

# List of Symbols

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## 1 Font notation

$a, b, c, \dots, A, B, C, \dots$	Scalars
$\mathbf{a}, \mathbf{b}, \mathbf{c}, \dots$	Vectors
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$	Matrices
$\mathcal{A}, \mathcal{B}, \mathcal{C}, \dots$	Tensors
$\mathcal{A}, \mathcal{B}, \mathcal{C}, \dots, \mathcal{A}, \mathcal{B}, \mathcal{C}, \dots$	Sets

## 2 Common symbols

$\nabla f, \mathbf{g}$	Gradient vector
$\nabla_x f, \mathbf{g}_x$	Gradient vector with respect $x$
$\mathbf{g}$ (or $\hat{\mathbf{g}}$ if the gradient vector is $\mathbf{g}$ )	Stochastic approximation of the gradient vector
$J(\cdot)$	Cost-function or objective function
$\Lambda(\cdot)$	Likelihood function
$\Lambda_l(\cdot)$	Log-likelihood function
$\mathcal{O}(\cdot), \mathcal{O}(\cdot)$	big-O notation
$Q(x)$	$Q$ -function
$\boldsymbol{\mu}_x, \mathbf{m}_x$	Mean vector
$\hat{\boldsymbol{\mu}}_x, \hat{\mathbf{m}}_x$	Sample mean vector
$r_x(\tau), R_x(\tau)$	Autocorrelation function of the signal $x(t)$ or $x[n]$
$\hat{r}_x(\tau), \hat{R}_x(\tau)$	Estimated autocorrelation function of the signal $x(t)$ or $x[n]$
$\mathbf{R}_x$	(Auto)correlation matrix of $\mathbf{x}$
$\hat{\mathbf{R}}_x$	Sample (auto)correlation matrix
$r_{x,d}(\tau), R_{x,d}(\tau)$	Cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$
$\hat{r}_{x,d}(\tau), \hat{R}_{x,d}(\tau)$	Estimated cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$

$\mathbf{R}_{xy}$	Cross-correlation matrix of $\mathbf{x}$ and $\mathbf{y}$
$\hat{\mathbf{R}}_{xy}$	Sample cross-correlation matrix of $\mathbf{R}_{xy}$
$\mathbf{p}_{xd}$	Cross-correlation vector
$\rho_{x,y}$	Pearson correlation coefficient between $x$ and $y$
$\hat{\rho}_{x,y}$	Estimated Pearson correlation coefficient between $x$ and $y$
$c_x(\tau), C_x(\tau)$	Autocovariance function of the signal $x(t)$ or $x[n]$
$\hat{c}_x(\tau), \hat{C}_x(\tau)$	Estimated autocovariance function of the signal $x(t)$ or $x[n]$
$\mathbf{C}_x, \mathbf{K}_x, \mathbf{\Sigma}_x$	(Auto)covariance matrix of $\mathbf{x}$
$\hat{\mathbf{C}}_x, \hat{\mathbf{K}}_x, \hat{\mathbf{\Sigma}}_x$	Sample (auto)covariance matrix
$c_{xy}(\tau), C_{xy}(\tau)$	Cross-covariance function of the signal $x(t)$ or $x[n]$
$\hat{c}_{xy}(\tau), \hat{C}_{xy}(\tau)$	Estimated cross-covariance function of the signal $x(t)$ or $x[n]$
$\mathbf{C}_{xy}, \mathbf{K}_{xy}, \mathbf{\Sigma}_{xy}$	Cross-covariance matrix of $\mathbf{x}$
$\hat{\mathbf{C}}_{xy}, \hat{\mathbf{K}}_{xy}, \hat{\mathbf{\Sigma}}_{xy}$	Sample cross-covariance matrix
$\delta(t)$	Delta function
$\delta[n]$	Kronecker function
$h(t), h[n]$	Impulse response (continuous and discrete time)
$\mathbf{C}$	Cofactor matrix
$\mathbf{W}, \mathbf{D}$	Diagonal matrix
$\mathbf{w}, \boldsymbol{\theta}$	Parameters, coefficients, or weights vector
$\mathbf{w}_o, \mathbf{w}^*, \boldsymbol{\theta}_o, \boldsymbol{\theta}^*$	Optimum value of the parameters, coefficients, or weights vector
$\mathbf{W}$	Matrix of the weights
$\mathbf{P}$	Projection matrix; Permutation matrix
$\boldsymbol{\Lambda}$	Eigenvalue matrix
$\mathbf{L}$	Lower matrix
$\mathbf{U}$	Upper matrix; Left singular vectors
$\mathbf{U}_r$	Left singular nondegenerated vectors
$\mathbf{\Sigma}$	Singular value matrix
$\mathbf{\Sigma}_r$	Singular value matrix with nonzero singular values in the main diagonal
$\mathbf{\Sigma}^+$	Singular value matrix of the pseudoinverse
$\mathbf{\Sigma}_r^+$	Singular value matrix of the pseudoinverse with nonzero singular values in the main diagonal

