

Notation

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1 Font notation

$a, b, c, \dots, A, B, C, \dots$	Scalars
$\mathbf{a}, \mathbf{b}, \mathbf{c}, \dots$	Vectors
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$	Matrices
$\mathcal{A}, \mathcal{B}, \mathcal{C}, \dots$	Tensors
$A, B, C, \dots, \mathcal{A}, \mathcal{B}, \mathcal{C}, \dots, \mathbb{A}, \mathbb{B}, \mathbb{C}, \dots$	Sets

2 Signals and functions

2.1 Time indexing

$x(t)$	Continuous-time t
$x[n], x[k], x[m], x[i], \dots$ $x_n, x_k, x_m, x_i, \dots$ $x(n), x(k), x(m), x(i), \dots$	Discrete-time n, k, m, i, \dots (parenthesis should be adopted only if there are no continuous-time signals in the context to avoid ambiguity)
$x[((n-m))_N]$ ^[30] , $x((n-m))_N$ ^[24]	Circular shift in m samples within a N -samples window

2.2 Common signals

$\delta(t)$	Delta function
$\delta[n], \delta_{i,j}$	Kronecker function ($n = i - j$)
$h(t), h[n]$	Impulse response (continuous and discrete time)
$\tilde{x}[n], \tilde{x}(t)$	Periodic discrete- or continuous-time signal
$\hat{x}[n], \hat{x}(t)$	Estimate of $x[n]$ or $x(t)$
$\dot{x}[m]$	Interpolation of $x[n]$

2.3 Common functions

$\mathcal{O}(\cdot), \mathcal{O}(\cdot)$	Big-O notation
$\Gamma(\cdot)$	Gamma function
$\mathcal{Q}(\cdot)$	Quantization function
$\text{sgn}(\cdot)$	Signum function
$\tanh(\cdot)$	Hyperbolic tangent function
$I_\alpha(\cdot)$	Modified Bessel function of the first kind and order α

$\binom{n}{k}$	Binomial coefficient
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2.4 Operations and symbols

$f : A \rightarrow B$	A function f whose domain is A and codomain is B
$\mathbf{f} : A \rightarrow \mathbb{R}^n$	A vector-valued function \mathbf{f} , i.e., $n \geq 2$
$f^n, x^n(t), x^n[k]$	n th power of the function f , $x[n]$ or $x(t)$
$f^{(n)}, x^{(n)}(t)$	n th derivative of the function f or $x(t)$
$f', f^{(1)}, x'(t)$	1th derivative of the function f or $x(t)$
$f'', f^{(2)}, x''(t)$	2th derivative of the function f or $x(t)$
$\arg \max_{x \in \mathcal{A}} f(x)$	Value of x that minimizes x
$\arg \min_{x \in \mathcal{A}} f(x)$	Value of x that minimizes x
$f(\mathbf{x}) = \inf_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Infimum, i.e., $f(\mathbf{x}) = \min \{g(\mathbf{x}, \mathbf{y}) \mid \mathbf{y} \in \mathcal{A} \wedge (\mathbf{x}, \mathbf{y}) \in \text{dom}(g)\}$, which is the greatest lower bound of this set [9]
$f(\mathbf{x}) = \sup_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Supremum, i.e., $f(\mathbf{x}) = \max \{g(\mathbf{x}, \mathbf{y}) \mid \mathbf{y} \in \mathcal{A} \wedge (\mathbf{x}, \mathbf{y}) \in \text{dom}(g)\}$, which is the least upper bound of this set [9]
$f \circ g$	Composition of the functions f and g
$*$	Convolution (discrete or continuous)
\otimes [16], $\textcircled{\text{N}}$ [30]	Circular convolution

2.5 Digital signal processing

W_N	Twiddle factor, $e^{-j\frac{2\pi}{N}}$ [24]
N	Number of samples in the DFT/FFT
Ω [24]	Continuous angular frequency (in rad/s)

ω	Discrete angular frequency. As ω is also used to denote continuous angular frequency outside the DSP context, it is always convenient to state that it denotes the discrete frequency when it does
f_c	Continuous linear frequency (in Hz)
f	Discrete linear frequency. As f is also used to denote continuous linear frequency outside the DSP context, it is always convenient to state that it denotes the discrete frequency when it does
$\mathcal{R}_N[n]$	Rectangular window used to cut off the discrete sequences [24]
$T[30], T_s$	Sampling period
f_s	Sampling frequency (in Hz), i.e., $1/T$
Ω_s	Sampling frequency (in rad/s), i.e., $2\pi f_s$
$\Omega_N [30], B$	One-sided effective bandwidth of the continuous-time signal spectrum
ω_s	Stop frequency [24]
ω_p	Pass frequency [24]
$\Delta\omega$	$\omega_s - \omega_p$ [24]
ω_c	Cutoff frequency [24]
$s(t)$	Impulse train
$\text{gdr} [H(e^{j\omega})] [30]$	Group delay of $H(e^{j\omega})$
$\angle H(e^{j\omega}) [30]$	Phase response of $H(e^{j\omega})$
$ H(e^{j\omega}) [30]$	Magnitude (or gain) of $H(e^{j\omega})$
$x_c(t) [30], x(t)$	Continuous-time signal
$x_s(t)$	Sampled version of $x(t)$, i.e., $x(t)s(t)$
$x_r(t)$	Reconstruction of $x(t)$ from interpolation
$\tilde{x}[n]$	Periodic extension of the the aperiodic signal $x[n]$

2.6 Transformations

$\mathcal{F}\{\cdot\}$	Fourier transform (FT)
DTFT $\{\cdot\}$, DFS $\{\cdot\}$, FFT $\{\cdot\}$	Discrete-time Fourier Transform (DTFT), Discrete Fourier Transform (DFT), Discrete Fourier Series (DFS), respectively

$\mathcal{L}\{\cdot\}$	Laplace transform
$\mathcal{Z}\{\cdot\}$	z-transform
$\hat{x}(t), \hat{x}[n]$	Hilbert transform of $x(t)$ or $x[n]$
$X(s)$	Laplace transform of $x(t)$
$X(f)$	Fourier transform (FT) (in linear frequency, Hz) of $x(t)$
$X(j\omega)$	Fourier transform (FT) (in angular frequency, rad/sec) of $x(t)$
$X(e^{j\omega})$	Discrete-time Fourier transform (DTFT) of $x[n]$
$X[k], X(k), X_k$	Discrete Fourier transform (DFT) or fast Fourier transform (FFT) of $x[n]$, or even the Fourier series (FS) of the periodic signal $x(t)$
$\tilde{X}[k], \tilde{X}(k), \tilde{X}_k$	Discrete Fourier series (DFS) of $\tilde{x}[n]$
$X(z)$	z-transform of $x[n]$

