

# Notation

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## 1 Font notation

$a, b, c, \dots, A, B, C, \dots$	Scalars
$\mathbf{a}, \mathbf{b}, \mathbf{c}, \dots$	Vectors
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$	Matrices
$\mathcal{A}, \mathcal{B}, \mathcal{C}, \dots$	Tensors
$A, B, C, \dots, \mathcal{A}, \mathcal{B}, \mathcal{C}, \dots, \mathbb{A}, \mathbb{B}, \mathbb{C}, \dots$	Sets

## 2 Signals and functions

### 2.1 Time indexing

$x(t)$	Continuous-time $t$
$x[n], x[k], x[m], x[i], \dots$	Discrete-time $n, k, m, i, \dots$ (parenthesis should be adopted only if there are no continuous-time signals in the context to avoid ambiguity)
$x_n, x_k, x_m, x_i, \dots$	
$x(n), x(k), x(m), x(i), \dots$	
$x[((n-m))_N], x((n-m))_N$	Circular shift in $m$ samples within a $N$ -samples window

### 2.2 Common functions

$\delta(t)$	Delta function
$\delta[n], \delta_{i,j}$	Kronecker function ( $n = i - j$ )
$h(t), h[n]$	Impulse response (continuous and discrete time)

$\tilde{x}[n], \tilde{x}(t)$	Periodic discrete- or continuous-time signal
$\hat{x}[n], \hat{x}(t)$	Estimate of $x[n]$ or $x(t)$
$\dot{x}[m]$	Interpolation of $x[n]$

## 2.3 Operations and symbols

$f : A \rightarrow B$	A function $f$ whose domain is $A$ and codomain is $B$
$f^n, x^n(t), x^n[k]$	$n$ th power of the function $f$ , $x[n]$ or $x(t)$
$f^{(n)}, x^{(n)}(t)$	$n$ th derivative of the function $f$ or $x(t)$
$f', f^{(1)}, x'(t)$	1th derivative of the function $f$ or $x(t)$
$f'', f^{(2)}, x''(t)$	2th derivative of the function $f$ or $x(t)$
$\arg \max_{x \in \mathcal{A}} f(x)$	Value of $x$ that minimizes $x$
$\arg \min_{x \in \mathcal{A}} f(x)$	Value of $x$ that minimizes $x$
$f(\mathbf{x}) = \inf_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Infimum, i.e., $f(\mathbf{x}) = \min \{g(\mathbf{x}, \mathbf{y}) \mid \mathbf{y} \in \mathcal{A} \wedge (\mathbf{x}, \mathbf{y}) \in \text{dom}(g)\}$ , which is the greatest lower bound of this set
$f(\mathbf{x}) = \sup_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Supremum, i.e., $f(\mathbf{x}) = \max \{g(\mathbf{x}, \mathbf{y}) \mid \mathbf{y} \in \mathcal{A} \wedge (\mathbf{x}, \mathbf{y}) \in \text{dom}(g)\}$ , which is the least upper bound of this set
$f \circ g$	Composition of the functions $f$ and $g$
$*$	Convolution (discrete or continuous)
$\otimes, \textcircled{\mathbb{N}}$	Circular convolution [13]

## 2.4 Transformations

$W_N$	Twiddle factor, $e^{-j\frac{2\pi}{N}}$ [9]
$\mathcal{F}\{\cdot\}$	Fourier transform
$\mathcal{L}\{\cdot\}$	Laplace transform
$\mathcal{Z}\{\cdot\}$	z-transform
$\hat{x}(t), \hat{x}[n]$	Hilbert transform of $x(t)$ or $x[n]$
$X(s)$	Laplace transform of $x(t)$
$X(f)$	Fourier transform (FT) (in linear frequency, Hz) of $x(t)$

$X(j\omega)$	Fourier transform (FT) (in angular frequency, rad/sec) of $x(t)$
$X(e^{j\omega})$	Discrete-time Fourier transform (DTFT) of $x[n]$
$X[k], X(k), X_k$	Discrete Fourier transform (DFT) or fast Fourier transform (FFT) of $x[n]$ , or even the Fourier series (FS) of the periodic signal $x(t)$
$\tilde{X}[k], \tilde{X}(k), \tilde{X}_k$	Discrete Fourier series (DFS) of $\tilde{x}[n]$
$X(z)$	$z$ -transform of $x[n]$