$\mathbf{V}$	Right singular vectors
$\mathbf{V}_r$	Right singular nondegenerated vectors
$\mathbf{J}$	Jordan matrix; Jacobian matrix
$\mathbf{S}$	Symmetric matrix
$\mathbf{Q}$	Orthogonal matrix
$\mathbf{I}_N$	$N \times N$ -dimensional identity matrix
$\mathbf{0}_{M \times N}$	$M \times N$ -dimensional null matrix
$\mathbf{0}_N$	$N$ -dimensional null vector
$\mathbf{0}$	Null matrix, vector, or tensor (dimensionality understood by context)
$\mathbf{1}_{M \times N}$	$M \times N$ -dimensional ones matrix
$\mathbf{1}_N$	$N$ -dimensional ones vector
$\mathbf{1}$	Ones matrix, vector, or tensor (dimensionality understood by context)
$j$	$\sqrt{-1}$

### 3 Linear Algebra operations

$\mathbf{A}^{-1}$	Inverse matrix
$\mathbf{A}^+, \mathbf{A}^\dagger$	Moore-Penrose pseudoinverse
$\mathbf{A}^\top$	Transpose
$\mathbf{A}^*$	Complex conjugate
$\mathbf{A}^H$	Hermitian
$\ \mathbf{A}\ _F$	Frobenius norm
$\ \mathbf{A}\ $	Matrix norm
$\ \mathbf{a}\ $	$l_1$ norm, 1-norm, or Manhattan norm
$\ \mathbf{a}\ , \ \mathbf{a}\ _2$	$l_2$ norm, 2-norm, or Euclidean norm
$\ \mathbf{a}\ _p$	$l_p$ norm, $p$ -norm, or Minkowski norm
$\ \mathbf{a}\ _\infty$	$l_\infty$ norm, $\infty$ -norm, or Chebyshev norm
$ \mathbf{A} , \det(\mathbf{A})$	Determinant
$\text{diag}(\mathbf{a}), \text{diag}(\mathbf{A})$	Diagonalization: a square, diagonal matrix with entries given by the vector $\mathbf{a}$ or the elements in the diagonal of $\mathbf{A}$
$\text{vec}(\mathbf{A})$	Vectorization: stacks the columns of the matrix $\mathbf{A}$ into a long column vector
$\text{vec}_d(\mathbf{A})$	Extracts the diagonal elements of a square matrix and returns them in a column vector

