# CT561: Systems Modelling & Simulation

## Lecture 5: Limits to Growth

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https://github.com/JimDuggan/SDMR



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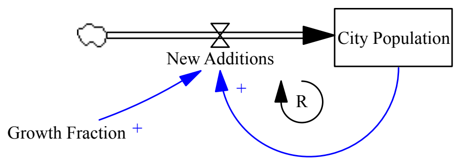
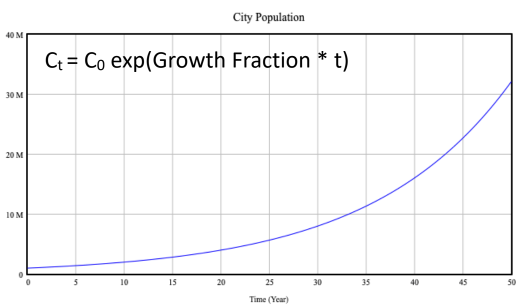
## Recap: Exponential Growth

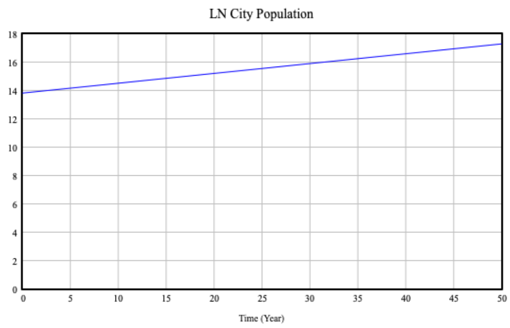



City Population = INTEG( New Additions , 1e+06)

Growth Fraction = 0.07

New Additions = City Population * Growth Fraction



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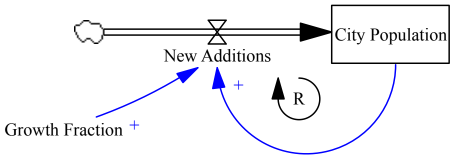
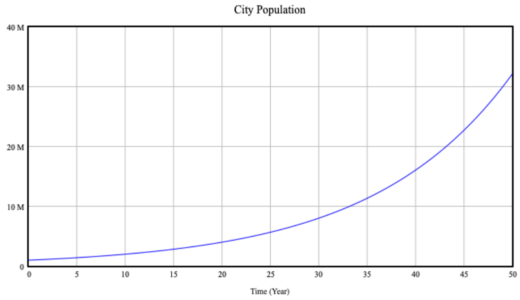
# Feedback Analysis

| Cause | Direction | Effect | Direction |
|---|---|---|---|
| City Population | ⇧ | New Additions | ⇧ |
| New Additions | ⇧ | City Population | ⇧ |



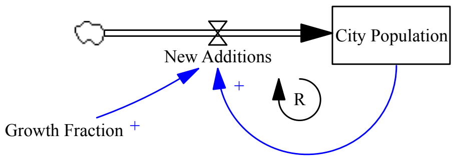



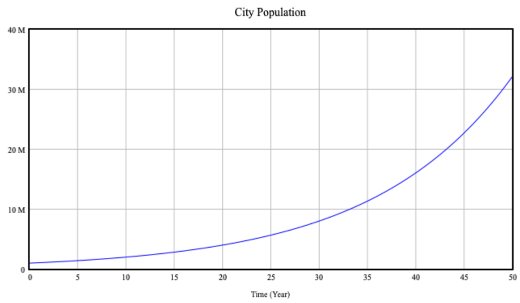





# Exponential Growth

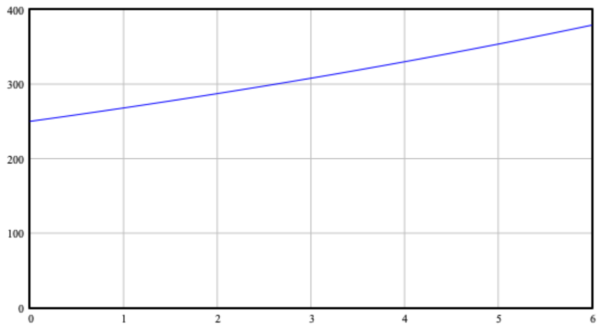
- Quantities that grow by a fixed percentage (e.g. 0.018) per time period exhibit exponential growth
- Exponential growth behaves according to a "doubling time"
- "Treacherous and misleading" Forrester (1971)
- Within one doubling time, the quantity goes from half its limit to its limit

