TIP8419 - Tensor Algebra
Homework 15

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Tensor Least Mean Square (Tensor-LMS) Filtering

Assuming 50 Monte Carlo experiments, generate the signal model

$$y_0(k) = \langle \mathcal{X}(k), \mathcal{W} \rangle,$$

for a randomly chosen input signal $\mathcal{X} \in \mathbb{R}^{4\times 32\times 8}$, and a rank-1 filter

$$\mathcal{W} = \mathbf{w}^{(1)} \circ \mathbf{w}^{(2)} \circ \mathbf{w}^{(3)},$$

where $\mathbf{w}^{(1)} \in \mathbb{R}^4$, is a vector whose elements are Gaussian random variables with zero mean and unitary variance, $\mathbf{w}^{(2)} = [0, \cdots, 0, 1]^T \in \mathbb{R}^{32}$, and $\mathbf{w}^{(3)} \in \mathbb{R}^8$, is a vector with elements $w_i^{(3)} = 0.9^{i-1}$, $1 \leq i \leq 8$. Set $k = 10000$ and generate $\mathbf{y}_0 = [y_0(1), \cdots, y_0(10000)]^T$. Let $\mathbf{y} = \mathbf{y}_0 + \alpha \mathbf{v}$ be a noisy version of $\mathbf{y}_0$, where $\mathbf{v}$ is the additive noise term, whose elements are drawn from a normal distribution. The parameter α controls the power (variance) of the noise term, and is defined as a function of the signal to noise ratio (SNR), in dB, as follows

$$\text{SNR}_{\text{dB}} = 10\text{log}_{10}\left(\frac{||\mathbf{y}_0||_2^2}{||\alpha\mathbf{v}||_2^2}\right). \tag{1}$$

Assuming the SNR=30dB, find the estimate $\hat{\mathcal{W}}$ obtained with the Tensor LMS algorithm. Let us define the normalized misalignment measure as follows

$$\text{NM} = \frac{1}{50}\sum_{i=1}^{50} \frac{\|\hat{\mathcal{W}}(i) - \mathcal{W}(i)\|_F^2}{\|\mathcal{W}(i)\|_F^2}, \tag{2}$$

where $\mathcal{W}(i)$ and $\hat{\mathcal{W}}(i)$ represent the original filter and the reconstructed one at the ith experiment, respectively. For the SNR value and configuration, plot the NM vs. iterations curve. Discuss the obtained results.

Note: For a given SNR (dB), the parameter α to be used in your experiment is determined from equation (1). Set $\epsilon = 10^{-15}$ and the step size parameter $\mu_{i,k} = \frac{\beta}{\|\hat{\mathbf{u}}_{-i}(k)\|_2^2}$ where $0 < \beta < 2$ and

$$\hat{\mathbf{u}}_{-i}(k) = \mathcal{X}(k) \times_1 \hat{\mathbf{w}}^{(1)} \cdots \times_{i-1} \hat{\mathbf{w}}^{(i-1)} \times_{i+1} \hat{\mathbf{w}}^{(i+1)} \cdots \times_M \hat{\mathbf{w}}^{(M)}.$$

Choose the best value of β.