Notation

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1 Font notation

$a, b, c, \ldots, A, B, C, \ldots$	Scalars
$\mathbf{a}, \mathbf{b}, \mathbf{c}, \ldots$	Vectors
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$	Matrices
$\boldsymbol{\mathcal{A}}, \boldsymbol{\mathcal{B}}, \boldsymbol{\mathcal{C}}, \ldots$	Tensors
$A, B, C, \ldots, \mathcal{A}, \mathcal{B}, \mathcal{C}, \ldots, \mathbb{A}, \mathbb{B}, \mathbb{C}, \ldots$	Sets

2 Signals and functions

2.1 Time indexing

$x(t)$	Continuous-time t
$x[n], x[k], x[m], x[i], \ldots$ $x_n, x_k, x_m, x_i, \ldots$ $x(n), x(k), x(m), x(i), \ldots$	Discrete-time $n, k, m, i, \ldots$ (parenthesis should be adopted only if there are no continuous-time signals in the context to avoid ambiguity)
$x\left[((n-m))_N\right], x\left((n-m)\right)_N$	Circular shift in m samples within a N-samples window [11, 15]

2.2 Common functions

$\delta(t)$	Delta function
$\delta[n], \delta_{i,j}$	Kronecker function $(n = i - j)$
$h(t), h[n]$	Impulse response (continuous and discrete time)

$\tilde{x}[n], \tilde{x}(t)$	Periodic discrete- or continuous-time signal
$\hat{x}[n], \hat{x}(t)$	Estimate of $x[n]$ or $x(t)$
$\dot{x}[m]$	Interpolation of $x[n]$

2.3 Operations and symbols

$f : A \to B$	A function f whose domain is A and codomain is B
$\mathbf{f} : A \to \mathbb{R}^n$	A vector-valued function $\mathbf{f}$, i.e., $n \geq 2$
$f^n, x^n(t), x^n[k]$	nth power of the function f, $x[n]$ or $x(t)$
$f^{(n)}, x^{(n)}(t)$	nth derivative of the function f or $x(t)$
$f', f^{(1)}, x'(t)$	1th derivative of the function f or $x(t)$
$f'', f^{(2)}, x''(t)$	2th derivative of the function f or $x(t)$
$\arg\max\limits_{x \in \mathcal{A}} f(x)$	Value of x that minimizes x
$\arg\min\limits_{x \in \mathcal{A}} f(x)$	Value of x that minimizes x
$f(\mathbf{x}) = \inf\limits_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Infimum, i.e., $f(\mathbf{x}) = \min\{g(\mathbf{x}, \mathbf{y}) \mid \mathbf{y} \in \mathcal{A} \wedge (\mathbf{x}, \mathbf{y}) \in \mathrm{dom}\,(g)\}$, which is the greatest lower bound of this set [3]
$f(\mathbf{x}) = \sup\limits_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Supremum, i.e., $f(\mathbf{x}) = \max\{g(\mathbf{x}, \mathbf{y}) \mid \mathbf{y} \in \mathcal{A} \wedge (\mathbf{x}, \mathbf{y}) \in \mathrm{dom}\,(g)\}$, which is the least upper bound of this set [3]
$f \circ g$	Composition of the functions f and g
$*$	Convolution (discrete or continuous)
$\circledast$, Ⓝ	Circular convolution [7, 15]

2.4 Transformations

W_N	Twiddle factor, $e^{-j\frac{2\pi}{N}}$ [11]
$\mathcal{F}\{\cdot\}$	Fourier transform
$\mathcal{L}\{\cdot\}$	Laplace transform
$\mathcal{Z}\{\cdot\}$	z-transform
$\hat{x}(t), \hat{x}[n]$	Hilbert transform of $x(t)$ or $x[n]$

$X(s)$	Laplace transform of $x(t)$
$X(f)$	Fourier transform (FT) (in linear frequency, Hz) of $x(t)$
$X(j\omega)$	Fourier transform (FT) (in angular frequency, rad/sec) of $x(t)$
$X(e^{j\omega})$	Discrete-time Fourier transform (DTFT) of $x[n]$
$X[k], X(k), X_k$	Discrete Fourier transform (DFT) or fast Fourier transform (FFT) of $x[n]$, or even the Fourier series (FS) of the periodic signal $x(t)$
$\tilde{X}[k], \tilde{X}(k), \tilde{X}_k$	Discrete Fourier series (DFS) of $\tilde{x}[n]$
$X(z)$	z-transform of $x[n]$

