## Notation

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## 1 Font notation

$a, b, c, \ldots, A, B, C, \ldots$	Scalars
$a, b, c, \dots$	Vectors
$A, B, C, \dots$	Matrices
$\mathcal{A}, \mathcal{B}, \mathcal{C}, \dots$	Tensors
$A, B, C, \ldots, A, B, C, \ldots, A, B, C, \ldots$	Sets

## 2 Signals and functions

## 2.1 Time indexing

x(t)	Continuous-time $t$
$x[n],x[k],x[m],x[i],\ldots$	Discrete-time $n, k, m, i, \dots$ (parenthe-
$x_n, x_k, x_m, x_i, \dots$	sis should be adopted only if there
$x(n), x(k), x(m), x(i), \dots$	are no continuous-time signals in the
	context to avoid ambiguity)
$x[((n-m))_N], x((n-m))_N$	Circular shift in $m$ samples within a
	N-samples window

#### 2.2 Common functions

$\delta(t)$	Delta function
$\delta[n], \delta_{i,j}$	Kronecker function $(n = i - j)$
h(t), h[n]	Impulse response (continuous and
	discrete time)

$\tilde{x}[n], \tilde{x}(t)$	Periodic discrete- or continuous-time
	signal
$\hat{x}[n], \hat{x}(t)$	Estimate of $x[n]$ or $x(t)$
$\dot{x}[m]$	Interpolation of $x[n]$

## 2.3 Operations and symbols

$f:A\to B$	A function $f$ whose domain is $A$ and
	codomain is $B$
$f^n, x^n(t), x^n[k]$	nth power of the function $f$ , $x[n]$ or
	x(t)
$f^{(n)}, x^{(n)}(t)$	nth derivative of the function $f$ or
	x(t)
$f', f^{(1)}, x'(t)$	1th derivative of the function $f$ or
	x(t)
$f'', f^{(2)}, x''(t)$	2th derivative of the function $f$ or
	x(t)
$f \circ g$	Composition of the functions $f$ and
	g
*	Convolution (discrete or continuous)
$(8, \overline{N})$	Circular convolution

## 2.4 Transformations

$\mathcal{F}\left\{ \cdot  ight\}$	Fourier transform
$\mathcal{L}\left\{ \cdot  ight\}$	Laplace transform
$\mathcal{Z}\left\{ \cdot  ight\}$	z-transform
$\hat{x}(t), \hat{x}[n]$	Hilbert transform of $x(t)$ or $x[n]$
X(s)	Laplace transform of $x(t)$
X(f)	Fourier transform (FT) (in linear fre-
	quency, Hz) of $x(t)$
$X(j\omega)$	Fourier transform (FT) (in angular
	frequency, rad/sec) of $x(t)$
$X(e^{j\omega})$	Discrete-time Fourier transform
	(DTFT) of $x[n]$
$X[k], X(k), X_k$	Discrete Fourier transform (DFT) or
	fast Fourier transform (FFT) of $x[n]$ ,
	or even the Fourier series (FS) of the
	periodic signal $x(t)$
$\tilde{X}[k], \tilde{X}(k), \tilde{X}_k$	Discrete Fourier series (DFS) of $\tilde{x}[n]$
X(z)	z-transform of $x[n]$

## 3 Probability, statistics, and stochastic processes

## 3.1 Operators and symbols

$E\left[\cdot\right]$ $E_{u}\left[\cdot\right]$	Statistical expectation Statistical expectation with respect
	to u
$\mu_{\scriptscriptstyle X}$	Mean of the random variable $x$
$\mu_x, m_x$	Mean vector of the random variable
	X
$\mu_n$	nth-order moment of a random vari-
	able
$\mathcal{K}_x, \mu_4$	Kurtosis (4th-order moment) of the
	random variable $x$
$VAR[\cdot]$	Variance operator
$VAR_u[\cdot]$	Variance operator with respect to $u$
$\kappa_n$	nth-order cumulant of a random vari-
	able
$\sigma_x, \kappa_2$	Variance of the random variable $x$
$\rho_{x,y}$	Pearson correlation coefficient be-
	tween $x$ and $y$
$a \sim P$	Random variable $a$ with distribution
	P
$\mathcal{R}$	Rayleigh's quotient

