# Notation

Rubem Vasconcelos Pacelli
rubem.engenharia@gmail.com

Department of Teleinformatics Engineering,
Federal University of Ceará.
Fortaleza, Ceará, Brazil.

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## Contents

# 1 Font notation

| | |
|---|---|
| $a, b, c, \ldots, A, B, C, \ldots$ | Scalars |
| $\mathbf{a}, \mathbf{b}, \mathbf{c}, \ldots$ | Vectors |
| $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ | Matrices |
| $\boldsymbol{\mathcal{A}}, \boldsymbol{\mathcal{B}}, \boldsymbol{\mathcal{C}}, \ldots$ | Tensors |
| $A, B, C, \ldots, \mathcal{A}, \mathcal{B}, \mathcal{C}, \ldots, \mathbb{A}, \mathbb{B}, \mathbb{C}, \ldots$ | Sets |

# 2 Signals and functions

## 2.1 Time indexing

| | |
|---|---|
| $x(t)$ | Continuous-time $t$ |
| $x[n], x[k], x[m], x[i], \ldots$<br>$x_n, x_k, x_m, x_i, \ldots$<br>$x(n), x(k), x(m), x(i), \ldots$ | Discrete-time $n, k, m, i, \ldots$ (parenthesis should be adopted only if there are no continuous-time signals in the context to avoid ambiguity) |
| $x\left[((n-m))_N\right]$[32], $x\left((n-m)\right)_N$[26] | Circular shift in $m$ samples within a $N$-samples window |

## 2.2 Common signals

| | |
|---|---|
| $\delta(t)$ | Delta function |
| $\delta[n], \delta_{i,j}$ | Kronecker function ($n = i - j$) |
| $h(t), h[n]$ | Impulse response (continuous and discrete time) |
| $\tilde{x}[n], \tilde{x}(t)$ | Periodic discrete- or continuous-time signal |
| $\hat{x}[n], \hat{x}(t)$ | Estimate of $x[n]$ or $x(t)$ |
| $\dot{x}[m]$ | Interpolation of $x[n]$ |

## 2.3 Common functions

| | |
|---|---|
| $\mathcal{O}(\cdot), O(\cdot)$ | Big-O notation |
| $\Gamma(\cdot)$ | Gamma function |
| $Q(\cdot)$ | Quantization function |
| $\mathrm{sgn}(\cdot)$ | Signum function |
| $\tanh(\cdot)$ | Hyperbolic tangent function |
| $I_\alpha(\cdot)$ | Modified Bessel function of the first kind and order $\alpha$ |

| | |
|---|---|
| $\binom{n}{k}$ | Binomial coefficient |

## 2.4 Operations and symbols

| | |
|---|---|
| $f : A \to B$ | A function $f$ whose domain is $A$ and codomain is $B$ |
| $\mathbf{f} : A \to \mathbb{R}^n$ | A vector-valued function $\mathbf{f}$, i.e., $n \geq 2$ |
| $f^n, x^n(t), x^n[k]$ | $n$th power of the function $f$, $x[n]$ or $x(t)$ |
| $f^{(n)}, x^{(n)}(t)$ | $n$th derivative of the function $f$ or $x(t)$ |
| $f', f^{(1)}, x'(t)$ | 1th derivative of the function $f$ or $x(t)$ |
| $f'', f^{(2)}, x''(t)$ | 2th derivative of the function $f$ or $x(t)$ |
| $\arg\max_{x \in \mathcal{A}} f(x)$ | Value of $x$ that minimizes $x$ |
| $\arg\min_{x \in \mathcal{A}} f(x)$ | Value of $x$ that minimizes $x$ |
| $f(\mathbf{x}) = \inf_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$ | Infimum, i.e., $f(\mathbf{x}) = \min\{g(\mathbf{x}, \mathbf{y}) \mid \mathbf{y} \in \mathcal{A} \wedge (\mathbf{x}, \mathbf{y}) \in \mathrm{dom}(g)\}$, which is the greatest lower bound of this set [10] |
| $f(\mathbf{x}) = \sup_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$ | Supremum, i.e., $f(\mathbf{x}) = \max\{g(\mathbf{x}, \mathbf{y}) \mid \mathbf{y} \in \mathcal{A} \wedge (\mathbf{x}, \mathbf{y}) \in \mathrm{dom}(g)\}$, which is the least upper bound of this set [10] |
| $f \circ g$ | Composition of the functions $f$ and $g$ |
| $*$ | Convolution (discrete or continuous) |
| $\circledast$ [17], $\textcircled{N}$ [32] | Circular convolution |

## 2.5 Digital signal processing

| | |
|---|---|
| $T_s$[26], $T$[32] | Sampling period |
| $f_s$, $F_s$[26] | Sampling frequency (in Hz or samples per secod [26, chapter 3]), i.e., $1/T_s$ |

| | |
|---|---|
| $f$ | Continuous linear frequency (in Hz). Apparently, there is no notation for the discrete linear frequency, we use $\omega$ only. However, in [26], the uppercase letters $F$ and $\Omega$ are used to denote the continuous-time frequency, while the lowercase $f$ and $\omega$ denote the discrete-time frequency (Oppenheim [32] does not do it!) |
| $\Omega$ [26] | Continuous angular frequency (in rad/s), that is, $2\pi f$. |
| $\Omega_s$ | Sampling frequency (in rad/s), i.e., $2\pi f_s$ |
| $\omega$ | Discrete angular frequency, i.e., $\Omega T_s$ [26, eq (3.27)]. As $\omega$ is also used to denote continuous angular frequency outside the DSP context, it is always convenient to state that it denotes the discrete frequency when it does |
| $W_N$ | Twiddle factor, $e^{-j\frac{2\pi}{N}}$ [26] |
| $N$ | Number of samples in the DFT/FFT |
| $\mathcal{R}_N[n]$ | Rectangular window used to cut off the discrete sequences [26] |
| $\Omega_N$ [32], $B$ | One-sided effective bandwidth of the continuous-time signal spectrum |
| $\omega_s$ [26] | Stop frequency |
| $\omega_p$ [26] | Pass frequency |
| $\Delta\omega$ [26] | $\omega_s - \omega_p$ |
| $\omega_c$ [26] | Cutoff frequency |
| $s(t)$ | Impulse train |
| $\text{gdr}\left[H(e^{j\omega})\right]$ [32] | Group delay of $H(e^{j\omega})$ |
| $\angle H(e^{j\omega})$ [32] | Phase response of $H(e^{j\omega})$ |
| $\left\lvert H(e^{j\omega})\right\rvert$ [32] | Magnitude (or gain) of $H(e^{j\omega})$ |
| $x_c(t)$ [32], $x(t)$ | Continuous-time signal |
| $x_s(t)$ | Sampled version of $x(t)$, i.e., $x(t)s(t)$ |
| $x_r(t)$ | Reconstruction of $x(t)$ from interpolation |
| $\tilde{x}[n]$ | Periodic extension of the the aperiodic signal $x[n]$ |

## 2.6 Transformations

| | |
|---|---|
| $\mathcal{F}\{\cdot\}$ [32, section 2.9] | Fourier transform (FT) |

| | |
|---|---|
| DTFT $\{\cdot\}$, DFS $\{\cdot\}$, FFT $\{\cdot\}$ | Discrete-time Fourier Transform (DTFT), Discrete Fourier Transform (DFT), Discrete Fourier Series (DFS), respectively |
| $\mathcal{L}\{\cdot\}$ | Laplace transform |
| $\mathcal{Z}\{\cdot\}$ | $z$-transform |
| $\hat{x}(t), \hat{x}[n]$ | Hilbert transform of $x(t)$ or $x[n]$ |
| $X(s)$ | Laplace transform of $x(t)$ |
| $X(f)$ | Fourier transform (FT) (in linear frequency, Hz) of $x(t)$ |
| $X(j\omega)$ | Fourier transform (FT) (in angular frequency, rad/sec) of $x(t)$ |
| $X(e^{j\omega})$ | Discrete-time Fourier transform (DTFT) of $x[n]$ |
| $X[k], X(k), X_k$ | Discrete Fourier transform (DFT) or fast Fourier transform (FFT) of $x[n]$, or even the Fourier series (FS) of the periodic signal $x(t)$ |
| $\tilde{X}[k], \tilde{X}(k), \tilde{X}_k$ | Discrete Fourier series (DFS) of $\tilde{x}[n]$ |
| $X(z)$ | $z$-transform of $x[n]$ |

