Tutorial 4

Detailed code measurements modelling

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August 2022

Detailed Computation of modeled pseudorange

Using files **UPC11490.050** and **UPC11490.05N** compute the SPP solution with **gLAB**.

Afterwards, calculate by hand the modelled C1 pseudo-range and the pre-fit residual for satellite PRN25 at time $\mathbf{t} = \mathbf{300}$ seconds of day 29 May 2005 (Day Of Year 149).

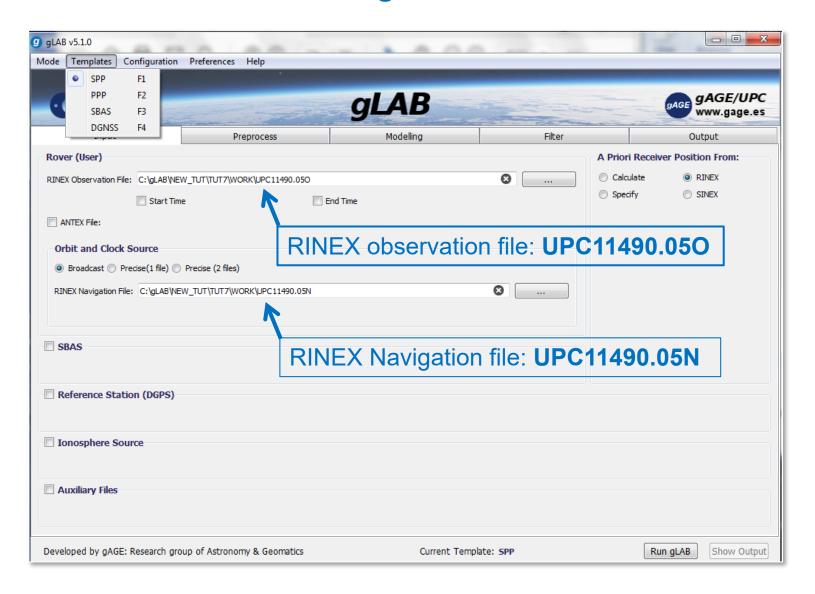
$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\overline{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

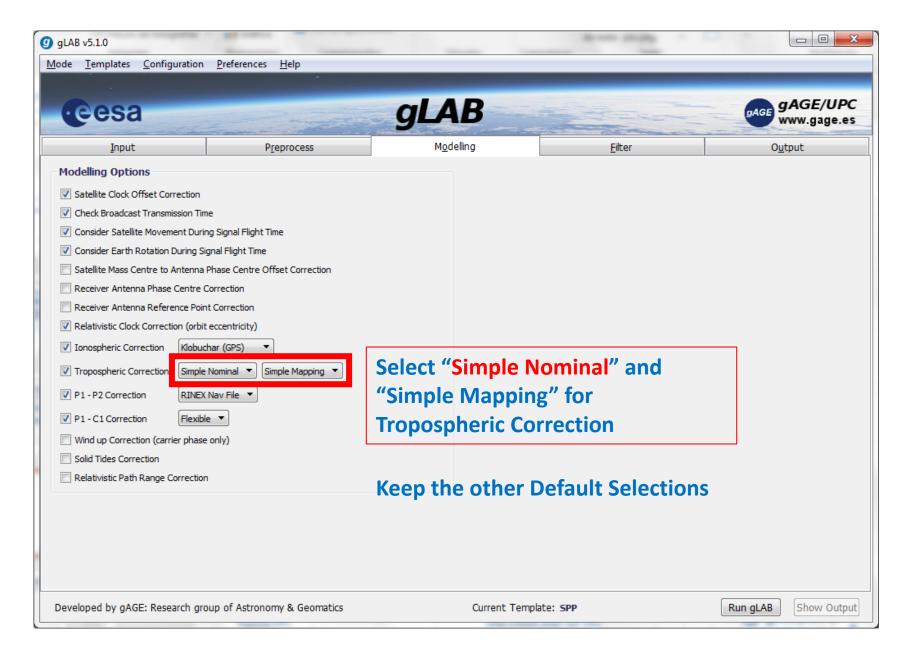
Compare the results with gLAB.

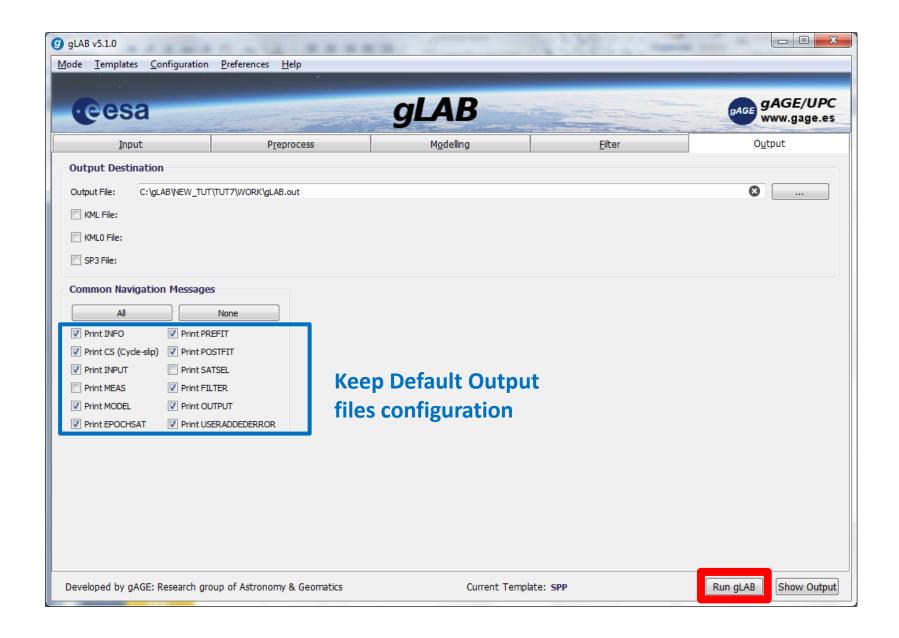
NOTE: use the Simple Nominal Model an Mapping for Tropospheric Correction.

Follow next steps:

Process the data files using the default SPP mode:



