MGB 206: Decision Making and Management Science

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Session 7

Lesson Plan: Session 7

1. Session 6 reprise
2. Optimizing portfolios
3. Optimizing under uncertainty
4. Decision trees

What We Discussed Last Time

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Portfolio Management

- Choosing among uncertain investments
  - Financial instruments
  - Real assets
  - Projects
- Portfolio selection is all about balancing
  - Reward
  - Risk

Financial Portfolio Allocation Under Uncertainty

- Harry Markowitz, *ca*. 1950 @UChicago
  - Recognized risk/return link
  - Use covariance as measure of stock movement, and thus risk

Basics: Covariance

- Recall definition of variance

$$\sigma_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

- For two vectors *x* and *y*, covariance is

$$\sigma_{xy} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_x)(y_i - \mu_y)$$

- Covariance tracks if *x* and *y* move together

Basics: Correlation

- Correlation measures the power of *x* as explanatory factor for *y*

$$R = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

  - R takes on values between 1 and -1
  - R^2 is the familiar measure from linear regression

Correlation And Risk

- Diversification reduces portfolio risk on its own (even before we consider any correlation between individual financial instruments)
- Negative correlation further lowers risk
- Positive correlation increases risk

Portfolio Risk (2 Instruments)

Instrument	**A**	**B**
Expected return (μ)	r_A	r_B
Variance (σ^2)	σ_A^2	σ_B^2

- Investment split

$$x_A + x_B = 1$$

- Expected return

$$E_P = r_A x_A + r_B x_B$$

- Portfolio risk

$$\sigma_P^2 = \sigma_A^2 x_A^2 + \sigma_B^2 x_B^2 + 2\sigma_{AB} x_A x_B$$

Portfolio Risk (n Instruments)

- Investment split $\sum_{i=1}^{n} x_i = 1$
- Expected return $E_P = \sum_{i=1}^{n} r_i x_i$
- Portfolio risk $\sigma_P^2 = x^T Q x$
  - x^T is the transpose of the n-dimensional portfolio allocation vector x
  - Q is the $n \; x \; n$ variance-covariance matrix

Markowitz Portfolio Allocation

- Nonlinear programming formulation

$$
\begin{aligned}
&\min \;\; x^T Q x \\
&\text{subject to} \\
&\quad \textstyle\sum_{i=1}^{n} r_i x_i \;\ge r_{expected} \\
&\quad \textstyle\sum_{i=1}^{n} x_i = 1 \\
&\quad x_i \ge 0, \quad \forall\, i = 1..n
\end{aligned}
$$

Markowitz Model Attributes

- Solution depends on choice of $r_{expected}$
  - How should we choose it?
- Is this optimizing under uncertainty?
- Objective is quadratic, constraints linear
  - QPs can be solved using LP methods
  - That's faster than general NLP methods
- Example: Stock Portfolio Optimization

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Stochastic Programming

- Uncertainty in LP/MIP/NLP
- Many ways to model unknowns
  - Multi-stage (2-stage) recourse models
  - Chance constraints
  - Robust optimization
- Different problem types require different solution technique
- RSPE example: chance constraints

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Exercise: Wertz Game & Toy

- Read your handout
- Discuss possible decision criteria

Optimistic Approach

Payoffs	States: Good	Fair	Poor	Max payoff
Conservative	100	60	-10	100
Hedged	200	50	-40	200
Bold	300	40	-100	300

- Maximum payoff criterion

Pessimistic Approach

Payoffs	States			
	Good	Fair	Poor	Min payoff
Conservative	100	60	-10	-10
Hedged	200	50	-40	-40
Bold	300	40	-100	-100

- Maximin payoff criterion

Regret-Avoiding Approach

Regret	Good	Fair	Poor	Max regret
	States			
Conservative	200	0	0	200
Hedged	100	10	30	100
Bold	0	20	90	90

- Minimax regret criterion

Probabilities ⇒ Expected Values

	States			
	Good	Fair	Poor	Expected payoff
Probability	0.2	0.5	0.3	
Conservative	100	60	-10	47
Hedged	200	50	-40	53
Bold	300	40	-100	50

What Does Uncertainty Cost?

- What if we can wait until the market response is known?
  - Market = Good ⇒ Bold (payoff = 300)
  - Market = Fair ⇒ Conservative (60)
  - Market = Poor ⇒ Conservative (-10)

Profit	300	60	-10
Probability	0.2	0.5	0.3

- Exp. payoff w/ perfect info = 87
- **Value of perfect info** = 87 – 53 = 34

Key Concepts

- Expected value
- Probability
- Utility vs. money
- Value of information
- Good decisions vs. good outcomes

Building Decision Trees

- Excel
  - Risk Solver has Decision Tree capability
    - Well integrated (e.g., parametrized analysis)
    - Limited (e.g., cannot flip a tree)
  - XLTree included with textbook
    - Comparatively inelegant, but functional
    - Can flip tree to compute value of information
- TreeAge, Precision Tree, etc. available
  - From small vendors

Using Decision Trees

- Framework for *coarse-grain* decisions under uncertainty
  - Computation is easy/minor

- Apparent ease of use may obscure knowledge pitfalls
- Powerful tool, but it requires considerable psychological buy-in

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