CT561: Systems Modelling & Simulation

Lecture 10: SIR Model Part 2

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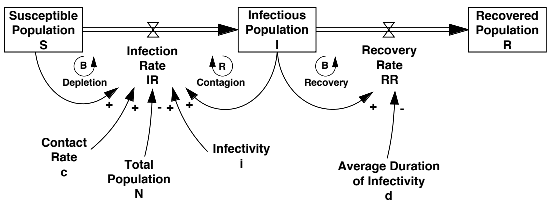
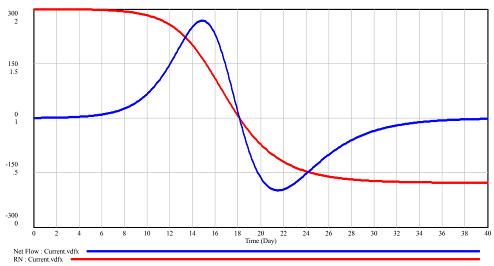
https://github.com/JimDuggan/SDMR

Overview

IR = Contact Rate * Infectious * (Susceptible/N) * Infectivity

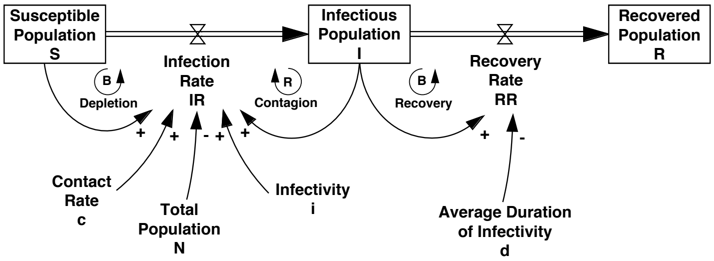
- Force of Infection (Lambda)
- Threshold Dynamics
- Net Reproduction Number and Herd Immunity
- Exploring downstream effects with the SEIR model (with clinical and sub-clinical streams)





(1) Force of Infection (Lambda)

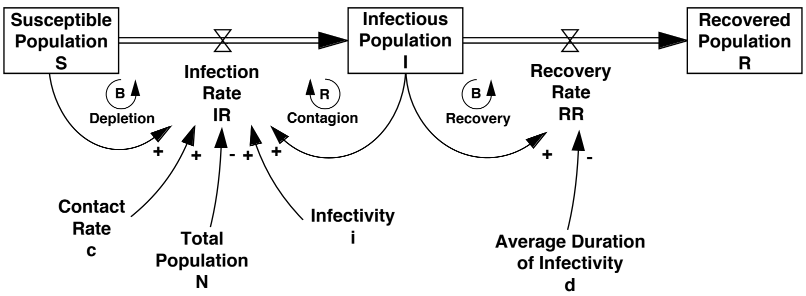
The rate at which susceptible individuals become infected per unit time. It is also known as the incidence rate or the hazard rate.



IR = Susceptible * Contact Rate * Infectivity / N * Infectious
= Susceptible * FOI

Beta

The per capita rate at which two specific individuals come into contact per unit time.



IR = Susceptible * Contact Rate * Infectivity / N * Infectious
= Susceptible * FOI
= Susceptible * Beta * Infectious

Challenge 9.1

- Suppose we have a town with 100,000 (=N) individuals, of which 1% were infectious with a novel pathogen, with R_0 = 14 and recovery delay (D) =7 days. Calculate the:
  - Effective per capital contact rate β
  - Force of infection λ

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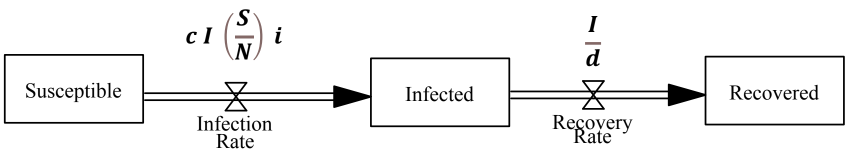
(2) Threshold Dynamics of SIR Model



For the number of infectious people to increase, the inflow must be greater than the outflow.
Assume S==N in a totally susceptible population

$$c\,I\left(\frac{S}{N}\right)i > \frac{I}{d} \longrightarrow c\left(\frac{S}{N}\right)i > \frac{1}{d} \longrightarrow c\,i\,d > 1 \longrightarrow R_0 > 1$$

