Notation

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1 Font notation

$a, b, c, \ldots, A, B, C, \ldots$	Scalars
a, b, c, \dots	Vectors
A, B, C, \dots	Matrices
$\mathcal{A}, \mathcal{B}, \mathcal{C}, \dots$	Tensors
$A, B, C, \ldots, A, B, C, \ldots, A, B, C, \ldots$	Sets

2 Signals and functions

2.1 Time indexing

x(t)	Continuous-time t
$x[n],x[k],x[m],x[i],\ldots$	Discrete-time n, k, m, i, \dots (parenthe-
$x_n, x_k, x_m, x_i, \dots$	sis should be adopted only if there
$x(n), x(k), x(m), x(i), \dots$	are no continuous-time signals in the
	context to avoid ambiguity)
$x[((n-m))_N], x((n-m))_N$	Circular shift in m samples within a
	N-samples window

2.2 Common functions

$x_I(t)$ or $x_I[n]$	Real or in-phase part of $x(t)$ or $x[n]$
$x_Q(t)$ or $x_Q[n]$	Imaginary or quadrature part of $x(t)$
	or $x[n]$
$\delta(t)$	Delta function
$\delta[n], \delta_{i,j}$	Kronecker function $(n = i - j)$

h(t), h[n]	Impulse response (continuous and
	discrete time)
$\tilde{x}[n], \tilde{x}(t)$	Periodic discrete- or continuous-time
	signal
$\hat{x}[n], \hat{x}(t)$	Estimate of $x[n]$ or $x(t)$
$\dot{x}[m]$	Interpolation of $x[n]$

2.3 Operations and symbols

$f:A\to B$	A function f whose domain is A and
	codomain is B
$f^n, x^n(t), x^n[k]$	nth power of the function f , $x[n]$ or
	x(t)
$f^{(n)}, x^{(n)}(t)$	nth derivative of the function f or
	x(t)
$f', f^{(1)}, x'(t)$	1th derivative of the function f or
	x(t)
$f^{\prime\prime\prime}, f^{(2)}, x^{\prime\prime\prime}(t)$	2th derivative of the function f or
	x(t)
$f \circ g$	Composition of the functions f and
	g
*	Convolution (discrete or continuous)
⊛, (N)	Circular convolution

2.4 Transformations

$\mathcal{F}\left\{ \cdot ight\}$	Fourier transform	
$\mathcal{L}\left\{ \cdot ight\}$	Laplace transform	
$\mathcal{Z}\left\{ \cdot ight\}$	z-transform	
$\hat{x}(t), \hat{x}[n]$	Hilbert transform of $x(t)$ or $x[n]$	
X(s)	Laplace transform of $x(t)$	
X(f)	Fourier transform (FT) (in linear fre-	
	quency, Hz) of $x(t)$	
$X(j\omega)$	Fourier transform (FT) (in angular	
	frequency, rad/sec) of $x(t)$	
$X(e^{j\omega})$	Discrete-time Fourier transform	
	(DTFT) of $x[n]$	
$X[k], X(k), X_k$	Discrete Fourier transform (DFT) or	
	fast Fourier transform (FFT) of $x[n]$,	
	or even the Fourier series (FS) of the	
	periodic signal $x(t)$	
$\tilde{X}[k], \tilde{X}(k), \tilde{X}_k$	Discrete Fourier series (DFS) of $\tilde{x}[n]$	
X(z)	z-transform of $x[n]$	

3 Probability, statistics, and stochastic processes

3.1 Operators and symbols

$E\left[\cdot ight]$	Statistical expectation
$E_u\left[\cdot\right]$	Statistical expectation with respect
	to u
$\mu_{\scriptscriptstyle X}$	Mean of the random variable x
μ_{x}, m_{x}	Mean vector of the random variable
	x
μ_n	nth-order moment of a random vari-
	able
\mathcal{K}_x, μ_4	Kurtosis (4th-order moment) of the
	random variable x
$VAR[\cdot]$	Variance operator
$VAR_u[\cdot]$	Variance operator with respect to u
κ_n	nth-order cumulant of a random vari-
	able
σ_x, κ_2	Variance of the random variable x
$\rho_{x,y}$	Pearson correlation coefficient be-
	tween x and y
$a \sim P$	Random variable a with distribution
	P