### 3 Probability, statistics, and stochastic processes

#### 3.1 Operators and symbols

$\mathbf{E}[\cdot], E[\cdot], \mathbb{E}[\cdot]$	Statistical expectation operator [5, 12]
$\mathbf{E}_u[\cdot], E_u[\cdot], \mathbb{E}_u[\cdot]$	Statistical expectation operator with respect to $u$
$\text{var}[\cdot], \text{VAR}[\cdot]$	Variance operator [1, 8, 11, 15]
$\text{var}_u[\cdot], \text{VAR}_u[\cdot]$	Variance operator with respect to $u$
$\mu_x$	Mean of the random variable $x$
$\mathbf{\mu}_x, \mathbf{m}_x$	Mean vector of the random variable $\mathbf{x}$ [3]
$\mu_n$	$n$ th-order moment of a random variable
$\sigma_x^2, \kappa_2$	Variance of the random variable $x$
$\mathcal{K}_x, \mu_4$	Kurtosis (4th-order moment) of the random variable $x$
$\kappa_n$	$n$ th-order cumulant of a random variable
$\rho_{x,y}$	Pearson correlation coefficient between $x$ and $y$
$a \sim P$	Random variable $a$ with distribution $P$
$\mathcal{R}$	Rayleigh's quotient

#### 3.2 Stochastic processes

$r_x(\tau), R_x(\tau)$	Autocorrelation function of the signal $x(t)$ or $x[n]$ [12]
$S_x(f), S_x(j\omega)$	Power spectral density (PSD) of $x(t)$ in linear ( $f$ ) or angular ( $\omega$ ) frequency

$S_{x,y}(f), S_{x,y}(j\omega)$	Cross PSD of $x(t)$ and $y(t)$ in linear or angular ( $\omega$ ) frequency
$\mathbf{R}_x$	(Auto)correlation matrix of $\mathbf{x}(n)$
$r_{x,d}(\tau), R_{x,d}(\tau)$	Cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$ [12]
$\mathbf{R}_{xy}$	Cross-correlation matrix of $\mathbf{x}(n)$ and $\mathbf{y}(n)$
$\mathbf{p}_{xd}$	Cross-correlation vector between $\mathbf{x}(n)$ and $d(n)$ [5]
$c_x(\tau), C_x(\tau)$	Autocovariance function of the signal $x(t)$ or $x[n]$ [12]
$\mathbf{C}_x, \mathbf{K}_x, \mathbf{\Sigma}_x, \text{cov}[\mathbf{x}]$	(Auto)covariance matrix of $\mathbf{x}$ [8, 11, 15, 19]
$c_{xy}(\tau), C_{xy}(\tau)$	Cross-covariance function of the signal $x(t)$ or $x[n]$ [12]
$\mathbf{C}_{xy}, \mathbf{K}_{xy}, \mathbf{\Sigma}_{xy}$	Cross-covariance matrix of $\mathbf{x}$ and $\mathbf{y}$

### 3.3 Functions

$Q(\cdot)$	$Q$ -function, i.e., $P[\mathcal{N}(0, 1) > x]$ [15]
$\text{erf}(\cdot)$	Error function [15]
$\text{erfc}(\cdot)$	Complementary error function i.e., $\text{erfc}(x) = 2Q(\sqrt{2}x) - \text{erf}(x)$ [15]
$P[A]$	Probability of the event or set $A$ [11]
$p(\cdot), f(\cdot)$	Probability density function (PDF) or probability mass function (PMF) [11]
$p(x   A)$	Conditional PDF or PMF [11]
$F(\cdot)$	Cumulative distribution function (CDF)
$\Phi_x(\omega), M_x(j\omega), E[e^{j\omega x}]$	First characteristic function (CF) of $x$ [15, 18]
$M_x(t), \Phi_x(-jt), E[e^{tx}]$	Moment-generating function (MGF) of $x$ [15, 18]
$\Psi_x(\omega), \ln \Phi_x(\omega), \ln E[e^{j\omega x}]$	Second characteristic function
$K_x(t), \ln E[e^{tx}], \ln M_x(t)$	Cumulant-generating function (CGF) of $x$ [8]

