

Notation

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1 Font notation

$a, b, c, \dots, A, B, C, \dots$	Scalars
$\mathbf{a}, \mathbf{b}, \mathbf{c}, \dots$	Vectors
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$	Matrices
$\mathcal{A}, \mathcal{B}, \mathcal{C}, \dots$	Tensors
$A, B, C, \dots, \mathcal{A}, \mathcal{B}, \mathcal{C}, \dots, \mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$	Sets

2 Signals and functions

2.1 Time indexing

$x(t)$	Continuous-time t
$x[n], x[k], x[m], x[i], \dots$ $x_n, x_k, x_m, x_i, \dots$ $x(n), x(k), x(m), x(i), \dots$	Discrete-time n, k, m, i, \dots (parenthesis should be adopted only if there are no continuous-time signals in the context to avoid ambiguity)
$x[((n-m))_N], x((n-m))_N$	Circular shift in m samples within a N -samples window [11, 16]

2.2 Common functions

$\delta(t)$	Delta function
$\delta[n], \delta_{i,j}$	Kronecker function ($n = i - j$)
$h(t), h[n]$	Impulse response (continuous and discrete time)

$\tilde{x}[n], \tilde{x}(t)$	Periodic discrete- or continuous-time signal
$\hat{x}[n], \hat{x}(t)$	Estimate of $x[n]$ or $x(t)$
$\dot{x}[m]$	Interpolation of $x[n]$

2.3 Operations and symbols

$f : A \rightarrow B$	A function f whose domain is A and codomain is B
$\mathbf{f} : A \rightarrow \mathbb{R}^n$	A vector-valued function \mathbf{f} , i.e., $n \geq 2$
$f^n, x^n(t), x^n[k]$	n th power of the function f , $x[n]$ or $x(t)$
$f^{(n)}, x^{(n)}(t)$	n th derivative of the function f or $x(t)$
$f', f^{(1)}, x'(t)$	1th derivative of the function f or $x(t)$
$f'', f^{(2)}, x''(t)$	2th derivative of the function f or $x(t)$
$\arg \max_{x \in \mathcal{A}} f(x)$	Value of x that minimizes x
$\arg \min_{x \in \mathcal{A}} f(x)$	Value of x that minimizes x
$f(\mathbf{x}) = \inf_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Infimum, i.e., $f(\mathbf{x}) = \min \{g(\mathbf{x}, \mathbf{y}) \mid \mathbf{y} \in \mathcal{A} \wedge (\mathbf{x}, \mathbf{y}) \in \text{dom}(g)\}$, which is the greatest lower bound of this set [3]
$f(\mathbf{x}) = \sup_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Supremum, i.e., $f(\mathbf{x}) = \max \{g(\mathbf{x}, \mathbf{y}) \mid \mathbf{y} \in \mathcal{A} \wedge (\mathbf{x}, \mathbf{y}) \in \text{dom}(g)\}$, which is the least upper bound of this set [3]
$f \circ g$	Composition of the functions f and g
$*$	Convolution (discrete or continuous)
$\otimes, \textcircled{\mathbf{N}}$	Circular convolution [7, 16]

2.4 Transformations

W_N	Twiddle factor, $e^{-j\frac{2\pi}{N}}$ [11]
$\mathcal{F}\{\cdot\}$	Fourier transform
$\mathcal{L}\{\cdot\}$	Laplace transform
$\mathcal{Z}\{\cdot\}$	z-transform
$\hat{x}(t), \hat{x}[n]$	Hilbert transform of $x(t)$ or $x[n]$

$X(s)$	Laplace transform of $x(t)$
$X(f)$	Fourier transform (FT) (in linear frequency, Hz) of $x(t)$
$X(j\omega)$	Fourier transform (FT) (in angular frequency, rad/sec) of $x(t)$
$X(e^{j\omega})$	Discrete-time Fourier transform (DTFT) of $x[n]$
$X[k], X(k), X_k$	Discrete Fourier transform (DFT) or fast Fourier transform (FFT) of $x[n]$, or even the Fourier series (FS) of the periodic signal $x(t)$
$\tilde{X}[k], \tilde{X}(k), \tilde{X}_k$	Discrete Fourier series (DFS) of $\tilde{x}[n]$
$X(z)$	z -transform of $x[n]$

3 Probability, statistics, and stochastic processes

3.1 Operators and symbols

$\mathbb{E}[\cdot], \mathbf{E}[\cdot], E[\cdot], \mathbb{E}[\cdot]$	Statistical expectation operator [6, 15]
$\mathbb{E}_u[\cdot], \mathbf{E}_u[\cdot], E_u[\cdot], \mathbb{E}_u[\cdot]$	Statistical expectation operator with respect to u
$\langle \cdot \rangle$	Ensamble average
$\text{var}[\cdot], \text{VAR}[\cdot]$	Variance operator [2, 10, 14, 18]
$\text{var}_u[\cdot], \text{VAR}_u[\cdot]$	Variance operator with respect to u
$\text{cov}[\cdot], \text{COV}[\cdot]$	Covariance operator [2]
$\text{cov}_u[\cdot], \text{COV}_u[\cdot]$	Covariance operator with respect to u
μ_x	Mean of the random variable x
$\mathbf{\mu}_x, \mathbf{m}_x$	Mean vector of the random variable \mathbf{x} [4]
μ_n	n th-order moment of a random variable
σ_x^2, κ_2	Variance of the random variable x
\mathcal{K}_x, μ_4	Kurtosis (4th-order moment) of the random variable x
κ_n	n th-order cumulant of a random variable
$\rho_{x,y}$	Pearson correlation coefficient between x and y
$a \sim P$	Random variable a with distribution P
\mathcal{R}	Rayleigh's quotient