3 Probability, statistics, and stochastic processes

3.1 Operators and symbols

$\mathbf{E}[\cdot], \mathbf{E}[\cdot] \text{ [29]}, E[\cdot], \mathbb{E}[\cdot]$	Statistical expectation operator [15]
$\mathbf{E}_u[\cdot], \mathbf{E}_u[\cdot] \text{ [29]}, E_u[\cdot], \mathbb{E}_u[\cdot]$	Statistical expectation operator with respect to u
$\langle \cdot \rangle$	Ensemble average
$\text{var}[\cdot], \text{VAR}[\cdot]$	Variance operator [8, 23, 28, 32]
$\text{var}_u[\cdot][\cdot], \text{VAR}_u[\cdot]$	Variance operator with respect to u
$\text{cov}[\cdot], \text{COV}[\cdot]$	Covariance operator [8]
$\text{cov}_u[\cdot], \text{COV}_u[\cdot]$	Covariance operator with respect to u
μ_x	Mean of the random variable x
$\mathbf{\mu}_x, \mathbf{m}_x$	Mean vector of the random variable \mathbf{x} [10]
μ_n	n th-order moment of a random variable
σ_x^2, κ_2	Variance of the random variable x
\mathcal{K}_x, μ_4	Kurtosis (4th-order moment) of the random variable x
κ_n	n th-order cumulant of a random variable
$\rho_{x,y}$	Pearson correlation coefficient between x and y

$a \sim P$	Random variable a with distribution P
\mathcal{R}	Rayleigh's quotient

3.2 Stochastic processes

$r_x(\tau), R_x(\tau)$	Autocorrelation function of the signal $x(t)$ or $x[n]$ [29]
$S_x(f), S_x(j\omega)$	Power spectral density (PSD) of $x(t)$ in linear (f) or angular (ω) frequency
$S_{x,y}(f), S_{x,y}(j\omega)$	Cross PSD of $x(t)$ and $y(t)$ in linear or angular (ω) frequency
\mathbf{R}_x	(Auto)correlation matrix of $\mathbf{x}(n)$
$r_{x,d}(\tau), R_{x,d}(\tau)$	Cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$ [29]
\mathbf{R}_{xy}	Cross-correlation matrix of $\mathbf{x}(n)$ and $\mathbf{y}(n)$
\mathbf{p}_{xd}	Cross-correlation vector between $\mathbf{x}(n)$ and $d(n)$ [dinizAdaptiveFiltering1997]
$c_x(\tau), C_x(\tau)$	Autocovariance function of the signal $x(t)$ or $x[n]$ [29]
$\mathbf{C}_x, \mathbf{K}_x, \mathbf{\Sigma}_x, \text{cov}[\mathbf{x}]$	(Auto)covariance matrix of \mathbf{x} [8, 23, 28, 32, 39]
$c_{xy}(\tau), C_{xy}(\tau)$	Cross-covariance function of the signal $x(t)$ or $x[n]$ [29]
$\mathbf{C}_{xy}, \mathbf{K}_{xy}, \mathbf{\Sigma}_{xy}$	Cross-covariance matrix of \mathbf{x} and \mathbf{y}

3.3 Functions

$Q(\cdot)$	Q -function, i.e., $P[\mathcal{N}(0, 1) > x]$ [32]
$\text{erf}(\cdot)$	Error function [32]
$\text{erfc}(\cdot)$	Complementary error function i.e., $\text{erfc}(x) = 2Q(\sqrt{2}x) - \text{erf}(x)$ [32]
$P[A]$	Probability of the event or set A [28]
$p(\cdot), f(\cdot)$	Probability density function (PDF) or probability mass function (PMF) [28]
$p(x A)$	Conditional PDF or PMF [28]
$F(\cdot)$	Cumulative distribution function (CDF)

$\Phi_x(\omega), M_x(j\omega), E[e^{j\omega x}]$	First characteristic function (CF) of x [32, 38]
$M_x(t), \Phi_x(-jt), E[e^{tx}]$	Moment-generating function (MGF) of x [32, 38]
$\Psi_x(\omega), \ln \Phi_x(\omega), \ln E[e^{j\omega x}]$	Second characteristic function
$K_x(t), \ln E[e^{tx}], \ln M_x(t)$	Cumulant-generating function (CGF) of x [23]

3.4 Distributions

$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution of a random variable with mean μ and variance σ^2
$\mathcal{CN}(\mu, \sigma^2)$	Complex Gaussian distribution of a random variable with mean μ and variance σ^2
$\mathcal{N}(\mathbf{\mu}, \mathbf{\Sigma})$	Gaussian distribution of a vector random variable with mean $\mathbf{\mu}$ and covariance matrix $\mathbf{\Sigma}$
$\mathcal{CN}(\mathbf{\mu}, \mathbf{\Sigma})$	Complex Gaussian distribution of a vector random variable with mean $\mathbf{\mu}$ and covariance matrix $\mathbf{\Sigma}$
$\mathcal{U}(a, b)$	Uniform distribution from a to b
$\chi^2(n), \chi_n^2$	Chi-square distribution with n degree of freedom (assuming that the Gaussians are $\mathcal{N}(0, 1)$)
$\text{Exp}(\lambda)$	Exponential distribution with rate parameter λ
$\Gamma(\alpha, \beta)$	Gamma distribution with shape parameter α and rate parameter β
$\Gamma(\alpha, \theta)$	Gamma distribution with shape parameter α and scale parameter $\theta = 1/\beta$
$\text{Nakagami}(m, \Omega)$	Nakagami-m distribution with shape parameter or fading figure m and spread, scale, or shape parameter Ω
$\text{Rayleigh}(\sigma)$	Rayleigh distribution with scale parameter σ
$\text{Rayleigh}(\Omega)$	Rayleigh distribution with the second moment $\Omega = E[x^2] = 2\sigma^2$
$\text{Rice}(s, \sigma)$	Rice distribution with noncentrality parameter s and σ . s^2 represent the specular component power

Rice(Ω, K), Rice(A, K)	Rice distribution with Rice factor $K = s^2/2\sigma^2$ and scale parameter $\Omega = A = s^2 + 2\sigma^2 = 2\sigma^2(K + 1)$ (Ω is preferred over A)
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4 Machine learning, optimization theory, and statistical signal processing

4.1 Matrix Calculus

$\mathbf{g}, \nabla f, \frac{\partial f}{\partial \mathbf{w}}$	Gradient descent vector, “used” in the steepest (or gradient) descent method
$\mathbf{g}_x, \nabla_{\mathbf{w}} f, \frac{\partial f}{\partial \mathbf{w}}$	Gradient descent vector with respect \mathbf{w} [8]
$\mathbf{J}, \frac{\partial \mathbf{y}^\top}{\partial \mathbf{x}}$	Jacobian matrix.
$\mathbf{H}, \frac{\partial^2 f}{\partial \mathbf{w}^2}, \nabla^2 f$ [22]	Hessian matrix. The notation ∇^2 is sometimes used in Matrix Calculus to denote the second-order vector derivative. However, it must be avoided since, in Vector Calculus, ∇^2 also denotes the Laplacian operator which in turn may be scalar or vector Laplacian operator depending on whether f is scalar- or vector-valued, respectively. Some discussion about can be found in [4–6]