## 3.2 Stochastic processes

$r_{X}(\tau), R_{X}(\tau)$	Autocorrelation function of the sig-
	$\operatorname{nal} x(t) \text{ or } x[n]$
$S_x(f), S_x(j\omega)$	Power spectral density (PSD) of $x(t)$
	in linear $(f)$ or angular $(\omega)$ frequency
$S_{x,y}(f), S_{x,y}(j\omega)$	Cross PSD of $x(t)$ and $y(t)$ in linear
	or angular $(\omega)$ frequency
$\mathbf{R}_{\mathbf{x}}$	(Auto)correlation matrix of $\mathbf{x}(n)$
$r_{x,d}(\tau), R_{x,d}(\tau)$	Cross-correlation between $x[n]$ and
	d[n] or $x(t)$ and $d(t)$
$\mathbf{R}_{\mathbf{x}\mathbf{y}}$	Cross-correlation matrix of $\mathbf{x}(n)$ and
-	$\mathbf{y}(n)$
$\mathbf{p}_{\mathbf{x}d}$	Cross-correlation vector between
	$\mathbf{x}(n)$ and $d(n)$
$c_X(\tau), C_X(\tau)$	Autocovariance function of the signal
	x(t) or $x[n]$
$\mathbf{C}_{\mathrm{x}},\mathbf{K}_{\mathrm{x}},\mathbf{\Sigma}_{\mathrm{x}}$	(Auto)covariance matrix of $\mathbf{x}$

$$c_{xy}(\tau), C_{xy}(\tau)$$

 $C_{xy}, K_{xy}, \Sigma_{xy}$ 

Cross-covariance function of the signal x(t) or x[n]

Cross-covariance matrix of  $\mathbf{x}$  and  $\mathbf{v}$ 

#### 3.3 Functions

 $Q(\cdot)$  erf $(\cdot)$ 

 $\operatorname{erfc}(\cdot)$ 

P[A]  $p(\cdot), f(\cdot)$ 

 $p(x \mid A)$  $F(\cdot)$ 

 $\Phi_{x}(\omega), M_{x}(j\omega), E\left[e^{j\omega x}\right]$ 

 $M_x(t), \Phi_x(-jt), E\left[e^{tx}\right]$ 

 $\Psi_X(\omega), \ln \Phi_X(\omega), \ln E\left[e^{j\omega x}\right]$  $K_X(t), \ln E\left[e^{tx}\right], \ln M_X(t)$  *Q*-function, i.e.,  $P[\mathcal{N}(0,1) > x]$ 

Error function

Complementary error function i.e.,

 $\operatorname{erfc}(x) = 2Q(\sqrt{2}x) - \operatorname{erf}(x)$ 

Probability of the event or set A

Probability density function (PDF) or probability mass function (PMF)

Conditional PDF or PMF

Cumulative distribution function

(CDF)

First characteristic function (CF) of

х

Moment-generating function (MGF)

of  $\lambda$ 

Second characteristic function

Cumulant-generating function

(CGF) of x

#### 3.4 Distributions

 $\mathcal{N}(\mu, \sigma^2)$ 

 $\mathcal{CN}(\mu, \sigma^2)$ 

 $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ 

 $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ 

 $\mathcal{U}(a,b)$  $\chi^2(n), \chi_n^2$ 

 $\text{Exp}(\lambda)$ 

Gaussian distribution of a random variable with mean  $\mu$  and variance  $\sigma^2$  Complex Gaussian distribution of a random variable with mean  $\mu$  and variance  $\sigma^2$ 

Gaussian distribution of a vector random variable with mean  $\mu$  and covariance matrix  $\Sigma$ 

Complex Gaussian distribution of a vector random variable with mean  $\mu$  and covariance matrix  $\Sigma$ 

and covariance matrix  $\Sigma$ Uniform distribution from a to b

Uniform distribution from a to bChi-square distribution with n degree of freedom (assuming that the Gaus-

sians are  $\mathcal{N}(0,1)$ )