3 Probability, statistics, and stochastic processes

3.1 Operators and symbols

$\mathrm{E}[\cdot], \mathbf{E}[\cdot], E[\cdot], \mathbb{E}[\cdot]$	Statistical expectation operator [6, 14]
$\mathrm{E}_u[\cdot], \mathbf{E}_u[\cdot], E_u[\cdot], \mathbb{E}_u[\cdot]$	Statistical expectation operator with respect to u
$\langle\cdot\rangle$	Ensamble average
$\mathrm{var}[\cdot], \mathrm{VAR}[\cdot]$	Variance operator [2, 10, 13, 17]
$\mathrm{var}_u[\cdot][\cdot], \mathrm{VAR}_u[\cdot]$	Variance operator with respect to u
$\mathrm{cov}[\cdot], \mathrm{COV}[\cdot]$	Covariance operator [2]
$\mathrm{cov}_u[\cdot], \mathrm{COV}_u[\cdot]$	Covariance operator with respect to u
μ_x	Mean of the random variable x
$\boldsymbol{\mu}_\mathbf{x}, \mathbf{m}_\mathbf{x}$	Mean vector of the random variable $\mathbf{x}$ [4]
μ_n	nth-order moment of a random variable
σ_x^2, κ_2	Variance of the random variable x
$\mathcal{K}_x, \mu_4$	Kurtosis (4th-order moment) of the random variable x
κ_n	nth-order cumulant of a random variable
$\rho_{x,y}$	Pearson correlation coefficient between x and y
$a \sim P$	Random variable a with distribution P
$\mathcal{R}$	Rayleigh's quotient

3.2 Stochastic processes

$r_x(\tau), R_x(\tau)$	Autocorrelation function of the signal $x(t)$ or $x[n]$ [14]
$S_x(f), S_x(j\omega)$	Power spectral density (PSD) of $x(t)$ in linear (f) or angular (ω) frequency
$S_{x,y}(f), S_{x,y}(j\omega)$	Cross PSD of $x(t)$ and $y(t)$ in linear or angular (ω) frequency
$\mathbf{R}_{\mathbf{x}}$	(Auto)correlation matrix of $\mathbf{x}(n)$
$r_{x,d}(\tau), R_{x,d}(\tau)$	Cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$ [14]
$\mathbf{R}_{\mathbf{xy}}$	Cross-correlation matrix of $\mathbf{x}(n)$ and $\mathbf{y}(n)$
$\mathbf{p}_{\mathbf{x}d}$	Cross-correlation vector between $\mathbf{x}(n)$ and $d(n)$ [**dinizAdaptiveFiltering1997**]
$c_x(\tau), C_x(\tau)$	Autocovariance function of the signal $x(t)$ or $x[n]$ [14]
$\mathbf{C}_{\mathbf{x}}, \mathbf{K}_{\mathbf{x}}, \mathbf{\Sigma}_{\mathbf{x}}, \operatorname{cov}[\mathbf{x}]$	(Auto)covariance matrix of $\mathbf{x}$ [10, 13, 17, 22]
$c_{xy}(\tau), C_{xy}(\tau)$	Cross-covariance function of the signal $x(t)$ or $x[n]$ [14]
$\mathbf{C}_{\mathbf{xy}}, \mathbf{K}_{\mathbf{xy}}, \mathbf{\Sigma}_{\mathbf{xy}}$	Cross-covariance matrix of $\mathbf{x}$ and $\mathbf{y}$

3.3 Functions

$Q(\cdot)$	Q-function, i.e., $P[\mathcal{N}(0,1) > x]$ [17]
$\operatorname{erf}(\cdot)$	Error function [17]
$\operatorname{erfc}(\cdot)$	Complementary error function i.e., $\operatorname{erfc}(x) = 2Q(\sqrt{2}x) - \operatorname{erf}(x)$ [17]
$P[A]$	Probability of the event or set A [13]
$p(\cdot), f(\cdot)$	Probability density function (PDF) or probability mass function (PMF) [13]
$p(x \mid A)$	Conditional PDF or PMF [13]
$F(\cdot)$	Cumulative distribution function (CDF)
$\Phi_x(\omega), M_x(j\omega), E\left[e^{j\omega x}\right]$	First characteristic function (CF) of x [**theodoridisMachineLearningBayesian2020a**, 17]

$M_x(t), \Phi_x(-jt), E\left[e^{tx}\right]$	Moment-generating function (MGF) of x [**theodoridisMachineLearningBayesian2020a**, 17]
$\Psi_x(\omega), \ln\Phi_x(\omega), \ln E\left[e^{j\omega x}\right]$	Second characteristic function
$K_x(t), \ln E\left[e^{tx}\right], \ln M_x(t)$	Cumulant-generating function (CGF) of x [10]

3.4 Distributions

$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution of a random variable with mean μ and variance σ^2. The same notation can be used to denote a real-valued white Gaussian process with mean equal to μ and power spectral density equal to $N_0/2$, e.g., $s(t) \sim \mathcal{N}(\mu, N_0/2)$
$\mathcal{CN}(\mu, \sigma^2)$	Complex Gaussian distribution of a random variable with mean μ and variance σ^2. The same notation can be used to denote a complex-valued white Gaussian process with mean equal to μ and power spectral density equal to N_0, e.g., $s(t) \sim \mathcal{CN}(\mu, N_0)$
$\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	Gaussian distribution of a vector random variable with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$
$\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	Complex Gaussian distribution of a vector random variable with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$
$\mathcal{U}(a, b)$	Uniform distribution from a to b
$\chi^2(n), \chi^2_n$	Chi-square distribution with n degree of freedom (assuming that the Gaussians are $\mathcal{N}(0, 1)$)
$\mathrm{Exp}(\lambda)$	Exponential distribution with rate parameter λ
$\Gamma(\alpha, \beta)$	Gamma distribution with shape parameter α and rate parameter β
$\Gamma(\alpha, \theta)$	Gamma distribution with shape parameter α and scale parameter $\theta = 1/\beta$
$\mathrm{Nakagami}(m, \Omega)$	Nakagami-m distribution with shape parameter m and spread parameter Ω