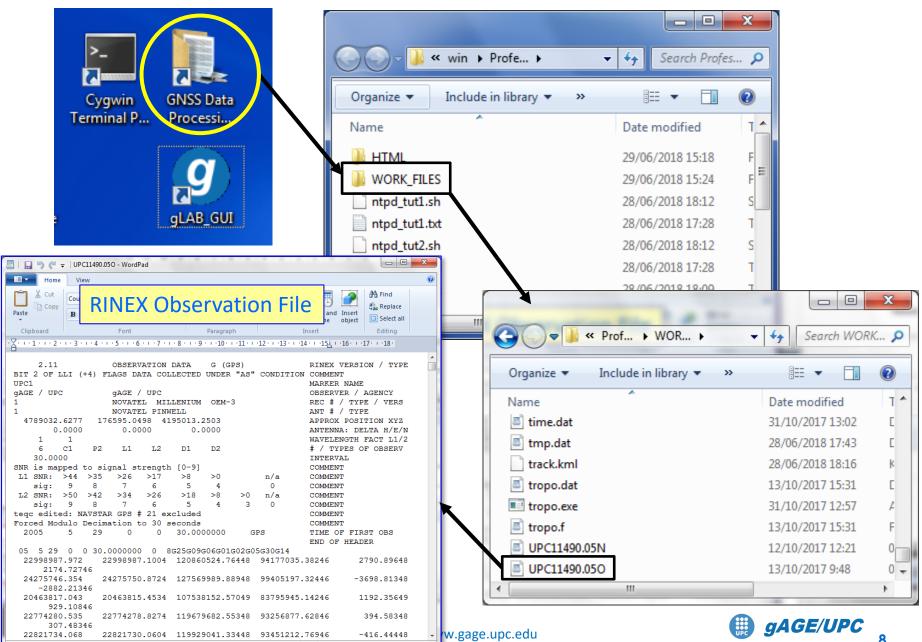




- O. Select pseudorange C1 for PRN25, at t=300 seconds.
- 1. Select orbital elements closest to t=300 seconds
- 2. Compute satellite clock offset
- 3. Compute satellite instrumental delay (TGD)
- 4. Compute satellite-receiver aprox. geometric range:
 - 4.1 Compute emission time from receiver (reception) time-tags and code pseudorange.
 - 4.2 Compute satellite coordinates at emission time
 - 4.3 Compute approximate geometric range.
- 5. Compute relativistic satellite clock correction
- 6. Compute ionospheric delay
- 7. Compute tropospheric delay
- 8. Compute modeled pseudorange from previous values:

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\overline{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

0. Select pseudorange C1 for PRN25, at t=300 seconds.



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0. Select pseudorange C1 for PRN25, at t=300 seconds.

From RINEX measurement file **UPC11490.050**, select the *C1* pseudorange measurement at receiver time-tag for PRN25:

from file UPC11490.050, C1 = 22857303.996 m at t = 300 s.

5 5 29 0 5	0.0000000 0 9	G <mark>25</mark> ; 9G 6G 1G21G	2G 5G30G14	
22857303.996	22857301.3054	120115969.49948	93596862.76546	2723.29048
2122.09146				
24466601.337	24466601.6684	128572940.94147	100186651.00844	-3729.38047
-2905.98944				
20405995.011	20405993.9894	107234297.78349	83559175.41846	1058.26649
824.62446				
22758443.914	22758442.9824	119596458.09448	93192027.40946	221.51848
172.61946				

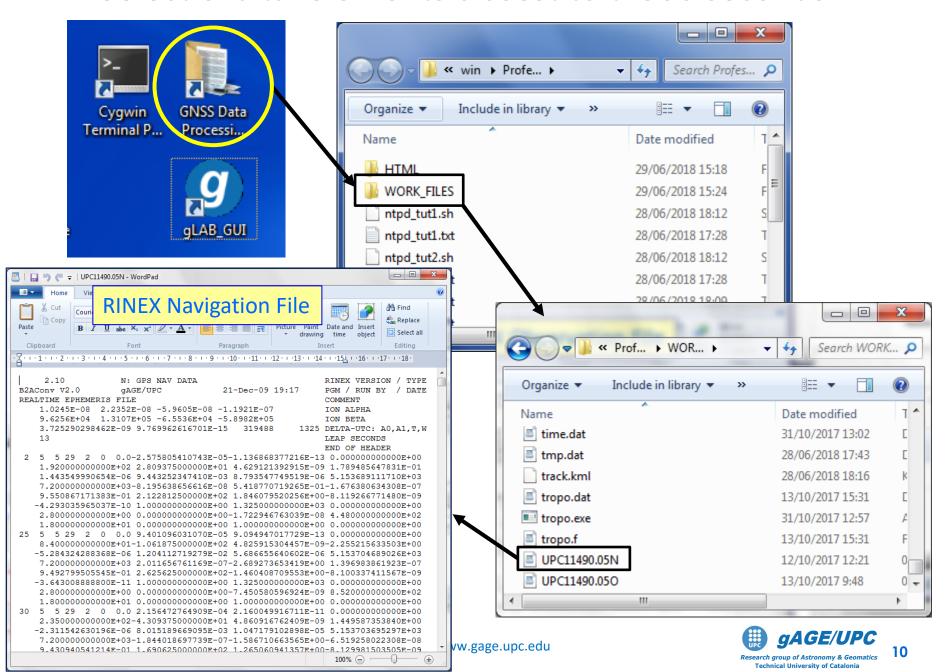
Thence:

Measurement file UPC11490.050



Pseudorange *C1* at receiver time-tag t=300: *C1*= 22857303.996 m

1.- Select orbital elements closest to t=300 seconds.



1. Selection of orbital elements:

For **PRN25**, select from file **UPC11490.05N** the <u>last transmitted</u> navigation message, <u>before t = 300 seconds</u> of DoY 149 of year 2005.

