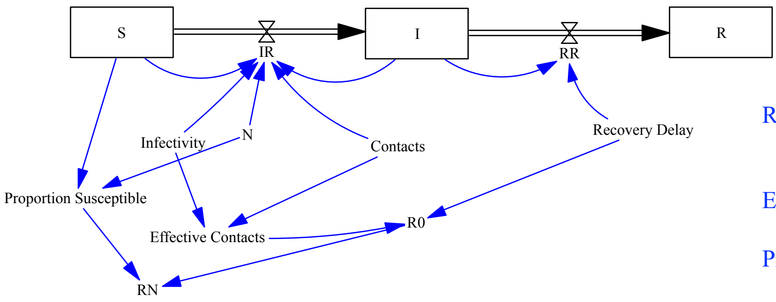
Challenge 9.2

- Estimate whether an epidemic will occur in the following scenario:
  - N = 100,000
  - It's a novel pathogen
  - The average contacts per day are 8
  - The probability of infection given contact between and infectious and susceptible person is 0.25
  - The duration of infectiousness is 1 day

Net Reproduction Number

- R_n is the net reproduction number
- Useful to evaluate as an epidemic proceeds
- **$R_n = (S/N) * R_0$**
- "The average number of secondary infectious persons resulting from one infectious person in a given population in which *some individuals may already be immune because of infection or vaccination*"
- When $R_N <= 1$, no epidemic occurs – the infectious stock goes to zero

Adding R_N to the SIR Model



RN=Proportion Susceptible*R0

Effective Contacts = Contacts * Infectivity

Proportion Susceptible = S / N

R0 = Effective Contacts * Recovery Delay

Recovery Delay = 2

RN = Proportion Susceptible * R0

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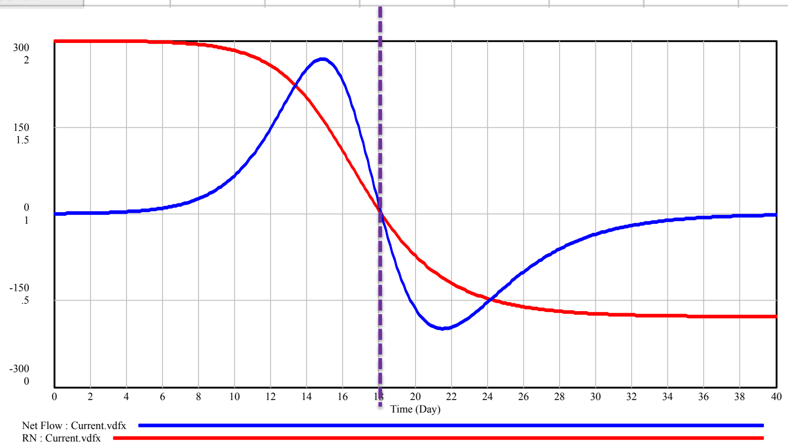
Exploring R_N and the dI/dt (Net Flow)

Time (Day)	18	18.125	18.25	18.375	18.5	18.625	18.75	18.875
Net Flow : Current	17.0731	2.18492	-12.431	-26.7112	-40.5977	-54.038	-66.9856	-79.3997
RN : Current	1.02227	1.00285	0.983794	0.965121	0.946842	0.928968	0.911509	0.894471



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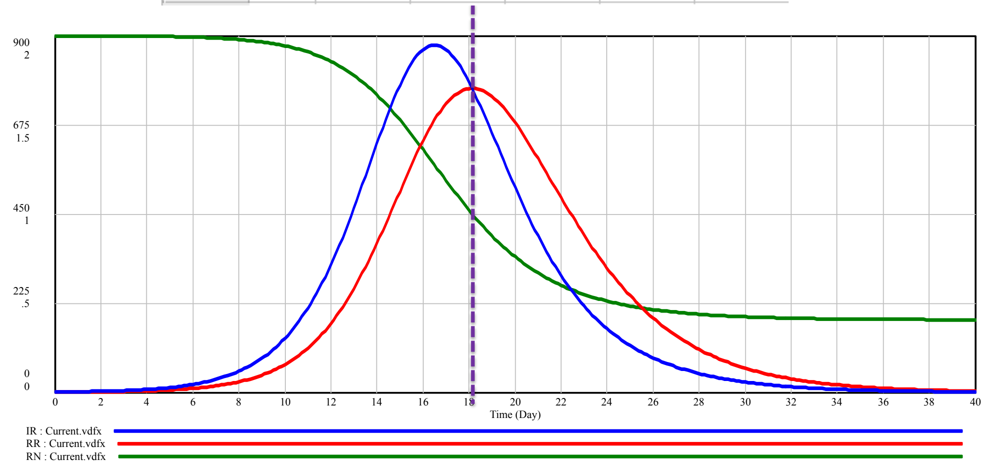
Exploring R_N and IR with RR