Exponential distribution with rate

parameter  $\lambda$ 

$\Gamma(\alpha, \beta)$ $\Gamma(\alpha, \theta)$	Gamma distribution with shape parameter $\alpha$ and rate parameter $\beta$ Gamma distribution with shape parameter $\alpha$ and scale parameter $\theta = 1/\beta$
$\operatorname{Nakagami}(m,\Omega)$	Nakagami-m distribution with shape parameter $m$ and spread parameter $\Omega$
$\operatorname{Rayleigh}(\sigma)$	Rayleigh distribution with scale parameter $\sigma$
$\operatorname{Rayleigh}(\Omega)$	Rayleigh distribution with the second moment $\Omega = E[x^2] = 2\sigma^2$
$\mathrm{Rice}(s,\sigma)$	Rice distribution with noncentrality parameter (specular component) $s$ and $\sigma$
$\mathrm{Rice}(A,K)$	Rice distribution with Rice factor $K=s^2/2\sigma^2$ and scale parameter $A=s^2+2\sigma^2$

# 4 Statistical signal processing

$\mathbf{\nabla} f, \mathbf{g}$	Gradient descent vector
$\nabla_x f, \mathbf{g}_x$	Gradient descent vector with respect
	x
$\mathbf{g}$ (or $\hat{\mathbf{g}}$ if the gradient vector is $\mathbf{g}$ )	Stochastic gradient descent (SGD)
$J(\cdot), \mathcal{E}(\cdot)$	Cost-function or objective function
$\Lambda(\cdot)$	Likelihood function
$\Lambda_l(\cdot)$	Log-likelihood function
$\hat{x}(t)$ or $\hat{x}[n]$	Estimate of $x(t)$ or $x[n]$
$\hat{\boldsymbol{\mu}}_{x},\hat{\mathbf{m}}_{x}$	Sample mean of $x[n]$ or $x(t)$
$\hat{\boldsymbol{\mu}}_{\mathbf{x}}, \hat{\mathbf{m}}_{\mathbf{x}}$	Sample mean vector of $\mathbf{x}[n]$ or $\mathbf{x}(t)$
$\hat{r}_{\scriptscriptstyle X}( au), \hat{R}_{\scriptscriptstyle X}( au)$	Estimated autocorrelation function
	of the signal $x(t)$ or $x[n]$
$\hat{S}_{x}(f), \hat{S}_{x}(j\omega)$	Estimated power spectral density
	(PSD) of $x(t)$ in linear $(f)$ or angular
	$(\omega)$ frequency
$\hat{\mathbf{R}}_{\mathbf{x}}$	Sample (auto)correlation matrix
$\hat{r}_{x,d}( au), \hat{R}_{x,d}( au)$	Estimated cross-correlation between
	x[n] and $d[n]$ or $x(t)$ and $d(t)$
$\hat{S}_{x,y}(f), \hat{S}_{x,y}(j\omega)$	Estimated cross PSD of $x(t)$ and $y(t)$
	in linear or angular $(\omega)$ frequency
$\hat{\mathbf{R}}_{\mathbf{x}\mathbf{v}}$	Sample cross-correlation matrix of
·	$R_{xy}$

$\hat{ ho}_{x,y}$	Estimated Pearson correlation coefficient between $x$ and $y$
$\hat{c}_{\scriptscriptstyle X}( au),\hat{C}_{\scriptscriptstyle X}( au)$	Estimated autocovariance function of
	the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_{\mathbf{x}}, \hat{\mathbf{K}}_{\mathbf{x}}, \hat{\mathbf{\Sigma}}_{\mathbf{x}}$	Sample (auto)covariance matrix
$\hat{c}_{xy}( au), \hat{C}_{xy}( au)$	Estimated cross-covariance function
	of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_{\mathbf{x}\mathbf{y}},\hat{\mathbf{K}}_{\mathbf{x}\mathbf{y}},\hat{\mathbf{\Sigma}}_{\mathbf{x}\mathbf{y}}$	Sample cross-covariance matrix
$\mathbf{w}, \mathbf{\theta}$	Parameters, coefficients, or weights
	vector
$\mathbf{w}_{o}, \mathbf{w}^{\star}, \mathbf{\theta}_{o}, \mathbf{\theta}^{\star}$	Optimum value of the parameters,
	coefficients, or weights vector
$\mathbf{W}$	Matrix of the weights
J	Jacobian matrix
H	Hessian matrix
$\hat{\mathbf{H}}$	Estimate of the Hessian matrix