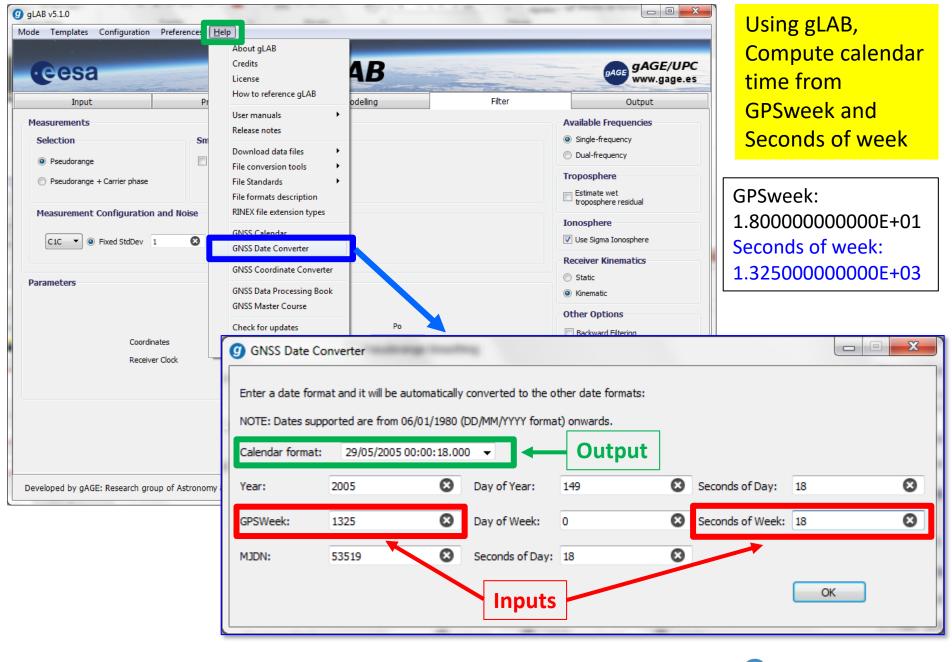
PRN

```
Transmission time:
```

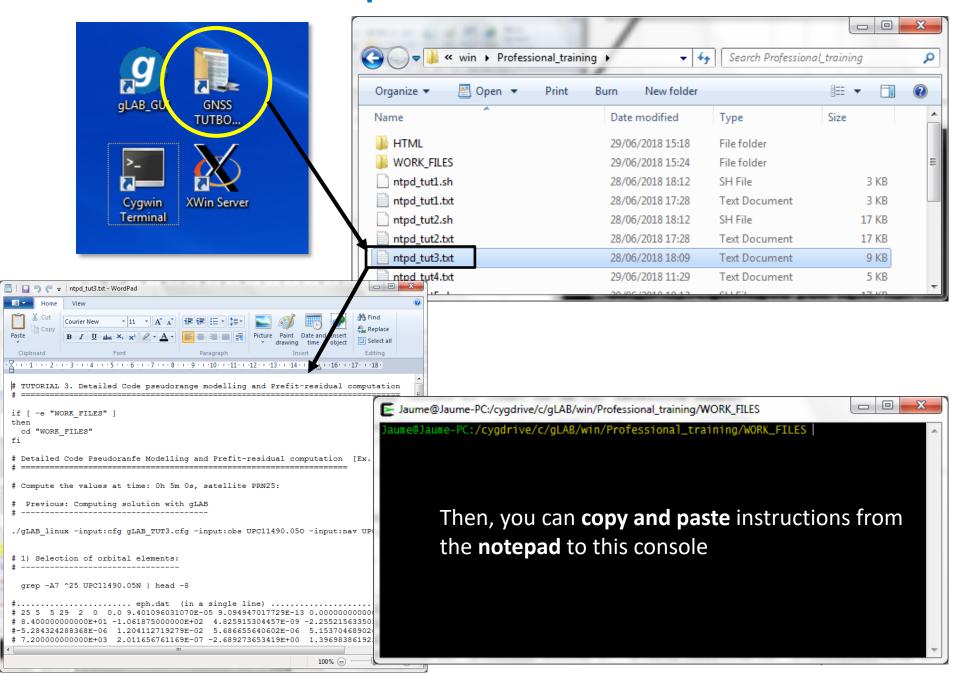
1325 18 → 2005/05/29 00:00:18

These data were transmitted by PRN25 at second 18 of GPS week 1325 (i.e. 1.80000000000E+01, 1.32500000000E+03 in the message).

The associated Y Y:MM:DD:hh:mm:ss with this transmission time can be computed using the GNSS Date Converter tool of gLAB as follows:



How to use the **notepad**?



2. Satellite clock offset computation:

From file **UPC11490.05N**, compute satellite clock offset at time **t=300 s** for **PRN25**:

PRN t_0 a_0 a_1 a_2

5 5 29 2 0 0.0 9.401096031070E-05 9.094947017729E-13 0.00000000000E+00 8.400000000000E+01-1.061875000000E+02 4.825915304457E-09-2.255215633503E+00 -5.284324288368E-06 1.204112719279E-02 5.686655640602E-06 5.153704689026E+03 7.20000000000E+03 2.011656761169E-07-2.689273653419E+00 1.396983861923E-07

9.492799505545E-01 2.625625000000E+02-1.460408709553E+00-8.100337411567E-09

-3.643008888800E-11 1.00000000000E+00 1.32500000000E+03 0.0000000000E+00

2.8000000000E+00 0.0000000000E+00-7.450580596924E-09 8.52000000000E+02

1.8000000000E+01 0.0000000000E+00 1.0000000000E+00 0.0000000000E+00

$$t = 300sec$$

 $t_0 = 2h \ Om \ Os = 7200 \ s$

<u>Last transmitted navigation</u> <u>message</u>, <u>before t = 300 seconds</u>

$$d\overline{t}^{sat} = a_0 + a_1(t - t_0) + a_2(t - t_0)^2 = 9.400 \cdot 10^{-5} \text{ s}$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(\overline{dt}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

```
gAGE@gAGE-PC:/cygdrive/c/gLAB/NEW_TUT/TUT7...
octave:1> format long
          c=299792458
octave:2>
c = 299792458
loctave:3> sec= 300
lsec = 300
octave:4> toc= 2*3600+ 0*60 +0
ltoc = 7200
octave:5> a0= 9.401096031070E-05
a0 = 9.40109603107000e-05
loctave:6> a1= 9.094947017729F-13
a1 = 9.09494701772900e-13
loctave:7> a2= 0
la2 = 0
octave:8> dt_sat0=a0+a1*(sec-toc)+a2*(sec-toc)**2
dt sat0 = 9.40046847972578e-05
loctave:9> c*dt sat0
ans = 28181.8955188851
octave:10>
```

Satellite clock offset computation with MATLAB (octave)

$$t_0 = 2 \text{ h } 0 \text{ min } 0 \text{ s} = 7200 \text{ s},$$

$$a_0 = 9.401096031070E-05$$
 $a_1 = 9.094947017729E-13$, $a_2 = 0.000000000000E+00$ (use also $c = 299792458 \,\mathrm{m/s}$).

$$d\overline{t}^{sat} = a_0 + a_1(t - t_0) + a_2(t - t_0)^2 = 9.400 \cdot 10^{-5} \text{ s}$$

Cross-checking results with gLAB

Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES

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To be a summary of the second of the second

$$d\overline{t}^{sat} = 9.40046848 \cdot 10^{-5} \,\mathrm{s} \implies -c \,d\overline{t}^{sat} = -28181.89551 \mathrm{m}.$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\overline{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

3. Satellite Instrumental delay (TGD)

From file UPC11490.05N, compute the Total Group Delay for PRN25:

PRN

TGD (in sec)

TGD= -7.450580596924E-09 * c= -2.23363m

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\overline{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

Cross-checking results with gLAB

TGD =
$$-7.450580596924E-09$$
 (in seconds)

Thus:
$$TGD = -7.450580596924E - 09 \times c = -2.23363 \,\mathrm{m}$$
.

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\overline{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

4. Satellite-receiver geometric range computation:

Use the following values (4789032.6277, 176595.0498, 4195013.2503) as approximate coordinates.

4.1: Emission time computation from receiver time-tag and code

pseudorange:

$$T[ems] = t_{rec}(T_R) - (C1/c + dt^{sat})$$

Measurement file UPC11490.050



Pseudorange *C1* at receiver time-tag t=300: *C1*= 22857303.996 m

Ephemeris file UPC11490.05N



Satellite clock offset at t=38230 s dt^{sat} = 9.40046848e-05 s

(see previous results)

Thence, the emission time in GPS system time is:

$$T[ems] = 300s - (22857303.996m /c + 9.40 10^{-5}s)$$

= 299.923662236054s (where c=299792458 m/s)

