Notation

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1 Font notation

$a, b, c, \ldots, A, B, C, \ldots$	Scalars
$\mathbf{a}, \mathbf{b}, \mathbf{c}, \ldots$	Vectors
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$	Matrices
$\boldsymbol{\mathcal{A}}, \boldsymbol{\mathcal{B}}, \boldsymbol{\mathcal{C}}, \ldots$	Tensors
$A, B, C, \ldots, \mathcal{A}, \mathcal{B}, \mathcal{C}, \ldots, \mathbb{A}, \mathbb{B}, \mathbb{C}, \ldots$	Sets

2 Signals and functions

2.1 Time indexing

$x(t)$	Continuous-time t
$x[n], x[k], x[m], x[i], \ldots$ $x_n, x_k, x_m, x_i, \ldots$ $x(n), x(k), x(m), x(i), \ldots$	Discrete-time $n, k, m, i, \ldots$ (parenthesis should be adopted only if there are no continuous-time signals in the context to avoid ambiguity)
$x\left[((n-m))_N\right], x\left((n-m)\right)_N$	Circular shift in m samples within a N-samples window

2.2 Common functions

$\delta(t)$	Delta function
$\delta[n], \delta_{i,j}$	Kronecker function ($n = i - j$)
$h(t), h[n]$	Impulse response (continuous and discrete time)

$\tilde{x}[n], \tilde{x}(t)$	Periodic discrete- or continuous-time signal
$\hat{x}[n], \hat{x}(t)$	Estimate of $x[n]$ or $x(t)$
$\dot{x}[m]$	Interpolation of $x[n]$

2.3 Operations and symbols

$f : A \rightarrow B$	A function f whose domain is A and codomain is B
$f^n, x^n(t), x^n[k]$	nth power of the function f, $x[n]$ or $x(t)$
$f^{(n)}, x^{(n)}(t)$	nth derivative of the function f or $x(t)$
$f', f^{(1)}, x'(t)$	1th derivative of the function f or $x(t)$
$f'', f^{(2)}, x''(t)$	2th derivative of the function f or $x(t)$
$f \circ g$	Composition of the functions f and g
$*$	Convolution (discrete or continuous)
$\circledast, \textcircled{N}$	Circular convolution

2.4 Transformations

$\mathcal{F}\{\cdot\}$	Fourier transform
$\mathcal{L}\{\cdot\}$	Laplace transform
$\mathcal{Z}\{\cdot\}$	z-transform
$\hat{x}(t), \hat{x}[n]$	Hilbert transform of $x(t)$ or $x[n]$
$X(s)$	Laplace transform of $x(t)$
$X(f)$	Fourier transform (FT) (in linear frequency, Hz) of $x(t)$
$X(j\omega)$	Fourier transform (FT) (in angular frequency, rad/sec) of $x(t)$
$X(e^{j\omega})$	Discrete-time Fourier transform (DTFT) of $x[n]$
$X[k], X(k), X_k$	Discrete Fourier transform (DFT) or fast Fourier transform (FFT) of $x[n]$, or even the Fourier series (FS) of the periodic signal $x(t)$
$\tilde{X}[k], \tilde{X}(k), \tilde{X}_k$	Discrete Fourier series (DFS) of $\tilde{x}[n]$
$X(z)$	z-transform of $x[n]$

3 Probability, statistics, and stochastic processes

3.1 Operators and symbols

$E[\cdot]$	Statistical expectation
$E_u[\cdot]$	Statistical expectation with respect to u
μ_x	Mean of the random variable x
$\boldsymbol{\mu}_{\mathbf{x}}, \mathbf{m}_{\mathbf{x}}$	Mean vector of the random variable $\mathbf{x}$
μ_n	nth-order moment of a random variable
$\mathcal{K}_x, \mu_4$	Kurtosis (4th-order moment) of the random variable x
$\mathrm{VAR}[\cdot]$	Variance operator
$\mathrm{VAR}_u[\cdot]$	Variance operator with respect to u
κ_n	nth-order cumulant of a random variable
σ_x, κ_2	Variance of the random variable x
$\rho_{x,y}$	Pearson correlation coefficient between x and y
$a \sim P$	Random variable a with distribution P
$\mathcal{R}$	Rayleigh's quotient