4.2 Estimated terms

\mathbf{g} (or $\hat{\mathbf{g}}$ if the gradient vector is \mathbf{g})	Stochastic gradient descent (SGD), i.e., instantaneous approximation of gradient descent vector
$\hat{x}(t)$ or $\hat{x}[n]$	Estimate of $x(t)$ or $x[n]$
$\hat{\boldsymbol{\mu}}_x, \hat{\mathbf{m}}_x$	Sample mean of $x[n]$ or $x(t)$
$\hat{\boldsymbol{\mu}}_{\mathbf{x}}, \hat{\mathbf{m}}_{\mathbf{x}}$	Sample mean vector of $\mathbf{x}[n]$ or $\mathbf{x}(t)$
$\hat{r}_x(\tau), \hat{R}_x(\tau)$	Estimated autocorrelation function of the signal $x(t)$ or $x[n]$ [29]
$\hat{S}_x(f), \hat{S}_x(j\omega)$	Estimated power spectral density (PSD) of $x(t)$ in linear (f) or angular (ω) frequency

$\hat{\mathbf{R}}_{\mathbf{x}}$	Sample (auto)correlation matrix
$\hat{r}_{x,d}(\tau), \hat{R}_{x,d}(\tau)$	Estimated cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$
$\hat{S}_{x,y}(f), \hat{S}_{x,y}(j\omega)$	Estimated cross PSD of $x(t)$ and $y(t)$ in linear or angular (ω) frequency
$\hat{\mathbf{R}}_{\mathbf{xy}}$	Sample cross-correlation matrix of $\mathbf{R}_{\mathbf{xy}}$
$\hat{c}_x(\tau), \hat{C}_x(\tau)$	Estimated autocovariance function of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_{\mathbf{x}}, \hat{\mathbf{K}}_{\mathbf{x}}, \hat{\mathbf{\Sigma}}_{\mathbf{x}}$	Sample (auto)covariance matrix
$\hat{c}_{xy}(\tau), \hat{C}_{xy}(\tau)$	Estimated cross-covariance function of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_{\mathbf{xy}}, \hat{\mathbf{K}}_{\mathbf{xy}}, \hat{\mathbf{\Sigma}}_{\mathbf{xy}}$	Sample cross-covariance matrix
$\hat{\mathbf{H}}$	Estimate of the Hessian matrix

4.3 Signals, (hyper)parameters, system performance, and criteria

N	Number of instances (or samples), i.e., $n \in \{1, 2, \dots, N\}$
N_{trn}	Number of instances in the training set, i.e., $n \in \{1, 2, \dots, N_{\text{trn}}\}$
N_{tst}	Number of instances in the test set, i.e., $n \in \{1, 2, \dots, N_{\text{tst}}\}$
N_{val}	Number of instances in the validation set, i.e., $n \in \{1, 2, \dots, N_{\text{val}}\}$
N_e	Number of epochs
N_a	Number of attributes
K [8]	Number of classes (which is the number of outputs in multiclass problems). Use k to iterate over it
L	Number of layers. Use l to iterate over it
m_l [8], M_l , J [8]	Number of neurons at the l th layer. You might prefer J in the case of the single-layer perceptron (use j to iterate over it). If you want to iterate through it, a sensible variation of Haykin notation is M_l , where m_l can be used as an iterator. m_0 is the length of the input vector without the bias.

$\mathbf{x}(n), \mathbf{x}_n$	Input signal (in \mathbb{R}^{N_a+1})
$x_0(n)$	Dummy input of the bias, which is usually ± 1 . +1 is preferred [8, 22].
$\varphi(\cdot)$ [22], $h(\cdot)$ [8]	Activation function
$\varphi'(v_{m_l}^{(l)}(n))$ [22], $\frac{\partial y_{m_l}^{(l)}(n)}{\partial v_{m_l}^{(l)}(n)}$ [22]	Partial derivative of the activation function with respect to $v_{m_l}^{(l)}(n)$ (m_l neuron at l th layer)
$y_{m_l}^{(l)}(n), \varphi(v_{m_l}^{(l)}(n))$	Output signal of the m_l th neuron at the l th layer
$\mathbf{y}^{(l)}(n)$	Output signal of the l th layer
$\mathbf{y}(n), \mathbf{y}^{(L)}(n)$	Output of the neural network
$\mathbf{d}(n), \mathbf{d}_n$	Desired label (in case of supervised learning). For multiclass classification, one-hot encoding is usually used. For binary (scalar) classification, however antipodal encoding, i.e., $\{-1, 1\}$ is more recommended [22].
$e_{m_l}(n)$	Error signal of the neuron m_l at the l th layer
$\mathbf{e}(n), \mathbf{d}(n) - \mathbf{y}(n)$	Error signal
$\mathbf{w}_{m_l}^{(l)}(n), \boldsymbol{\theta}_{m_l}^{(l)}(n)$ $\begin{bmatrix} w_{m_l,0}^{(l)}(n) & w_{m_l,1}^{(l)}(n) & \dots & w_{m_l,m_{l-1}}^{(l)}(n) \end{bmatrix}$	Parameters, coefficients, or weights vector in the l th layer. In the case of Single Layer Perceptrons or adaptive filters, the superscript is omitted
$w_{m_l,0}^{(l)}(n), b_{m_l}^{(l)}(n)$	Bias (the first term of the weight vector) of the l th layer
$\mathbf{W}(n), [\mathbf{w}(1) \quad \mathbf{w}(2) \quad \dots \quad \mathbf{w}(N)]^T$	Matrix of the weights
$\mathbf{W}(n)$	Matrix of the weights, but without the bias
$v_{m_l}^{(l)}(n), \mathbf{w}_{m_l}^{(l)\top}(n) \mathbf{y}_{m_{l-1}}^{(l-1)}(n)$	Induced local field or activation potential. At the first layer $\mathbf{y}_{m_0}^{(0)}(n) = \mathbf{x}(n)$ [8]
$\mathbf{v}^{(l)}(n), \mathbf{W}^{(l)}(n) \mathbf{y}_{m_{l-1}}^{(l-1)}(n)$	Vector of the local fields at the l th layer
$\mathbf{w}^*, \mathbf{w}_o, \boldsymbol{\theta}^*, \boldsymbol{\theta}_o$	Optimum value of the parameters, coefficients, or weights vector (\mathbf{w}^* is also used [8] but it is not recommended as it may be confused with the conjugation operator)
$\delta_{m_l}^{(l)}(n), \frac{\partial \mathcal{E}(n)}{\partial v_{m_l}^{(l)}(n)}$	Local gradient of the m_l th neuron of the l th layer.