3.2 Stochastic processes

$r_x(\tau), R_x(\tau)$	Autocorrelation function of the signal $x(t)$ or $x[n]$ [15]
$S_x(f), S_x(j\omega)$	Power spectral density (PSD) of $x(t)$ in linear (f) or angular (ω) frequency
$S_{x,y}(f), S_{x,y}(j\omega)$	Cross PSD of $x(t)$ and $y(t)$ in linear or angular (ω) frequency
\mathbf{R}_x	(Auto)correlation matrix of $\mathbf{x}(n)$
$r_{x,d}(\tau), R_{x,d}(\tau)$	Cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$ [15]
\mathbf{R}_{xy}	Cross-correlation matrix of $\mathbf{x}(n)$ and $\mathbf{y}(n)$
\mathbf{p}_{xd}	Cross-correlation vector between $\mathbf{x}(n)$ and $d(n)$ [dinizAdaptiveFiltering1997]
$c_x(\tau), C_x(\tau)$	Autocovariance function of the signal $x(t)$ or $x[n]$ [15]
$\mathbf{C}_x, \mathbf{K}_x, \mathbf{\Sigma}_x, \text{cov}[\mathbf{x}]$	(Auto)covariance matrix of \mathbf{x} [10, 14, 18, 24]
$c_{xy}(\tau), C_{xy}(\tau)$	Cross-covariance function of the signal $x(t)$ or $x[n]$ [15]
$\mathbf{C}_{xy}, \mathbf{K}_{xy}, \mathbf{\Sigma}_{xy}$	Cross-covariance matrix of \mathbf{x} and \mathbf{y}

3.3 Functions

$Q(\cdot)$	Q -function, i.e., $P[\mathcal{N}(0, 1) > x]$ [18]
$\text{erf}(\cdot)$	Error function [18]
$\text{erfc}(\cdot)$	Complementary error function i.e., $\text{erfc}(x) = 2Q(\sqrt{2}x) - \text{erf}(x)$ [18]
$P[A]$	Probability of the event or set A [14]
$p(\cdot), f(\cdot)$	Probability density function (PDF) or probability mass function (PMF) [14]
$p(x A)$	Conditional PDF or PMF [14]
$F(\cdot)$	Cumulative distribution function (CDF)
$\Phi_x(\omega), M_x(j\omega), E[e^{j\omega x}]$	First characteristic function (CF) of x [theodoridisMachineLearningBayesian2020a, 18]

$M_x(t), \Phi_x(-jt), E[e^{tx}]$	Moment-generating function (MGF) of x [theodoridisMachineLearningBayesian2020a, 18]
$\Psi_x(\omega), \ln \Phi_x(\omega), \ln E[e^{j\omega x}]$	Second characteristic function
$K_x(t), \ln E[e^{tx}], \ln M_x(t)$	Cumulant-generating function (CGF) of x [10]

3.4 Distributions

$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution of a random variable with mean μ and variance σ^2 . The same notation can be used to denote a real-valued white Gaussian process with mean equal to μ and power spectral density equal to $N_0/2$, e.g., $s(t) \sim \mathcal{N}(\mu, N_0/2)$
$\mathcal{CN}(\mu, \sigma^2)$	Complex Gaussian distribution of a random variable with mean μ and variance σ^2 . The same notation can be used to denote a complex-valued white Gaussian process with mean equal to μ and power spectral density equal to N_0 , e.g., $s(t) \sim \mathcal{CN}(\mu, N_0)$
$\mathcal{N}(\mathbf{\mu}, \mathbf{\Sigma})$	Gaussian distribution of a vector random variable with mean $\mathbf{\mu}$ and covariance matrix $\mathbf{\Sigma}$
$\mathcal{CN}(\mathbf{\mu}, \mathbf{\Sigma})$	Complex Gaussian distribution of a vector random variable with mean $\mathbf{\mu}$ and covariance matrix $\mathbf{\Sigma}$
$\mathcal{U}(a, b)$	Uniform distribution from a to b
$\chi^2(n), \chi_n^2$	Chi-square distribution with n degree of freedom (assuming that the Gaussians are $\mathcal{N}(0, 1)$)
$\text{Exp}(\lambda)$	Exponential distribution with rate parameter λ
$\Gamma(\alpha, \beta)$	Gamma distribution with shape parameter α and rate parameter β
$\Gamma(\alpha, \theta)$	Gamma distribution with shape parameter α and scale parameter $\theta = 1/\beta$
$\text{Nakagami}(m, \Omega)$	Nakagami-m distribution with shape parameter m and spread parameter Ω

Rayleigh(σ)	Rayleigh distribution with scale parameter σ
Rayleigh(Ω)	Rayleigh distribution with the second moment $\Omega = E[x^2] = 2\sigma^2$
Rice(s, σ)	Rice distribution with noncentrality parameter (specular component) s and σ
Rice(A, K)	Rice distribution with Rice factor $K = s^2/2\sigma^2$ and scale parameter $A = s^2 + 2\sigma^2$

