CT561: Systems Modelling & Simulation

Lecture 7: Formulating Delays

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https://github.com/JimDuggan/SDMR

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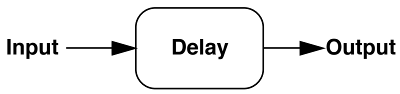
(1) Delays

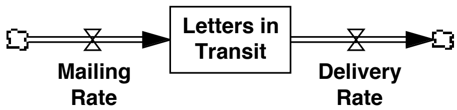
- "Delays are pervasive.
  - It takes time to measure and report information.
  - It takes time to make decisions.
  - It takes time for decisions to affect the state of the system" (Sterman 2000)
- We need to use delays in many of our models

The output of a delay lags behind the input:



General structure of a material delay:

The post office as a delay:



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Example of a delay… incubation period

Open access Original research

BMJ Open **Incubation period of COVID-19: a rapid systematic review and meta-analysis of observational research**

Conor McAloon [1] Áine Collins,[2] Kevin Hunt,[3] Ann Barber,[2] Andrew W Byrne [4] Francis Butler,[3] Miriam Casey [2] John Griffin [5] Elizabeth Lane,[6] David McEvoy [7] Patrick Wall,[7] Martin Green,[8] Luke O'Grady,[1,8] Simon J More[2]

Table 2 Percentiles of the pooled lognormal distribution after simulating all possible combinations of mu and sigma within the 95% CIs of the pooled estimates of both parameters

Percentile	Median (days)	Min	Max	Difference (max – min)
2.5th	1.92	1.54	2.38	0.84
5th	2.24	1.83	2.75	0.92
10th	2.69	2.24	3.23	0.99
25th	3.64	3.12	4.25	1.13
50th	5.10	4.53	5.75	1.22
75th	7.15	6.13	8.34	2.21
90th	9.69	8.06	11.60	3.54
95th	11.60	9.49	14.20	4.71
97.5th	13.60	10.9	16.90	6.00

The median days for each percentile are shown along with the minimum and maximum values for that percentile.

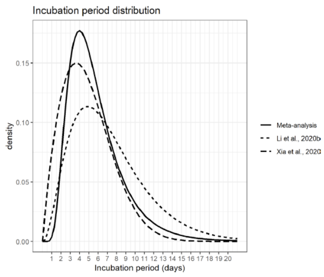

Figure 6 Probability density function of pooled lognormal distribution for incubation period and studies (n=2) not included in the meta-analysis because of the distribution used.

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Delay Distributions

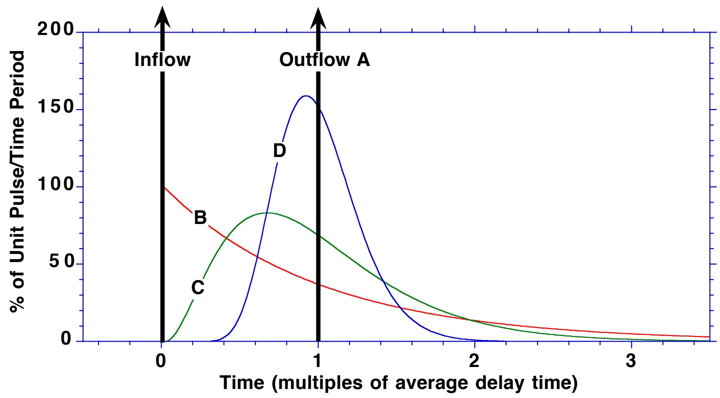

Figure 11-2 Some distributions of the outflow from a delay

The input I all cases is a unit pulse at time zero. Outflow A is a pipeline delay in which all items arrive together exactly 1 delay time after they enter. Outflow distributions B-D exhibit different degrees of variation in processing times for individual items so some arrive before and some after the average delay time. In all cases the average delay time is the same and the areas under each distribution are equal.