### 3.4 Distributions

$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution of a random variable with mean $\mu$ and variance $\sigma^2$ . The same notation can be used to denote a real-valued white Gaussian process with mean equal to $\mu$ and power spectral density equal to $N_0/2$ , e.g., $s(t) \sim \mathcal{N}(\mu, N_0/2)$
$\mathcal{CN}(\mu, \sigma^2)$	Complex Gaussian distribution of a random variable with mean $\mu$ and variance $\sigma^2$ . The same notation can be used to denote a complex-valued white Gaussian process with mean equal to $\mu$ and power spectral density equal to $N_0$ , e.g., $s(t) \sim \mathcal{CN}(\mu, N_0)$
$\mathcal{N}(\mathbf{\mu}, \mathbf{\Sigma})$	Gaussian distribution of a vector random variable with mean $\mathbf{\mu}$ and covariance matrix $\mathbf{\Sigma}$
$\mathcal{CN}(\mathbf{\mu}, \mathbf{\Sigma})$	Complex Gaussian distribution of a vector random variable with mean $\mathbf{\mu}$ and covariance matrix $\mathbf{\Sigma}$
$\mathcal{U}(a, b)$	Uniform distribution from $a$ to $b$
$\chi^2(n), \chi_n^2$	Chi-square distribution with $n$ degree of freedom (assuming that the Gaussians are $\mathcal{N}(0, 1)$ )
$\text{Exp}(\lambda)$	Exponential distribution with rate parameter $\lambda$
$\Gamma(\alpha, \beta)$	Gamma distribution with shape parameter $\alpha$ and rate parameter $\beta$
$\Gamma(\alpha, \theta)$	Gamma distribution with shape parameter $\alpha$ and scale parameter $\theta = 1/\beta$
$\text{Nakagami}(m, \Omega)$	Nakagami-m distribution with shape parameter $m$ and spread parameter $\Omega$
$\text{Rayleigh}(\sigma)$	Rayleigh distribution with scale parameter $\sigma$
$\text{Rayleigh}(\Omega)$	Rayleigh distribution with the second moment $\Omega = E[x^2] = 2\sigma^2$
$\text{Rice}(s, \sigma)$	Rice distribution with noncentrality parameter (specular component) $s$ and $\sigma$
$\text{Rice}(A, K)$	Rice distribution with Rice factor $K = s^2/2\sigma^2$ and scale parameter $A = s^2 + 2\sigma^2$

## 4 Statistical signal processing

$\nabla f, \mathbf{g}$	Gradient descent vector
$\nabla_x f, \mathbf{g}_x$	Gradient descent vector with respect to $x$
$\mathbf{g}$ (or $\hat{\mathbf{g}}$ if the gradient vector is $\mathbf{g}$ )	Stochastic gradient descent (SGD)
$J(\cdot), \mathcal{E}(\cdot)$	Cost-function or objective function
$\Lambda(\cdot)$	Likelihood function
$\Lambda_l(\cdot)$	Log-likelihood function
$\hat{x}(t)$ or $\hat{x}[n]$	Estimate of $x(t)$ or $x[n]$
$\hat{\mu}_x, \hat{\mathbf{m}}_x$	Sample mean of $x[n]$ or $x(t)$
$\hat{\boldsymbol{\mu}}_x, \hat{\mathbf{m}}_x$	Sample mean vector of $\mathbf{x}[n]$ or $\mathbf{x}(t)$
$\hat{r}_x(\tau), \hat{R}_x(\tau)$	Estimated autocorrelation function of the signal $x(t)$ or $x[n]$
$\hat{S}_x(f), \hat{S}_x(j\omega)$	Estimated power spectral density (PSD) of $x(t)$ in linear ( $f$ ) or angular ( $\omega$ ) frequency
$\hat{\mathbf{R}}_x$	Sample (auto)correlation matrix
$\hat{r}_{x,d}(\tau), \hat{R}_{x,d}(\tau)$	Estimated cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$
$\hat{S}_{x,y}(f), \hat{S}_{x,y}(j\omega)$	Estimated cross PSD of $x(t)$ and $y(t)$ in linear or angular ( $\omega$ ) frequency
$\hat{\mathbf{R}}_{xy}$	Sample cross-correlation matrix of $\mathbf{R}_{xy}$
$\hat{\rho}_{x,y}$	Estimated Pearson correlation coefficient between $x$ and $y$
$\hat{c}_x(\tau), \hat{C}_x(\tau)$	Estimated autocovariance function of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_x, \hat{\mathbf{K}}_x, \hat{\boldsymbol{\Sigma}}_x$	Sample (auto)covariance matrix
$\hat{c}_{xy}(\tau), \hat{C}_{xy}(\tau)$	Estimated cross-covariance function of the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_{xy}, \hat{\mathbf{K}}_{xy}, \hat{\boldsymbol{\Sigma}}_{xy}$	Sample cross-covariance matrix
$\mathbf{w}, \boldsymbol{\theta}$	Parameters, coefficients, or weights vector
$\mathbf{w}_o, \mathbf{w}^*, \boldsymbol{\theta}_o, \boldsymbol{\theta}^*$	Optimum value of the parameters, coefficients, or weights vector
$\mathbf{W}$	Matrix of the weights
$\mathbf{J}$	Jacobian matrix
$\mathbf{H}$	Hessian matrix
$\hat{\mathbf{H}}$	Estimate of the Hessian matrix

