Functions		
Function	Convex?	Proof
$\mathbf{y} = \max(f_1, f_2)$	Yes, if f_1 and f_2 are convex functions	
$\mathbf{y} = \min(f_1, f_2)$	Not always	
$C = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{A} \in \mathbb{R}^{m\times n}, \mathbf{b} \in \mathbb{R}^m\}$	It is an affine set (all affine set is a convex set)	
$y = \mathbf{c}^\mathsf{T}\mathbf{x}$ (linear function)	Yes (but not strictly convex)	
$y = \lVert\mathbf{x}\rVert_p$ (p-norm)	Yes (for any $p \in \mathbb{N}_+$)	$\lVert\theta\mathbf{x} + (1-\theta)\mathbf{y}\rVert \le \theta \lVert\mathbf{x}\rVert + (1-\theta)\lVert\mathbf{y}\rVert$ (triangular inequality)
$f(g(\mathbf{x}))$	Yes, if f, g are convex	

Function	Domain	Codomain	Comments
System of linear equation: $\mathbf{b} = f(\mathbf{x}) = \mathbf{A}\mathbf{x}$	$D = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{A}\mathbf{x} = \mathbf{b} \in C, \mathbf{A} \in \mathbb{R}^{m\times n}\}$	$C = \{\mathbf{b} \in \mathbb{R}^m \mid \mathbf{b} = \mathbf{A}\mathbf{x}, \forall\, \mathbf{x} \in D\}$	If D is an affine set, so C is also affine set which, in turn, is a convex set.

