Notation

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1 Font notation

$a, b, c, \ldots, A, B, C, \ldots$	Scalars
a, b, c, \dots	Vectors
A, B, C, \dots	Matrices
$\mathcal{A}, \mathcal{B}, \mathcal{C}, \dots$	Tensors
$A, B, C, \ldots, A, B, C, \ldots, A, B, C, \ldots$	Sets

2 Signals and functions

2.1 Time indexing

x(t)	Continuous-time t
$x[n],x[k],x[m],x[i],\ldots$	Discrete-time n, k, m, i, \dots (parenthe-
$x_n, x_k, x_m, x_i, \dots$	sis should be adopted only if there
$x(n), x(k), x(m), x(i), \dots$	are no continuous-time signals in the
	context to avoid ambiguity)

2.2 Common functions

$x_I(t)$ or $x_I[n]$	Real or in-phase part of $x(t)$ or $x[n]$
$x_Q(t)$ or $x_Q[n]$	Imaginary or quadrature part of $x(t)$
	or $x[n]$
$\delta(t)$	Delta function
$\delta[n]$	Kronecker function
h(t), h[n]	Impulse response (continuous and
	discrete time)

$\tilde{x}[n], \tilde{x}(t)$	Periodic discrete- or continuous-time
	signal
$\hat{x}[n], \hat{x}(t)$	Estimate of $x[n]$ or $x(t)$
$\dot{x}[m]$	Interpolation of $x[n]$

2.3 Operations and symbols

$f:A\to B$	A function f whose domain is A and
	codomain is B
f^n	nth power of the function f
$x^{k}(t), x^{k}[n]$ f^{-1}	kth power of $x[n]$ or $x(t)$
	Inverse function of f
$x^{-1}(t), x^{-1}[n]$	Inverse of $x[n]$ or $x(t)$
$f^{(n)}$	nth derivative of the function f
$x^{(n)}(t)$	nth derivative of $x(t)$
$f', f^{(1)}$	1th derivative of the function f
x'(t)	1th derivative of $x(t)$
$f'', f^{(2)}$	2th derivative of the function f
x''(t)	2th derivative of $x(t)$
$\inf_{\mathbf{y}\in\mathcal{A}}g(\mathbf{x},\mathbf{y})$	Infimum
$\sup g(\mathbf{x},\mathbf{y})$	Supremum
$y \in A$ $f \circ g$	Composition of the functions f and
J ~ 8	g
*	8 Convolution
*(N)	Circular convolution
$x \left[((n-m))_N \right], x \left((n-m) \right)_N$	
$x \left[((n-m))_N \right], x \left((n-m) \right)_N$	Circular shift in <i>m</i> samples within a
	N-samples window

2.4 Transformations

$\mathcal{F}\left\{ \cdot \right\}$	Fourier transform
$\mathcal{L}\left\{ \cdot \right\}$	Laplace transform
$\mathcal{Z}\left\{ \cdot \right\}$	z-transform
$\hat{x}(t)$ or $\hat{x}[n]$	Hilbert transform of $x(t)$ or $x[n]$
X(s)	Laplace transform of $x(t)$
X(f)	Fourier transform (FT) (in linear fre-
	quency, Hz) of $x(t)$
$X(j\omega)$	Fourier transform (FT) (in angular
	frequency, rad/sec) of $x(t)$
$X(e^{j\omega})$	Discrete-time Fourier transform
	(DTFT) of $x[n]$

 $X[k], X(k), X_k$ Discrete Fourier transform (DFT) or fast Fourier transform (FFT) of x[n], or even the Fourier series (FS) of the periodic signal x(t) Discrete Fourier series (DFS) of $\tilde{x}[n]$ X(z) z-transform of x[n]