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(1) First Order Material Delay

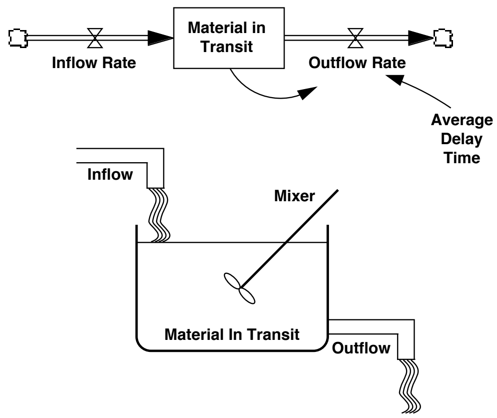
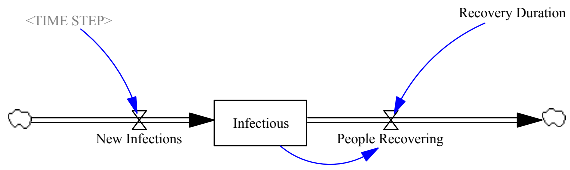


Figure 11-4 First-order material delay: structure

The outflow is proportional to the stock of material in transit. The contents of the stock are perfectly mixed at all times, so all items in the stock have the same probability of exit, independent of their arrival time.

Business Dynamics

Example 1: Infectious Disease Recovery Delay



The outflow is proportional to the stock of material in transit. The contents of the stock are perfectly mixed at all times, so all items in the stock have the same probability of exit, independent of their arrival time.

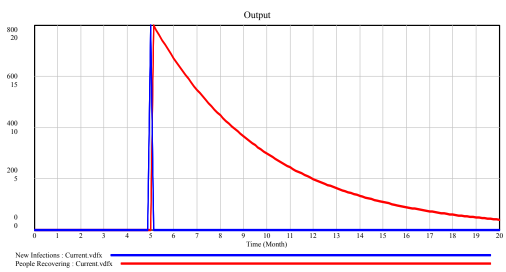
Infectious = INTEG(New Infections - People Recovering , 0)

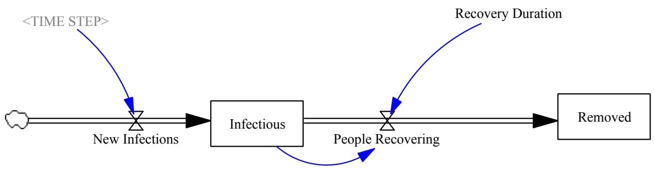
New Infections = PULSE (5, TIME STEP) * 100 / TIME STEP

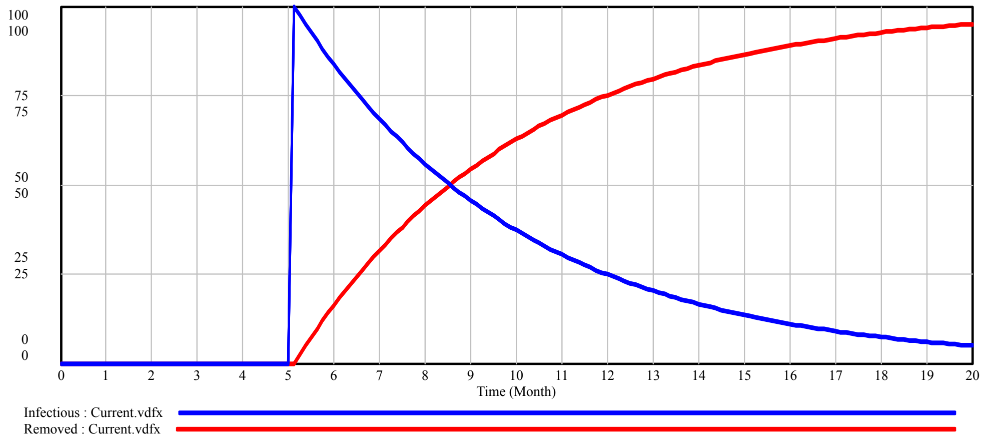
People Recovering = Infectious / Recovery Duration

Recovery Duration = 5

TIME STEP = 0.125
The time step for the simulation.