$\delta^{(l)}(n)$	Vector of the local gradients of all neurons at the l th layer
$\mathbf{X}, [\mathbf{x}(1) \ \mathbf{x}(2) \ \cdots \ \mathbf{x}(N)]$	Data matrix
$\eta(n)$	Learning rate hyperparameter [8]
\mathcal{R}	Bayes risk or average risk [8]
c_{ij}, C_{ij}	Misclassification cost in deciding in favor of class \mathcal{C}_i (represented in the subspace \mathcal{H}_i) when the \mathcal{C}_j is the true class (used in Bayes classifiers/detectors) [8, 11]
\mathcal{C}_k	k th class [8]
\mathcal{T}	Training set, i.e., the set $\{\mathbf{x}(n), d(n)\}$ that is used in the training phase [8]
\mathcal{H}_k	Subspace of the training vector belonging to the class \mathcal{C}_k
\mathcal{H}	Complete space of the input vector, i.e., $\mathcal{H}_1 \cup \mathcal{H}_2 \cup \cdots \mathcal{H}_K$
\mathcal{X} [22]	Set of all vectors in the training, batch, validation, or test dataset that was misclassified
$\mathcal{E}(\mathbf{w}), \mathcal{E}(\mathbf{w}(n)), \mathcal{E}(n)$	Cost function or objective function (the way it is written depends on the purpose of the text)
$J(\mathbf{w}), J(\mathbf{w}(n)), J(n)$	Alternative to the cost function
$\Delta\mathcal{E}(\mathbf{w}(n)), \Delta\mathcal{E}(n), \mathcal{E}(\mathbf{w}(n+1)) - \mathcal{E}(\mathbf{w}(n))$	Cost function or objective function (the way it is written depends on the purpose of the text)
$\mathcal{E}_{\text{av}}(\cdot)$	Error energy averaged over the training sample or the empirical risk [8]
$\Lambda(\cdot)$	Likelihood function
$\Lambda_l(\cdot)$	Log-likelihood function
$\hat{\rho}_{x,y}$	Estimated Pearson correlation coefficient between x and y
ρ	Distance of the margin of separation between two classes (Support Vector Machine, SVM)
$g(\cdot)$	Discriminant function, i.e., $g(\mathbf{w}^\star) = 0$

5 Linear Algebra

5.1 Common matrices and vectors

W, D	Diagonal matrix
P	Projection matrix; Permutation matrix
J	Jordan matrix
L	Lower matrix
U	Upper matrix
C	Cofactor matrix
C_A, cof (A)	Cofactor matrix of A
S	Symmetric matrix
Q	Orthogonal matrix
I_N	$N \times N$ -dimensional identity matrix
0_{M×N}	$M \times N$ -dimensional null matrix
0_N	N -dimensional null vector
1_{M×N}	$M \times N$ -dimensional ones matrix
1_N	N -dimensional ones vector
0	Null matrix, vector, or tensor (dimensionality understood by context)
1	Ones matrix, vector, or tensor (dimensionality understood by context)

5.2 Indexing

$x_{i_1, i_2, \dots, i_N}, [\mathcal{X}]_{i_1, i_2, \dots, i_N}$	Element in the position (i_1, i_2, \dots, i_N) of the tensor \mathcal{X}
$\mathcal{X}^{(n)}$	n th tensor of a nontemporal sequence
$\mathbf{x}_n, \mathbf{X}_{:n}$	n th column of the matrix X
\mathbf{x}_n	n th row of the matrix X
$\mathbf{x}_{i_1, \dots, i_{n-1}, :, i_{n+1}, \dots, i_N}$	Mode- n fiber of the tensor \mathcal{X}
$\mathbf{x}_{:, i_2, i_3}$	Column fiber (mode-1 fiber) of the thrid-order tensor \mathcal{X}
$\mathbf{x}_{i_1, :, i_3}$	Row fiber (mode-2 fiber) of the thrid-order tensor \mathcal{X}
$\mathbf{x}_{i_1, i_2, :}$	Tube fiber (mode-3 fiber) of the thrid-order tensor \mathcal{X}
$\mathbf{X}_{i_1, :, :}$	Horizontal slice of the thrid-order tensor \mathcal{X}
$\mathbf{X}_{:, i_2, :}$	Lateral slices slice of the thrid-order tensor \mathcal{X}
$\mathbf{X}_{i_3}, \mathbf{X}_{:, :, i_3}$	Frontal slices slice of the thrid-order tensor \mathcal{X}

5.3 General operations

$\langle \mathbf{a}, \mathbf{b} \rangle, \mathbf{a}^\top \mathbf{b}, \mathbf{a} \cdot \mathbf{b}$	Inner or dot product
$\mathbf{a} \circ \mathbf{b}, \mathbf{a} \mathbf{b}^\top$	Outer product
\otimes	Kronecker product
\odot	Hadamard (or Schur) (elementwise) product
$\cdot^{\odot n}$	n th-order Hadamard power
$\cdot^{\odot \frac{1}{n}}$	n th-order Hadamard root
\oslash	Hadamard (or Schur) (elementwise) division
\diamond	Khatri-Rao product
\otimes	Kronecker Product
\times_n	n -mode product

5.4 Operations with matrices and tensors

\mathbf{A}^{-1}	Inverse matrix
$\mathbf{A}^+, \mathbf{A}^\dagger$	Moore-Penrose left pseudoinverse
$\mathbf{A}^\top, \mathbf{A}^T, \mathbf{A}^t, \mathbf{A}'$ [35]	Transpose
$\mathbf{A}^{-\top}$	Transpose of the inverse, i.e., $(\mathbf{A}^{-1})^\top = (\mathbf{A}^\top)^{-1}$ [20, 31]
\mathbf{A}^*	Complex conjugate
\mathbf{A}^H	Hermitian
$\ \mathbf{A}\ _F$	Frobenius norm
$\ \mathbf{A}\ $	Matrix norm
$ \mathbf{A} , \det(\mathbf{A})$	Determinant
$\text{diag}(\mathbf{A})$	The elements in the diagonal of \mathbf{A}
$\mathbf{E}[\mathbf{A}]$	Vectorization: stacks the columns of the matrix \mathbf{A} into a long column vector
$\mathbf{E}_d[\mathbf{A}]$	Extracts the diagonal elements of a square matrix and returns them in a column vector
$\mathbf{E}_l[\mathbf{A}]$	Extracts the elements strictly below the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\mathbf{E}_u[\mathbf{A}]$	Extracts the elements strictly above the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\mathbf{E}_b[\mathbf{A}]$	Block vectorization operator: stacks square block matrices of the input into a long block column matrix