3 Probability, statistics, and stochastic processes

3.1 Operators and symbols

 $E\left[\cdot\right]$ Statistical expectation $E_u[\cdot]$ Statistical expectation with respect Mean of the random variable x μ_x Mean vector of the random variable $\boldsymbol{\mu}_x, \boldsymbol{m}_x$ nth-order moment of a random vari- μ_n \mathcal{K}_x, μ_4 Kurtosis (4th-order moment) of the random variable x $VAR[\cdot]$ Variance operator nth-order cumulant of a random vari- κ_n Variance of the random variable x σ_x, κ_2 Pearson correlation coefficient be- $\rho_{x,y}$ tween x and y $a \sim P$ Random variable a with distribution

3.2 Stochastic processes

$r_X(\tau), R_X(\tau)$	Autocorrelation function of the signal $x(t)$ or $x[n]$
$S_x(f)$	Power spectral density of $x(t)$ in linear frequency
$S_x(j\omega)$	Power spectral density of $x(t)$ in angular frequency
$\mathbf{R_x}$	(Auto)correlation matrix of $\mathbf{x}(n)$
$r_{x,d}(\tau), R_{x,d}(\tau)$	Cross-correlation between $x[n]$ and
	d[n] or $x(t)$ and $d(t)$
R_{xy}	Cross-correlation matrix of $\mathbf{x}(n)$ and
	$\mathbf{y}(n)$

$\mathbf{p}_{\mathbf{x}d}$	Cross-correlation vector between
	$\mathbf{x}(n)$ and $d(n)$
$c_x(\tau), C_x(\tau)$	Autocovariance function of the signal
	x(t) or $x[n]$
C_x, K_x, Σ_x	(Auto)covariance matrix of \mathbf{x}
$c_{xy}(\tau), C_{xy}(\tau)$	Cross-covariance function of the sig-
	$\operatorname{nal} x(t) \text{ or } x[n]$
$\mathrm{C}_{\mathrm{xy}}, \mathrm{K}_{\mathrm{xy}}, \Sigma_{\mathrm{xy}}$	Cross-covariance matrix of ${\bf x}$ and ${\bf y}$

3.3 Functions

$Q(\cdot)$	Q-function, i.e., $P[\mathcal{N}(0,1) > x]$
$\operatorname{erf}(\cdot)$	Error function
$\operatorname{erfc}(\cdot)$	Complementary error function i.e.,
	$\operatorname{erfc}(x) = 2Q(\sqrt{2}x) - \operatorname{erf}(x)$
P[A]	Probability of the event or set A
$p(\cdot), f(\cdot)$	Probability density function (PDF)
	or probability mass function (PMF)
$p(x \mid A)$	Conditional PDF or PMF
$F(\cdot)$	Cumulative distribution function
	(CDF)
$\Phi_X(\omega), M_X(j\omega), E\left[e^{j\omega x}\right]$	First characteristic function (CF) of
	X
$M_X(t), \Phi_X(-jt), E[e^{tX}]$	Moment-generating function (MGF)
	of x
$\Psi_x(\omega), \ln \Phi_x(\omega), \ln E\left[e^{j\omega x}\right]$	Second characteristic function
$K_X(t), \ln E\left[e^{tx}\right], \ln M_X(t)$	Cumulant-generating function (CGF) of x

3.4 Distributions

$\mathcal{N}(\mu,\sigma^2)$	Gaussian distribution of a random
	variable with mean μ and variance σ^2
$\mathcal{CN}(\mu, \sigma^2)$	Complex Gaussian distribution of a
	random variable with mean μ and
	variance σ^2
$\mathcal{N}(oldsymbol{\mu}, oldsymbol{\Sigma})$	Gaussian distribution of a vector ran-
	dom variable with mean μ and co-
	variance matrix Σ
$\mathcal{CN}(oldsymbol{\mu},oldsymbol{\Sigma})$	Complex Gaussian distribution of a
	vector random variable with mean μ
	and covariance matrix Σ
$\mathcal{U}(a,b)$	Uniform distribution from a to b