3.2 Stochastic processes

() P ()	A
$r_{x}(\tau), R_{x}(\tau)$	Autocorrelation function of the sig-
	$\operatorname{nal} x(t) \text{ or } x[n]$
$S_x(f), S_x(j\omega)$	Power spectral density (PSD) of $x(t)$
$-\chi(y) = \chi(y)$	in linear (f) or angular (ω) frequency
g (s) g (;)	(0)
$S_{x,y}(f), S_{x,y}(j\omega)$	Cross PSD of $x(t)$ and $y(t)$ in linear
	or angular (ω) frequency
R_x	(Auto)correlation matrix of $\mathbf{x}(n)$
$r_{x,d}(\tau), R_{x,d}(\tau)$	Cross-correlation between $x[n]$ and
.,	d[n] or $x(t)$ and $d(t)$
$\mathbf{R}_{\mathbf{x}\mathbf{v}}$	Cross-correlation matrix of $\mathbf{x}(n)$ and
	$\mathbf{y}(n)$
$\mathbf{p}_{\mathbf{x}d}$	Cross-correlation vector between
Pxa	
	$\mathbf{x}(n)$ and $d(n)$
$c_x(\tau), C_x(\tau)$	Autocovariance function of the signal
	x(t) or $x[n]$
C_x, K_x, Σ_x	(Auto)covariance matrix of \mathbf{x}
A/ A/ A	,
$c_{xy}(\tau), C_{xy}(\tau)$	Cross-covariance function of the sig-
	$\operatorname{nal} x(t) \text{ or } x[n]$

$\mathbf{C}_{\mathbf{x}\mathbf{v}}$	\mathbf{K}_{xy} ,	Σ_{xy}
		,

Cross-covariance matrix of ${\bf x}$ and ${\bf y}$

3.3 Functions

2()	0.6
$Q(\cdot)$	Q-function, i.e., $P[\mathcal{N}(0,1) > x]$
$\operatorname{erf}(\cdot)$	Error function
$\operatorname{erfc}(\cdot)$	Complementary error function i.e.,
	$\operatorname{erfc}(x) = 2Q(\sqrt{2}x) - \operatorname{erf}(x)$
P[A]	Probability of the event or set A
$p(\cdot), f(\cdot)$	Probability density function (PDF)
	or probability mass function (PMF)
$p(x \mid A)$	Conditional PDF or PMF
$F(\cdot)$	Cumulative distribution function
	(CDF)
$\Phi_X(\omega), M_X(j\omega), E\left[e^{j\omega x}\right]$	First characteristic function (CF) of
	X
$M_X(t), \Phi_X(-jt), E[e^{tX}]$	Moment-generating function (MGF)
	of x
$\Psi_{x}(\omega), \ln \Phi_{x}(\omega), \ln E\left[e^{j\omega x}\right]$	Second characteristic function
$K_x(t), \ln E\left[e^{tx}\right], \ln M_x(t)$	Cumulant-generating function
	(CGF) of x

3.4 Distributions

$\mathcal{N}(\mu,\sigma^2)$	Gaussian distribution of a random
$\mathcal{N}(\mu, \sigma^2)$ $\mathcal{C}\mathcal{N}(\mu, \sigma^2)$	variable with mean μ and variance σ^2 Complex Gaussian distribution of a random variable with mean μ and variance σ^2
$\mathcal{N}(\pmb{\mu},\pmb{\Sigma})$	Gaussian distribution of a vector random variable with mean μ and covariance matrix Σ
$\mathcal{CN}(\pmb{\mu},\pmb{\Sigma})$	Complex Gaussian distribution of a vector random variable with mean μ and covariance matrix Σ
$\mathcal{U}(a,b) \\ \chi^2(n), \chi_n^2$	Uniform distribution from a to b Chi-square distribution with n degree of freedom (assuming that the Gaus- sians are $\mathcal{N}(0,1)$)
$\operatorname{Exp}(\lambda)$	Exponential distribution with rate parameter λ
$\Gamma(lpha,oldsymbol{eta})$	Gamma distribution with shape parameter α and rate parameter β