4 Statistical signal processing

$\nabla f, \mathbf{g}$	Gradient descent vector
$\nabla_x f, \mathbf{g}_x$	Gradient descent vector with respect x
\mathbf{g} (or $\hat{\mathbf{g}}$ if the gradient vector is \mathbf{g})	Stochastic gradient descent (SGD)
$J(\cdot), \mathcal{E}(\cdot)$	Cost-function or objective function
$\Lambda(\cdot)$	Likelihood function
$\Lambda_l(\cdot)$	Log-likelihood function
$\hat{x}(t)$ or $\hat{x}[n]$	Estimate of $x(t)$ or $x[n]$
$\hat{\mu}_x, \hat{\mathbf{m}}_x$	Sample mean of $x[n]$ or $x(t)$
$\hat{\boldsymbol{\mu}}_x, \hat{\mathbf{m}}_x$	Sample mean vector of $\mathbf{x}[n]$ or $\mathbf{x}(t)$
$\hat{r}_x(\tau), \hat{R}_x(\tau)$	Estimated autocorrelation function of the signal $x(t)$ or $x[n]$
$\hat{S}_x(f), \hat{S}_x(j\omega)$	Estimated power spectral density (PSD) of $x(t)$ in linear (f) or angular (ω) frequency
$\hat{\mathbf{R}}_x$	Sample (auto)correlation matrix
$\hat{r}_{x,d}(\tau), \hat{R}_{x,d}(\tau)$	Estimated cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$
$\hat{S}_{x,y}(f), \hat{S}_{x,y}(j\omega)$	Estimated cross PSD of $x(t)$ and $y(t)$ in linear or angular (ω) frequency
$\hat{\mathbf{R}}_{xy}$	Sample cross-correlation matrix of \mathbf{R}_{xy}
$\hat{\rho}_{x,y}$	Estimated Pearson correlation coefficient between x and y
$\hat{c}_x(\tau), \hat{C}_x(\tau)$	Estimated autocovariance function of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_x, \hat{\mathbf{K}}_x, \hat{\boldsymbol{\Sigma}}_x$	Sample (auto)covariance matrix

$\hat{c}_{xy}(\tau), \hat{C}_{xy}(\tau)$	Estimated cross-covariance function of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_{xy}, \hat{\mathbf{K}}_{xy}, \hat{\boldsymbol{\Sigma}}_{xy}$	Sample cross-covariance matrix
$\mathbf{w}, \boldsymbol{\theta}$	Parameters, coefficients, or weights vector
$\mathbf{w}_o, \mathbf{w}^*, \boldsymbol{\theta}_o, \boldsymbol{\theta}^*$	Optimum value of the parameters, coefficients, or weights vector
\mathbf{W}	Matrix of the weights
\mathbf{J}	Jacobian matrix
\mathbf{H}	Hessian matrix
$\hat{\mathbf{H}}$	Estimate of the Hessian matrix

5 Linear Algebra

5.1 Common matrices and vectors

\mathbf{W}, \mathbf{D}	Diagonal matrix
\mathbf{P}	Projection matrix; Permutation matrix
\mathbf{J}	Jordan matrix
\mathbf{L}	Lower matrix
\mathbf{U}	Upper matrix
\mathbf{C}	Cofactor matrix
$\mathbf{C}_A, \text{cof}(\mathbf{A})$	Cofactor matrix of \mathbf{A}
\mathbf{S}	Symmetric matrix
\mathbf{Q}	Orthogonal matrix
\mathbf{I}_N	$N \times N$ -dimensional identity matrix
$\mathbf{0}_{M \times N}$	$M \times N$ -dimensional null matrix
$\mathbf{0}_N$	N -dimensional null vector
$\mathbf{1}_{M \times N}$	$M \times N$ -dimensional ones matrix
$\mathbf{1}_N$	N -dimensional ones vector
$\mathbf{0}$	Null matrix, vector, or tensor (dimensionality understood by context)
$\mathbf{1}$	Ones matrix, vector, or tensor (dimensionality understood by context)

5.2 Indexing

$x_{i_1, i_2, \dots, i_N}, [\mathcal{X}]_{i_1, i_2, \dots, i_N}$	Element in the position (i_1, i_2, \dots, i_N) of the tensor \mathcal{X}
$\mathcal{X}^{(n)}$	n th tensor of a nontemporal sequence

$\mathbf{x}_n, \mathbf{X}_{:,n}$	n th column of the matrix \mathbf{X}
\mathbf{x}_n	n th row of the matrix \mathbf{X}
$\mathbf{x}_{i_1, \dots, i_{n-1}, :, i_{n+1}, \dots, i_N}$	Mode- n fiber of the tensor \mathcal{X}
$\mathbf{x}_{:, i_2, i_3}$	Column fiber (mode-1 fiber) of the thrid-order tensor \mathcal{X}
$\mathbf{x}_{i_1, :, i_3}$	Row fiber (mode-2 fiber) of the thrid-order tensor \mathcal{X}
$\mathbf{x}_{i_1, i_2, :}$	Tube fiber (mode-3 fiber) of the thrid-order tensor \mathcal{X}
$\mathbf{X}_{i_1, :, :}$	Horizontal slice of the thrid-order tensor \mathcal{X}
$\mathbf{X}_{:, i_2, :}$	Lateral slices slice of the thrid-order tensor \mathcal{X}
$\mathbf{X}_{i_3}, \mathbf{X}_{:, :, i_3}$	Frontal slices slice of the thrid-order tensor \mathcal{X}