# 3 Probability, statistics, and stochastic processes

## 3.1 Operators and symbols

| | |
|---|---|
| $\mathrm{E}[\cdot]$, $\mathbf{E}[\cdot]$ [31], $E[\cdot]$, $\mathbb{E}[\cdot]$ [16] | Statistical expectation operator |
| $\mathrm{E}_u[\cdot]$, $\mathbf{E}_u[\cdot]$ [31], $E_u[\cdot]$, $\mathbb{E}_u[\cdot]$ | Statistical expectation operator with respect to $u$ |
| $\langle\cdot\rangle$ | Ensemble average |
| $\mathrm{var}[\cdot]$ [31], $\mathrm{VAR}[\cdot]$ [9, 25, 30, 34] | Variance operator |
| $\mathrm{var}_u[\cdot][\cdot], \mathrm{VAR}_u[\cdot]$ | Variance operator with respect to $u$ |
| $\mathrm{cov}[\cdot], \mathrm{COV}[\cdot]$ | Covariance operator [9] |
| $\mathrm{cov}_u[\cdot], \mathrm{COV}_u[\cdot]$ | Covariance operator with respect to $u$ |
| $\mu_x$ | Mean of the random variable $x$ |
| $\boldsymbol{\mu}_{\mathbf{x}}, \mathbf{m}_{\mathbf{x}}$ | Mean vector of the random variable $\mathbf{x}$ [11] |
| $\mu_n$ | $n$th-order moment of a random variable |
| $\sigma_x^2, \kappa_2$ | Variance of the random variable $x$ |
| $\mathcal{K}_x, \mu_4$ | Kurtosis (4th-order moment) of the random variable $x$ |

| | |
|---|---|
| $\kappa_n$ | $n$th-order cumulant of a random variable |
| $\rho_{x,y}$ | Pearson correlation coefficient between $x$ and $y$ |
| $a \sim P$ | Random variable $a$ with distribution $P$ |
| $\mathcal{R}$ | Rayleigh's quotient |

## 3.2 Stochastic processes

PS: Estimated terms come with a hat above of it

| | |
|---|---|
| $r_x(\tau)$ [31], $R_x(\tau)$ | Autocorrelation function of the signal $x(t)$ or $x[n]$ |
| $S_x(f), S_x(j\omega)$ | Power spectral density (PSD) of $x(t)$ in linear $(f)$ or angular $(\omega)$ frequency |
| $S_{x,y}(f), S_{x,y}(j\omega)$ | Cross PSD of $x(t)$ and $y(t)$ in linear or angular $(\omega)$ frequency |
| $\mathbf{R_x}$ | (Auto)correlation matrix of $\mathbf{x}(n)$ |
| $r_{x,d}(\tau), R_{x,d}(\tau)$ | Cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$ [31] |
| $\mathbf{R_{xy}}$ | Cross-correlation matrix of $\mathbf{x}(n)$ and $\mathbf{y}(n)$ |
| $\mathbf{r}_{\mathbf{x}d}$ [24], $\mathbf{p}_{\mathbf{x}d}$ [16] | Cross-correlation vector between $\mathbf{x}(n)$ and $d(n)$ |
| $c_x(\tau), C_x(\tau)$ | Autocovariance function of the signal $x(t)$ or $x[n]$ [31] |
| $\mathbf{C_x}, \mathbf{K_x}, \mathbf{\Sigma_x}, \mathrm{cov}\,[\mathbf{x}]$ | (Auto)covariance matrix of $\mathbf{x}$ [9, 25, 30, 34, 41] |
| $\tilde{\mathbf{C}}_\mathbf{x}$[34] | Pseudocovariance matrix of $\mathbf{x}$ |
| $c_{xy}(\tau), C_{xy}(\tau)$ | Cross-covariance function of the signal $x(t)$ or $x[n]$ [31] |
| $\mathbf{C_{xy}}, \mathbf{K_{xy}}, \mathbf{\Sigma_{xy}}$ | Cross-covariance matrix of $\mathbf{x}$ and $\mathbf{y}$ |

## 3.3 Functions

| | |
|---|---|
| $Q(\cdot)$ | $Q$-function, i.e., $P\left[\mathcal{N}(0,1) > x\right]$ [34] |
| $\mathrm{erf}(\cdot)$ | Error function [34] |
| $\mathrm{erfc}(\cdot)$ | Complementary error function i.e., $\mathrm{erfc}(x) = 2Q(\sqrt{2}x) - \mathrm{erf}(x)$ [34] |
| $P[A]$ | Probability of the event or set $A$ [30] |

| | |
|---|---|
| $p(\cdot), f(\cdot)$ | Probability density function (PDF) or probability mass function (PMF) [30] |
| $p(x \mid A)$ | Conditional PDF or PMF [30] |
| $F(\cdot)$ | Cumulative distribution function (CDF) |
| $\Phi_x(\omega), M_x(j\omega), E\left[e^{j\omega x}\right]$ | First characteristic function (CF) of $x$ [34, 40] |
| $M_x(t), \Phi_x(-jt), E\left[e^{tx}\right]$ | Moment-generating function (MGF) of $x$ [34, 40] |
| $\Psi_x(\omega), \ln\Phi_x(\omega), \ln E\left[e^{j\omega x}\right]$ | Second characteristic function |
| $K_x(t), \ln E\left[e^{tx}\right], \ln M_x(t)$ | Cumulant-generating function (CGF) of $x$ [25] |

## 3.4 Distributions

| | |
|---|---|
| $\mathcal{N}(\mu, \sigma^2)$ | Gaussian distribution of a random variable with mean $\mu$ and variance $\sigma^2$ |
| $\mathcal{CN}(\mu, \sigma^2)$ | Complex Gaussian distribution of a random variable with mean $\mu$ and variance $\sigma^2$ |
| $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ | Gaussian distribution of a vector random variable with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ |
| $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ | Complex Gaussian distribution of a vector random variable with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ |
| $\mathcal{U}(a, b)$ | Uniform distribution from $a$ to $b$ |
| $\chi^2(n), \chi^2_n$ | Chi-square distribution with $n$ degree of freedom (assuming that the Gaussians are $\mathcal{N}(0, 1)$) |
| $\mathrm{Exp}(\lambda)$ | Exponential distribution with rate parameter $\lambda$ |
| $\Gamma(\alpha, \beta)$ | Gamma distribution with shape parameter $\alpha$ and rate parameter $\beta$ |
| $\Gamma(\alpha, \theta)$ | Gamma distribution with shape parameter $\alpha$ and scale parameter $\theta = 1/\beta$ |
| $\mathrm{Nakagami}(m, \boldsymbol{\Omega})$ | Nakagami-m distribution with shape parameter or fading figure $m$ and spread, scale, or shape parameter $\boldsymbol{\Omega}$ |
| $\mathrm{Rayleigh}(\sigma)$ | Rayleigh distribution with scale parameter $\sigma$ |

| | |
|---|---|
| Rayleigh($\Omega$) | Rayleigh distribution with the second moment $\Omega = E\left[x^2\right] = 2\sigma^2$ |
| Rice($s, \sigma$) | Rice distribution with noncentrality parameter $s$ and $\sigma$. $s^2$ represent the specular component power |
| Rice($\Omega, K$), Rice($A, K$) | Rice distribution with Rice factor $K = s^2/2\sigma^2$ and scale parameter $\Omega = A = s^2 + 2\sigma^2 = 2\sigma^2(K+1)$ ($\Omega$ is prefered over $A$) |

# 4 Machine learning, optimization theory, and statistical signal processing

## 4.1 Matrix Calculus (in denominator layout)

PS: Estimated terms come with a hat above of it

| | |
|---|---|
| $\mathbf{g}, \nabla f, \dfrac{\partial f}{\partial \mathbf{w}}$ | Gradient descent vector, used in the steepest descent method, also called gradient descent method or deterministic gradient method. |
| $\mathbf{g}$ if the gradient vector is $\nabla f$ (or $\hat{\mathbf{g}}$ if the gradient vector is $\mathbf{g}$ [24]) | Stochastic gradient descent (SGD) vector, i.e., instantaneous approximation of gradient descent vector |
| $\mathbf{g}_{\mathbf{x}}, \nabla_{\mathbf{w}} f, \dfrac{\partial f}{\partial \mathbf{w}}$ | Gradient descent vector with respect $\mathbf{w}$ [9] |
| $\mathbf{J}, \dfrac{\partial \mathbf{y}^\top}{\partial \mathbf{x}}, \nabla \mathbf{y}^\top$ [24] | Jacobian matrix. |
| $\mathbf{H}, \dfrac{\partial^2 f}{\partial \mathbf{w}^2}, \nabla^2 f$ [24], $\nabla\nabla f$ [9] | Hessian matrix. The notation $\nabla^2$ is sometimes used in Matrix Calculus to denote the second-order vector derivative. However, it must be avoided since, in Vector Calculus, $\nabla^2$ also denotes the Laplacian operator which in turn may be scalar or vector Laplacian operator depending on whether $f$ is scalar- or vector-valued, respectively. Some discussion about can be found in [5–7] |

