Notation

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1 Font notation

$a, b, c, \ldots, A, B, C, \ldots$	Scalars
a, b, c, \dots	Vectors
A, B, C, \dots	Matrices
$\mathcal{A}, \mathcal{B}, \mathcal{C}, \dots$	Tensors
$A, B, C, \ldots, A, B, C, \ldots, A, B, C, \ldots$	Sets

2 Signals and functions

2.1 Time indexing

x(t)	Continuous-time t
$x[n], x[k], x[m], x[i], \dots$	Discrete-time n, k, m, i, \dots (parenthe-
$x_n, x_k, x_m, x_i, \dots$	sis should be adopted only if there
$x(n), x(k), x(m), x(i), \dots$	are no continuous-time signals in the
	context to avoid ambiguity)
$x\left[\left((n-m)\right)_{N}\right], x\left((n-m)\right)_{N}$	Circular shift in m samples within a
	N-samples window

2.2 Common functions

$\delta(t)$	Delta function
$\delta[n], \delta_{i,j}$	Kronecker function $(n = i - j)$
h(t), h[n]	Impulse response (continuous and
	discrete time)

$\tilde{x}[n], \tilde{x}(t)$	Periodic discrete- or continuous-time
	signal
$\hat{x}[n], \hat{x}(t)$	Estimate of $x[n]$ or $x(t)$
$\dot{x}[m]$	Interpolation of $x[n]$

2.3 Operations and symbols

$f:A\to B$	A function f whose domain is A and
	codomain is B
$f^n, x^n(t), x^n[k]$	nth power of the function f , $x[n]$ or
	x(t)
$f^{(n)}, x^{(n)}(t)$	nth derivative of the function f or
	x(t)
$f', f^{(1)}, x'(t)$	1th derivative of the function f or
	x(t)
$f'', f^{(2)}, x''(t)$	2th derivative of the function f or
	x(t)
$f \circ g$	Composition of the functions f and
	g
*	Convolution (discrete or continuous)
$(8, \overline{N})$	Circular convolution

2.4 Transformations

$\mathcal{F}\left\{ \cdot ight\}$	Fourier transform
$\mathcal{L}\left\{ \cdot ight\}$	Laplace transform
$\mathcal{Z}\left\{ \cdot ight\}$	z-transform
$\hat{x}(t), \hat{x}[n]$	Hilbert transform of $x(t)$ or $x[n]$
X(s)	Laplace transform of $x(t)$
X(f)	Fourier transform (FT) (in linear fre-
	quency, Hz) of $x(t)$
$X(j\omega)$	Fourier transform (FT) (in angular
	frequency, rad/sec) of $x(t)$
$X(e^{j\omega})$	Discrete-time Fourier transform
	(DTFT) of $x[n]$
$X[k], X(k), X_k$	Discrete Fourier transform (DFT) or
	fast Fourier transform (FFT) of $x[n]$,
	or even the Fourier series (FS) of the
	periodic signal $x(t)$
$\tilde{X}[k], \tilde{X}(k), \tilde{X}_k$	Discrete Fourier series (DFS) of $\tilde{x}[n]$
X(z)	z-transform of $x[n]$

3 Probability, statistics, and stochastic processes

3.1 Operators and symbols

$E\left[\cdot\right]$ $E_{u}\left[\cdot\right]$	Statistical expectation Statistical expectation with respect
	to u
$\mu_{\scriptscriptstyle X}$	Mean of the random variable x
μ_x, m_x	Mean vector of the random variable
	X
μ_n	nth-order moment of a random vari-
	able
\mathcal{K}_x, μ_4	Kurtosis (4th-order moment) of the
	random variable x
$VAR[\cdot]$	Variance operator
$VAR_u[\cdot]$	Variance operator with respect to u
κ_n	nth-order cumulant of a random vari-
	able
σ_x, κ_2	Variance of the random variable x
$\rho_{x,y}$	Pearson correlation coefficient be-
	tween x and y
$a \sim P$	Random variable a with distribution
	P
\mathcal{R}	Rayleigh's quotient