$\Gamma(lpha, heta)$	Gamma distribution with shape parameter α and scale parameter $\theta = 1/\beta$
$\operatorname{Nakagami}(m,\Omega)$	Nakagami-m distribution with shape parameter m and spread parameter Ω
$\operatorname{Rayleigh}(\sigma)$	Rayleigh distribution with scale parameter σ
$\operatorname{Rayleigh}(\Omega)$	Rayleigh distribution with the second moment $\Omega = E[x^2] = 2\sigma^2$
$\mathrm{Rice}(s,\sigma)$	Rice distribution with noncentrality parameter (specular component) s and σ
$\mathrm{Rice}(A,K)$	Rice distribution with Rice factor $K = s^2/2\sigma^2$ and scale parameter $A = s^2 + 2\sigma^2$

4 Statistical signal processing

$\mathbf{\nabla} f, \mathbf{g}$	Gradient descent vector
$\nabla_x f, \mathbf{g}_x$	Gradient descent vector with respect
	x
\mathbf{g} (or $\hat{\mathbf{g}}$ if the gradient vector is \mathbf{g})	Stochastic gradient descent (SGD)
$J(\cdot), \mathcal{E}(\cdot)$	Cost-function or objective function
$\Lambda(\cdot)$	Likelihood function
$\Lambda_l(\cdot)$	Log-likelihood function
$\hat{x}(t)$ or $\hat{x}[n]$	Estimate of $x(t)$ or $x[n]$
$\hat{\boldsymbol{\mu}}_{x},\hat{\mathbf{m}}_{x}$	Sample mean of $x[n]$ or $x(t)$
$\hat{\boldsymbol{\mu}}_{\mathbf{x}},\hat{\mathbf{m}}_{\mathbf{x}}$	Sample mean vector of $\mathbf{x}[n]$ or $\mathbf{x}(t)$
$\hat{r}_{x}(au),\hat{R}_{x}(au)$	Estimated autocorrelation function
	of the signal $x(t)$ or $x[n]$
$\hat{S}_{x}(f), \hat{S}_{x}(j\omega)$	Estimated power spectral density
	(PSD) of $x(t)$ in linear (f) or angular
	(ω) frequency
$\hat{\mathbf{R}}_{\mathbf{x}}$	Sample (auto)correlation matrix
$\hat{r}_{x,d}(au), \hat{R}_{x,d}(au)$	Estimated cross-correlation between
	x[n] and $d[n]$ or $x(t)$ and $d(t)$
$\hat{S}_{x,y}(f), \hat{S}_{x,y}(j\omega)$	Estimated cross PSD of $x(t)$ and $y(t)$
	in linear or angular (ω) frequency
$\hat{\mathbf{R}}_{\mathbf{x}\mathbf{y}}$	Sample cross-correlation matrix of
	$\mathbf{R}_{\mathbf{x}\mathbf{y}}$
$\hat{ ho}_{x,y}$	Estimated Pearson correlation coeffi-
•	cient between x and y