$\mathrm{Rayleigh}(\sigma)$	Rayleigh distribution with scale parameter σ
$\mathrm{Rayleigh}(\boldsymbol{\Omega})$	Rayleigh distribution with the second moment $\boldsymbol{\Omega} = E\left[x^2\right] = 2\sigma^2$
$\mathrm{Rice}(s, \sigma)$	Rice distribution with noncentrality parameter (specular component) s and σ
$\mathrm{Rice}(A, K)$	Rice distribution with Rice factor $K = s^2/2\sigma^2$ and scale parameter $A = s^2 + 2\sigma^2$

4 Statistical signal processing

$\boldsymbol{\nabla} f, \mathbf{g}$	Gradient descent vector
$\boldsymbol{\nabla}_x f, \mathbf{g}_x$	Gradient descent vector with respect x
$\mathbf{g}$ (or $\hat{\mathbf{g}}$ if the gradient vector is $\mathbf{g}$)	Stochastic gradient descent (SGD)
$J(\cdot), \mathcal{E}(\cdot)$	Cost-function or objective function
$\Lambda(\cdot)$	Likelihood function
$\Lambda_l(\cdot)$	Log-likelihood function
$\hat{x}(t)$ or $\hat{x}[n]$	Estimate of $x(t)$ or $x[n]$
$\hat{\boldsymbol{\mu}}_x, \hat{\mathbf{m}}_x$	Sample mean of $x[n]$ or $x(t)$
$\hat{\boldsymbol{\mu}}_{\mathbf{x}}, \hat{\mathbf{m}}_{\mathbf{x}}$	Sample mean vector of $\mathbf{x}[n]$ or $\mathbf{x}(t)$
$\hat{r}_x(\tau), \hat{R}_x(\tau)$	Estimated autocorrelation function of the signal $x(t)$ or $x[n]$
$\hat{S}_x(f), \hat{S}_x(j\omega)$	Estimated power spectral density (PSD) of $x(t)$ in linear (f) or angular (ω) frequency
$\hat{\mathbf{R}}_{\mathbf{x}}$	Sample (auto)correlation matrix
$\hat{r}_{x,d}(\tau), \hat{R}_{x,d}(\tau)$	Estimated cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$
$\hat{S}_{x,y}(f), \hat{S}_{x,y}(j\omega)$	Estimated cross PSD of $x(t)$ and $y(t)$ in linear or angular (ω) frequency
$\hat{\mathbf{R}}_{\mathbf{xy}}$	Sample cross-correlation matrix of $\mathbf{R}_{\mathbf{xy}}$
$\hat{\rho}_{x,y}$	Estimated Pearson correlation coefficient between x and y
$\hat{c}_x(\tau), \hat{C}_x(\tau)$	Estimated autocovariance function of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_{\mathbf{x}}, \hat{\mathbf{K}}_{\mathbf{x}}, \hat{\boldsymbol{\Sigma}}_{\mathbf{x}}$	Sample (auto)covariance matrix

$\hat{c}_{xy}(\tau), \hat{C}_{xy}(\tau)$	Estimated cross-covariance function of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_{\mathbf{xy}}, \hat{\mathbf{K}}_{\mathbf{xy}}, \hat{\mathbf{\Sigma}}_{\mathbf{xy}}$	Sample cross-covariance matrix
$\mathbf{w}, \boldsymbol{\theta}$	Parameters, coefficients, or weights vector
$\mathbf{w}_o, \mathbf{w}^\star, \boldsymbol{\theta}_o, \boldsymbol{\theta}^\star$	Optimum value of the parameters, coefficients, or weights vector
$\mathbf{W}$	Matrix of the weights
$\mathbf{J}$	Jacobian matrix
$\mathbf{H}$	Hessian matrix
$\hat{\mathbf{H}}$	Estimate of the Hessian matrix

5 Linear Algebra

5.1 Common matrices and vectors

$\mathbf{W}, \mathbf{D}$	Diagonal matrix
$\mathbf{P}$	Projection matrix; Permutation matrix
$\mathbf{J}$	Jordan matrix
$\mathbf{L}$	Lower matrix
$\mathbf{U}$	Upper matrix
$\mathbf{C}$	Cofactor matrix
$\mathbf{C}_{\mathbf{A}}, \mathrm{cof}\,(\mathbf{A})$	Cofactor matrix of $\mathbf{A}$
$\mathbf{S}$	Symmetric matrix
$\mathbf{Q}$	Orthogonal matrix
$\mathbf{I}_N$	$N \times N$-dimensional identity matrix
$\mathbf{0}_{M\times N}$	$M \times N$-dimensional null matrix
$\mathbf{0}_N$	N-dimensional null vector
$\mathbf{1}_{M\times N}$	$M \times N$-dimensional ones matrix
$\mathbf{1}_N$	N-dimensional ones vector
$\mathbf{0}$	Null matrix, vector, or tensor (dimensionality understood by context)
$\mathbf{1}$	Ones matrix, vector, or tensor (dimensionality understood by context)