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Challenge 7.1 – First Order Delay

- Draw a stock and flow model of a software engineering team
- Assume there are 100 rookies to start and 0 experienced
- Assume the average delay for progression is 12 months
- Simulate for 36 months, and show the two stocks together
- Discuss the advanages/limitations of the model

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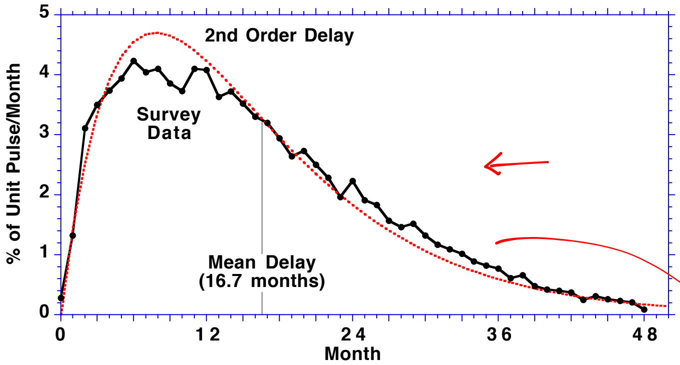
(2) Second Order Material Delay (Sterman 2000)



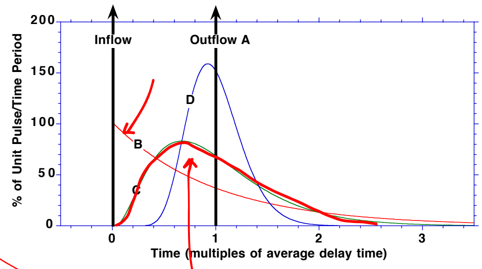


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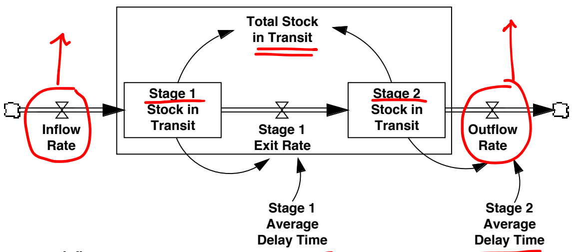
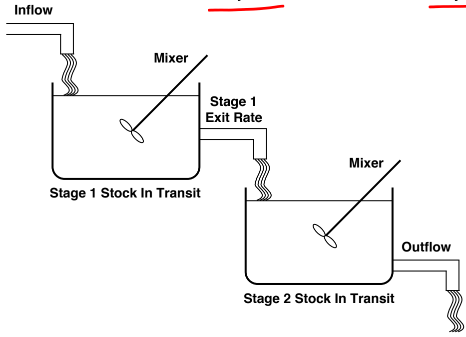
Figure 11-17 The construction lag for capital plant: data vs. model

Data: Distribution of construction completion times for US private nonresidential structures, 1961-1991, as estimated by Montgomery (1995) from US Dept. of Commerce survey data. The mean lag is 16.7 months. *Model:* Second-order material delay with average delay time of 16.7 months.

Figure 11-6 Higher-order delays are formed by cascading first-order delays together.

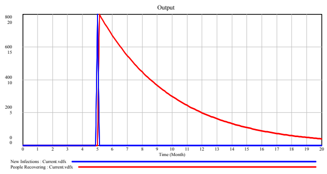
Delay time "shared" by outflows

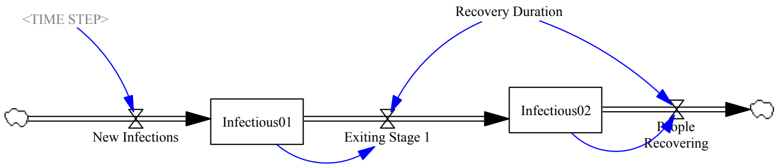




Stage 1 Exit Rate = Stage 1 Stock in Transit/Stage 1 Average Delay Time
Outflow Rate = Stage 2 Stock in Transit/Stage 2 Average Delay Time