Time (Day)	18	18.125	18.25	18.375	18.5	18.625
IR : Current	783.845	769.557	754.619	739.114	723.122	706.722
RN : Current	1.02227	1.00285	0.983794	0.965121	0.946842	0.928968
RR : Current	766.771	767.372	767.05	765.825	763.719	760.76



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Challenge 9.3

- Calculate the net reproduction number in the following scenario
  - N = 100,000
  - 40,000 People are immune
  - The average contacts per day are 8
  - The probability of infection given contact between and infectious and susceptible person is 0.25
  - The duration of infectiousness is 2 days

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Exploring the R_N equation

$$R_n = R_0 * \left(\frac{S}{N}\right)$$

$$\text{If} \left(\frac{S}{N}\right) = \frac{1}{R_0} \; then \; R_n = 1$$

$$HIT = 1 - \frac{1}{R_0}$$

- When S/N = $1/R_0$, then each infectious person will lead to a single transmission (R_n=1)
- If the proportion susceptible is less than this, incidence will decrease
- This allows us to define a critical threshold for S, under which a disease will not spread

(3) Herd Immunity

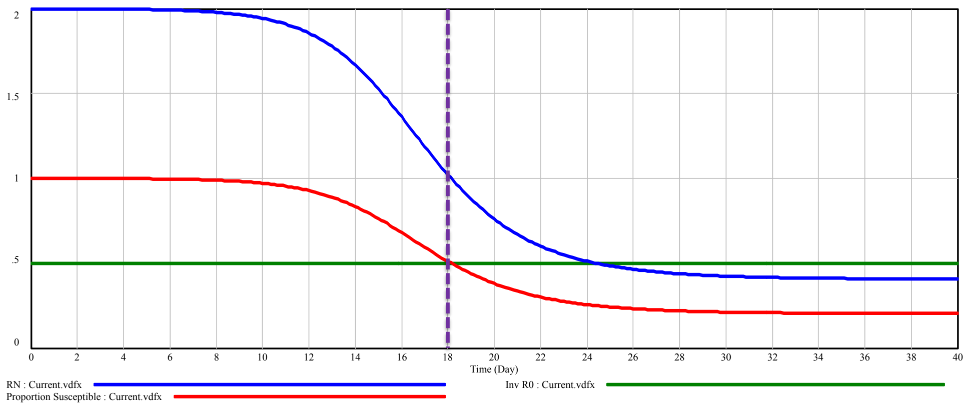
- If (1- $1/R_0$) proportion of the population can be vaccinated, the disease will not spread.
- Why? Because R_N = (S/N) * R_0
- If R_0 = 2, we vaccinate 50% of the population
- R_N = (5000/10000) * 2 = 1, one person infects only one, so no spread.

$$HIT = 1 - \frac{1}{R_0}$$

Infection	Serial Interval (Range)	R_o	Herd Immunity
Diphtheria	2-30 Days	6-7	85
Influenza	2-4 Days	2-4	50-75
Malaria	20 Days	5-100	80-99
Measles	7-16 Days	12-18	83-94
Pertussis	5-35 Days	12-17	92-94

Exploring R_N and $1/R_0$

Time (Day)	17.75	17.875	18	18.125	18.25	18.375	18.5	18.625
Inv R0 : Current	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Proportion Susceptible : Current	0.531065	0.521017	0.511133	0.501424	0.491897	0.482561	0.473421	0.464484
RN : Current	1.06213	1.04203	1.02227	1.00285	0.983794	0.965121	0.946842	0.928968



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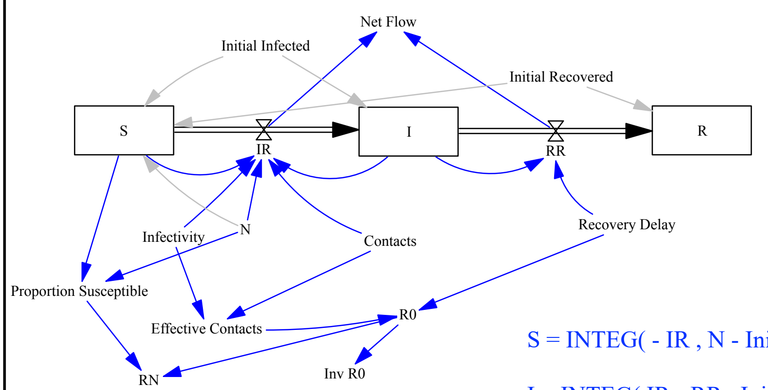
SIR with Vaccinations - Adjusted Model



