Assignment 1: Linear Algebra, Probability and Statistics

Tapaswi Satyapanthi
satyapanthi.t@northeastern.edu

Linear Algebra

Question 1

1. [10pt] Create a matrix array A and vector array in Numpy with **random** integers using random method. For example

$A = \begin{bmatrix} 5 & -2 \\ -3 & -1 \\ 7 & 9 \end{bmatrix}, x = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$. Your A matrix is **5x2** and your x vector is **2x1**. Show your code and result.

Create a diagonal matrix B, with 1,2,3,4,5 in the diagonal. The order doesn't matter. You can have

$B = \begin{bmatrix} 3 & & \\ & 4 & \\ & & 1 \end{bmatrix}$. Note your diagonal matrix is 5x5.

Calculate Ax, and BA. Show your code and result.

Code with Result:

Importing necessary library

In [1]:
```
import numpy as np
from numpy.linalg import matrix_rank
```

Matrix initialisation

In [2]:
```
# Creating matrix A or 2d array with the help of numpy's
# randint function by passing tuple, containing matrix dimentions
# to the size parameter
matrix_A = np.random.randint(-10, 10, size=(5, 2))

# Creating vector X by using numpy's randint function
vector_x = np.random.randint(-10, 10, size=(2,1))

# Diagonal matrix B with dimentions 5x5 having 1 to 5 on diagonal
matrix_B = np.diag([2, 1, 3, 4, 5])

print(f"Matrix A: \n{matrix_A}")
print(f"\nMatrix B: \n{matrix_B}")
print(f"\nVector X: \n{vector_x}")
```

```
Matrix A:
[[ 4  0]
 [ 4  6]
 [ 2  6]
 [-8 -4]
 [-4  9]]

Matrix B:
[[2 0 0 0 0]
 [0 1 0 0 0]
 [0 0 3 0 0]
 [0 0 0 4 0]
 [0 0 0 0 5]]

Vector X:
[[-2]
 [ 2]]
```

Matrix-Vector product: Ax

In [3]:
```
Ax_product = np.dot(matrix_A, vector_x)

print(f"Matrix - Vector(Ax) product: \n{Ax_product}")
```

```
Matrix - Vector(Ax) product:
[[-8]
 [ 4]
 [ 8]
 [ 8]
 [26]]
```

Matrix-Matrix product: BA

In [4]:
```
BA_product = np.dot(matrix_B, matrix_A)

print(f"Matrix - Matrix(BA) product: \n{BA_product}")
```

```
Matrix - Matrix(BA) product:
[[  8   0]
 [  4   6]
 [  6  18]
 [-32 -16]
 [-20  45]]
```

Question 2

2. [10pt] Calculate the rank of your matrix A and the rank of your matrix B. What about the rank of BA? Are they different? Why?

Matric Rank Calculation - Code with Result:

In [5]:
```
# Calculating rank of matrix A
rank_A = matrix_rank(matrix_A)

# Calculating rank of matrix B
rank_B = matrix_rank(matrix_B)

# Calculating rank of matrix BA
rank_BA = matrix_rank(np.dot(matrix_B,matrix_A))

print(f"Rank of matrix A: {rank_A}")
print(f"Rank of matrix B: {rank_B}")
print(f"Rank of matrix BA: {rank_BA}")
```

```
Rank of matrix A: 2
Rank of matrix B: 5
Rank of matrix BA: 2
```

What about the rank of BA? Are they different? Why?

Here rank of the matrix *BA* is not equal to the rank of matrix *B* but equal to the rank of matrix *A* because the *rank(BA)* is determined by the following relationship:

rank(BA) is minimum of the ranks of the matrices *B* and *A* i.e. *rank(BA) <= minimum(rank(B), rank(A))*

Hence, the rank of matrix *BA* will not be *5*, same as matrix *B*, because the minimum of the ranks of *B* and *A* is *2*, the rank of the matrix *A*.

So, the rank of the matrix *BA <= rank of A*, in this case equal to *A*, i.e. *2*.