$\text{unvec}(\mathbf{A})$	Reshapes a column vector into a matrix
$\text{tr}\{\mathbf{A}\}$	trace
$\mathbf{X}_{(n)}$	n -mode matricization of the tensor \mathcal{X}

5.5 Operations with vectors

$\ \mathbf{a}\ $	l_1 norm, 1-norm, or Manhattan norm
$\ \mathbf{a}\ , \ \mathbf{a}\ _2$	l_2 norm, 2-norm, or Euclidean norm
$\ \mathbf{a}\ _p$	l_p norm, p -norm, or Minkowski norm
$\ \mathbf{a}\ _\infty$	l_∞ norm, ∞ -norm, or Chebyshev norm
$\text{diag}(\mathbf{a})$	Diagonalization: a square, diagonal matrix with entries given by the vector \mathbf{a}

5.6 Decompositions

$\mathbf{\Lambda}$	Eigenvalue matrix [37]
\mathbf{Q}	Eigenvectors matrix; Orthogonal matrix of the QR decomposition[37]
\mathbf{R}	Upper triangular matrix of the QR decomposition[37]
\mathbf{U}	Left singular vectors[37]
\mathbf{U}_r	Left singular nondegenerated vectors
$\mathbf{\Sigma}$	Singular value matrix
$\mathbf{\Sigma}_r$	Singular value matrix with nonzero singular values in the main diagonal
$\mathbf{\Sigma}^+$	Singular value matrix of the pseudoinverse [37]
$\mathbf{\Sigma}_r^+$	Singular value matrix of the pseudoinverse with nonzero singular values in the main diagonal
\mathbf{V}	Right singular vectors [37]
\mathbf{V}_r	Right singular nondegenerated vectors
$\text{eig}(\mathbf{A})$	Set of the eigenvalues of \mathbf{A} [12, 28, 31]
$\llbracket \mathbf{A}, \mathbf{B}, \mathbf{C}, \dots \rrbracket$	CANDECOMP/PARAFAC (CP) decomposition of the tensor \mathcal{X} from the outer product of column vectors of \mathbf{A} , \mathbf{B} , \mathbf{C}, \dots

$[[\boldsymbol{\lambda}; \mathbf{A}, \mathbf{B}, \mathbf{C}, \dots]]$	Normalized CANDECOMP/PARAFAC (CP) decomposition of the tensor \mathcal{X} from the outer product of column vectors of $\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$
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5.7 Spaces and sets

5.7.1 Common spaces and sets

\mathbb{R}	Set of real numbers
$[a, b]$	Closed interval of a real set from a to b
(a, b)	Opened interval of a real set from a to b
$[a, b), (a, b]$	Half-opened intervals of a real set from a to b
\mathbb{C}	Set of complex numbers
\mathbb{Z}	Set of integer number
$\{1, 2, \dots, n\}$	Discrete set containing the integer elements $1, 2, \dots, n$
$\mathbb{B} = \{0, 1\}$	Boolean set
\emptyset	Empty set
\mathbb{N}	Set of natural numbers
$\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$	Real or complex space (field)
$\mathbb{K}^{I_1 \times I_2 \times \dots \times I_N}$	$I_1 \times I_2 \times \dots \times I_N$ -dimensional real (or complex) space
\mathbb{K}_+	Nonnegative real (or complex) space [9]
\mathbb{K}_{++}	Positive real (or complex) space, i.e., $\mathbb{K}_{++} = \mathbb{K}_+ \setminus \{\mathbf{0}\}$ [9]
U	Universe
2^A	Power set of A

5.7.2 Convex sets (or spaces)

\mathbb{S}^n [14], \mathcal{S}^n [9]	Conic set of the symmetric matrices in $\mathbb{R}^{n \times n}$
$\mathbb{S}_+^n, \mathcal{S}_+^n$	Conic set of the symmetric positive semidefinite matrices in $\mathbb{R}^{n \times n}$ [9]
$\mathbb{S}_{++}^n, \mathcal{S}_{++}^n$	Conic set of the symmetric positive definite matrices in $\mathbb{R}^{n \times n}$, i.e., $\mathbb{S}_{++}^n = \mathbb{S}_+^n \setminus \{\mathbf{0}\}$ [9]

\mathbb{H}^n	Set of all hermitian matrices in $\mathbb{C}^{n \times n}$
$\text{conv } C$	Convex hull
$\text{aff } C$	Affine hull
\mathcal{R}	Ray
\mathcal{H}	Hyperplane
$\mathcal{H}_+, \mathcal{H}_-$	Positive/negative halfspace
$B(\mathbf{x}_c, r)$	Euclidean ball with radius r and centered at \mathbf{x}_c
\mathcal{E}	Ellipsoid
C	Norm cone
K	Proper cone
K^*	Dual cone
\mathcal{P}	Polyhedra
S	Simplex
C_α	α -sublevel set
$\text{epi } f$	Epigraph of the function f
$\text{hypo } f$	Hypograph of the function f

5.7.3 Spaces from matrices or vectors

$\text{span}\{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n\}$	Vector space spanned by the argument vectors [20]
$C(\mathbf{A}), \text{columnspace}(\mathbf{A}), \text{range}(\mathbf{A}), \text{span}\{\mathbf{A}\}, \text{image}(\mathbf{A})$	Columnspace, range or image, i.e., the space $\text{span}\{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n\}$, where \mathbf{a}_i is the i th column vector of the matrix \mathbf{A} [29, 37]
$C(\mathbf{A}^H)$	Row space (also called left column space) [29, 37]
$N(\mathbf{A}), \text{nullspace}(\mathbf{A}), \text{null}(\mathbf{A}), \text{kernel}(\mathbf{A})$	Nullspace (or kernel space) [29, 37, 38]
$N(\mathbf{A}^H)$	Left nullspace
$\text{rank } \mathbf{A}$	Rank, that is, $\dim(\text{span}\{\mathbf{A}\}) = \dim(C(\mathbf{A}))$ [29]
$\text{nullity}(\mathbf{A})$	Nullity of \mathbf{A} , i.e., $\dim(N(\mathbf{A}))$

5.8 Set operations

$A + B$	Set addition (Minkowski sum), i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} = \mathbf{x} + \mathbf{y}, \forall \mathbf{x} \in \mathcal{X} \wedge \mathbf{y} \in \mathcal{Y}\}$ [26]
$A - B$	Minkowski difference, i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} = \mathbf{x} - \mathbf{y}, \forall \mathbf{x} \in \mathcal{X} \wedge \mathbf{y} \in \mathcal{Y}\}$