5.3 General operations

$\langle \mathbf{a}, \mathbf{b} \rangle, \mathbf{a}^\top \mathbf{b}, \mathbf{a} \cdot \mathbf{b}$	Inner or dot product
$\mathbf{a} \circ \mathbf{b}, \mathbf{a} \mathbf{b}^\top$	Outer product
\otimes	Kronecker product
\odot	Hadamard (or Schur) (elementwise) product
$\cdot^{\odot n}$	n th-order Hadamard power
$\cdot^{\odot \frac{1}{n}}$	n th-order Hadamard root
\oslash	Hadamard (or Schur) (elementwise) division
\diamond	Khatri-Rao product
\otimes	Kronecker Product
\times_n	n -mode product

5.4 Operations with matrices and tensors

\mathbf{A}^{-1}	Inverse matrix
$\mathbf{A}^+, \mathbf{A}^\dagger$	Moore-Penrose left pseudoinverse
\mathbf{A}^\top	Transpose
$\mathbf{A}^{-\top}$	Transpose of the inverse, i.e., $(\mathbf{A}^{-1})^\top = (\mathbf{A}^\top)^{-1}$ [8, 17]
\mathbf{A}^*	Complex conjugate
\mathbf{A}^H	Hermitian
$\ \mathbf{A}\ _F$	Frobenius norm
$\ \mathbf{A}\ $	Matrix norm

$ \mathbf{A} , \det(\mathbf{A})$	Determinant
$\text{diag}(\mathbf{A})$	The elements in the diagonal of \mathbf{A}
$\mathbf{E}[\mathbf{A}]$	Vectorization: stacks the columns of the matrix \mathbf{A} into a long column vector
$\mathbf{E}_d[\mathbf{A}]$	Extracts the diagonal elements of a square matrix and returns them in a column vector
$\mathbf{E}_l[\mathbf{A}]$	Extracts the elements strictly below the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\mathbf{E}_u[\mathbf{A}]$	Extracts the elements strictly above the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\mathbf{E}_b[\mathbf{A}]$	Block vectorization operator: stacks square block matrices of the input into a long block column matrix
$\text{unvec}(\mathbf{A})$	Reshapes a column vector into a matrix
$\text{tr}\{\mathbf{A}\}$	trace
$\mathbf{X}_{(n)}$	n -mode matricization of the tensor \mathcal{X}

5.5 Operations with vectors

$\ \mathbf{a}\ $	l_1 norm, 1-norm, or Manhattan norm
$\ \mathbf{a}\ , \ \mathbf{a}\ _2$	l_2 norm, 2-norm, or Euclidean norm
$\ \mathbf{a}\ _p$	l_p norm, p -norm, or Minkowski norm
$\ \mathbf{a}\ _\infty$	l_∞ norm, ∞ -norm, or Chebyshev norm
$\text{diag}(\mathbf{a})$	Diagonalization: a square, diagonal matrix with entries given by the vector \mathbf{a}

5.6 Decompositions

$\mathbf{\Lambda}$	Eigenvalue matrix [22]
\mathbf{Q}	Eigenvectors matrix; Orthogonal matrix of the QR decomposition[22]
\mathbf{R}	Upper triangular matrix of the QR decomposition[22]

\mathbf{U}	Left singular vectors [22]
\mathbf{U}_r	Left singular nondegenerated vectors
$\mathbf{\Sigma}$	Singular value matrix
$\mathbf{\Sigma}_r$	Singular value matrix with nonzero singular values in the main diagonal
$\mathbf{\Sigma}^+$	Singular value matrix of the pseudoinverse [22]
$\mathbf{\Sigma}_r^+$	Singular value matrix of the pseudoinverse with nonzero singular values in the main diagonal
\mathbf{V}	Right singular vectors [22]
\mathbf{V}_r	Right singular nondegenerated vectors
$\text{eig}(\mathbf{A})$	Set of the eigenvalues of \mathbf{A} [5, 14, 17]
$\llbracket \mathbf{A}, \mathbf{B}, \mathbf{C}, \dots \rrbracket$	CANDECOMP/PARAFAC (CP) decomposition of the tensor \mathcal{X} from the outer product of column vectors of $\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$
$\llbracket \boldsymbol{\lambda}; \mathbf{A}, \mathbf{B}, \mathbf{C}, \dots \rrbracket$	Normalized CANDECOMP/PARAFAC (CP) decomposition of the tensor \mathcal{X} from the outer product of column vectors of $\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$

5.7 Spaces

$\text{span}\{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n\}$	Vector space spanned by the argument vectors [8]
$\mathbf{C}(\mathbf{A}), \text{columnspace}(\mathbf{A}), \text{range}(\mathbf{A}), \text{span}\{\mathbf{A}\}, \text{image}(\mathbf{A})$	Columnspace, range or image, i.e., the space $\text{span}\{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n\}$, where \mathbf{a}_i is the i th column vector of the matrix \mathbf{A} [15, 22]
$\mathbf{C}(\mathbf{A}^H)$	Row space (also called left column space) [15, 22]
$\mathbf{N}(\mathbf{A}), \text{nullspace}(\mathbf{A}), \text{null}(\mathbf{A}), \text{kernel}(\mathbf{A})$	Nullspace (or kernel space) [15, 22, 23]
$\mathbf{N}(\mathbf{A}^H)$	Left nullspace
$\text{rank } \mathbf{A}$	Rank, that is, $\dim(\text{span}\{\mathbf{A}\}) = \dim(\mathbf{C}(\mathbf{A}))$ [15]
$\text{nullity}(\mathbf{A})$	Nullity of \mathbf{A} , i.e., $\dim(\mathbf{N}(\mathbf{A}))$
$\mathbf{a} \perp \mathbf{b}$	\mathbf{a} is orthogonal to \mathbf{b}
$\mathbf{a} \not\perp \mathbf{b}$	\mathbf{a} is not orthogonal to \mathbf{b}