Question 3

$$A = \begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 0 & -7 \\ 6 & 3 & -9 \\ -2 & -2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 3 & -1 \\ 2 & 4 & 5 \\ -1 & -1 & 8 \end{bmatrix}$$

3. [10pt] Find the following expressions by hand. Show your steps.
   (a) AB
   (b) BA
   (c) AB – BA
   (d) ABC

a, b, c, & d

Assignment 1 : Linear Algebra

Date: 12-01-24

Q-3

$A = \begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ +3 & 0 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 0 & -7 \\ 6 & 3 & -9 \\ -2 & -2 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 6 & 3 & -1 \\ 2 & 4 & 5 \\ -1 & -1 & 8 \end{bmatrix}$

a) $AB = \begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ -3 & 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 & -7 \\ 6 & 3 & -9 \\ -2 & -2 & 0 \end{bmatrix}$

$$= \begin{bmatrix} \begin{bmatrix}-2\\1\\8\end{bmatrix}\cdot\begin{bmatrix}5\\6\\-2\end{bmatrix} & \begin{bmatrix}-2\\1\\8\end{bmatrix}\cdot\begin{bmatrix}0\\3\\-2\end{bmatrix} & \begin{bmatrix}-2\\1\\8\end{bmatrix}\cdot\begin{bmatrix}-7\\-9\\0\end{bmatrix} \\ \begin{bmatrix}-1\\-1\\7\end{bmatrix}\cdot\begin{bmatrix}5\\6\\-2\end{bmatrix} & \begin{bmatrix}-1\\-1\\7\end{bmatrix}\cdot\begin{bmatrix}0\\3\\-2\end{bmatrix} & \begin{bmatrix}-1\\-1\\7\end{bmatrix}\cdot\begin{bmatrix}-7\\-9\\0\end{bmatrix} \\ \begin{bmatrix}3\\0\\4\end{bmatrix}\cdot\begin{bmatrix}5\\6\\-2\end{bmatrix} & \begin{bmatrix}3\\0\\4\end{bmatrix}\cdot\begin{bmatrix}0\\3\\-2\end{bmatrix} & \begin{bmatrix}3\\0\\4\end{bmatrix}\cdot\begin{bmatrix}-7\\-9\\0\end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -10+6+(-16) & 0+3+(-16) & 14+(-9)+0 \\ -5+(-6)+(-14) & 0+(-3)+(-14) & 7+9+0 \\ 15+0+(-8) & 0+0+(-8) & -21+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} -20 & -13 & 5 \\ -25 & -17 & 16 \\ 7 & -8 & -21 \end{bmatrix}$$

b) $BA = \begin{bmatrix} 5 & 0 & -7 \\ 6 & 3 & -9 \\ -2 & -2 & 0 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ +3 & 0 & 4 \end{bmatrix}$

$$= \begin{bmatrix} \begin{bmatrix}5\\0\\-7\end{bmatrix}\cdot\begin{bmatrix}-2\\-1\\3\end{bmatrix} & \begin{bmatrix}5\\0\\-7\end{bmatrix}\cdot\begin{bmatrix}1\\-1\\0\end{bmatrix} & \begin{bmatrix}5\\0\\-7\end{bmatrix}\cdot\begin{bmatrix}8\\7\\4\end{bmatrix} \\ \begin{bmatrix}6\\3\\-9\end{bmatrix}\cdot\begin{bmatrix}-2\\-1\\3\end{bmatrix} & \begin{bmatrix}6\\3\\-9\end{bmatrix}\cdot\begin{bmatrix}1\\-1\\0\end{bmatrix} & \begin{bmatrix}6\\3\\-9\end{bmatrix}\cdot\begin{bmatrix}8\\7\\4\end{bmatrix} \\ \begin{bmatrix}-2\\-2\\0\end{bmatrix}\cdot\begin{bmatrix}-2\\-1\\3\end{bmatrix} & \begin{bmatrix}-2\\-2\\0\end{bmatrix}\cdot\begin{bmatrix}1\\-1\\0\end{bmatrix} & \begin{bmatrix}-2\\-2\\0\end{bmatrix}\cdot\begin{bmatrix}8\\7\\4\end{bmatrix} \end{bmatrix}$$