$\text{vec}_l(\mathbf{A})$	Extracts the elements strictly below the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\text{vec}_u(\mathbf{A})$	Extracts the elements strictly above the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\text{vec}_b(\mathbf{A})$	Block vectorization operator: stacks square block matrices of the input into a long block column matrix
$\text{unvec}(\mathbf{A})$	Reshapes a column vector into a matrix
$\text{cof}(\mathbf{A})$	Cofactor matrix of $\mathbf{A}$
$\text{eig}(\mathbf{A})$	Set of the eigenvalues of $\mathbf{A}$
$\llbracket \mathbf{A}, \mathbf{B}, \mathbf{C}, \dots \rrbracket$	CANDECOMP/PARAFAC (CP) decomposition of the tensor $\mathcal{X}$ from the outer product of column vectors of $\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$
$\llbracket \boldsymbol{\lambda}; \mathbf{A}, \mathbf{B}, \mathbf{C}, \dots \rrbracket$	Normalized CANDECOMP/PARAFAC (CP) decomposition of the tensor $\mathcal{X}$ from the outer product of column vectors of $\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$
$\text{N}(\mathbf{A}), \text{nullspace}(\mathbf{A}), \text{kernel}(\mathbf{A})$	Nullspace (or kernel)
$\text{C}(\mathbf{A}), \text{columnspace}(\mathbf{A}), \text{range}(\mathbf{A})$	Columnspace (or range), i.e., the space span $(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$ , where $\mathbf{a}_i$ is the $i$ th column vector of the matrix $\mathbf{A}$
$\text{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$	Vector space spanned by the argument vectors
$\text{span}(\mathbf{A})$	Vector space spanned by the column vectors of $\mathbf{A}$ , which gives the column space of $\mathbf{A}$
$\text{rank}(\mathbf{A})$	Rank, that is, $\dim(\text{span}(\mathbf{A})) = \dim(\text{C}(\mathbf{A}))$
$\text{nullity}(\mathbf{A})$	Nullity of $\mathbf{A}$ , i.e., $\dim(\text{N}(\mathbf{A}))$
$\text{tr}(\mathbf{A})$	trace
$\mathbf{a} \perp \mathbf{b}$	$\mathbf{a}$ is orthogonal to $\mathbf{b}$
$\mathbf{a} \not\perp \mathbf{b}$	$\mathbf{a}$ is not orthogonal to $\mathbf{b}$
$\langle \mathbf{a}, \mathbf{b} \rangle$	Inner product, i.e., $\mathbf{a}^\top \mathbf{b}$
$\mathbf{a} \circ \mathbf{b}$	Outer product, i.e., $\mathbf{a} \mathbf{b}^\top$
$\otimes$	Kronecker product
$\odot$	Hadamard (elementwise) product
$\diamond$	Khatri-Rao product
$\boxtimes$	Kronecker Product

$\times_n$	$n$ -mode product
$\mathbf{X}_{(n)}$	$n$ -mode matricization of the tensor $\mathcal{X}$
$\mathcal{X} \leq 0$	Nonnegative tensor
$\mathbf{a} \preceq_K \mathbf{b}$	Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the conic subset $K$ in the space $\mathbb{R}^n$
$\mathbf{a} \prec_K \mathbf{b}$	Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the interior of the conic subset $K$ in the space $\mathbb{R}^n$
$\mathbf{a} \leq \mathbf{b}$	Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the nonnegative orthant conic subset, $\mathbb{R}_+^n$ , in the space $\mathbb{R}^n$
$\mathbf{a} < \mathbf{b}$	Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the positive orthant conic subset, $\mathbb{R}_{++}^n$ , in the space $\mathbb{R}^n$
$\mathbf{A} \preceq_K \mathbf{B}$	Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the conic subset $K$ in the space $\mathbb{S}^n$
$\mathbf{A} \prec_K \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the interior of the conic subset $K$ in the space $\mathbb{S}^n$
$\mathbf{A} \leq \mathbf{B}$	Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive semidefinite conic subset, $\mathbb{S}_+^n$ , in the space $\mathbb{S}^n$
$\mathbf{A} < \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive orthant conic subset, $\mathbb{S}_{++}^n$ , in the space $\mathbb{S}^n$

### 3.1 Indexing

$x_{i_1, i_2, \dots, i_N}$	Element in the position $(i_1, i_2, \dots, i_N)$ of the tensor $\mathcal{X}$
$\mathcal{X}^{(n)}$	$n$ th tensor in a nontemporal sequence
$[\mathcal{X}]_{i_1, i_2, \dots, i_N}$	Element $x_{i_1, i_2, \dots, i_N}$
$\mathbf{x}_n, \mathbf{x}_{:n}$	$n$ th column of the matrix $X$
$\mathbf{x}_{n:}$	$n$ th row of the matrix $X$
$\mathbf{x}_{i_1, \dots, i_{n-1}, :, i_{n+1}, \dots, i_N}$	Mode- $n$ fiber of the tensor $\mathcal{X}$
$\mathbf{x}_{:, i_2, i_3}$	Column fiber (mode-1 fiber) of the thrid-order tensor $\mathcal{X}$
$\mathbf{x}_{i_1, :, i_3}$	Row fiber (mode-2 fiber) of the thrid-order tensor $\mathcal{X}$