$\chi^2(n), \chi_n^2$	Chi-square distribution with n degree of freedom (assuming that the Gaussians are $\mathcal{N}(0,1)$)
$\operatorname{Exp}(\lambda)$	Exponential distribution with rate parameter λ
$\Gamma(lpha,eta)$	Gamma distribution with shape parameter α and rate parameter β
$\Gamma(lpha, heta)$	Gamma distribution with shape parameter α and scale parameter $\theta = 1/\beta$
$\operatorname{Nakagami}(m,\Omega)$	Nakagami-m distribution with shape parameter m and spread parameter Ω
$\operatorname{Rayleigh}(\sigma)$	Rayleigh distribution with scale parameter σ
$\operatorname{Rayleigh}(\Omega)$	Rayleigh distribution with the second moment $\Omega = E[x^2] = 2\sigma^2$
$\mathrm{Rice}(s,\sigma)$	Rice distribution with noncentrality parameter (specular component) s and σ
$\mathrm{Rice}(A,K)$	Rice distribution with Rice factor $K=s^2/2\sigma^2$ and scale parameter $A=s^2+2\sigma^2$

4 Statistical signal processing

Gradient descent vector
Gradient descent vector with respect
x
Stochastic gradient descent (SGD)
Cost-function or objective function
Likelihood function
Log-likelihood function
Estimate of $x(t)$ or $x[n]$
Sample mean of $x[n]$ or $x(t)$
Sample mean vector of $\mathbf{x}[n]$ or $\mathbf{x}(t)$
Estimated autocorrelation function
of the signal $x(t)$ or $x[n]$
Sample (auto)correlation matrix
Estimated cross-correlation between
x[n] and $d[n]$ or $x(t)$ and $d(t)$
Sample cross-correlation matrix of
$\mathbf{R}_{\mathbf{x}\mathbf{y}}$

$\hat{ ho}_{x,y}$	Estimated Pearson correlation coefficient between x and y
$\hat{c}_X(au), \hat{C}_X(au)$	Estimated autocovariance function of the signal $x(t)$ or $x[n]$
$\hat{ ext{C}}_{ ext{x}}, \hat{ ext{K}}_{ ext{x}}, \hat{\Sigma}_{ ext{x}}$	Sample (auto)covariance matrix
$\hat{c}_{xy}(au),\hat{C}_{xy}(au)$	Estimated cross-covariance function of the signal $x(t)$ or $x[n]$
$\hat{ extbf{C}}_{ ext{xy}}, \hat{ extbf{K}}_{ ext{xy}}, \hat{oldsymbol{\Sigma}}_{ ext{xy}}$	Sample cross-covariance matrix
$\mathbf{w}, \mathbf{\theta}$	Parameters, coefficients, or weights vector
$\mathbf{w}_{\scriptscriptstyle O}, \mathbf{w}^{\star}, \mathbf{\theta}_{\scriptscriptstyle O}, \mathbf{\theta}^{\star}$	Optimum value of the parameters, coefficients, or weights vector
W	Matrix of the weights
J	Jacobian matrix
H	Hessian matrix

5 Linear Algebra

5.1 Common matrices and vectors

\mathbf{W}, \mathbf{D}	Diagonal matrix
P	Projection matrix; Permutation ma-
	trix
J	Jordan matrix
${f L}$	Lower matrix
\mathbf{U}	Upper matrix
\mathbf{C}	Cofactor matrix
$\mathbf{C}_{\mathbf{A}}, \operatorname{cof}\left(\mathbf{A}\right)$	Cofactor matrix of A
\mathbf{S}	Symmetric matrix
Q	Orthogonal matrix
\mathbf{I}_N	$N \times N$ -dimensional identity matrix
$0_{M imes N}$	$M \times N$ -dimensional null matrix
0_N	N-dimensional null vector
$1_{M imes N}$	$M \times N$ -dimensional ones matrix
1_N	N-dimensional ones vector
0	Null matrix, vector, or tensor (di-
	mensionality understood by context)
1	Ones matrix, vector, or tensor (di-
	mensionality understood by context)
	· ,