5.8 Inequalities

$\mathbf{X} \leq 0$	Nonnegative tensor
$\mathbf{a} \preceq_K \mathbf{b}$	Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the conic subset K in the space $\mathbb{R}^n[3]$
$\mathbf{a} \prec_K \mathbf{b}$	Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the interior of the conic subset K in the space $\mathbb{R}^n[3]$
$\mathbf{a} \preceq \mathbf{b}$	Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the nonnegative orthant conic subset, \mathbb{R}_+^n , in the space $\mathbb{R}^n[3]$
$\mathbf{a} \prec \mathbf{b}$	Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the positive orthant conic subset, \mathbb{R}_{++}^n , in the space $\mathbb{R}^n[3]$
$\mathbf{A} \preceq_K \mathbf{B}$	Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the conic subset K in the space $\mathbb{S}^n[3]$
$\mathbf{A} \prec_K \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the interior of the conic subset K in the space $\mathbb{S}^n[3]$
$\mathbf{A} \preceq \mathbf{B}$	Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive semidefinite conic subset, \mathbb{S}_+^n , in the space $\mathbb{S}^n[3]$
$\mathbf{A} \prec \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive orthant conic subset, \mathbb{S}_{++}^n , in the space $\mathbb{S}^n[3]$

6 Communication systems

B	One-sided bandwidth of the transmitted signal, in Hz
W	One-sided bandwidth of the transmitted signal, in rad/s
x_i	Real or in-phase part of x
x_q	Imaginary or quadrature part of x
f_c, f_{RF}	Carrier frequency (in Hertz)
f_L	Carrier frequency in L-band (in Hertz)

f_{IF}	Intermediate frequency (in Hertz)
f_s	Sampling frequency or sampling rate (in Hertz)
T_s	Sampling time interval/duration/period
R	Bit rate
T	Bit interval/duration/period
T_c	Chip interval/duration/period
T_{sy}, T_{sym}	Symbol/signaling[18] interval/duration/period
s_{RF}	Transmitted signal in RF
s_{FI}	Transmitted signal in FI
s, s_l	Lowpass (or baseband) equivalent signal or envelope complex of transmitted signal
r_{RF}	Received signal in RF
r_{FI}	Received signal in FI
r, r_l	Lowpass (or baseband) equivalent signal or envelope complex of received signal
ϕ	Signal phase
ϕ_0	Initial phase
η_{RF}, w_{RF}	Noise in RF
η_{FI}, w_{FI}	Noise in FI
η, w	Noise in baseband
τ	Timing delay
$\Delta\tau$	Timing error (delay - estimated)
φ	Phase offset
$\Delta\varphi$	Phase error (offset - estimated)
f_d	Linear Doppler frequency
Δf_d	Frequency error (Doppler frequency - estimated)
ν	Angular Doppler frequency
$\Delta\nu$	Frequency error (Doppler frequency - estimated)
γ, A	Transmitted signal amplitude
γ_0, A_0	Combined effect of the path loss and antenna gain

7 Discrete mathematics

7.1 Set theory

$A + B$	Set addition (Minkowski sum), i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} = \mathbf{x} + \mathbf{y}, \forall \mathbf{x} \in \mathcal{X} \wedge \mathbf{y} \in \mathcal{Y}\}$ [12]
$A - B$	Minkowski difference, i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} = \mathbf{x} - \mathbf{y}, \forall \mathbf{x} \in \mathcal{X} \wedge \mathbf{y} \in \mathcal{Y}\}$
$A \ominus B$	Pontryagin difference, i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} + \mathbf{y} \in \mathcal{X}, \forall \mathbf{y} \in \mathcal{Y}\}$ [12]
$A \setminus B, A - B$	Set difference or set subtraction, i.e., $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ the set containing the elements of A that are not in B [20]
$A \cup B$	Set of union
$A \cap B$	Set of intersection
$A \times B$	Cartesian product
A^n	$\underbrace{A \times A \times \cdots \times A}_{n \text{ times}}$
A^\perp	Orthogonal complement of A , e.g., $\mathbf{N}(\mathbf{A}) = \mathbf{C}(\mathbf{A}^\top)^\perp$ [3]
$A \oplus B$	Direct sum, i.e., each $\mathbf{v} \in \{\sum \mathbf{a}_i \mid \mathbf{a}_i \in S_i, i = 1, \dots, k\}$ has a unique representation of $\sum \mathbf{a}_i$ with $\mathbf{a}_i \in S_i$. That is, they expand to a space. Note that $\{S_i\}$ might not be orthogonal each other [8]
$A \overset{\perp}{\oplus} B$	Direct sum of two space that are orthogonal and span a n -dimensional space, e.g., $\mathbf{C}(\mathbf{A}^\top) \overset{\perp}{\oplus} \mathbf{C}(\mathbf{A}^\top)^\perp = \mathbb{R}^n$ (this decomposition of \mathbb{R}^n is called the orthogonal decomposition induced by \mathbf{A}) [3]
A, A^c	Complement set (given U)
$\#A, A $	Cardinality
$a \in A$	a is element of A
$a \notin A$	a is not element of A
$\{1, 2, \dots, n\}$	Discrete set containing the integer elements $1, 2, \dots, n$
U	Universe
2^A	Power set of A
\mathbb{R}	Set of real numbers
\mathbb{C}	Set of complex numbers
\mathbb{Z}	Set of integer number
$\mathbb{B} = \{0, 1\}$	Boolean set
\emptyset	Empty set
\mathbb{N}	Set of natural numbers