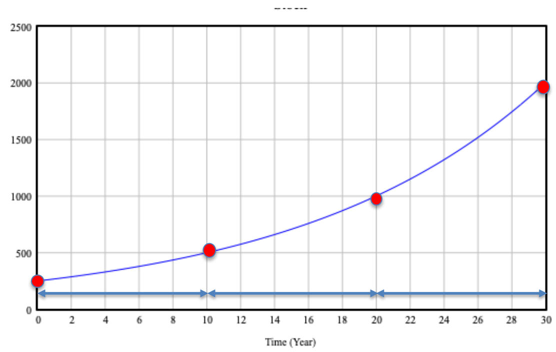


# Misperceptions of Exponential Growth

- Studies have shown that people grossly underestimate the rate of growth, by extrapolating *linearly* instead of *exponentially*.
- *Doubling time is* a valuable way to understand exponential growth



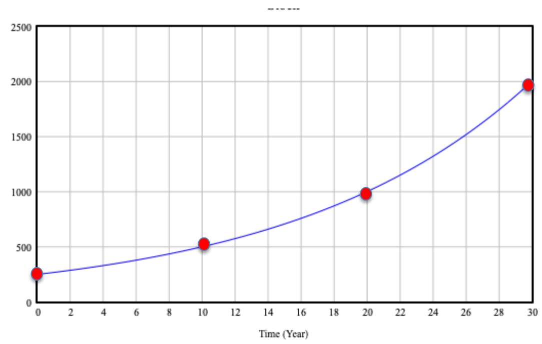








# Doubling Time Calculation

- S(t) = S(0) exp(growth rate * t)
  - 2 * S(0) = S(0) exp (growth rate * tD)
  - 2 = exp(growth rate * tD)
  - Ln (2) = growth rate * tD
  - tD = Ln(2)/growth rate = (0.6931)/growth Rate
- "The Rule of 70"
  - Doubling time independent of stock size
  - tD = 70 /(100*growth rate)
  - An investment earning 7%/year doubles after 10 years




Growth Fraction = 0.07

New Additions = Stock * Growth Fraction

Stock = INTEG( New Additions , 250)

## Challenge 5.1

- The growth rate of an epidemic in its early stage is estimated at 15% per day. From that, estimate the doubling time.
- If there are 100 people infected on day 1, estimate (using the growth rate and the integral equation solution for exponential growth), how many have been infected after 30 days.
- Implement a simple model in Vensim and compare the results



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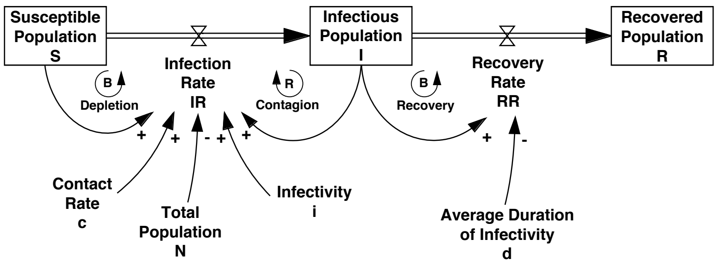
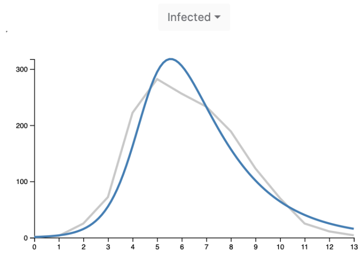
## Limits to Growth Model

*There will always be limits to growth. They can be self-imposed. If they aren't, they will be system-imposed.*

Donella H. Meadows, Thinking in Systems: A Primer (2008), p.103



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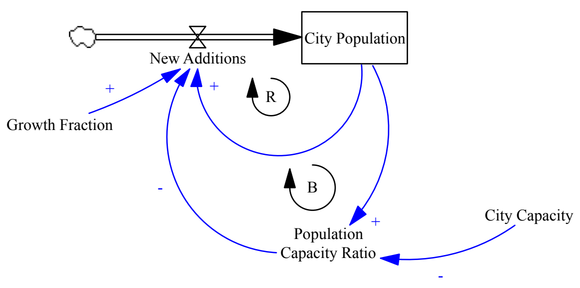
# Limit to Growth –
# *note extension to net flow equation*