# 5 Linear Algebra

## 5.1 Common matrices and vectors

$\mathbf{W}, \mathbf{D}$	Diagonal matrix
P	Projection matrix; Permutation ma-
	trix
J	Jordan matrix
${f L}$	Lower matrix
$\mathbf{U}$	Upper matrix
$\mathbf{C}$	Cofactor matrix
$\mathbf{C}_{\mathbf{A}}, \operatorname{cof}\left(\mathbf{A}\right)$	Cofactor matrix of <b>A</b>
$\mathbf{S}$	Symmetric matrix
Q	Orthogonal matrix
$\mathbf{I}_N$	$N \times N$ -dimensional identity matrix
$0_{M imes N}$	$M \times N$ -dimensional null matrix
$0_N$	N-dimensional null vector
$1_{M imes N}$	$M \times N$ -dimensional ones matrix
$1_N$	N-dimensional ones vector
0	Null matrix, vector, or tensor (di-
	mensionality understood by context)
1	Ones matrix, vector, or tensor (di-
	mensionality understood by context)

## 5.2 Indexing

 $x_{i_1,i_2,...,i_N}, [\mathcal{X}]_{i_1,i_2,...,i_N}$ Element the position in  $(i_1, i_2, \ldots, i_N)$  of the tensor  $\boldsymbol{\mathcal{X}}$  $\mathcal{X}^{(n)}$ nth tensor of a nontemporal sequence nth column of the matrix X $\mathbf{x}_n, \mathbf{x}_{:n}$ nth row of the matrix X $\mathbf{x}_{n}$ : Mode-n fiber of the tensor  $\boldsymbol{\mathcal{X}}$  $\mathbf{X}_{i_1,...,i_{n-1},:,i_{n+1},...,i_N}$ Column fiber (mode-1 fiber) of the  $\mathbf{X}_{:,i_2,i_3}$ thrid-order tensor  $\boldsymbol{\mathcal{X}}$ Row fiber (mode-2 fiber) of the thrid- $\mathbf{x}_{i_1,:,i_3}$ order tensor  $\mathcal{X}$ Tube fiber (mode-3 fiber) of the  $\mathbf{x}_{i_1,i_2,:}$ thrid-order tensor  $\boldsymbol{\mathcal{X}}$  $X_{i_1,:,:}$ Horizontal slice of the thrid-order tensor  $\boldsymbol{\mathcal{X}}$  $\mathbf{X}_{:,i_2,:}$ Lateral slices slice of the thrid-order tensor  $\mathcal{X}$  $X_{i_3}, X_{:,:,i_3}$ Frontal slices slice of the thrid-order tensor  $\boldsymbol{\mathcal{X}}$ 

### 5.3 General operations

 $\langle \cdot, \cdot \rangle$ Inner product, e.g.,  $\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^{\mathsf{T}} \mathbf{b}$ Outer product, e.g.,  $\mathbf{a} \circ \mathbf{b} = \mathbf{a} \mathbf{b}^{\mathsf{T}}$ Kronecker product  $\otimes$ Hadamard (or Schur) (elementwise)  $\odot$ product .⊙nnth-order Hadamard power  $\cdot \circ \frac{1}{n}$ nth-order Hadamard root 0 Hadamard (or Schur) (elementwise) division Khatri-Rao product  $\Diamond$ Kronecker Product  $\otimes$ *n*-mode product  $\times_n$ 

#### 5.4 Operations with matrices and tensors

 $\mathbf{A}^{-1}$ Inverse matrix  $A^+, A^\dagger$ Moore-Penrose left pseudoinverse  $\mathbf{A}^{\top}$ Transpose  $\mathbf{A}^{-\top}$ Transpose of the inverse  $\mathbf{A}^*$ Complex conjugate  $\mathbf{A}^\mathsf{H}$ Hermitian Frobenius norm  $\|\mathbf{A}\|_{\mathrm{F}}$  $\|\mathbf{A}\|$ Matrix norm