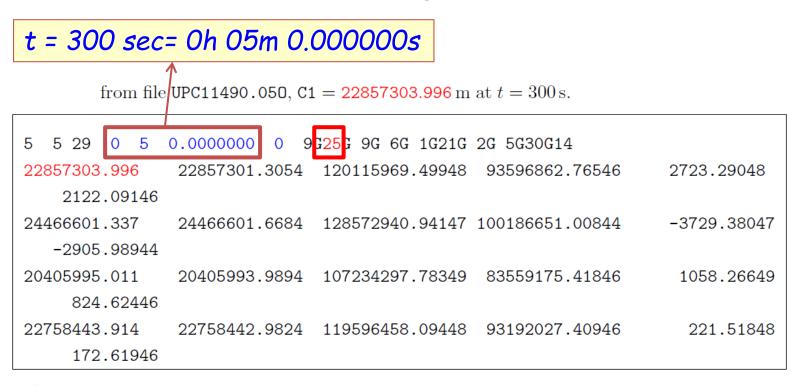
Measurement file UPC11490.050

t = 300 sec = 0h 05m 0.000000s

■ Jaume@Portatil_Jaume:/cygdrive/c/gLAB/win/Professional_training/W	/O □ ×				
05 5 29 0 5 0.0000000 0 G25 09G06G01G02G05G30G14	2722 20040				
22857303.996 22857301.3054 120115969.49948 93596862.76546	2723.29048				
2122.09146	2720 28047				
	-3729.38047				
-2905.98944 20405995.011 20405993.9894 107234297.78349 83559175.41846	1059 26640				
824.62446	1058.26649				
22758443.914 22758442.9824 119596458.09448 93192027.40946	221.51848				
172.61946	221.31040				
22847797.979 22847793.9524 120066006.91748 93557939.31646	-597.92448				
-465.90346	-337 : 32440				
	-2309.52148				
-1799.62646	2303132210				
20171035.530 20171033.5794 105999650.93349 82597114.84546	-377.07249				
-293.81446					
	-2193.61648				
-1709.30346					
05 5 29 0 5 30.0000000 0 8G25G09G06G01G02G05G30G14					
22841780.362 22841777.9824 120034393.69248 93533297.24445	2715.16248				
2115.71845					
24487903.545 24487901.6274 128684880.96348 100273876.88342	-3732.94148				
	V				

Note:

From RINEX measurement file **UPC11490.050**, select the *C1* pseudorange measurement at receiver time-tag for PRN25:



Thence:

Measurement file UPC11490.050



Pseudorange *C1* at receiver time-tag t=300: *C1*= 22857303.996 m

$T[ems] = t_{rec}(T_R) - (C1/c + dt^{sat})$

Measurement file **UPC11490.050**



Pseudorange *C1* at receiver time-tag t=300: *C1*= 22857303.996 m

Ephemeris file **UPC11490.05N**



Satellite clock offset at t=300 s

 dt^{sat} = 9.40046848e-05 s

Thence, the emission time in GPS satellite clock is:

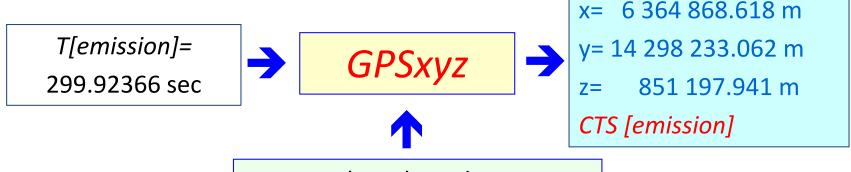
 $T[ems] = 300s - (22857303.996 \text{ m}/c + 9.40 10^{-5}\text{s})$

= 299.923662236054s (where c=299792458 m/s)

```
octave:1> format long
octave:2> c=299792458
c = 299792458
octave:3> sec= 300
sec = 300
octave:4> dt_sat0= 9.40046848e-05
dt_sat0 = 9.40046848000000e-05
octave:5> C1=22857303.996
C1 = 22857303.9960000
octave:6> sec_ems=sec-C1/c-dt_sat0
sec_ems = 299.923662236054
octave:7>
```

Emission time computation with MATLAB (octave)

4.2: Satellite coordinates at emission time pseudorange:

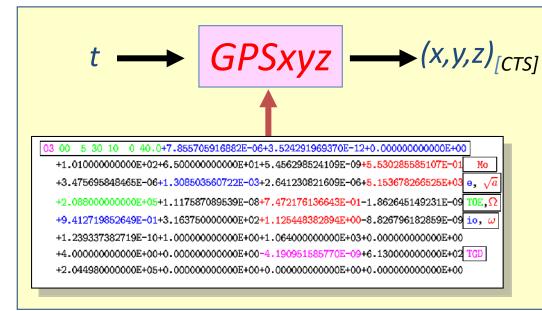


Use the selected ephemeris for PRN25 (from file UPC11490.05N)

The obtained coordinates are given in an Earth-fixed reference frame (CTS) at t=T[emission]=299.92366 s.

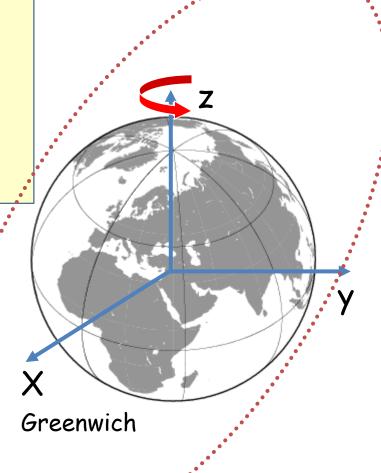
This reference frame rotates by un amount " $\omega_E \Delta t$ " during traveling time $\Delta t = T[reception] - T[emission]$.

$$(x^{\text{sat}}, y^{\text{sat}}, z^{\text{sat}})_{\text{CTS[reception]}} = R_3(\omega_E \Delta t).(x^{\text{sat}}, y^{\text{sat}}, z^{\text{sat}})_{\text{CTS[emission]}}$$



Conventional Terrestrial Reference System (CTS):

Earth Centered, Earth-Fixed (ECEF) → the reference system rotates with Earth.



(x,y,z)

Computation of satellite coordinates from navigation message (GPSxyz.f)

• Computation of t_k time since ephemerids reference epoch t_{oe} (t and t_{oe} are given in GPS seconds of week):

$$t_k = t - t_{oe}$$

• Computation of mean anomaly M_k for t_k ,

$$M_k = M_0 + \left(\frac{\sqrt{\mu}}{\sqrt{a^3}} + \Delta n\right) t_k$$

• Iterative resolution of Kepler's equation in order to compute eccentric anomaly E_{ν} :

$$M_k = E_k - e \sin E_k$$

• Calculation of true anomaly v_k :

$$v_k = \arctan\left(\frac{\sqrt{1 - e^2} \sin E_k}{\cos E_k - e}\right)$$

• Computation of latitude argument u_k from perigee argument W, true anomaly v_k and corrections c_{uc} and c_{us} :

$$u_k = \omega + v_k + c_{uc} \cos 2(\omega + v_k) + c_{us} \sin 2(\omega + v_k)$$

Computation of satellite coordinates from navigation message (GPSxyz.f)

• Computation of radial distance r_k , taking into consideration corrections c_{rc} and c_{rs} :

$$r_k = a(1 - 2\cos E_k) + c_{rc}\cos 2(\omega + v_k) + c_{rs}\sin 2(\omega + v_k)$$

• Calculation of orbital plane inclination i_k from inclination i_o at reference epoch t_{oe} and corrections c_{ic} and c_{is} :

$$i_k = i_0 + it_k + c_{ic}\cos 2(\omega + v_k) + c_{is}\sin 2(\omega + v_k)$$

• Computation of ascending node longitude Ω_k (Greenwich), from longitude Ω_0 at start of GPS week, corrected from apparent variation of sidereal time at Greenwich between start of week and and reference time t_k =t- t_{oe} , and also corrected from change of ascending node longitude since reference epoch t_{oe} .

