Notation

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1 Font notation

$a, b, c, \ldots, A, B, C, \ldots$	Scalars
a, b, c, \dots	Vectors
A, B, C, \dots	Matrices
$\mathcal{A}, \mathcal{B}, \mathcal{C}, \dots$	Tensors
$A, B, C, \ldots, A, B, C, \ldots, A, B, C, \ldots$	Sets

2 Signals and functions

2.1 Time indexing

x(t)	Continuous-time t
$x[n], x[k], x[m], x[i], \dots$	Discrete-time n, k, m, i, \dots (parenthe-
$x_n, x_k, x_m, x_i, \dots$	sis should be adopted only if there
$x(n), x(k), x(m), x(i), \dots$	are no continuous-time signals in the
	context to avoid ambiguity)
$x\left[\left((n-m)\right)_{N}\right], x\left((n-m)\right)_{N}$	Circular shift in m samples within a
	N-samples window

2.2 Common functions

$\delta(t)$	Delta function
$\delta[n], \delta_{i,j}$	Kronecker function $(n = i - j)$
h(t), h[n]	Impulse response (continuous and
	discrete time)

$\tilde{x}[n], \tilde{x}(t)$	Periodic discrete- or continuous-time
	signal
$\hat{x}[n], \hat{x}(t)$	Estimate of $x[n]$ or $x(t)$
$\dot{x}[m]$	Interpolation of $x[n]$

2.3 Operations and symbols

$f:A\to B$	A function f whose domain is A and
	codomain is B
$f^n, x^n(t), x^n[k]$	nth power of the function f , $x[n]$ or
	x(t)
$f^{(n)}, x^{(n)}(t)$	nth derivative of the function f or
	x(t)
$f', f^{(1)}, x'(t)$	1th derivative of the function f or
	x(t)
$f'', f^{(2)}, x''(t)$	2th derivative of the function f or
	x(t)
$f \circ g$	Composition of the functions f and
	g
*	Convolution (discrete or continuous)
$(8, \overline{N})$	Circular convolution

2.4 Transformations

$\mathcal{F}\left\{ \cdot ight\}$	Fourier transform
$\mathcal{L}\left\{ \cdot ight\}$	Laplace transform
$\mathcal{Z}\left\{ \cdot ight\}$	z-transform
$\hat{x}(t), \hat{x}[n]$	Hilbert transform of $x(t)$ or $x[n]$
X(s)	Laplace transform of $x(t)$
X(f)	Fourier transform (FT) (in linear fre-
	quency, Hz) of $x(t)$
$X(j\omega)$	Fourier transform (FT) (in angular
	frequency, rad/sec) of $x(t)$
$X(e^{j\omega})$	Discrete-time Fourier transform
	(DTFT) of $x[n]$
$X[k], X(k), X_k$	Discrete Fourier transform (DFT) or
	fast Fourier transform (FFT) of $x[n]$,
	or even the Fourier series (FS) of the
	periodic signal $x(t)$
$\tilde{X}[k], \tilde{X}(k), \tilde{X}_k$	Discrete Fourier series (DFS) of $\tilde{x}[n]$
X(z)	z-transform of $x[n]$

3 Probability, statistics, and stochastic processes

3.1 Operators and symbols

$E\left[\cdot\right]$ $E_{u}\left[\cdot\right]$	Statistical expectation Statistical expectation with respect
	to u
$\mu_{\scriptscriptstyle X}$	Mean of the random variable x
μ_x, m_x	Mean vector of the random variable
	X
μ_n	nth-order moment of a random vari-
	able
\mathcal{K}_x, μ_4	Kurtosis (4th-order moment) of the
	random variable x
$VAR[\cdot]$	Variance operator
$VAR_u[\cdot]$	Variance operator with respect to u
κ_n	nth-order cumulant of a random vari-
	able
σ_x, κ_2	Variance of the random variable x
$\rho_{x,y}$	Pearson correlation coefficient be-
	tween x and y
$a \sim P$	Random variable a with distribution
	P
\mathcal{R}	Rayleigh's quotient

3.2 Stochastic processes

$r_{X}(\tau), R_{X}(\tau)$	Autocorrelation function of the sig-
	$\operatorname{nal} x(t) \text{ or } x[n]$
$S_x(f), S_x(j\omega)$	Power spectral density (PSD) of $x(t)$
	in linear (f) or angular (ω) frequency
$S_{x,y}(f), S_{x,y}(j\omega)$	Cross PSD of $x(t)$ and $y(t)$ in linear
	or angular (ω) frequency
$\mathbf{R}_{\mathbf{x}}$	(Auto)correlation matrix of $\mathbf{x}(n)$
$r_{x,d}(\tau), R_{x,d}(\tau)$	Cross-correlation between $x[n]$ and
	d[n] or $x(t)$ and $d(t)$
$\mathbf{R}_{\mathbf{x}\mathbf{y}}$	Cross-correlation matrix of $\mathbf{x}(n)$ and
-	$\mathbf{y}(n)$
$\mathbf{p}_{\mathbf{x}d}$	Cross-correlation vector between
	$\mathbf{x}(n)$ and $d(n)$
$c_X(\tau), C_X(\tau)$	Autocovariance function of the signal
	x(t) or $x[n]$
$\mathbf{C}_{\mathrm{x}},\mathbf{K}_{\mathrm{x}},\mathbf{\Sigma}_{\mathrm{x}}$	(Auto)covariance matrix of \mathbf{x}

$$c_{xy}(\tau), C_{xy}(\tau)$$

 $C_{xy}, K_{xy}, \Sigma_{xy}$

Cross-covariance function of the signal x(t) or x[n]