City Capacity = 5e+06

City Population = INTEG( New Additions , 50000)

Growth Fraction = 0.15

New Additions = City Population * Growth Fraction *
**(1 - Population Capacity Ratio)**

Population Capacity Ratio = City Population / City Capacity



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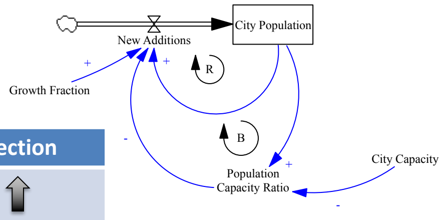
# Feedback Analysis – 2 Loops




| Cause | Direction | Effect | Direction |
|---|---|---|---|
| City Population | ↑ | New Additions | ↑ |
| New Additions | ↑ | City Population | ↑ |

| Cause | Direction | Effect | Direction |
|---|---|---|---|
| City Population | ↑ | Population Capacity Ratio | ↑ |
| Population Capacity Ratio | ↑ | New Additions | ↓ |
| New Additions | ↓ | City Population | ↓ |

New Additions = City Population * Growth Fraction * **(1 - Population Capacity Ratio)**



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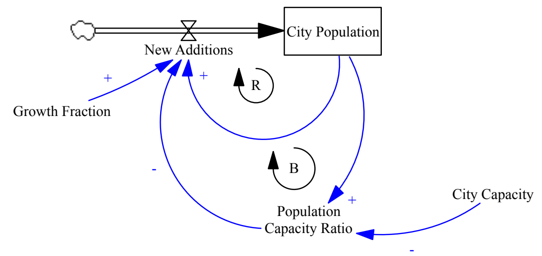
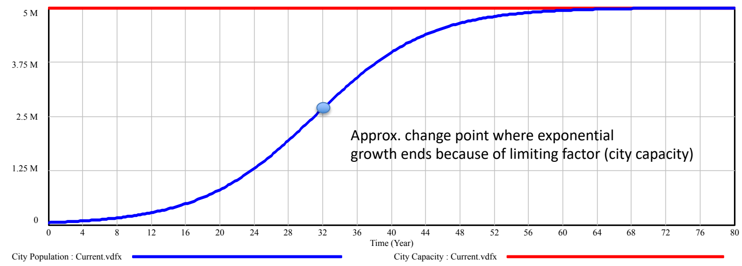
# Model Outputs

| Cause | Direction | Effect | Direction |
|---|---|---|---|
| City Population | ↑ | New Additions | ↑ |
| New Additions | ↑ | City Population | ↑ |

| Cause | Direction | Effect | Direction |
|---|---|---|---|
| City Population | ↑ | Population Capacity Ratio | ↑ |
| Population Capacity Ratio | ↑ | New Additions | ↓ |
| New Additions | ↓ | City Population | ↓ |



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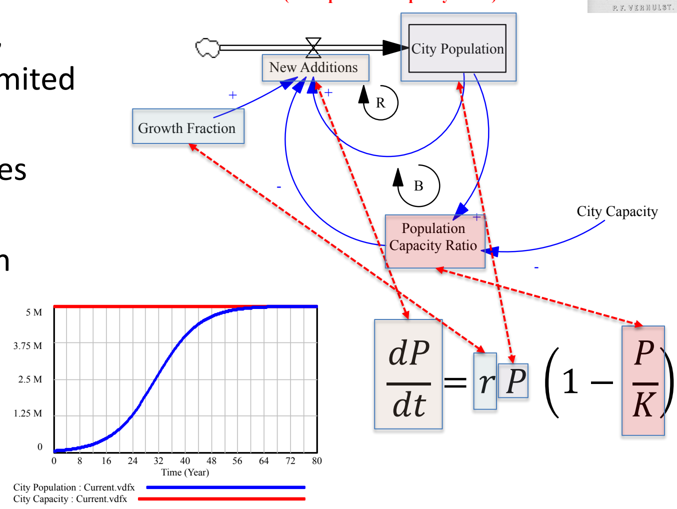
# Limits to Growth - Verhulst Model