$ \mathbf{A} , \det{(\mathbf{A})}$	Determinant
$\operatorname{diag}\left(\mathbf{A}\right)$	The elements in the diagonal of $\bf A$
$\text{vec}\left(\mathbf{A}\right)$	Vectorization: stacks the columns of
	the matrix A into a long column vec-
	tor
$\mathrm{vec_d}\left(\mathbf{A}\right)$	Extracts the diagonal elements of a
	square matrix and returns them in a
	column vector
$\operatorname{vec}_{\operatorname{l}}\left(\mathbf{A} ight)$	Extracts the elements strictly below
	the main diagonal of a square matrix
	in a column-wise manner and returns
	them into a column vector
$\operatorname{vec}_{\mathrm{u}}\left(\mathbf{A}\right)$	Extracts the elements strictly above
	the main diagonal of a square matrix
	in a column-wise manner and returns
	them into a column vector
$\operatorname{vec_b}\left(\mathbf{A}\right)$	Block vectorization operator: stacks
	square block matrices of the input
	into a long block column matrix
$\operatorname{unvec}\left(\mathbf{A}\right)$	Reshapes a column vector into a ma-
	trix
$\mathrm{tr}\left(\mathbf{A} ight)$	trace
$\mathbf{X}_{(n)}$	$n\text{-}\mathrm{mode}$ matricization of the tensor $oldsymbol{\mathcal{X}}$

## 5.5 Operations with vectors

$\ \mathbf{a}\ $	$l_1$ norm, 1-norm, or Manhatan norm
$\ \mathbf{a}\ , \ \mathbf{a}\ _2$	$l_2$ norm, 2-norm, or Euclidean norm
$\ \mathbf{a}\ _p$	$l_p$ norm, $p$ -norm, or Minkowski norm
$\ \mathbf{a}\ _{\infty}$	$l_{\infty}$ norm, $\infty$ -norm, or Chebyshev
	norm
$\operatorname{diag}\left(\mathbf{a}\right)$	Diagonalization: a square, diagonal
	matrix with entries given by the vec-
	tor a

## 5.6 Decompositions

Λ	Eigenvalue matrix
Q	Eigenvectors matrix; Orthogonal ma-
	trix of the QR decomposition
R	Upper triangular matrix of the QR
	decomposition
U	Left singular vectors
$\mathbf{U}_r$	Left singular nondegenerated vectors

$\Sigma$	Singular value matrix
$\Sigma_r$	Singular value matrix with nonzero
	singular values in the main diagonal
$\Sigma^+$	Singular value matrix of the pseu-
	doinverse
$\Sigma_r^+$	Singular value matrix of the pseu-
	doinverse with nonzero singular val-
	ues in the main diagonal
V	Right singular vectors
$\mathbf{V}_r$	Right singular nondegenerated vec-
	tors
$\operatorname{eig}\left(\mathbf{A}\right)$	Set of the eigenvalues of <b>A</b>
$[\![\mathbf{A},\mathbf{B},\mathbf{C},\ldots]\!]$	CANDECOMP/PARAFAC (CP) de-
	composition of the tensor ${\cal X}$ from the
	outer product of column vectors of <b>A</b> ,
	B, C,
$[\![\boldsymbol{\lambda};\mathbf{A},\mathbf{B},\mathbf{C},\ldots]\!]$	Normalized CANDE-
	COMP/PARAFAC (CP) decom-
	position of the tensor ${\cal X}$ from the
	outer product of column vectors of
	$A, B, C, \dots$

## 5.7 Spaces

$\mathrm{span}\left(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_n\right)$	Vector space spanned by the argument vectors
$C(\mathbf{A})$ , columnspace( $\mathbf{A}$ ), range( $\mathbf{A}$ ), span ( $\mathbf{A}$ ), image( $\mathbf{A}$ )	Columnspace, range or image, i.e., the space span $(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$ , where $\mathbf{a}_i$ is the ith column vector of the ma-
	trix A
$C(\mathbf{A}^{H})$	Row space
$N(\mathbf{A})$ , $null space(\mathbf{A})$ , $kernel(\mathbf{A})$	Nullspace (or kernel space)
$N\left(\mathbf{A}^{H}\right)$	Left nullspace
$\operatorname{rank}\left(\mathbf{\hat{A}}\right)$	Rank, that is, $\dim(\text{span}(\mathbf{A})) =$
	$\dim (C (\mathbf{A}))$
nullity ( <b>A</b> )	Nullity of $\mathbf{A}$ , i.e., dim $(N(\mathbf{A}))$
$\mathbf{a} \perp \mathbf{b}$	<b>a</b> is orthogonal to <b>b</b>
a ⊥ b	$\mathbf{a}$ is not orthogonal to $\mathbf{b}$