## 4.2 Statistics: estimation and detection theory

| | |
|---|---|
| $\mathbf{x}$ | output |
| $\mathbf{w}$ | Parameters |
| $p(\mathbf{x} \mid \mathbf{w})$, $l(\mathbf{x} \mid \mathbf{w})$[30] | Likelihood function |
| $\ln p(\mathbf{x} \mid \mathbf{w})$ | Log-likelihood function |
| $\Lambda(\mathbf{x})$[30], $\frac{p(\mathbf{x}\mid H_1)}{p(\mathbf{x}\mid H_0)}$ [27, 30], $L(\mathbf{x})$ [12, 27] | Likelihood ratio function (also called likelihood ratio test (LRT) [27]) |
| $\Lambda_l(\mathbf{x})$, $\mathcal{L}(\mathbf{x})$ [12], $l(\mathbf{x})$ [27] | Log-likelihood ratio (LLR [27]) function |
| $\hat{\rho}_{x,y}$ | Estimated Pearson correlation coefficient between $x$ and $y$ |
| $\mathcal{R}_k$ | $k$th Decision region |
| $x(t) \stackrel{m.s.e}{=} y(t)$ | $x(t)$ equals $y(t)$ is the mean square error sense, that is $\mathrm{E}\left[\lvert x(t) - y(t)\rvert^2\right] = 0$ |
| $x(t) = \underset{N\to\infty}{\mathrm{l.\,i.\,m.}} \sum_{i=1}^{N} x_i \phi_i(t)$[42] | $\lim_{N\to\infty} \mathrm{E}\left[\left\lvert x(t) - \sum_{i=1}^{N} x_i \phi_i(t)\right\rvert^2\right] = 0$ (l.i.m stands for "limit in the mean"). It is analogous to the $\stackrel{m.s.e}{=}$ notation, but denoting that they equal in the MSE sense only when $N \to \infty$ |

## 4.3 Signals, (hyper)parameters, system performance, and criteria

| | |
|---|---|
| $N$ | Number of instances (or samples), i.e., $n \in \{1, 2, \ldots, N\}$ |
| $N_{\mathrm{trn}}$ | Number of instances in the training set, i.e., $n \in \{1, 2, \ldots, N_{\mathrm{trn}}\}$ |
| $N_{\mathrm{tst}}$ | Number of instances in the test set, i.e., $n \in \{1, 2, \ldots, N_{\mathrm{tst}}\}$ |
| $N_{\mathrm{val}}$ | Number of instances in the validation set, i.e., $n \in \{1, 2, \ldots, N_{\mathrm{val}}\}$ |
| $N_e$ | Number of epochs |
| $N_a$ | Number os attributes |
| $K$ [24] | Number of classes (which is the number of outputs in multiclass problems). Use $k$ to iterate over it |
| $L$ | Number of layers, i.e., the depth of the network. Use $l$ to iterate over it |

| | |
|---|---|
| $M_l$, $m_l$ [24], $J$ [24] | Number of neurons at the $l$th layer. You might prefer $J$ in the case of the single-layer perceptron (use $j$ to iterate over it). If you want to iterate through it, a sensible variation of Haykin notation is $M_l$, where $m_l$ can be used as an iterator. $m_0$ is the length of the input vector without the bias. |
| $\mathbf{x}(n), \mathbf{x}_n$ | Input signal (in $\mathbb{R}^{N_a+1}$) |
| $x_0(n)$ | Dummy input of the bais, which is usually $\pm 1$. $+1$ is prefered [9, 24]. |
| $\varphi(\cdot)$[24], $h(\cdot)$[9] | Activation function |
| $\varphi'(v_{m_l}^{(l)}(n))$[24], $\frac{\partial y_{m_l}^{(l)}(n)}{\partial v_{m_l}^{(l)}(n)}$ [24] | Partial derivative of the activation function with respect to $v_{m_l}^{(l)}(n)$ ($m_l$ neuron at $l$th layer) |
| $y_{m_l}^{(l)}(n), \varphi\left(v_{m_l}^{(l)}(n)\right)$[24], $\mathbf{t}_{m_l}^{(l)}(n)$[9] | Output signal (target) of the $m_l$th neuron at the $l$th layer |
| $\mathbf{y}^{(l)}(n)$ | Output signal of the $l$th layer |
| $\mathbf{y}(n)$, $\mathbf{y}^{(L)}(n)$ | Output of the neural network |
| $\mathbf{d}(n), \mathbf{d}_n$ | Desired label (in case of supervised learning). For multiclass classification, one-hot encoding is usually used. For binary (scalar) classification, however antipodal encoding, i.e., $\{-1, 1\}$ is more recommended [24]. |
| $e_{m_l}(n)$ | Error signal of the neuron $m_l$ at the $l$th layer |
| $\mathbf{e}(n)$, $\mathbf{d}(n) - \mathbf{y}(n)$ | Error signal |
| $\mathbf{w}_{m_l}^{(l)}(n), \boldsymbol{\theta}_{m_l}^{(l)}(n)$ $\begin{bmatrix} w_{m_l,0}^{(l)}(n) & w_{m_l,1}^{(l)}(n) & \dots & w_{m_l,m_{l-1}}^{(l)}(n) \end{bmatrix}$ | Parameters, coefficients, or synaptic weights vector in the $l$th layer. In the case of Single Layer Perceptrons or adaptive filters, the superscript is omitted |
| $w_{m_l,0}^{(l)}(n), b_{m_l}^{(l)}(n)$ | Bias (the first term of the weight vector) of the $l$th layer |
| $\mathbf{W}(n)$, $\begin{bmatrix} \mathbf{w}(1) & \mathbf{w}(2) & \cdots & \mathbf{w}(N) \end{bmatrix}^\top$ | Matrix of the synaptic weights |
| $\tilde{\mathbf{W}}(n)$ | Matrix of the synaptic weights, but without the bias |

| | |
|---|---|
| $v_{m_l}^{(l)}(n)$, $\mathbf{w}_{m_l}^{(l)\top}(n)\mathbf{y}_{m_{l-1}}^{(l-1)}(n)$ | Induced local field or activation potential. At the first layer $\mathbf{y}_{m_0}^{(0)}(n) = \mathbf{x}(n)$ [9] |
| $\mathbf{v}^{(l)}(n)$, $\mathbf{W}^{(l)}(n)\mathbf{y}_{m_{l-1}}^{(l-1)}(n)$ | Vector of the local fields at the $l$th layer |
| $\mathbf{w}^\star, \mathbf{w}_o, \boldsymbol{\theta}^\star, \boldsymbol{\theta}_o$ | Optimum value of the parameters, coefficients, or synaptic weights vector ($\mathbf{w}^*$ is also used [9] but it is not recommended as it may be confused with the conjugation operator) |
| $\delta_{m_l}^{(l)}(n)$, $\frac{\partial \mathscr{E}(n)}{\partial v_{m_l}^{(l)}(n)}$ | Local gradient of the $m_l$th neuron of the $l$th layer. |
| $\boldsymbol{\delta}^{(l)}(n)$ | Vector of the local gradients of all neurons at the $l$th layer |
| $\mathbf{X}$, $\begin{bmatrix}\mathbf{x}(1) & \mathbf{x}(2) & \cdots & \mathbf{x}(N)\end{bmatrix}$ | Data matrix [24] |
| $\eta(n)$ | Learning rate hyperparameter [24] |
| $\mathscr{R}$ | Bayes risk or average risk [24] |
| $c_{ij}, C_{ij}$ | Misclassification cost in deciding in favor of class $\mathscr{C}_i$ (represented in the subspace $\mathscr{H}_i$) when the $\mathscr{C}_j$ is the true class (used in Bayes classifiers/detectors) [12, 24] |
| $\mathscr{C}_k$[24], $\mathcal{C}_k$ [9] | $k$th class |
| $\mathscr{T}$ [24], $\mathbb{X}$ [22] | Training set, i.e., the set $\{\mathbf{x}(n), d(n)\}$ that is used in the training phase. |
| $\mathscr{H}_k$ | Subspace of the training vector belonging to the class $\mathscr{C}_k$ |
| $\mathscr{H}$ | Complete space of the input vector, i.e., $\mathscr{H}_1 \cup \mathscr{H}_2 \cup \cdots \mathscr{H}_K$ |
| $\mathscr{X}$ [24] | Set of all vectors in the training, batch, validation, or test dataset that were misclassified |
| $\mathscr{E}(\mathbf{w}), \mathscr{E}(\mathbf{w}(n)), \mathscr{E}(n)$ | Cost function or objective function (the way it is written depends on the purpose of the text) |
| $J(\mathbf{w}), J(\mathbf{w}(n)), J(n)$ | Alternative to the cost function |
| $\Delta\mathscr{E}(\mathbf{w}(n)), \Delta\mathscr{E}(n), \mathscr{E}(\mathbf{w}(n+1)) - \mathscr{E}(\mathbf{w}(n))$ | Cost function or objective function (the way it is written depends on the purpose of the text) |
| $\mathscr{E}_{\text{av}}(\cdot)$[24] | Error energy averaged over the training sample or the empirical risk |