$$\Omega_{k} = \Omega_{0} + (\Omega - \omega_{E})t_{k} - \omega_{E}t_{oe}$$

• Calculation of coordinates in CTS system, applying three rotations (around u_k , i_k , Ω_k):

$$\begin{bmatrix} \mathbf{X}_k \\ \mathbf{Y}_k \\ \mathbf{Z}_k \end{bmatrix} = \mathbf{R}_3(-\Omega_k)\mathbf{R}_1(-i_k)\mathbf{R}_3(-u_k) \begin{bmatrix} r_k \\ 0 \\ 0 \end{bmatrix}$$

Computation of satellite coordinates in an Earth-Fixed reference frame

(CTS) at t=T[emission]=299.92366 s.

```
echo "2005 149 299.92366224" > time.dat cat time.dat eph.dat | GPSxyz
```

Note: use the file "eph . dat" (with the selected Nav. Message)

Result: [25. 299.92366224 ← time

 $6364868.618075 - 14298233.062153 21851197.940638 \leftarrow coord$

3.022976 ← Excentric anomaly Ek

```
Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES

Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES echo "2005 149 299.92366224" > time.dat Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES cat time.dat eph.dat | GPSxyz 25. 299.92366224 6364868.618075 -14298233.062153 21851197.940638 3.022976

Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES |
```

These coordinates are given in an t=T[emission] = 299.92366 s, i.e CTS [emission]

Next step is to transform these coordinates to CTS [reception]

$$(x^{\text{sat}}, y^{\text{sat}}, z^{\text{sat}})_{\text{CTS[reception]}} = R_3(\omega_E \Delta t).(x^{\text{sat}}, y^{\text{sat}}, z^{\text{sat}})_{\text{CTS[emission]}}$$

x= 6 364 868.618 m

y= -14 298 233.062 m

7= 21851 197.941 m

CTS [emission]

$$(x^{\text{sat}}, y^{\text{sat}}, z^{\text{sat}})_{\text{CTS[reception]}} = R_3(\omega_E \Delta t).(x^{\text{sat}}, y^{\text{sat}}, z^{\text{sat}})_{\text{CTS[emission]}}$$

$$\begin{pmatrix} 6364789.025 \\ -14298268.493 \\ 21851197.941 \end{pmatrix} = \begin{pmatrix} \cos(\omega_E \Delta t) & \sin(\omega_E \Delta t) & 0 \\ -\sin(\omega_E \Delta t) & \cos(\omega_E \Delta t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 6 & 364 & 868.618 \\ -14 & 298 & 233.062 \\ 21851 & 197.941 \end{pmatrix}$$

$$CTS[reception]$$

$$CTS[emission]$$

$$(x, y, z)^{satellite} \approx (6364868.618 , -14298233.062 , 21851197.940)$$

$$(x_0, y_0, z_0)_{receiver} \approx (4789032.628, 176595.050, 4195013.250)$$

$$\rho_{0,rec}^{sat} = \sqrt{\left(x^{sat} - x_{0,rec}\right)^2 + \left(y^{sat} - y_{0,rec}\right)^2 + \left(z^{sat} - z_{0,rec}\right)^2} \approx 22885470.626m$$

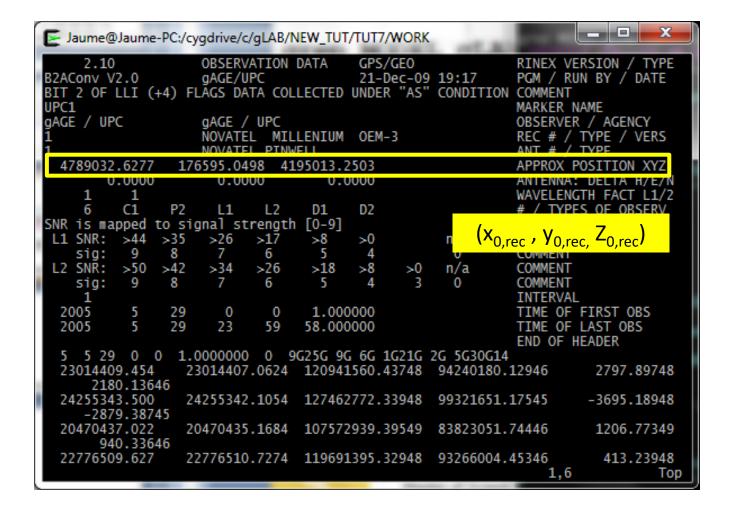
$$\Delta t = \frac{\rho_{0,rec}^{sat}}{c} = 0.0763 \, \text{sec}$$

$$\omega_E \Delta t = 5.56 \cdot 10^{-6} \, rad. \quad (where \quad \omega_E = 7.2921151467 \cdot 10^{-5} \, rad \, / \, \text{sec})$$

An approximate value is enough to compute Δt .

Note: Both satellite and receiver coordinates must be given in the same reference system!

→ the CTS[reception] will be used to build navigation equations.



Approximate Receiver coordinates given in the RINEX file header

 $x_{0,rec} = 4789032.6277$ $y_{0,rec} = 176595.0498$ $z_{0,rec} = 4195013.2503$

Transformation of satellite coordinates form CTS [emission] to CTS [reception]

with MATLAB (octave)

```
r0 rcv=[ 4789032.6277
                                                                    _ D X
                                                                                                            176595,0498
gAGE@gAGE-PC:/cygdrive/c/gLAB/NEW TUT/TUT7/WORK
                                                                                                            4195013.2503 ]
octave:1> format long
octave:2> r0_rcv=[4789032.6277
                                                          4195013.2503]
                                         176595.0498
                                                                                            (from RINEX header)
r0 rcv =
   4789032.627700000
                           176595.049800000
                                                4195013.250300000
                                                                                            r sat= [ 6364868.61807
octave:3> r_sat= [6364868.61807 -14298233.06215 21851197.94064]
                                                                                                        -14298233.06215
r_sat =
                                                                                                        21851197.94064]
    6364868.61807000 -14298233.06215000
                                                21851197.94064000
                                                                                           CTS [emission]
octave:4> c=299792458
c = 299792458
octave:5> dt_fight=norm(r_sat-r0_rcv,2)/c
                                                                    \rho_{0,receiver}^{\text{satellite}} = \sqrt{\left(x^{\text{sat}} - x_{0,rec}\right)^2 + \left(y^{\text{sat}} - y_{0,rec}\right)^2 + \left(z^{\text{sat}} - z_{0,rec}\right)^2}
dt_fight = 0.0763377130243576
             we= 7.2921151467e-5
      7.29211514670000e-05
octave:7> theta=we*dt_fight
            5.56663393409356e-06
octave:8> R=[cos(theta) sin(theta) 0 ; -sin(theta) cos(theta) 0 ; 0 0 1]
   0.99999999984506
                         0.000005566633934
                                                0.0000000000
                                                                   6364789.025
                                                                                                                       6 364 868.618
                                                                                      \cos(\omega_{\scriptscriptstyle F}\Delta t)
                                                                                                    \sin(\omega_{\scriptscriptstyle F} \Delta t)
  -0.000005566633934
                         0.99999999984506
                                                0.00000000000
   0.000000000000000
                                                1.000000000000
                         0.000000000000000
                                                                -14298268.493
                                                                                      -\sin(\omega_E \Delta t) \cos(\omega_E \Delta t) = 0 | -14 298 233.062
octave:9> r_sat_ems=(R*r_sat')'
                                                                   21851197.941
r_sat_ems =
                                                                    CTS[reception]
    6364789.02494202 -14298268.49282210
                                                21851197.94064000
                                                                                      \omega_r = 7.2921151467 \cdot 10^{-5} \, rad \, / \, sec
octave:10>
```

