TIP8419 - Tensor Algebra
Homework 2

Prof. André de Almeida
andre@gtel.ufc.br

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Khatri-Rao Product

Problem 1 Generate $\mathbf{X} = \mathbf{A} \diamond \mathbf{B} \in \mathbb{R}^{I \times R}$, for randomly chosen $\mathbf{A} \in \mathbb{C}^{I \times R}$ and $\mathbf{B} \in \mathbb{R}^{I \times R}$. Compute the left pseudo-inverse of $\mathbf{X}$ and obtain a graph that shows the run time vs. number of rows (I) for the following methods

(a) Method 1: Matlab/Octave function $pinv(\mathbf{X}) = pinv(\mathbf{A} \diamond \mathbf{B})$

(b) Method 2: $\mathbf{X}^{\dagger} = (\mathbf{X}^{\mathsf{T}} \mathbf{X})^{-1} \mathbf{X}^{\mathsf{T}} = [(\mathbf{A} \diamond \mathbf{B})^{\mathsf{T}} (\mathbf{A} \diamond \mathbf{B})]^{-1} (\mathbf{A} \diamond \mathbf{B})^{\mathsf{T}}$

(c) Method 3: $\mathbf{X}^{\dagger} = [(\mathbf{A} \diamond \mathbf{B})^{\mathsf{T}} (\mathbf{A} \diamond \mathbf{B})]^{-1} (\mathbf{A} \diamond \mathbf{B})^{\mathsf{T}} = [(\mathbf{A}^{\mathsf{T}} \mathbf{A}) \odot (\mathbf{B}^{\mathsf{T}} \mathbf{B})]^{-1} (\mathbf{A} \diamond \mathbf{B})^{\mathsf{T}}$

Note: Consider the range of values $I \in \{2, 4, 8, 16, 32, 64, 128, 256\}$ and plot the curves for $R = 2$ and $R = 4$.

Problem 2 Generate $\mathbf{X} = \overset{N}{\underset{n=1}{\diamond}} \mathbf{A}_{(n)} = \mathbf{A}_{(1)} \diamond \cdots \diamond \mathbf{A}_{(N)}$, where every $\mathbf{A}_{(n)}$ has dimensions 4×2, $n = 1, \ldots, N$. Evaluate the run time associated with the computation of the Khatri-Rao product as a function of the number N of matrices for the above methods.

Note: Consider the range of values $N \in \{2, 4, 6, 8, 10\}$.

⊙ Denotes the Hadamard Product.
◇ Denotes the Khatri-Rao Product.