| | |
|---|---|
| $\rho$ | Distance of the margin of separation between two classes (Support Vector Machine, SVM) |
| $g(\cdot)$ | Discriminant function, i.e., $g(\mathbf{w}^\star) = 0$ |

# 5 Linear Algebra

## 5.1 Common matrices and vectors

| | |
|---|---|
| $\mathbf{W}, \mathbf{D}$ | Diagonal matrix |
| $\mathbf{P}$ | Projection matrix; Permutation matrix |
| $\mathbf{J}$ | Jordan matrix |
| $\mathbf{L}$ | Lower matrix |
| $\mathbf{U}$ | Upper matrix |
| $\mathbf{C}$ | Cofactor matrix |
| $\mathbf{C}_{\mathbf{A}}, \mathrm{cof}\,(\mathbf{A})$ | Cofactor matrix of $\mathbf{A}$ |
| $\mathbf{S}$ | Symmetric matrix |
| $\mathbf{Q}$ | Orthogonal matrix |
| $\mathbf{I}_N$ | $N \times N$-dimensional identity matrix |
| $\mathbf{0}_{M\times N}$ | $M \times N$-dimensional null matrix |
| $\mathbf{0}_N$ | $N$-dimensional null vector |
| $\mathbf{1}_{M\times N}$ | $M \times N$-dimensional ones matrix |
| $\mathbf{1}_N$ | $N$-dimensional ones vector |
| $\mathbf{0}$ | Null matrix, vector, or tensor (dimensionality understood by context) |
| $\mathbf{1}$ | Ones matrix, vector, or tensor (dimensionality understood by context) |

## 5.2 Indexing

| | |
|---|---|
| $x_{i_1,i_2,\ldots,i_N}, [\mathcal{X}]_{i_1,i_2,\ldots,i_N}$ | Element in the position $(i_1, i_2, \ldots, i_N)$ of the tensor $\mathcal{X}$ |
| $\mathcal{X}^{(n)}$ | $n$th tensor of a nontemporal sequence |
| $\mathbf{x}_n, \mathbf{x}_{:n}$ | $n$th column of the matrix $X$ |
| $\mathbf{x}_{n:}$ | $n$th row of the matrix $X$ |
| $\mathbf{x}_{i_1,\ldots,i_{n-1},:,i_{n+1},\ldots,i_N}$ | Mode-$n$ fiber of the tensor $\mathcal{X}$ |
| $\mathbf{x}_{:,i_2,i_3}$ | Column fiber (mode-1 fiber) of the thrid-order tensor $\mathcal{X}$ |
| $\mathbf{x}_{i_1,:,i_3}$ | Row fiber (mode-2 fiber) of the thrid-order tensor $\mathcal{X}$ |

| | |
|---|---|
| $\mathbf{x}_{i_1,i_2,:}$ | Tube fiber (mode-3 fiber) of the thrid-order tensor $\mathcal{X}$ |
| $\mathbf{X}_{i_1,:,:}$ | Horizontal slice of the thrid-order tensor $\mathcal{X}$ |
| $\mathbf{X}_{:,i_2,:}$ | Lateral slices slice of the thrid-order tensor $\mathcal{X}$ |
| $\mathbf{X}_{i_3}, \mathbf{X}_{:,:,i_3}$ | Frontal slices slice of the thrid-order tensor $\mathcal{X}$ |

## 5.3 General operations

| | |
|---|---|
| $\langle \mathbf{a}, \mathbf{b} \rangle, \mathbf{a}^{\top}\mathbf{b}, \mathbf{a} \cdot \mathbf{b}$ | Inner or dot product |
| $\mathbf{a} \circ \mathbf{b}, \mathbf{a}\mathbf{b}^{\top}$ | Outer product |
| $\otimes$ | Kronecker product |
| $\odot$ | Hadamard (or Schur) (elementwise) product |
| $.^{\odot n}$ | $n$th-order Hadamard power |
| $.^{\odot \frac{1}{n}}$ | $n$th-order Hadamard root |
| $\oslash$ | Hadamard (or Schur) (elementwise) division |
| $\diamond$ | Khatri-Rao product |
| $\otimes$ | Kronecker Product |
| $\times_n$ | $n$-mode product |

## 5.4 Operations with matrices and tensors

| | |
|---|---|
| $\mathbf{A}^{-1}$ | Inverse matrix |
| $\mathbf{A}^{+}, \mathbf{A}^{\dagger}$ | Moore-Penrose left pseudoinverse |
| $\mathbf{A}^{\top}, \mathbf{A}^{T}, \mathbf{A}^{t}, \mathbf{A}^{\prime}$ [37] | Transpose |
| $\mathbf{A}^{-\top}$ | Transpose of the inverse, i.e., $\left(\mathbf{A}^{-1}\right)^{\top} = (\mathbf{A}^{\top})^{-1}$ [21, 33] |
| $\mathbf{A}^{*}$ | Complex conjugate |
| $\mathbf{A}^{\mathsf{H}}$ | Hermitian |
| $\Vert\mathbf{A}\Vert_{\mathrm{F}}$ | Frobenius norm |
| $\Vert\mathbf{A}\Vert$ | Matrix norm |
| $\vert\mathbf{A}\vert, \det(\mathbf{A})$ | Determinant |
| $\operatorname{diag}(\mathbf{A})$ | The elements in the diagonal of $\mathbf{A}$ |
| $\mathbf{E}[\mathbf{A}]$ | Vectorization: stacks the columns of the matrix $\mathbf{A}$ into a long column vector |

| | |
|---|---|
| $\mathbf{E}_d\left[\mathbf{A}\right]$ | Extracts the diagonal elements of a square matrix and returns them in a column vector |
| $\mathbf{E}_l\left[\mathbf{A}\right]$ | Extracts the elements strictly below the main diagonal of a square matrix in a column-wise manner and returns them into a column vector |
| $\mathbf{E}_u\left[\mathbf{A}\right]$ | Extracts the elements strictly above the main diagonal of a square matrix in a column-wise manner and returns them into a column vector |
| $\mathbf{E}_b\left[\mathbf{A}\right]$ | Block vectorization operator: stacks square block matrices of the input into a long block column matrix |
| $\mathrm{unvec}\left(\mathbf{A}\right)$ | Reshapes a column vector into a matrix |
| $\mathrm{tr}\{\mathbf{A}\}$ | trace |
| $\mathbf{X}_{(n)}$ | $n$-mode matricization of the tensor $\mathcal{X}$ |

## 5.5 Operations with vectors

| | |
|---|---|
| $\lVert\mathbf{a}\rVert$ | $l_1$ norm, 1-norm, or Manhattan norm |
| $\lVert\mathbf{a}\rVert$, $\lVert\mathbf{a}\rVert_2$ | $l_2$ norm, 2-norm, or Euclidean norm |
| $\lVert\mathbf{a}\rVert_p$ | $l_p$ norm, $p$-norm, or Minkowski norm |
| $\lVert\mathbf{a}\rVert_\infty$ | $l_\infty$ norm, $\infty$-norm, or Chebyshev norm |
| $\mathrm{diag}\left(\mathbf{a}\right)$ | Diagonalization: a square, diagonal matrix with entries given by the vector $\mathbf{a}$ |

