MGB 206: Decision Making and Management Science

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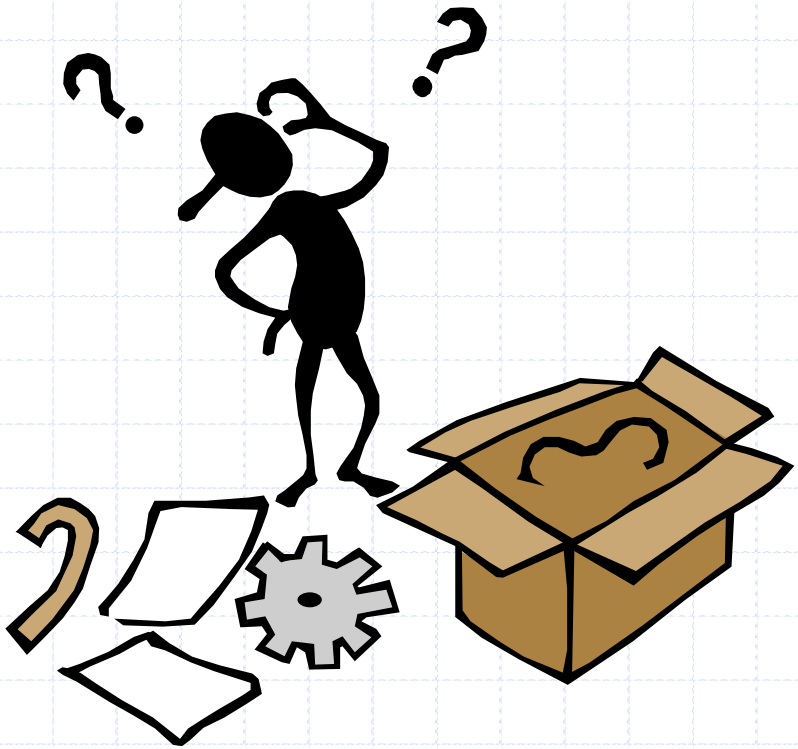
Lesson Plan: Session 6

1. Session 5 reprise
2. Integer programs
3. Nonlinear programs

What We Discussed Last Time



LP: Simplex Method

- Illustrative animations
  - A tiny geometric example: here
  - How the simplex method works: here
- The simplex method is fast & accurate
  - Practical LPs with tens of thousands of variables solved in a few minutes
  - LPs with many millions of variables can be solved in under an hour

http://optlab-server.sce.carleton.ca/POAnimations2007/Default.html

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Exercise: Extending Boats LP

1. ~~What if we have multiple objectives?~~
2. ~~What if demand is limited?~~
3. ~~What if we need to analyze multiple time periods in one model?~~
4. What if we need to upgrade manufacturing?

Exercise: Evaluating Upgrade

- Producing large sailboats requires an upgrade of existing production facilities
  - Cost: $14,000
- How to model such a bump in cost?

Integer Programs

- An LP in which some of the variables cannot take on fractional values
  - A mix of continuous and integer variables, is called Mixed-Integer Programming (or, MIP)
- In RSPE, simply add constraint limiting a variable to be either *int* or *bin*
  - *bin* stands for binary, i.e., 0 or 1

Integer Programs (2)

- Consequences of integrality constraints
  - Longer solution times
    - Turning LP into MIP may make it unsolvable!
    - For especially difficult problem may require settling for suboptimal solution
  - Dual values no longer meaningful

Modeling With Integers

- Excluding fractional answers
- Yes/no decisions
- Logical conditions

Exercise: Project Portfolio

You need to choose among five projects shown below (units, million$)

	Project				
	P1	**P2**	**P3**	**P4**	**P5**
NPV	10	17	16	8	14
Cost	48	96	80	32	64

Your budget is $160 million

- Pick the highest-yielding projects
- Selecting P5 requires you to also select P3. Does this impact your total NPV?

MIP: Branch-And-Bound Method

- Solve the 'LP relaxation' of the MIP
  - In effect, ignore integrality of variables
  - For any variable with fractional solution value, create and solve two new LPs
  - Explore this 'tree' until
    - All variables are integer-valued or
    - You run out of time/patience or
    - You can prove you found the optimal solution
- Illustrative example: here

MIP: B&B Properties

- Essentially exponential search
  - No guarantees it'll converge fast
    - You may run out of patience
    - The computer may run out of memory
  - It (usually) provides an optimality bound
    - You may opt to stop at 'good enough' solution
- In practice it works quite well
  - Problems with tens of thousands of variables solved within a few hours

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What Is A Nonlinear Program?

- No expectation of linearity
  - Either in the objective or in constraints
- Integrality could be present, or not
  - But typically, variables in nonlinear programs are continuous

Exercise: Homesteading

- You have been given a 600’ roll of barbed wire to fence off a rectangular parcel of land of your choosing
- Create a model to maximize your land holding
  - Outline on paper
  - Model and solve in Excel

NLP: Solution Methods

- Unlike LP/MIP
  - Many flavors of problems
  - Many distinct solution methods
    - E.g., augmented lagrangian, reduced gradient, sequential quadratic, quasi-Newton, gradient projection,…
    - Each works best on specific NLP flavor(s)
  - Not so robust: see here

http://vis.supstat.com/2013/03/gradient-descent-algorithm-with-r/

NLP: Solution Method Properties

- Exponential local search methods
  - Subject to slow convergence
    - Floating point arithmetic-related instabilities
  - Local search => global min/max missed
    - Start near optimal and you'll be ok, else not
  - Convergence criteria can be flaky
- In practice methods need to be highly tailored to class of problems of interest

NLP: Main Takeaways

- Far less robust than LP/MIP
  - You need much problem *and* method knowledge to solve NLPs
  - RSPE not great at alerting you that your model likely won't converge
    - Even its error messages can't be 100% trusted
- Big message: to solve NLPs of any size, find an NLP expert

Example: Markowitz Model

- Basic concepts
  - Instrument (stocks, bonds, etc.)
  - Portfolio
  - Expected return (instrument vs portfolio)
  - Risk (= covariance)
- This notion of risk leads to a quadratic objective, with linear constraints
- Example: Stock Portfolio Optimization

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