Midterm Test

1. Find $\inf, \sup, \min, \max$ and the accumulation points for $\{\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \ldots\}$. **1p (5×0.2p)**

2. Study the convergence of the following series:

   (a) $\sum_{n\geq 1} \frac{2^n n!}{n^n}$. **1p**

   (b) $\sum_{n\geq 0} \frac{n!}{a(a+1)\ldots(a+n)}$, $a > 0$. **1p**

   (c) $\sum_{n\geq 1} \frac{1}{n\sqrt[n]{n}}$. **0.5p**

   (d) $\sum_{n\geq 2} \frac{1}{n(\ln n)^p}$, $p > 0$. **1p**

   (e) $\sum_{n\geq 1} (-1)^n \sin\frac{1}{n^3}$. **1p**

   (f) $\sum_{n\geq 1} \sin\left(\pi\sqrt{n^2+1}\right)$. **0.5p**

3. Find the sum of the following series:

   (a) $\sum_{n\geq 1} \frac{1}{(n+\alpha)(n+\alpha+1)}$, $\alpha \in (0,1)$. **1p**

   (b) $\sum_{n\geq 2} \frac{n(n+1)}{2^n}$. **1p**

4. Find the sum and the convergence set for the power series $\sum_{n\geq 1} nx^n$. **1p**

5. Find the Taylor series around 0 and its radius of convergence for $\sinh(x) := \frac{1}{2}(e^x - e^{-x})$. **1p**

Row: 1. Time: 1h50min. Total number of points: 10.