$A \ominus B$	Pontryagin difference, i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} + \mathbf{y} \in \mathcal{X}, \forall \mathbf{y} \in \mathcal{Y}\}$ [26]
$A \setminus B, A - B$	Set difference or set subtraction, i.e., $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ the set containing the elements of A that are not in B [34]
$A \cup B$	Set of union
$A \cap B$	Set of intersection
$A \times B$	Cartesian product
A^n	$\underbrace{A \times A \times \cdots \times A}_{n \text{ times}}$
A^\perp	Orthogonal complement of A , e.g., $N(\mathbf{A}) = C(\mathbf{A}^\top)^\perp$ [9]
$\mathbf{a} \perp \mathbf{b}$	\mathbf{a} is orthogonal to \mathbf{b}
$\mathbf{a} \not\perp \mathbf{b}$	\mathbf{a} is not orthogonal to \mathbf{b}
$A \oplus B$	Direct sum, i.e., each $\mathbf{v} \in \{\sum \mathbf{a}_i \mid \mathbf{a}_i \in S_i, i = 1, \dots, k\}$ has a unique representation of $\sum \mathbf{a}_i$ with $\mathbf{a}_i \in S_i$. That is, they expand to a space. Note that $\{S_i\}$ might not be orthogonal each other [20]
$A \overset{\perp}{\oplus} B$	Direct sum of two spaces that are orthogonal and span a n -dimensional space, e.g., $C(\mathbf{A}^\top) \overset{\perp}{\oplus} C(\mathbf{A}^\top)^\perp = \mathbb{R}^n$ (this decomposition of \mathbb{R}^n is called the orthogonal decomposition induced by \mathbf{A}) [9]
\bar{A}, A^c	Complement set (given U)
$\#A, A $	Cardinality of A
$a \in A$	a is element of A
$a \notin A$	a is not element of A

5.9 Inequalities

$\mathcal{X} \leq 0$	Nonnegative tensor
$\mathbf{a} \preceq_K \mathbf{b}$	Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the conic subset K in the space \mathbb{R}^n [9]
$\mathbf{a} \prec_K \mathbf{b}$	Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the interior of the conic subset K in the space \mathbb{R}^n [9]

$\mathbf{a} \preceq \mathbf{b}$	Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the nonnegative orthant conic subset, \mathbb{R}_+^n , in the space \mathbb{R}^n . [9]
$\mathbf{a} \prec \mathbf{b}$	Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the positive orthant conic subset, \mathbb{R}_{++}^n , in the space \mathbb{R}^n [9]
$\mathbf{A} \preceq_K \mathbf{B}$	Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the conic subset K in the space \mathbb{S}^n [9]
$\mathbf{A} \prec_K \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the interior of the conic subset K in the space \mathbb{S}^n [9]
$\mathbf{A} \preceq \mathbf{B}$	Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive semidefinite conic subset, \mathbb{S}_+^n , in the space \mathbb{S}^n [9]
$\mathbf{A} \prec \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive orthant conic subset, \mathbb{S}_{++}^n , in the space \mathbb{S}^n [9]

6 Communication systems

6.1 Common symbols

B	One-sided bandwidth of the transmitted signal, in Hz
W	One-sided bandwidth of the transmitted signal, in rad/s
x_i	Real or in-phase part of x
x_q	Imaginary or quadrature part of x
f_C, f_{RF}	Carrier frequency (in Hertz)
f_L	Carrier frequency in L-band (in Hertz)
f_{IF}	Intermediate frequency (in Hertz)
f_s	Sampling frequency or sampling rate (in Hertz)
T_s	Sampling time interval/duration/period
R	Bit rate
T	Bit interval/duration/period

T_c	Chip interval/duration/period
T_{sy}, T_{sym}	Symbol/signaling[32] interval/duration/period
s_{RF}	Transmitted signal in RF
s_{FI}	Transmitted signal in FI
s, s_l	Lowpass (or baseband) equivalent signal or envelope complex of transmitted signal
r_{RF}	Received signal in RF
r_{FI}	Received signal in FI
r, r_l	Lowpass (or baseband) equivalent signal or envelope complex of received signal
ϕ	Signal phase
ϕ_0	Initial phase
η_{RF}, w_{RF}	Noise in RF
η_{FI}, w_{FI}	Noise in FI
η, w	Noise in baseband
τ	Timing delay
$\Delta\tau$	Timing error (delay - estimated)
φ	Phase offset
$\Delta\varphi$	Phase error (offset - estimated)
f_d	Linear Doppler frequency
Δf_d	Frequency error (Doppler frequency - estimated)
ν	Angular Doppler frequency
$\Delta\nu$	Frequency error (Doppler frequency - estimated)
γ, A	Transmitted signal amplitude
γ_0, A_0	Combined effect of the path loss and antenna gain

6.2 Fading multipath channels

$t \xleftrightarrow{\mathcal{F}} \lambda$ [32]	Support temporal of the signal. λ is obtained after taking the Fourier transform on t .
$\tau \xleftrightarrow{\mathcal{F}} f$ [32]	Second support temporal of the signal ($c(t)$ varies with the input at the time τ). f is obtained after taking the Fourier transform on τ .

$c(t, \tau)$ [32]	Complex envelope of the channel response at the time t due to an impulse applied at the $t - \tau$
$C(f, t)$ [32]	Transfer function of $c(t, \tau)$ in τ
$\alpha(t, \tau)$ [32]	Attenuation of $c(t, \tau)$, i.e., $c(t, \tau) = \alpha(t, \tau)e^{e\pi f_c \tau}$
$R_c(\tau_1, \tau_2, \Delta t)$ [32]	Autocorrelation function of $c(t, \tau)$, i.e., $R_c(\tau_1, \tau_2, \Delta t) = E [c^*(t, \tau_1), c^*(t + \Delta t, \tau_2)]$
$R_c(\tau, \Delta t)$ [32]	Autocorrelation function of $c(t, \tau)$ assuming uncorrelated scattering
$R_c(\tau), R_c(\tau, \Delta t) _{\Delta t=0}$ [32]	Multipath intensity profile or delay power spectrum
$R_C(\Delta f, \Delta t), R_C(f_1, f_2; \Delta t), E [C(f_1, t), C(f_2, t + \Delta t)], \mathcal{F}_\tau \{R_c(\tau, \Delta t)\}$ [19]	Spaced-frequency, spaced-time correlation function ($\Delta f = f_2 - f_1$)
$R_C(\Delta f), R_C(\Delta f, \Delta t) _{\Delta t=0} [32], \mathcal{F} \{R_c(\tau)\}$ [19]	Spaced-frequency correlation function
$(\Delta f)_c$	Coherence bandwidth of $c(t)$, that is, the frequency interval in which $R_C(\Delta f)$ is nonzero [32]
T_m	Multipath spread of the channel, that is, the time interval in which $R_c(\tau)$ is nonzero ($T_m \approx 1/(\Delta f)_c$) [32]
$R_C(\Delta t), R_C(\Delta f, \Delta t) _{\Delta f=0}$	Spaced-time correlation function [32]
$S_C(\lambda)$ [32], $\mathcal{F} \{R_C(\Delta t)\}$ [19]	Doppler power spectrum
$(\Delta t)_c$	Coherence time of $c(t)$, that is, the time interval in which $R_C(\Delta t)$ is nonzero [32]
B_m	Multipath spread of the channel, that is, the frequency interval in which $S_c(\lambda)$ is nonzero ($B_d \approx 1/(\Delta t)_c$) [32]
$S_C(\tau, \lambda)$ [32], $\mathcal{F}_{\Delta f, \Delta t} \{R_C(\Delta f, \Delta t)\}$ [19]	Scattering function