5.2 Indexing

$x_{i_1,i_2,\ldots,i_N}, [\boldsymbol{\mathcal{X}}]_{i_1,i_2,\ldots,i_N}$	Element in the position $(i_1, i_2, \ldots, i_N)$ of the tensor $\boldsymbol{\mathcal{X}}$
$\boldsymbol{\mathcal{X}}^{(n)}$	nth tensor of a nontemporal sequence

$\mathbf{x}_n, \mathbf{x}_{:n}$	nth column of the matrix X
$\mathbf{x}_{n:}$	nth row of the matrix X
$\mathbf{x}_{i_1,\dots,i_{n-1},:,i_{n+1},\dots,i_N}$	Mode-n fiber of the tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{x}_{:,i_2,i_3}$	Column fiber (mode-1 fiber) of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{x}_{i_1,:,i_3}$	Row fiber (mode-2 fiber) of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{x}_{i_1,i_2,:}$	Tube fiber (mode-3 fiber) of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{i_1,:,:}$	Horizontal slice of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{:,i_2,:}$	Lateral slices slice of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{i_3}, \mathbf{X}_{:,:,i_3}$	Frontal slices slice of the thrid-order tensor $\boldsymbol{\mathcal{X}}$

5.3 General operations

$\langle \mathbf{a}, \mathbf{b} \rangle, \mathbf{a}^\top \mathbf{b}, \mathbf{a} \cdot \mathbf{b}$	Inner product or dot product
$\circ$	Outer product, e.g., $\mathbf{a} \circ \mathbf{b} = \mathbf{a}\mathbf{b}^\top$
$\otimes$	Kronecker product
$\odot$	Hadamard (or Schur) (elementwise) product
$.^{\odot n}$	nth-order Hadamard power
$.^{\odot \frac{1}{n}}$	nth-order Hadamard root
$\oslash$	Hadamard (or Schur) (elementwise) division
$\diamond$	Khatri-Rao product
$\otimes$	Kronecker Product
$\times_n$	n-mode product

5.4 Operations with matrices and tensors

$\mathbf{A}^{-1}$	Inverse matrix
$\mathbf{A}^+, \mathbf{A}^\dagger$	Moore-Penrose left pseudoinverse
$\mathbf{A}^\top$	Transpose
$\mathbf{A}^{-\top}$	Transpose of the inverse, i.e., $\left(\mathbf{A}^{-1}\right)^\top = \left(\mathbf{A}^\top\right)^{-1}$ [8, 16]
$\mathbf{A}^*$	Complex conjugate
$\mathbf{A}^\mathsf{H}$	Hermitian
$\|\mathbf{A}\|_\mathrm{F}$	Frobenius norm
$\|\mathbf{A}\|$	Matrix norm

$\lvert\mathbf{A}\rvert$, $\det(\mathbf{A})$	Determinant
$\operatorname{diag}(\mathbf{A})$	The elements in the diagonal of $\mathbf{A}$
$\mathbf{E}[\mathbf{A}]$	Vectorization: stacks the columns of the matrix $\mathbf{A}$ into a long column vector
$\mathbf{E}_d[\mathbf{A}]$	Extracts the diagonal elements of a square matrix and returns them in a column vector
$\mathbf{E}_l[\mathbf{A}]$	Extracts the elements strictly below the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\mathbf{E}_u[\mathbf{A}]$	Extracts the elements strictly above the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\mathbf{E}_b[\mathbf{A}]$	Block vectorization operator: stacks square block matrices of the input into a long block column matrix
$\operatorname{unvec}(\mathbf{A})$	Reshapes a column vector into a matrix
$\operatorname{tr}\{\mathbf{A}\}$	trace
$\mathbf{X}_{(n)}$	n-mode matricization of the tensor $\boldsymbol{\mathcal{X}}$

5.5 Operations with vectors

$\lVert\mathbf{a}\rVert$	l_1 norm, 1-norm, or Manhatan norm
$\lVert\mathbf{a}\rVert$, $\lVert\mathbf{a}\rVert_2$	l_2 norm, 2-norm, or Euclidean norm
$\lVert\mathbf{a}\rVert_p$	l_p norm, p-norm, or Minkowski norm
$\lVert\mathbf{a}\rVert_\infty$	l_∞ norm, ∞-norm, or Chebyshev norm
$\operatorname{diag}(\mathbf{a})$	Diagonalization: a square, diagonal matrix with entries given by the vector $\mathbf{a}$

5.6 Decompositions

$\mathbf{\Lambda}$	Eigenvalue matrix [20]
$\mathbf{Q}$	Eigenvectors matrix; Orthogonal matrix of the QR decomposition[20]
$\mathbf{R}$	Upper triangular matrix of the QR decomposition[20]

