$$A \xrightarrow{k_1} X \xrightarrow{k_2} Z$$

At $t=0$ $\quad [A]_0 \quad 0 \quad 0$

$t=t \quad C_A \quad C_X \quad C_Z$

$[A] = C_A = [A]_0\, e^{-k_1 t}$ (Already derived)

$$\frac{d[X]}{dt} = k_1[A] - k_2[X]$$

$$\frac{d[X]}{dt} + k_2[X] = k_1[A] = k_1[A]_0\, e^{-k_1 t}$$

$$\frac{d[X]}{dt} + k_2[X] = k_1[A]_0\, e^{-k_1 t}$$

Multiplying both sides with the integrating factor $e^{k_2 t}$, we get

$$\frac{d[X]}{dt} e^{k_2 t} + k_2[B]\, e^{k_2 t} = k_1[A]_0\, e^{(k_2-k_1)t}$$

This equation can be written as

$$\frac{d([X]e^{k_2 t})}{dt} = k_1[A]_0\, e^{(k_2-k_1)t}$$

or, $\displaystyle\int d([X]e^{k_2 t}) = \int k_1[A]_0\, e^{(k_2-k_1)t}\, dt$

or, $\displaystyle [X]e^{k_2 t} = \frac{k_1[A]_0\, e^{(k_2-k_1)t}}{(k_2-k_1)} + I$

I is the constant of Integration.

When $t = 0$, then $[X] = 0$, this gives

$$0 \times e^{k_2 \times 0} = \frac{k_1 [A]_0 \, e^{(k_2 - k_1) \times 0}}{k_2 - k_1} + I$$

$$0 = \frac{k_1 [A]_0}{k_2 - k_1} + I$$

or, $$I = -\frac{k_1 [A]_0}{k_2 - k_1}$$

Putting this value in the expression of $[X]$, we get

$$[X] e^{k_2 t} = \frac{k_1 [A]_0 \, e^{(k_2 - k_1) t}}{k_2 - k_1} - \frac{k_1 [A]_0}{k_2 - k_1}$$

$$= \frac{k_1 [A]_0}{(k_2 - k_1)} \left[e^{(k_2 - k_1)} - 1 \right]$$

$$[X] = \frac{k_1 [A]_0}{k_2 - k_1} \left[\cancel{e^{-k_2 t}} \, \cancel{e}^{(\cancel{k_2} - k_1) t} - e^{-k_2 t} \right]$$

$$= \frac{k_1 [A]_0}{k_2 - k_1} \left[e^{-k_1 t} - e^{-k_2 t} \right]$$