$\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$	Real or complex space (field)
$\mathbb{K}^{I_1 \times I_2 \times \dots \times I_N}$	$I_1 \times I_2 \times \dots \times I_N$ -dimensional real (or complex) space
\mathbb{K}_+	Nonnegative real (or complex) space [3]
\mathbb{K}_{++}	Positive real (or complex) space, i.e., $\mathbb{K}_{++} = \mathbb{K}_+ \setminus \{\mathbf{0}\}$ [3]
$\mathbb{S}^n, \mathcal{S}^n$	Conic set of the symmetric matrices in $\mathbb{R}^{n \times n}$ [3]
$\mathbb{S}_+^n, \mathcal{S}_+^n$	Conic set of the symmetric positive semidefinite matrices in $\mathbb{R}^{n \times n}$ [3]
$\mathbb{S}_{++}^n, \mathcal{S}_{++}^n$	Conic set of the symmetric positive definite matrices in $\mathbb{R}^{n \times n}$, i.e., $\mathbb{S}_{++}^n = \mathbb{S}_+^n \setminus \{\mathbf{0}\}$ [3]
\mathbb{H}^n	Set of all hermitian matrices in $\mathbb{C}^{n \times n}$
$[a, b]$	Closed interval of a real set from a to b
(a, b)	Opened interval of a real set from a to b
$[a, b), (a, b]$	Half-opened intervals of a real set from a to b

7.2 Quantifiers, inferences

\forall	For all (universal quantifier) [9]
\exists	There exists (existential quantifier) [9]
\nexists	There does not exist [9]
$\exists!$	There exist an unique [9]
\in	Belongs to [9]
\notin	Does not belong to [9]
\because	Because [9]
$, :$	Such that, sometimes that parantheses is used [9]
$,, (\cdot)$	Used to separate the quantifier with restricted domain from the its scope, e.g., $\forall x < 0 (x^2 > 0)$ or $\forall x < 0, x^2 > 0$ [9]
\therefore	Therefore [9]

7.3 Propositional Logic

$\neg a$	Logical negation of a [20]
$a \wedge b$	Conjunction (logical AND) operator between a and b [20]
$a \vee b$	Disjunction (logical OR) operator between a and b [20]
$a \oplus b$	Exclusive OR (logical XOR) operator between a and b [20]
$a \rightarrow b$	Implication (or conditional) statement[20]
$a \leftrightarrow b$	Bi-implication (or biconditional) statement, i.e., $(a \rightarrow b) \wedge (b \rightarrow a)$ [20]
$a \equiv b, a \iff b, a \Leftrightarrow b$	Logical equivalence, i.e., $a \leftrightarrow b$ is a tautology[20]

7.4 Operations

$ a $	Absolute value of a
\log	Base-10 logarithm or decimal logarithm
\ln	Natural logarithm
$\operatorname{Re}\{x\}$	Real part of x
$\operatorname{Im}\{x\}$	Imaginary part of x
$\angle \cdot$	Phase (complex argument)
$x \bmod y$	Remainder, i.e., $x - y[x/y]$, for $y \neq 0$
$x \operatorname{div} y$	Quotient [20]
$x \equiv y \pmod{m}$	Congruent, i.e., $m \mid (x - y)$ [20]
$\operatorname{frac}(x)$	Fractional part, i.e., $x \bmod 1$ [9]
$a \setminus b, a \mid b$	b is a positive integer multiple of a , i.e., $\exists n \in \mathbb{Z}_{++} \mid b = na$ [9, 20]
$a \nmid b, a \nmid b$	b is not a positive integer multiple of a , i.e., $\nexists n \in \mathbb{Z}_{++} \mid b = na$ [9, 20]
$\lceil \cdot \rceil$	Ceiling operation [9]
$\lfloor \cdot \rfloor$	Floor operation [9]

8 Physics

Φ	Electric flux (scalar) (in V m)
\mathbf{D}	Electric flux density, electric displacement, or electric induction vector (in C/m ²)

J	Electric current density vector (in A/m ²)
H	Magnetic field vector (in A/m)
B	Magnetic flux density vector (in Wb/m ² = T)
q	Electric charge strength/magnitude (in C)
ρ	Electric charge density (for volumes) (in C/m ³)
ρ_s	Electric charge density (for surface) (in C/m ²)
ρ_l	Electric charge density (for volumes) (in C/m)
f	Electrostatic force (Coulomb force), (in kg m/s ²)
ε	Electric permittivity(in F/m) [19]
ε_r	Relative electric permittivity or dielectric constant (in F/m) [19]
ε_0	Electric permittivity in vacuum, 8.854×10^{-12} F/m [19]
E	Electric field vector (in V/m)
μ	Magnetic permeability
μ_0	Magnetic permeability in vacuum