Initial Infected = 1

Initial Recovered = 5000

N = 10000

S = INTEG(- IR , N - Initial Infected - Initial Recovered)

I = INTEG(IR - RR , Initial Infected)

R = INTEG(RR , Initial Recovered)

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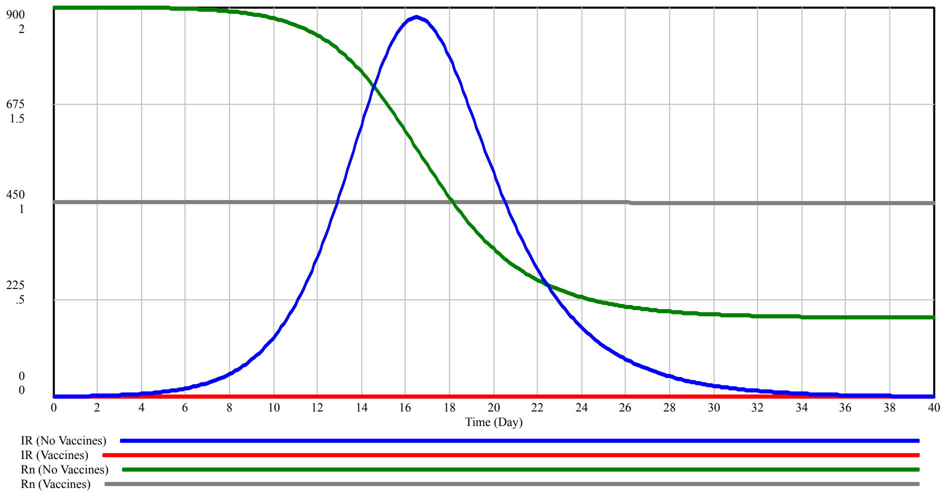
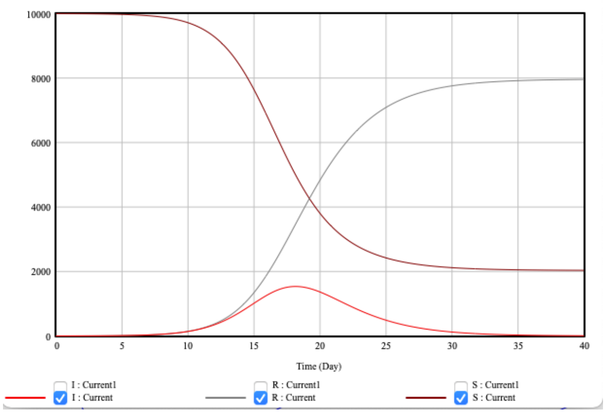
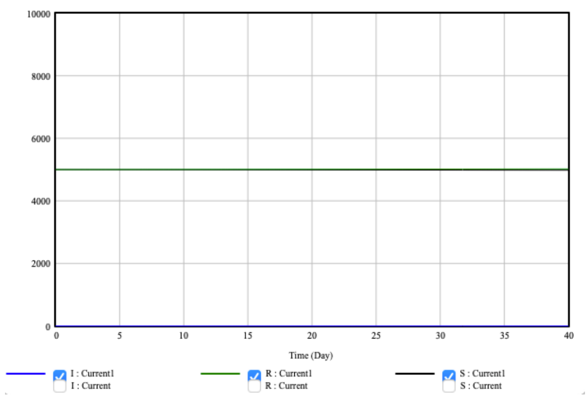
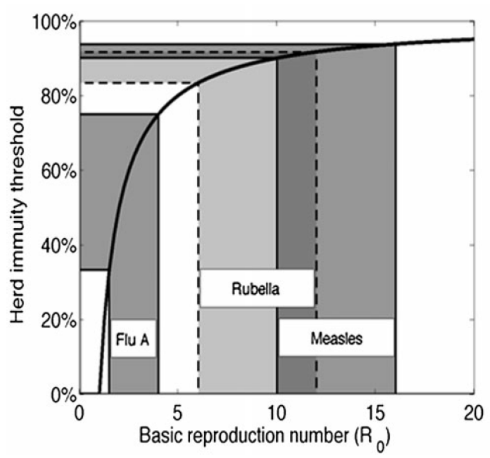
Scenarios

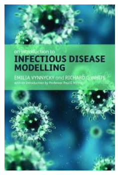


SIR Analysis









"In general, vaccination programmes aim to achieve a coverage which is above the herd Immunity threshold, since if the proportion of the population that is immune is above this value, substantial outbreaks are unlikely to occur."