5.2 Indexing

 $x_{i_1,i_2,...,i_N}, [\mathcal{X}]_{i_1,i_2,...,i_N}$ Element the position in (i_1, i_2, \dots, i_N) of the tensor $\boldsymbol{\mathcal{X}}$ $\mathcal{X}^{(n)}$ nth tensor of a nontemporal sequence nth column of the matrix X $\mathbf{x}_n, \mathbf{x}_{:n}$ nth row of the matrix X \mathbf{x}_{n} : Mode-n fiber of the tensor $\boldsymbol{\mathcal{X}}$ $\mathbf{x}_{i_1,...,i_{n-1},:,i_{n+1},...,i_N}$ Column fiber (mode-1 fiber) of the $\mathbf{X}_{:,i_2,i_3}$ thrid-order tensor $\boldsymbol{\mathcal{X}}$ Row fiber (mode-2 fiber) of the thrid- $\mathbf{x}_{i_1,:,i_3}$ order tensor \mathcal{X} Tube fiber (mode-3 fiber) of the $\mathbf{x}_{i_1,i_2,:}$ thrid-order tensor $\boldsymbol{\mathcal{X}}$ $X_{i_1,:,:}$ Horizontal slice of the thrid-order $\mathbf{X}_{:,i_2,:}$ Lateral slices slice of the thrid-order tensor $\boldsymbol{\mathcal{X}}$ $\mathbf{X}_{i_3}, \mathbf{X}_{:,:,i_3}$ Frontal slices slice of the thrid-order tensor $\boldsymbol{\mathcal{X}}$

5.3 Operations with tensors

 $\mathbf{X}_{(n)}$ n-mode matricization of the tensor $\boldsymbol{\mathcal{X}}$

5.4 Operations with matrices

\mathbf{A}^{-1}	Inverse matrix
${f A}^+,{f A}^\dagger$	Moore-Penrose pseudoinverse
$\mathbf{A}^ op$	Transpose
\mathbf{A}^*	Complex conjugate
\mathbf{A}^H	Hermitian
$\ \mathbf{A}\ _{\mathrm{F}}$	Frobenius norm
$\ \mathbf{A}\ $	Matrix norm
$ \mathbf{A} , \det{(\mathbf{A})}$	Determinant
$\operatorname{diag}\left(\mathbf{A}\right)$	The elements in the diagonal of $\bf A$
$\text{vec}(\mathbf{A})$	Vectorization: stacks the columns of
	the matrix A into a long column vec-
	tor
$\operatorname{vec}_{\operatorname{d}}\left(\mathbf{A}\right)$	Extracts the diagonal elements of a
	square matrix and returns them in a
	column vector
$\mathrm{vec}_{\mathrm{l}}\left(\mathbf{A} ight)$	Extracts the elements strictly below
	the main diagonal of a square matrix
	in a column-wise manner and returns
	them into a column vector

$\mathrm{vec_u}\left(\mathbf{A}\right)$	Extracts the elements strictly above the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\mathrm{vec_b}\left(\mathbf{A}\right)$	Block vectorization operator: stacks square block matrices of the input into a long block column matrix
$\mathrm{unvec}\left(\mathbf{A}\right)$	Reshapes a column vector into a matrix
$\mathrm{tr}\left(\mathbf{A} ight)$	trace
\otimes	Kronecker product
⊙	Hadamard (or Schur) (elementwise) product
$\mathbf{A}^{\odot n}$	n th-order Hadamard power of the matrix \mathbf{A}
$\mathbf{A}^{\odot rac{1}{n}}$	nth-order Hadamard root of the matrix A
∅	Hadamard (or Schur) (elementwise)
	division
♦	Khatri-Rao product
\otimes	Kronecker Product
\times_n	<i>n</i> -mode product

5.5 Operations with vectors

$\ \mathbf{a}\ $	l_1 norm, 1-norm, or Manhatan norm
$\ \mathbf{a}\ , \ \mathbf{a}\ _2$	l_2 norm, 2-norm, or Euclidean norm
$\ \mathbf{a}\ _p$	l_p norm, p -norm, or Minkowski norm
$\ \mathbf{a}\ _{\infty}^{2}$	l_{∞} norm, ∞ -norm, or Chebyshev
	norm
$\operatorname{diag}\left(\mathbf{a}\right)$	Diagonalization: a square, diagonal
	matrix with entries given by the vec-
	tor a
$\langle \mathbf{a}, \mathbf{b} angle$	Inner product, i.e., $\mathbf{a}^{T}\mathbf{b}$
$\mathbf{a} \circ \mathbf{b}$	Outer product, i.e., $\mathbf{a}\mathbf{b}^{T}$

