Galileo Masterclass Brazil (GMB) 2022

Lecture 3 - Chip Pulse Shapes and Multiplexing

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Outline

Rectangular Chip Pulse Shape

Binary Offset Carrier (BOC) Signals

Composite BOC and Time-Multiplex BOC

Signal Mapping/Multiplexing Methods

Signal Interplex

Alternate BOC (AltBOC)

Chip Pulse Shape (1)

The rectangular chip pulse shape can be described by

$$p_{\sqcap}(t) = \frac{1}{\sqrt{T_c}} \left(U(t + \frac{T_c}{2}) - U(t - \frac{T_c}{2}) \right),$$

where $U(t)$ denotes the unit step or Heaviside's unit step function

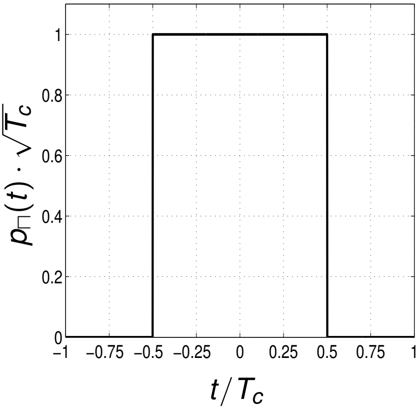
$$U(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases},$$

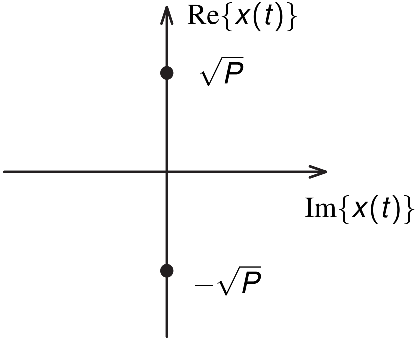
and

$$\int_{-\infty}^{\infty} |p_{\sqcap}(t)|^2 \, dt = 1$$

- The rectangular chip pulse shape can be considered as the *classical* chip pulse shape, which originally was used for early spread spectrum signals
- We get a binary phase shift keying (BPSK) signal

Chip Pulse Shape (2)





Fourier Transform and Autocorrelation (1)

The Fourier transform of $p_\sqcap(t)$ reads

$$P_\sqcap(f) = \frac{\sqrt{T_c}\,\sin(\pi f T_c)}{\pi f T_c} = \sqrt{T_c}\,\mathrm{sinc}(f T_c)$$

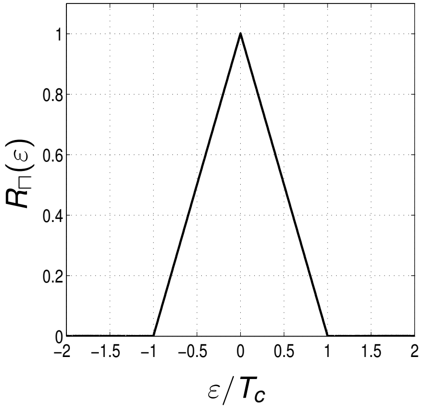
Here, the sinc function is defined as

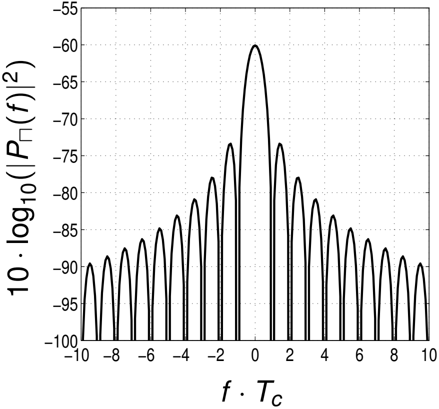
$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

The autocorrelation function can be given as

$$\begin{aligned} R_\sqcap(\varepsilon) &= \int_{-\infty}^{\infty} |P_\sqcap(f)|^2 \, \mathrm{e}^{\mathrm{j}2\pi f \varepsilon} \, df = \int_{-\infty}^{\infty} T_c \, \mathrm{sinc}^2(f T_c) \, \mathrm{e}^{\mathrm{j}2\pi f \varepsilon} \, df \\ &= \int_{-\infty}^{\infty} p_\sqcap(t) p_\sqcap(t+\varepsilon) \, dt \end{aligned}$$

Autocorrelation and Power Spectral Density (PSD)





$$B \to \infty$$

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Chip Pulse Shapes (1)

BOC signals became the standard of GNSS signal design besides using rectangular chip pulse shapes. Their chip pulse shapes are formed by the product of a rectangular pulse

$$p_{n_c}(t) = \sqrt{n_c f_r}\left(U(t + \frac{1}{2n_c f_r}) - U(t - \frac{1}{2n_c f_r})\right)$$

and a sine or a cosine square wave subcarrier which is given as

$$g_{n_s}(t) = \begin{cases} \mathrm{sgn}\left(-\sin(2\pi n_s f_r t)\right) \\ \mathrm{sgn}\left(-\cos(2\pi n_s f_r t)\right) \end{cases}$$

- n_c: Chip rate (chip duration $\frac{1}{n_c f_r}$)
- n_s: Subcarrier rate (subcarrier frequency $n_s f_r$)
- f_r: Reference frequency ($f_r = 1.023$ MHz)
- $\mathrm{BOC}(n_s, n_c)$: BOC signals with sine subcarrier
- $\mathrm{BOC_{cos}}(n_s, n_c)$: BOC signals with cosine subcarrier

Chip Pulse Shapes (2)