"Herd Immunity": A Rough Guide

Paul Fine, Ken Eames, and David L. Heymann

Department of Infectious Disease Epidemiology, London School of Hygiene and Tropical Medicine, London, United Kingdom

Challenge 9.5 – Downstream Effects

- Build on the workshop three problem (see overleaf)
- Add a hospitalization stream to the model
- Assume that 5% of clinical people are hospitalized 6 days after they are no longer infectious
- Assume that, on average, they stay in hospital for 10 days. Assume it's a second order delay.
- When people leave hospital they are assumed to be recovered.
- Show the hospitalization rates, and the total number in hospital

CT561 – Workshop #3

Extending the SIR Model

The aim of this workshop is to extend the SIR model in the following ways:

1. Add an exposed stock that models people who have become infected but are not yet infectious. Assume the duration of exposure is 3 days.

2. There are now two kinds of infectious people (assume an infectious delay of 5 days):
    a. Sub-clinical, where they do not show symptoms. Sub-clinical people are half as infectious as clinically infectious people.

    b. Clinical, where people show symptoms

3. The breakdown between the two types of infectious people is determined by a constant called *clinical fraction*.

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4. Add an estimate of R_0 to the model, given that R_0 is defined as “the average number of secondary infectious persons resulting from a typical infectious person following their introduction to a totally susceptible population.”

5. Run for a population of 1M people, where 10 people are initially infectious. Assume people have, on average, 10 contacts per day, and the infectivity for clinical people is 10%. Assume that 40% of the population do not show any symptoms.

6. Run the model for different values of the clinical fraction (0, .2, .4, .6, .8, 1.0) and explain the results. How do these value impact R_0?

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CT561: Systems Modelling & Simulation

Lecture 11: Revision Part 1

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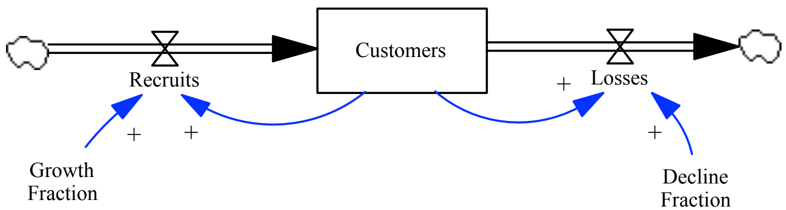
https://github.com/JimDuggan/SDMR

Note!

- In examination questions, differential equation terminology may be used to specify systems. You should be familiar with both types of specification.

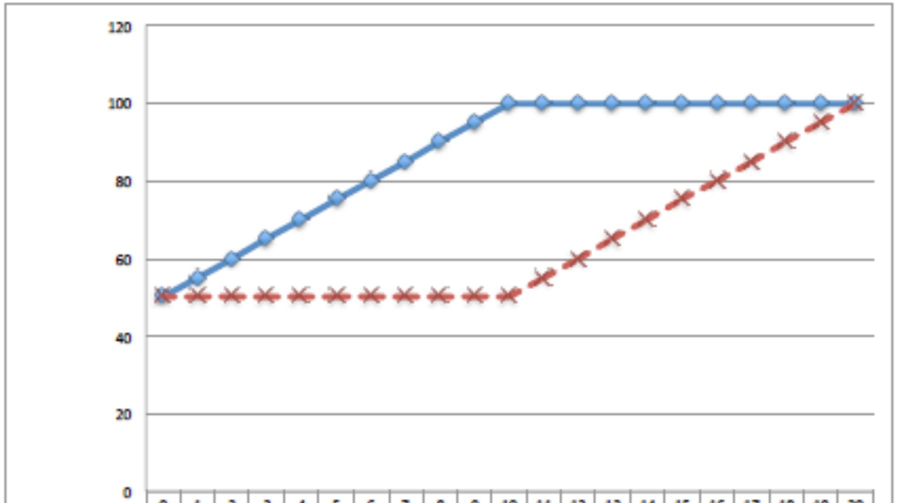


$$\frac{dC}{dt} = gC - dC$$

2018 Q1

1. (a) Based on the flow variables shown in the diagram, and assuming that the initial value of the stock is 100, perform the following:

- Calculate, and show, the net flow.
- Use graphical integration to plot the stock over time, clearly showing the behaviour mode for each segment (e.g. *increasing at an increasing rate, etc.)*



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Inflow	50	55	60	65	70	75	80	85	90	95	100	100	100	100	100	100	100	100	100	100	100
Outflow	50	50	50	50	50	50	50	50	50	50	50	55	60	65	70	75	80	85	90	95	100

[10]

(b) Calculate the stock using Euler's equation where DT=5. Estimate the integration error for each time step of 5.