$\mathbf{x}_{i_1, i_2, :}$	Tube fiber (mode-3 fiber) of the thrid-order tensor $\mathcal{X}$
$\mathbf{X}_{i_1, :, :}$	Horizontal slice of the thrid-order tensor $\mathcal{X}$
$\mathbf{X}_{:, i_2, :}$	Lateral slices slice of the thrid-order tensor $\mathcal{X}$
$\mathbf{X}_{i_3}, \mathbf{X}_{:, :, i_3}$	Frontal slices slice of the thrid-order tensor $\mathcal{X}$

## 4 Sets

$A + B$	Set addition (Minkowski sum)
$A - B$	Minkowski difference
$A \setminus B, A - B$	Set difference or set subtraction, i.e., the set containing the elements of $A$ that are not in $B$
$A \cup B$	Set of union
$A \cap B$	Set of intersection
$A \times B$	Cartesian product
$A^\perp$	Orthogonal complement of $A$ , e.g., $\mathbf{N}(\mathbf{A}) = \mathbf{C}(\mathbf{A}^\top)^\perp$
$A \oplus B$	Direct sum, e.g., $\mathbf{C}(\mathbf{A}^\top) \oplus \mathbf{C}(\mathbf{A}^\top)^\perp = \mathbb{R}^n$
$A^c, \bar{A}$	Complement set
$\#A,  A $	Cardinality
$a \in A$	$a$ is element of $A$
$a \notin A$	$a$ is not element of $A$
$\{1, 2, \dots, n\}$	Discrete set containing the integer elements $1, 2, \dots, n$
$\mathbb{R}$	Set of real numbers
$\mathbb{C}$	Set of complex numbers
$\mathbb{Z}$	Set of integer number
$\mathbb{B} = \{0, 1\}$	Boolean set
$\emptyset$	Empty set
$\mathbb{N}$	Set of natural numbers
$\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$	???
$\mathbb{K}^{I_1 \times I_2 \times \dots \times I_N}$	$I_1 \times I_2 \times \dots \times I_N$ -dimensional real (or complex) space
$\mathbb{K}_+$	Nonnegative real (or complex) space
$\mathbb{K}_{++}$	Positive real (or complex) space, i.e., $\mathbb{K}_{++} = \mathbb{K}_+ \setminus \{\mathbf{0}\}$
$\mathbb{S}^n, \mathcal{S}^n$	Conic set of the symmetric matrices in $\mathbb{R}^{n \times n}$

$\mathbb{S}_+^n, \mathcal{S}_+^n$	Conic set of the symmetric positive semidefinite matrices in $\mathbb{R}^{n \times n}$
$\mathbb{S}_{++}^n, \mathcal{S}_{++}^n$	Conic set of the symmetric positive definite matrices in $\mathbb{R}^{n \times n}$ , i.e., $\mathbb{S}_{++}^n = \mathbb{S}_+^n \setminus \{\mathbf{0}\}$
$\mathbb{H}^n$	Set of all hermitian matrices in $\mathbb{C}^{n \times n}$
$[a, b]$	Closed interval of a real set from $a$ to $b$
$(a, b)$	Opened interval of a real set from $a$ to $b$
$[a, b), (a, b]$	Half-opened intervals of a real set from $a$ to $b$

