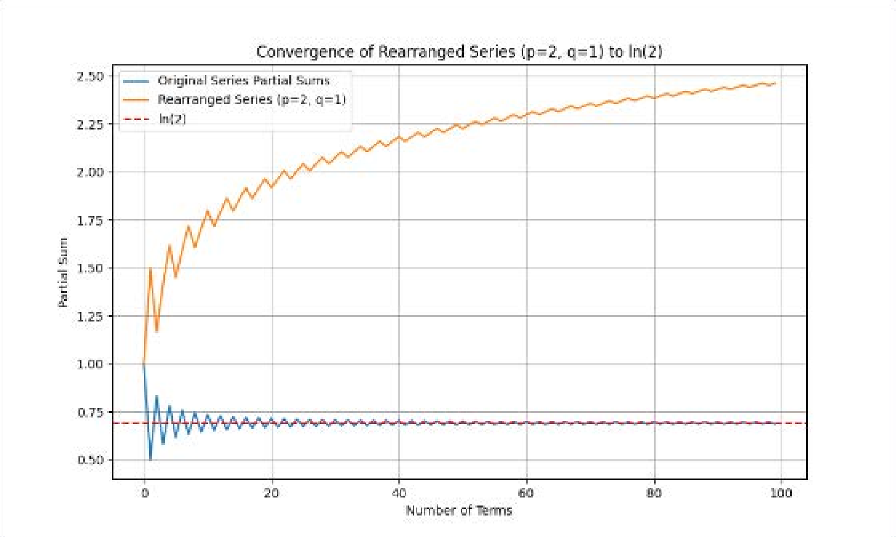
```python
import numpy as np
import matplotlib.pyplot as plt

# Constants
num_terms = 100  # Number of terms for the partial sum
target_value = np.log(2)  # The target value for the series sum

# Part 1: Calculate partial sums for the original series
original_partial_sums = []
current_sum = 0
for n in range(1, num_terms + 1):
    term = (-1) ** (n + 1) / n
    current_sum += term
    original_partial_sums.append(current_sum)

# Plot the convergence of the original series
plt.figure(figsize=(10, 6))
plt.plot(original_partial_sums, label='Original Series Partial Sums')
plt.axhline(target_value, color='red', linestyle='--', label='ln(2)')
plt.xlabel('Number of Terms')
plt.ylabel('Partial Sum')
plt.title('Convergence of Original Series to ln(2)')
plt.legend()
plt.grid()
plt.show()

```

```python
# Part 2: Calculate partial sums with rearranged terms (p positive, q negative)
def rearranged_partial_sums(p, q, num_terms):
    partial_sums = []
    current_sum = 0
    n = 1
    while len(partial_sums) < num_terms:
        # Add p positive terms
        for _ in range(p):
            if len(partial_sums) >= num_terms:
                break
            current_sum += 1 / n
            partial_sums.append(current_sum)
            n += 1

        # Add q negative terms
        for _ in range(q):
            if len(partial_sums) >= num_terms:
                break
            current_sum -= 1 / n
            partial_sums.append(current_sum)
            n += 1

    return partial_sums
```

```python
# Example values of p and q
p = 2
q = 1
rearranged_sums = rearranged_partial_sums(p, q, num_terms)

# Plot the convergence of the rearranged series
plt.figure(figsize=(10, 6))
plt.plot(original_partial_sums, label='Original Series Partial Sums')
plt.plot(rearranged_sums, label=f'Rearranged Series (p={p}, q={q})')
plt.axhline(target_value, color='red', linestyle='--', label='ln(2)')
plt.xlabel('Number of Terms')
plt.ylabel('Partial Sum')
plt.title(f'Convergence of Rearranged Series (p={p}, q={q}) to ln(2)')
plt.legend()
plt.grid()
plt.show()

```



Convergence of Rearranged Series (p=2, q=1) to ln(2)
Original Series Partial Sums
Rearranged Series (p=2, q=1)
ln(2)
Partial Sum
Number of Terms
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0
20
40
60
80
100

08.11.24

Mathematical Analysis
Homework 4
Tapordei Maia

Q: Ex 1

Find radius of convergence and convergence set of these series:

a) $\sum_{n\geq 0} \frac{nx^n}{2^n} = \sum_{n\geq 0} \frac{n}{2^n}(x-0)^n$, $a_n = \frac{n}{2^n}$, $c=0$

$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n^{1/n}}{2} = \frac{1}{2} \lim_{n\to\infty} n^{1/n} \underset{\text{criterium}}{\overset{\text{Radical}}{=}}$

$= \frac{1}{2} \lim_{n\to\infty} \frac{n+1}{n} = \frac{1}{2} = L \in (0,+\infty)$

$R = \frac{1}{1/2} = 2 \Rightarrow$ Series is absolutely convergent on $(-2,2)$ and divergent on $(-\infty;-2) \cup (2;+\infty)$

If $x=-2 \Rightarrow \sum_{n\geq 0} \frac{n(-2)^n}{2^n} = \sum_{n\geq 0} (-1)^n \cdot n = 0-1+2-3+4\ldots \to$ divergent