BOC signal pulse shapes are given as

$$p_{\mathrm{BOC}(n_s,n_c)}(t) = \begin{cases} p_{n_c}(t) \cdot g_{n_s}(t) & |t| \leq \frac{1}{2n_c f_r} \\ 0 & \text{else} \end{cases}$$

- $\mathrm{BOC}(1,1)$ with sine square wave subcarrier is also known as biphase Manchester pulse
- We get a BPSK signal
- BOC signals achieve FD/CDMA
- $n = 2\,\frac{n_s}{n_c}$: Number of half periods of the subcarrier within the duration of one chip
- The higher n, the higher the Gabor bandwidth of the signal

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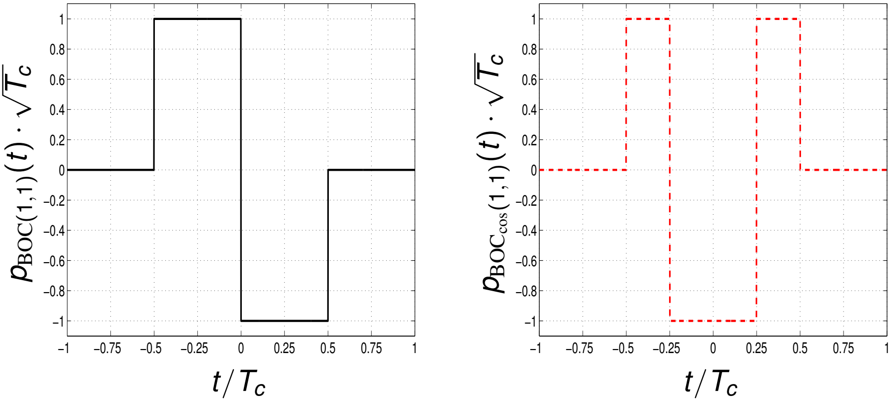
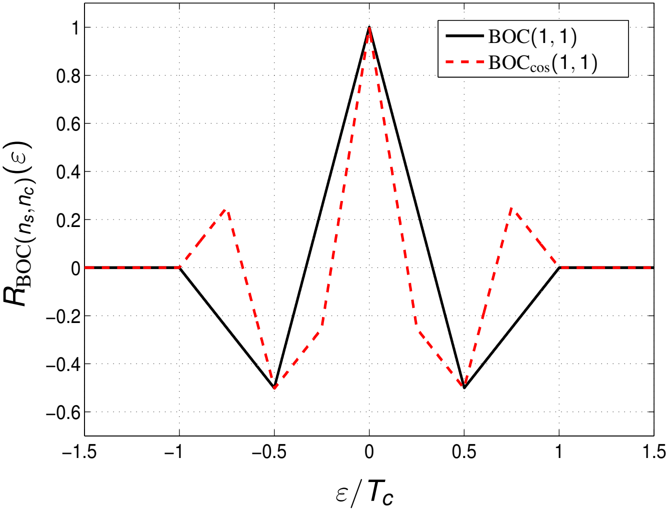
$\mathrm{BOC}(1,1)$ and $\mathrm{BOC}_{\cos}(1,1)$ Chip Pulse Shapes