## 5.6 Decompositions

| | |
|---|---|
| $\mathbf{\Lambda}$ | Eigenvalue matrix [39] |
| $\mathbf{Q}$ | Eigenvectors matrix; Orthogonal matrix of the QR decomposition[39] |
| $\mathbf{R}$ | Upper triangular matrix of the QR decomposition[39] |
| $\mathbf{U}$ | Left singular vectors[39] |
| $\mathbf{U}_r$ | Left singular nondegenerated vectors |
| $\mathbf{\Sigma}$ | Singular value matrix |
| $\mathbf{\Sigma}_r$ | Singular value matrix with nonzero singular values in the main diagonal |

| | |
|---|---|
| $\boldsymbol{\Sigma}^{+}$ | Singular value matrix of the pseudoinverse [39] |
| $\boldsymbol{\Sigma}_r^{+}$ | Singular value matrix of the pseudoinverse with nonzero singular values in the main diagonal |
| $\mathbf{V}$ | Right singular vectors [39] |
| $\mathbf{V}_r$ | Right singular nondegenerated vectors |
| $\mathrm{eig}\,(\mathbf{A})$ | Set of the eigenvalues of $\mathbf{A}$ [13, 30, 33] |
| $[\![\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots]\!]$ | CANDECOMP/PARAFAC (CP) decomposition of the tensor $\mathcal{X}$ from the outer product of column vectors of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}, \ldots$ |
| $[\![\boldsymbol{\lambda}; \mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots]\!]$ | Normalized CANDECOMP/PARAFAC (CP) decomposition of the tensor $\mathcal{X}$ from the outer product of column vectors of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}, \ldots$ |

## 5.7 Spaces and sets

### 5.7.1 Common spaces and sets

| | |
|---|---|
| $\mathbb{R}$ | Set of real numbers |
| $[a, b]$ | Closed interval of a real set from $a$ to $b$ |
| $(a, b)$ | Opened interval of a real set from $a$ to $b$ |
| $[a, b), (a, b]$ | Half-opened intervals of a real set from $a$ to $b$ |
| $\mathbb{C}$ | Set of complex numbers |
| $\mathbb{I}, j\mathbb{R}$ | Set of imaginary numbers |
| $\mathbb{Q}$ | Set of rational number |
| $\mathbb{R} \setminus \mathbb{Q}$ | Set of irrational number |
| $\mathbb{Z}$ | Set of integer number |
| $\mathbb{N}$ | Set of natural numbers |
| $\{1, 2, \ldots, n\}$ | Discrete set containing the integer elements $1, 2, \ldots, n$ |
| $\mathbb{B} = \{0, 1\}$ | Boolean set |
| $\varnothing$ | Empty set |
| $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$ | Real or complex space (field) |

| | |
|---|---|
| $\mathbb{K}^{I_1 \times I_2 \times \cdots \times I_N}$ | $I_1 \times I_2 \times \cdots \times I_N$-dimensional real (or complex) space |
| $\mathbb{K}_+$ | Nonnegative real (or complex) space [10] |
| $\mathbb{K}_{++}$ | Positive real (or complex) space, i.e., $\mathbb{K}_{++} = \mathbb{K}_+ \setminus \{\mathbf{0}\}$ [10] |
| $U$ | Universe |
| $2^A$ | Power set of $A$ |

### 5.7.2 Convex sets (or spaces)

| | |
|---|---|
| $\mathbb{S}^n$ [15], $\mathcal{S}^n$ [10] | Conic set of the symmetric matrices in $\mathbb{R}^{n\times n}$ |
| $\mathbb{S}^{n\perp}$ [15] | Conic set of the skew-symmetric (also called antisymmetric) matrices in $\mathbb{R}^{n\times n}$ |
| $\mathbb{S}^n_+$, $\mathcal{S}^n_+$ | Conic set of the symmetric positive semidefinite matrices in $\mathbb{R}^{n\times n}$ [10] |
| $\mathbb{S}^n_{++}, \mathcal{S}^n_{++}$ | Conic set of the symmetric positive definite matrices in $\mathbb{R}^{n\times n}$, i.e., $\mathbb{S}^n_{++} = \mathbb{S}^n_+ \setminus \{\mathbf{0}\}$ [10] |
| $\mathbb{H}^n$ | Set of all hermitian matrices in $\mathbb{C}^{n\times n}$ |
| conv $C$ | Convex hull |
| aff $C$ | Affune hull |
| $\mathcal{R}$ | Ray |
| $\mathcal{H}$ | Hyperplane |
| $\mathcal{H}_+, \mathcal{H}_-$ | Positive/negative halfspace |
| $B(\mathbf{x}_c, r)$ | Euclidean ball with radium $r$ and centered at $\mathbf{x}_c$ |
| $\mathcal{E}$ | Ellipsoid |
| $C$ | Norm cone |
| $K$ | Proper cone |
| $K^*$ | Dual cone |
| $\mathcal{P}$ | Polyhedra |
| $S$ | Simplex |
| $C_\alpha$ | $\alpha$-sublevel set |
| epi $f$ | Epigraph of the function $f$ |
| hypo $f$ | Hypograph of the function $f$ |

### 5.7.3 Spaces from matrices or vectors

| | |
|---|---|
| $\operatorname{span}\{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n\}$ | Vector space spanned by the argument vectors [21] |
| $\mathrm{C}(\mathbf{A})$, columnspace$(\mathbf{A})$, range$(\mathbf{A})$, span $\{\mathbf{A}\}$, image$(\mathbf{A})$ | Columnspace, range or image, i.e., the space span $\{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n\}$, where $\mathbf{a}_i$ is the ith column vector of the matrix $\mathbf{A}$ [31, 39] |
| $\mathrm{C}(\mathbf{A}^{\mathsf{H}})$ | Row space (also called left columnspace) [31, 39] |
| $\mathrm{N}(\mathbf{A})$, nullspace$(\mathbf{A})$, null$(\mathbf{A})$, kernel$(\mathbf{A})$ | Nullspace (or kernel space) [31, 39, 40] |
| $\mathrm{N}(\mathbf{A}^{\mathsf{H}})$ | Left nullspace |
| rank $\mathbf{A}$ | Rank, that is, $\dim(\operatorname{span}\{\mathbf{A}\}) = \dim(\mathrm{C}(\mathbf{A}))$ [31] |
| nullity $(\mathbf{A})$ | Nullity of $\mathbf{A}$, i.e., $\dim(\mathrm{N}(\mathbf{A}))$ |

## 5.8 Set operations

| | |
|---|---|
| $A + B$ | Set addition (Minkowski sum), i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} = \mathbf{x} + \mathbf{y},\ \forall\ \mathbf{x} \in \mathcal{X} \land \mathbf{y} \in \mathcal{Y}\}$ [28] |
| $A - B$ | Minkowski difference, i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} = \mathbf{x} - \mathbf{y},\ \forall\ \mathbf{x} \in \mathcal{X} \land \mathbf{y} \in \mathcal{Y}\}$ |
| $A \ominus B$ | Pontryagin difference, i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} + \mathbf{y} \in \mathcal{X},\ \forall\ \mathbf{y} \in \mathcal{Y}\}$ [28] |
| $A \setminus B$, $A - B$ | Set difference or set subtraction, i.e., $A \setminus B = \{x \vert x \in A \land x \notin B\}$ the set containing the elements of $A$ that are not in $B$ [36] |
| $A \cup B$ | Set of union |
| $A \cap B$ | Set of intersection |
| $A \times B$ | Cartesian product |
| $A^n$ | $\underbrace{A \times A \times \cdots \times A}_{n \text{ times}}$ |
| $A^\perp$ | Orthogonal complement of $A$, e.g., $\mathrm{N}(\mathbf{A}) = \mathrm{C}(\mathbf{A}^\top)^\perp$ [10] |
| $\mathbf{a} \perp \mathbf{b}$ | $\mathbf{a}$ is orthogonal to $\mathbf{b}$ |
| $\mathbf{a} \not\perp \mathbf{b}$ | $\mathbf{a}$ is not orthogonal to $\mathbf{b}$ |
| $A \oplus B$ | Direct sum, i.e., each $\mathbf{v} \in \{\sum \mathbf{a}_i \mid \mathbf{a}_i \in S_i, i = 1, \ldots, k\}$ has a unique representation of $\sum \mathbf{a}_i$ with $\mathbf{a}_i \in S_i$. That is, they expand to a space. Note that $\{S_i\}$ might not be orthogonal each other [21] |

| | |
|---|---|
| $A \overset{\perp}{\oplus} B$ | Direct sum of two spaces that are orthogonal and span a $n$-dimensional space, e.g., $\mathrm{C}(\mathbf{A}^\top) \overset{\perp}{\oplus} \mathrm{C}(\mathbf{A}^\top)^\perp = \mathbb{R}^n$ (this decomposition of $\mathbb{R}^n$ is called the orthogonal decomposition induced by $\mathbf{A}$) [10] |
| $\bar{A}, A^c$ | Complement set (given $U$) |
| $\#A, \lvert A \rvert$ | Cardinality of $A$ |
| $a \in A$ | $a$ is element of $A$ |
| $a \notin A$ | $a$ is not element of $A$ |