## 5 Signals and functions operations and indexing

$f : A \rightarrow B$	A function $f$ whose domain is $A$ and codomain is $B$
$f^{(n)}$	$n$ th derivative of the function $f$
$f^{-1}$	Inverse function of $f$
$f \circ g$	Composition of the functions $f$ and $g$
$\inf_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Infimum
$\sup_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Supremum
$*$	Convolution
$\otimes, \textcircled{\mathbb{N}}$	Circular convolution
$x(t)$	Continuous-time $t$
$x[n], x[k], x[m], x[i], \dots$	Discrete-time $n, k, m, i, \dots$
$x(n), x(k), x(m), x(i), \dots$	Discrete-time $n, k, m, i, \dots$ (it should be used only if there are no continuous-time signals in the context to avoid ambiguity)
$\hat{x}(t)$ or $\hat{x}[n]$	Estimate of $x(t)$ or $x[n]$ ; the Hilbert transform of $x(t)$ or $x[n]$
$\tilde{x}[n]$	Periodic discrete-time signal
$x[((n-m))_N], x((n-m))_N$	Circular shift in $m$ samples within a $N$ -samples window
$x_I(t)$ or $x_I[n]$	Real or in-phase part of $x(t)$ or $x[n]$
$x_Q(t)$ or $x_Q[n]$	Imaginary or quadrature part of $x(t)$ or $x[n]$
$\mathcal{F}\{\cdot\}$	Fourier transform
$\mathcal{L}\{\cdot\}$	Laplace transform
$\mathcal{Z}\{\cdot\}$	$z$ -transform
$X(s)$	Laplace transform of $x(t)$

$X(f)$	Fourier transform (FT) (in linear frequency, Hz) of $x(t)$
$X(j\omega)$	Fourier transform (FT) (in angular frequency, rad/sec) of $x(t)$
$X(e^{j\omega})$	Discrete-time Fourier transform (DTFT) of $x[n]$
$X[k], X(k)$	Discrete Fourier transform (DFT) or fast Fourier transform (FFT) of $x[n]$ , or even the Fourier series (FS) of the periodic signal $x(t)$
$\tilde{X}[k], \tilde{X}(k)$	Discrete Fourier series (DFS) of $\tilde{x}[n]$
$X(z)$	$z$ -transform of $x[n]$
$S_x(f)$	Power spectral density of $x(t)$ in linear frequency
$S_x(j\omega)$	Power spectral density of $x(t)$ in angular frequency

## 6 Probability and stochastic processes

$E[\cdot]$	Statistical expectation
$E_u[\cdot]$	Statistical expectation with respect to $u$
$\text{var}(x)$	Variance of the random variable $x$
$\text{erfc}(\cdot)$	Complementary error function
$P(A)$	Probability of the event or set $A$
$p(\cdot)$	Probability density function
$p(x   A)$	Conditional probability density function
$a \sim P$	Random variable $a$ with distribution $P$
$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution of a random variable with mean $\mu$ and variance $\sigma^2$
$\mathcal{CN}(\mu, \sigma^2)$	Complex Gaussian distribution of a random variable with mean $\mu$ and variance $\sigma^2$
$\mathcal{N}(\mathbf{\mu}, \mathbf{\Sigma})$	Gaussian distribution of a vector random variable with mean $\mathbf{\mu}$ and covariance matrix $\mathbf{\Sigma}$
$\mathcal{CN}(\mathbf{\mu}, \mathbf{\Sigma})$	Complex Gaussian distribution of a vector random variable with mean $\mathbf{\mu}$ and covariance matrix $\mathbf{\Sigma}$
$\mathcal{U}(a, b)$	Uniform distribution from $a$ to $b$



## 7 General notations

$a \wedge b$	Logical AND of $a$ and $b$
$a \vee b$	Logical OR of $a$ and $b$
$\neg a$	Logical negation of $a$
$\exists$	There exists
$\nexists$	There does not exist
$\exists!$	There exist an unique
$\forall$	For all
$ $	Such that
$\therefore$	Therefore
$\Longleftrightarrow$	Logical equivalence
$\triangleq$	Equal by definition
$\neq$	Not equal
$\infty$	Infinity
$ a $	Absolute value of $a$
$\log$	Base-10 logarithm or decimal logarithm
$\ln$	Natural logarithm
$\operatorname{Re}\{x\}$	Real part of $x$
$\operatorname{Im}\{x\}$	Imaginary part of $x$
$\lceil \cdot \rceil$	Ceiling operation
$\lfloor \cdot \rfloor$	Floor operation
$\angle \cdot$	phase (complex argument)
$x \bmod y$	Remainder, i.e., $x - y\lfloor x/y \rfloor$
$\operatorname{frac}(x)$	Fractional part, i.e., $x \bmod 1$

## 8 Abbreviations

wrt.	With respect to
st.	Subject to
iff.	If and only if
EVD	Eigenvalue decomposition, or eigen-decomposition
SVD	Singular value decomposition
CP	CANDECOMP/PARAFAC