Sets		
Set	Convex?	Commens
Convex hull: conv $C = \left\{\sum_{i=1}^k \theta_i\mathbf{x}_i \mid \mathbf{x}_i \in C, 0 \le \theta_i \le 1 \text{ for } i = 1, \cdots, k, \mathbf{1}^\mathsf{T}\boldsymbol{\theta} = 1\right\}$	Yes.	• A will be the smallest convex set that contains C. • conv C will be a finite set as long as C is also finite.
Affine hull: aff $C = \left\{\sum_{i=1}^k \theta_i\mathbf{x}_i \mid \mathbf{x}_i \in C \text{ for } i = 1, \cdots, k, \mathbf{1}^\mathsf{T}\boldsymbol{\theta} = 1\right\}$	Yes.	• A will be the smallest affine set that contains C. • Different from the convex set, θ_i is not restricted between 0 and 1 • aff C will always be an infinite set. If aff C contains the origin, it is also a subspace.
Conic hull: $A = \left\{\sum_{i=1}^k \theta_i\mathbf{x}_i \mid \mathbf{x}_i \in C, \theta_i > 0 \text{ for } i = 1, \cdots, k\right\}$	Yes.	• A will be the smallest convex conic that contains C. • Different from the convex and affine sets, θ_i does not need to sum up 1.
Hyperplane: $\mathcal{H} = \left\{\mathbf{x} \mid \mathbf{a}^\mathsf{T}\mathbf{x} = b\right\}$ $\mathcal{H} = \left\{\mathbf{x} \mid \mathbf{a}^\mathsf{T}(\mathbf{x} - \mathbf{x}_0) = \mathbf{0}\right\}$ $\mathcal{H} = \mathbf{x}_0 + a^\perp$	Yes.	• It is an infinite set $\mathbb{R}^{n-1} \subset \mathbb{R}^n$ that divides the space into two halfspaces. • $a^\perp = \left\{\mathbf{v} \mid \mathbf{a}^\mathsf{T}\mathbf{v} = 0\right\}$ is the set of vectors perpendicular to $\mathbf{a}$. It passes through the origin. • $a^\perp$ is offset from the origin by $\mathbf{x}_0$, which is any vector in $\mathcal{H}$.
Halfspace: $\mathcal{H}_-$ or $\mathcal{H}_+$ $\left\{\mathbf{x} \mid \mathbf{a}^\mathsf{T}\mathbf{x} \lesseqgtr b\right\}$	Yes.	• They are infinite sets of the parts divided by $\mathcal{H}$.
Euclidean ball: $B(\mathbf{x}_c, r) = \{\mathbf{x} \mid \lVert\mathbf{x} - \mathbf{x}_c\rVert_2 \le r\}$ $B(\mathbf{x}_c, r) = \left\{\mathbf{x} \mid (\mathbf{x} - \mathbf{x}_c)^\mathsf{T}(\mathbf{x} - \mathbf{x}_c) \le r\right\}$ $B(\mathbf{x}_c, r) = \{\mathbf{x}_c + r\lVert\mathbf{u}\rVert \mid \lVert\mathbf{u}\rVert \le 1\}$	Yes	• $B(\mathbf{x}_c, r)$ as long as $r < \infty$. • $\mathbf{x}_c$ is the center of the ball. • r is its radius.
Ellipsoid: $\mathcal{E} = \left\{\mathbf{x} \mid (\mathbf{x} - \mathbf{x}_c)^\mathsf{T}\mathbf{P}^{-1}(\mathbf{x} - \mathbf{x}_c) \le 1\right\}$ $\mathcal{E} = \{\mathbf{x}_c + \mathbf{A}\mathbf{u} \mid \lVert\mathbf{u}\rVert \le 1\}$	Yes	• $\mathcal{E}$ is a finite set as long as $\mathbf{P}$ is a finite matrix. • $\mathbf{P}$ is symmetric and positive definite, that is, $\mathbf{P} = \mathbf{P}^\mathsf{T} \succ 0$. • $\mathbf{x}_c$ is the center of the ellipsoid. • The lengths of the semi-axes are given by $\sqrt{\lambda_i}$. • $\mathbf{A} = \mathbf{P}^{1/2}$. • $\mathbf{A}$ is invertible. When it is not, we say that $\mathcal{E}$ is a degenerated ellipsoid (degenerated ellipsoids are also convex).
Norm cone: $C = \left\{[x_1, x_2, \cdots, x_n, t]^\mathsf{T} \in \mathbb{R}^{n+1} \mid \mathbf{x} \in \mathbb{R}^n, \lVert\mathbf{x}\rVert_p \le t\right\} \subseteq \mathbb{R}^{n+1}$	Yes.	• Although it is named "Norm cone", it is a set, not a scalar. • The cone norm increases the dimension of $\mathbf{x}$ in 1. • For $p = 2$, it is called the second-order cone, quadratic cone, Lorentz cone or ice-cream cone.
Polyhedra: $\mathcal{P} = \left\{\mathbf{x} \mid \mathbf{a}_j^\mathsf{T}\mathbf{x} \le b_j, j = 1, \ldots, m, \mathbf{a}_j^\mathsf{T}\mathbf{x} = d_j, j = 1, \cdots, p\right\}$ $\mathcal{P} = \{\mathbf{x} \mid \mathbf{A}\mathbf{x} \preceq \mathbf{b}, \mathbf{C}\mathbf{x} = \mathbf{d}\}$, where $\mathbf{A} = \begin{bmatrix}\mathbf{a}_1 & \mathbf{a}_2 & \ldots & \mathbf{a}_m\end{bmatrix}^\mathsf{T}$ and $\mathbf{C} = \begin{bmatrix}\mathbf{c}_1 & \mathbf{c}_2 & \ldots & \mathbf{c}_m\end{bmatrix}^\mathsf{T}$	Yes.	• Polyhedron is the intersection of m halfspaces. • The polyhedron may or may not be an infinite set. • Subspaces, hyperplanes, lines, rays line segments, and halfspaces are all polyhedra. • The *nonnegative orthant*, $\mathbb{R}^n_+ = \{\mathbf{x} \in \mathbb{R}^n \mid x_i \le 0 \text{ for } i = 1, \ldots n\} = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{I}\mathbf{x} \succeq \mathbf{0}\}$, is a special polyhedron • *Simplexes*, , are another family of polyhedra, where $\{\mathbf{v}_m\}_{m=0}^k$ is a affinely independent set. It forms a k-dimensional shape in $\mathbb{R}^n$, thus being called k-dimensional simplex in $\mathbb{R}^n$.
Simplex: • $\mathcal{S} = \text{conv } \{\mathbf{v}_m\}_{m=0}^k = \left\{\sum_{i=0}^k \theta_i\mathbf{v}_i \mid \mathbf{0} \preceq \boldsymbol{\theta} \preceq \mathbf{1}, \mathbf{1}^\mathsf{T}\boldsymbol{\theta} = 1\right\}$ • $\mathcal{S} = \left\{\mathbf{x} \mid \mathbf{A}_1\mathbf{x} \preceq \mathbf{A}_1\mathbf{v}_0, \mathbf{A}_2\mathbf{x} = \mathbf{A}_2\mathbf{v}_0, \mathbf{1}^\mathsf{T}\mathbf{A}_1\mathbf{x} \le 1 + \mathbf{1}^\mathsf{T}\mathbf{A}_1\mathbf{v}_0\right\}$ • $\mathcal{S} = \{\mathbf{x} \mid \mathbf{A}_1\mathbf{x} \prec \mathbf{b}, \mathbf{C}\mathbf{x} = \mathbf{d}\}$ (Polyhedra form), where $\mathbf{b} = \mathbf{A}_1\mathbf{v}_0, \mathbf{C} = \mathbf{A}_2, \mathbf{d} = \mathbf{A}_2\mathbf{v}_0$	Yes.	• Also called k-dimensional Simplex in $\mathbb{R}^n$.
$C = A \cup B$	Not always.	
$C = A \cap B$	Yes, if A and B are convex sets.	

- All convex set is quasiconvex, but not all quasiconvex is convex.
- It is possible to solve quasiconvex functions, even if it is not convex (see Algorithm 4.1). But not all quasiconvex functions that are nonconvex can be solved(?).
- Superlevel set (a set) 3.3.6, all convex functions have all convex α sub-level set, but not all functions that have convex α sub-level set are convex (see slide 3.11).