[8]

(c) Draw a stock and flow model of the variables. Show how Little's Law can be used to show that the steady state delay value (from time 20 onwards) will be 6. What kind of delay distribution is suggested by the behaviour of the outflow when compared to that of the inflow?

[7]

2018 Q2

2. (a) Construct a stock and flow model from the following description of a Health Care system. Specify the stock equation (there is no need to document equations for flows or auxiliaries). Clearly show the link polarities on the diagram.

- Patients are **scheduled** for treatment, and this increases the number of **patients waiting**. An increase in **treatments completed**, causes a decrease in **patients waiting.**
- As the number of **patients waiting** increases, so too does the **health care system pressure.**
- An increase in **health care system pressure** leads to two system responses:
  - The **time per procedure** is reduced.
  - The **workweek** is increased.
- An increase in **time per procedure** leads to a decrease in **treatments completed.**
- A decrease in **time per procedure** leads to an increase in **follow-up visits**, which in turn, will increase **patients waiting.**
- An increase in **workweek** leads to an increase in **treatments completed.**
- An increase in **workweek** leads to an increase in **fatigue** (after a delay), and increasing **fatigue** leads to an increase in **time per procedure.**

[8]

(b) Based on the stock and flow model in part(a):

- Show the following feedback loops on the model: *"Midnight Oil"*, *"Burnout"*, *"Corner Cutting"*, and *"Haste Makes Waste"*.

- Starting with the proposition that the stock (**patients waiting**) is increasing, calculate the polarity for each of these loops, and comment on how the loops would impact the stock over time.

[12]

(c) What are the advantages of building this feedback model? How can its insights be use to identify ways to improve the Health Care System?

[5]

2018 Q3

3. (a) Given a boarding school with 500 students (N=500), calculate the force of infection if 10 students are infectious, and 480 are susceptible. Assume $R_0 = 4$, and the recovery delay D = 2.

Given this scenario, show why an epidemic would spread through the school.

[5]

(b) Show the Stocks, Flows and Feedbacks for the SIR Model.

Show the structure and polarity of the following loops: *"Contagion", "Depletion" and "Recovery".*

Explain the formulation of the number of infections (inflow to Infectious). Also, answer the following:

- What are the plausible range of values for the parameters of the equation?
- How do the parameters of the SIR model relate to the reproduction number?
- If the model was to be extended to include people wearing personal protective equipment (PPE), for example, face masks during an influenza outbreak, what parameter could be modified? How might PPE resources be added to the SIR model?

[12]

[12]

(c) Show that when an outbreak is at its peak ($dI/dt = 0$), the proportion susceptible equals $1/R_0$.

Explain why the SIR model conforms to the following statement: "The chain of transmission eventually breaks down due to a decline in infectives, not due to a complete lack of susceptibles."

[6]

2018 Q4

(b) Create a stock and flow model (with full equations) for the following description. It should contain a structure for modeling the effect of the **student staff ratio** on the **student attrition fraction**, and also model staff recruitment as a *stock management structure.*

- **Students** are increased by **enrollments**, and decreased by **graduations** and **attritions**.
- All inflows & outflows to students are governed by fractional increase/decrease rates.
- The initial value of students is 10000. The enrollment fraction for students is 25%, and the graduation fraction is 24%. The **attrition fraction** depends on the student to staff ratio.
- The normal attrition fraction is 1%, which applies when the **student staff ratio** is 20. If the student staff ratio increases over 20, then the attrition fraction should increase (up to a maximum of 10%). Should the student staff ratio fall below 20, then the attrition fraction should decline to a minimum of 0.1%. Show the appropriate effect function on a graph.
- **Staff** are increased by **hires** and reduced by **retirements** and **turnover**.
- The **desired student to staff ratio** is 20. This then determines the **desired staff number**.
- The inflow to staff (hires) is governed by the stock management structure. The **adjustment time** is 1. **Expected losses** are initially 10/year, where the retirement fraction is 8% and the turnover fraction is 2%. The **expectation adjustment time** is 3.

(c) Consider the following scenario. The model starts running in 2015. Assume it runs in equilibrium. After three years, the **desired student to staff ratio** is reduced to 10. Assume the model will then run until 2030.

