

Notation

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1 Font notation

$a, b, c, \dots, A, B, C, \dots$	Scalars
$\mathbf{a}, \mathbf{b}, \mathbf{c}, \dots$	Vectors
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$	Matrices
$\mathcal{A}, \mathcal{B}, \mathcal{C}, \dots$	Tensors
$A, B, C, \dots, \mathcal{A}, \mathcal{B}, \mathcal{C}, \dots, \mathbb{A}, \mathbb{B}, \mathbb{C}, \dots$	Sets

2 Signals and functions operations

$f : A \rightarrow B$	A function f whose domain is A and codomain is B
$f^{(n)}$	n th derivative of the function f
f^{-1}	Inverse function of f
$f \circ g$	Composition of the functions f and g
$\inf_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Infimum
$\sup_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Supremum
$*$	Convolution
$\otimes, \textcircled{\mathbb{N}}$	Circular convolution
$\delta(t)$	Delta function
$\delta[n]$	Kronecker function
$h(t), h[n]$	Impulse response (continuous and discrete time)
$x(t)$	Continuous-time t
$x[n], x[k], x[m], x[i], \dots$	Discrete-time n, k, m, i, \dots
$x(n), x(k), x(m), x(i), \dots$	Discrete-time n, k, m, i, \dots (it should be used only if there are no continuous-time signals in the context to avoid ambiguity)
$\hat{x}(t)$ or $\hat{x}[n]$	Hilbert transform of $x(t)$ or $x[n]$

$\tilde{x}[n]$	Periodic discrete-time signal
$x[((n-m))_N], x((n-m))_N$	Circular shift in m samples within a N -samples window
$x_I(t)$ or $x_I[n]$	Real or in-phase part of $x(t)$ or $x[n]$
$x_Q(t)$ or $x_Q[n]$	Imaginary or quadrature part of $x(t)$ or $x[n]$

2.1 Transformations

$\mathcal{F}\{\cdot\}$	Fourier transform
$\mathcal{L}\{\cdot\}$	Laplace transform
$\mathcal{Z}\{\cdot\}$	z -transform
$X(s)$	Laplace transform of $x(t)$
$X(f)$	Fourier transform (FT) (in linear frequency, Hz) of $x(t)$
$X(j\omega)$	Fourier transform (FT) (in angular frequency, rad/sec) of $x(t)$
$X(e^{j\omega})$	Discrete-time Fourier transform (DTFT) of $x[n]$
$X[k], X(k)$	Discrete Fourier transform (DFT) or fast Fourier transform (FFT) of $x[n]$, or even the Fourier series (FS) of the periodic signal $x(t)$
$\tilde{X}[k], \tilde{X}(k)$	Discrete Fourier series (DFS) of $\tilde{x}[n]$
$X(z)$	z -transform of $x[n]$
$S_x(f)$	Power spectral density of $x(t)$ in linear frequency
$S_x(j\omega)$	Power spectral density of $x(t)$ in angular frequency

3 Statistical signal processing

$\nabla f, \mathbf{g}$	Gradient vector
$\nabla_x f, \mathbf{g}_x$	Gradient vector with respect x
\mathbf{g} (or $\hat{\mathbf{g}}$ if the gradient vector is \mathbf{g})	Stochastic approximation of the gradient vector
$J(\cdot)$	Cost-function or objective function
$\Lambda(\cdot)$	Likelihood function
$\Lambda_l(\cdot)$	Log-likelihood function
$\hat{x}(t)$ or $\hat{x}[n]$	Estimate of $x(t)$ or $x[n]$
$\hat{\mu}_x, \hat{\mathbf{m}}_x$	Sample mean of $x[n]$ or $x(t)$
$\hat{\boldsymbol{\mu}}_x, \hat{\mathbf{m}}_x$	Sample mean vector of $\mathbf{x}[n]$ or $\mathbf{x}(t)$