3.2 Stochastic processes

$r_x(\tau), R_x(\tau)$	Autocorrelation function of the signal $x(t)$ or $x[n]$
$S_x(f), S_x(j\omega)$	Power spectral density (PSD) of $x(t)$ in linear (f) or angular (ω) frequency
$S_{x,y}(f), S_{x,y}(j\omega)$	Cross PSD of $x(t)$ and $y(t)$ in linear or angular (ω) frequency
$\mathbf{R}_{\mathbf{x}}$	(Auto)correlation matrix of $\mathbf{x}(n)$
$r_{x,d}(\tau), R_{x,d}(\tau)$	Cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$
$\mathbf{R}_{\mathbf{xy}}$	Cross-correlation matrix of $\mathbf{x}(n)$ and $\mathbf{y}(n)$
$\mathbf{p}_{\mathbf{x}d}$	Cross-correlation vector between $\mathbf{x}(n)$ and $d(n)$
$c_x(\tau), C_x(\tau)$	Autocovariance function of the signal $x(t)$ or $x[n]$
$\mathbf{C}_{\mathbf{x}}, \mathbf{K}_{\mathbf{x}}, \boldsymbol{\Sigma}_{\mathbf{x}}$	(Auto)covariance matrix of $\mathbf{x}$

$c_{xy}(\tau), C_{xy}(\tau)$	Cross-covariance function of the signal $x(t)$ or $x[n]$
$\mathbf{C_{xy}}, \mathbf{K_{xy}}, \mathbf{\Sigma_{xy}}$	Cross-covariance matrix of $\mathbf{x}$ and $\mathbf{y}$

3.3 Functions

$Q(\cdot)$	Q-function, i.e., $P\left[\mathcal{N}(0,1) > x\right]$
$\mathrm{erf}(\cdot)$	Error function
$\mathrm{erfc}(\cdot)$	Complementary error function i.e., $\mathrm{erfc}(x) = 2Q(\sqrt{2}x) - \mathrm{erf}(x)$
$P[A]$	Probability of the event or set A
$p(\cdot), f(\cdot)$	Probability density function (PDF) or probability mass function (PMF)
$p(x \mid A)$	Conditional PDF or PMF
$F(\cdot)$	Cumulative distribution function (CDF)
$\Phi_x(\omega), M_x(j\omega), E\left[e^{j\omega x}\right]$	First characteristic function (CF) of x
$M_x(t), \Phi_x(-jt), E\left[e^{tx}\right]$	Moment-generating function (MGF) of x
$\Psi_x(\omega), \ln \Phi_x(\omega), \ln E\left[e^{j\omega x}\right]$	Second characteristic function
$K_x(t), \ln E\left[e^{tx}\right], \ln M_x(t)$	Cumulant-generating function (CGF) of x

3.4 Distributions

$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution of a random variable with mean μ and variance σ^2. The same notation can be used to denote a real-valued white Gaussian process with mean equal to μ and power spectral density equal to $N_0/2$, e.g., $s(t) \sim \mathcal{N}(\mu, N_0/2)$
$\mathcal{CN}(\mu, \sigma^2)$	Complex Gaussian distribution of a random variable with mean μ and variance σ^2. The same notation can be used to denote a complex-valued white Gaussian process with mean equal to μ and power spectral density equal to N_0, e.g., $s(t) \sim \mathcal{CN}(\mu, N_0)$
$\mathcal{N}(\boldsymbol{\mu}, \mathbf{\Sigma})$	Gaussian distribution of a vector random variable with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{\Sigma}$