5.6 Decompositions

Eigenvalue matrix
Eigenvectors matrix; Orthogonal ma-
trix of the QR decomposition
Upper triangular matrix of the QR
decomposition
Left singular vectors

\mathbf{U}_r	Left singular nondegenerated vectors
Σ	Singular value matrix
Σ_r	Singular value matrix with nonzero
	singular values in the main diagonal
Σ^+	Singular value matrix of the pseu-
	doinverse
Σ_r^+	Singular value matrix of the pseu-
,	doinverse with nonzero singular val-
	ues in the main diagonal
V	Right singular vectors
\mathbf{V}_r	Right singular nondegenerated vec-
	tors
$\operatorname{eig}\left(\mathbf{A}\right)$	Set of the eigenvalues of A
$\llbracket \mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots bracket$	CANDECOMP/PARAFAC (CP) de-
	composition of the tensor ${\cal X}$ from the
	outer product of column vectors of A ,
	B, C,
$\llbracket \boldsymbol{\lambda}; \mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots bracket$	Normalized CANDE-
<u>.</u> , <u> </u>	COMP/PARAFAC (CP) decom-
	position of the tensor \mathcal{X} from the
	outer product of column vectors of
	A, B, C,
	11, 12, 12,

5.7 Spaces

$N(\mathbf{A})$, nullspace(\mathbf{A}), kernel(\mathbf{A})	Nullspace (or kernel space)
$C(\mathbf{A})$, columnspace(\mathbf{A}), range(\mathbf{A})	Columnspace (or range), i.e., the
	space span $(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$, where \mathbf{a}_i is
	the ith column vector of the matrix
	\mathbf{A}
$\mathrm{span}\left(\mathbf{a}_{1},\mathbf{a}_{2},\ldots,\mathbf{a}_{n}\right)$	Vector space spanned by the argument vectors
$\mathrm{span}\left(\mathbf{A}\right)$	Vector space spanned by the col-
• , ,	umn vectors of A , which gives the
	columnspace of A
$\operatorname{rank}\left(\mathbf{A}\right)$	Rank, that is, $\dim(\text{span}(\mathbf{A})) =$
, ,	$\dim (C(\mathbf{A}))$
nullity (A)	Nullity of \mathbf{A} , i.e., dim $(N(\mathbf{A}))$
$\mathbf{a} \perp \mathbf{b}$	a is orthogonal to b
a ⊥ b	a is not orthogonal to b

5.8 Inequalities

$\mathcal{X} \leq 0$	Nonnegative tensor
$\mathbf{a} \leq_K \mathbf{b}$	Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the conic subset K in
$\mathbf{a} \prec_K \mathbf{b}$	the space \mathbb{R}^n Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the interior of
$\mathbf{a} \leq \mathbf{b}$	the conic subset K in the space \mathbb{R}^n Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the nonnegative orthant conic subset, \mathbb{R}^n_+ , in the space
$\mathbf{a} < \mathbf{b}$	\mathbb{R}^n Strict generalized inequality meaning
	that $\mathbf{b} - \mathbf{a}$ belongs to the positive orthant conic subset, \mathbb{R}^n_{++} , in the space \mathbb{R}^n
$\mathbf{A} \leq_K \mathbf{B}$	Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the conic subset K
$\mathbf{A} \prec_K \mathbf{B}$	in the space \mathbb{S}^n Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the interior of
$A \leq B$	the conic subset K in the space \mathbb{S}^n Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive semidef-
A < B	inite conic subset, \mathbb{S}_{+}^{n} , in the space \mathbb{S}^{n} Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive or-
	thant conic subset, \mathbb{S}_{++}^n , in the space \mathbb{S}^n