## 5.9 Inequalities

| $\mathcal{X} \preceq 0$ | Nonnegative tensor |
|---|---|
| $\mathbf{a} \preceq_K \mathbf{b}$ | Generalized inequality meaning that $\mathbf{b}-\mathbf{a}$ belongs to the conic subset $K$ in the space $\mathbb{R}^n$[10] |
| $\mathbf{a} \prec_K \mathbf{b}$ | Strict generalized inequality meaning that $\mathbf{b}-\mathbf{a}$ belongs to the interior of the conic subset $K$ in the space $\mathbb{R}^n$[10] |
| $\mathbf{a} \preceq \mathbf{b}$ | Generalized inequality meaning that $\mathbf{b}-\mathbf{a}$ belongs to the nonnegative orthant conic subset, $\mathbb{R}^n_+$, in the space $\mathbb{R}^n$.[10] |
| $\mathbf{a} \prec \mathbf{b}$ | Strict generalized inequality meaning that $\mathbf{b}-\mathbf{a}$ belongs to the positive orthant conic subset, $\mathbb{R}^n_{++}$, in the space $\mathbb{R}^n$[10] |
| $\mathbf{A} \preceq_K \mathbf{B}$ | Generalized inequality meaning that $\mathbf{B}-\mathbf{A}$ belongs to the conic subset $K$ in the space $\mathbb{S}^n$[10] |
| $\mathbf{A} \prec_K \mathbf{B}$ | Strict generalized inequality meaning that $\mathbf{B}-\mathbf{A}$ belongs to the interior of the conic subset $K$ in the space $\mathbb{S}^n$[10] |
| $\mathbf{A} \preceq \mathbf{B}$ | Generalized inequality meaning that $\mathbf{B}-\mathbf{A}$ belongs to the positive semidefinite conic subset, $\mathbb{S}^n_+$, in the space $\mathbb{S}^n$[10] |
| $\mathbf{A} \prec \mathbf{B}$ | Strict generalized inequality meaning that $\mathbf{B}-\mathbf{A}$ belongs to the positive orthant conic subset, $\mathbb{S}^n_{++}$, in the space $\mathbb{S}^n$[10] |

# 6 Communication systems

## 6.1 Common symbols

| | |
|---|---|
| $B$ | One-sided bandwidth of the baseband signal, in Hz |
| $W$ | One-sided bandwidth of the baseband signal, in rad/s |
| $N_0$ | Noise density, in ??? |
| $x_i$ | Real or in-phase part of $x$ |
| $x_q$ | Imaginary or quadrature part of $x$ |
| $f_c, f_{RF}$ | Carrier frequency (in Hertz) |
| $f_L$ | Carrier frequency in L-band (in Hertz) |
| $f_{IF}$ | Intermediate frequency (in Hertz) |
| $f_s$ | Sampling frequency or sampling rate (in Hertz) |
| $T_s$ | Sampling time interval/duration/period |
| $R$ | Bit rate |
| $T$ | Bit interval/duration/period |
| $T_c$ | Chip interval/duration/period |
| $T_{sy}, T_{sym}$ | Symbol/signaling[34] interval/duration/period |
| $s_{RF}$ | Transmitted signal in RF |
| $s_{FI}$ | Transmitted signal in FI |
| $s, s_l$ | Lowpass (or baseband) equivalent signal or envelope complex of transmitted signal |
| $r_{RF}$ | Received signal in RF |
| $r_{FI}$ | Received signal in FI |
| $r, r_l$ | Lowpass (or baseband) equivalent signal or envelope complex of received signal |
| $\phi$ | Signal phase |
| $\phi_0$ | Initial phase |
| $\eta_{RF}, w_{RF}$ | Noise in RF |
| $\eta_{FI}, w_{FI}$ | Noise in FI |
| $\eta, w$ | Noise in baseband |
| $\tau$ | Timing delay |
| $\Delta\tau$ | Timing error (delay - estimated) |
| $\varphi$ | Phase offset |
| $\Delta\varphi$ | Phase error (offset - estimated) |
| $f_d$ | Linear Doppler frequency |

| | |
|---|---|
| $\Delta f_d$ | Frequency error (Doppler frequency - estimated) |
| $\nu$ | Angular Doppler frequency |
| $\Delta\nu$ | Frequency error (Doppler frequency - estimated) |
| $\gamma, A$ | Transmitted signal amplitude |
| $\gamma_0, A_0$ | Combined effect of the path loss and antenna gain |

## 6.2 Fading multipath channels

| | |
|---|---|
| $t \overset{\mathcal{F}}{\leftrightarrow} \lambda$ [34] | Support temporal of the signal. $\lambda$ is obtained after taking the Fourier transform on $t$. |
| $\tau \overset{\mathcal{F}}{\leftrightarrow} f$ [34] | Second support temporal of the signal ($c(t)$ varies with with the input at the time $\tau$). $f$ is obtained after taking the Fourier transform on $\tau$. |
| $c(t,\tau)$ [34] | Complex envelope of the channel response at the time $t$ due to an impulse applied at the $t-\tau$ |
| $C(f,t)$ [34] | Transfer function of $c(t,\tau)$ in $\tau$ |
| $\alpha(t,\tau)$ [34] | Attenuation of $c(t,\tau)$, i.e., $c(t,\tau) = \alpha(t,\tau)e^{e\pi f_c \tau}$ |
| $R_c(\tau_1,\tau_2,\Delta t)$ [34] | Autocorrelation function of $c(t,\tau)$, i.e., $R_c(\tau_1,\tau_2,\Delta t) = \mathrm{E}\left[c^*(t,\tau_1), c^*(t+\Delta t,\tau_2)\right]$ |
| $R_c(\tau,\Delta t)$ [34] | Autocorrelation function of $c(t,\tau)$ assuming uncorrelated scattering |
| $R_c(\tau)$, $R_c(\tau,\Delta t)\big\rvert_{\Delta t=0}$ [34] | Multipath intensity profile or delay power spectrum |
| $R_C(\Delta f,\Delta t), R_C(f_1,f_2;\Delta t)$, $\mathrm{E}\left[C(f_1,t), C(f_2,t+\Delta t)\right]$, $\mathcal{F}_\tau\left\{R_c(\tau,\Delta t)\right\}$ [20] | Spaced-frequency, spaced-time correlation function ($\Delta f = f_2 - f_1$) |
| $R_C(\Delta f)$, $R_C(\Delta f,\Delta t)\big\rvert_{\Delta t=0}$ [34], $\mathcal{F}\left\{R_c(\tau)\right\}$ [20] | Spaced-frequency correlation function |
| $(\Delta f)_c$ | Coherence bandwidth of $c(t)$, that is, the frequency interval in which $R_C(\Delta f)$ is nonzero [34] |
| $T_m$ | Multipath spread of the channel, that is, the time interval in which $R_c(\tau)$ is nonzero ($T_m \approx 1/(\Delta f)_c$) [34] |

| | |
|---|---|
| $R_C(\Delta t)$, $R_C(\Delta f, \Delta t)\big\|_{\Delta f=0}$ | Spaced-time correlation function [34] |
| $S_C(\lambda)$ [34], $\mathcal{F}\{R_C(\Delta t)\}$ [20] | Doppler power spectrum |
| $(\Delta t)_c$ | Coherence time of $c(t)$, that is, the time interval in which $R_C(\Delta t)$ is nonzero [34] |
| $B_m$ | Multipath spread of the channel, that is, the frequency interval in which $S_c(\lambda)$ is nonzero ($B_d \approx 1/(\Delta t)_c$) [34] |
| $S_C(\tau, \lambda)$ [34], $\mathcal{F}_{\Delta f, \Delta t}\{R_C(\Delta f, \Delta t)\}$ [20] | Scattering function |