3.2 Stochastic processes

$r_{X}(\tau), R_{X}(\tau)$	Autocorrelation function of the sig-
	$\operatorname{nal} x(t) \text{ or } x[n]$
$S_x(f), S_x(j\omega)$	Power spectral density (PSD) of $x(t)$
	in linear (f) or angular (ω) frequency
$S_{x,y}(f), S_{x,y}(j\omega)$	Cross PSD of $x(t)$ and $y(t)$ in linear
	or angular (ω) frequency
$\mathbf{R}_{\mathbf{x}}$	(Auto)correlation matrix of $\mathbf{x}(n)$
$r_{x,d}(\tau), R_{x,d}(\tau)$	Cross-correlation between $x[n]$ and
	d[n] or $x(t)$ and $d(t)$
$\mathbf{R}_{\mathbf{x}\mathbf{y}}$	Cross-correlation matrix of $\mathbf{x}(n)$ and
-	$\mathbf{y}(n)$
$\mathbf{p}_{\mathbf{x}d}$	Cross-correlation vector between
	$\mathbf{x}(n)$ and $d(n)$
$c_X(\tau), C_X(\tau)$	Autocovariance function of the signal
	x(t) or $x[n]$
$\mathbf{C}_{\mathrm{x}},\mathbf{K}_{\mathrm{x}},\mathbf{\Sigma}_{\mathrm{x}}$	(Auto)covariance matrix of \mathbf{x}

$$c_{xy}(\tau), C_{xy}(\tau)$$

 $C_{xy}, K_{xy}, \Sigma_{xy}$

Cross-covariance function of the signal x(t) or x[n]

Cross-covariance matrix of \mathbf{x} and \mathbf{v}

3.3 **Functions**

 $Q(\cdot)$ $\operatorname{erf}(\cdot)$

 $\operatorname{erfc}(\cdot)$

P[A]

 $p(\cdot), f(\cdot)$

 $p(x \mid A)$ $F(\cdot)$

 $\Phi_X(\omega), M_X(j\omega), E\left[e^{j\omega x}\right]$

 $M_x(t), \Phi_x(-jt), E[e^{tx}]$

 $\Psi_x(\omega), \ln \Phi_x(\omega), \ln E\left[e^{j\omega x}\right]$ $K_x(t), \ln E\left[e^{tx}\right], \ln M_x(t)$

Q-function, i.e., $P[\mathcal{N}(0,1) > x]$

Error function

Complementary error function i.e.,

 $\operatorname{erfc}(x) = 2Q(\sqrt{2}x) - \operatorname{erf}(x)$

Probability of the event or set A

Probability density function (PDF) or probability mass function (PMF)

Conditional PDF or PMF

Cumulative distribution function

(CDF)

First characteristic function (CF) of

Moment-generating function (MGF)

Second characteristic function

Cumulant-generating function

(CGF) of x

3.4 **Distributions**

 $\mathcal{N}(\mu, \sigma^2)$

 $\mathcal{CN}(\mu, \sigma^2)$

 $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

Gaussian distribution of a random variable with mean μ and variance σ^2 . The same notation can be used to denote a real-valued white Gaussian process with mean equal to μ and power spectral density equal to $N_0/2$, e.g., $s(t) \sim \mathcal{N}(\mu, N_0/2)$