7 Discrete mathematics

7.1 Quantifiers, inferences

\forall	For all (universal quantifier) [21]
\exists	There exists (existential quantifier) [21]

\nexists	There does not exist [21]
$\exists!$	There exists an unique [21]
\exists_n	There exists exactly n [34]
\in	Belongs to [21]
\notin	Does not belong to [21]
\because	Because [21]
$, :$	Such that, sometimes that parentheses is used [21]
$,, (\cdot)$	Used to separate the quantifier with restricted domain from its scope, e.g., $\forall x < 0 (x^2 > 0)$ or $\forall x < 0, x^2 > 0$ [21]
\therefore	Therefore [21]

7.2 Propositional Logic

$\neg a$	Logical negation of a [34]
$a \wedge b$	Conjunction (logical AND) operator between a and b [34]
$a \vee b$	Disjunction (logical OR) operator between a and b [34]
$a \oplus b$	Exclusive OR (logical XOR) operator between a and b [34]
$a \rightarrow b$	Implication (or conditional) statement [34]
$a \leftrightarrow b$	Bi-implication (or biconditional) statement, i.e., $(a \rightarrow b) \wedge (b \rightarrow a)$ [34]
$a \equiv b, a \iff b, a \Leftrightarrow b$	Logical equivalence, i.e., $a \leftrightarrow b$ is a tautology [34]

7.3 Operations

$ a $	Absolute value of a
\log	Base-10 logarithm or decimal logarithm
\ln	Natural logarithm
$\operatorname{Re}\{x\}$	Real part of x
$\operatorname{Im}\{x\}$	Imaginary part of x
$\angle \cdot$	Phase (complex argument)
$x \bmod y$	Remainder, i.e., $x - y \lfloor x/y \rfloor$, for $y \neq 0$
$x \operatorname{div} y$	Quotient [34]

$x \equiv y \pmod{m}$	Congruent, i.e., $m \mid (x - y)$ [34]
$\text{frac}(x)$	Fractional part, i.e., $x \bmod 1$ [21]
$a \setminus b, a \mid b$	b is a positive integer multiple of a , i.e., $\exists n \in \mathbb{Z}_{++} \mid b = na$ [21, 34]
$a \nmid b, a \nsetminus b$	b is not a positive integer multiple of a , i.e., $\nexists n \in \mathbb{Z}_{++} \mid b = na$ [21, 34]
$\lceil \cdot \rceil$	Ceiling operation [21]
$\lfloor \cdot \rfloor$	Floor operation [21]

8 Vector Calculus

∇f [36], $\text{grad} f$ [33]	Vector differential operator (Nabla symbol), i.e., ∇f is the gradient of the scalar-valued function f , i.e., $f : \mathbb{R}^n \rightarrow \mathbb{R}$
$t, (u, v)$	Parametric variables commonly used, t for one variable, (u, v) for two variables[36]
$\mathbf{l}(x, y, z)$ [33], $\mathbf{r}(x, y, z)$ [36], $x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$	Vector position, i.e., (x, y, z) .
$\mathbf{l}(t)$	Vector position parametrized by t , i.e., $(x(t), y(t), z(t))$ [33, 36]
$\mathbf{l}'(t), d\mathbf{l}/dt$	First derivative of $\mathbf{l}(t)$, i.e., the tangent vector of the curve $(x(t), y(t), z(t))$ [36]
$\mathbf{u}(t)$ [27] $\mathbf{T}(t)$ [36], $d\mathbf{l}(t)$ [33]	Tangent unit vector of $\mathbf{l}(t)$, i.e., $\mathbf{u}(t) = \mathbf{l}'(t)/ \mathbf{l}'(t) $
$\mathbf{n}(t), \left(\frac{y'(t)}{ \mathbf{l}'(t) }, -\frac{x'(t)}{ \mathbf{l}'(t) } \right)$	Normal vector of $\mathbf{l}(t)$, i.e., $\mathbf{n}(t) \perp \mathbf{T}(t)$ [36]
C	Contour that traveled by $\mathbf{l}(t)$, for $a \leq t \leq b$ [36]
$L, L(C)$	Total length of the contour C (which can be defined the vector \mathbf{l} , parametrized by t), i.e., $L_C = \int_a^b \mathbf{l}'(t) dt$ [36]
$s(t)$	Length of the arc, which can be defined by the vector \mathbf{l} and t , that is, $s(t) = \int_a^t \mathbf{l}'(u) du$ ($s(b) = L$) [36]
ds	Differential operator of the length of the contour C , i.e., $ds = \mathbf{l}'(t) dt$ [36]

$\int_C f(\mathbf{l}) \, ds, \int_a^b f(\mathbf{l}(t)) \mathbf{l}'(t) \, dt$	Line integral of the function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ along the contour C . In the context of integrals in the complex plane, it is also called “contour integral”
θ [33]	Angle between the contour C and the vector field \mathbf{F}
$\int_C \mathbf{F} \cdot d\mathbf{l}, \int_a^b \mathbf{F}(\mathbf{l}(t)) \cdot \mathbf{l}'(t) \, dt$ [7, 36], $\int_C \mathbf{F} \cdot \mathbf{u} \, ds, \int_C \mathbf{F} \cos \theta \, ds$ [33] $\int_C \mathbf{F} \cdot d\mathbf{u}$ [33]	Line integral of vector field \mathbf{F} along the contour C In the field of electromagnetics, it is common to apply the line integral between the vector field \mathbf{F} and the unit vector $\mathbf{u}(t)$. Therefore, this line integral may appear as well
$\int_a^b \mathbf{F}, \int_a^b \mathbf{F} \cdot d\mathbf{l}$	Alternative notation to the line integral, where the parametric variable t goes from a to b , making r goes from $\mathbf{l}(a) = \mathbf{a}$ to $\mathbf{l}(b) = \mathbf{b}$ [7]
\oint_C, \oint_C	Line integral along the closed contour C . The arrow indicates the contour integral orientation, which is counter-clockwise, by default. In the context of integrals in the complex plane, it is also called “closed contour integral”.
\oint_S	Surface integral over the closed surface S
$\mathbf{l}(u, v)$	Vector position ($x(u, v), y(u, v), z(u, v)$) parametrized by (u, v)
\mathbf{l}_u	$(\partial x / \partial u, \partial y / \partial u, \partial z / \partial u)$
\mathbf{l}_v	$(\partial x / \partial v, \partial y / \partial v, \partial z / \partial v)$
dA	Differential operator of a 2D area (denoted by D or R) in the \mathbb{R}^2 domain. This differential operator can be solved in different ways (rectangular, polar, cylindric, etc) [36]
D, R	Integration domain in which dA is integrated, i.e., $\iint_D f \, dA$ [36]
S	Smooth surface S , i.e., a 2D area in a 3D space (\mathbb{R}^3 domain)