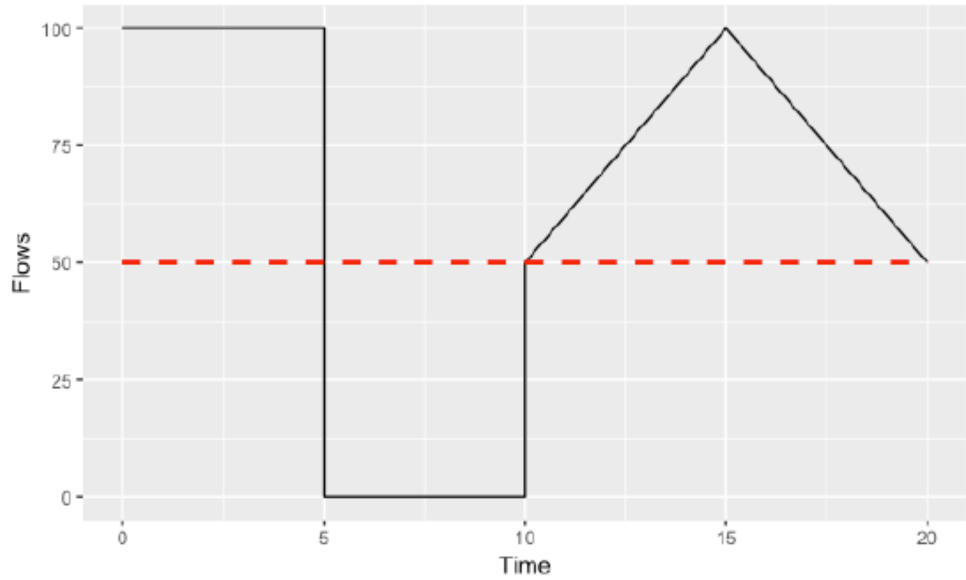
Show your understanding of how the system will respond by sketching the responses of following variables (for simplification, assume that the number of students will remain constant). **Student attrition fraction**, **Student to staff ratio**, **Number of staff** and **Expected staff losses.**

[8]

2019 Q1

1. (a) Based on the flow variables shown in the diagram (dashed line is the outflow, solid line the inflow), and assuming that the initial value of the stock is 100, perform the following:

- Draw a stock and flow model (including the net flow).
- Calculate, and show, the net flow.
- Use graphical integration to plot the stock over time, clearly showing the different behaviour modes for each time interval



[12]

(b) Describe Euler's method of integration, and state the assumptions.

[5]

(c) Calculate the stock using Euler's equation where DT=5. Estimate the integration error for each time step of 5.

[8]

2019 Q2

2. (a) Construct a stock and flow model from the following description of a service industry company. Specify the stock equation (there is no need to document equations for flows or auxiliaries), and clearly show the link polarities, and feedbacks, on the diagram.

- **Customer Orders** increases the **Service Backlog (Stock),** while **Order Fulfillment** decreases the Service Backlog
- An increase in **Service Backlog** leads to an increase in **Desired Service Capacity**.
- An increase in **Service Standard** leads to an increase in **Desired Service Capacity**
- An increase in **Desired Service Capacity** leads to an increase in **Work Pressure**.
- An increase in **Work Pressure** leads to: (1) a reduction in **Time Per Order,** and (2) an increase in **Work Intensity**.
- A reduction in **Time Per Order** leads to a reduction in **Service Standard**. A fall in **Service Standard** leads to a corresponding decline in **Time Per Order**.
- An increase in **Time Per Order** leads to a reduction in **Order Fulfillment**.
- An increase in **Work Intensity** leads to (1) an increase in **Order Fulfillment** and (2) an increase in **Fatigue.**
- An increase in **Fatigue** leads to a decrease in **Order Fulfillment.**

[12]

(b) Based on the stock and flow model in part(a):

- Show the following feedback loops on the model: *"Goal Erosion"*, *"Burnout"*, *"Corner Cutting"*, and *"Overtime"*.
- Starting with the proposition that the Customer Orders are increasing, confirm the polarity for each of these four loops.

[8]

(c) What are the advantages of building this feedback model? How can its insights be use to identify ways to improve the Service Industry?

[5]

2019 Q3

3. (a) Describe the stock and flow structure (with equations) of the following model structures, and show their expected behaviour over time.

- Fractional Increase Rate
- Fractional Decrease Rate
- Adjustment to a Goal

[4]

(b) Consider the following description of a University model. Construct a stock and flow model, showing the link polarities and the feedback loops (with polarities).

- *Enrollments* (Enrollment Fraction = 0.25) increase the stock of **Students** (10000), while *Graduations* (Graduation Fraction = 0.23) and *Attritions* (Reference Attrition Fraction = 0.02) reduce the student stock.
- The *Desired Number of Staff* is the ratio of **Students** to *Desired Student Staff Ratio* (initially 20).
- *Change in Staff* (Net Flow) increases/reduces **Staff** (500), based on an adjustment of the *Discrepancy in Staff* (AT=2).
- The *Student to Staff Ratio* is based on the ratio of **Students** to **Staff**.
- The *Attrition Fraction* is formulated as an effects structure that depends on the *Student Staff Ratio*. Assume the *Reference Student Staff Ratio* = 0.02, and that the *Effect of Student Staff Ratio on the Attrition Fraction* has a minimum value of 0.01, and a maximum value of 0.10.