## 5.8 Inequalities

 $\mathcal{X} \leq 0$ 

Nonnegative tensor

$\mathbf{a} \leq_K \mathbf{b}$	Generalized inequality meaning that
	$\mathbf{b} - \mathbf{a}$ belongs to the conic subset $K$ in
a / h	the space $\mathbb{R}^n$
$\mathbf{a} \prec_K \mathbf{b}$	Strict generalized inequality meaning
	that $\mathbf{b} - \mathbf{a}$ belongs to the interior of the conic subset $K$ in the space $\mathbb{R}^n$
$\mathbf{a} \leq \mathbf{b}$	Generalized inequality meaning that
a = b	$\mathbf{b} - \mathbf{a}$ belongs to the nonnegative or-
	thant conic subset, $\mathbb{R}^n_+$ , in the space
	$\mathbb{R}^n$ .
$\mathbf{a} \prec \mathbf{b}$	Strict generalized inequality meaning
	that $\mathbf{b} - \mathbf{a}$ belongs to the positive or-
	thant conic subset, $\mathbb{R}^n_{++}$ , in the space
	$\mathbb{R}^n$
$\mathbf{A} \leq_K \mathbf{B}$	Generalized inequality meaning that
	$\mathbf{B} - \mathbf{A}$ belongs to the conic subset $K$
	in the space $\mathbb{S}^n$
$\mathbf{A} \prec_K \mathbf{B}$	Strict generalized inequality meaning
	that $\mathbf{B} - \mathbf{A}$ belongs to the interior of
A ~ D	the conic subset $K$ in the space $\mathbb{S}^n$
$A \leq B$	Generalized inequality meaning that <b>B-A</b> belongs to the positive semidef-
	inite conic subset, $\mathbb{S}_{+}^{n}$ , in the space $\mathbb{S}^{n}$
A < B	Strict generalized inequality meaning
	that $\mathbf{B} - \mathbf{A}$ belongs to the positive or-
	thant conic subset, $\mathbb{S}^n_{++}$ , in the space
	$\mathbb{S}^n$

## 6 Sets

A+B	Set addition (Minkowski sum)
A - B	Minkowski difference
$A \setminus B, A - B$	Set difference or set subtraction,
	i.e., the set containing the elements
	of $A$ that are not in $B$
$A \cup B$	Set of union
$A \cap B$	Set of intersection
$A \times B$	Cartesian product
$A^n$	$A \times A \times \cdots \times A$
	n  times
$A^{\perp}$	Orthogonal complement of $A$ , e.g.,
	$N(\mathbf{A}) = C(\mathbf{A}^{T})^{\perp}$
$A \oplus B$	Direct sum, e.g., $C(\mathbf{A}^{T}) \oplus C(\mathbf{A}^{T})^{\perp} =$
	$\mathbb{R}^n$

