

TIP8419 - Tensor Algebra

Homework 9

Prof. André de Almeida
andre@gtel.ufc.br

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Multidimensional Least-Squares Khatri-Rao Factorization (MLS-KRF)

Problem 1 On practice 3 we implement the LS-KRF (Least Square Khatri-Rao Factorization) algorithm, now we will go to implement the MLS-KRF (Multidimensional Least Square Khatri-Rao Factorization) algorithm. Then, Let $\mathbf{X} \approx \mathbf{A}^{(1)} \diamond \mathbf{A}^{(2)} \dots \diamond \mathbf{A}^{(N)} \in \mathbb{C}^{I_1 I_2 \dots I_N \times R}$ be a matrix composed by Khatri-Rao product of N matrices $\mathbf{A}^{(n)} \in \mathbb{C}^{I_n \times R}$, with $n = 1, 2, \dots, N$. For $N = 3$ and R and I_n arbitrary implement the MLS-KRF for that estimate $\mathbf{A}^{(1)}, \mathbf{A}^{(2)}$ and $\mathbf{A}^{(3)}$ by solving the following problem.

$$(\hat{\mathbf{A}}^{(1)}, \hat{\mathbf{A}}^{(2)}, \hat{\mathbf{A}}^{(3)}) = \min_{\mathbf{A}, \mathbf{B}} \|\mathbf{X} - \mathbf{A}^{(1)} \diamond \mathbf{A}^{(2)} \diamond \mathbf{A}^{(3)}\|_F^2.$$

Compare the estimate matrices $\hat{\mathbf{A}}^{(1)}, \hat{\mathbf{A}}^{(2)}$ and $\hat{\mathbf{A}}^{(3)}$ with the original ones. What can you conclude? Explain the results.

Hint: Use the file “krf_matrix_3D.mat” to validate your result.

Problem 2 Assuming 1000 Monte Carlo experiments, generate $\mathbf{X}_0 = \mathbf{A} \diamond \mathbf{B} \diamond \mathbf{C} \in \mathbb{C}^{I_1 I_2 I_3 \times R}$, for randomly chosen $\mathbf{A} \in \mathbb{C}^{I_1 \times R}$, $\mathbf{B} \in \mathbb{C}^{I_2 \times R}$ and $\mathbf{C} \in \mathbb{C}^{I_3 \times R}$, with $R = 4$, whose elements are drawn from a normal distribution. Let $\mathbf{X} = \mathbf{X}_0 + \alpha \mathbf{V}$ be a noisy version of \mathbf{X}_0 , where \mathbf{V} is the additive noise term, whose elements are drawn from a normal distribution. The parameter α controls the power (variance) of the noise term, and is defined as a function of the signal to noise ratio (SNR), in dB, as follows

$$\text{SNR}_{\text{dB}} = 10 \log_{10} \left(\frac{\|\mathbf{X}_0\|_F^2}{\|\alpha \mathbf{V}\|_F^2} \right). \quad (1)$$

Assuming the SNR range $[0, 5, 10, 15, 20, 25, 30]$ dB, find the estimates $\hat{\mathbf{A}}, \hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$ obtained with the MLS-KRF algorithm for the configurations $I_1 = 2, I_2 = 3$ and $I_3 = 4$.

Let us define the normalized mean square error (NMSE) measure as follows

$$\text{NMSE}(\mathbf{X}_0) = \frac{1}{1000} \sum_{i=1}^{1000} \frac{\|\hat{\mathbf{X}}_0(i) - \mathbf{X}_0(i)\|_F^2}{\|\mathbf{X}_0(i)\|_F^2}, \quad (2)$$

where $\mathbf{X}_0(i)$ e $\hat{\mathbf{X}}_0(i)$ represent the original data matrix and the reconstructed one at the i th experiment, respectively. For each SNR value and configuration, plot the NMSE vs. SNR curve. Discuss the obtained results.

Note: For a given SNR (dB), the parameter α to be used in your experiment is determined from equation (1).