## 5 Linear Algebra

### 5.1 Common matrices and vectors

<b>W, D</b>	Diagonal matrix
<b>P</b>	Projection matrix; Permutation matrix
<b>J</b>	Jordan matrix
<b>L</b>	Lower matrix
<b>U</b>	Upper matrix
<b>C</b>	Cofactor matrix
<b>C<sub>A</sub>, cof(A)</b>	Cofactor matrix of <b>A</b>
<b>S</b>	Symmetric matrix
<b>Q</b>	Orthogonal matrix
<b>I<sub>N</sub></b>	$N \times N$ -dimensional identity matrix
<b>0<sub>M×N</sub></b>	$M \times N$ -dimensional null matrix
<b>0<sub>N</sub></b>	$N$ -dimensional null vector
<b>1<sub>M×N</sub></b>	$M \times N$ -dimensional ones matrix
<b>1<sub>N</sub></b>	$N$ -dimensional ones vector
<b>0</b>	Null matrix, vector, or tensor (dimensionality understood by context)
<b>1</b>	Ones matrix, vector, or tensor (dimensionality understood by context)

### 5.2 Indexing

$x_{i_1, i_2, \dots, i_N}, [\mathcal{X}]_{i_1, i_2, \dots, i_N}$	Element in the position $(i_1, i_2, \dots, i_N)$ of the tensor $\mathcal{X}$
$\mathcal{X}^{(n)}$	$n$ th tensor of a nontemporal sequence
$\mathbf{x}_n, \mathbf{X}_{:n}$	$n$ th column of the matrix $X$
$\mathbf{x}_n$	$n$ th row of the matrix $X$
$\mathbf{x}_{i_1, \dots, i_{n-1}, :, i_{n+1}, \dots, i_N}$	Mode- $n$ fiber of the tensor $\mathcal{X}$
$\mathbf{x}_{:, i_2, i_3}$	Column fiber (mode-1 fiber) of the thrid-order tensor $\mathcal{X}$
$\mathbf{x}_{i_1, :, i_3}$	Row fiber (mode-2 fiber) of the thrid-order tensor $\mathcal{X}$
$\mathbf{x}_{i_1, i_2, :}$	Tube fiber (mode-3 fiber) of the thrid-order tensor $\mathcal{X}$
$\mathbf{X}_{i_1, :, :}$	Horizontal slice of the thrid-order tensor $\mathcal{X}$
$\mathbf{X}_{:, i_2, :}$	Lateral slices slice of the thrid-order tensor $\mathcal{X}$
$\mathbf{X}_{i_3}, \mathbf{X}_{:, :, i_3}$	Frontal slices slice of the thrid-order tensor $\mathcal{X}$

