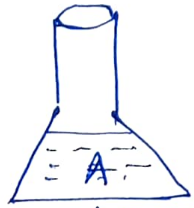
$$A \xrightarrow{k_1} X \xrightarrow{k_2} Z$$

At $t=0$, $[A]_0$ 0 0

$t=t'$, $[A]$ or C_A $[X]$ or C_X $[Z]$ or C_Z



is present
no X and Z

differential rate equation for

$A \longrightarrow X$ is $-\dfrac{d[A]}{dt} = k_1[A]$

or, $$\frac{d[A]}{[A]} = -k_1\,dt$$

$$\int_{[A]_0}^{C_A} \frac{d[A]}{[A]} = -k_1 \int_{t=0}^{t'} dt$$

$$\ln[A]\Big|_{[A]_0}^{C_A} = -k_1[t]_0^{t'}$$

Note: k_1 is temperature independent

$$\int \frac{dx}{x} = \ln x + C$$

$$\int dt = t + C'$$

$$\ln\frac{C_A}{[A]_0} = -k_1 t'$$

t' is a dummy variable, therefore, one can replace t' with t.

$$\ln\frac{C_A}{[A]_0} = -k_1 t'$$

$$C_A = [A]_0\, e^{-k_1 t}$$

$$\ln C_A = \ln[A]_0 - k_1 t$$

$$A \xrightarrow{k_1} X \xrightarrow{k_2} Z$$

At $t=0$, $[A]_0$ $\quad 0 \quad 0$

$t=t'$, $[A]$ $\quad [X]$ $\quad [Z]$

$C_A \quad C_X \quad C_Z$

not $\cancel{[C_A]}$ $\cancel{[C_X]}$ $\cancel{[C_Z]}$

$$\frac{d[X]}{dt} = k_1[A] - k_2[X] \qquad [A] = [A]_0 e^{-k_1 t}$$

$$\frac{d[X]}{dt} + k_2[X] = k_1[A] = k_1[A]_0 e^{-k_1 t}$$

Multiplying $e^{k_2 t}$ both side of the equation, we obtain

$$e^{k_2 t}\frac{d[X]}{dt} + k_2[X]e^{k_2 t} = k_1[A]_0 e^{(k_2-k_1)t}$$

$$\frac{d\{[X]e^{k_2 t}\}}{dt} = k_1[A]_0 e^{(k_2-k_1)t}$$

$$\int d\{[X]e^{k_2 t}\} = k_1[A]_0 \int e^{(k_2-k_1)t} dt$$

$$[X]e^{k_2 t} = k_1[A]_0 \frac{e^{(k_2-k_1)t}}{k_2-k_1} + C$$

$\cancel{A \xrightarrow{k_1} B \xrightarrow{k_2}}$

At $t=0$, $[X]=0$

therefore

$$0\, e^{k_2 \times 0} = k_1 [A]_0 \frac{e^{(k_2-k_1)\times 0}}{k_2-k_1} + C$$

$$0 = \frac{k_1[A]_0}{k_2-k_1} + C$$

$$C = -\frac{k_1[A]_0}{k_2-k_1}$$

$$[X]e^{k_2 t} = \frac{k_1[A]_0}{k_2-k_1} e^{(k_2-k_1)t} - \frac{k_1[A]_0}{k_2-k_1}$$

$$[X] = \frac{k_1[A]_0}{k_2-k_1} \left\{ e^{-k_2 t} e^{(k_2-k_1)t} - e^{-k_2 t} \right\}$$

$$= \frac{k_1[A]_0}{k_2-k_1} \left\{ e^{-k_1 t} - e^{-k_2 t} \right\}$$