A Gentle Introduction to the Phase Screen Model

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Abstract

This note provides a basic introduction to the two-dimensional two-component power-law phase screen model for equatorial regions, developed by Charles L. Rino, Charles S. Carrano, Y.T. Jade Morton, *et. al.*. The state of the art of the model is quite complex and requires deep knowledge in the field of electromagnetic wave propagation and vector calculus. On the one hand, students who have such a background and are interested in a down-top learning approach are encouraged to read the author's book [1]. On the other hand, students with a mild background in these fields or who are interested in a top-down learning and implementation-focused approach may find this note interesting. In the top-down approach, we start with the simplifications and assumptions that lead to the phase screen model and then go deeper into the theoretical details of electromagnetic propagation as much as necessary. Moreover, we take special care about how the theoretical concepts are implemented in code. As references, we will use the various versions of the phase screen model implemented by the team to understand how the theory is implemented in practice. Most of the code is implemented in Matlab. However, some knowledge of GNSS signal processing and stochastic processing is assumed.

1 Modeling the scintillation impairment in GNSS measurements

Let us start with the most fundamental concept in GNSS: the time τ that the satellite-transmitted signal takes to reach the receiver. We first model the code and carrier phase pseudorange measurements are modelled separately.

1.1 Code pseudorange measurement

Considering that the GNSS signals are propagating at the speed of light [2, p. 5], we have:

$$\rho(t) = c\,[t_u(t) - t^s(t-\tau)] \tag{1}$$

The emission time (in seconds), which is observed with respect to the satellite clock.

The arrival time (in seconds), which is observed with respect to the receiver clock.

the speed of light, $300 \cdot 10^6\,\mathrm{m\,s^{-1}}$.

the pseudorange (in meters): The apparent distance from the satellite to the receiver. It is so called since the travelling time is biased by many impairing factors, such as receiver/satellite clock bias, tropospheric/ionospheric delay, hardware bias, etc.

By putting the time scales in terms of the reference timing, defined and broadcasted by the control segment, we have [3, p. 148]:

$$t_u(t) = t + \delta t_u(t) \tag{2}$$

Receiver clock error

and

$$t^s(t-\tau) = (t-\tau) + \delta t^s(t) \tag{3}$$

Satellite clock error

Therefore:

$$\begin{aligned} \rho(t) &= c\tau + c[\delta t_u(t) - \delta t^s(t-\tau)] \\ &= c\tau + c\delta t_u^s(\tau), \end{aligned} \tag{4}$$

- $\delta t_u^s(\tau) \triangleq \delta t_u(t) - \delta t^s(t-\tau)$: the net clock error bias with regard to the reference timing.

The main idea is to model the interference sources that impair $c\tau$. The theory of scintillation generally assumes the absence of noise, clock error bias (so $\delta t_u^s(\tau) = 0$), and other error sources but the ionosphere [4]. Therefore,

$$\boxed{c\tau = r + I + \frac{\lambda_c}{2\pi}\phi_s(t)} \tag{5}$$

in rad: scintillation phase. The factor $\frac{\lambda_c}{2\pi}$ converts rad to meters.

in meters: ionosphere delay error model

in meters: the true range

where $\lambda_c = c/f_c$ is the wavelength (in meters) and f_c is the carrier frequency (in Hz). The ionospheric delay model is given by [3, p. 163]

$$I = \frac{40.3TEC}{f^2} \tag{6}$$

Where TEC is the total content electron, defined as the number of electrons in a tube of $1\,\mathrm{m}^2$ cross-section from the receiver to the satellite and measured in 1 TECU $= 10^{16} e^-/m^2$.

1.2 Carrier phase measurement

Likewise, one can measure the travelling time considering the carrier phase, that is, [3, p. 153]

$$\phi(t) = \phi_u(t) - \phi^s(t-\tau) + 2\pi\, N \tag{7}$$

Dimensionless: integer ambiguity

in rad: satellite carrier phase at the emission time (with regard to the satellite timing)

in rad: receiver carrier phase generated by the local NCO (numerically-controlled oscillator) at the reception time (with regard to the receiver timing).