$\hat{r}_x(\tau), \hat{R}_x(\tau)$	Estimated autocorrelation function of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{R}}_x$	Sample (auto)correlation matrix
$\hat{r}_{x,d}(\tau), \hat{R}_{x,d}(\tau)$	Estimated cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$
$\hat{\mathbf{R}}_{xy}$	Sample cross-correlation matrix of \mathbf{R}_{xy}
$\hat{\rho}_{x,y}$	Estimated Pearson correlation coefficient between x and y
$\hat{c}_x(\tau), \hat{C}_x(\tau)$	Estimated autocovariance function of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_x, \hat{\mathbf{K}}_x, \hat{\Sigma}_x$	Sample (auto)covariance matrix
$\hat{c}_{xy}(\tau), \hat{C}_{xy}(\tau)$	Estimated cross-covariance function of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_{xy}, \hat{\mathbf{K}}_{xy}, \hat{\Sigma}_{xy}$	Sample cross-covariance matrix
$\mathbf{w}, \boldsymbol{\theta}$	Parameters, coefficients, or weights vector
$\mathbf{w}_o, \mathbf{w}^*, \boldsymbol{\theta}_o, \boldsymbol{\theta}^*$	Optimum value of the parameters, coefficients, or weights vector
\mathbf{W}	Matrix of the weights
\mathbf{J}	Jacobian matrix

4 Linear Algebra

4.1 Common matrices and vectors

\mathbf{W}, \mathbf{D}	Diagonal matrix
\mathbf{P}	Projection matrix; Permutation matrix
\mathbf{J}	Jordan matrix
\mathbf{L}	Lower matrix
\mathbf{U}	Upper matrix
\mathbf{C}	Cofactor matrix
$\mathbf{C}_A, \text{cof}(\mathbf{A})$	Cofactor matrix of \mathbf{A}
\mathbf{S}	Symmetric matrix
\mathbf{Q}	Orthogonal matrix
\mathbf{I}_N	$N \times N$ -dimensional identity matrix
$\mathbf{0}_{M \times N}$	$M \times N$ -dimensional null matrix
$\mathbf{0}_N$	N -dimensional null vector
$\mathbf{1}_{M \times N}$	$M \times N$ -dimensional ones matrix
$\mathbf{1}_N$	N -dimensional ones vector
$\mathbf{0}$	Null matrix, vector, or tensor (dimensionality understood by context)

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Ones matrix, vector, or tensor (dimensionality understood by context)

4.2 Operations with matrices

\mathbf{A}^{-1}	Inverse matrix
$\mathbf{A}^+, \mathbf{A}^\dagger$	Moore-Penrose pseudoinverse
\mathbf{A}^\top	Transpose
\mathbf{A}^*	Complex conjugate
\mathbf{A}^H	Hermitian
$\ \mathbf{A}\ _F$	Frobenius norm
$\ \mathbf{A}\ $	Matrix norm
$ \mathbf{A} , \det(\mathbf{A})$	Determinant
$\text{diag}(\mathbf{A})$	The elements in the diagonal of \mathbf{A}
$\text{vec}(\mathbf{A})$	Vectorization: stacks the columns of the matrix \mathbf{A} into a long column vector
$\text{vec}_d(\mathbf{A})$	Extracts the diagonal elements of a square matrix and returns them in a column vector
$\text{vec}_l(\mathbf{A})$	Extracts the elements strictly below the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\text{vec}_u(\mathbf{A})$	Extracts the elements strictly above the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\text{vec}_b(\mathbf{A})$	Block vectorization operator: stacks square block matrices of the input into a long block column matrix
$\text{unvec}(\mathbf{A})$	Reshapes a column vector into a matrix
$\text{tr}(\mathbf{A})$	trace
\otimes	Kronecker product
\odot	Hadamard (or Schur) (elementwise) product
$\mathbf{A}^{\odot n}$	n th-order Hadamard power of the matrix \mathbf{A}
$\mathbf{A}^{\odot \frac{1}{n}}$	n th-order Hadamard root of the matrix \mathbf{A}
\oslash	Hadamard (or Schur) (elementwise) division
\diamond	Khatri-Rao product