9 Calculus

∇	Vector differential operator (Nabla symbol), i.e., ∇f is the gradient of the scalar-valued function f , i.e., $f : \mathbb{R}^n \rightarrow \mathbb{R}$
$t, (u, v)$	Parametric variables commonly used, t for one variable, (u, v) for two variables[21]
$\mathbf{r}(t)$	Vector position $(x(t), y(t), z(t))$ parametrized by t [21]
$\mathbf{r}'(t)$	First derivative of $\mathbf{r}(t)$, i.e., the tangent vector of the curve $(x(t), y(t), z(t))$ [21]
$\mathbf{T}(t), \mathbf{u}(t)$	Tangent unit vector of $\mathbf{r}(t)$, i.e., $\mathbf{u}(t) = \mathbf{r}'(t)/ \mathbf{r}'(t) $ [13, 21]
$\mathbf{n}(t), \left(\frac{y'(t)}{ \mathbf{r}'(t) }, -\frac{x'(t)}{ \mathbf{r}'(t) } \right)$	Normal vector of $\mathbf{r}(t)$, i.e., $\mathbf{n}(t) \perp \mathbf{T}(t)$ [21]

C	Contour that traveled by $\mathbf{r}(t)$, for $a \leq t \leq b$ [21]
$L, L(C)$	Total length of the contour C (which can be defined the vector \mathbf{r} , parametrized by t), i.e., $L_C = \int_a^b \mathbf{r}'(t) dt$ [21]
$s(t)$	Length of the arc, which can be defined by the vector \mathbf{r} and t , that is, $s(t) = \int_a^t \mathbf{r}'(u) du$ ($s(b) = L$) [21]
ds	Differential operator of the length of the contour C , i.e., $ds = \mathbf{r}'(t) dt$
$\int_C f(\mathbf{r}) ds, \int_a^b f(\mathbf{r}(t)) \mathbf{r}'(t) dt$	Line integral of the function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ along the contour C [1, 21]
$\int_C \mathbf{F} \cdot d\mathbf{r}, \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt, \int_C \mathbf{F} \cdot \mathbf{T} ds$	Line integral of vector field \mathbf{F} along the contour C [1, 21]
$\int_a^b \mathbf{F}, \int_a^b \mathbf{F} \cdot d\mathbf{r}$	Alternative notation to the line integral, where the parametric variable t goes from a to b , making r goes from $\mathbf{r}(a) = \mathbf{a}$ to $\mathbf{r}(b) = \mathbf{b}$ [1]
\oint_C, \oint_C	Closed line integral along the contour C
$\mathbf{r}(u, v)$	Vector position $(x(u, v), y(u, v), z(u, v))$ parametrized by (u, v)
\mathbf{r}_u	$(\partial x / \partial u, \partial y / \partial u, \partial z / \partial u)$
\mathbf{r}_v	$(\partial x / \partial v, \partial y / \partial v, \partial z / \partial v)$
dA	Differential operator of a 2D area (denoted by D or R) in the \mathbb{R}^2 domain. This differential operator can be solved in different ways (rectangular, polar, cylindric, etc) [21]
D, R	Integration domain in which dA is integrated, i.e., $\iint_D f dA$ [21]
S	Smooth surface S , i.e., a 2D area in a 3D space (\mathbb{R}^3 domain)
$dS, \mathbf{r}_u \times \mathbf{r}_v dA$	Differential operator of a 2D area in a 3D domain (an surface). Note that $dS = \mathbf{r}_u \times \mathbf{r}_v dA$ should be accompanied with the change of the integration interval (from S to D)

$A(S), \iint_S dS, \iint_D \mathbf{r}_u \times \mathbf{r}_v dA$	Area of the surface S parametrized by (u, v) , in which dA is the area defined in the D domain (which is form by the u -by- v graph)
dV	Differential operator of a shape volume (denoted by E) in \mathbb{R}^3 domain, i.e., $\iiint_E dV = V$
E	Integration domain in which dV is integrated, i.e., $\iiint_E f dV$ [21]
$V, \iint_D f dA, \iiint_E f dV$	Volume of the function f over the regions D (in the case of double integrals) or E (in the case of triple integrals)
$\iint_S f dS, \iint_D f \mathbf{r}_u \times \mathbf{r}_v dA$	Surface integral over S
$\mathbf{n}(u, v), \frac{\mathbf{r}_u(u, v) \times \mathbf{r}_v(u, v)}{ \mathbf{r}_u(u, v) \times \mathbf{r}_v(u, v) }$	Normal vector of of the smooth surface S
$\iint_S \mathbf{F} \cdot \mathbf{n} dS, \iint_S \mathbf{F} \cdot d\mathbf{S}, \iint_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) dA$	Flux integral of vector field \mathbf{F} through the smooth surface S ($\mathbf{n} dS \triangleq d\mathbf{S}$)
$\nabla \times \mathbf{F}, \text{curl } \mathbf{F}$	Curl (rotacional) of the vector field \mathbf{F}
$\nabla \cdot \mathbf{F}, \text{div } \mathbf{F}$	Divergence of the vector field \mathbf{F}
$\nabla^2 f, \nabla \cdot (\nabla f), \Delta f, \partial^2 f / \partial x^2 + \partial^2 f / \partial y^2 + \partial^2 f / \partial z^2$	Scalar Laplacian operator (performed on a scalar-valued function $f : \mathbb{R}^n \rightarrow \mathbb{R}$)
$\nabla^2 \mathbf{F}, \nabla \times \nabla \times \mathbf{F} - \nabla(\nabla \cdot \mathbf{F}), \Delta \mathbf{F}, (\partial^2 \mathbf{F} / \partial x^2, \partial^2 \mathbf{F} / \partial y^2, \partial^2 \mathbf{F} / \partial z^2)$	Vector Laplacian operator (performed on a vector field, i.e., a vector-valued function, $\mathbf{F} : \mathbb{R}^n \rightarrow \mathbb{R}^n$). ∇^2 denotes the scalar (vector) Laplacian if the function is scalar-valued (vector-valued)