Example 2: Recovery Delay (2nd Order)





Exiting Stage 1 = Infectious01 / (Recovery Duration / 2)

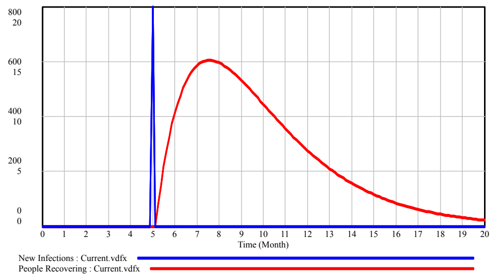
Infectious01 = INTEG(New Infections - Exiting Stage 1 , 0)

Infectious02 = INTEG(Exiting Stage 1 - People Recovering , 0)

New Infections = PULSE (5, TIME STEP) * 100 / TIME STEP

People Recovering = Infectious02 / (Recovery Duration / 2)

Recovery Duration = 5



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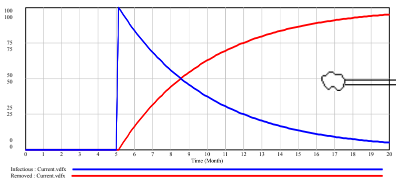

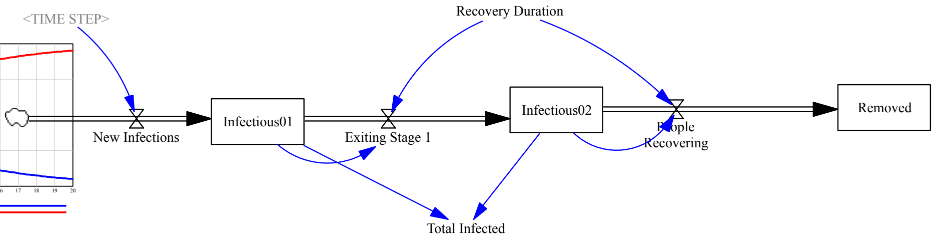

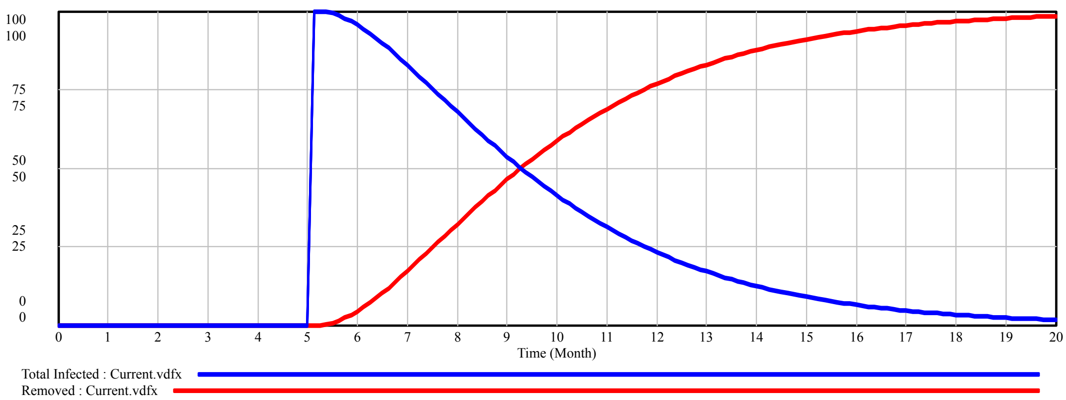

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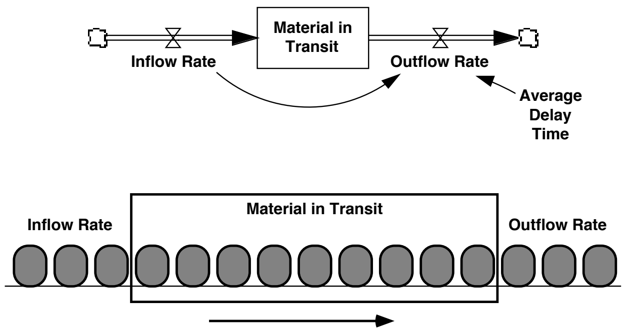
(3) Pipeline Delay