New Additions = City Population * Growth Fraction *
**(1 - Population Capacity Ratio)**

- A model of population growth, where the rate of increase is limited by the carrying capacity (K)
- When P is small, it approximates exponential growth
- Model compared to population growth in:
  - France (1817-1831)
  - Belgium (1815-1833)
  - Essex (1811-1831)




$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$



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# Challenge 5.2

- A fixed amount of land is designated for a new trading estate in order to encourage business development in a town.
- Initially, the trading estate grows rapidly as more businesses attract more business developments
- However, as growth continues, land availability falls, and business construction is reduced
- Draw a Stock and Flow Model and formulate the equations
- Variables include: **Business Structures (S)**, Construction Rate, **Business Construction (F)**, Land Availability, Land Area, Land Per Business Structure



# Overshoot and Collapse (Sterman p 123)

- The Verhulst model assumes that the carrying capacity is fixed
- Often, however, the ability of an environment to support a growing population is eroded or consumed by the population itself
- Example: Population of deer rises, leading to overbrowsing, which consumes the vegetation, leading to a decline in the deer population
- Real world examples include overfishing of St George's Bank, population of Easter Island

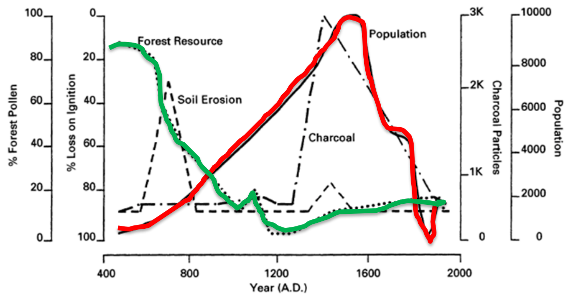
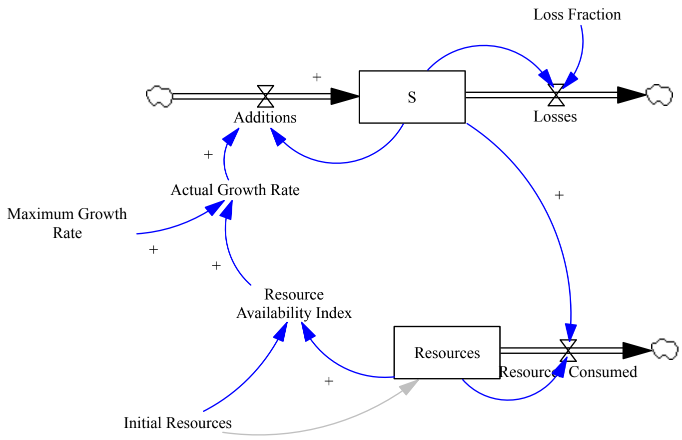
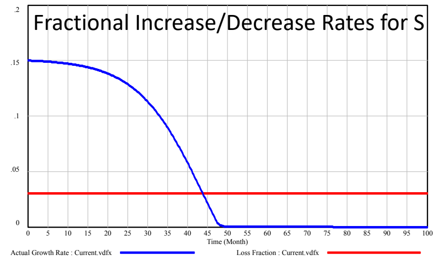


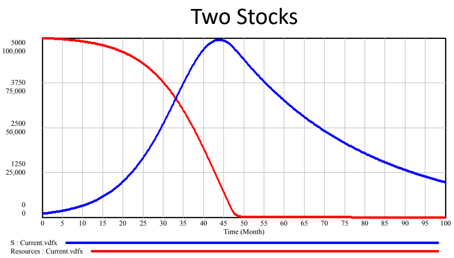

Figure 1. Easter Island dynamics from archaeological study by Bahn and Flenley (1992)

Published in 2012
System Dynamics Implementation of a Model of Population and Resource Dynamics with Adaptation
T. Uehara, Yoko Nagase, W. Wakeland

