Babeș-Bolyai University
Bachelor in Computer Science

Seminar 7

1. Compute the following limits using a Riemann sum:

   (a) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \frac{1}{n+k}$.

   (b) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \frac{k^p}{n^{p+1}}$, $p \in \mathbb{N}$.

   (c) $\lim\limits_{n\to\infty} \frac{\sqrt[n]{n!}}{n}$.

2. Study the Riemann integrability of the function $f : [0,1] \to \mathbb{R}$,

$$f(x) = \begin{cases} 1, & \text{if } x \in \mathbb{Q}, \\ 0, & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

3. Compute the following improper integrals:

   (a) $\displaystyle\int_1^2 \frac{1}{x(x-2)}\,\mathrm{d}x$.

   (b) $\displaystyle\int_0^\infty x e^{-x^2}\,\mathrm{d}x$.

   (c) $\displaystyle\int_0^1 \frac{\ln x}{\sqrt{x}}\,\mathrm{d}x$.

   (d) $\displaystyle\int_0^\infty e^{-x} \sin x\,\mathrm{d}x$.

4. Study the convergence of the following improper integrals:

   (a) $\displaystyle\int_1^\infty \frac{1}{x\sqrt{1+x^2}}\,\mathrm{d}x$.

   (b) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{1}{\cos x}\,\mathrm{d}x$.

   (c) $\displaystyle\int_1^\infty \frac{\ln x}{x\sqrt{x^2-1}}\,\mathrm{d}x$.

5. Using the integral test, study the convergence of the following series:

   (a) $\sum\limits_{n\geq 2} \frac{1}{n(\ln n)^2}$.

   (b) $\sum\limits_{n\geq 2} \frac{\ln n}{n^2}$.

   (c) $\sum\limits_{n\geq 2} \frac{1}{n \ln n \ln(\ln n)}$.