 $\hat{c}_{x}(\tau), \hat{C}_{x}(\tau)$ Estimated autocovariance function of the signal x(t) or x[n] $\hat{\mathbf{C}}_{\mathbf{x}}, \hat{\mathbf{K}}_{\mathbf{x}}, \hat{\boldsymbol{\Sigma}}_{\mathbf{x}}$ $\hat{c}_{xy}(\tau), \hat{C}_{xy}(\tau)$ Sample (auto)covariance matrix Estimated cross-covariance function of the signal x(t) or x[n] $\begin{array}{l} \hat{C}_{xy}, \hat{K}_{xy}, \hat{\Sigma}_{xy} \\ w, \theta \end{array}$ Sample cross-covariance matrix Parameters, coefficients, or weights vector $\mathbf{w}_o, \mathbf{w}^{\star}, \mathbf{\theta}_o, \mathbf{\theta}^{\star}$ Optimum value of the parameters, coefficients, or weights vector W Matrix of the weights J Jacobian matrix \mathbf{H} Hessian matrix $\hat{\mathbf{H}}$ Estimate of the Hessian matrix \mathcal{R} Rayleigh's quotient

5 Linear Algebra

5.1 Common matrices and vectors

W, D P	Diagonal matrix Projection matrix; Permutation ma-
	trix
J	Jordan matrix
\mathbf{L}	Lower matrix
\mathbf{U}	Upper matrix
\mathbf{C}	Cofactor matrix
$\mathbf{C}_{\mathbf{A}}, \operatorname{cof}\left(\mathbf{A}\right)$	Cofactor matrix of A
S	Symmetric matrix
Q	Orthogonal matrix
\mathbf{I}_N	$N \times N$ -dimensional identity matrix
$0_{M imes N}$	$M \times N$ -dimensional null matrix
0_N	N-dimensional null vector
$1_{M imes N}$	$M \times N$ -dimensional ones matrix
1_N	N-dimensional ones vector
0	Null matrix, vector, or tensor (di-
	mensionality understood by context)
1	Ones matrix, vector, or tensor (di-
	mensionality understood by context)

5.2 Indexing

 $x_{i_1,i_2,...,i_N}, [\mathcal{X}]_{i_1,i_2,...,i_N}$ Element the position in (i_1, i_2, \ldots, i_N) of the tensor $\boldsymbol{\mathcal{X}}$ $\mathcal{X}^{(n)}$ nth tensor of a nontemporal sequence nth column of the matrix X $\mathbf{x}_n, \mathbf{x}_{:n}$ nth row of the matrix X \mathbf{x}_{n} : Mode-n fiber of the tensor $\boldsymbol{\mathcal{X}}$ $\mathbf{X}_{i_1,...,i_{n-1},:,i_{n+1},...,i_N}$ Column fiber (mode-1 fiber) of the $\mathbf{X}_{:,i_2,i_3}$ thrid-order tensor $\boldsymbol{\mathcal{X}}$ Row fiber (mode-2 fiber) of the thrid- $\mathbf{x}_{i_1,:,i_3}$ order tensor \mathcal{X} Tube fiber (mode-3 fiber) of the $\mathbf{x}_{i_1,i_2,:}$ thrid-order tensor $\boldsymbol{\mathcal{X}}$ $X_{i_1,:,:}$ Horizontal slice of the thrid-order tensor $\boldsymbol{\mathcal{X}}$ $\mathbf{X}_{:,i_2,:}$ Lateral slices slice of the thrid-order tensor \mathcal{X} $X_{i_3}, X_{:,:,i_3}$ Frontal slices slice of the thrid-order tensor $\boldsymbol{\mathcal{X}}$

5.3 General operations

 $\langle \cdot, \cdot \rangle$ Inner product, e.g., $\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^{\mathsf{T}} \mathbf{b}$ Outer product, e.g., $\mathbf{a} \circ \mathbf{b} = \mathbf{a} \mathbf{b}^{\mathsf{T}}$ Kronecker product \otimes Hadamard (or Schur) (elementwise) \odot product .⊙nnth-order Hadamard power $\cdot \circ \frac{1}{n}$ nth-order Hadamard root 0 Hadamard (or Schur) (elementwise) division Khatri-Rao product \Diamond Kronecker Product \otimes *n*-mode product \times_n