$\mathbf{U}$	Left singular vectors[20]
$\mathbf{U}_r$	Left singular nondegenerated vectors
$\mathbf{\Sigma}$	Singular value matrix
$\mathbf{\Sigma}_r$	Singular value matrix with nonzero singular values in the main diagonal
$\mathbf{\Sigma}^+$	Singular value matrix of the pseudoinverse [20]
$\mathbf{\Sigma}_r^+$	Singular value matrix of the pseudoinverse with nonzero singular values in the main diagonal
$\mathbf{V}$	Right singular vectors [20]
$\mathbf{V}_r$	Right singular nondegenerated vectors
$\mathrm{eig}\left(\mathbf{A}\right)$	Set of the eigenvalues of $\mathbf{A}$ [5, 13, 16]
$[\![\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots]\!]$	CANDECOMP/PARAFAC (CP) decomposition of the tensor $\boldsymbol{\mathcal{X}}$ from the outer product of column vectors of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}, \ldots$
$[\![\boldsymbol{\lambda}; \mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots]\!]$	Normalized CANDECOMP/PARAFAC (CP) decomposition of the tensor $\boldsymbol{\mathcal{X}}$ from the outer product of column vectors of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}, \ldots$

5.7 Spaces

$\mathrm{span}\left\{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n\right\}$	Vector space spanned by the argument vectors [8]
$\mathrm{C}\left(\mathbf{A}\right)$, $\mathrm{columnspace}(\mathbf{A})$, $\mathrm{range}(\mathbf{A})$, $\mathrm{span}\left\{\mathbf{A}\right\}$, $\mathrm{image}(\mathbf{A})$	Columnspace, range or image, i.e., the space $\mathrm{span}\left\{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n\right\}$, where $\mathbf{a}_i$ is the ith column vector of the matrix $\mathbf{A}$ [14, 20]
$\mathrm{C}\left(\mathbf{A}^{\mathsf{H}}\right)$	Row space (also called left columnspace) [14, 20]
$\mathrm{N}\left(\mathbf{A}\right), \mathrm{nullspace}(\mathbf{A}), \mathrm{null}(\mathbf{A}), \mathrm{kernel}(\mathbf{A})$	Nullspace (or kernel space) [14, 20, 21]
$\mathrm{N}\left(\mathbf{A}^{\mathsf{H}}\right)$	Left nullspace
$\mathrm{rank}\,\mathbf{A}$	Rank, that is, $\dim\left(\mathrm{span}\left\{\mathbf{A}\right\}\right) = \dim\left(\mathrm{C}\left(\mathbf{A}\right)\right)$ [14]
$\mathrm{nullity}\left(\mathbf{A}\right)$	Nullity of $\mathbf{A}$, i.e., $\dim\left(\mathrm{N}\left(\mathbf{A}\right)\right)$
$\mathbf{a} \perp \mathbf{b}$	$\mathbf{a}$ is orthogonal to $\mathbf{b}$
$\mathbf{a} \not\perp \mathbf{b}$	$\mathbf{a}$ is not orthogonal to $\mathbf{b}$

5.8 Inequalities

$\mathcal{X} \leq 0$	Nonnegative tensor
$\mathbf{a} \preceq_K \mathbf{b}$	Generalized inequality meaning that $\mathbf{b}-\mathbf{a}$ belongs to the conic subset K in the space $\mathbb{R}^n$[3]
$\mathbf{a} \prec_K \mathbf{b}$	Strict generalized inequality meaning that $\mathbf{b}-\mathbf{a}$ belongs to the interior of the conic subset K in the space $\mathbb{R}^n$[3]
$\mathbf{a} \preceq \mathbf{b}$	Generalized inequality meaning that $\mathbf{b}-\mathbf{a}$ belongs to the nonnegative orthant conic subset, $\mathbb{R}^n_+$, in the space $\mathbb{R}^n$.[3]
$\mathbf{a} \prec \mathbf{b}$	Strict generalized inequality meaning that $\mathbf{b}-\mathbf{a}$ belongs to the positive orthant conic subset, $\mathbb{R}^n_{++}$, in the space $\mathbb{R}^n$[3]
$\mathbf{A} \preceq_K \mathbf{B}$	Generalized inequality meaning that $\mathbf{B}-\mathbf{A}$ belongs to the conic subset K in the space $\mathbb{S}^n$[3]
$\mathbf{A} \prec_K \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B}-\mathbf{A}$ belongs to the interior of the conic subset K in the space $\mathbb{S}^n$[3]
$\mathbf{A} \preceq \mathbf{B}$	Generalized inequality meaning that $\mathbf{B}-\mathbf{A}$ belongs to the positive semidefinite conic subset, $\mathbb{S}^n_+$, in the space $\mathbb{S}^n$[3]
$\mathbf{A} \prec \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B}-\mathbf{A}$ belongs to the positive orthant conic subset, $\mathbb{S}^n_{++}$, in the space $\mathbb{S}^n$[3]