$\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	Complex Gaussian distribution of a vector random variable with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$
$\mathcal{U}(a, b)$	Uniform distribution from a to b
$\chi^2(n), \chi_n^2$	Chi-square distribution with n degree of freedom (assuming that the Gaussians are $\mathcal{N}(0, 1)$)
$\mathrm{Exp}(\lambda)$	Exponential distribution with rate parameter λ
$\Gamma(\alpha, \beta)$	Gamma distribution with shape parameter α and rate parameter β
$\Gamma(\alpha, \theta)$	Gamma distribution with shape parameter α and scale parameter $\theta = 1/\beta$
$\mathrm{Nakagami}(m, \boldsymbol{\Omega})$	Nakagami-m distribution with shape parameter m and spread parameter $\boldsymbol{\Omega}$
$\mathrm{Rayleigh}(\sigma)$	Rayleigh distribution with scale parameter σ
$\mathrm{Rayleigh}(\boldsymbol{\Omega})$	Rayleigh distribution with the second moment $\boldsymbol{\Omega} = E\left[x^2\right] = 2\sigma^2$
$\mathrm{Rice}(s, \sigma)$	Rice distribution with noncentrality parameter (specular component) s and σ
$\mathrm{Rice}(A, K)$	Rice distribution with Rice factor $K = s^2/2\sigma^2$ and scale parameter $A = s^2 + 2\sigma^2$

4 Statistical signal processing

$\boldsymbol{\nabla} f, \mathbf{g}$	Gradient descent vector
$\boldsymbol{\nabla}_x f, \mathbf{g}_x$	Gradient descent vector with respect x
$\mathbf{g}$ (or $\hat{\mathbf{g}}$ if the gradient vector is $\mathbf{g}$)	Stochastic gradient descent (SGD)
$J(\cdot), \mathcal{E}(\cdot)$	Cost-function or objective function
$\Lambda(\cdot)$	Likelihood function
$\Lambda_l(\cdot)$	Log-likelihood function
$\hat{x}(t)$ or $\hat{x}[n]$	Estimate of $x(t)$ or $x[n]$
$\hat{\boldsymbol{\mu}}_x, \hat{\mathbf{m}}_x$	Sample mean of $x[n]$ or $x(t)$
$\hat{\boldsymbol{\mu}}_{\mathbf{x}}, \hat{\mathbf{m}}_{\mathbf{x}}$	Sample mean vector of $\mathbf{x}[n]$ or $\mathbf{x}(t)$
$\hat{r}_x(\tau), \hat{R}_x(\tau)$	Estimated autocorrelation function of the signal $x(t)$ or $x[n]$
$\hat{S}_x(f), \hat{S}_x(j\omega)$	Estimated power spectral density (PSD) of $x(t)$ in linear (f) or angular (ω) frequency

$\hat{\mathbf{R}}_{\mathbf{x}}$	Sample (auto)correlation matrix
$\hat{r}_{x,d}(\tau), \hat{R}_{x,d}(\tau)$	Estimated cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$
$\hat{S}_{x,y}(f), \hat{S}_{x,y}(j\omega)$	Estimated cross PSD of $x(t)$ and $y(t)$ in linear or angular (ω) frequency
$\hat{\mathbf{R}}_{\mathbf{xy}}$	Sample cross-correlation matrix of $\mathbf{R}_{\mathbf{xy}}$
$\hat{\rho}_{x,y}$	Estimated Pearson correlation coefficient between x and y
$\hat{c}_x(\tau), \hat{C}_x(\tau)$	Estimated autocovariance function of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_{\mathbf{x}}, \hat{\mathbf{K}}_{\mathbf{x}}, \hat{\boldsymbol{\Sigma}}_{\mathbf{x}}$	Sample (auto)covariance matrix
$\hat{c}_{xy}(\tau), \hat{C}_{xy}(\tau)$	Estimated cross-covariance function of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_{\mathbf{xy}}, \hat{\mathbf{K}}_{\mathbf{xy}}, \hat{\boldsymbol{\Sigma}}_{\mathbf{xy}}$	Sample cross-covariance matrix
$\mathbf{w}, \boldsymbol{\theta}$	Parameters, coefficients, or weights vector
$\mathbf{w}_o, \mathbf{w}^\star, \boldsymbol{\theta}_o, \boldsymbol{\theta}^\star$	Optimum value of the parameters, coefficients, or weights vector
$\mathbf{W}$	Matrix of the weights
$\mathbf{J}$	Jacobian matrix
$\mathbf{H}$	Hessian matrix
$\hat{\mathbf{H}}$	Estimate of the Hessian matrix