By considering a stable oscillator, one can assume that [3, p. 153]

$$\phi_u(t) - \phi^s(t-\tau) \approx 2\pi f\tau. \tag{8}$$

Similarly to $c\tau$, we should model the error sources that impair $f\tau$. By considering only the ionosphere effects and ignoring the clock bias, we have that

$$f\tau = \frac{r-I}{\lambda_c} + \frac{\phi_s(t)}{2\pi}, \tag{9}$$

where the minus in the ionospheric interference is due to the code-carrier divergence [3, p. 153 *et seq.*]. Note that, since the left-hand side of the equation is in the number of cycles (adimensional), the modelling error in the right-hand side needs to be normalized by the carrier wavelength, λ_c (in meters), and 2π.

Substituting Equation 9 and Equation 8 into Equation 7 leads to

$$\phi(t) = \frac{2\pi}{\lambda_c}(r-I) + \phi_s(t) + 2\pi N \tag{10}$$

By converting from rad to meters, we have

$$\boxed{\frac{\lambda_c}{2\pi}\phi(t) = r - I + \frac{\lambda_c}{2\pi}\phi_s(t) + \lambda_c N} \tag{11}$$

1.3 Scintillation signal

By considering only the satellite-user range variation and the ionospheric effects, the received signal power can be modelled as [4]

$$\underbrace{I(t)}_{\text{Received signal power}} = \underbrace{P(t)}_{\text{LOS (line-of-sight) signal power variation. The path loss and antenna gain are also taken into account.}}\ \underbrace{|h(t)|^2}_{\text{Scintillation amplitude.}} \tag{12}$$

Finally, the scintillation signal can be modelled as

$$h(t) = |h(t)|\ e^{j\phi_s(t)} \tag{13}$$

2 The phase screen theory

2.1 Fundamentals of electromagnetic wave propagation

2.2 Phase screen realization

Let us assume that

- The phase screen model is used to model equatorial scintillation, which is caused by irregularities that are highly elongated along the geomagnetic field lines [5];
- The electromagnetic wave is simplified to a plane wave propagating through the phase screen realization, which is defined in a two-dimensional space [5]:
    - x (in meters): distance from the phase screen in the propagation direction [5], [6];
    - y (in meters): geometric eastward direction [5], i.e., the field-aligned direction [6]. It is assumed that this direction is transverse to x [7, p. 52]

Let $\psi(x, y)$ be the complex field representing principal (i.e., scalar) component of the electromagnetic wave [4]. Considering that the propagation of the electromagnetic wave in the Earth's ionosphere is transparent for the GNSS frequency band, one can state that it is governed parabolic wave equation (PWE). Once the scalar form of the PWE is sufficient to characterize the complex modulation impairing the plane wave [4], we have that:

$$\boxed{\frac{\partial\, \overbrace{\psi(x,y)}^{\text{Complex field}}}{\underbrace{\partial x}_{\text{Differentiate on the propagation axis.}}} = \overbrace{\Theta_{\rho_f}\psi(x,y)}^{\text{Free-space propagation [8]}} + \overbrace{j\ \underbrace{k}_{\text{Carrier wavenumber}}\ \underbrace{\Delta n(x,y)}_{\text{Local refractive index in the point }(x,y)}\ \psi(x,y)}^{\text{Interaction with the propagation medium [8]}}} \tag{14}$$

The free-space propagation term is given by [8]

Fresnel scale.

The wavenumber on the direction y [5]

$$\Theta_{\rho_f}\psi(x,y) = \int \Psi(x,q_y)\, e^{-j\frac{(q_y\,\rho_F)^2}{2}} e^{jyq_y} \frac{\mathrm{d}q_y}{2\pi} \tag{15}$$

Fourier transformation of the complex field, $\varphi(x,y)$ in y

where

$$\rho_F = \sqrt{x/k} \tag{16}$$

and

$$\Psi(x,q_y) = \mathcal{F}_y\{\psi(x,y)\} = \int \psi(x,y)e^{-jq_y y}\,\mathrm{d}y \tag{17}$$

In the absence of diffraction, the complex field defined by PWE has an analytical solution, given by [4, Equation (11)]

$$\psi(x+\Delta x, y) = \psi(x,y)\,\exp\left\{ j\, k \int_x^{x+\Delta x} \Delta n(\eta, y)\,\mathrm{d}\eta \right\} \tag{18}$$

The phase contribution due to the refractive part in the point y after the wave has passed Δx through the phase screen

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