Date: 12-01-24

$$= \begin{bmatrix} -10+0+(-21) & 5+0+0 & 40+0+(-28) \\ -12+(-3)+(-27) & 6+(-3)+0 & 48+21+(-36) \\ 4+2+0 & -2+2+0 & -16+(-14)+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} -31 & 5 & 12 \\ -42 & 3 & 33 \\ 6 & 0 & -30 \end{bmatrix}$$

c) $AB = \begin{bmatrix} -20 & -13 & 5 \\ -25 & -17 & 16 \\ 7 & -8 & -21 \end{bmatrix}$ $BA = \begin{bmatrix} -31 & 5 & 12 \\ -42 & 3 & 33 \\ 6 & 0 & -30 \end{bmatrix}$

$$AB - BA = \begin{bmatrix} -20 & -13 & 5 \\ -25 & -17 & 16 \\ 7 & -8 & -21 \end{bmatrix} - \begin{bmatrix} -31 & 5 & 12 \\ -42 & 3 & 33 \\ 6 & 0 & -30 \end{bmatrix}$$

$$= \begin{bmatrix} -20-(-31) & -13-5 & 5-12 \\ -25-(-42) & -17-3 & 16-33 \\ 7-6 & -8-0 & -21-(-30) \end{bmatrix}$$

$$= \begin{bmatrix} 11 & -18 & -7 \\ 17 & -20 & -17 \\ 1 & -8 & 9 \end{bmatrix}$$

d) $ABC = AB(C)$, $AB = \begin{bmatrix} -20 & -13 & 5 \\ -25 & -17 & 16 \\ 7 & -8 & -21 \end{bmatrix}$, $C = \begin{bmatrix} 6 & 3 & -1 \\ 2 & 4 & 5 \\ -1 & -1 & 8 \end{bmatrix}$

$$ABC = \begin{bmatrix} -20 & -13 & 5 \\ -25 & -17 & 16 \\ 7 & -8 & -21 \end{bmatrix} \cdot \begin{bmatrix} 6 & 3 & -1 \\ 2 & 4 & 5 \\ -1 & -1 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix}-20\\-13\\5\end{bmatrix}\cdot\begin{bmatrix}6\\2\\-1\end{bmatrix} & \begin{bmatrix}-20\\-13\\5\end{bmatrix}\cdot\begin{bmatrix}3\\4\\-1\end{bmatrix} & \begin{bmatrix}-20\\-13\\5\end{bmatrix}\cdot\begin{bmatrix}-1\\5\\8\end{bmatrix} \\ \begin{bmatrix}-25\\-17\\16\end{bmatrix}\cdot\begin{bmatrix}6\\2\\-1\end{bmatrix} & \begin{bmatrix}-25\\-17\\16\end{bmatrix}\cdot\begin{bmatrix}3\\4\\-1\end{bmatrix} & \begin{bmatrix}-25\\-17\\16\end{bmatrix}\cdot\begin{bmatrix}-1\\5\\8\end{bmatrix} \\ \begin{bmatrix}7\\-8\\-21\end{bmatrix}\cdot\begin{bmatrix}6\\2\\-1\end{bmatrix} & \begin{bmatrix}7\\-8\\-21\end{bmatrix}\cdot\begin{bmatrix}3\\4\\-1\end{bmatrix} & \begin{bmatrix}7\\-8\\-21\end{bmatrix}\cdot\begin{bmatrix}-1\\5\\8\end{bmatrix} \end{bmatrix}$$

d) contd…

Date: 12-01-24

$$ABC = \begin{bmatrix} -120+(-26)+(-5) & -60+(-52)+(-5) & 20+(-65)+40 \\ -150+(-34)+(-16) & -75+(-68)+(-16) & 25+(-85)+128 \\ 42+(-16)+(21) & 21+(-32)+21 & -7+(-40)+(-168) \end{bmatrix}$$

$$ABC = \begin{bmatrix} -151 & -117 & -5 \\ -200 & -159 & 68 \\ 47 & 10 & -215 \end{bmatrix}$$

Question 4

4. [10pt] Calculate the eigenvalues and eigenvectors of matrix A above by hand.
   a. Show your steps below. You can use external tools to solve for a polynomial equation only.
   b. Show the trace of matrix A.

a) Calculate Eigenvalues and Eigenvectors

Q-4 $A = \begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix}$ Calculate eigenvalues and eigenvectors of matrix A. Also, show the trace of matrix A.