5 Linear Algebra

5.1 Common matrices and vectors

$\mathbf{W}, \mathbf{D}$	Diagonal matrix
$\mathbf{P}$	Projection matrix; Permutation matrix
$\mathbf{J}$	Jordan matrix
$\mathbf{L}$	Lower matrix
$\mathbf{U}$	Upper matrix
$\mathbf{C}$	Cofactor matrix
$\mathbf{C}_{\mathbf{A}}, \operatorname{cof}(\mathbf{A})$	Cofactor matrix of $\mathbf{A}$
$\mathbf{S}$	Symmetric matrix
$\mathbf{Q}$	Orthogonal matrix
$\mathbf{I}_N$	$N \times N$-dimensional identity matrix
$\mathbf{0}_{M \times N}$	$M \times N$-dimensional null matrix
$\mathbf{0}_N$	N-dimensional null vector
$\mathbf{1}_{M \times N}$	$M \times N$-dimensional ones matrix
$\mathbf{1}_N$	N-dimensional ones vector

$\mathbf{0}$	Null matrix, vector, or tensor (dimensionality understood by context)
$\mathbf{1}$	Ones matrix, vector, or tensor (dimensionality understood by context)

5.2 Indexing

$x_{i_1,i_2,\ldots,i_N}, [\boldsymbol{\mathcal{X}}]_{i_1,i_2,\ldots,i_N}$	Element in the position $(i_1, i_2, \ldots, i_N)$ of the tensor $\boldsymbol{\mathcal{X}}$
$\boldsymbol{\mathcal{X}}^{(n)}$	nth tensor of a nontemporal sequence
$\mathbf{x}_n, \mathbf{x}_{:n}$	nth column of the matrix X
$\mathbf{x}_{n:}$	nth row of the matrix X
$\mathbf{x}_{i_1,\ldots,i_{n-1},:,i_{n+1},\ldots,i_N}$	Mode-n fiber of the tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{x}_{:,i_2,i_3}$	Column fiber (mode-1 fiber) of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{x}_{i_1,:,i_3}$	Row fiber (mode-2 fiber) of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{x}_{i_1,i_2,:}$	Tube fiber (mode-3 fiber) of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{i_1,:,:}$	Horizontal slice of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{:,i_2,:}$	Lateral slices slice of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{i_3}, \mathbf{X}_{:,:,i_3}$	Frontal slices slice of the thrid-order tensor $\boldsymbol{\mathcal{X}}$

5.3 General operations

$\langle \cdot, \cdot \rangle$	Inner product, e.g., $\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^\top \mathbf{b}$
$\circ$	Outer product, e.g., $\mathbf{a} \circ \mathbf{b} = \mathbf{a}\mathbf{b}^\top$
$\otimes$	Kronecker product
$\odot$	Hadamard (or Schur) (elementwise) product
$.^{\odot n}$	nth-order Hadamard power
$.^{\odot \frac{1}{n}}$	nth-order Hadamard root
$\oslash$	Hadamard (or Schur) (elementwise) division
$\diamond$	Khatri-Rao product
$\otimes$	Kronecker Product
$\times_n$	n-mode product