(c) Specify the full set of stock and flow equations, including the effects structure (the effect relationship should be shown on an x-y plot).

[8]

(d) Assume that the model starts in equilibrium in 2019, and runs to 2030. At time 2024, the *Desired Student Staff Ratio* increases to 25. Show how the *Effect of Student Staff Ratio on Attrition Fraction* will change as a result of this increase.

[5]

2019 Q4

4. (a) Give a definition of reproduction number R_0, and describe why it is an important measurement to help control the spread of infectious diseases. What is the relationship between R_0 and the goal of reaching *herd immunity* in a population? Show a practical example, where an influenza outbreak is assumed to have $R_0=2$.

[5]

(b) Show the Stocks, Flows and Feedbacks for the SIR Model. Assume the following values.

- $S_0 = 9999$, $I_0=1$, $R_0=0$
- Contact Rate = 6/day
- Infectivity = 0.25
- Recovery delay = 2 days

Extend the model by including the possibility of vaccinations, where vaccines are a finite resource, and that the average vaccination delay is 2. If there are no vaccines, no vaccinations can take place.

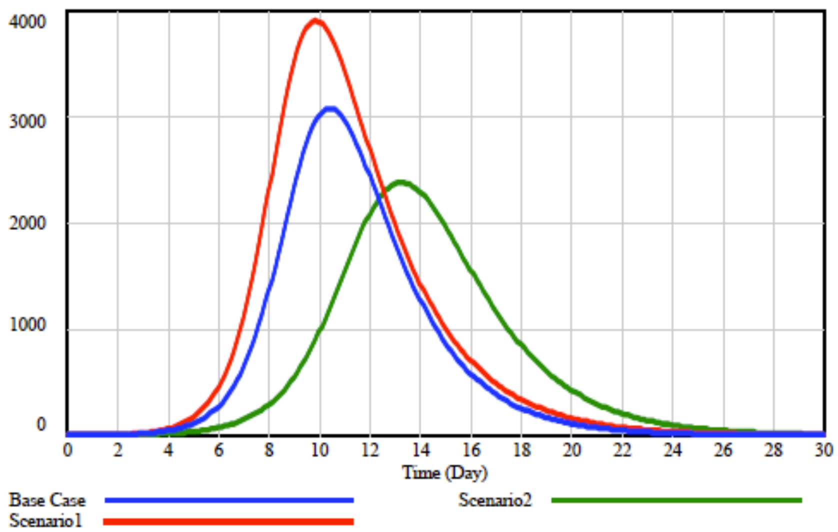
[12]

(c) Discuss how the parameters *vaccination delay*, *contact rate* and *infectivity* can impact the infected curve. How might public health policies be formulated to impact these three parameters?

[4]

(d) Based on the following graph, where the *Base Case* (blue) is the middle curve, and the highest peak is *Scenario 1* (red), and the lowest peak corresponds to *Scenario 2* (green), identify – providing explanations - which curve corresponds to the following scenarios.

- The recovery delay increases from 2 to 2.5
- The contact rate drops from 6 to 5



CT561: Systems Modelling & Simulation

Lecture 13: Dimensional Analysis

Prof. Jim Duggan,

School of Engineering & Informatics

National University of Ireland Galway.

https://github.com/JimDuggan/SDMR

Dimensional Analysis

- In the physical sciences and engineering, any equation representing a real-world process needs to have the units (i.e. dimensions) balanced on each side of the '='
- This checking – also known as **dimensional analysis** – is also an important activity in system dynamics, as it provides a validation mechanism for the model being designed.
- As a starting point, the units for system stocks are identified, and examples from a range of modeling are now shown

Stock Units - Example

Application Area	Stock	Units
Business	Inventory	Stock Keeping Unit (SKU)
Financial Planning	Cash	€, $
Education Planning	Students	People
Epidemiology	Infected	People
Demographics	Population	People
Climate Change	Carbon in the Atmosphere	Metric Tons

Table 1.2 Sample stock variables along with indicative values for units

Flow Units

- Stocks change over time through their flows, and therefore, in order to maintain dimensional consistency, a flow must have units of the stock it feeds, *divided by the units in which time is measured* (Coyle 1996).
- The selection of time unit depends on the problem being explored, for example, planning in a higher education context has annual student intakes, therefore the most suitable time unit would be *year*.