6 Communication systems

B	One-sided bandwidth of the transmitted signal, in Hz
W	One-sided bandwidth of the transmitted signal, in rad/s
x_i	Real or in-phase part of x
x_q	Imaginary or quadrature part of x
f_c, f_{RF}	Carrier frequency (in Hertz)
f_L	Carrier frequency in L-band (in Hertz)

f_{IF}	Intermediate frequency (in Hertz)
f_s	Sampling frequency or sampling rate (in Hertz)
T_s	Sampling time interval/duration/period
R	Bit rate
T	Bit interval/duration/period
T_c	Chip interval/duration/period
T_{sy}, T_{sym}	Symbol/signaling[17] interval/duration/period
s_{RF}	Transmitted signal in RF
s_{FI}	Transmitted signal in FI
s, s_l	Lowpass (or baseband) equivalent signal or envelope complex of transmitted signal
r_{RF}	Received signal in RF
r_{FI}	Received signal in FI
r, r_l	Lowpass (or baseband) equivalent signal or envelope complex of received signal
ϕ	Signal phase
ϕ_0	Initial phase
η_{RF}, w_{RF}	Noise in RF
η_{FI}, w_{FI}	Noise in FI
η, w	Noise in baseband
τ	Timing delay
$\Delta\tau$	Timing error (delay - estimated)
φ	Phase offset
$\Delta\varphi$	Phase error (offset - estimated)
f_d	Linear Doppler frequency
Δf_d	Frequency error (Doppler frequency - estimated)
ν	Angular Doppler frequency
$\Delta\nu$	Frequency error (Doppler frequency - estimated)
γ, A	Transmitted signal amplitude
γ_0, A_0	Combined effect of the path loss and antenna gain

7 Discrete mathematics

7.1 Set theory

$A + B$	Set addition (Minkowski sum), i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} = \mathbf{x} + \mathbf{y},\ \forall\ \mathbf{x} \in \mathcal{X} \wedge \mathbf{y} \in \mathcal{Y}\}$ [12]
$A - B$	Minkowski difference, i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} = \mathbf{x} - \mathbf{y},\ \forall\ \mathbf{x} \in \mathcal{X} \wedge \mathbf{y} \in \mathcal{Y}\}$
$A \ominus B$	Pontryagin difference, i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} + \mathbf{y} \in \mathcal{X},\ \forall\ \mathbf{y} \in \mathcal{Y}\}$ [12]
$A \setminus B, A - B$	Set difference or set subtraction, i.e., $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ the set containing the elements of A that are not in B [18]
$A \cup B$	Set of union
$A \cap B$	Set of intersection
$A \times B$	Cartesian product
A^n	$\underbrace{A \times A \times \cdots \times A}_{n \text{ times}}$
$A^\perp$	Orthogonal complement of A, e.g., $\mathrm{N}(\mathbf{A}) = \mathrm{C}(\mathbf{A}^\top)^\perp$ [3]
$A \oplus B$	Direct sum, i.e., each $\mathbf{v} \in \{\sum \mathbf{a}_i \mid \mathbf{a}_i \in S_i, i = 1, \ldots, k\}$ has a unique representation of $\sum \mathbf{a}_i$ with $\mathbf{a}_i \in S_i$. That is, they expand to a space. Note that $\{S_i\}$ might not be orthogonal each other [8]
$A \overset{\perp}{\oplus} B$	Direct sum of two space that are orthogonal and span a n-dimensional space, e.g., $\mathrm{C}(\mathbf{A}^\top) \overset{\perp}{\oplus} \mathrm{C}(\mathbf{A}^\top)^\perp = \mathbb{R}^n$ (this decomposition of $\mathbb{R}^n$ is called the orthogonal decomposition induced by $\mathbf{A}$) [3]
$\bar{A}, A^c$	Complement set (given U)
$\#A, \lvert A \rvert$	Cardinality
$a \in A$	a is element of A
$a \notin A$	a is not element of A
$\{1, 2, \ldots, n\}$	Discrete set containing the integer elements $1, 2, \ldots, n$
U	Universe
2^A	Power set of A
$\mathbb{R}$	Set of real numbers
$\mathbb{C}$	Set of complex numbers
$\mathbb{Z}$	Set of integer number
$\mathbb{B} = \{0, 1\}$	Boolean set
$\varnothing$	Empty set
$\mathbb{N}$	Set of natural numbers

$\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$	Real or complex space (field)
$\mathbb{K}^{I_1 \times I_2 \times \cdots \times I_N}$	$I_1 \times I_2 \times \cdots \times I_N$-dimensional real (or complex) space
$\mathbb{K}_+$	Nonnegative real (or complex) space [3]
$\mathbb{K}_{++}$	Positive real (or complex) space, i.e., $\mathbb{K}_{++} = \mathbb{K}_+ \setminus \{\mathbf{0}\}$ [3]
$\mathbb{S}^n, \mathcal{S}^n$	Conic set of the symmetric matrices in $\mathbb{R}^{n\times n}$ [3]
$\mathbb{S}^n_+, \mathcal{S}^n_+$	Conic set of the symmetric positive semidefinite matrices in $\mathbb{R}^{n\times n}$ [3]
$\mathbb{S}^n_{++}, \mathcal{S}^n_{++}$	Conic set of the symmetric positive definite matrices in $\mathbb{R}^{n\times n}$, i.e., $\mathbb{S}^n_{++} = \mathbb{S}^n_+ \setminus \{\mathbf{0}\}$ [3]
$\mathbb{H}^n$	Set of all hermitian matrices in $\mathbb{C}^{n\times n}$
$[a, b]$	Closed interval of a real set from a to b
(a, b)	Opened interval of a real set from a to b
$[a, b), (a, b]$	Half-opened intervals of a real set from a to b