\otimes	Kronecker Product
\times_n	n -mode product

4.3 Operations with vectors

$\ \mathbf{a}\ $	l_1 norm, 1-norm, or Manhattan norm
$\ \mathbf{a}\ , \ \mathbf{a}\ _2$	l_2 norm, 2-norm, or Euclidean norm
$\ \mathbf{a}\ _p$	l_p norm, p -norm, or Minkowski norm
$\ \mathbf{a}\ _\infty$	l_∞ norm, ∞ -norm, or Chebyshev norm
$\text{diag}(\mathbf{a})$	Diagonalization: a square, diagonal matrix with entries given by the vector \mathbf{a}
$\langle \mathbf{a}, \mathbf{b} \rangle$	Inner product, i.e., $\mathbf{a}^\top \mathbf{b}$
$\mathbf{a} \circ \mathbf{b}$	Outer product, i.e., $\mathbf{a} \mathbf{b}^\top$

4.4 Decompositions

$\mathbf{\Lambda}$	Eigenvalue matrix
\mathbf{Q}	Eigenvectors matrix; Orthogonal matrix of the QR
\mathbf{R}	Upper triangular matrix of the QR decomposition
\mathbf{U}	Left singular vectors
\mathbf{U}_r	Left singular nondegenerated vectors
$\mathbf{\Sigma}$	Singular value matrix
$\mathbf{\Sigma}_r$	Singular value matrix with nonzero singular values in the main diagonal
$\mathbf{\Sigma}^+$	Singular value matrix of the pseudoinverse
$\mathbf{\Sigma}_r^+$	Singular value matrix of the pseudoinverse with nonzero singular values in the main diagonal
\mathbf{V}	Right singular vectors
\mathbf{V}_r	Right singular nondegenerated vectors
$\text{eig}(\mathbf{A})$	Set of the eigenvalues of \mathbf{A}
$\llbracket \mathbf{A}, \mathbf{B}, \mathbf{C}, \dots \rrbracket$	CANDECOMP/PARAFAC (CP) decomposition of the tensor \mathcal{X} from the outer product of column vectors of $\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$

$\llbracket \boldsymbol{\lambda}; \mathbf{A}, \mathbf{B}, \mathbf{C}, \dots \rrbracket$

Normalized CANDECOMP/PARAFAC (CP) decomposition of the tensor $\boldsymbol{\mathcal{X}}$ from the outer product of column vectors of $\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$

4.5 Spaces

$\mathcal{N}(\mathbf{A})$, nullspace(\mathbf{A}), kernel(\mathbf{A})
 $\mathcal{C}(\mathbf{A})$, columnspace(\mathbf{A}), range(\mathbf{A})

Nullspace (or kernel)
 Columnspace (or range), i.e., the space $\text{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$, where \mathbf{a}_i is the i th column vector of the matrix \mathbf{A}

$\text{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$

Vector space spanned by the argument vectors

$\text{span}(\mathbf{A})$

Vector space spanned by the column vectors of \mathbf{A} , which gives the columnspace of \mathbf{A}

$\text{rank}(\mathbf{A})$

Rank, that is, $\dim(\text{span}(\mathbf{A})) = \dim(\mathcal{C}(\mathbf{A}))$

nullity(\mathbf{A})

Nullity of \mathbf{A} , i.e., $\dim(\mathcal{N}(\mathbf{A}))$

$\mathbf{a} \perp \mathbf{b}$

\mathbf{a} is orthogonal to \mathbf{b}

$\mathbf{a} \not\perp \mathbf{b}$

\mathbf{a} is not orthogonal to \mathbf{b}

$\mathbf{X}_{(n)}$

n -mode matricization of the tensor $\boldsymbol{\mathcal{X}}$

4.6 Inequalities

$\boldsymbol{\mathcal{X}} \succeq 0$

Nonnegative tensor

$\mathbf{a} \preceq_K \mathbf{b}$

Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the conic subset K in the space \mathbb{R}^n