$Ax = \lambda x$

$Ax - \lambda I x = 0 \quad \therefore (A - \lambda I)x = 0$

$\det(A - \lambda I) = 0$

$$\begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} -2-\lambda & 1 & 8 \\ -1 & -1-\lambda & 7 \\ 3 & 0 & 4-\lambda \end{bmatrix}$$

Finding the determinant

$$(-2-\lambda)\left[(-1-\lambda)(4-\lambda) - (7\times 0)\right] - 1\left[(-1)(4-\lambda) - (7)(3)\right] + 8\left[(-1)(0) - (-1-\lambda)(3)\right] = 0$$

$\therefore \lambda^3 + \lambda^2 + 33\lambda + 57 = 0$

$\therefore \lambda = -2.192, \quad \lambda = -3.747, \quad \lambda = 6.939$

Hence, the eigenvalues (λ) are $\lambda = -2.192$, $\lambda = -3.747$, $\lambda = 6.939$

* Now, find the eigenvector with eigenvalue $\lambda = -2.192$

a) contd...

Date: 17/01/24

$(A-\lambda I)x = 0$

$$\left[\begin{bmatrix}-2 & 1 & 8\\ -1 & -1 & 7\\ 3 & 0 & 4\end{bmatrix} - (-2.192)\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\right] x = 0$$

$$\left[\begin{bmatrix}-2 & 1 & 8\\ -1 & -1 & 7\\ 3 & 0 & 4\end{bmatrix} - \begin{bmatrix}-2.192 & 0 & 0\\ 0 & -2.192 & 0\\ 0 & 0 & -2.192\end{bmatrix}\right] x = 0$$

$$\begin{bmatrix}0.192 & 1 & 8\\ -1 & 1.192 & 7\\ 3 & 0 & 6.192\end{bmatrix} \cdot \begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = 0$$

$0.192x_1 + x_2 + 8x_3 = 0$

$-x_1 + 1.192x_2 + 7x_3 = 0$

$3x_1 + 0 + 6.192x_3 = 0$

Solving system of equations using cramer's rule

x_1: $\begin{vmatrix}1.192 & 7\\ 0 & 6.192\end{vmatrix}$ $\quad -x_2$: $\begin{vmatrix}-1 & 7\\ 3 & 6.192\end{vmatrix}$ $\quad x_3$: $\begin{vmatrix}-1 & 1.192\\ 3 & 0\end{vmatrix}$

$(1.192\times 6.192) - (7\times 0) = 7.38$

$(-1\times 6.192) - (3\times 7) = 27.192$

$(-1\times 0) - (1.192\times 3) = -3.576$

Hence the eigenvector for the eigenvalue -2.192 is

$\begin{bmatrix}7.38\\ 27.192\\ -3.576\end{bmatrix}$ ← eigenvector for $\lambda = -2.192$

Date: 17-01-24

* Eigenvector with eigenvalue $\lambda = -3.747$

$(A-\lambda I)x = 0$

$$\left(\begin{bmatrix}-2 & 1 & 8\\ -1 & -1 & 7\\ 3 & 0 & 4\end{bmatrix} - \begin{bmatrix}-3.747 & 0 & 0\\ 0 & -3.747 & 0\\ 0 & 0 & -3.747\end{bmatrix}\right) x = 0$$

$$\begin{bmatrix}1.747 & 1 & 8\\ -1 & 2.747 & 7\\ 3 & 0 & 7.747\end{bmatrix} \cdot \begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = 0$$

$1.747x_1 + x_2 + 8x_3 = 0$

$-1x_1 + 2.747x_2 + 7x_3 = 0$

$3x_1 + 0x_2 + 7.747x_3 = 0$

x_1: $\begin{vmatrix}2.747 & 7\\ 0 & 7.747\end{vmatrix} = 21.28$ $\quad -x_2$: $\begin{vmatrix}-1 & 7\\ 3 & 7.747\end{vmatrix} = 28.747$ $\quad x_3$: $\begin{vmatrix}-1 & 2.747\\ 3 & 0\end{vmatrix} = -8.241$