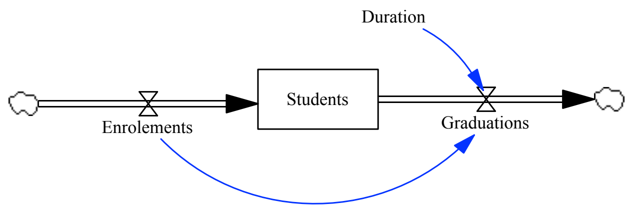
Figure 11-3 Pipeline delay: structure

In a pipeline delay individual items exit the delay in the same order and after exactly the same time, like widgets moving down an assembly line at a constant speed.



Material in Transit(t) = INTEGTRAL(Inflow(t) – Outflow(t), Material in Transit(0))
Outflow(t) = Inflow(t – Average Delay Time)

Student Example – Step Input



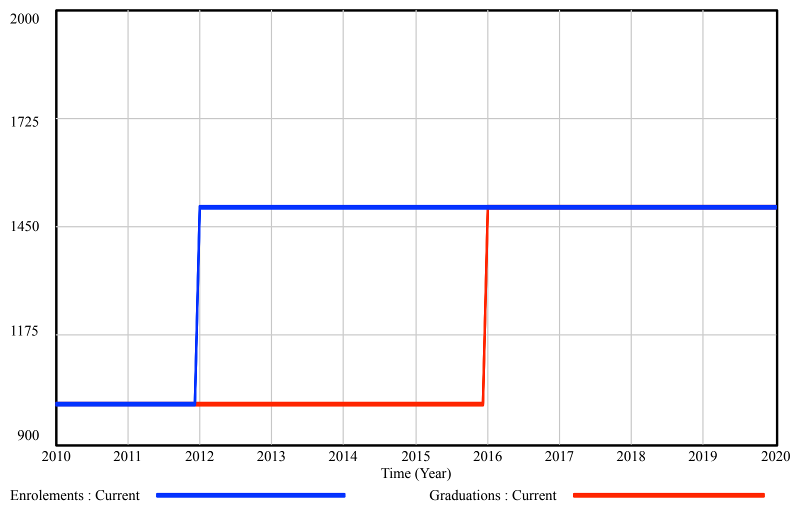
Duration = 4

Enrolements = 1000 + step (500, 2012)

Graduations = DELAY FIXED (Enrolements ,Duration , 1000)

Students = INTEG(Enrolements - Graduations , 4000)

Pipeline Delay Response

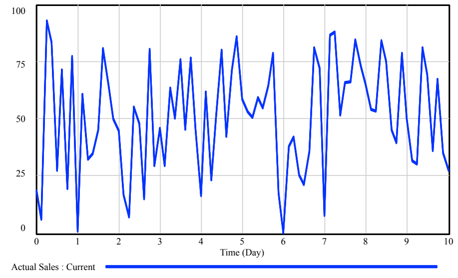


Challenge 7.2 – Second Order Delay

- Draw a stock and flow model of a software engineering team
- Assume there are 100 rookies to start and 0 experienced
- Assume the average delay for progression is 12 months
- Simulate for 36 months, and show the two stocks together
- Discuss the advantages/limitations of the model

Information Delay (Smoothing)

- Separates the signal from the noise
- Used mainly to model expectations (usually of flows)
  - Model of decision maker's expectation (what value might a variable take on?)
  - Similar to a forecast (exponential smoothing)