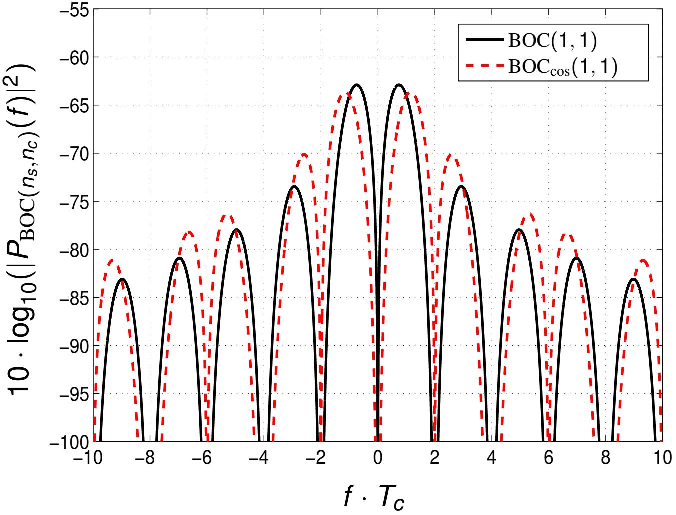


BOC(1, 1) and $\mathrm{BOC}_{\cos}(1, 1)$ Autocorrelation

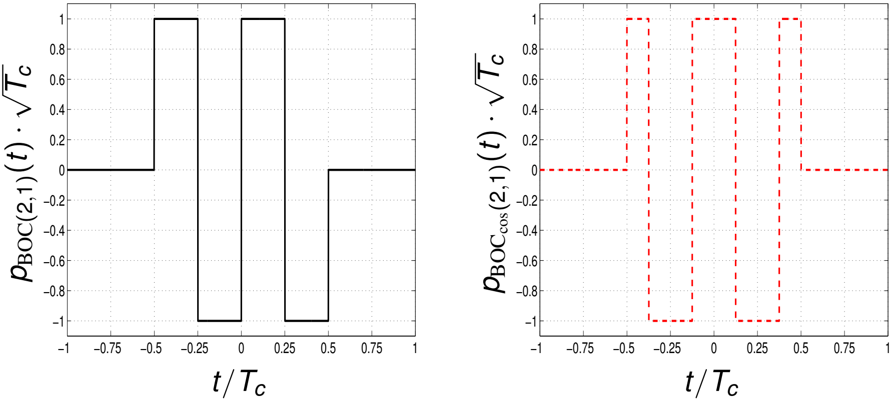


$\kappa = 0.5$

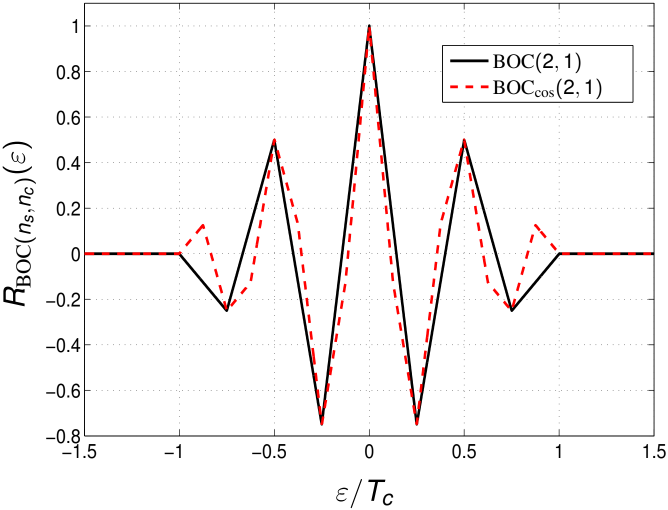
BOC$(1,1)$ and BOC$_{\cos}(1,1)$ PSD



BOC(2, 1) and $\mathrm{BOC_{cos}}$(2, 1) Chip Pulse Shapes

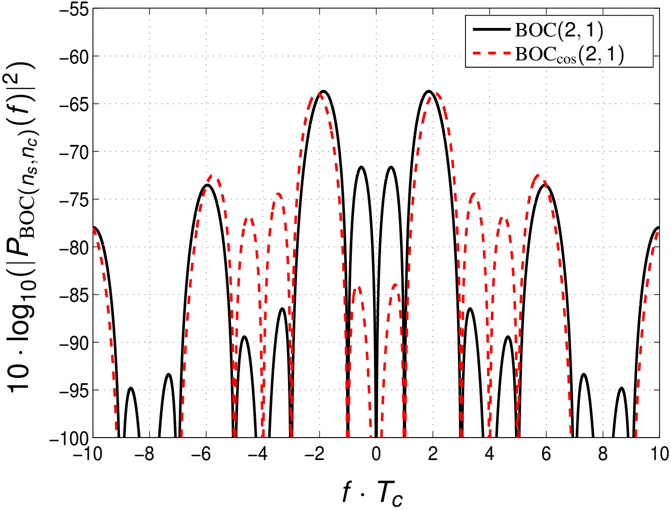


BOC(2, 1) and $\mathrm{BOC_{cos}}(2, 1)$ Autocorrelation

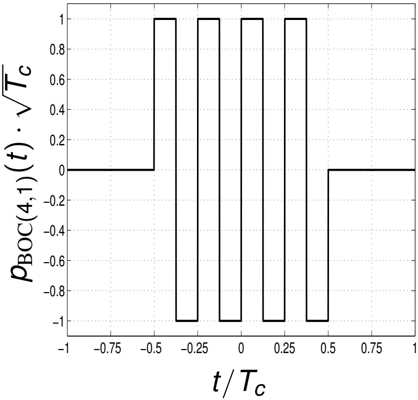


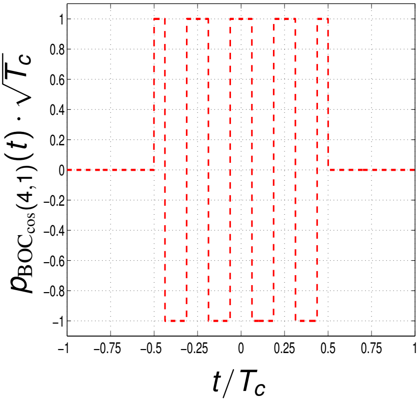
$\kappa = 0.7$

$\mathrm{BOC}(2,1)$ and $\mathrm{BOC_{cos}}(2,1)$ PSD

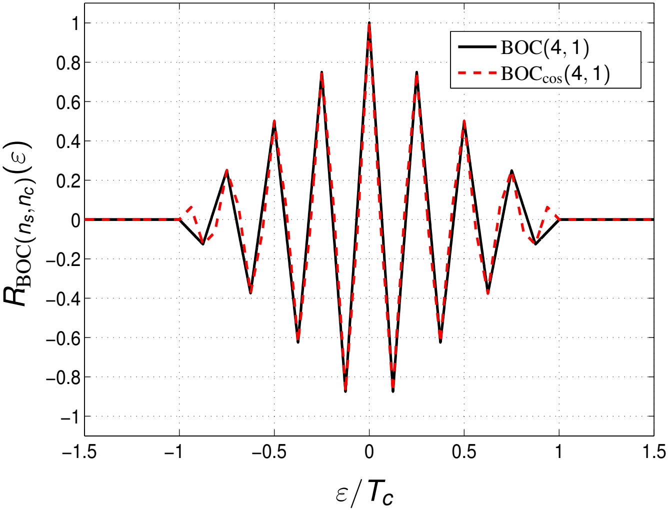


BOC(4, 1) and $\mathrm{BOC_{cos}}(4, 1)$ Chip Pulse Shapes



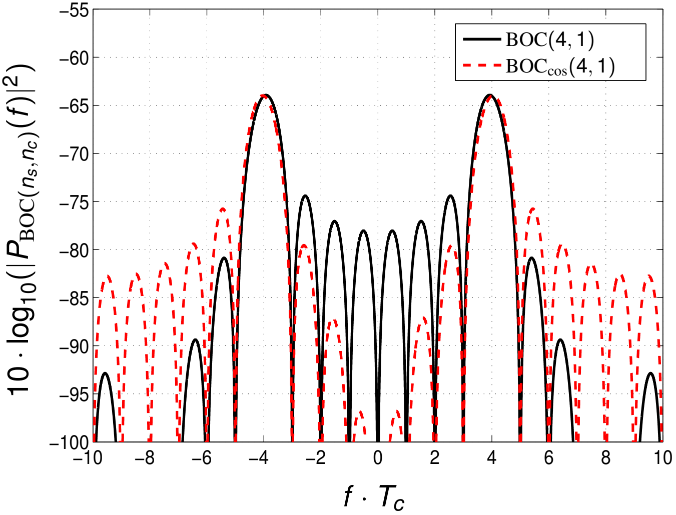


BOC(4, 1) and $\mathrm{BOC}_{\cos}(4, 1)$ Autocorrelation



$$\kappa = 0.876$$

$\mathrm{BOC}(4,1)$ and $\mathrm{BOC}_{\cos}(4,1)$ PSD



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Composite BOC (CBOC) (1)

CBOC signals are composed of a linear combination of several BOC signals. A CBOC signal with two BOC signals can be given as

$$p_{\text{CBOC}}(t) = \sqrt{\xi}\,\left(\sqrt{\omega}\,p_{\text{BOC}(a,b)}(t) \pm \sqrt{1-\omega}\,p_{\text{BOC}(c,d)}(t)\right), \quad \xi \in \mathbb{R}_0^+$$

while $0 \leq \omega \leq 1$, the Fourier transform is given as

$$P_{\text{CBOC}}(f) = \sqrt{\xi}\,\left(\sqrt{\omega}\,P_{\text{BOC}(a,b)}(f) \pm \sqrt{1-\omega}\,P_{\text{BOC}(c,d)}(f)\right)$$

and the PSD is given as

$$\begin{aligned} |P_{\text{CBOC}}(f)|^2 &= \xi\,\Big(\omega\,|P_{\text{BOC}(a,b)}(f)|^2 + (1-\omega)\,|P_{\text{BOC}(c,d)}(f)|^2 \\ &\pm 2\cdot\sqrt{\omega-\omega^2}\,\text{Re}\left\{P_{\text{BOC}(a,b)}(f)P^*_{\text{BOC}(c,d)}(f)\right\}\Big) \end{aligned}$$