6 Sets

A + B	Set addition (Minkowski sum)
A - B	Minkowski difference
$A \setminus B, A - B$	Set difference or set subtraction,
	i.e., the set containing the elements
	of A that are not in B
$A \cup B$	Set of union
$A \cap B$	Set of intersection
$A \times B$	Cartesian product
A^n	$A \times A \times \cdots \times A$
	n times
A^{\perp}	Orthogonal complement of A , e.g.,
	$N(\mathbf{A}) = C(\mathbf{A}^{\top})^{\perp}$

A o D	Direct sum of $C(AT) \circ C(AT)^{\perp}$
$A \oplus B$	Direct sum, e.g., $C(\mathbf{A}^{\top}) \oplus C(\mathbf{A}^{\top})^{\perp} = \mathbb{R}^{n}$
$A^c, ar{A}$	Complement set (given U)
#A, A	Cardinality
$a \in A$	a is element of A
$a \notin A$	a is not element of A
$\{1,2,\ldots,n\}$	Discrete set containing the integer el-
$\{1,2,\ldots,n\}$	ements $1, 2, \ldots, n$
U	Universe
2^A	Power set of A
\mathbb{R}	Set of real numbers
C	Set of real numbers Set of complex numbers
\mathbb{Z}	Set of complex numbers Set of integer number
$\mathbb{B} = \{0, 1\}$	Boolean set
Ø = {0,1}	Empty set
Ŋ	Set of natural numbers
$\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$	Real or complex space (field)
$\mathbb{K}^{I_1 \times I_2 \times \cdots \times I_N}$	$I_1 \times I_2 \times \cdots \times I_N$ -dimensional real (or
112	$r_1 \wedge r_2 \wedge \cdots \wedge r_N$ -dimensional real (of complex) space
K ₊	Nonnegative real (or complex) space
K++	Positive real (or complex) space, i.e.,
1/2++	$\mathbb{K}_{++} = \mathbb{K}_+ \setminus \{0\}$
$\mathbb{S}^n, \mathcal{S}^n$	Conic set of the symmetric matrices
,,,	in $\mathbb{R}^{n \times n}$
$\mathbb{S}^n_+, \mathcal{S}^n_+$	Conic set of the symmetric positive
w+, v+	semidefinite matrices in $\mathbb{R}^{n \times n}$
$\mathbb{S}^n_{++}, \mathcal{S}^n_{++}$	Conic set of the symmetric positive
~++, ·· ++	definite matrices in $\mathbb{R}^{n \times n}$, i.e., $\mathbb{S}^{n}_{++} =$
	$\mathbb{S}_{+}^{n}\setminus\{0\}$
\mathbb{H}^n	Set of all hermitian matrices in $\mathbb{C}^{n\times n}$
[a,b]	Closed interval of a real set from a to
L	b
(a,b)	Opened interval of a real set from a
	to b
[a,b),(a,b]	Half-opened intervals of a real set
	from a to b

7 Other notations

7.1 Mathematical symbols

3	There exists
∄	There does not exist
3!	There exist an unique

7.2 Operations

arg max f(x)Value of x that minimizes x $x \in A$ arg min f(x)Value of x that minimizes x $x \in \mathcal{A}$ |a|Absolute value of alog Base-10 logarithm or decimal logarithm lnNatual logarithm $\operatorname{Re}\left\{ x\right\}$ Real part of x $\operatorname{Im}\left\{ x\right\}$ Imaginary part of x۷٠ phase (complex argument) $x \mod y$ Remainder, i.e., $x - y \lfloor x/y \rfloor$ Fractional part, i.e., $x \mod 1$ $\operatorname{frac}(x)$ $a \wedge b$ Logical AND of a and b $a \lor b$ Logical OR of a and bLogical negation of a $\neg a$ $\lceil \cdot \rceil$ Ceiling operation Floor operation $\lfloor \cdot \rfloor$

7.3 Functions

 $\mathcal{O}(\cdot), O(\cdot)$ Big-O notation $\Gamma(\cdot)$ Gamma function

8 Abbreviations

wrt. With respect to
st. Subject to
iff. If and only if
EVD Eigenvalue decomposition, or eigendecomposition
SVD Singular value decomposition