$\mathbf{a} \prec_K \mathbf{b}$

Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the interior of the conic subset K in the space \mathbb{R}^n

$\mathbf{a} \preceq \mathbf{b}$

Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the nonnegative orthant conic subset, \mathbb{R}_+^n , in the space \mathbb{R}^n

$\mathbf{a} \prec \mathbf{b}$

Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the positive orthant conic subset, \mathbb{R}_{++}^n , in the space \mathbb{R}^n

$\mathbf{A} \preceq_K \mathbf{B}$

Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the conic subset K in the space \mathbb{S}^n

$\mathbf{A} \prec_K \mathbf{B}$

Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the interior of the conic subset K in the space \mathbb{S}^n

$\mathbf{A} \preceq \mathbf{B}$

Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive semidefinite conic subset, \mathbb{S}_+^n , in the space \mathbb{S}^n

$\mathbf{A} \prec \mathbf{B}$

Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive orthant conic subset, \mathbb{S}_{++}^n , in the space \mathbb{S}^n

4.7 Indexing

$x_{i_1, i_2, \dots, i_N}, [\mathcal{X}]_{i_1, i_2, \dots, i_N}$

Element in the position (i_1, i_2, \dots, i_N) of the tensor \mathcal{X}

$\mathcal{X}^{(n)}$

n th tensor of a nontemporal sequence

$\mathbf{x}_n, \mathbf{x}_{:n}$

n th column of the matrix X

\mathbf{x}_n

n th row of the matrix X

$\mathbf{x}_{i_1, \dots, i_{n-1}, :, i_{n+1}, \dots, i_N}$

Mode- n fiber of the tensor \mathcal{X}

$\mathbf{x}_{:, i_2, i_3}$

Column fiber (mode-1 fiber) of the thrid-order tensor \mathcal{X}

$\mathbf{x}_{i_1, :, i_3}$

Row fiber (mode-2 fiber) of the thrid-order tensor \mathcal{X}

$\mathbf{x}_{i_1, i_2, :}$

Tube fiber (mode-3 fiber) of the thrid-order tensor \mathcal{X}

$\mathbf{X}_{i_1, :, :}$

Horizontal slice of the thrid-order tensor \mathcal{X}

$\mathbf{X}_{:, i_2, :}$

Lateral slices slice of the thrid-order tensor \mathcal{X}

$\mathbf{X}_{i_3}, \mathbf{X}_{:, :, i_3}$

Frontal slices slice of the thrid-order tensor \mathcal{X}

5 Sets

$A + B$

Set addition (Minkowski sum)

$A - B$

Minkowski difference

$A \setminus B, A - B$

Set difference or set subtraction, i.e., the set containing the elements of A that are not in B