Cross-checking results with gLAB

```
grep MODEL gLAB.out | grep -v INFO
```

0

Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES grep MODEL gLAB.out | grep -v INFO | gawk '{if (\$6==25) print \$4,\$6,\$11,\$12,\$13}'|head -1 300.00 25 6364789.0249 -14298268.4928 21851197.9406 aume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES |

4.3: Geometric range computation

The geometric range between satellite coordinates at emission time and the "approximate position of the receiver" at reception time both coordinates given in the same reference system [for instance the CTS system at reception time]) is computed by:

$$\rho_{0,rec}^{sat} = \sqrt{\left(x^{sat} - x_{0,rec}\right)^2 + \left(y^{sat} - y_{0,rec}\right)^2 + \left(z^{sat} - z_{0,rec}\right)^2} \approx 22885487.555m$$

$$(x, y, z)^{satellite} = (6364789.0249 - 14298268.4928 \ 21851197.9406)_{CTS[reception]}$$

$$(x_0, y_0, z_0)_{receiver} = (4789032.6277 \ 176595.0498 \ 4195013.2503)_{CTS[reception]}$$

"Approximate" receiver coordinates at reception time.

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\overline{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

Geometric range computation with octave (MATLAB)

from previous computations

```
gAGE@gAGE-PC:/cygdrive/c/gLAB/NEW_TUT/TUT7/WORK
octave:1> format long
octave:2> r_sat_ems=[6364789.02494205 -14298268.49282209
                                                                             21851197.940640007
r sat ems =
     6364789.02494205 -14298268.49282209
                                                       21851197.94064000
octave:3> r0_rcv=[4789032.6277 176595.0498
                                                                  4195013.2503
r0 rcv =
    4789032.627700000
                              176595.049800000
                                                       4195013.250300000
octave:4> rho=norm(r_sat_ems-r0_rcv,2)
                                                                      \rho_{0,receiver}^{\text{satellite}} = \sqrt{\left(x^{\text{sat}} - x_{0,rec}^{\text{o}}\right)^{2} + \left(y^{\text{sat}} - y_{0,rec}^{\text{o}}\right)^{2} + \left(z^{\text{sat}} - z_{0,rec}^{\text{o}}\right)^{2}}
rho = 22885487.5547884
octave:5>
```

Cross-checking results with gLAB

```
grep MODEL gLAB.out | grep -v INFO | gawk '{if ($6==25) print $4,$6,$17}' |head -1
```

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c(d\overline{t}^{sat} + \Delta rel^{sat}) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

5. Relativistic clock correction:

PRN

sqrt(a)

25

5 5 29 2 0 0.0 9.401096 31070E-05 9.094947017729E-13 0.000000 000000E+00

8.40000000000E+01-1.061875(00000E+02 4.825915304457E-09-2.255215533503E+00

-5.284324288368E-06 1.204112719279E-02 5.686655640602E-06 5.153704689026E+03

7.20000000000E+03 2.011656761169E-07-2.689273653419E+00 1.396983861923E-07

9.492799505545E-01 2.625625000000E+02-1.460408709553E+00-8.100337411567E-09

-3.643008888800E-11 1.00000000000E+00 1.32500000000E+03 0.0000000000E+00

2.8000000000E+00 0.0000000000E+00-7.450580596924E-09 8.52000000000E+02

1.8000000000E+01 0.0000000000E+00 1.0000000000E+00 0.0000000000E+00

T[emission] = 299.92366224 s



GPSxyz



E = 3.022976 rad. (eccentric anomaly)

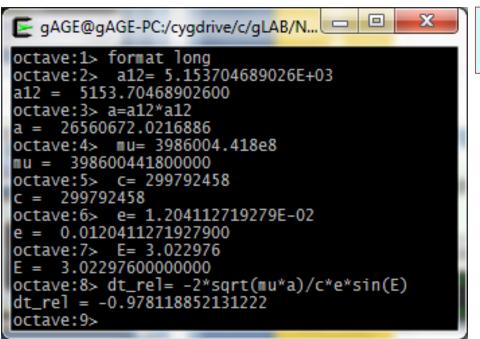
$$\Delta rel^{sat} = -2\frac{\sqrt{\mu a}}{c^2} e \sin(E) = -3.28 \cdot 10^{-9} s$$

$$\mu = 3.986005 \cdot 10^{14} \quad m^3 s^{-2}$$

$$c = 299792458 \quad m s^{-1}$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\overline{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

Relativistic clock correction computation with MATLAB (octave)



```
\Delta rel^{sat} = -2\frac{\sqrt{\mu a}}{c^2} e \sin(E) = -3.28 \cdot 10^{-9} s
```

sqrt(a)= 5.153704689026E+03

e= 1.204112719279E-02

```
E = 3.022976 \text{ rad.}
(eccentric anomaly)
From previous computations
```

$$\mu = 3.986005 \cdot 10^{14} \quad m^3 s^{-2}$$

$$c = 299792458 \quad m \ s^{-1}$$

Cross-checking results with gLAB

```
Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES

Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES grep MODEL gLAB.out | grep -v INFO | gawk '{if ($6==25) print $4,$6,$22}' | head -1 300.00 25 0.98343

Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES |
```

Note: gLAB computes this relativistic correction using a different algorithm: **dt_rel= -2*r_sat_ems*v'/c/c** (see GNSS book). Where the velocity is estimated from coordinates variation from two close epochs. This is the reason of the small discrepancy.

6. Ionospheric correction

(time,
$$r_{sta}$$
, r^{sat} $\alpha 0, \alpha 1, \alpha 2, \alpha 3, \beta 0, \beta 1, \beta 2, \beta 3$ \Rightarrow [iono] \Rightarrow lono=2.49m

2.10 N: GPS NAV DATA RINEX VERSION / TYPE

B2AConv V2.0 gAGE/UPC 21-Dec-09 19:17 PGM / RUN BY / DATE

REALTIME EPHEMERIS FILE COMMENT

1.0245E-08 2.2352E-08 -5.9605E-08 -1.1921E-07 ION ALPHA
9.6256E+04 1.3107E+05 -6.5536E+04 -5.8982E+05 ION BETA

3.725290298462E-09 9.769962616701E-15 319488 1325 DELTA-UTC: A0,A1,T,W

13 LEAP SECONDS
END OF HEADER

$$t = 300 \,\text{sec}$$

 $(x, y, z)^{satellite} = (6364789.0249 - 14298268.4928 \ 21851197.9406)_{CTS[reception]}$
 $(x_0, y_0, z_0)_{receiver} = (4789032.6277 \ 176595.0498 \ 4195013.2503)_{CTS[reception]}$

Approximate values for receiver or satellite coordinates are enough

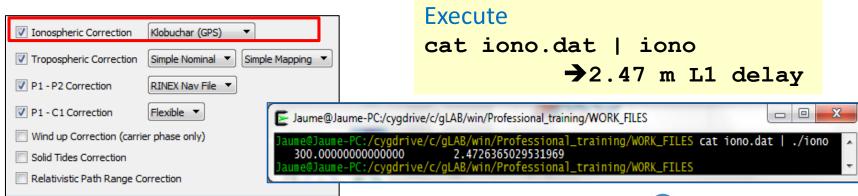
$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\overline{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

6. Ionospheric correction

The FORTRAN program iono.f implements the Klobuchar ionospheric model selected by default in gLAB.