Composite BOC (CBOC) (2)

The autocorrelation is given as

$$R_{\mathrm{CBOC}}(\varepsilon) = \xi \left(\omega\, R_{\mathrm{BOC}(a,b)}(\varepsilon) + (1-\omega)\, R_{\mathrm{BOC}(c,d)}(\varepsilon) \right.$$
$$\left. \pm\, 2 \cdot \sqrt{\omega - \omega^2} \int_{-\infty}^{\infty} \mathrm{Re}\left\{ P_{\mathrm{BOC}(a,b)}(f) P^*_{\mathrm{BOC}(c,d)}(f) \right\} \mathrm{e}^{\mathrm{j}2\pi f \varepsilon}\, df \right)$$

The normalization factor ξ is

$$\xi = \frac{1}{\int_{-\infty}^{\infty} \left|\sqrt{\omega}\, P_{\mathrm{BOC}(a,b)}(f) + \sqrt{1-\omega}\, P_{\mathrm{BOC}(c,d)}(f)\right|^2 df}$$
$$= \frac{1}{\int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} \left|\sqrt{\omega}\, p_{\mathrm{BOC}(a,b)}(t) + \sqrt{1-\omega}\, p_{\mathrm{BOC}(c,d)}(t)\right|^2 dt}$$

Thus,

$$\int_{-\infty}^{\infty} |P_{\mathrm{CBOC}}(f)|^2\, df = \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} |p_{\mathrm{CBOC}}(t)|^2\, dt = 1$$

Galileo OS CBOC Signal

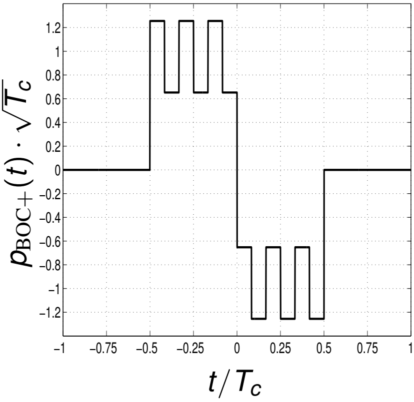
For the Galileo Open Service (OS) data ($\mathrm{BOC}+$) and pilot ($\mathrm{BOC}-$) signal the following two CBOC signals are composed:

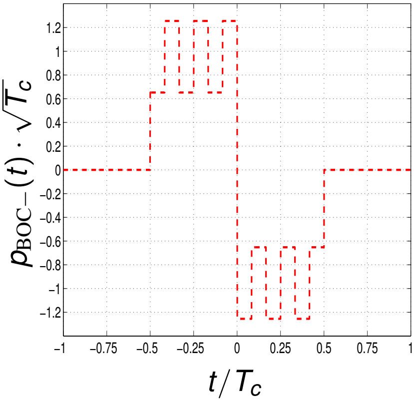
$$\begin{aligned} p_{\mathrm{BOC}+}(t) &= \sqrt{\frac{10}{11}}\, p_{\mathrm{BOC}(1,1)}(t) + \sqrt{\frac{1}{11}}\, p_{\mathrm{BOC}(6,1)}(t) \\ p_{\mathrm{BOC}-}(t) &= \sqrt{\frac{10}{11}}\, p_{\mathrm{BOC}(1,1)}(t) - \sqrt{\frac{1}{11}}\, p_{\mathrm{BOC}(6,1)}(t) \end{aligned}$$

