TIP8419 - Tensor Algebra
Homework 3

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Least-Squares Khatri-Rao Factorization (LSKRF)

Generate $\mathbf{X} = \mathbf{A} \diamond \mathbf{B} \in \mathbb{C}^{24 \times 2}$, for randomly chosen $\mathbf{A} \in \mathbb{C}^{4 \times 2}$ and $\mathbf{B} \in \mathbb{C}^{6 \times 2}$. Then, implement the Least-Squares Khatri-Rao Factorization (LSKRF) algorithm that estimate $\mathbf{A}$ and $\mathbf{B}$ by solving the following problem

$$(\hat{\mathbf{A}}, \hat{\mathbf{B}}) = \min_{\mathbf{A}, \mathbf{B}} \|\mathbf{X} - \mathbf{A} \diamond \mathbf{B}\|_F^2.$$

Compare the estimated matrices $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ with the original ones. What can you conclude? Explain the results.

Hint: Use the file "krf_matrix.mat" to validate your result.

Assuming 1000 Monte Carlo experiments, generate $\mathbf{X}_0 = \mathbf{A} \diamond \mathbf{B} \in \mathbb{C}^{IJ \times R}$, for randomly chosen $\mathbf{A} \in \mathbb{C}^{I \times R}$ and $\mathbf{B} \in \mathbb{C}^{J \times R}$, with $R = 4$, whose elements are drawn from a normal distribution. Let $\mathbf{X} = \mathbf{X}_0 + \alpha \mathbf{V}$ be a noisy version of $\mathbf{X}_0$, where $\mathbf{V}$ is the additive noise term, whose elements are drawn from a normal distribution. The parameter α controls the power (variance) of the noise term, and is defined as a function of the signal to noise ratio (SNR), in dB, as follows

$$\text{SNR}_{\text{dB}} = 10 \log_{10} \left(\frac{\|\mathbf{X}_0\|_F^2}{\|\alpha \mathbf{V}\|_F^2} \right). \tag{1}$$

Assuming the SNR range [0, 5, 10, 15, 20, 25, 30] dB, find the estimates $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ obtained with the LSKRF algorithm for the configurations $(I, J) = (10, 10)$ and $(I, J) = (30, 10)$.

Let us define the normalized mean square error (NMSE) measure as follows

$$\text{NMSE}(\mathbf{X}_0) = \frac{1}{1000} \sum_{i=1}^{1000} \frac{\|\hat{\mathbf{X}}_0(i) - \mathbf{X}_0(i)\|_F^2}{\|\mathbf{X}_0(i)\|_F^2}, \tag{2}$$

where $\mathbf{X}_0(i)$ e $\hat{\mathbf{X}}_0(i)$ represent the original data matrix and the reconstructed one at the ith experiment, respectively. For each SNR value and configuration, plot the NMSE vs. SNR curve. Discuss the obtained results.

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Note: For a given SNR (dB), the parameter α to be used in your experiment is determined from equation (1).