Actual Sales= RANDOM NORMAL(0, 100 , 50, 30 , 1)

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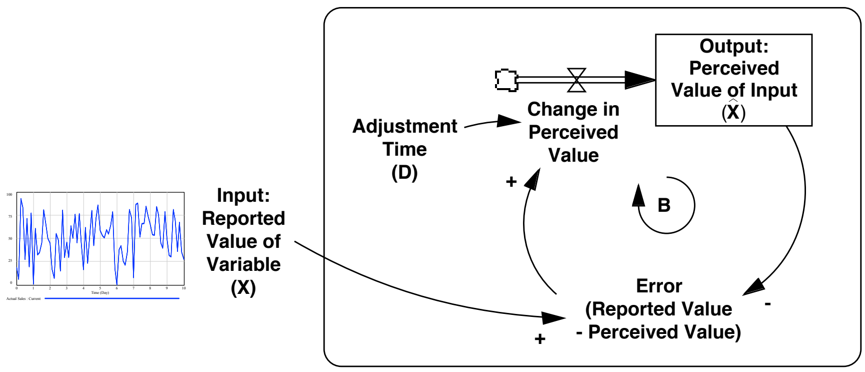
Information Delay - Structure



$\hat{X}$ = INTEGRAL(Change in Perceived Value, $\hat{X}(0)$)

Change in Perceived Value = Error/D = $(X - \hat{X})/D$

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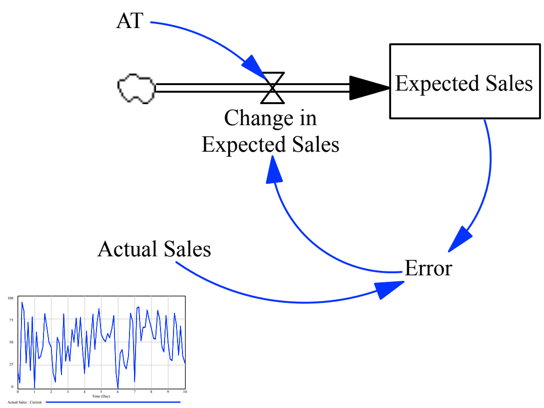
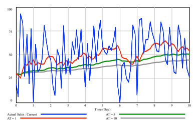
The Stock and Flow Model





Actual Sales= RANDOM NORMAL(0, 100 , 50, 30 , 1)

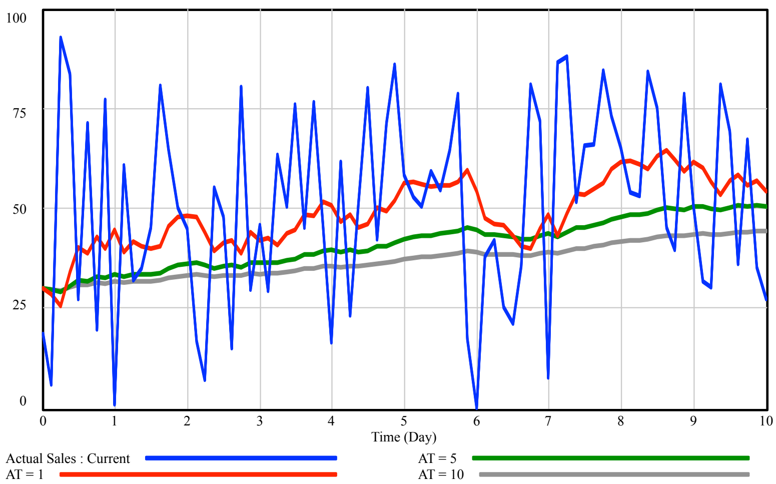
AT= {1,5,10}

Change in Expected Sales = Error/AT

Error=Actual Sales-Expected Sales

Expected Sales= INTEG (Change in Expected Sales, 30)

Impact of Adjustment Times



Challenge 7.3

- For the Student model, add a new variable:
  - Expected Enrollments
- See how this behaves for a step increase in enrollments