10 Generic mathematical symbols

■	Q.E.D.
\triangleq	Equal by definition
$:=, \leftarrow$	Assignment [20]
\neq	Not equal
∞	Infinity
j	$\sqrt{-1}$

11 Generic mathematical functions

$\mathcal{O}(\cdot), \mathcal{O}(\cdot)$	Big-O notation
$\Gamma(\cdot)$	Gamma function
$\mathcal{Q}(\cdot)$	Quantization function

12 Abbreviations

wrt.	With respect to
st.	Subject to
iff.	If and only if
EVD	Eigenvalue decomposition, or eigen-decomposition [15]
SVD	Singular value decomposition
CP	CANDECOMP/PARAFAC

References

- [1] TM Apostol. *Calculus, 2nd Edn., Vol. 2*. 1967.
- [2] Christopher M Bishop and Nasser M Nasrabadi. *Pattern Recognition and Machine Learning*. Vol. 4. 4. Springer, 2006.
- [3] Stephen Boyd, Stephen P. Boyd, and Lieven Vandenberghe. *Convex Optimization*. Cambridge university press, 2004.
- [4] Robert Grover Brown and Patrick YC Hwang. “Introduction to Random Signals and Applied Kalman Filtering: With MATLAB Exercises and Solutions”. In: *Introduction to random signals and applied Kalman filtering: with MATLAB exercises and solutions* (1997).
- [5] Rama Chellappa and Sergios Theodoridis. *Signal Processing Theory and Machine Learning*. Academic Press, 2014. ISBN: 0-12-396502-0.
- [6] Paulo SR Diniz. *Adaptive Filtering: Algorithms and Practical Implementation*. Nowell, MA: Kluwer Academic Publishers, 2002.
- [7] Paulo SR Diniz, Eduardo AB Da Silva, and Sergio L Netto. *Digital Signal Processing: System Analysis and Design*. Cambridge University Press, 2010. ISBN: 1-139-49157-1.
- [8] Gene H Golub and Charles F Van Loan. *Matrix Computations*. JHU press, 2013. ISBN: 1-4214-0859-7.
- [9] Ronald L Graham et al. “Concrete Mathematics: A Foundation for Computer Science”. In: *Computers in Physics* 3.5 (1989), pp. 106–107. ISSN: 0894-1866.
- [10] Simon S Haykin. *Adaptive Filter Theory*. Pearson Education India, 2002. ISBN: 81-317-0869-1.

- [11] Vinay K Ingle and John G Proakis. *Digital Signal Processing Using MATLAB*. Cole Publishing Company, 2000.
- [12] Basil Kouvaritakis and Mark Cannon. “Model Predictive Control”. In: *Switzerland: Springer International Publishing* 38 (2016).
- [13] Erwin Kreyszig, K Stroud, and G Stephenson. *Advanced Engineering Mathematics*. Vol. 9. John Wiley & Sons, Inc. 9 th edition, 2006 Page 2 of 6 Teaching methods ..., 2008.
- [14] Alberto Leon-Garcia. *Probability, Statistics, and Random Processes for Electrical Engineering*. 3rd ed. edição. Upper Saddle River, NJ: Prentice Hall, 2007. ISBN: 978-0-13-147122-1.
- [15] Josef Nossek. *Adaptive and Array Signal Processing*. 2015.
- [16] Alan V. Oppenheim and Ronald W. Schaffer. *Discrete-Time Signal Processing: International Edition*. 3^a edição. Upper Saddle River Munich: Pearson, Nov. 12, 2009. ISBN: 978-0-13-206709-6.
- [17] Kaare Brandt Petersen and Michael Syskind Pedersen. “The Matrix Cookbook”. In: *Technical University of Denmark* 7.15 (2008), p. 510.
- [18] John Proakis and Masoud Salehi. *Digital Communications*. 5th ed. edição. Boston: Mc Graw Hill, Jan. 1, 2007. ISBN: 978-0-07-295716-7.
- [19] Simon Ramo, John R Whinnery, and Theodore Van Duzer. *Fields and Waves in Communication Electronics*. John Wiley & Sons, 1994. ISBN: 81-265-1525-2.
- [20] Kenneth H Rosen. “Discrete Mathematics and Its Applications (7Th Edition)”. In: *William C Brown Pub* (2011).
- [21] James Stewart. *Calculus*. Cengage Learning, 2011. ISBN: 1-133-17069-2.
- [22] Gilbert Strang et al. *Introduction to Linear Algebra*. Vol. 3. Wellesley-Cambridge Press Wellesley, MA, 1993.
- [23] Sergios Theodoridis. *Machine Learning: A Bayesian and Optimization Perspective*. 2nd ed. Academic Pr, 2020. ISBN: 978-0-12-818803-3.
- [24] Harry L Van Trees. *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory*. John Wiley & Sons, 2002. ISBN: 0-471-09390-4.