5.4 Operations with matrices and tensors

 \mathbf{A}^{-1} Inverse matrix A^+, A^\dagger Moore-Penrose left pseudoinverse \mathbf{A}^{\top} Transpose $\mathbf{A}^{-\top}$ Transpose of the inverse \mathbf{A}^* Complex conjugate \mathbf{A}^{H} Hermitian Frobenius norm $\|\mathbf{A}\|_{\mathrm{F}}$ $\|\mathbf{A}\|$ Matrix norm

$ \mathbf{A} , \det{(\mathbf{A})}$	Determinant
$\operatorname{diag}\left(\mathbf{A}\right)$	The elements in the diagonal of $\bf A$
$\text{vec}\left(\mathbf{A}\right)$	Vectorization: stacks the columns of
	the matrix A into a long column vec-
	tor
$\mathrm{vec_d}\left(\mathbf{A}\right)$	Extracts the diagonal elements of a
	square matrix and returns them in a
	column vector
$\operatorname{vec}_{\operatorname{l}}\left(\mathbf{A} ight)$	Extracts the elements strictly below
	the main diagonal of a square matrix
	in a column-wise manner and returns
	them into a column vector
$\operatorname{vec}_{\mathrm{u}}\left(\mathbf{A}\right)$	Extracts the elements strictly above
	the main diagonal of a square matrix
	in a column-wise manner and returns
	them into a column vector
$\operatorname{vec_b}\left(\mathbf{A}\right)$	Block vectorization operator: stacks
	square block matrices of the input
	into a long block column matrix
$\operatorname{unvec}\left(\mathbf{A}\right)$	Reshapes a column vector into a ma-
	trix
$\mathrm{tr}\left(\mathbf{A} ight)$	trace
$\mathbf{X}_{(n)}$	$n\text{-}\mathrm{mode}$ matricization of the tensor $oldsymbol{\mathcal{X}}$

5.5 Operations with vectors

$\ \mathbf{a}\ $	l_1 norm, 1-norm, or Manhatan norm
$\ \mathbf{a}\ , \ \mathbf{a}\ _2$	l_2 norm, 2-norm, or Euclidean norm
$\ \mathbf{a}\ _p$	l_p norm, p -norm, or Minkowski norm
$\ \mathbf{a}\ _{\infty}$	l_{∞} norm, ∞ -norm, or Chebyshev
	norm
$\operatorname{diag}\left(\mathbf{a}\right)$	Diagonalization: a square, diagonal
	matrix with entries given by the vec-
	tor a

5.6 Decompositions

Λ	Eigenvalue matrix
Q	Eigenvectors matrix; Orthogonal ma-
	trix of the QR decomposition
R	Upper triangular matrix of the QR
	decomposition
U	Left singular vectors
\mathbf{U}_r	Left singular nondegenerated vectors

Σ	Singular value matrix
Σ_r	Singular value matrix with nonzero
	singular values in the main diagonal
Σ^+	Singular value matrix of the pseu-
	doinverse
Σ_r^+	Singular value matrix of the pseu-
	doinverse with nonzero singular val-
	ues in the main diagonal
V	Right singular vectors
\mathbf{V}_r	Right singular nondegenerated vec-
	tors
$\operatorname{eig}\left(\mathbf{A}\right)$	Set of the eigenvalues of A
$[\![\mathbf{A},\mathbf{B},\mathbf{C},\ldots]\!]$	CANDECOMP/PARAFAC (CP) de-
	composition of the tensor ${\cal X}$ from the
	outer product of column vectors of A ,
	B, C,
$[\![\boldsymbol{\lambda};\mathbf{A},\mathbf{B},\mathbf{C},\ldots]\!]$	Normalized CANDE-
	COMP/PARAFAC (CP) decom-
	position of the tensor ${\cal X}$ from the
	outer product of column vectors of
	A, B, C, \dots