∴ Eigenvector for eigenvalue $\lambda = -3.747$ is $\begin{bmatrix}21.28\\ 28.74\\ -8.24\end{bmatrix}$

* Finding eigenvector with eigenvalue $\lambda = 6.939$

$$\left[\begin{bmatrix}-2 & 1 & 8\\ -1 & -1 & 7\\ 3 & 0 & 4\end{bmatrix} - \begin{bmatrix}6.939 & 0 & 0\\ 0 & 6.939 & 0\\ 0 & 0 & 6.939\end{bmatrix}\right] x = 0$$

$$\begin{bmatrix}-8.939 & 1 & 8\\ -1 & -7.939 & 7\\ 3 & 0 & -2.939\end{bmatrix} \cdot \begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = 0$$

$-8.939x_1 + x_2 + 8x_3 = 0$

$-x_1 - 7.939x_2 + 7x_3 = 0$

$3x_1 + 0x_2 - 2.939x_3 = 0$

Date: 17-01-24

x_1: $\begin{vmatrix}-7.939 & 7\\ 0 & -2.939\end{vmatrix} = 23.33$ $\quad -x_2$: $\begin{vmatrix}-1 & 7\\ 3 & -2.939\end{vmatrix} = 18.061$ $\quad x_3$: $\begin{vmatrix}-1 & -7.939\\ 3 & 0\end{vmatrix} = 23.817$

∴ Eigenvector resulting from eigenvalue $\lambda = 6.939$ is $\begin{bmatrix}23.33\\ 18.06\\ 23.817\end{bmatrix}$

b) Trace of Matrix

b. Trace of a matrix A:

trace is the sum of all the eigenvalues of a matrix.
Here eigenvalues are $\lambda = -2.192$, $\lambda = -3.747$, $\lambda = 6.939$

Hence the trace of matrix A $= (-2.192) + (-3.747) + (6.939) = 1$

Hence, the trace of a matrix A is 1

Question 5

5. [10pt] Use Numpy to concatenate A, B, C to a 9 x 3 matrix. The order doesn't matter. Let's call the new matrix D. Create b=[3, -10, 2]^T. Find the least squares solution x that minimize $||D^T x - b||^2$. Show your code and results.

Initializing matrices for question 5

In [6]:
```
# Initializing matrices A, B & C
A = np.array([[-2,  1,  8],
              [-1, -1,  7],
              [ 3,  0,  4]])

B = np.array([[ 5,  0, -7],
              [ 6,  3, -9],
              [-2, -2,  0]])

C = np.array([[ 6,  3, -1],
              [ 2,  4,  5],
              [-1, -1,  8]])
```

Matrix Concatenation and vector b initialization

In [7]:
```
# Concatenating matrices A, B & C
D = np.concatenate((A, B, C))
print(f"Concatenated matrix D: \n{D}")

# Initializing vector b and transposing it
b = np.array([[3, -10, 2]]).T
print(f"\nVector b after transposing: \n{b}")
```

```
Concatenated matrix D:
[[-2  1  8]
 [-1 -1  7]
 [ 3  0  4]
 [ 5  0 -7]
 [ 6  3 -9]
 [-2 -2  0]
 [ 6  3 -1]
 [ 2  4  5]
 [-1 -1  8]]

Vector b after transposing:
[[  3]
 [-10]
 [  2]]
```

Least Square solution

In [8]:
```
# Finding the least square solution
least_square = np.linalg.lstsq(D.T,b, rcond=None)[0]

print(f"Least Square solution: \n{least_square}")
```

```
Least Square solution:
[[-0.6588419 ]
 [ 0.79125205]
 [ 1.2848211 ]
 [ 1.12195654]
 [-0.49220611]
 [ 0.53822844]
 [ 0.10452284]
 [-1.36113905]
 [ 0.86584317]]
```

Statistics

Question 1

1. [10pt] Roll a six-sided die 5 times. What is the probability of rolling a six in all 5 rolls? If rolling the die 5 times is considered one trial, perform 500 trials. What is the probability of rolling a six in all 5 rolls in exactly one of these 500 trials? What about rolling a six in all 5 rolls in at least one of the 500 trials?