Note: the algorithms are given in the GNSS book, Volume-1.

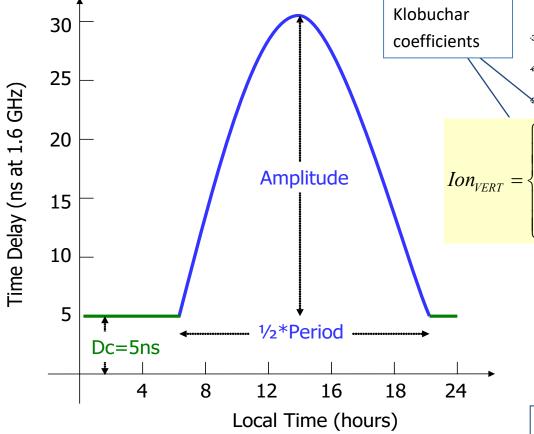
The Fortran code iono.f and C code Model.c are available in the CD-ROM, Volume -2



(time, r_{sta} , r^{sat} , $\alpha 0$, $\alpha 1$, $\alpha 2$, $\alpha 3$, $\beta 0$, $\beta 1$, $\beta 2$, $\beta 3$) \rightarrow iono elev, ϕ RINEX VERSION / TYPE NAVIGATION DATA 24-MAR- 0 00:23 PGM / RUN BY / DATE CCRINEXN V1.5.2 UX CDDIS COMMENT TGS BROADCAST FPHFMFRTS FTLF 0.3167D-07 0.4051D-07 -0.2347D-06 0.1732D-06 **ION ALPHA ION BETA** 0.2842D+05 -0.2150D+05 -0.1096D+06 0.4301D+06 -0.121071934700D-07-0.488498130835D-13 1002 DELTA-UTC: A0,A1,T,W 319488 13 LEAP SECONDS END OF HEADER 0.0 0.783577561379D-04 0.113686837722D-11 0.000000000000D+00 0.19100000000D+03-0.10625000000D+01 0.487163149444D-08-0.123716752769D+01 -0.540167093277D-07 0.476544268895D-02 0.713579356670D-05 0.515433833885D+040.17280000000D+06-0.260770320892D-07-0.850753478531D+00 0.763684511185D-07 0.957259887797D+00 0.241437500000D+03-0.167990552187D+01-0.823998608564D-08 0.174650132022D-09 0.10000000000D+01 0.1002000000D+04 0.00000000000D+00 0.32000000000D+02 0.0000000000D+00 0.465661287308D-09 0.19100000000D+03

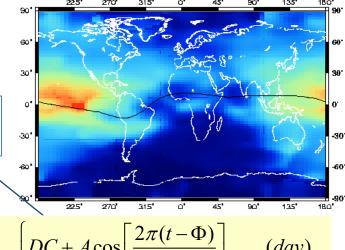
0.17280000000D+06 0.0000000000D+00 0.000000000D+00 0.0000000D+00

Klobuchar model



$$Ion_{SLANT} = Ion_{VERT} \ m(elev)$$

$$m(elev) = \left[1 - \left(\frac{R_E}{R_E + h} \cos(elev)\right)^2\right]^{-1/2}$$



Being φ = Geomagnetic Latitude

Where:

$$DC = 5$$
ns

$$\Phi$$
= 14 (ctt. phase offset)

$$t = Local Time$$

Cross-checking results with gLAB

cat iono.dat | iono

Solution:

 $I_1 = 2.47264 \,\mathrm{m}$ of L1 delay.

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\overline{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

7. Tropospheric correction

$$Trop_{rec}^{sat} = (d_{dry} + d_{wet})m(elev) = 4.319m$$

$$d_{dry} = 2.3 e^{-0.116 \cdot 10^{-3} H}$$

$$d_{wet} = 0.1 m$$

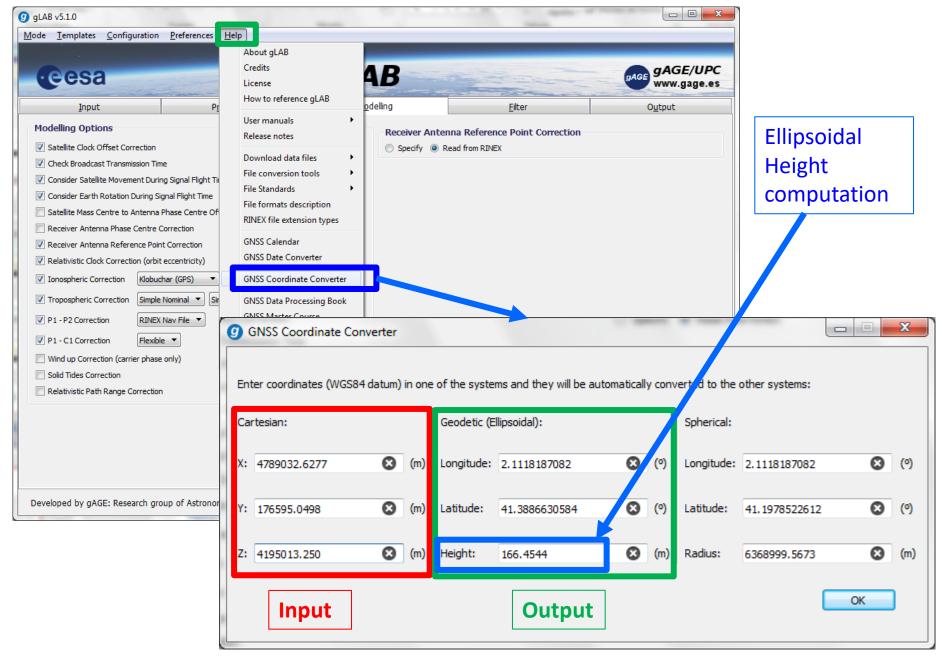
$$m(elev) = \frac{1.001}{\sqrt{0.002001 + \sin^2(elev)}}$$

See next slides

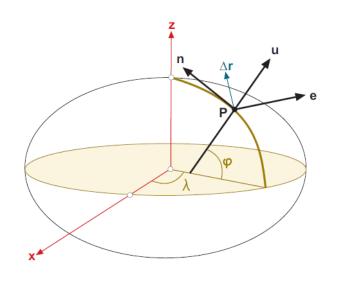
elev : satellite elevation

H = height over the ellipsoid

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\overline{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$