5.4 Operations with matrices and tensors

$\mathbf{A}^{-1}$	Inverse matrix
$\mathbf{A}^{+}, \mathbf{A}^{\dagger}$	Moore-Penrose left pseudoinverse
$\mathbf{A}^{\top}$	Transpose
$\mathbf{A}^{-\top}$	Transpose of the inverse
$\mathbf{A}^{*}$	Complex conjugate
$\mathbf{A}^{\mathsf{H}}$	Hermitian
$\\|\mathbf{A}\\|_{\mathrm{F}}$	Frobenius norm
$\\|\mathbf{A}\\|$	Matrix norm
$\lvert\mathbf{A}\rvert, \det(\mathbf{A})$	Determinant
$\operatorname{diag}(\mathbf{A})$	The elements in the diagonal of $\mathbf{A}$
$\operatorname{vec}(\mathbf{A})$	Vectorization: stacks the columns of the matrix $\mathbf{A}$ into a long column vector
$\operatorname{vec}_{\mathrm{d}}(\mathbf{A})$	Extracts the diagonal elements of a square matrix and returns them in a column vector
$\operatorname{vec}_{\mathrm{l}}(\mathbf{A})$	Extracts the elements strictly below the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\operatorname{vec}_{\mathrm{u}}(\mathbf{A})$	Extracts the elements strictly above the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\operatorname{vec}_{\mathrm{b}}(\mathbf{A})$	Block vectorization operator: stacks square block matrices of the input into a long block column matrix
$\operatorname{unvec}(\mathbf{A})$	Reshapes a column vector into a matrix
$\operatorname{tr}(\mathbf{A})$	trace
$\mathbf{X}_{(n)}$	n-mode matricization of the tensor $\boldsymbol{\mathcal{X}}$

5.5 Operations with vectors

$\\|\mathbf{a}\\|$	l_1 norm, 1-norm, or Manhatan norm
$\\|\mathbf{a}\\|, \\|\mathbf{a}\\|_2$	l_2 norm, 2-norm, or Euclidean norm
$\\|\mathbf{a}\\|_p$	l_p norm, p-norm, or Minkowski norm
$\\|\mathbf{a}\\|_\infty$	l_∞ norm, ∞-norm, or Chebyshev norm
$\operatorname{diag}(\mathbf{a})$	Diagonalization: a square, diagonal matrix with entries given by the vector $\mathbf{a}$

5.6 Decompositions

$\mathbf{\Lambda}$	Eigenvalue matrix
$\mathbf{Q}$	Eigenvectors matrix; Orthogonal matrix of the QR decomposition
$\mathbf{R}$	Upper triangular matrix of the QR decomposition
$\mathbf{U}$	Left singular vectors
$\mathbf{U}_r$	Left singular nondegenerated vectors
$\mathbf{\Sigma}$	Singular value matrix
$\mathbf{\Sigma}_r$	Singular value matrix with nonzero singular values in the main diagonal
$\mathbf{\Sigma}^+$	Singular value matrix of the pseudoinverse
$\mathbf{\Sigma}_r^+$	Singular value matrix of the pseudoinverse with nonzero singular values in the main diagonal
$\mathbf{V}$	Right singular vectors
$\mathbf{V}_r$	Right singular nondegenerated vectors
$\mathrm{eig}\left(\mathbf{A}\right)$	Set of the eigenvalues of $\mathbf{A}$
$[\![\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots]\!]$	CANDECOMP/PARAFAC (CP) decomposition of the tensor $\boldsymbol{\mathcal{X}}$ from the outer product of column vectors of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}, \ldots$
$[\![\boldsymbol{\lambda}; \mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots]\!]$	Normalized CANDECOMP/PARAFAC (CP) decomposition of the tensor $\boldsymbol{\mathcal{X}}$ from the outer product of column vectors of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}, \ldots$