# 7 Discrete mathematics

## 7.1 Quantifiers, inferences

| | |
|---|---|
| $\forall$ | For all (universal quantifier) [23] |
| $\exists$ | There exists (existential quantifier) [23] |
| $\nexists$ | There does not exist [23] |
| $\exists!$ | There exists an unique [23] |
| $\exists_n$ | There exists exactly $n$ [36] |
| $\in$ | Belongs to [23] |
| $\notin$ | Does not belong to [23] |
| $\because$ | Because [23] |
| $\|, :$ | Such that, sometimes that parentheses is used [23] |
| $,\ , (\cdot)$ | Used to separate the quantifier with restricted domain from its scope, e.g., $\forall\ \ x < 0\left(x^2 > 0\right)$ or $\forall\ \ x < 0, x^2 > 0$ [23] |
| $\therefore$ | Therefore [23] |

## 7.2 Propositional Logic

| | |
|---|---|
| $\neg a$ | Logical negation of $a$ [36] |
| $a \wedge b$ | Conjunction (logical AND) operator between $a$ and $b$[36] |
| $a \vee b$ | Disjunction (logical OR) operator between $a$ and $b$[36] |
| $a \oplus b$ | Exclusive OR (logical XOR) operator between $a$ and $b$[36] |

| | |
|---|---|
| $a \to b$ | Implication (or conditional) statement[36] |
| $a \leftrightarrow b$ | Bi-implication (or biconditional) statement, i.e., $(a \to b) \wedge (b \to a)$ [36] |
| $a \equiv b, a \iff b, a \Leftrightarrow b$ | Logical equivalence, i.e., $a \leftrightarrow b$ is a tautology[36] |

### 7.3 Operations

| | |
|---|---|
| $\lvert a \rvert$ | Absolute value of $a$ |
| log | Base-10 logarithm or decimal logarithm |
| ln | Natual logarithm |
| $\operatorname{Re}\{x\}$ | Real part of $x$ |
| $\operatorname{Im}\{x\}$ | Imaginary part of $x$ |
| $\angle\cdot$ | Phase (complex argument) |
| $x \bmod y$ | Remainder, i.e., $x - y\lfloor x/y \rfloor$, for $y \neq 0$ |
| $x \operatorname{div} y$ | Quotient [36] |
| $x \equiv y \pmod{m}$ | Congruent, i.e., $m \backslash (x - y)$ [36] |
| $\operatorname{frac}(x)$ | Fractional part, i.e., $x \bmod 1$ [23] |
| $a \backslash b$ [23, Section 4.1], $a \mid b$ [36] | $b$ is a positive integer multiple of $a \in \mathbb{Z}$, i.e., $\exists!\, n \in \mathbb{Z}_{++} \mid b = na$ |
| $a \not\backslash b$ [23, Section 4.1], $a \nmid b$ [36] | $b$ is not a positive integer multiple of $a \in \mathbb{Z}$, i.e., $\nexists\, n \in \mathbb{Z}_{++} \mid b = na$ |
| $\lceil\cdot\rceil$ | Ceiling operation [23] |
| $\lfloor\cdot\rfloor$ | Floor operation [23] |

## 8 Vector Calculus

| | |
|---|---|
| $\nabla f$[38], $\operatorname{grad} f$[35] | Vector differential operator (Nabla symbol), i.e., $\nabla f$ is the gradient of the scalar-valued function $f$, i.e., $f : \mathbb{R}^n \to \mathbb{R}$ |
| $t, (u, v)$ | Parametric variables commonly used, $t$ for one variable, $(u, v)$ for two variables[38] |
| $\mathbf{l}(x, y, z)$ [35], $\mathbf{r}(x, y, z)$ [38], $x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ | Vector position, i.e., $(x, y, z)$. |
| $\mathbf{l}(t)$ | Vector position parametrized by $t$, i.e., $(x(t), y(t), z(t))$ [35, 38] |

| | |
|---|---|
| $\mathbf{l}'(t), \mathrm{d}\mathbf{l}/\mathrm{d}t$ | First derivative of $\mathbf{l}(t)$, i.e., the tangent vector of the curve $(x(t), y(t), z(t))$ [38] |
| $\mathbf{u}(t)$[29] $\mathbf{T}(t)$[38], $\mathrm{d}\mathbf{l}\,(t)$[35] | Tangent unit vector of $\mathbf{l}(t)$, i.e., $\mathbf{u}(t) = \mathbf{l}'(t)/\lvert\mathbf{l}'(t)\rvert$ |
| $\mathbf{n}(t), \left(\frac{y'(t)}{\lvert\mathbf{l}'(t)\rvert}, -\frac{x'(t)}{\lvert\mathbf{l}'(t)\rvert}\right)$ | Normal vector of $\mathbf{l}(t)$, i.e., $\mathbf{n}(t) \perp \mathbf{T}(t)$[38] |
| $C$ | Contour that traveled by $\mathbf{l}(t)$, for $a \leq t \leq b$ [38] |
| $L, L(C)$ | Total length of the contour $C$ (which can be defined the vector $\mathbf{l}$, parametrized by $t$), i.e., $L_C = \int_a^b \lvert\mathbf{l}'(t)\rvert\,\mathrm{d}t$[38] |
| $s(t)$ | Length of the arc, which can be defined by the vector $\mathbf{l}$ and $t$, that is, $s(t) = \int_a^t \lvert\mathbf{l}'(u)\rvert\,\mathrm{d}u$ $(s(b) = L)$[38] |
| $\mathrm{d}s$ | Differential operator of the length of the contour $C$, i.e., $\mathrm{d}s = \lvert\mathbf{l}'(t)\rvert\,\mathrm{d}t$ [38] |
| $\int_C f(\mathbf{l})\,\mathrm{d}s$, $\int_a^b f(\mathbf{l}(t))\lvert\mathbf{l}'(t)\rvert\,\mathrm{d}t$ | Line integral of the function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ along the contour $C$. In the context of integrals in the complex plane, it is also called "contour integral" |
| $\theta$ [35] | Angle between the contour $C$ and the vector field $\mathbf{F}$ |
| $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{l}$, $\int_a^b \mathbf{F}(\mathbf{l}(t)) \cdot \mathbf{l}'(t)\,\mathrm{d}t$ [8, 38], $\int_C \mathbf{F} \cdot \mathbf{u}\,\mathrm{d}s$, $\int_C \mathbf{F}\cos\theta\,\mathrm{d}s$ [35] | Line integral of vector field $\mathbf{F}$ along the contour $C$ |
| $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{u}$ [35] | In the field of electromagnetics, it is common to apply the line integral between the vector field $\mathbf{F}$ and the unit vector $\mathbf{u}(t)$. Therefore, this line integral may appear as well |
| $\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{F}, \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{F} \cdot \mathrm{d}\mathbf{l}$ | Alternative notation to the line integral, where the parametric variable $t$ goes from $a$ to $b$, making $r$ goes from $\mathbf{l}(a) = \mathbf{a}$ to $\mathbf{l}(b) = \mathbf{b}$ [8] |
| $\oint_C, \circlearrowleft\!\!\!\!\!\int_C$ | Line integral along the closed contour $C$. The arrow indicates the contour integral orientation, which is counterclockwise, by default. In the context of integrals in the complex plane, it is also called "closed contour integral". |