Satellite Elevation and Azimuth computation



The unit vectors in the local east, north and up directions as expressed in ECEF Cartesian coordinates are given by

$$\hat{\mathbf{e}} = (-\sin \lambda, \cos \lambda, 0)
\hat{\mathbf{n}} = (-\cos \lambda \sin \varphi, -\sin \lambda \sin \varphi, \cos \varphi)
\hat{\mathbf{u}} = (\cos \lambda \cos \varphi, \sin \lambda \cos \varphi, \sin \varphi)$$

$$\hat{\boldsymbol{\rho}} = \frac{\mathbf{r}^{sat} - \mathbf{r}_{rcv}}{\|\mathbf{r}^{sat} - \mathbf{r}_{rcv}\|}$$

BACKUP SLIDE

$$\hat{\boldsymbol{\rho}} \cdot \hat{\mathbf{e}} = \cos E \sin A$$
$$\hat{\boldsymbol{\rho}} \cdot \hat{\mathbf{n}} = \cos E \cos A$$
$$\hat{\boldsymbol{\rho}} \cdot \hat{\mathbf{u}} = \sin E$$

$$E = \arcsin(\hat{\boldsymbol{\rho}} \cdot \hat{\mathbf{u}})$$
$$A = \arctan\left(\frac{\hat{\boldsymbol{\rho}} \cdot \hat{\mathbf{e}}}{\hat{\boldsymbol{\rho}} \cdot \hat{\mathbf{n}}}\right)$$

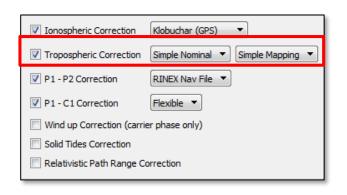
Computation of satellite elevation

```
# Using Octave or MATLAB compute:
octave
 format long
 1=2.1118187082
 f= 41.3886630584
 l=1*pi/180
 f=f*pi/180
 u=[cos(1)*cos(f);sin(1)*cos(f);sin(f)]
    r0\_rcv=[4789032.6277 	 176595.0498 	 4195013.250]
  r_sat_ems=[6364789.0249 -14298268.4928 21851197.9406]
  rho=r_sat_ems-r0_rcv
  rho=rho/norm(rho)
 elev=asin(rho*u)
 # ==> elev=0.575464444394506 (rad)
```

Computation of Tropospheric delay

```
# Using Octave or MATLAB compute:
octave
 format long
 H=166.4544
 elev=0.575464444394506
 dry=2.3*exp(-0.116e-3*H)
 wet=0.1
 m=1.001/sqrt(0.002001+sin(elev)**2)
 Tropo=(dry+wet)*m
 # ==> Tropo= 4.31889 (metres)
 exit
```

Cross-checking results with gLAB



grep MODEL gLAB.out | grep -v INFO |
 gawk '{if (\$6==25) print \$4,\$6,\$24}' |head -1

cat tropo.dat | tropo

Solution:

 $T = 4.46583 \,\mathrm{m}$.

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\overline{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

8. Compute the modeled pseudorange

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c\left(d\overline{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

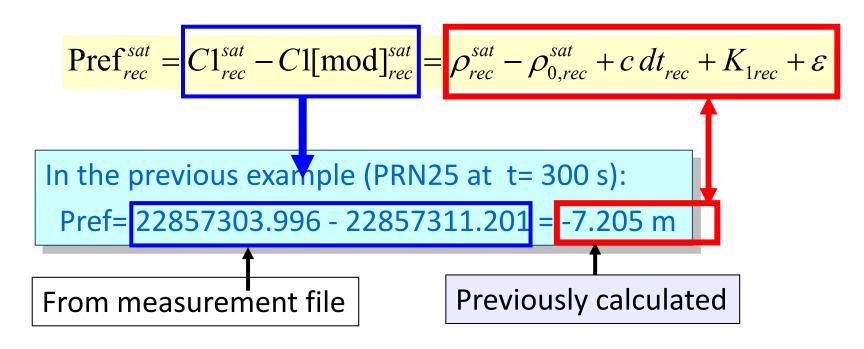
$$\rho_{0,rec}^{sat} = 22885487.554 \,\mathrm{m}$$
 $d\overline{t}^{sat} = 9.400 \cdot 10^{-5} \, c = 28181.896 \,\mathrm{m}$
 $c \Delta rel^{sat} = -3.28 \cdot 10^{-9} \, c = -0.0983 \,\mathrm{m}$
 $Trop_{rec}^{sat} = 4.319 \,\mathrm{m}$
 $Ion_{1rec}^{sat} = 2.473 \,\mathrm{m}$
 $TGD^{sat} = -2.234 \,\mathrm{m}$

 $\rightarrow C1_{rec}^{sat}$ [modelled] = 22857311.201m

Cross-checking results with gLAB

9. Pre-fit residual:

Is the difference between measured and modeled pseudorange



Cross-checking results with gLAB

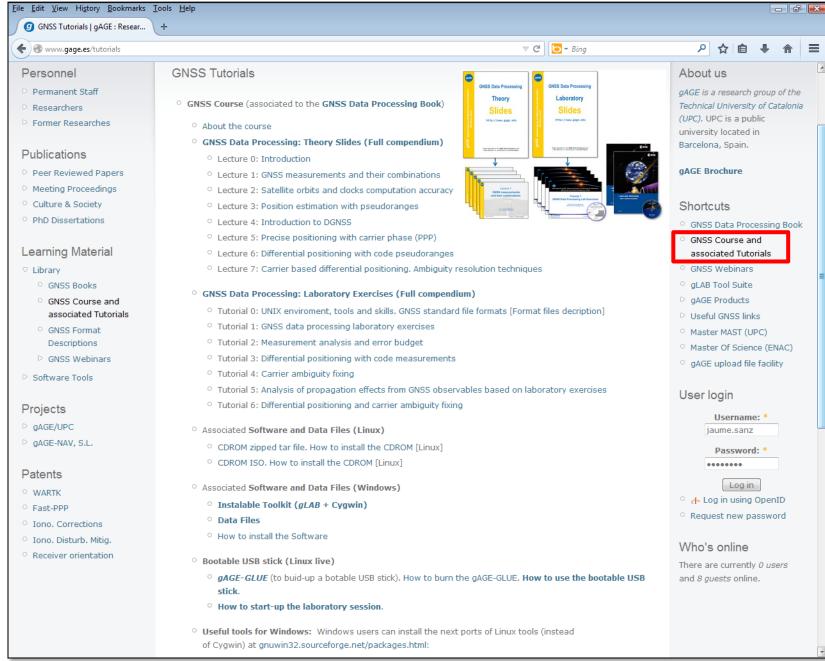
Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES

Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES grep PREFIT gLAB.out | grep -v INFO | gawk '{if (\$6==25) print \$4,\$6,\$8}' | head -1 aume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES

Thank you

References

- [RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.
- [RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.
- [RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga –Jamuna Press, 2004.
- [RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.



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- The other data files used in this study were acquired as part of NASA's Earth Science Data Systems and archived and distributed by the Crustal Dynamics Data Information System (CDDIS).
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- To Deimos Ibáñez for his contribution to gLAB updating and making the Windows, Mac and LINUX installable versions for this tutorial.