Complex Gaussian distribution of a random variable with mean μ and variance σ^2 . The same notation can be used to denote a complex-valued white Gaussian process with mean equal to μ and power spectral density equal to N_0 , e.g., $s(t) \sim \mathcal{CN}(\mu, N_0)$ Gaussian distribution of a vector ran-

dom variable with mean μ and co-

variance matrix Σ

$\mathcal{CN}(\mu,\Sigma)$	Complex Gaussian distribution of a vector random variable with mean μ and covariance matrix Σ
$\mathcal{U}(a,b) \\ \chi^2(n), \chi_n^2$	Uniform distribution from a to b Chi-square distribution with n degree of freedom (assuming that the Gaus- sians are $\mathcal{N}(0,1)$)
$\operatorname{Exp}(\lambda)$	Exponential distribution with rate parameter λ
$\Gamma(lpha,eta)$	Gamma distribution with shape parameter α and rate parameter β
$\Gamma(lpha, heta)$	Gamma distribution with shape parameter α and scale parameter θ =
$\mathrm{Nakagami}(m,\Omega)$	$1/\beta$ Nakagami-m distribution with shape parameter m and spread parameter Ω
$\operatorname{Rayleigh}(\sigma)$	Rayleigh distribution with scale parameter σ
$\operatorname{Rayleigh}(\Omega)$	Rayleigh distribution with the second moment $\Omega = E[x^2] = 2\sigma^2$
$\mathrm{Rice}(s,\sigma)$	Rice distribution with noncentrality parameter (specular component) s and σ
$\mathrm{Rice}(A,K)$	Rice distribution with Rice factor $K = s^2/2\sigma^2$ and scale parameter $A = s^2 + 2\sigma^2$

4 Statistical signal processing

$oldsymbol{ abla} f, \mathbf{g} \ oldsymbol{ abla}_x f, \mathbf{g}_x$	Gradient descent vector Gradient descent vector with respect
$ \begin{array}{l} \mathbf{g} \text{ (or } \hat{\mathbf{g}} \text{ if the gradient vector is } \mathbf{g}) \\ J(\cdot), \mathcal{E}(\cdot) \\ \Lambda(\cdot) \\ \Lambda_l(\cdot) \\ \hat{x}(t) \text{ or } \hat{x}[n] \\ \hat{\boldsymbol{\mu}}_{\boldsymbol{\chi}}, \hat{\mathbf{m}}_{\boldsymbol{\chi}} \\ \hat{\boldsymbol{\mu}}_{\boldsymbol{\chi}}, \hat{\mathbf{m}}_{\boldsymbol{\chi}} \end{array} $	x Stochastic gradient descent (SGD) Cost-function or objective function Likelihood function Log-likelihood function Estimate of $x(t)$ or $x[n]$ Sample mean of $x[n]$ or $x(t)$ Sample mean vector of $x[n]$ or $x(t)$
$\hat{r}_X(au), \hat{R}_X(au)$ $\hat{S}_X(f), \hat{S}_X(j\omega)$	Estimated autocorrelation function of the signal $x(t)$ or $x[n]$ Estimated power spectral density (PSD) of $x(t)$ in linear (f) or angular (ω) frequency

$\hat{\mathbf{R}}_{\mathbf{x}}$	Sample (auto)correlation matrix
$\hat{r}_{x,d}(au), \hat{R}_{x,d}(au)$	Estimated cross-correlation between
	x[n] and $d[n]$ or $x(t)$ and $d(t)$
$\hat{S}_{x,y}(f), \hat{S}_{x,y}(j\omega)$	Estimated cross PSD of $x(t)$ and $y(t)$
	in linear or angular (ω) frequency
$\hat{\mathbf{R}}_{\mathbf{x}\mathbf{y}}$	Sample cross-correlation matrix of
	$\mathbf{R}_{\mathbf{x}\mathbf{y}}$
$\hat{ ho}_{x,y}$	Estimated Pearson correlation coeffi-
•	cient between x and y
$\hat{c}_{x}(au),\hat{C}_{x}(au)$	Estimated autocovariance function of
	the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_{\mathbf{x}},\hat{\mathbf{K}}_{\mathbf{x}},\hat{\mathbf{\Sigma}}_{\mathbf{x}}$	Sample (auto)covariance matrix
$\hat{c}_{xy}(au), \hat{C}_{xy}(au)$	Estimated cross-covariance function
	of the signal $x(t)$ or $x[n]$
$\hat{ extbf{C}}_{ extbf{xy}}, \hat{ extbf{K}}_{ extbf{xy}}, \hat{ extbf{\Sigma}}_{ extbf{xy}}$	Sample cross-covariance matrix
$\mathbf{w}, \mathbf{\theta}$	Parameters, coefficients, or weights
	vector
$\mathbf{w}_{o}, \mathbf{w}^{\star}, \mathbf{\theta}_{o}, \mathbf{\theta}^{\star}$	Optimum value of the parameters,
	coefficients, or weights vector
\mathbf{W}	Matrix of the weights
J	Jacobian matrix
H	Hessian matrix
$\hat{\mathbf{H}}$	Estimate of the Hessian matrix