Q-1 The probability of rolling a six-sided die is $1/6$

- Probability of rolling a six in all 5 roll

$$= (1/6)^5 = \cancel{1}\,(1/7776) = 0.000129$$

- What is Probability of rolling a six in all 5 rolls in exactly 1 of these 500 trials?

* Rolling the die 5 times is considered as 1 trial *

Binomial Probability formula :

$$P(x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

n = 500 trials

x = no. of success = 1

p = Prob. of Success on a Single trial = $(1/6)^5$

q = Prob. of faliure on a Single trial = $1 - p = 1 - (1/6)^5$

Que 1 contd…

Date: 23-01-24

$\therefore$ P(exactly 1 success in 500 trials) =

$$\frac{500}{1} \times \left(\frac{1}{6}\right)^5 \times \left(1-\left(\frac{1}{6}\right)^5\right)^{500-1}$$

$$= 500 \times \frac{1}{7776} \times \left(1-\frac{1}{7776}\right)^{499}$$

$$= 0.06$$

Hence, probability of getting a six in ~~exactly~~ all 5 rolls in exactly 1 of 500 trials is 6%.

- Probability of rolling a six in ~~at least~~ all 5 rolls in atleast one of the 500 trials.

P(at least one success) = 1 − P(no success)

$$= \left(1-\left(1/6\right)^5\right)^{500}$$

$$= \left(1 - 1/7776\right)^{500}$$

$$= \left(7775/7776\right)^{500} = 0.93$$

Hence, Probability of rolling a six in all 5 rolls in at least one of the 500 trials is 93%.

Question 2

2. [10pt] A study found that the average amount of coffee consumed by college students is 3 cups per day. Assuming this consumption follows a normal distribution with a standard deviation of 0.8 cups, what is the probability that a randomly selected college student drinks between 2.2 and 3.8 cups of coffee per day?

M T W T F S S
Page No.:
Date: 23-01-24
YOUVA

Q-2 mean $\bar{x} = 3$ cups , std. deviation $\sigma = 0.8$

x = amount of coffee consumed per day

$x = 2.2$ & $x = 3.8$

* calculating z-score for $x = 2.2$ & $x = 3.8$

$Z_x = \frac{x - \bar{x}}{\sigma}$ $\therefore$ $Z_{2.2} = \frac{2.2 - 0.8}{0.8} = -1$

$Z_{3.8} = \frac{3.8 - 0.8}{0.8} = 1$

* Now, looking z-scores in a standard normal distribution table.

for $Z_{2.2} = -1$, the z-score is 0.15866 &

for $Z_{3.8} = 1$, the z-score is 0.84134

$\therefore P(Z \leq z_{2.2}) = 0.15866$

$P(Z \leq z_{3.8}) = 0.84134$

* Now, Probability of a randomly selected student drinks between 2.2 & 3.8 cups

$P(3.8 \geq X \geq 2.2) = P(Z \leq z_{3.8}) - P(Z \leq z_{2.2})$

$= 0.84134 - 0.15866$

$= 0.683$

* Hence, Probability that a randomly selected student drinks between 2.2 & 3.8 cups of coffee is 68%.

Question 3

3. [10pt] 6 Digital Camera Prices The prices (in dollars) for a particular model of digital camera with 18.0 megapixels and a f/3.5–5.6 zoom lens are shown here for 10 randomly selected online retailers. Estimate the true mean price for this particular model with 95% confidence.

Prices: [999, 1499, 1997, 398, 591, 498, 798, 849, 449, 348]

Page No.: | Date: 27-01-24 | YOUVA

Q-3 $n = 10$ confidence level = 95%

Prices: [999, 1499, 1997, 398, 591, 498, 798, 849, 449, 348]

* $\bar{x} = \dfrac{999 + 1499 + 1997 + 398 + 591 + 498 + 798 + 849 + 449 + 348}{10}$

$\bar{x} = 842.6$

$s = \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{n-1}}$

$= \sqrt{\dfrac{2569262.4}{9}}$

$= 534.29$

$(x_i - \bar{x})^2$
24460.96
430860.96
1332634.36
197669.16
63302.56
118749.16
1989.16
40.96
154920.96
244629.16
2569262.4

t-value for 95% confidence level with degree of freedom $(n-1)$ i.e. 9 is 2.262

* Now, to find true mean:

$\mu = \bar{x} \pm t\left(\frac{s}{\sqrt{n}}\right)$ i.e.