The overall PSD of the sum of both signals with 50% power for the data and 50% power for the pilot signal can be given as

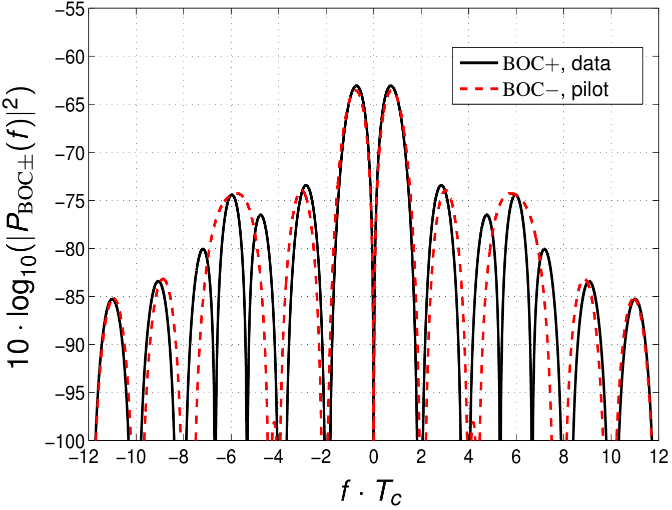
$$\begin{aligned} |P_{\mathrm{OS}}|^2 &= \frac{1}{2}\,|P_{\mathrm{BOC}+}(f)|^2 + \frac{1}{2}\,|P_{\mathrm{BOC}-}(f)|^2 \\ &= \frac{10}{11}\,|P_{\mathrm{BOC}(1,1)}(f)|^2 + \frac{1}{11}\,|P_{\mathrm{BOC}(6,1)}(f)|^2 \end{aligned}$$

Galileo OS CBOC Chip Pulse Shapes

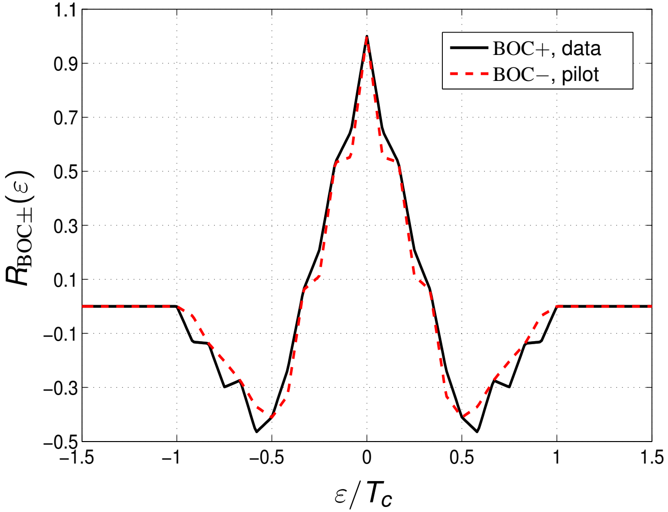




Galileo OS CBOC PSD



Galileo OS CBOC Autocorrelation



Time-Multiplexed BOC (TMBOC)

For TMBOC different chip pulse shapes are used for different chips of the PR sequence. A TMBOC chip pulse shape with two different BOC chip pulse shapes which are emitted each T_c seconds can be given as

$$p_{\mathrm{TMBOC}}(t) = \begin{cases} p_{\mathrm{BOC}(a,b)}(t) & \text{with probability } p \\ p_{\mathrm{BOC}(c,d)}(t) & \text{with probability } 1-p \end{cases}$$

In case the signal source is negative equally probable (NEP), the PSD of a TMBOC signal can be given as

$$|P_{\mathrm{TMBOC}}(f)|^2 = p\,|P_{\mathrm{BOC}(a,b)}(f)|^2 + (1-p)\,|P_{\mathrm{BOC}(c,d)}(f)|^2$$

and the autocorrelation function is given as

$$R_{\mathrm{TMBOC}}(\varepsilon) = p\,R_{\mathrm{BOC}(a,b)}(\varepsilon) + (1-p)\,R_{\mathrm{BOC}(c,d)}(\varepsilon)$$

GPS L1C TMBOC Signal

In the sequel of GPS system modernization the L1C signal was defined as the future open service signal in L1 frequency band. For the L1C signal two signal components, pilot and data, are defined:

$$
\begin{aligned}
|P_{\text{pilot}}(f)|^2 &= \frac{29}{33}\,|P_{\text{BOC}(1,1)}(f)|^2 + \frac{4}{33}\,|P_{\text{BOC}(6,1)}(f)|^2 \\
|P_{\text{data}}(f)|^2 &= |P_{\text{BOC}(1,1)}(f)|^2
\end{aligned}
$$

With a 75% to 25% power sharing between pilot and data we get the combined PSD

$$
\begin{aligned}
|P_{\text{L1C}}(f)|^2 &= \frac{3}{4}\,|P_{\text{pilot}}(f)|^2 + \frac{1}{4}\,|P_{\text{data}}(f)|^2 \\
&= \frac{10}{11}\,|P_{\text{BOC}(1,1)}(f)|^2 + \frac{1}{11}\,|P_{\text{BOC}(6,1)}(f)|^2
\end{aligned}
$$

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Signal Mapping/Multiplexing Methods

Signal Interplex

Alternate BOC (AltBOC)