5.7 Spaces

$\mathrm{span}\left(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_n\right)$	Vector space spanned by the argument vectors
$C(\mathbf{A})$, columnspace(\mathbf{A}), range(\mathbf{A}), span (\mathbf{A}), image(\mathbf{A})	Columnspace, range or image, i.e., the space span $(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$, where \mathbf{a}_i is the ith column vector of the ma-
	trix A
$C(\mathbf{A}^{H})$	Row space
$N(\mathbf{A})$, $null space(\mathbf{A})$, $kernel(\mathbf{A})$	Nullspace (or kernel space)
$N\left(\mathbf{A}^{H}\right)$	Left nullspace
$\operatorname{rank}\left(\mathbf{\hat{A}}\right)$	Rank, that is, $\dim(\text{span}(\mathbf{A})) =$
	$\dim (C (\mathbf{A}))$
nullity (A)	Nullity of \mathbf{A} , i.e., dim $(N(\mathbf{A}))$
$\mathbf{a} \perp \mathbf{b}$	a is orthogonal to b
a ⊥ b	\mathbf{a} is not orthogonal to \mathbf{b}

5.8 Inequalities

 $\mathcal{X} \leq 0$

Nonnegative tensor

$\mathbf{a} \leq_K \mathbf{b}$	Generalized inequality meaning that
	$\mathbf{b} - \mathbf{a}$ belongs to the conic subset K in
a / h	the space \mathbb{R}^n
$\mathbf{a} \prec_K \mathbf{b}$	Strict generalized inequality meaning
	that $\mathbf{b} - \mathbf{a}$ belongs to the interior of the conic subset K in the space \mathbb{R}^n
$\mathbf{a} \leq \mathbf{b}$	Generalized inequality meaning that
a = b	$\mathbf{b} - \mathbf{a}$ belongs to the nonnegative or-
	thant conic subset, \mathbb{R}^n_+ , in the space
	\mathbb{R}^n .
$\mathbf{a} \prec \mathbf{b}$	Strict generalized inequality meaning
	that $\mathbf{b} - \mathbf{a}$ belongs to the positive or-
	thant conic subset, \mathbb{R}^n_{++} , in the space
	\mathbb{R}^n
$\mathbf{A} \leq_K \mathbf{B}$	Generalized inequality meaning that
	$\mathbf{B} - \mathbf{A}$ belongs to the conic subset K
	in the space \mathbb{S}^n
$\mathbf{A} \prec_K \mathbf{B}$	Strict generalized inequality meaning
	that $\mathbf{B} - \mathbf{A}$ belongs to the interior of
A ~ D	the conic subset K in the space \mathbb{S}^n
$A \leq B$	Generalized inequality meaning that B-A belongs to the positive semidef-
	inite conic subset, \mathbb{S}_{+}^{n} , in the space \mathbb{S}^{n}
A < B	Strict generalized inequality meaning
	that $\mathbf{B} - \mathbf{A}$ belongs to the positive or-
	thant conic subset, \mathbb{S}^n_{++} , in the space
	\mathbb{S}^n

6 Sets

A + B	Set addition (Minkowski sum)
A - B	Minkowski difference
$A \setminus B, A - B$	Set difference or set subtraction,
	i.e., the set containing the elements
	of A that are not in B
$A \cup B$	Set of union
$A \cap B$	Set of intersection
$A \times B$	Cartesian product
A^n	$A \times A \times \cdots \times A$
	n times
A^{\perp}	Orthogonal complement of A , e.g.,
	$N(\mathbf{A}) = C(\mathbf{A}^{T})^{\perp}$
$A \oplus B$	Direct sum, e.g., $C(\mathbf{A}^{\top}) \oplus C(\mathbf{A}^{\top})^{\perp} =$
	\mathbb{R}^n