$\bar{x} + t\left(\frac{s}{\sqrt{n}}\right) \leq \mu \leq \bar{x} + t\left(\frac{s}{\sqrt{n}}\right)$

$\therefore \mu = 842.6 \pm 2.262\left(\frac{534.29}{\sqrt{10}}\right)$

$= 842.6 \pm 168.95 \times 2.262$

$= 842.6 \pm 382.16$

$[460.44 \leq \mu \leq 1224.76]$

* Hence, the true mean price of this camera model with 95% confidence lies between 460.44 & 1224.76.

Question 4

4. [10pt] The average number of books read by a person in a year is reported to be 12. A 'reader' is defined as a person who reads at least one book in a year. A random sample of 50 readers from a local community library showed that the average number of books read per person was 13.4. The population standard deviation is 4.5 books. At the 0.01 level of significance, can it be concluded that this sample represents a significant difference from the national average?

Page No.:
Date: 27-01-24
YOUVA

Q-4 $\mu = 12$ → Avg. no. of books read by a person in a year.
$\bar{x} = 13.4$ → Sample mean
$n = 50$ → Sample size
$\sigma =$ ~~Sam~~ 4.5 → Population Standard Deviation
Significance Level $= 0.01$ → α

* Null Hypothesis (H_0): The average no. of books read by a person in a year is 12.
Alt. Hypothesis (H_1): The average ~~not~~ no. of books read by a person in a year is 13.4.

* Here, $n > 30$ & population standard deviation is known so, I will use a z-test.

$z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ $\therefore$ $z = \dfrac{13.4 - 12}{4.5/\sqrt{50}} = 2.19 \approx 2.20$

* Critical Value based on alpha (α) = ~~0.05~~ 0.01 is 2.58

* Since the calculated z-value, 2.2, is less than critical value 2.58, we ~~reject the~~ fail to reject the null hypothesis.

Hence, with 0.01 level of significance i.e. with 99% confidence we can conclude that this sample doesn't represent a significant difference from the national average.

Question 5

5. [10pt] A statistics professor is used to having a variance in his class grades of no more than 100. He feels that his current group of students is different, and so he examines a random sample of midterm grades as shown. At α = 0.05, can it be concluded that the variance in grades exceeds 100?

The grades: [92.3, 89.4, 76.9, 65.2, 49.1, 96.7, 69.5, 72.8, 67.5, 52.8, 88.5, 79.2, 72.9, 68.7, 75.8]

M T W T F S S
Page No.:
Date: 27-01-24
YOUVA

Q-5 $n = 15$, Significance level $(\alpha) = 0.05$

$\sigma^2 = 100 \rightarrow$ Population Variance

grades: [92.3, 89.4, 76.9, 65.2, 49.1, 96.7, 69.5, 72.8, 67.5, 52.8, 88.5, 79.2, 72.9, 68.7, 75.8]

* Null Hypothesis (H_0): Variance in class grades is no more than 100.

Alt. Hypothesis (H_1): Variance in class grades is more than 100.

* $$\bar{x} = \frac{92.3 + 89.4 + 76.9 + 65.2 + 49.1 + 96.7 + 69.5 + 72.8 + 67.5 + 52.8 + 88.5 + 79.2 + 72.9 + 68.7 + 75.8}{15}$$

$\therefore \bar{x} = 74.48$

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = \frac{2572.85}{14} = 183.77$$

* The critical chi-squared value for degree of freedom = 15 & significance level of 0.05 is $\underline{23.68} \rightarrow$ Critical Val.

Now, calculate the test stats.

$$\chi^2 = \frac{(n-1) \cdot s^2}{\sigma^2} = \frac{14 \times 183.77}{100} = 25.72$$

* Since, $25.72 > 23.68$, we reject the null Hypothesis.

Hence, ~~th~~ with 0.05 significance level we can conclude that variance in grades exceeds 100.

$(x_i - \bar{x})^2$
317.31
222.40
5.82
86.24
644.48
493.43
24.86
2.84
48.81
470.31
196.37
22.21
2.51
33.48
1.72
2572.85