5 Linear Algebra

5.1 Common matrices and vectors

W, D	Diagonal matrix
P	Projection matrix; Permutation ma-
	trix
J	Jordan matrix
${f L}$	Lower matrix
\mathbf{U}	Upper matrix
\mathbf{C}	Cofactor matrix
$\mathbf{C}_{\mathbf{A}}, \operatorname{cof}\left(\mathbf{A}\right)$	Cofactor matrix of $\bf A$
\mathbf{S}	Symmetric matrix
Q	Orthogonal matrix
\mathbf{I}_N	$N \times N$ -dimensional identity matrix
$0_{M imes N}$	$M \times N$ -dimensional null matrix
0_N	N-dimensional null vector
$1_{M imes N}$	$M \times N$ -dimensional ones matrix
1_N	N-dimensional ones vector

0	Null matrix, vector, or tensor (di-
	mensionality understood by context)
1	Ones matrix, vector, or tensor (di-
	mensionality understood by context)

5.2 Indexing

$x_{i_1,i_2,,i_N}, [\mathcal{X}]_{i_1,i_2,,i_N}$	Element in the position
1, 2, , , 1,	(i_1,i_2,\ldots,i_N) of the tensor $\boldsymbol{\mathcal{X}}$
$\mathcal{X}^{(n)}$	nth tensor of a nontemporal sequence
$\mathbf{x}_n, \mathbf{x}_{:n}$	nth column of the matrix X
\mathbf{x}_{n} :	nth row of the matrix X
$\mathbf{X}_{i_1,,i_{n-1},:,i_{n+1},,i_N}$	Mode- n fiber of the tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{:,i_2,i_3}$	Column fiber (mode-1 fiber) of the
	thrid-order tensor \mathcal{X}
$\mathbf{x}_{i_1,:,i_3}$	Row fiber (mode-2 fiber) of the thrid-
	order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{x}_{i_1,i_2,:}$	Tube fiber (mode-3 fiber) of the
	thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{i_1,:,:}$	Horizontal slice of the thrid-order
	tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{:,i_2,:}$	Lateral slices slice of the thrid-order
	tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{i_3},\mathbf{X}_{:,:,i_3}$	Frontal slices slice of the thrid-order
	tensor \mathcal{X}

5.3 General operations

$\langle \cdot, \cdot angle$	Inner product, e.g., $\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^{T} \mathbf{b}$
0	Outer product, e.g., $\mathbf{a} \circ \mathbf{b} = \mathbf{a} \mathbf{b}^{T}$
\otimes	Kronecker product
⊙	Hadamard (or Schur) (elementwise)
	product
$.\odot n$	nth-order Hadamard power
$\cdot \circ \frac{1}{n}$	nth-order Hadamard root
\oslash	Hadamard (or Schur) (elementwise)
	division
♦	Khatri-Rao product
\otimes	Kronecker Product
\times_n	<i>n</i> -mode product