$A \cup B$

Set of union

$A \cap B$	Set of intersection
$A \times B$	Cartesian product
A^n	$\underbrace{A \times A \times \cdots \times A}_{n \text{ times}}$
A^\perp	Orthogonal complement of A , e.g., $N(\mathbf{A}) = C(\mathbf{A}^\top)^\perp$
$A \oplus B$	Direct sum, e.g., $C(\mathbf{A}^\top) \oplus C(\mathbf{A}^\top)^\perp = \mathbb{R}^n$
A^c, \bar{A}	Complement set (given U)
$\#A, A $	Cardinality
$a \in A$	a is element of A
$a \notin A$	a is not element of A
$\{1, 2, \dots, n\}$	Discrete set containing the integer elements $1, 2, \dots, n$
U	Universe
2^A	Power set of A
\mathbb{R}	Set of real numbers
\mathbb{C}	Set of complex numbers
\mathbb{Z}	Set of integer number
$\mathbb{B} = \{0, 1\}$	Boolean set
\emptyset	Empty set
\mathbb{N}	Set of natural numbers
$\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$???
$\mathbb{K}^{I_1 \times I_2 \times \cdots \times I_N}$	$I_1 \times I_2 \times \cdots \times I_N$ -dimensional real (or complex) space
\mathbb{K}_+	Nonnegative real (or complex) space
\mathbb{K}_{++}	Positive real (or complex) space, i.e., $\mathbb{K}_{++} = \mathbb{K}_+ \setminus \{\mathbf{0}\}$
$\mathbb{S}^n, \mathcal{S}^n$	Conic set of the symmetric matrices in $\mathbb{R}^{n \times n}$
$\mathbb{S}_+^n, \mathcal{S}_+^n$	Conic set of the symmetric positive semidefinite matrices in $\mathbb{R}^{n \times n}$
$\mathbb{S}_{++}^n, \mathcal{S}_{++}^n$	Conic set of the symmetric positive definite matrices in $\mathbb{R}^{n \times n}$, i.e., $\mathbb{S}_{++}^n = \mathbb{S}_+^n \setminus \{\mathbf{0}\}$
\mathbb{H}^n	Set of all hermitian matrices in $\mathbb{C}^{n \times n}$
$[a, b]$	Closed interval of a real set from a to b
(a, b)	Opened interval of a real set from a to b
$[a, b), (a, b]$	Half-opened intervals of a real set from a to b

6 Probability, statistics, and stochastic processes

6.1 Operators and symbols

$E[\cdot]$	Statistical expectation
$E_u[\cdot]$	Statistical expectation with respect to u
μ_x	Mean of the random variable x
$\mathbf{\mu}_x, \mathbf{m}_x$	Mean vector of the random variable \mathbf{x}
μ_n	n th-order moment of a random variable
\mathcal{K}_x, μ_4	Kurtosis (4th-order moment) of the random variable x
$\text{VAR}[\cdot]$	Variance operator
σ_x	Variance of the random variable x
$\rho_{x,y}$	Pearson correlation coefficient between x and y
$a \sim P$	Random variable a with distribution P

6.2 Stochastic processes

$r_x(\tau), R_x(\tau)$	Autocorrelation function of the signal $x(t)$ or $x[n]$
\mathbf{R}_x	(Auto)correlation matrix of $\mathbf{x}(n)$
$r_{x,d}(\tau), R_{x,d}(\tau)$	Cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$
\mathbf{R}_{xy}	Cross-correlation matrix of $\mathbf{x}(n)$ and $\mathbf{y}(n)$
\mathbf{p}_{xd}	Cross-correlation vector between $\mathbf{x}(n)$ and $d(n)$
$c_x(\tau), C_x(\tau)$	Autocovariance function of the signal $x(t)$ or $x[n]$
$\mathbf{C}_x, \mathbf{K}_x, \mathbf{\Sigma}_x$	(Auto)covariance matrix of \mathbf{x}
$c_{xy}(\tau), C_{xy}(\tau)$	Cross-covariance function of the signal $x(t)$ or $x[n]$
$\mathbf{C}_{xy}, \mathbf{K}_{xy}, \mathbf{\Sigma}_{xy}$	Cross-covariance matrix of \mathbf{x}

6.3 Functions

$Q(\cdot)$	Q -function, i.e., $P[\mathcal{N}(0, 1) > x]$
$\text{erf}(\cdot)$	Error function