### 5.3 General operations

$\langle \cdot, \cdot \rangle$	Inner product, e.g., $\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^\top \mathbf{b}$
$\circ$	Outer product, e.g., $\mathbf{a} \circ \mathbf{b} = \mathbf{a} \mathbf{b}^\top$
$\otimes$	Kronecker product
$\odot$	Hadamard (or Schur) (elementwise) product
$\cdot^{\odot n}$	$n$ th-order Hadamard power
$\cdot^{\odot \frac{1}{n}}$	$n$ th-order Hadamard root
$\oslash$	Hadamard (or Schur) (elementwise) division
$\diamond$	Khatri-Rao product
$\otimes$	Kronecker Product
$\times_n$	$n$ -mode product

### 5.4 Operations with matrices and tensors

$\mathbf{A}^{-1}$	Inverse matrix
$\mathbf{A}^+, \mathbf{A}^\dagger$	Moore-Penrose left pseudoinverse
$\mathbf{A}^\top$	Transpose
$\mathbf{A}^{-\top}$	Transpose of the inverse, i.e., $(\mathbf{A}^{-1})^\top = (\mathbf{A}^\top)^{-1}$ [6, 14]
$\mathbf{A}^*$	Complex conjugate
$\mathbf{A}^H$	Hermitian
$\ \mathbf{A}\ _F$	Frobenius norm
$\ \mathbf{A}\ $	Matrix norm
$ \mathbf{A} , \det(\mathbf{A})$	Determinant
$\text{diag}(\mathbf{A})$	The elements in the diagonal of $\mathbf{A}$
$\text{vec}(\mathbf{A})$	Vectorization: stacks the columns of the matrix $\mathbf{A}$ into a long column vector
$\text{vec}_d(\mathbf{A})$	Extracts the diagonal elements of a square matrix and returns them in a column vector
$\text{vec}_l(\mathbf{A})$	Extracts the elements strictly below the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\text{vec}_u(\mathbf{A})$	Extracts the elements strictly above the main diagonal of a square matrix in a column-wise manner and returns them into a column vector



$\text{vec}_b(\mathbf{A})$	Block vectorization operator: stacks square block matrices of the input into a long block column matrix
$\text{unvec}(\mathbf{A})$	Reshapes a column vector into a matrix
$\text{tr}(\mathbf{A})$	trace
$\mathbf{X}_{(n)}$	$n$ -mode matricization of the tensor $\mathcal{X}$

## 5.5 Operations with vectors

$\ \mathbf{a}\ $	$l_1$ norm, 1-norm, or Manhattan norm
$\ \mathbf{a}\ , \ \mathbf{a}\ _2$	$l_2$ norm, 2-norm, or Euclidean norm
$\ \mathbf{a}\ _p$	$l_p$ norm, $p$ -norm, or Minkowski norm
$\ \mathbf{a}\ _\infty$	$l_\infty$ norm, $\infty$ -norm, or Chebyshev norm
$\text{diag}(\mathbf{a})$	Diagonalization: a square, diagonal matrix with entries given by the vector $\mathbf{a}$

## 5.6 Decompositions

$\mathbf{A}$	Eigenvalue matrix [17]
$\mathbf{Q}$	Eigenvectors matrix; Orthogonal matrix of the QR decomposition[17]
$\mathbf{R}$	Upper triangular matrix of the QR decomposition[17]
$\mathbf{U}$	Left singular vectors[17]
$\mathbf{U}_r$	Left singular nondegenerated vectors
$\mathbf{\Sigma}$	Singular value matrix
$\mathbf{\Sigma}_r$	Singular value matrix with nonzero singular values in the main diagonal
$\mathbf{\Sigma}^+$	Singular value matrix of the pseudoinverse [17]
$\mathbf{\Sigma}_r^+$	Singular value matrix of the pseudoinverse with nonzero singular values in the main diagonal
$\mathbf{V}$	Right singular vectors [17]
$\mathbf{V}_r$	Right singular nondegenerated vectors
$\text{eig}(\mathbf{A})$	Set of the eigenvalues of $\mathbf{A}$ [4, 11, 14]
$\llbracket \mathbf{A}, \mathbf{B}, \mathbf{C}, \dots \rrbracket$	CANDECOMP/PARAFAC (CP) decomposition of the tensor $\mathcal{X}$ from the outer product of column vectors of $\mathbf{A}$ , $\mathbf{B}$ , $\mathbf{C}, \dots$