5.7 Spaces

$\mathrm{span}\left(\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n\right)$	Vector space spanned by the argument vectors
$\mathrm{C}\left(\mathbf{A}\right)$, $\mathrm{columnspace}(\mathbf{A})$, $\mathrm{range}(\mathbf{A})$, $\mathrm{span}\left(\mathbf{A}\right)$, $\mathrm{image}(\mathbf{A})$	Columnspace, range or image, i.e., the space $\mathrm{span}\left(\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n\right)$, where $\mathbf{a}_i$ is the ith column vector of the matrix $\mathbf{A}$
$\mathrm{C}\left(\mathbf{A}^{\mathsf{H}}\right)$	Row space
$\mathrm{N}\left(\mathbf{A}\right), \mathrm{nullspace}(\mathbf{A}), \mathrm{kernel}(\mathbf{A})$	Nullspace (or kernel space)
$\mathrm{N}\left(\mathbf{A}^{\mathsf{H}}\right)$	Left nullspace
$\mathrm{rank}\left(\mathbf{A}\right)$	Rank, that is, $\dim\left(\mathrm{span}\left(\mathbf{A}\right)\right) = \dim\left(\mathrm{C}\left(\mathbf{A}\right)\right)$
$\mathrm{nullity}\left(\mathbf{A}\right)$	Nullity of $\mathbf{A}$, i.e., $\dim\left(\mathrm{N}\left(\mathbf{A}\right)\right)$
$\mathbf{a} \perp \mathbf{b}$	$\mathbf{a}$ is orthogonal to $\mathbf{b}$
$\mathbf{a} \not\perp \mathbf{b}$	$\mathbf{a}$ is not orthogonal to $\mathbf{b}$

5.8 Inequalities

$\boldsymbol{\mathcal{X}} \preceq 0$	Nonnegative tensor
$\mathbf{a} \preceq_K \mathbf{b}$	Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the conic subset K in the space $\mathbb{R}^n$
$\mathbf{a} \prec_K \mathbf{b}$	Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the interior of the conic subset K in the space $\mathbb{R}^n$
$\mathbf{a} \preceq \mathbf{b}$	Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the nonnegative orthant conic subset, $\mathbb{R}^n_+$, in the space $\mathbb{R}^n$.
$\mathbf{a} \prec \mathbf{b}$	Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the positive orthant conic subset, $\mathbb{R}^n_{++}$, in the space $\mathbb{R}^n$
$\mathbf{A} \preceq_K \mathbf{B}$	Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the conic subset K in the space $\mathbb{S}^n$
$\mathbf{A} \prec_K \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the interior of the conic subset K in the space $\mathbb{S}^n$
$\mathbf{A} \preceq \mathbf{B}$	Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive semidefinite conic subset, $\mathbb{S}^n_+$, in the space $\mathbb{S}^n$
$\mathbf{A} \prec \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive orthant conic subset, $\mathbb{S}^n_{++}$, in the space $\mathbb{S}^n$

6 Sets

$A + B$	Set addition (Minkowski sum)
$A - B$	Minkowski difference
$A \setminus B, A - B$	Set difference or set subtraction, i.e., the set containing the elements of A that are not in B
$A \cup B$	Set of union
$A \cap B$	Set of intersection
$A \times B$	Cartesian product
A^n	$\underbrace{A \times A \times \cdots \times A}_{n \text{ times}}$