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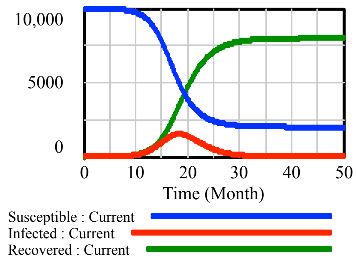
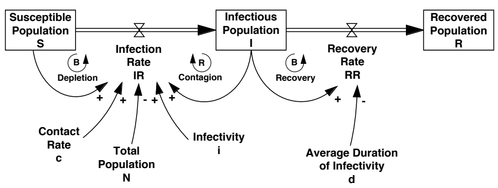
R_0 Threshold Phenomenon (Keeling & Rohani 2008)

- Assuming everyone in the population is initially susceptible, a pathogen can only invade if $R_0 > 1$
- Due to differences in demographic rates, rural-urban gradients, and contact structures, different populations may be associated with different values of R_0 for the same disease.
- *R_0 depends on the disease and the population*

Observation

"The chain of transmission eventually breaks due to a decline in **Infectives**, NOT due to a complete lack of **Suceptibles"**





IR = Contact Rate * Susceptible* (Infectious/N) * Infectivity

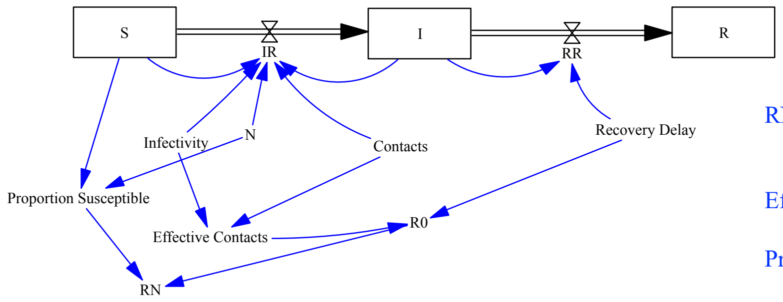
Challenge 9.2

- Estimate whether an epidemic will occur in the following scenario:
  - N = 100,000
  - It's a novel pathogen
  - The average contacts per day are 8
  - The probability of infection given contact between and infectious and susceptible person is 0.25
  - The duration of infectiousness is 1 day

Net Reproduction Number

- R_n is the net reproduction number
- Useful to evaluate as an epidemic proceeds
- **$R_n = (S/N) * R_0$**
- "The average number of secondary infectious persons resulting from one infectious person in a given population in which *some individuals may already be immune because of infection or vaccination*"
- When $R_N <= 1$, no epidemic occurs – the infectious stock goes to zero

Adding R_N to the SIR Model



RN=Proportion Susceptible*R0

Effective Contacts = Contacts * Infectivity

Proportion Susceptible = S / N

R0 = Effective Contacts * Recovery Delay

Recovery Delay = 2

RN = Proportion Susceptible * R0

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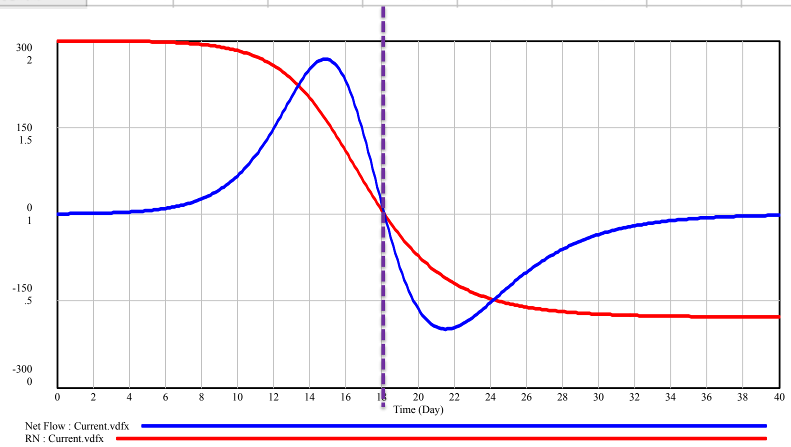
Exploring R_N and the dI/dt (Net Flow)

