Homework 1 - MultiLinear Algebra

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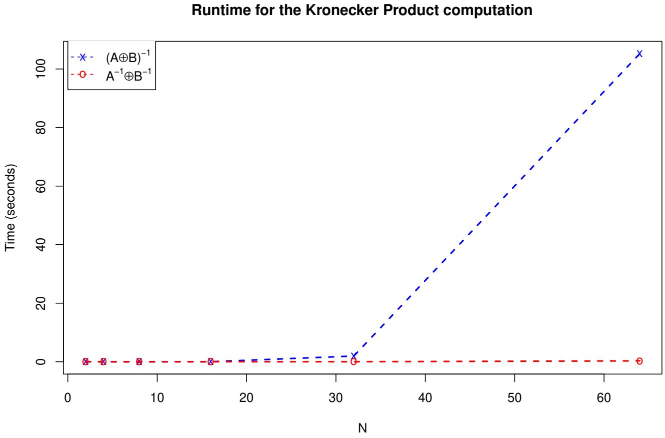
Problem 01 - Item a

Let us define $A, B \in \mathbb{C}^{N \times N}$. We have the following property

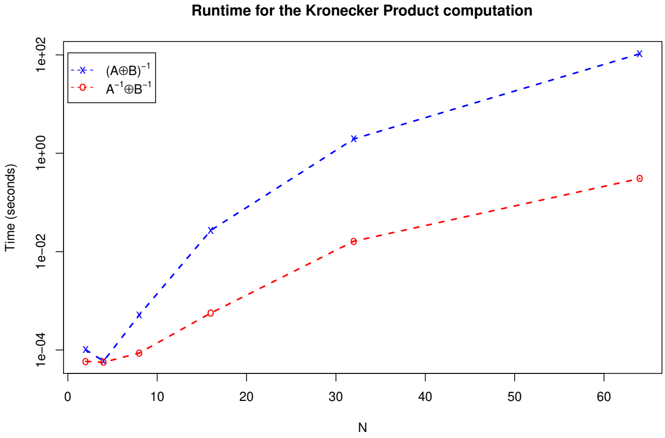
$$(A \otimes B)^{-1} = A^{-1} \otimes B^{-1} \tag{1}$$

It important to notice the computational efficiency in both approach. Consider the task of calculating Eq.(1) for $N \in \{2, 4, 8, 16, 32, 64\}$.

Problem 01 - Item a



Problem 01 - Item a



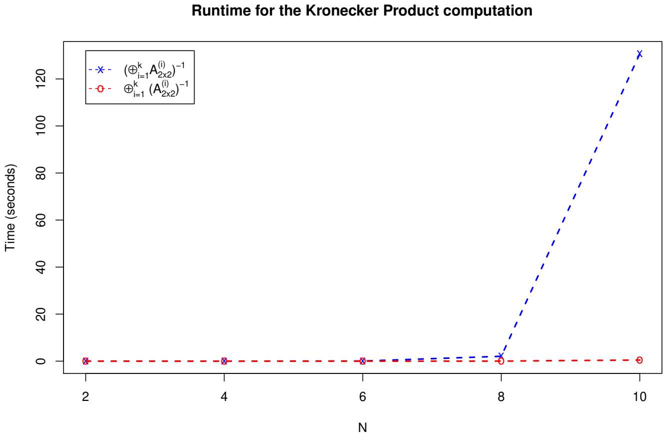
Problem 01 - Item b

Now let us define $N = 2$ and vary the number of elements in the kronecker operation, that is

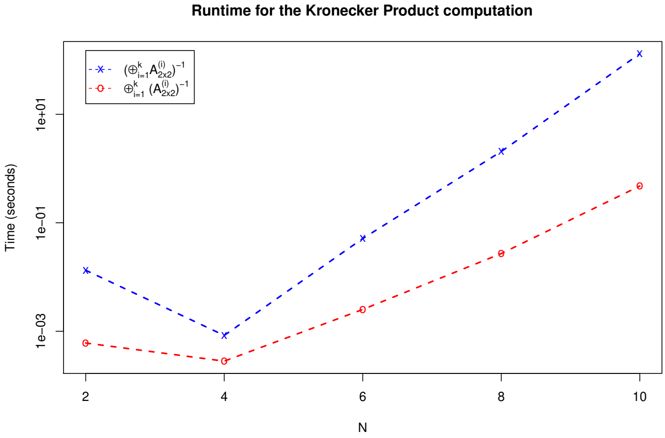
$$\left(\bigotimes_{i=1}^{k} A_{2\times 2} \right)^{-1} = \bigotimes_{i=1}^{k} \left(A_{2\times 2} \right)^{-1}, \tag{2}$$

where $k \in \{2, 4, 6, 8, 10\}$.

Problem 01 - Item b



Problem 01 - Item b



Problem 02

Proof that $eig(\mathbf{A} \otimes \mathbf{B}) = eig(\mathbf{A}) \otimes eig(\mathbf{B})$. If $\mathbf{A}$ and $\mathbf{B}$ are diagonalizable and square matrices, the eigenvalues of $\mathbf{A} \otimes \mathbf{B}$ is given by

$$\begin{aligned}
\mathbf{A} \otimes \mathbf{B} &= \mathbf{V}_A eig(A) \mathbf{V}_A^{-1} \otimes \mathbf{V}_B eig(B) \mathbf{V}_B^{-1} && (3) \\
&= (\mathbf{V}_A \otimes \mathbf{V}_B)(eig(A) \otimes eig(B))(\mathbf{V}_A \otimes \mathbf{V}_B)^{-1} && (4) \\
&= \mathbf{V}_{\mathbf{A} \otimes \mathbf{B}} eig(\mathbf{A} \otimes \mathbf{B}) \mathbf{V}_{\mathbf{A} \otimes \mathbf{B}}^{-1} && (5)
\end{aligned}$$

where $\mathbf{V}_A$ and $\mathbf{V}_B$ is the modal matrix of $\mathbf{A}$ and $\mathbf{B}$, respectively. It is possible to notice the que matrix containing the eigenvalues of $\mathbf{A} \otimes \mathbf{B}$ is composed by the kronecker product of $eig(A)$ and $eig(B)$.s