$[\![\boldsymbol{\lambda}; \mathbf{A}, \mathbf{B}, \mathbf{C}, \dots]\!]$

Normalized CANDECOMP/PARAFAC (CP) decomposition of the tensor  $\boldsymbol{\mathcal{X}}$  from the outer product of column vectors of  $\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$

## 5.7 Spaces

$\text{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$

Vector space spanned by the argument vectors [6]

$\mathbf{C}(\mathbf{A})$ ,  $\text{columnspace}(\mathbf{A})$ ,  $\text{range}(\mathbf{A})$ ,  $\text{span}(\mathbf{A})$ ,  $\text{image}(\mathbf{A})$

Columnspace, range or image, i.e., the space  $\text{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$ , where  $\mathbf{a}_i$  is the  $i$ th column vector of the matrix  $\mathbf{A}$  [12, 17]

$\mathbf{C}(\mathbf{A}^H)$

Row space (also called left column space) [12, 17]

$\mathbf{N}(\mathbf{A})$ ,  $\text{nullspace}(\mathbf{A})$ ,  $\text{kernel}(\mathbf{A})$

Nullspace (or kernel space) [12, 17]

$\mathbf{N}(\mathbf{A}^H)$

Left nullspace

$\text{rank}(\mathbf{A})$

Rank, that is,  $\dim(\text{span}(\mathbf{A})) = \dim(\mathbf{C}(\mathbf{A}))$  [12]

$\text{nullity}(\mathbf{A})$

Nullity of  $\mathbf{A}$ , i.e.,  $\dim(\mathbf{N}(\mathbf{A}))$

$\mathbf{a} \perp \mathbf{b}$

$\mathbf{a}$  is orthogonal to  $\mathbf{b}$

$\mathbf{a} \not\perp \mathbf{b}$

$\mathbf{a}$  is not orthogonal to  $\mathbf{b}$

## 5.8 Inequalities

$\boldsymbol{\mathcal{X}} \leq 0$

Nonnegative tensor

$\mathbf{a} \preceq_K \mathbf{b}$

Generalized inequality meaning that  $\mathbf{b} - \mathbf{a}$  belongs to the conic subset  $K$  in the space  $\mathbb{R}^n[2]$

$\mathbf{a} \prec_K \mathbf{b}$

Strict generalized inequality meaning that  $\mathbf{b} - \mathbf{a}$  belongs to the interior of the conic subset  $K$  in the space  $\mathbb{R}^n[2]$

$\mathbf{a} \preceq \mathbf{b}$

Generalized inequality meaning that  $\mathbf{b} - \mathbf{a}$  belongs to the nonnegative orthant conic subset,  $\mathbb{R}_+^n$ , in the space  $\mathbb{R}^n[2]$

$\mathbf{a} \prec \mathbf{b}$

Strict generalized inequality meaning that  $\mathbf{b} - \mathbf{a}$  belongs to the positive orthant conic subset,  $\mathbb{R}_{++}^n$ , in the space  $\mathbb{R}^n[2]$

$\mathbf{A} \preceq_K \mathbf{B}$

Generalized inequality meaning that  $\mathbf{B} - \mathbf{A}$  belongs to the conic subset  $K$  in the space  $\mathbb{S}^n[2]$

$\mathbf{A} <_K \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the interior of the conic subset $K$ in the space $\mathbb{S}^n[2]$
$\mathbf{A} \preceq \mathbf{B}$	Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive semidefinite conic subset, $\mathbb{S}_+^n$ , in the space $\mathbb{S}^n[2]$
$\mathbf{A} < \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive or-thant conic subset, $\mathbb{S}_{++}^n$ , in the space $\mathbb{S}^n[2]$