$\text{erfc}(\cdot)$	Complementary error function i.e., $\text{erfc}(x) = 2Q(\sqrt{2}x) - \text{erf}(x)$
$P[A]$	Probability of the event or set A
$p(\cdot), f(\cdot)$	Probability density function (PDF)
$p(x A)$	Conditional probability density function
$F(\cdot)$	Cumulative distribution function (CDF)
$\Phi_x(\omega), M_x(j\omega), E[e^{j\omega x}]$	First characteristic function (CF) of x
$M_x(t), \Phi_x(-jt), E[e^{tx}]$	Moment-generating function (MGF) of x ($M_x(t) = \Phi_x(-jt)$ and $\Phi(\omega) = M_x(j\omega)$)
$\Psi_x(\omega), \ln \Phi_x(\omega), \ln E[e^{j\omega x}]$	Second characteristic function, i.e., $\ln E[e^{j\omega x}]$
$K_x(t), \ln E[e^{tx}], \ln M_x(t)$	Cumulant-generating function (CGF) of x

6.4 Distributions

$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution of a random variable with mean μ and variance σ^2
$\mathcal{CN}(\mu, \sigma^2)$	Complex Gaussian distribution of a random variable with mean μ and variance σ^2
$\mathcal{N}(\mathbf{\mu}, \mathbf{\Sigma})$	Gaussian distribution of a vector random variable with mean $\mathbf{\mu}$ and covariance matrix $\mathbf{\Sigma}$
$\mathcal{CN}(\mathbf{\mu}, \mathbf{\Sigma})$	Complex Gaussian distribution of a vector random variable with mean $\mathbf{\mu}$ and covariance matrix $\mathbf{\Sigma}$
$\mathcal{U}(a, b)$	Uniform distribution from a to b
$\chi^2(n), \chi_n^2$	Chi-square distribution with n degree of freedom (assuming that the Gaussians are $\mathcal{N}(0, 1)$)
$\text{Exp}(\lambda)$	Exponential distribution with rate parameter λ
$\Gamma(\alpha, \beta)$	Gamma distribution with shape parameter α and rate parameter β
$\Gamma(\alpha, \theta)$	Gamma distribution with shape parameter α and scale parameter $\theta = 1/\beta$
Nakagami(m, Ω)	Nakagami-m distribution with shape parameter m and spread parameter Ω

$\text{Rayleigh}(\sigma)$	Rayleigh distribution with scale parameter σ
$\text{Rayleigh}(\Omega)$	Rayleigh distribution with the second moment $\Omega = E[x^2] = 2\sigma^2$
$\text{Rice}(s, \sigma)$	Rice distribution with noncentrality parameter (specular component) s and σ
$\text{Rice}(A, K)$	Rice distribution with Rice factor $K = s^2/2\sigma^2$ and scale parameter $A = s^2 + 2\sigma^2$

7 Other notations

7.1 Mathematical symbols

\exists	There exists
\nexists	There does not exist
$\exists!$	There exist an unique
\forall	For all
$, :$	Such that
\therefore	Therefore
\Longleftrightarrow	Logical equivalence
\triangleq	Equal by definition
\neq	Not equal
∞	Infinity
j	$\sqrt{-1}$

7.2 Operations

$ a $	Absolute value of a
\log	Base-10 logarithm or decimal logarithm
\ln	Natural logarithm
$\text{Re}\{x\}$	Real part of x
$\text{Im}\{x\}$	Imaginary part of x
\angle	phase (complex argument)
$x \bmod y$	Remainder, i.e., $x - y[x/y]$
$\text{frac}(x)$	Fractional part, i.e., $x \bmod 1$
$a \wedge b$	Logical AND of a and b
$a \vee b$	Logical OR of a and b
$\neg a$	Logical negation of a
$\lceil \cdot \rceil$	Ceiling operation
$\lfloor \cdot \rfloor$	Floor operation

7.3 Functions

$\mathcal{O}(\cdot), O(\cdot)$
 $\Gamma(\cdot)$

Big-O notation
Gamma function

8 Abbreviations

wrt.
st.
iff.
EVD

SVD
CP

With respect to
Subject to
If and only if
Eigenvalue decomposition, or eigen-
decomposition
Singular value decomposition
CANDECOMP/PARAFAC