Objectives of Multiplexing Methods

- Goal: multiplex/map several signals $s_n(t)$ onto one carrier frequency
- Achieve minimal cross-talk and maximal power and bandwidth efficiency
- Each signal $s_n(t)$ provides a different service
- In general there are two main strategies:
  1. time multiplex
  2. frequency multiplex

A constant envelope of a baseband signal $s(t)$ is given if the peak-to-average-power ratio (PAPR)

$$\mathrm{PAPR} = \frac{\max |s(t)|^2}{\mathrm{E}[|s(t)|^2]} = 1$$

Preserving a constant envelope of the signal $s(t)$ is very beneficial for the amplification of the signal by the high power amplifier (HPA) on board the satellite payload $\Rightarrow$ Out-of band emissions and power inefficiencies are mostly avoided

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Interplex

A phase-modulated radio frequency passband signal in a phase-shift-keyed/phase modulated (PSK/PM) system can be given as

$$\tilde{s}(t) = \sqrt{2P}\sin(2\pi f_c t + \Theta(t))$$

with the N-channel interplex phase modulation

$$\Theta(t) = \left[\beta_1 + \sum_{n=2}^{N} \beta_n s_n(t)\right] s_1(t),$$

- P: Total average power
- f_c: Carrier frequency
- N: Number of channels
- β_n: Modulation angles
- $s_n(t) = \in \{-1, 1\}$: Binary data streams or binary GNSS signals

Two-Channel Interplex ($N = 2$) (1)

A two-channel interplex signal can be given as

$$\tilde{s}(t) = \sqrt{2P}\sin(2\pi f_c t + \beta_1 s_1(t) + \beta_2 s_1(t) s_2(t))$$

We use

$$\begin{aligned}
\sin(\alpha \pm \beta) &= \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) \\
\cos(\alpha \pm \beta) &= \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) \\
\cos(\beta_n s_n(t)) &= \cos(\beta_n) \\
\sin(\beta_n s_n(t)) &= s_n(t)\sin(\beta_n)
\end{aligned}$$

and we get

$$\begin{aligned}
\tilde{s}(t) &= \sqrt{2P}\sin(2\pi f_c t)\ [\cos(\beta_1)\cos(\beta_2) - s_2(t)\sin(\beta_1)\sin(\beta_2)] \\
&+ \sqrt{2P}\cos(2\pi f_c t)\ [s_1(t)\sin(\beta_1)\cos(\beta_2) \\
&\quad + s_1(t)s_2(t)\cos(\beta_1)\sin(\beta_2)]
\end{aligned}$$

Two-Channel Interplex ($N = 2$) (2)

$$P_c = P\ \cos^2(\beta_1)\cos^2(\beta_2)$$
$$P_1 = P\ \sin^2(\beta_1)\cos^2(\beta_2)$$
$$P_2 = P\ \sin^2(\beta_1)\sin^2(\beta_2)$$
$$P_{im} = P\ \cos^2(\beta_1)\sin^2(\beta_2)$$

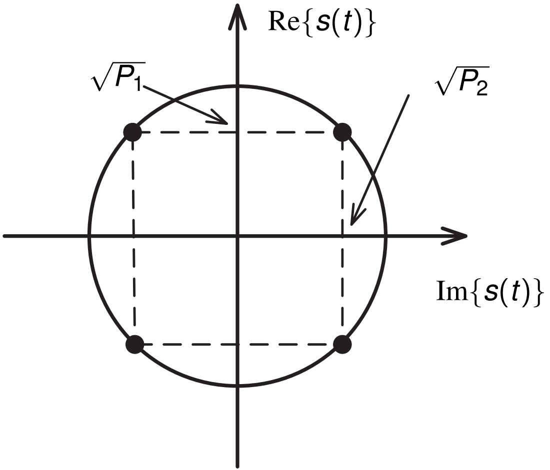
- P_c: Carrier power
- P_1: Power in channel 1
- P_2: Power in channel 2
- P_{im}: Power of the inter-modulation product

If we choose $\beta_1 = \pi/2$ and $\beta_2 = \pi/4$ we get

$$P_c = 0$$
$$P_2 = P/2$$
$$P_1 = P/2$$
$$P_{im} = 0$$

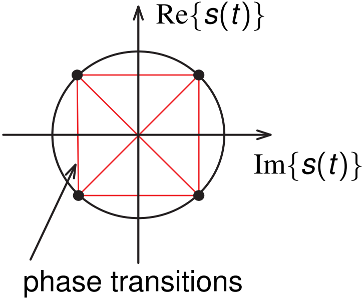
$\Rightarrow$ Quadrature binary shift keying (QPSK) modulation

Two-Channel Interplex ($N = 2$) (3)

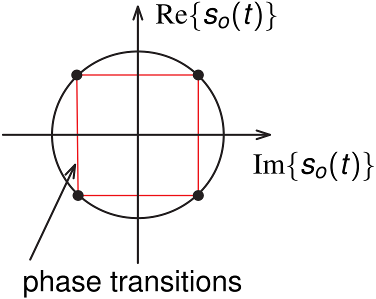


Equivalent baseband signal $s(t) = \sqrt{P_1} s_1(t) + \mathrm{j}\sqrt{P_2} s_2(t)$ with
$\tilde{s}(t) = \sqrt{2}\mathrm{Re}\{s(t)\mathrm{e}^{\mathrm{j}2\pi f_c t}\}$

Staggering - Offset QPSK (OQPSK)



QPSK



OQPSK

$$s_o(t) = \sqrt{P_1} s_1(t) + \mathrm{j}\sqrt{P_2} s_2(t - T_o)$$

Three-Channel Interplex ($N = 3$) (1)