# Exploratory Model



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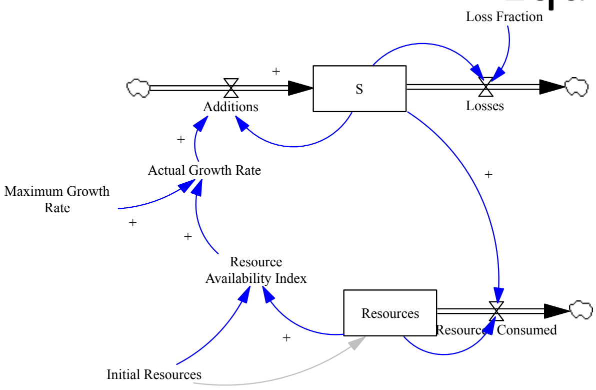
# Equations




S = INTEG( Additions - Losses , 100)

Additions = Actual Growth Rate * S

Losses = Loss Fraction * S

Loss Fraction = 0.03

Actual Growth Rate = Maximum Growth Rate * Resource Availability Index

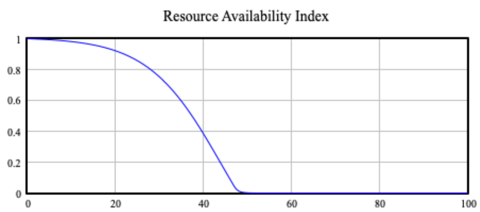
Maximum Growth Rate = 0.15

Resources = INTEG( - Resources Consumed , Initial Resources )

Initial Resources = 100000

Resource Availability Index = Resources / Initial Resources

Resources Consumed = min ( S , Resources )



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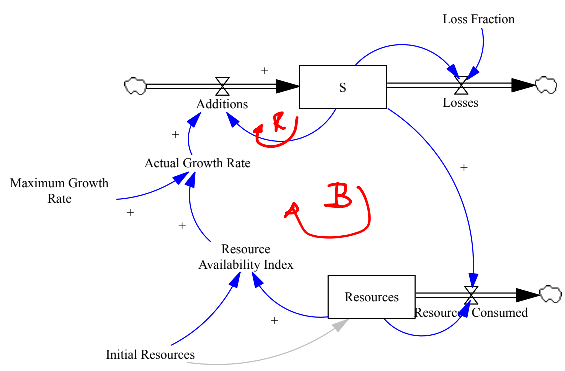
# Balancing Loop

| Cause | Direction | Effect | Direction |
|---|---|---|---|
| S | ↑ | Resources Consumed | ↑ |
| Resources Consumed | ↑ | Resources | ↓ |
| Resources | ↓ | Resource Availability Index | ↓ |
| Resource Availability Index | ↓ | Actual Growth Rate | ↓ |
| Actual Growth Rate | ↓ | Additions | ↓ |
| Additions | ↓ | S | ↓ |



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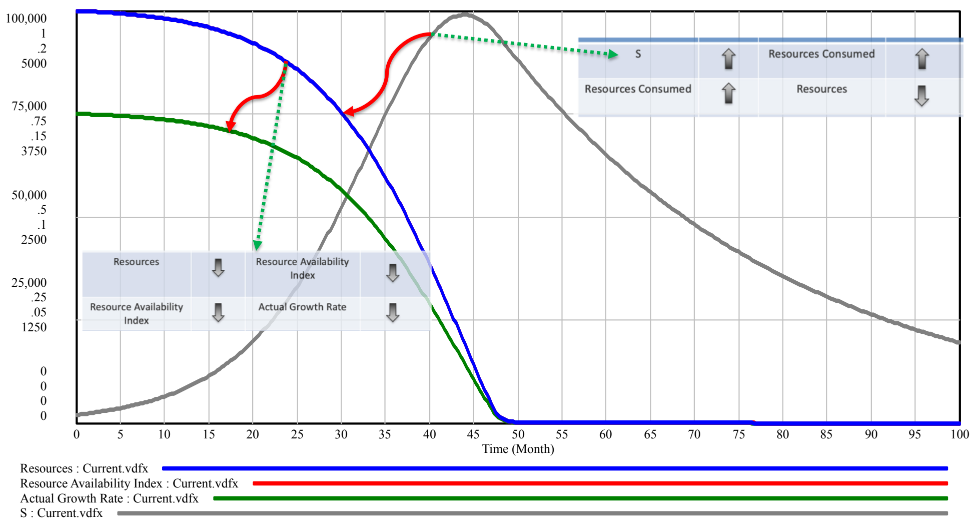
# Key Dynamics



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