$dS, \mathbf{l}_u \times \mathbf{l}_v dA$	Differential operator of a 2D area in a 3D domain (an surface). Note that $dS = \mathbf{l}_u \times \mathbf{l}_v dA$ should be accompanied with the change of the integration interval(from S to D)
$A(S), \iint_S dS, \iint_D \mathbf{l}_u \times \mathbf{l}_v dA$	Area of the surface S parametrized by (u, v) , in which dA is the area defined in the D domain (which is form by the u -by- v graph)
dV	Differential operator of a shape volume (denoted by E) in \mathbb{R}^3 domain, i.e., $\iiint_E dV = V$
E	Integration domain in which dV is integrated, i.e., $\iiint_E f dV$ [36]
$V, \iint_D f dA, \iiint_E f dV$	Volume of the function f over the regions D (in the case of double integrals) or E (in the case of triple integrals)
$\iint_S f dS, \iint_D f \mathbf{l}_u \times \mathbf{l}_v dA$	Surface integral over S
$\mathbf{n}(u, v), \frac{\mathbf{l}_u(u, v) \times \mathbf{l}_v(u, v)}{ \mathbf{l}_u(u, v) \times \mathbf{l}_v(u, v) }$	Normal vector of of the smooth surface S
$\iint_S \mathbf{F} \cdot \mathbf{n} dS, \iint_S \mathbf{F} \cdot d\mathbf{S}, \iint_D \mathbf{F} \cdot (\mathbf{l}_u \times \mathbf{l}_v) dA$	Flux integral of vector field \mathbf{F} through the smooth surface S ($\mathbf{n} dS \triangleq d\mathbf{S}$)
$\oint_S \mathbf{F} \cdot \mathbf{n} dS, \oint_S \mathbf{F} \cdot d\mathbf{S}, \iint_D \mathbf{F} \cdot (\mathbf{l}_u \times \mathbf{l}_v) dA$	Flux integral of vector field \mathbf{F} through the smooth and closed surface S ($\mathbf{n} dS \triangleq d\mathbf{S}$)
$\nabla \times \mathbf{F}, \text{curl } \mathbf{F}$	Curl (rotacional) of the vector field \mathbf{F}
$\nabla \cdot \mathbf{F}, \text{div } \mathbf{F}$	Divergence of the vector field \mathbf{F}
$\nabla^2 f, \nabla \cdot (\nabla f), \Delta f, \partial^2 f / \partial x^2 + \partial^2 f / \partial y^2 + \partial^2 f / \partial z^2$	Scalar Laplacian operator (performed on a scalar-valued function $f : \mathbb{R}^n \rightarrow \mathbb{R}$)
$\nabla^2 \mathbf{F}, \nabla \times \nabla \times \mathbf{F} - \nabla(\nabla \cdot \mathbf{F}), \Delta \mathbf{F}, (\partial^2 \mathbf{F} / \partial x^2, \partial^2 \mathbf{F} / \partial y^2, \partial^2 \mathbf{F} / \partial z^2)$	Vector Laplacian operator (performed on a vector field, i.e., a vector-valued function, $\mathbf{F} : \mathbb{R}^n \rightarrow \mathbb{R}^n$). ∇^2 denotes the scalar (vector) Laplacian if the function is scalar-valued (vector-valued). The notation Δ must be avoided as it is overused in many contexts

9 Electromagnetic waves

Φ	Electric flux (scalar) (in V m)
\mathbf{J}	Electric current density vector (in A/m ²)
\mathbf{J}_m [13]	Magnetization current density vector (in A/m ²)
\mathbf{H}	Magnetic field vector (in A/m)
\mathbf{B}	Magnetic flux density vector (in Wb/m ² = T)
Φ [13]	Magnetic flux
q_{free}	Free electric charge (in C)
q_{bound}	Bound electric charge (in C)
$q, q_{\text{free}} + q_{\text{bound}}$	Electric charge (in C)
ρ_{free}	Free electric charge density
ρ_{bound}	Electric charge density
$\rho, \rho_{\text{free}} + \rho_{\text{bound}}$	Electric charge density (it can be in C/m ³ , C/m ² or C/m depending whether it is a volume, surface, or line shapes)
\mathbf{f} [33], \mathbf{F} [1]	Electrostatic force (Coulomb force), (in kg m/s ²).
ε	Electric permittivity(in F/m). If the medium is isotropic, it is a scalar. If it is anisotropic, it is a tensor. [33]
ε_r	Relative electric permittivity or dielectric constant (in F/m) [33]
ε_0	Electric permittivity in vacuum, 8.854×10^{-12} F/m [33]
\mathbf{E}	Electric field vector (in V/m)
\mathbf{D}	Electric flux density, electric displacement, or electric induction vector (in C/m ²)
U	Electric potential energy
V [2], Φ [33]	Electric potential (in voltage, V). However, keep in mind that there is a subtle difference between both definitions [3]
Φ_D [17], Ψ [33], $\oint_S \mathbf{D} \cdot d\mathbf{S}$	Electric flux (D -field flux)
Φ_E [18], $\oint_S \mathbf{E} \cdot d\mathbf{S}$	Electric flux (E -field flux)
\mathbf{P}	Electric polarization of the material (in C/m ²)
χ_e	Electric susceptibility (for linear and isotropic materials)
μ	Magnetic permeability
μ_0	Magnetic permeability in vacuum

10 Generic mathematical symbols

■	Q.E.D.
\triangleq	Equal by definition
$:=, \leftarrow$	Assignment [34]
\neq	Not equal
∞	Infinity
j	$\sqrt{-1}$

11 Abbreviations

PS: Only names of techniques and algorithms or usual abbreviations are considered.

wrt.	With respect to
st.	Subject to
iff.	If and only if
EVD	Eigenvalue decomposition, or eigen-decomposition [29]
SVD	Singular value decomposition
CP	CANDECOMP/PARAFAC
SGD	Stochastic gradient descent
SVM	Support vector machine
BPNN	Backpropagation neural network [25]
RBF	Radial basis function

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