A three-channel interplex signal can be given as

$$\tilde{s}(t) = \sqrt{2P}\sin(2\pi f_c t + \beta_1 s_1(t) + \beta_2 s_1(t)s_2(t) + \beta_3 s_1(t)s_3(t)),$$

and we get

$$\begin{aligned}\tilde{s}(t) &= \sqrt{2P}\sin(2\pi f_c t)\underbrace{\cos(\beta_1 s_1(t) + \beta_2 s_1(t)s_2(t) + \beta_3 s_1(t)s_3(t))}_{=A_1} \\ &+ \sqrt{2P}\cos(2\pi f_c t)\underbrace{\sin(\beta_1 s_1(t) + \beta_2 s_1(t)s_2(t) + \beta_3 s_1(t)s_3(t))}_{=A_2}\end{aligned}$$

Now, we can write

$$\begin{aligned}A_1 = &\cos(\beta_1)\cos(\beta_2)\cos(\beta_3) - s_2(t)s_3(t)\cos(\beta_1)\sin(\beta_2)\sin(\beta_3) \\ &- s_2(t)\sin(\beta_1)\sin(\beta_2)\cos(\beta_3) - s_3(t)\sin(\beta_1)\cos(\beta_2)\sin(\beta_3)\end{aligned}$$

Three-Channel Interplex ($N = 3$) (2)

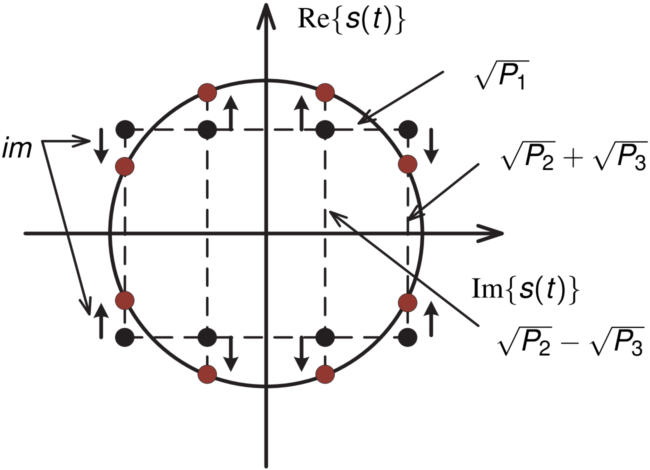
and

$$\begin{aligned} A_2 &= s_1(t)\sin(\beta_1)\cos(\beta_2)\cos(\beta_3) \\ &- s_1(t)s_2(t)s_3(t)\sin(\beta_1)\sin(\beta_2)\sin(\beta_3) \\ &+ s_1(t)s_2(t)\cos(\beta_1)\sin(\beta_2)\cos(\beta_3) \\ &+ s_1(t)s_3(t)\cos(\beta_1)\cos(\beta_2)\sin(\beta_3) \end{aligned}$$

In order to eliminate most of the inter-modulation terms we can choose $\beta_1 = \pi/2$ and we get

$$P_1 = P\ \cos^2(\beta_2)\cos^2(\beta_3)$$
$$P_2 = P\ \sin^2(\beta_2)\cos^2(\beta_3)$$
$$P_3 = P\ \cos^2(\beta_2)\sin^2(\beta_3)$$
$$P_{im} = P\ \sin^2(\beta_2)\sin^2(\beta_3)$$

Three-Channel Interplex ($N = 3$) (3)



$$s(t) = \sqrt{P_1} s_1(t) - \sqrt{P_{im}} s_1(t) s_2(t) s_3(t) + \mathrm{j} \left(\sqrt{P_2} s_2(t) + \sqrt{P_3} s_3(t) \right)$$

Three-Channel Interplex ($N = 3$) (4)

Example for $P_1 = 2 \cdot P_2$ and $P_2 = P_3$:

$$\begin{aligned} P_1 &= \cos^2(\beta_2)\cos^2(\beta_3) = 2 \cdot \sin^2(\beta_2)\cos^2(\beta_3) \\ P_2 &= P_3 = \sin^2(\beta_2)\cos^2(\beta_3) = \cos^2(\beta_2)\sin^2(\beta_3) \end{aligned}$$

We get

$$\cos^2(\beta_2) = 2 \cdot \sin^2(\beta_2)$$

and

$$\begin{aligned} \cos^2(\beta_2) = 2 \cdot (1 - \cos^2(\beta_2)) \\ \cos^2(\beta_3) = 2 \cdot (1 - \cos^2(\beta_3)) \end{aligned}$$

and

$$\beta_2 = \beta_3 = \arccos(\sqrt{\frac{2}{3}})$$

$\Rightarrow P_1 = \frac{4}{9}\,P$, $P_2 = P_3 = \frac{2}{9}\,P$, and $P_{im} = \frac{1}{9}\,P$