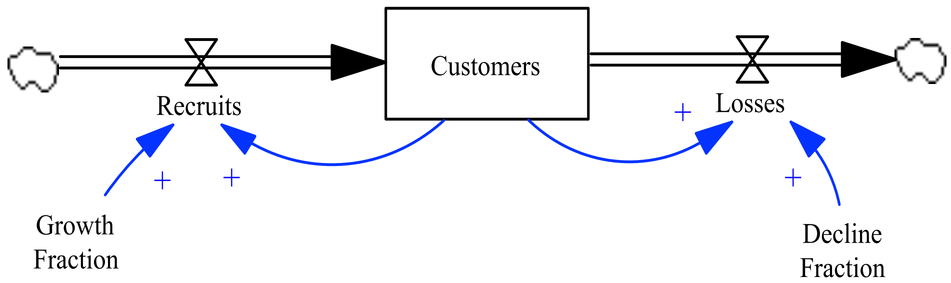
Flow Units - Example

Stock	Inflow	Outflow	Flow Units
Inventory	Arrivals	Shipments	SKU/week
Cash	Deposits	Withdrawals	€/day, $/day
Student	Registrations	Graduations	People/year
Infected	Incidence	Recovery	People/day
Population	Births	Deaths	People/year
Carbon in the Atmosphere	Emissions	Absorptions	Metric Tons/year

Table 1.3 Sample flow variables along with indicative values for units

Dimensional Analysis

- Once the units for stocks and flows are identified, dimensional analysis can be performed, where both sides of an equation are simplified to their basic units.
- If the two sides of the dimensional equation are equal, then the equation is dimensionally consistent



Eulers Equation (the Stock)

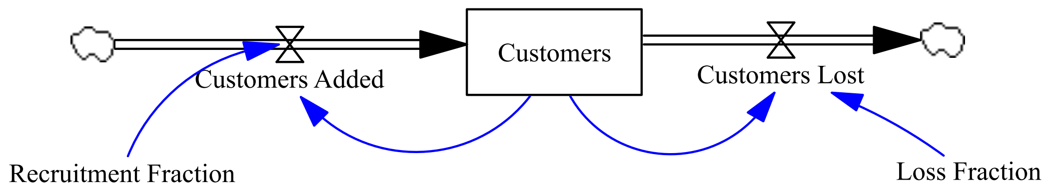
$$Customers_t \quad = \quad Customers_{t-dt} + (Recruits - Losses) * DT$$

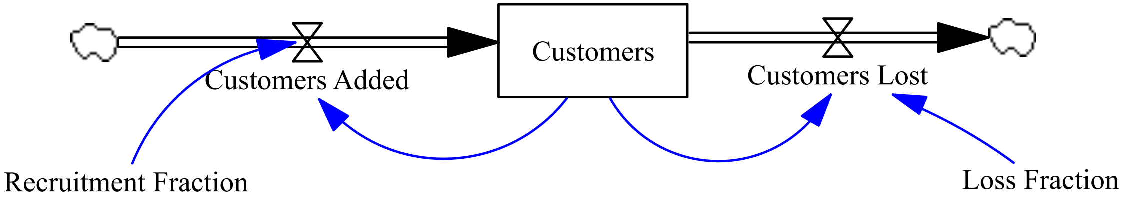
$$\text{people} \quad = \quad \text{people} \quad + (\text{people/year} - \text{people/year}) * \text{year}$$

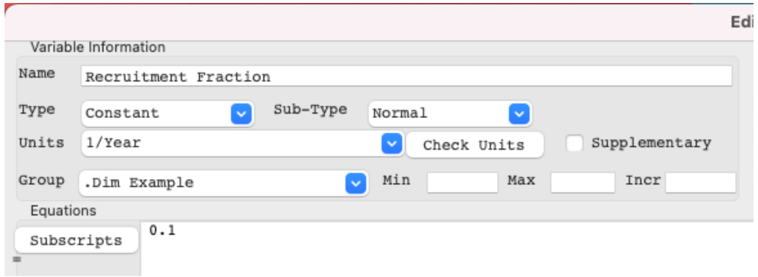
- This equation is dimensionally consistent, as the inflow and outflow denominator (year) cancels with the dimensions of DT (year) to arrive at the dimension (people).

Fraction Increase and Decrease Rates

- The units for these are 1/time







Customers = INTEG (Customers Added-Customers Lost, 1000)
Units: People

Customers Added = Customers*Recruitment Fraction
Units: People/Year

Customers Lost = Customers*Loss Fraction
Units: People/Year

Loss Fraction = 0.03
Units: 1/Year

Recruitment Fraction = 0.1
Units: 1/Year