Seminar 12

1. Use Lagrange multipliers to find the extrema of the following functions subject to constraints:

   (a) $x^2 + y^2$ subject to $x - y + 1 = 0$.

   (b) $(x+y)^2$ subject to $x^2 + y^2 = 1$.

   (c) $x + 2y + 3z$ subject to $x^2 + y^2 + z^2 = 1$.

   (d) $2x^2 + y^2 + 3z^2$ subject to $x^2 + y^2 + z^2 = 1$.

2. Find the minimum value of $\frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$ subject to the following constraints:

   (a) $x_1 + x_2 + x_3 = 3$.

   (b) $x_1 + x_2 + x_3 = 3$ and $x_1 + 2x_2 + 3x_3 = 12$.

3. Compute the following double integrals:

   (a) $\displaystyle\iint_R \cos x \sin y \,\mathrm{d}x\,\mathrm{d}y$, where $R = [0, \pi/2] \times [0, \pi/2]$.

   (b) $\displaystyle\iint_R \frac{1}{(x+y)^2} \,\mathrm{d}x\,\mathrm{d}y$ and $\displaystyle\iint_R y e^{xy} \,\mathrm{d}x\,\mathrm{d}y$, where $R = [1, 2] \times [0, 1]$.

4. Let $D \subseteq \mathbb{R}^2$ be the subset bounded by the parabola $y = x^2$ and the lines $x = 2$ and $y = 0$.

   (a) Express D as a simple set, first w.r.t. the y-axis and then w.r.t. the x-axis.

   (b) Compute $\displaystyle\iint_D xy \, dx \, dy$ in two ways, by changing the order of integration.

5. By changing the order of integration, compute the following double integrals:

   (a) $\displaystyle\int_0^1 \int_y^1 \sin(x^2) \,\mathrm{d}x\,\mathrm{d}y$.

   (b) $\displaystyle\int_0^1 \int_x^1 e^{y^2} \,\mathrm{d}y\,\mathrm{d}x$.

6. Compute the following integrals by a change of variables:

   (a) $\displaystyle\iint_D e^{\frac{x-y}{x+y}} \,\mathrm{d}x\,\mathrm{d}y$, $D = \{(x, y) \in \mathbb{R}^2 \mid x \geq 0,\ y \geq 0, x + y \leq 1\}$. Use $u = x - y$, $v = x + y$.

   (b) $\displaystyle\iint_D \left(\frac{x-y}{x+y+2}\right)^2 \mathrm{d}x\,\mathrm{d}y$, $D = \{(x, y) \in \mathbb{R}^2 \mid |x| + |y| \leq 1\}$. Use $u = x - y$, $v = x + y$.

   (c) $\displaystyle\iint_D \frac{y^2}{x} \,\mathrm{d}x\,\mathrm{d}y$, D is bounded by $x = 1 - y^2$ and $x = 3(1 - y^2)$. Use $y = u$, $x = (1 - u^2)v$.