Time (Day)	18	18.125	18.25	18.375	18.5	18.625	18.75	18.875
Net Flow : Current	17.0731	2.18492	-12.431	-26.7112	-40.5977	-54.038	-66.9856	-79.3997
RN : Current	1.02227	1.00285	0.983794	0.965121	0.946842	0.928968	0.911509	0.894471



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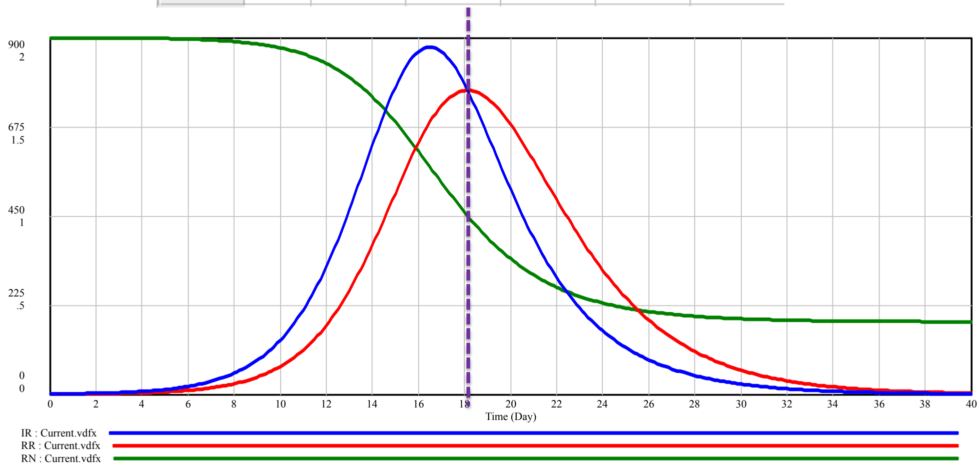
Exploring R_N and IR with RR

Time (Day)	18	18.125	18.25	18.375	18.5	18.625
IR : Current	783.845	769.557	754.619	739.114	723.122	706.722
RN : Current	1.02227	1.00285	0.983794	0.965121	0.946842	0.928968
RR : Current	766.771	767.372	767.05	765.825	763.719	760.76



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Challenge 9.3

- Calculate the net reproduction number in the following scenario
  - N = 100,000
  - 40,000 People are immune
  - The average contacts per day are 8
  - The probability of infection given contact between and infectious and susceptible person is 0.25
  - The duration of infectiousness is 2 days

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Exploring the R_N equation

$$R_n = R_0 * \left(\frac{S}{N}\right)$$

$$\text{If} \left(\frac{S}{N}\right) = \frac{1}{R_0} \; then \; R_n = 1$$

$$HIT = 1 - \frac{1}{R_0}$$

- When $S/N = 1/R_0$, then each infectious person will lead to a single transmission (R_n=1)
- If the proportion susceptible is less than this, incidence will decrease
- This allows us to define a critical threshold for S, under which a disease will not spread

(3) Herd Immunity

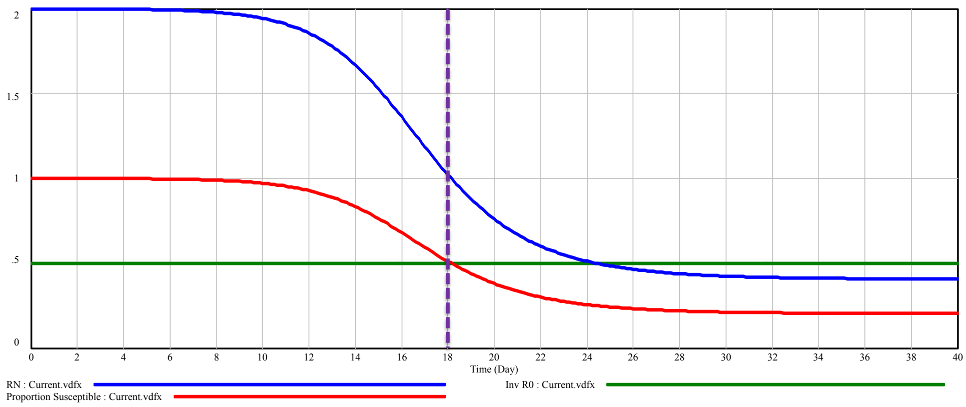
- If (1- $1/R_0$) proportion of the population can be vaccinated, the disease will not spread.
- Why? Because $R_N = (S/N) * R_0$
- If R_0 = 2, we vaccinate 50% of the population
- R_N = (5000/10000) * 2 = 1, one person infects only one, so no spread.