## 6 Communication systems

$B$	One-sided bandwidth of the transmitted signal, in Hz
$W$	One-sided bandwidth of the transmitted signal, in rad/s
$x_i$	Real or in-phase part of $x$
$x_q$	Imaginary or quadrature part of $x$
$f_c, f_{RF}$	Carrier frequency (in Hertz)
$f_L$	Carrier frequency in L-band (in Hertz)
$f_{IF}$	Intermediate frequency (in Hertz)
$f_s$	Sampling frequency or sampling rate (in Hertz)
$T_s$	Sampling time interval/duration/period
$R$	Bit rate
$T$	Bit interval/duration/period
$T_c$	Chip interval/duration/period
$T_{sy}, T_{sym}$	Symbol interval/duration/period
$s_{RF}$	Transmitted signal in RF
$s_{FI}$	Transmitted signal in FI
$s, s_l$	Low-pass equivalent signal or envelope complex of transmitted signal
$r_{RF}$	Received signal in RF
$r_{FI}$	Received signal in FI
$r, r_l$	Low-pass equivalent signal or envelope complex of received signal
$\phi$	Signal phase
$\phi_0$	Initial phase
$\eta_{RF}, w_{RF}$	Noise in RF
$\eta_{FI}, w_{FI}$	Noise in FI

$\eta, w$	Noise in baseband
$\tau$	Timing delay
$\Delta\tau$	Timing error (delay - estimated)
$\varphi$	Phase offset
$\Delta\varphi$	Phase error (offset - estimated)
$f_d$	Linear Doppler frequency
$\Delta f_d$	Frequency error (Doppler frequency - estimated)
$\nu$	Angular Doppler frequency
$\Delta\nu$	Frequency error (Doppler frequency - estimated)
$\gamma, A$	Transmitted signal amplitude
$\gamma_0, A_0$	Combined effect of the path loss and antenna gain

## 7 Discrete mathematics

### 7.1 Set theory

$A + B$	Set addition (Minkowski sum), i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} = \mathbf{x} + \mathbf{y}, \forall \mathbf{x} \in \mathcal{X} \wedge \mathbf{y} \in \mathcal{Y}\}$ [10]
$A - B$	Minkowski difference, i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} = \mathbf{x} - \mathbf{y}, \forall \mathbf{x} \in \mathcal{X} \wedge \mathbf{y} \in \mathcal{Y}\}$
$A \ominus B$	Pontryagin difference, i.e., $\{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} + \mathbf{y} \in \mathcal{X}, \forall \mathbf{y} \in \mathcal{Y}\}$ [10]
$A \setminus B, A - B$	Set difference or set subtraction, i.e., $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ the set containing the elements of $A$ that are not in $B$ [16]
$A \cup B$	Set of union
$A \cap B$	Set of intersection
$A \times B$	Cartesian product
$A^n$	$\underbrace{A \times A \times \cdots \times A}_{n \text{ times}}$
$A^\perp$	Orthogonal complement of $A$ , e.g., $\mathbf{N}(\mathbf{A}) = \mathbf{C}(\mathbf{A}^\top)^\perp$ [2]
$A \oplus B$	Direct sum, i.e., each $\mathbf{v} \in \{\sum \mathbf{a}_i \mid \mathbf{a}_i \in S_i, i = 1, \dots, k\}$ has a unique representation of $\sum \mathbf{a}_i$ with $\mathbf{a}_i \in S_i$ . That is, they expand to a space. Note that $\{S_i\}$ might not be orthogonal each other [6]

$A \overset{\perp}{\oplus} B$	Direct sum of two space that are orthogonal and span a $n$ -dimensional space, e.g., $C(\mathbf{A}^\top) \overset{\perp}{\oplus} C(\mathbf{A}^\top)^\perp = \mathbb{R}^n$ (this decomposition of $\mathbb{R}^n$ is called the orthogonal decomposition induced by $\mathbf{A}$ ) [2]
$\bar{A}, A^c$	Complement set (given $U$ )
$\#A,  A $	Cardinality
$a \in A$	$a$ is element of $A$
$a \notin A$	$a$ is not element of $A$
$\{1, 2, \dots, n\}$	Discrete set containing the integer elements $1, 2, \dots, n$
$U$	Universe
$2^A$	Power set of $A$
$\mathbb{R}$	Set of real numbers
$\mathbb{C}$	Set of complex numbers
$\mathbb{Z}$	Set of integer number
$\mathbb{B} = \{0, 1\}$	Boolean set
$\emptyset$	Empty set
$\mathbb{N}$	Set of natural numbers
$\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$	Real or complex space (field)
$\mathbb{K}^{I_1 \times I_2 \times \dots \times I_N}$	$I_1 \times I_2 \times \dots \times I_N$ -dimensional real (or complex) space
$\mathbb{K}_+$	Nonnegative real (or complex) space [2]
$\mathbb{K}_{++}$	Positive real (or complex) space, i.e., $\mathbb{K}_{++} = \mathbb{K}_+ \setminus \{\mathbf{0}\}$ [2]
$\mathbb{S}^n, \mathcal{S}^n$	Conic set of the symmetric matrices in $\mathbb{R}^{n \times n}$ [2]
$\mathbb{S}_+^n, \mathcal{S}_+^n$	Conic set of the symmetric positive semidefinite matrices in $\mathbb{R}^{n \times n}$ [2]
$\mathbb{S}_{++}^n, \mathcal{S}_{++}^n$	Conic set of the symmetric positive definite matrices in $\mathbb{R}^{n \times n}$ , i.e., $\mathbb{S}_{++}^n = \mathbb{S}_+^n \setminus \{\mathbf{0}\}$ [2]
$\mathbb{H}^n$	Set of all hermitian matrices in $\mathbb{C}^{n \times n}$
$[a, b]$	Closed interval of a real set from $a$ to $b$
$(a, b)$	Opened interval of a real set from $a$ to $b$
$[a, b), (a, b]$	Half-opened intervals of a real set from $a$ to $b$