If $x=2 \Rightarrow \sum_{n\geq 0} \frac{n\cdot 2^n}{2^n} = \sum_{n\geq 0} n \to$ divergent $\Rightarrow C=(-2,2)$

b) $\sum_{n\geq 1} \frac{x^{2n}}{\sqrt{n}}$, $y=x^2 \Rightarrow \sum_{n\geq 1} \frac{y^n}{\sqrt{n}} = \sum_{n\geq 1} \frac{1}{\sqrt{n}}\cdot(y-0)^n$, $\frac{1}{\sqrt{n}} = a_n$, $c=0$

$\lim_{n\to\infty} \sqrt[n]{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{1}{n^{1/2n}} = \lim_{n\to\infty} n^{-1/2n} = \lim_{n\to\infty} e^{-\frac{\ln n}{2n}} =$

$= e^{-1/2 \cdot \lim_{n\to\infty} \ln(n)/n} = e^0 = 1 = L \in (0,\infty)$

$R=1 \Rightarrow$ series is abs. conv. on $(-1,1)$ and div. on $(-\infty;-1)\cup(1;+\infty)$

If $y=-1 \Rightarrow \sum_{n\geq 1} \frac{(-1)^n}{\sqrt{n}}$, $\frac{1}{\sqrt{n}}, n\in N = \{1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} \ldots\} \to$ decreasing (1)

$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ (2), (1),(2) $\underset{\text{test}}{\overset{\text{Leibniz}}{\Longrightarrow}} \sum_{n\geq 1} \frac{(-1)^n}{\sqrt{n}} \to$ convergent

If $y=1 \Rightarrow \sum_{n\geq 1} \frac{1^n}{\sqrt{n}} = \sum_{n\geq 1} \frac{1}{\sqrt{n}} \to$ divergent $\Rightarrow C=[-1,1)$

c) $\sum_{n\geq 1} \frac{(-1)^n(x-1)^n}{n}$, $a_n = \frac{(-1)^n}{n}$, $c=1$

$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(-1)^n}{n}\right|} = \lim_{n\to\infty} \frac{1}{n^{1/n}} = \lim_{n\to\infty} \sqrt[n]{\frac{1}{n}} \underset{\text{criterium}}{\overset{\text{Radical}}{=}} \lim_{n\to\infty} \frac{1/n+1}{1/n} = 1$

$R=1 \Rightarrow$ Series is abs. conv. on $(0,2)$ and div. on $(-\infty,0)\cup(2;+\infty)$

If $x=0 \Rightarrow \sum_{n\geq 1} \frac{(-1)^{2n}}{n} = \sum_{n\geq 1} \frac{1}{n} \to$ divergent (harmonic series)

If $x=2 \Rightarrow \sum_{n\geq 1} \frac{(-1)^n}{n}$, $x_n = \frac{1}{n} \to$ decreasing, $\lim_{n\to\infty} \frac{1}{n} = 0 \underset{\text{test}}{\overset{\text{Leibniz}}{\Longrightarrow}}$

$\Rightarrow$ series is convergent $\Rightarrow C=(0,2]$

Q: Ex 2

Study convergence and compute sum for series:

$\sum_{n\geq 2} \frac{x^n}{n(n-1)}$

Ratio test: $\left|\frac{x^{n+1}}{(n+1)n} \cdot \frac{n(n-1)}{x^n}\right| = \left|\frac{x\cdot(n-1)}{n+1}\right| \Rightarrow |x| \lim_{n\to\infty} \frac{n-1}{n+1} = |x| \Rightarrow$

$\Rightarrow$ the series converges absolutely for $|x|<1$

If $x=1 \Rightarrow \sum_{n\geq 2} \frac{1}{n(n-1)}$, $y_n = \frac{1}{n}$, $\frac{1}{n(n+1)} < \frac{1}{n} \overset{C.T.I}{\Longrightarrow} \sum_{n\geq 2} \frac{1}{n(n+1)} \to$ convergent

If $x=-1 \Rightarrow \sum_{n\geq 2} \frac{(-1)^n}{n(n-1)}$, $x_n = \frac{1}{n(n+1)} \to$ decreasing, $\lim_{n\to\infty} \frac{1}{n(n-1)} = 0 \Rightarrow$

$\Rightarrow$ the series converges conditionally $\Rightarrow$ the series converges for $|x|\leq 1$

$\sum_{n\geq 2} \frac{x^n}{n(n-1)} = \sum_{n\geq 1} x^n\left(\frac{1}{n-1} - \frac{1}{n}\right) = \sum_{n\geq 2} \frac{x^n}{n-1} - \sum_{n\geq 2} \frac{x^n}{n} = x\sum_{n\geq 1} \frac{x^n}{n} - \sum_{n\geq 2} \frac{x^n}{n} =$

$= -x\ln(1-x) - (\ln(1-x) - x) = -\ln(1-x)(1-x)$

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