$A^c, ar{A}$	Complement set (given $U$ )
#A,  A	Cardinality
$a \in A$	a is element of $A$
$a \notin A$	a is not element of $A$
$\{1,2,\ldots,n\}$	Discrete set containing the integer el-
$(1, 2, \dots, n)$	ements $1, 2, \ldots, n$
U	Universe
$\frac{C}{2^A}$	Power set of A
$\mathbb{R}$	Set of real numbers
C	Set of real numbers Set of complex numbers
7.	Set of complex numbers Set of integer number
$\mathbb{B} = \{0, 1\}$	Boolean set
Ø = {0, 1}	
N	Empty set Set of natural numbers
-,	
$\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}\$ $\mathbb{K}^{I_1 \times I_2 \times \dots \times I_N}$	Real or complex space (field)
	$I_1 \times I_2 \times \cdots \times I_N$ -dimensional real (or
TP	complex) space
K <sub>+</sub>	Nonnegative real (or complex) space
$\mathbb{K}_{++}$	Positive real (or complex) space, i.e.,
$\mathbb{S}^n, \mathcal{S}^n$	$\mathbb{K}_{++} = \mathbb{K}_{+} \setminus \{0\}$
$S^n, S^n$	Conic set of the symmetric matrices
©n Cn	in $\mathbb{R}^{n \times n}$
$\mathbb{S}^n_+, \mathcal{S}^n_+$	Conic set of the symmetric positive $\mathbb{R}^{n \times n}$
on on	semidefinite matrices in $\mathbb{R}^{n \times n}$
$\mathbb{S}^n_{++}, \mathcal{S}^n_{++}$	Conic set of the symmetric positive $\mathbb{R}^{n \times n}$ .
	definite matrices in $\mathbb{R}^{n \times n}$ , i.e., $\mathbb{S}_{++}^n = \mathbb{S}_{++}^n$
ппи	$\mathbb{S}^n_+ \setminus \{0\}$
$\mathbb{H}^n$	Set of all hermitian matrices in $\mathbb{C}^{n\times n}$
[a,b]	Closed interval of a real set from $a$ to
	<i>b</i>
(a,b)	Opened interval of a real set from a
[ 1\	to b
[a,b),(a,b]	Half-opened intervals of a real set
	from $a$ to $b$

# 7 Communication systems

$x_i$	Real or in-phase part of $x$
$x_q$	Imaginary or quadrature part of $x$
S	Transmitted signal
$\phi$	Signal phase
$s_l$	Low-pass equivalent signal or enve-
	lope complex of $s$
$\eta, w$	Gaussian noise

r	Received signal
τ	Timing delay
$\Delta  au$	Timing error (delay - estimated)
arphi	Phase offset
$\Delta arphi$	Phase error (offset - estimated)
$f_d$	Doppler frequency
A	Received signal amplitude
γ	Combined effect of the path loss and
	antenna gain

## 8 Other notations

## 8.1 Mathematical symbols

3	There exists
∄	There does not exist
∃!	There exist an unique
€	Belongs to
∉	Does not belong to
	Q.E.D.
:	Therefore
:	Because
$\forall$	For all
<b> </b> ,:	Such that
$\iff$	Logical equivalence
≜,:=	Equal by definition
<b>≠</b>	Not equal
$\infty$	Infinity
j	$\sqrt{-1}$
$W_N$	Twiddle factor, $e^{-j\frac{2\pi}{N}}$

## 8.2 Operations

$arg \max_{x} f(x)$	Value of $x$ that minimizes $x$
$\underset{x \in \mathcal{A}}{\operatorname{argmin}} f(x)$	Value of $x$ that minimizes $x$
$\inf_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Infimum
$\sup g(\mathbf{x}, \mathbf{y})$	Supremum
$y \in A$ $ a $	Absolute value of $a$
log	Base-10 logarithm or decimal loga-
	$\operatorname{rithm}$
$\ln$	Natual logarithm

 $\operatorname{Re}\left\{ x\right\}$ Real part of x $\operatorname{Im}\left\{ x\right\}$ Imaginary part of x۷٠ phase (complex argument) Remainder, i.e.,  $x - y \lfloor x/y \rfloor$  $x \mod y$ Fractional part, i.e.,  $x \mod 1$  $\operatorname{frac}(x)$  $a \wedge b$ Logical AND of a and b $a \lor b$ Logical OR of a and bLogical negation of a $\neg a$  $\lceil \cdot \rceil$ Ceiling operation  $\lfloor \cdot \rfloor$ Floor operation

#### 8.3 **Functions**

 $\mathcal{O}(\cdot), O(\cdot)$ Big-O notation  $\Gamma(\cdot)$ Gamma function

#### Abbreviations 9

With respect to wrt. st. Subject to iff. If and only if EVD

Eigenvalue decomposition, or eigen-

decomposition

 $\operatorname{SVD}$ Singular value decomposition  $\operatorname{CP}$ CANDECOMP/PARAFAC