$$HIT = 1 - \frac{1}{R_0}$$

Infection	Serial Interval (Range)	R_o	Herd Immunity
Diphtheria	2-30 Days	6-7	85
Influenza	2-4 Days	2-4	50-75
Malaria	20 Days	5-100	80-99
Measles	7-16 Days	12-18	83-94
Pertussis	5-35 Days	12-17	92-94

Exploring R_N and $1/R_0$

Time (Day)	17.75	17.875	18	18.125	18.25	18.375	18.5	18.625
Inv R0 : Current	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Proportion Susceptible : Current	0.531065	0.521017	0.511133	0.501424	0.491897	0.482561	0.473421	0.464484
RN : Current	1.06213	1.04203	1.02227	1.00285	0.983794	0.965121	0.946842	0.928968



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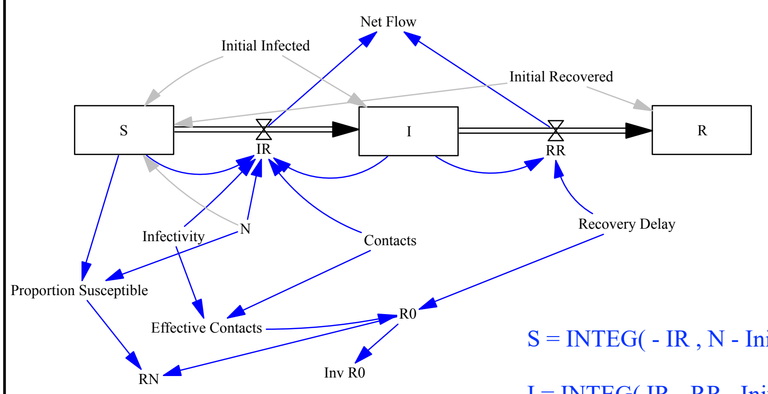
SIR with Vaccinations - Adjusted Model



Initial Infected = 1

Initial Recovered = 5000

N = 10000

S = INTEG(- IR , N - Initial Infected - Initial Recovered)

I = INTEG(IR - RR , Initial Infected)

R = INTEG(RR , Initial Recovered)

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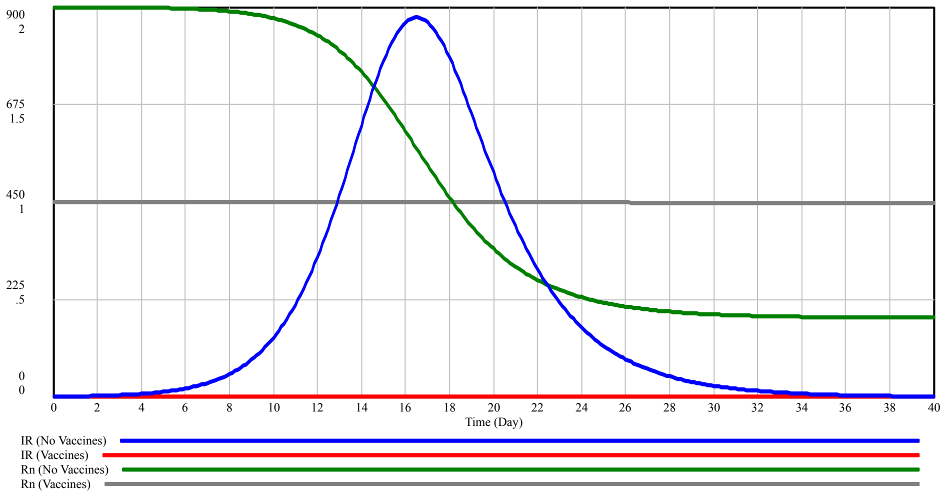
Scenarios