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Constant Envelope AltBOC

In order to achieve a constant envelope four signal components can be multiplexed on one carrier frequency with

$$s(t) = \frac{1}{2\sqrt{2}}\left[(s_1(t)+\mathrm{j}s_2(t))\,\psi'_M(t)+(s_3(t)+\mathrm{j}s_4(t))\,\psi_M(t)\right.$$
$$\left.+\,(\bar{s}_1(t)+\mathrm{j}\bar{s}_2(t))\,\bar{\psi}'_M(t)+(\bar{s}_3(t)+\mathrm{j}\bar{s}_4(t))\,\bar{\psi}_M(t)\right]$$

with the inter-modulation products

$$\begin{aligned}
\bar{s}_1(t) &= s_2(t)s_3(t)s_4(t)\\
\bar{s}_2(t) &= s_1(t)s_3(t)s_4(t)\\
\bar{s}_3(t) &= s_1(t)s_2(t)s_4(t)\\
\bar{s}_4(t) &= s_1(t)s_2(t)s_3(t)
\end{aligned}$$

and the multi-level complex subcarriers

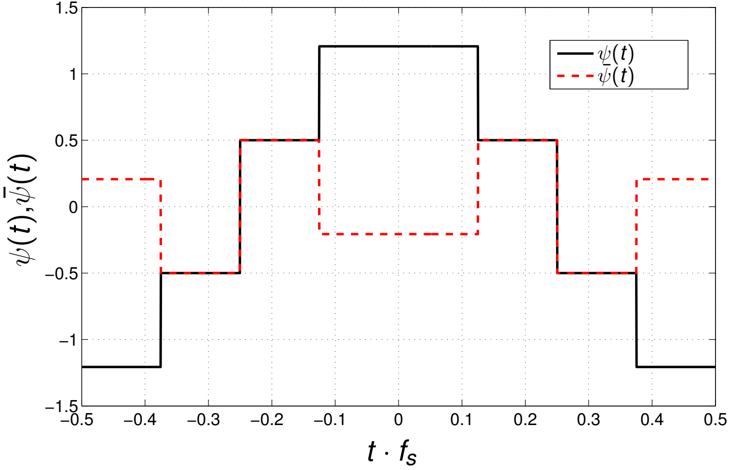
$$\psi_M(t) = \psi(t)+\mathrm{j}\psi(t-\frac{1}{4f_s})\,,\ \psi'_M(t) = \psi(t)-\mathrm{j}\psi(t-\frac{1}{4f_s})$$
$$\bar{\psi}_M(t) = \bar{\psi}(t)+\mathrm{j}\bar{\psi}(t-\frac{1}{4f_s})\,,\ \bar{\psi}'_M(t) = \bar{\psi}(t)-\mathrm{j}\bar{\psi}(t-\frac{1}{4f_s})$$

Constant Envelope AltBOC Subcarriers (1)

The two four-valued subcarriers are given as

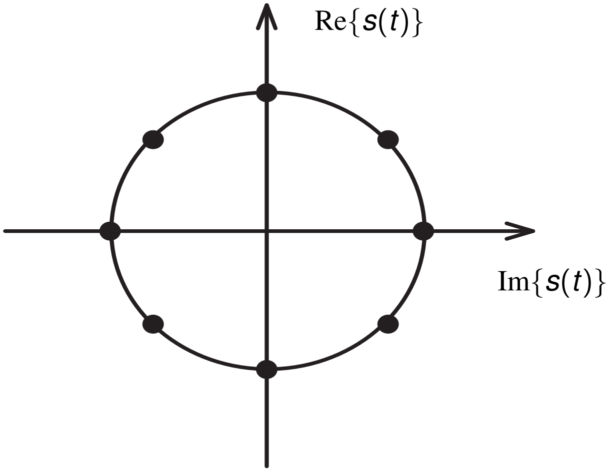
$$
\begin{aligned}
\psi(t) &= \frac{\sqrt{2}}{4}\mathrm{sgn}(\cos(2\pi f_s t - \frac{\pi}{4})) + \frac{1}{2}\mathrm{sgn}(\cos(2\pi f_s t)) \\
&+ \frac{\sqrt{2}}{4}\mathrm{sgn}(\cos(2\pi f_s t + \frac{\pi}{4})) \\
\bar{\psi}(t) &= -\frac{\sqrt{2}}{4}\mathrm{sgn}(\cos(2\pi f_s t - \frac{\pi}{4})) + \frac{1}{2}\mathrm{sgn}(\cos(2\pi f_s t)) \\
&- \frac{\sqrt{2}}{4}\mathrm{sgn}(\cos(2\pi f_s t + \frac{\pi}{4}))
\end{aligned}
$$

Constant Envelope AltBOC Subcarriers (2)

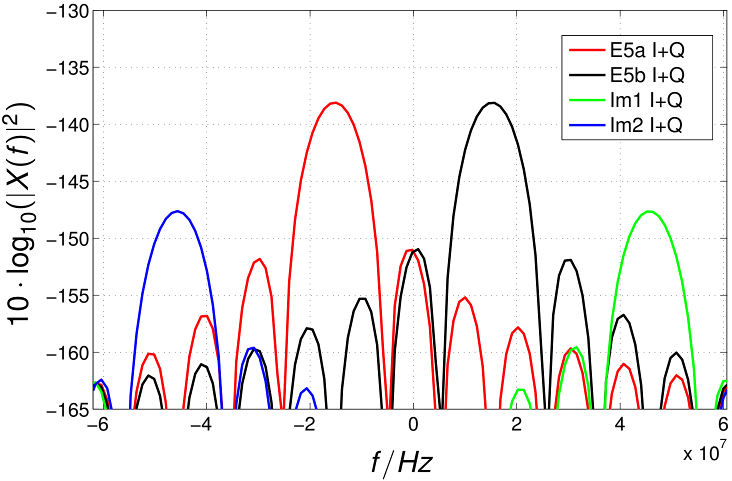


GALILEO INFORMATION CENTRE Brazil GALILEO EGNOS

Constant Envelope AltBOC Signal Constellation



Constant Envelope AltBOC for Galileo



$f_s = 15 \cdot 1.023$ MHz, four signals with $\frac{1}{T_c} = 10 \cdot 1.023$ Mcps and rectangular chip pulse shapes