$A^{\perp}$	Orthogonal complement of A, e.g., $\mathrm{N}(\mathbf{A}) = \mathrm{C}(\mathbf{A}^{\top})^{\perp}$
$A \oplus B$	Direct sum, e.g., $\mathrm{C}(\mathbf{A}^{\top}) \oplus \mathrm{C}(\mathbf{A}^{\top})^{\perp} = \mathbb{R}^n$
$A^c, \bar{A}$	Complement set (given U)
$\#A, \|A\|$	Cardinality
$a \in A$	a is element of A
$a \notin A$	a is not element of A
$\{1, 2, \ldots, n\}$	Discrete set containing the integer elements $1, 2, \ldots, n$
U	Universe
2^A	Power set of A
$\mathbb{R}$	Set of real numbers
$\mathbb{C}$	Set of complex numbers
$\mathbb{Z}$	Set of integer number
$\mathbb{B} = \{0, 1\}$	Boolean set
$\varnothing$	Empty set
$\mathbb{N}$	Set of natural numbers
$\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$	Real or complex space (field)
$\mathbb{K}^{I_1 \times I_2 \times \cdots \times I_N}$	$I_1 \times I_2 \times \cdots \times I_N$-dimensional real (or complex) space
$\mathbb{K}_+$	Nonnegative real (or complex) space
$\mathbb{K}_{++}$	Positive real (or complex) space, i.e., $\mathbb{K}_{++} = \mathbb{K}_+ \setminus \{\mathbf{0}\}$
$\mathbb{S}^n, \mathcal{S}^n$	Conic set of the symmetric matrices in $\mathbb{R}^{n \times n}$
$\mathbb{S}^n_+, \mathcal{S}^n_+$	Conic set of the symmetric positive semidefinite matrices in $\mathbb{R}^{n \times n}$
$\mathbb{S}^n_{++}, \mathcal{S}^n_{++}$	Conic set of the symmetric positive definite matrices in $\mathbb{R}^{n \times n}$, i.e., $\mathbb{S}^n_{++} = \mathbb{S}^n_+ \setminus \{\mathbf{0}\}$
$\mathbb{H}^n$	Set of all hermitian matrices in $\mathbb{C}^{n \times n}$
$[a, b]$	Closed interval of a real set from a to b
(a, b)	Opened interval of a real set from a to b
$[a, b), (a, b]$	Half-opened intervals of a real set from a to b

7 Communication systems

x_i	Real or in-phase part of x
x_q	Imaginary or quadrature part of x
s	Transmitted signal

ϕ	Signal phase
s_l	Low-pass equivalent signal or envelope complex of s
η, w	Gaussian noise
r	Received signal
τ	Timing delay
$\Delta\tau$	Timing error (delay - estimated)
φ	Phase offset
$\Delta\varphi$	Phase error (offset - estimated)
f_d	Doppler frequency
A	Received signal amplitude
γ	Combined effect of the path loss and antenna gain

8 Other notations

8.1 Mathematical symbols

$\exists$	There exists
$\nexists$	There does not exist
$\exists!$	There exist an unique
$\in$	Belongs to
$\notin$	Does not belong to
$\blacksquare$	Q.E.D.
$\therefore$	Therefore
$\because$	Because
$\forall$	For all
$\|, :$	Such that
$\iff$	Logical equivalence
$\triangleq, :=$	Equal by definition
$\neq$	Not equal
∞	Infinity
j	$\sqrt{-1}$
W_N	Twiddle factor, $e^{-j\frac{2\pi}{N}}$

8.2 Operations

$\arg\max\limits_{x\in\mathcal{A}} f(x)$	Value of x that minimizes x
$\arg\min\limits_{x\in\mathcal{A}} f(x)$	Value of x that minimizes x
$\inf\limits_{\mathbf{y}\in\mathcal{A}} g(\mathbf{x},\mathbf{y})$	Infimum
$\sup\limits_{\mathbf{y}\in\mathcal{A}} g(\mathbf{x},\mathbf{y})$	Supremum

$\lvert a \rvert$	Absolute value of a
log	Base-10 logarithm or decimal logarithm
ln	Natual logarithm
$\mathrm{Re}\{x\}$	Real part of x
$\mathrm{Im}\{x\}$	Imaginary part of x
$\angle\cdot$	phase (complex argument)
$x \bmod y$	Remainder, i.e., $x - y\lfloor x/y \rfloor$
$\mathrm{frac}(x)$	Fractional part, i.e., $x \bmod 1$
$a \wedge b$	Logical AND of a and b
$a \vee b$	Logical OR of a and b
$\neg a$	Logical negation of a
$\lceil\cdot\rceil$	Ceiling operation
$\lfloor\cdot\rfloor$	Floor operation

8.3 Functions

$\mathcal{O}(\cdot), O(\cdot)$	Big-O notation
$\Gamma(\cdot)$	Gamma function

9 Abbreviations

wrt.	With respect to
st.	Subject to
iff.	If and only if
EVD	Eigenvalue decomposition, or eigendecomposition
SVD	Singular value decomposition
CP	CANDECOMP/PARAFAC