5.4 Operations with matrices and tensors

 \mathbf{A}^{-1} Inverse matrix A^+, A^\dagger Moore-Penrose left pseudoinverse $\mathbf{A}^{ op}$ Transpose $\mathbf{A}^{-\top}$ Transpose of the inverse \mathbf{A}^* Complex conjugate \mathbf{A}^H Hermitian $\|\mathbf{A}\|_{\mathrm{F}}$ Frobenius norm $\|\mathbf{A}\|$ Matrix norm $|\mathbf{A}|, \det(\mathbf{A})$ Determinant $\operatorname{diag}\left(\mathbf{A}\right)$ The elements in the diagonal of A vec(A)Vectorization: stacks the columns of the matrix A into a long column vec $vec_d(\mathbf{A})$ Extracts the diagonal elements of a square matrix and returns them in a column vector $\text{vec}_{l}\left(\mathbf{A}\right)$ Extracts the elements strictly below the main diagonal of a square matrix in a column-wise manner and returns them into a column vector $vec_u(\mathbf{A})$ Extracts the elements strictly above the main diagonal of a square matrix in a column-wise manner and returns them into a column vector $\text{vec}_{\text{b}}\left(\mathbf{A}\right)$ Block vectorization operator: stacks square block matrices of the input into a long block column matrix unvec (A) Reshapes a column vector into a matrix $\mathrm{tr}\left(\mathbf{A}\right)$ trace n-mode matricization of the tensor ${\cal X}$ $\mathbf{X}_{(n)}$

5.5 Operations with vectors

$\ \mathbf{a}\ $	l_1 norm, 1-norm, or Manhatan norm
$\ \mathbf{a}\ , \ \mathbf{a}\ _2$	l_2 norm, 2-norm, or Euclidean norm
$\ \mathbf{a}\ _p$	l_p norm, p -norm, or Minkowski norm
$\ \mathbf{a}\ _{\infty}^{r}$	l_{∞} norm, ∞ -norm, or Chebyshev
	norm
$\operatorname{diag}\left(\mathbf{a}\right)$	Diagonalization: a square, diagonal matrix with entries given by the vec-
	tor a

5.6 Decompositions

$rac{oldsymbol{\Lambda}}{oldsymbol{Q}}$	Eigenvalue matrix Eigenvectors matrix; Orthogonal ma-
R	trix of the QR decomposition Upper triangular matrix of the QR decomposition
\mathbf{U}	Left singular vectors
_	e e e e e e e e e e e e e e e e e e e
$egin{array}{c} \mathbf{U}_r \ \mathbf{\Sigma} \end{array}$	Left singular nondegenerated vectors
	Singular value matrix
Σ_r	Singular value matrix with nonzero
Σ^+	singular values in the main diagonal
L .	Singular value matrix of the pseu-
V +	doinverse
Σ_r^+	Singular value matrix of the pseu-
	doinverse with nonzero singular val-
	ues in the main diagonal
V	Right singular vectors
\mathbf{V}_r	Right singular nondegenerated vec-
	tors
$\operatorname{eig}\left(\mathbf{A} ight)$	Set of the eigenvalues of A
$[\![\mathbf{A},\mathbf{B},\mathbf{C},\ldots]\!]$	CANDECOMP/PARAFAC (CP) de-
	composition of the tensor $\boldsymbol{\mathcal{X}}$ from the
	outer product of column vectors of \mathbf{A} ,
	B, C,
$[\![\boldsymbol{\lambda};\mathbf{A},\mathbf{B},\mathbf{C},\ldots]\!]$	Normalized CANDE-
	COMP/PARAFAC (CP) decom-
	position of the tensor \mathcal{X} from the
	outer product of column vectors of
	A, B, C, \dots
	,,,,