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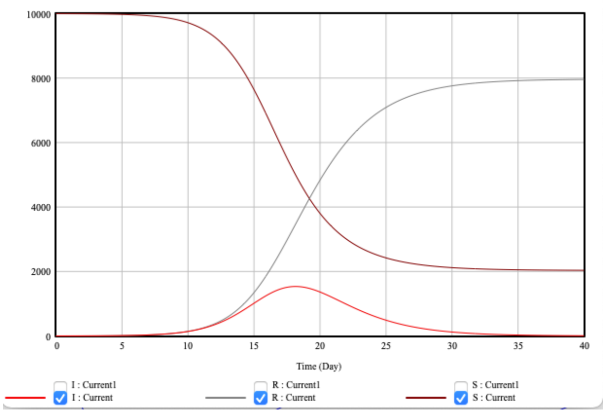
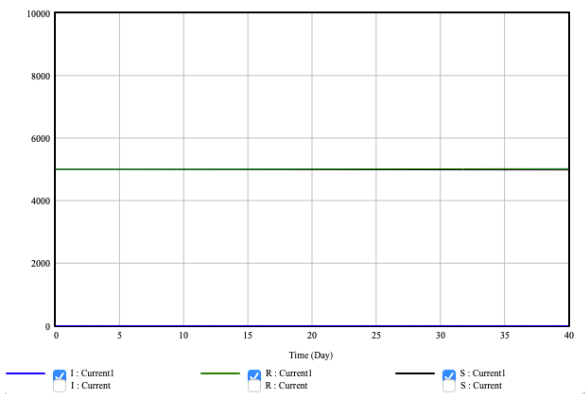
SIR Analysis





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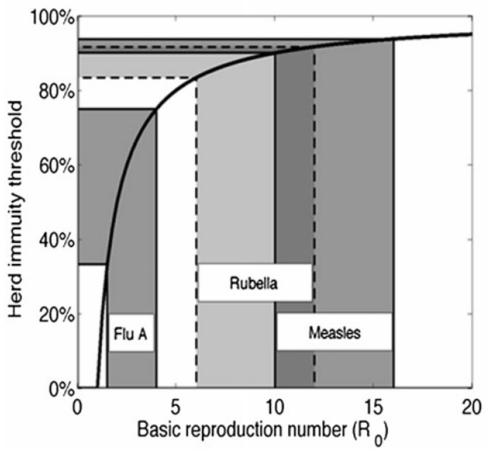

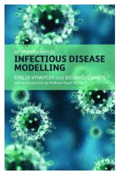

"In general, vaccination programmes aim to achieve a coverage which is above the herd Immunity threshold, since if the proportion of the population that is immune is above this value, substantial outbreaks are unlikely to occur."

"Herd Immunity": A Rough Guide

Paul Fine, Ken Eames, and David L. Heymann

Department of Infectious Disease Epidemiology, London School of Hygiene and Tropical Medicine, London, United Kingdom

Challenge 9.5 – Downstream Effects

- Build on the workshop three problem (see overleaf)
- Add a hospitalization stream to the model
- Assume that 5% of clinical people are hospitalized 6 days after they are no longer infectious
- Assume that, on average, they stay in hospital for 10 days. Assume it's a second order delay.
- When people leave hospital they are assumed to be recovered.
- Show the hospitalization rates, and the total number in hospital

CT561 – Workshop #3

Extending the SIR Model

The aim of this workshop is to extend the SIR model in the following ways:

1. Add an exposed stock that models people who have become infected but are not yet infectious. Assume the duration of exposure is 3 days.

2. There are now two kinds of infectious people (assume an infectious delay of 5 days):
   a. Sub-clinical, where they do not show symptoms. Sub-clinical people are half as infectious as clinically infectious people.

   b. Clinical, where people show symptoms

3. The breakdown between the two types of infectious people is determined by a constant called *clinical fraction*.

4. Add an estimate of R_0 to the model, given that R_0 is defined as "the average number of secondary infectious persons resulting from a typical infectious person following their introduction to a totally susceptible population."

5. Run for a population of 1M people, where 10 people are initially infectious. Assume people have, on average, 10 contacts per day, and the infectivity for clinical people is 10%. Assume that 40% of the population do not show any symptoms.

6. Run the model for different values of the clinical fraction (0, .2, .4, .6, .8, 1.0) and explain the results. How do these value impact R_0?