## 7.2 Quantifiers, inferences

$\forall$	For all (universal quantifier) [7]
$\exists$	There exists (existential quantifier) [7]
$\nexists$	There does not exist [7]
$\exists!$	There exist an unique [7]
$\in$	Belongs to [7]
$\notin$	Does not belong to [7]
$\because$	Because [7]
$ , :$	Such that, sometimes that parantheses is used [7]
$,, (\cdot)$	Used to separate the quantifier with restricted domain from the propositional function, e.g., $\forall x < 0 (x^2 > 0)$ or $\forall x < 0, x^2 > 0$ [7]
$\therefore$	Therefore [7]

### 7.3 Propositional Logic

$\neg a$	Logical negation of $a$ [16]
$a \wedge b$	Conjunction (logical AND) operator between $a$ and $b$ [16]
$a \vee b$	Disjunction (logical OR) operator between $a$ and $b$ [16]
$a \oplus b$	Exclusive OR (logical XOR) operator between $a$ and $b$ [16]
$a \rightarrow b$	Implication (or conditional) statement [16]
$a \leftrightarrow b$	Bi-implication (or biconditional) statement, i.e., $(a \rightarrow b) \wedge (b \rightarrow a)$ [16]
$a \equiv b, a \iff b, a \Leftrightarrow b$	Logical equivalence, i.e., $a \leftrightarrow b$ is a tautology [16]

## 8 Number theory, algorithm theory, and other notations

### 8.1 Mathematical symbols

■	Q.E.D.
$\triangleq, :=$	Equal by definition
$\neq$	Not equal
$\infty$	Infinity
$j$	$\sqrt{-1}$

## 8.2 Operations

$ a $	Absolute value of $a$
$\log$	Base-10 logarithm or decimal logarithm
$\ln$	Natural logarithm
$\operatorname{Re}\{x\}$	Real part of $x$
$\operatorname{Im}\{x\}$	Imaginary part of $x$
$\angle \cdot$	phase (complex argument)
$x \bmod y$	Remainder, i.e., $x - y\lfloor x/y \rfloor$ , for $y \neq 0$
$\operatorname{frac}(x)$	Fractional part, i.e., $x \bmod 1$ [7]
$a \setminus b$	$b$ is a positive integer multiple of $a$ , i.e., $\exists n \in \mathbb{Z}_{++} \mid b = na$ [7]
$a \nsetminus b$	$b$ is not a positive integer multiple of $a$ , i.e., $\nexists n \in \mathbb{Z}_{++} \mid b = na$ [7]
$\lceil \cdot \rceil$	Ceiling operation [7]
$\lfloor \cdot \rfloor$	Floor operation [7]

## 8.3 Functions

$\mathcal{O}(\cdot), \mathcal{O}(\cdot)$	Big-O notation
$\Gamma(\cdot)$	Gamma function

## 9 Abbreviations

wrt.	With respect to
st.	Subject to
iff.	If and only if
EVD	Eigenvalue decomposition, or eigendecomposition [12]
SVD	Singular value decomposition
CP	CANDECOMP/PARAFAC

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