5.7 Spaces

$\mathrm{span}\left(\mathbf{a}_{1},\mathbf{a}_{2},\ldots,\mathbf{a}_{n}\right)$	Vector space spanned by the argument vectors
C(A), columnspace(A), range(A), span(A), image(A)	Columnspace, range or image, i.e., the space span $(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$, where \mathbf{a}_i is the ith column vector of the ma-
	trix A
$C(\mathbf{A}^{H})$	Row space
$N(\mathbf{A})$, nullspace(\mathbf{A}), kernel(\mathbf{A})	Nullspace (or kernel space)
$N(\mathbf{A}^{H})$	Left nullspace
$\operatorname{rank}(\mathbf{A})$	Rank, that is, $\dim(\operatorname{span}(\mathbf{A})) =$
` '	$\dim (C(\mathbf{A}))$
nullity (A)	Nullity of \mathbf{A} , i.e., dim (N (\mathbf{A}))
$\mathbf{a} \perp \mathbf{b}$	a is orthogonal to b
a ≠ b	${f a}$ is not orthogonal to ${f b}$

5.8 Inequalities

\mathcal{X}	\leq	0
a :	≤ _K	b

 $\mathbf{a} \prec_K \mathbf{b}$

 $\mathbf{a} \leq \mathbf{b}$

 $\mathbf{a} \prec \mathbf{b}$

 $\mathbf{A} \leq_K \mathbf{B}$

 $\mathbf{A} \prec_K \mathbf{B}$

 $A \leq B$

A < B

Nonnegative tensor

Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the conic subset K in the space \mathbb{R}^n

Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the interior of the conic subset K in the space \mathbb{R}^n Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the nonnegative orthant conic subset, \mathbb{R}^n_+ , in the space \mathbb{R}^n .

Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the positive orthant conic subset, \mathbb{R}^n_{++} , in the space \mathbb{R}^n

Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the conic subset K in the space \mathbb{S}^n

Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the interior of the conic subset K in the space \mathbb{S}^n Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive semidefinite conic subset, \mathbb{S}^n_+ , in the space \mathbb{S}^n Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive orthant conic subset, \mathbb{S}^n_{++} , in the space \mathbb{S}^n

6 Sets

A + B
A - B
$A \setminus B, A - B$

 $A \cup B$ $A \cap B$ $A \times B$ A^n

Set addition (Minkowski sum)

Minkowski difference

Set difference or set subtraction, i.e., the set containing the elements

of A that are not in B

Set of union Set of intersection Cartesian product $A \times A \times \cdots \times A$

n times

A^{\perp}	Orthogonal complement of A , e.g., $N(\mathbf{A}) = C(\mathbf{A}^{\top})^{\perp}$
$A \oplus B$	Direct sum, e.g., $C(\mathbf{A}^{T}) \oplus C(\mathbf{A}^{T})^{\perp} =$
	\mathbb{R}^n
A^c, \bar{A}	Complement set (given U)
#A, A	Cardinality
$a \in A$	a is element of A
$a \notin A$	a is not element of A
$\{1,2,\ldots,n\}$	Discrete set containing the integer el-
	ements $1, 2, \ldots, n$
U	Universe
2^A	Power set of A
\mathbb{R}	Set of real numbers
\mathbb{C}	Set of complex numbers
$\mathbb Z$	Set of integer number
$\mathbb{B} = \{0, 1\}$	Boolean set
Ø	Empty set
N	Set of natural numbers
$\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$	Real or complex space (field)
$\mathbb{K}^{I_1 \times I_2 \times \cdots \times I_N}$	$I_1 \times I_2 \times \cdots \times I_N$ -dimensional real (or
	complex) space
\mathbb{K}_{+}	Nonnegative real (or complex) space
K++	Positive real (or complex) space, i.e., $\mathbb{K}_{++} = \mathbb{K}_{+} \setminus \{0\}$
$\mathbb{S}^n, \mathcal{S}^n$	Conic set of the symmetric matrices
, , ,	in $\mathbb{R}^{n \times n}$
$\mathbb{S}^n_+, \mathcal{S}^n_+$	Conic set of the symmetric positive
	semidefinite matrices in $\mathbb{R}^{n \times n}$
$\mathbb{S}^n_{++}, \mathcal{S}^n_{++}$	Conic set of the symmetric positive
	definite matrices in $\mathbb{R}^{n\times n}$, i.e., \mathbb{S}^n_{++}
	$\mathbb{S}^n_+\setminus\{0\}$
\mathbb{H}^n	Set of all hermitian matrices in $\mathbb{C}^{n\times n}$
[a,b]	Closed interval of a real set from a to
	b
(a,b)	Opened interval of a real set from a
	to b
[a,b),(a,b]	Half-opened intervals of a real set
	from a to b