$A^c, ar{A}$	Complement set (given U)
#A, A	Cardinality
$a \in A$	a is element of A
$a \notin A$	a is not element of A
$\{1,2,\ldots,n\}$	Discrete set containing the integer el-
	ements $1, 2, \ldots, n$
U	Universe
2^A	Power set of A
\mathbb{R}	Set of real numbers
\mathbb{C}	Set of complex numbers
\mathbb{Z}	Set of integer number
$\mathbb{B} = \{0, 1\}$	Boolean set
Ø	Empty set
\mathbb{N}	Set of natural numbers
$\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$	Real or complex space (field)
$\mathbb{K}^{I_1 imes I_2 imes \cdots imes I_N}$	$I_1 \times I_2 \times \cdots \times I_N$ -dimensional real (or
	complex) space
\mathbb{K}_{+}	Nonnegative real (or complex) space
\mathbb{K}_{++}	Positive real (or complex) space, i.e.,
	$\mathbb{K}_{++} = \mathbb{K}_+ \setminus \{0\}$
$\mathbb{S}^n,\mathcal{S}^n$	Conic set of the symmetric matrices
	in $\mathbb{R}^{n \times n}$
$\mathbb{S}^n_+, \mathcal{S}^n_+$	Conic set of the symmetric positive
	semidefinite matrices in $\mathbb{R}^{n \times n}$
$\mathbb{S}^n_{++}, \mathcal{S}^n_{++}$	Conic set of the symmetric positive
	definite matrices in $\mathbb{R}^{n \times n}$, i.e., $\mathbb{S}^n_{++} =$
	$\mathbb{S}^n_+\setminus\{0\}$
\mathbb{H}^n	Set of all hermitian matrices in $\mathbb{C}^{n\times n}$
[a,b]	Closed interval of a real set from a to
	b
(a,b)	Opened interval of a real set from a
	to b
[a,b),(a,b]	Half-opened intervals of a real set
	from a to b

7 Communication systems

S	Trasmitted signal
ϕ	Signal phase
s_l	Low-pass equivalent signal or enve-
	lope complex of s
η, w	Gaussian noise
r	Received signal
τ	Timming delay

Δau	Timming error (delay - estimated)
arphi	Phase offset
$\Delta arphi$	Phase error (offset - estimated)
f_d	Doppler frequency
A	Received signal amplitude
γ	Combined effect of the path loss and
	antenna gain

8 Other notations

$\bf 8.1 \quad Mathematical \ symbols$

3	There exists
∄	There does not exist
∃!	There exist an unique
€	Belongs to
∉	Does not belong to
	Q.E.D.
∴	Therefore
:	Because
A	For all
ļ,:	Such that
\iff	Logical equivalence
≜,:=	Equal by definition
≠	Not equal
∞	Infinity
j	$\sqrt{-1}$
W_N	Twiddle factor, $e^{-j\frac{2\pi}{N}}$

8.2 Operations

$\underset{x \in A}{\operatorname{argmax}} f(x)$	Value of x that minimizes x
$\underset{x \in \mathcal{A}}{\operatorname{argmin}} f(x)$	Value of x that minimizes x
$\inf_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Infimum
$\sup g(\mathbf{x}, \mathbf{y})$	Supremum
$y \in \mathcal{A}$ $ a $	Absolute value of a
log	Base-10 logarithm or decimal logarithm
ln	Natual logarithm
$\operatorname{Re}\left\{ x\right\}$	Real part of x
$\operatorname{Im}\left\{ x\right\}$	Imaginary part of x

۷٠ phase (complex argument) $x \mod y$ Remainder, i.e., $x - y \lfloor x/y \rfloor$ $\operatorname{frac}(x)$ Fractional part, i.e., $x \mod 1$ $a \wedge b$ Logical AND of a and b $a \lor b$ Logical OR of a and b $\neg a$ Logical negation of a $\lceil \cdot \rceil$ Ceiling operation $\lfloor \cdot \rfloor$ Floor operation

8.3 Functions

 $\begin{array}{ll} \mathcal{O}(\cdot), O(\cdot) & \text{Big-O notation} \\ \Gamma(\cdot) & \text{Gamma function} \end{array}$

9 Abbreviations

wrt. With respect to st. Subject to iff. If and only if

EVD Eigenvalue decomposition, or eigen-

 ${\it decomposition}$

SVD Singular value decomposition CP CANDECOMP/PARAFAC