21.11.24

Logical computation
Homework 2

Q: Ex 6.7
Using lock resolution prove the inconsistency of the following sets of clauses. Choose two different indexing for the literals. For one indexing combine lock resolution with level-saturation strategy.
$S_7 = \{ r \vee p,\ r \vee \neg p \vee \neg q,\ \neg q \vee \neg r,\ q \}$

I. $C_1 = (1)\, r \vee (2)\, p$
$C_2 = (3)\, r \vee (3)\, \neg p \vee (4)\, \neg q$
$C_3 = (5)\, \neg q \vee (5)\, \neg r$
$C_4 = (6)\, q$
Lock resolution with level saturation strategy:
$S^0 = \{ C_1, C_2, C_3, C_4 \}$
$C_5 = Res_p^{lock}(C_1, C_2) = (1)\, r \vee (4)\, \neg q$
$C_6 = Res_q^{lock}(C_3, C_4) = (5)\, \neg r$
$S^1 = \{ C_5, C_6 \}$
$C_7 = Res_r^{lock}(C_5, C_6) = (4)\, \neg q$
$S^2 = \{ C_7 \}$
$C_8 = Res_q^{lock}(C_7, C_4) = \bot$ (contradiction)
$S_3 = \{ C_8 \}$ => S is inconsistent

II. $C_1 = (2)\, r \vee (1)\, p$
$C_2 = (2)\, r \vee (4)\, \neg p \vee (3)\, \neg q$
$C_3 = (3)\, \neg q \vee (5)\, \neg r$
$C_4 = (6)\, q$
$C_5 = Res_p^{lock}(C_1, C_2) = (2)\, r \vee (3)\, \neg q$
$C_6 = Res_r^{lock}(C_3, C_5) = (3)\, \neg q$
$C_7 = Res_q^{lock}(C_6, C_6) = \bot$ (contradiction) => S is inconsistent.

Theory:
Resolution → rule that produces a new clause implied by 2 clauses containing complementary literals.
Lock resolution → a restricted form of resolution where we only resolve clauses containing a locked literal, which must remain in at least one of the resolvents
Level-saturation strategy → generates levels of resolvents corresponding to exploration of the whole search space which contains all possible resolvents.