7.2 Quantifiers, inferences

$\forall$	For all (universal quantifier) [9]
$\exists$	There exists (existential quantifier) [9]
$\nexists$	There does not exist [9]
$\exists!$	There exist an unique [9]
$\in$	Belongs to [9]
$\notin$	Does not belong to [9]
$\because$	Because [9]
$\vert, :$	Such that, sometimes that parantheses is used [9]
$,, (\cdot)$	Used to separate the quantifier with restricted domain from the its scope, e.g., $\forall \quad x < 0\left(x^2 > 0\right)$ or $\forall \quad x < 0, x^2 > 0$ [9]
$\therefore$	Therefore [9]

7.3 Propositional Logic

$\neg a$	Logical negation of a [18]
$a \wedge b$	Conjunction (logical AND) operator between a and b[18]
$a \vee b$	Disjunction (logical OR) operator between a and b[18]
$a \oplus b$	Exclusive OR (logical XOR) operator between a and b[18]
$a \to b$	Implication (or conditional) statement[18]
$a \leftrightarrow b$	Bi-implication (or biconditional) statement, i.e., $(a \to b) \wedge (b \to a)$ [18]
$a \equiv b, a \iff b, a \Leftrightarrow b$	Logical equivalence, i.e., $a \leftrightarrow b$ is a tautology[18]

8 Physics

$\mathbf{E}$	Electric feild vector (in V/m)
$\mathbf{\Phi}$	Electric flux (scalar) (in V m)
$\mathbf{D}$	Electric flux density, electric displacement, or electric induction vector (in $\mathrm{C/m^2}$)
$\mathbf{J}$	Electric current density vector (in $\mathrm{A/m^2}$)
$\mathbf{H}$	Magnetic feild vector (in A/m)
$\mathbf{B}$	Magnetic flux density vector (in $\mathrm{Wb/m^2} = \mathrm{T}$)
ϵ	Electric permittivity
μ	Magnetic permeability
μ_0	Magnetic permeability in vacuum

9 Number theory, algorithm theory, and other notations

9.1 Mathematical symbols

$\blacksquare$	Q.E.D.
$\triangleq$	Equal by definition
$:=, \leftarrow$	Assignment [18]
$\neq$	Not equal
∞	Infinity
j	$\sqrt{-1}$

10 Calculus

∇	Nabla operator (vector differential operator), i.e., ∇f is the gradient of the scalar-valued function f, i.e., $f : \mathbb{R}^n \to \mathbb{R}$
$t, (u, v)$	Parametric variables commonly used, t for one variable, (u, v) for two variables
$\mathbf{r}(t)$	Vector position $(x(t), y(t), z(t))$ parametrized by t
$\mathbf{r}'(t)$	First derivative of $\mathbf{r}(t)$, i.e., the tangent vector of the curve $(x(t), y(t), z(t))$
$\mathbf{t}(t), \mathbf{T}(t)$	Unit tangent vector of $\mathbf{r}(t)$, i.e., $\mathbf{t}(t) = \mathbf{r}'(t)/\lvert\mathbf{r}'(t)\rvert$
$\mathbf{n}(t), \left(\frac{y'(t)}{\lvert\mathbf{r}'(t)\rvert}, -\frac{x'(t)}{\lvert\mathbf{r}'(t)\rvert}\right)$	Normal vector of $\mathbf{r}(t)$, i.e., $\mathbf{n}(t) \perp \mathbf{T}(t)$
C	Contour that traveled by $\mathbf{r}(t)$, for $a \leq t \leq b$
$L, L(C)$	Total length of the contour C (which can be defined the vector $\mathbf{r}$, parametrized by t), i.e., $L_C = \int_a^b \lvert\mathbf{r}'(t)\rvert\,\mathrm{d}t$
$s(t)$	Length of the arc, which can be defined by the vector $\mathbf{r}$ and t, that is, $s(t) = \int_a^t \lvert\mathbf{r}'(t)\rvert\,\mathrm{d}t$ $(s(b) = L)$[19]
$\mathrm{d}s$	Differential operator of the length of the contour C, i.e., $\mathrm{d}s = \lvert\mathbf{r}'(t)\rvert\,\mathrm{d}t$
$\int_C f(\mathbf{r})\,\mathrm{d}s$, $\int_a^b f(\mathbf{r}(t))\lvert\mathbf{r}'(t)\rvert\,\mathrm{d}t$	Line integral of the function $f : \mathbb{R}^n \to \mathbb{R}$ along the contour C [1, 19]
$\int_C \mathbf{F}\cdot\mathrm{d}\mathbf{r}$, $\int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,\mathrm{d}t$, $\int_C \mathbf{F}\cdot\mathbf{T}\,\mathrm{d}s$	Line integral of vector field $\mathbf{F}$ along the contour C [1, 19]
$\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{F}, \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{F}\cdot\mathrm{d}\mathbf{r}$	Alternative notation to the line integral, where the parametric variable t goes from a to b, making r goes from $\mathbf{r}(a) = \mathbf{a}$ to $\mathbf{r}(b) = \mathbf{b}$ [1]
$\oint_C, \oint_C$	Closed line integral along the contour C
$\mathbf{r}(u, v)$	Vector position $(x(u, v), y(u, v), z(u, v))$ parametrized by (u, v)
$\mathbf{r}_u$	$(\partial x/\partial u, \partial y/\partial u, \partial z/\partial u)$

