TIP8419 - Tensor Algebra
Homework 13

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Tensor Train Singular Value Decomposition (TT-SVD)

Problem 1 Implement the TT-SVD algorithm, which, given an input tensor $\mathcal{X}$ and the tuple of TT-ranks $(R_1, \ldots, R_{N-1})$, estimates the TT-cores $\{\mathbf{G}_1, \mathcal{G}_2, \mathcal{G}_3, \mathbf{G}_4\}$ of $\mathcal{X}$ by approximately solving the following problem

$$\min_{\mathbf{G}_1, \mathcal{G}_2, \mathcal{G}_3, \mathbf{G}_4} \| \mathcal{X} - \mathbf{G}_1 \bullet_2^1 \mathcal{G}_2 \bullet_3^1 \mathcal{G}_3 \bullet_3^1 \mathbf{G}_4 \|_F, \tag{1}$$

using sequential SVD truncations (see the slides for a detailed description).

Next, test this algorithm with a randomly generated tensor having exact TT form. Specifically, generate cores $\mathbf{G}_1$, $\mathcal{G}_2$, $\mathcal{G}_3$ and $\mathbf{G}_4$ with standard Gaussian elements, and then construct a tensor $\mathcal{X}_0 \in \mathbb{C}^{5\times5\times5\times5}$ given by

$$\mathcal{X} = \mathbf{G}_1 \bullet_2^1 \mathcal{G}_2 \bullet_3^1 \mathcal{G}_3 \bullet_3^1 \mathbf{G}_4.$$

The TT-ranks of $\mathcal{X}$ must be (3,3,3).

Then, run your algorithm on this tensor, providing the correct TT-ranks (3,3,3) as input. Compare the estimated tensor $\hat{\mathcal{X}}$ (reconstructed from the computed cores) with the original one. What can you conclude? Now, what happens if you run the algorithm again on $\mathcal{X}$, but this time specifying the TT-ranks (2,2,2)? Explain the results.

Problem 2 Assuming 1000 Monte Carlo experiments, generate cores $\mathbf{G}_1$, $\mathcal{G}_2$, $\mathcal{G}_3$ and $\mathbf{G}_4$ with standard Gaussian elements, and then construct a tensor $\mathcal{X}_0 \in \mathbb{C}^{5\times5\times5\times5}$ given by

$$\mathcal{X}_0 = \mathbf{G}_1 \bullet_2^1 \mathcal{G}_2 \bullet_3^1 \mathcal{G}_3 \bullet_3^1 \mathbf{G}_4.$$

The TT-ranks of $\mathcal{X}_0$ must be (3,3,3). Let $\mathcal{X} = \mathcal{X}_0 + \alpha\mathcal{V}$ be a noisy version of $\mathcal{X}_0$, where $\mathcal{V}$ is the additive noise term, whose elements are drawn from a normal distribution. The parameter α controls the power (variance) of the noise term, and is defined as a function of the signal to noise ratio (SNR), in dB, as follows

$$\text{SNR}_{\text{dB}} = 10\log_{10}\left(\frac{||\mathcal{X}_0||_F^2}{||\alpha\mathcal{V}||_F^2}\right). \tag{2}$$

Assuming the SNR range [0, 5, 10, 15, 20, 25, 30] dB, find the estimated tensor $\hat{\mathcal{X}}$ reconstructed with the TT-SVD algorithm. Let us define the normalized mean square error (NMSE) measure as follows

$$\text{NMSE}(\mathcal{X}_0) = \frac{1}{1000} \sum_{i=1}^{1000} \frac{\|\hat{\mathcal{X}}_0(i) - \mathcal{X}_0(i)\|_F^2}{\|\mathcal{X}_0(i)\|_F^2}, \tag{3}$$

where $\mathcal{X}_0(i)$ and $\hat{\mathcal{X}}_0(i)$ represent the original (generated) data tensor and the reconstructed one (computed from the estimated factors) at the ith experiment, respectively. For each SNR value, plot the NMSE vs. SNR curve. Discuss the obtained results.

Note: For a given SNR (dB), the parameter α to be used in your experiment is determined from equation (1).