| | |
|---|---|
| $\oiint_S$ | Surface integral over the closed surface $S$ |
| $\mathbf{l}(u, v)$ | Vector position $(x(u, v), y(u, v), z(u, v))$ parametrized by $(u, v)$ |
| $\mathbf{l}_u$ | $(\partial x/\partial u, \partial y/\partial u, \partial z/\partial u)$ |
| $\mathbf{l}_v$ | $(\partial x/\partial v, \partial y/\partial v, \partial z/\partial v)$ |
| $\mathrm{d}A$ | Differential operator of a 2D area (denoted by $D$ or $R$) in the $\mathbb{R}^2$ domain. This differential operator can be solved in different ways (rectangular, polar, cylindric, etc) [38] |
| $D, R$ | Integration domain in which $\mathrm{d}A$ is integrated, i.e., $\iint_D f \,\mathrm{d}A$. $R$ is preferred when the integration domain is a rectangle, while $D$ is used when it has nonrectangular shape [38] |
| $S$ | Smooth surface $S \subset \mathbb{R}^3$, i.e., a 2D area in a 3D space |
| $\mathrm{d}S$ , $\lvert\mathbf{l}_u \times \mathbf{l}_v\rvert \,\mathrm{d}A$ | Differential operator of a 2D area in a 3D domain (an surface). Note that $\mathrm{d}S = \lvert\mathbf{l}_u \times \mathbf{l}_v\rvert \,\mathrm{d}A$ should be accompanied with the change of the integration interval(from $S$ to $D$) |
| $A(S)$, $\iint_S \mathrm{d}S$ , $\iint_D \lvert\mathbf{l}_u \times \mathbf{l}_v\rvert \,\mathrm{d}A$ | Area of the surface $S$ parametrized by $(u, v)$, in which $\mathrm{d}A$ is the area defined in the $D$ domain (which is form by the $u$-by-$v$ graph) |
| $\mathrm{d}V$ | Differential operator of a shape volume (denoted by $E$) in $\mathbb{R}^3$ domain, i.e., $\iiint_E \mathrm{d}V = V$ |
| $E$ | Integration domain in which $\mathrm{d}V$ is integrated, i.e., $\iiint_E f \,\mathrm{d}V$ [38] |
| $V$, $\iint_D f \,\mathrm{d}A$ , $\iiint_E f \,\mathrm{d}V$ | Volume of the function $f$ over the regions $D$ (in the case of double integrals) or $E$ (in the case of triple integrals) |
| $\iint_S f \,\mathrm{d}S$ , $\iint_D f\lvert\mathbf{l}_u \times \mathbf{l}_v\rvert \,\mathrm{d}A$ | Surface integral over $S$ |
| $\mathbf{n}(u, v)$, $\frac{\mathbf{l}_u(u,v)\times\mathbf{l}_v(u,v)}{\lvert\mathbf{l}_u(u,v)\times\mathbf{l}_v(u,v)\rvert}$ | Normal vector of of the smooth surface $S$ |
| $\iint_S \mathbf{F} \cdot \mathbf{n} \,\mathrm{d}S$ , $\iint_S \mathbf{F} \cdot \mathrm{d}\mathbf{S}$ , $\iint_D \mathbf{F} \cdot (\mathbf{l}_u \times \mathbf{l}_v) \,\mathrm{d}A$ | Flux integral of vector field $\mathbf{F}$ through the smooth surface $S$ ($\mathbf{n} \,\mathrm{d}S \triangleq \mathrm{d}\mathbf{S}$) |

| | |
|---|---|
| $\oiint_S \mathbf{F}\cdot\mathbf{n}\,\mathrm{d}S$, $\oiint_S \mathbf{F}\cdot\mathrm{d}\mathbf{S}$, $\iint_D \mathbf{F}\cdot(\mathbf{l}_u\times\mathbf{l}_v)\,\mathrm{d}A$ | Flux integral of vector field $\mathbf{F}$ through the smooth and closed surface $S$ ($\mathbf{n}\,\mathrm{d}S \triangleq \mathrm{d}\mathbf{S}$) |
| $\nabla\times\mathbf{F}$, curl $\mathbf{F}$ | Curl (rotacional) of the vector field $\mathbf{F}$ |
| $\nabla\cdot\mathbf{F}$, div $\mathbf{F}$ | Divercence of the vector field $\mathbf{F}$ |
| $\nabla^2 f, \nabla\cdot(\nabla f), \Delta f$, $\partial^2 f/\partial x^2 + \partial^2 f/\partial y^2 + \partial^2 f/\partial z^2$ | Scalar Laplacian operator (performed on a scalar-valued function $f:\mathbb{R}^n\to\mathbb{R}$) |
| $\nabla^2\mathbf{F}, \nabla\times\nabla\times\mathbf{F}-\nabla(\nabla\cdot\mathbf{F}), \Delta\mathbf{F}$, $(\partial^2\mathbf{F}/\partial x^2, \partial^2\mathbf{F}/\partial y^2, \partial^2\mathbf{F}/\partial z^2)$ | Vector Laplacian operator (performed on a vector field, i.e., a vector-valued function, $\mathbf{F}:\mathbb{R}^n\to\mathbb{R}^n$). $\nabla^2$ denotes the scalar (vector) Laplacian if the function is scalar-valued (vector-valued). The notation $\Delta$ must be avoided as it is overused in many contexts |

# 9 Electromagnetic waves

| | |
|---|---|
| $\mathbf{\Phi}$ | Electric flux (scalar) (in V m) |
| $\mathbf{H}$ | Magnetic field vector (in A/m) |
| $\mathbf{B}$ | Magnetic flux density vector (in Wb/m$^2$ = T) |
| $\mathbf{\Phi}$[14] | Magnetic flux |
| $q_\mathrm{f}$, $q_\mathrm{free}$, $Q_\mathrm{free}$[18] | Free electric charge (in C) |
| $q_\mathrm{b}$, $q_\mathrm{bound}$, $Q_\mathrm{bound}$[18] | Bound electric charge (in C) |
| $q$, $q_\mathrm{f}+q_\mathrm{b}$ | Electric charge (in C) |
| $\rho_\mathrm{f}$[1],$\rho_\mathrm{free}$ [18] | Free electric charge density |
| $\rho_\mathrm{b}$[1], $\rho_\mathrm{bound}$ [18] | Electric charge density |
| $\rho, \rho_\mathrm{f}+\rho_\mathrm{b}$ | Electric charge density (it can be in C/m$^3$, C/m$^2$ or C/m depending whether it is a volume, surface, or line shapes) |
| $\mathbf{f}$[35], $\mathbf{F}$[2] | Electrostatic force (Coulomb force), (in kg m/s$^2$). |
| $\varepsilon$ | Electric permittivity(in F/m). If the medium is isotropic, it is a scalar. If it is anisotropic, it is a tensor. [35] |
| $\varepsilon_r$ | Relative electric permittivity or dielectric constant (in F/m) [35] |
| $\varepsilon_0$ | Electric permittivity in vacuum, $8.854\times10^{-12}$ F/m [35] |

| | |
|---|---|
| $\mathbf{E}$ | Electric field vector (in V/m) |
| $\sigma$ | Electric conductivity (in S/m) |
| $\mathbf{J}$ | Electric current density vector (in $A/m^2$) |
| $\mathbf{J}_m$[14] | Magnetization current density vector (in $A/m^2$) |
| $\mathbf{D}$ | Electric flux density, electric displacement, or electric induction vector (in $C/m^2$) |
| $U$ | Electric potential energy |
| $V$[3, 14], $\mathbf{\Phi}$[35] | Electric potential (in voltage, V). However, keep in mind that there is a subtle difference between both definitions [4] |
| $\mathbf{\Phi}_E$[19], $\oiint_S \mathbf{E}\,\mathrm{d}\mathbf{S}$ | Electric flux (in V m) |
| $\mathbf{\Phi}_D$[18], $\Psi$[35], $\oiint_S \mathbf{D}\,\mathrm{d}\mathbf{S}$ | Electric flux (**D**-field flux) |
| $\mathbf{P}$ | Electric polarization of the material (in $C/m^2$) |
| $\chi_e$ | Electric susceptibility (for linear and isotropic materials) |
| $\mu$ | Magnetic permeability |
| $\mu_0$ | Magnetic permeability in vacuum |

# 10 Generic mathematical symbols

| | |
|---|---|
| $\blacksquare$ | Q.E.D. |
| $\triangleq$ | Equal by definition |
| $:=, \leftarrow$ | Assignment [36] |
| $\neq$ | Not equal |
| $\infty$ | Infinity |
| $j$ | $\sqrt{-1}$ |

# 11 Abbreviations

| | |
|---|---|
| wrt. | With respect to |
| st. | Subject to |
| iff. | If and only if |
| EVD | Eigenvalue decomposition, or eigendecomposition [31] |
| SVD | Singular value decomposition |
| CP | CANDECOMP/PARAFAC |

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