$\mathbf{r}_v$	$(\partial x/\partial v, \partial y/\partial v, \partial z/\partial v)$
$\mathrm{d}A$	Differential operator of 2D area (D or R) in 2D space ($\mathbb{R}^2$ domain). This differential operator can be solved in different ways (rectangular, polar, cylindric, etc) [19]
D, R	Integration domain in which $\mathrm{d}A$ is integrated, i.e., $\iint_D f \,\mathrm{d}A$ [19]
S	Smooth surface S, i.e., a 2D area in a 3D space ($\mathbb{R}^3$ domain)
$\mathrm{d}S$, $\lvert\mathbf{r}_u \times \mathbf{r}_v\rvert \,\mathrm{d}A$	Differential operator of a 2D area in a 3D domain (an surface). Note that $\mathrm{d}S = \lvert\mathbf{r}_u \times \mathbf{r}_v\rvert \,\mathrm{d}A$ should be accompanied with the change of the integration interval(from S to D)
$A(S)$, $\iint_S \mathrm{d}S$, $\iint_D \lvert\mathbf{r}_u \times \mathbf{r}_v\rvert \,\mathrm{d}A$	Area of the surface S parametrized by (u, v), in which $\mathrm{d}A$ is the area defined in the D domain (which is form by the u-by-v graph)
V, $\iint_D f \,\mathrm{d}A$	Volume of the function f over the region D
$\iint_S f \,\mathrm{d}S$, $\iint_D f \lvert\mathbf{r}_u \times \mathbf{r}_v\rvert \,\mathrm{d}A$	Surface integral over S
$\mathbf{n}(u, v)$, $\frac{\mathbf{r}_u(u,v)\times\mathbf{r}_v(u,v)}{\lvert\mathbf{r}_u(u,v)\times\mathbf{r}_v(u,v)\rvert}$	Normal vector of of the smooth surface S
$\iint_S \mathbf{F}\cdot\mathbf{n}\,\mathrm{d}S$, $\iint_S \mathbf{F}\cdot\mathrm{d}\mathbf{S}$, $\iint_D \mathbf{F}\cdot(\mathbf{r}_u \times \mathbf{r}_v)\,\mathrm{d}A$	Flux integral of vector field $\mathbf{F}$ through the smooth surface S ($\mathbf{n}\,\mathrm{d}S \triangleq \mathrm{d}\mathbf{S}$)
$\nabla \times \mathbf{F}$, curl $\mathbf{F}$	Curl (rotacional) of the vector field $\mathbf{F}$
$\nabla \cdot \mathbf{F}$, div $\mathbf{F}$	Divercence of the vector field $\mathbf{F}$
$\nabla^2\mathbf{F}$	Laplacian operator

10.1 Operations

$\lvert a\rvert$	Absolute value of a
log	Base-10 logarithm or decimal logarithm
ln	Natual logarithm
$\mathrm{Re}\{x\}$	Real part of x
$\mathrm{Im}\{x\}$	Imaginary part of x
$\angle\cdot$	Phase (complex argument)
$x \bmod y$	Remainder, i.e., $x - y\lfloor x/y\rfloor$, for $y \neq 0$
x div y	Quotient [18]
$x \equiv y \pmod m$	Congruent, i.e., $m\backslash(x-y)$ [18]
$\mathrm{frac}\,(x)$	Fractional part, i.e., $x \bmod 1$ [9]

$a \backslash b$, $a \mid b$	b is a positive integer multiple of a, i.e., $\exists\, n \in \mathbb{Z}_{++} \mid b = na$ [9, 18]
$a \not\backslash b$, $a \nmid b$	b is not a positive integer multiple of a, i.e., $\nexists\, n \in \mathbb{Z}_{++} \mid b = na$ [9, 18]
$\lceil \cdot \rceil$	Ceiling operation [9]
$\lfloor \cdot \rfloor$	Floor operation [9]

10.2 Functions

$\mathcal{O}(\cdot), O(\cdot)$	Big-O notation
$\Gamma(\cdot)$	Gamma function
$Q(\cdot)$	Quantization function

11 Abbreviations

wrt.	With respect to
st.	Subject to
iff.	If and only if
EVD	Eigenvalue decomposition, or eigendecomposition [14]
SVD	Singular value decomposition
CP	CANDECOMP/PARAFAC

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