7 Communication systems

One-sided bandwidth of the transmitted signal, in ${\rm Hz}$

W	One-sided bandwidth of the trans-
	mitted signal, in rad/s
x_i	Real or in-phase part of x
x_q	Imaginary or quadrature part of x
f_c, f_{RF}	Carrier frequency (in Hertz)
f_L	Carrier frequency in L-band (in
•	Hertz)
f_{IF}	Intermediate frequency (in Hertz)
f_s	Sampling frequency or sampling rate
	(in Hertz)
T_{s}	Sampling time interval
R	Bit rate
T	Bit duration
T_c	Chip duration
T_{sy}, T_{sym}	Symbol duration
SRF	Transmitted signal in RF
SFI	Transmitted signal in FI
S, S_l	Low-pass equivalent signal or enve-
•	lope complex of transmitted signal
r_{RF}	Received signal in RF
r_{FI}	Received signal in FI
r, r_l	Low-pass equivalent signal or enve-
	lope complex of received signal
ϕ	Signal phase
ϕ_0	Initial phase
η_{RF}, w_{RF}	Noise in RF
η_{FI}, w_{FI}	Noise in FI
η, w	Noise in baseband
τ	Timing delay
Δau	Timing error (delay - estimated)
arphi	Phase offset
$\Delta arphi$	Phase error (offset - estimated)
f_d	Linear Doppler frequency
Δf_d	Frequency error (Doppler frequency -
	estimated)
ν	Angular Doppler frequency
Δu	Frequency error (Doppler frequency -
	estimated)
γ, A	Transmitted signal amplitude
	Combined effect of the path loss and
γ_0, A_0	Combined effect of the path loss and

8 Other notations

8.1 Mathematical symbols

There exists
There does not exist
There exist an unique
Belongs to
Does not belong to
Q.E.D.
Therefore
Because
For all
Such that
Logical equivalence
Equal by definition
Not equal
Infinity
$\sqrt{-1}$
Twiddle factor, $e^{-j\frac{2\pi}{N}}$

8.2 Operations

$arg \max f(x)$	Value of x that minimizes x
$ \underset{x \in \mathcal{A}}{\operatorname{arg min}} f(x) $	Value of x that minimizes x
$\inf_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Infimum
$\sup_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Supremum
a	Absolute value of a
log	Base-10 logarithm or decimal loga-
	rithm
ln	Natual logarithm
$\operatorname{Re}\left\{ x\right\}$	Real part of x
$\operatorname{Im}\left\{ x\right\}$	Imaginary part of x
۷٠	phase (complex argument)
$x \mod y$	Remainder, i.e., $x - y \lfloor x/y \rfloor$
$\operatorname{frac}(x)$	Fractional part, i.e., x mod 1
$a \wedge b$	Logical AND of a and b
$a \lor b$	Logical OR of a and b
$\neg a$	Logical negation of a
[·]	Ceiling operation
[·]	Floor operation

8.3 Functions

 $\mathcal{O}(\cdot), O(\cdot)$ Big-O notation $\Gamma(\cdot)$ Gamma function

9 Abbreviations

wrt. With respect to st. Subject to iff. If and only if

EVD Eigenvalue decomposition, or eigen-

 ${\it decomposition}$

 $\begin{array}{cc} {\rm SVD} & {\rm Singular\ value\ decomposition} \\ {\rm CP} & {\rm CANDECOMP/PARAFAC} \end{array}$