Cross-covariance matrix of \mathbf{x} and \mathbf{v}

3.3 **Functions**

 $Q(\cdot)$ $\operatorname{erf}(\cdot)$

 $\operatorname{erfc}(\cdot)$

P[A]

 $p(\cdot), f(\cdot)$

 $p(x \mid A)$ $F(\cdot)$

 $\Phi_X(\omega), M_X(j\omega), E\left[e^{j\omega x}\right]$

 $M_x(t), \Phi_x(-jt), E[e^{tx}]$

 $\Psi_x(\omega), \ln \Phi_x(\omega), \ln E\left[e^{j\omega x}\right]$ $K_x(t), \ln E\left[e^{tx}\right], \ln M_x(t)$

Q-function, i.e., $P[\mathcal{N}(0,1) > x]$

Error function

Complementary error function i.e.,

 $\operatorname{erfc}(x) = 2Q(\sqrt{2}x) - \operatorname{erf}(x)$

Probability of the event or set A

Probability density function (PDF) or probability mass function (PMF)

Conditional PDF or PMF

Cumulative distribution function

(CDF)

First characteristic function (CF) of

Moment-generating function (MGF)

Second characteristic function

Cumulant-generating function

(CGF) of x

3.4 **Distributions**

 $\mathcal{N}(\mu, \sigma^2)$

 $\mathcal{CN}(\mu, \sigma^2)$

 $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

Gaussian distribution of a random variable with mean μ and variance σ^2 . The same notation can be used to denote a real-valued white Gaussian process with mean equal to μ and power spectral density equal to $N_0/2$, e.g., $s(t) \sim \mathcal{N}(\mu, N_0/2)$

Complex Gaussian distribution of a random variable with mean μ and variance σ^2 . The same notation can be used to denote a complex-valued white Gaussian process with mean equal to μ and power spectral density equal to N_0 , e.g., $s(t) \sim \mathcal{CN}(\mu, N_0)$ Gaussian distribution of a vector ran-

dom variable with mean μ and co-

variance matrix Σ

$\mathcal{CN}(\mu,\Sigma)$	Complex Gaussian distribution of a vector random variable with mean μ and covariance matrix Σ
$\mathcal{U}(a,b) \\ \chi^2(n), \chi_n^2$	Uniform distribution from a to b Chi-square distribution with n degree of freedom (assuming that the Gaus- sians are $\mathcal{N}(0,1)$)
$\operatorname{Exp}(\lambda)$	Exponential distribution with rate parameter λ
$\Gamma(lpha,eta)$	Gamma distribution with shape parameter α and rate parameter β
$\Gamma(lpha, heta)$	Gamma distribution with shape parameter α and scale parameter θ =
$\mathrm{Nakagami}(m,\Omega)$	$1/\beta$ Nakagami-m distribution with shape parameter m and spread parameter Ω
$\operatorname{Rayleigh}(\sigma)$	Rayleigh distribution with scale parameter σ
$\operatorname{Rayleigh}(\Omega)$	Rayleigh distribution with the second moment $\Omega = E[x^2] = 2\sigma^2$
$\mathrm{Rice}(s,\sigma)$	Rice distribution with noncentrality parameter (specular component) s and σ
$\mathrm{Rice}(A,K)$	Rice distribution with Rice factor $K = s^2/2\sigma^2$ and scale parameter $A = s^2 + 2\sigma^2$

4 Statistical signal processing

$oldsymbol{ abla} f, \mathbf{g} \ oldsymbol{ abla}_x f, \mathbf{g}_x$	Gradient descent vector Gradient descent vector with respect
$ \begin{array}{l} \mathbf{g} \text{ (or } \hat{\mathbf{g}} \text{ if the gradient vector is } \mathbf{g}) \\ J(\cdot), \mathcal{E}(\cdot) \\ \Lambda(\cdot) \\ \Lambda_l(\cdot) \\ \hat{x}(t) \text{ or } \hat{x}[n] \\ \hat{\boldsymbol{\mu}}_{\boldsymbol{\chi}}, \hat{\mathbf{m}}_{\boldsymbol{\chi}} \\ \hat{\boldsymbol{\mu}}_{\boldsymbol{\chi}}, \hat{\mathbf{m}}_{\boldsymbol{\chi}} \end{array} $	x Stochastic gradient descent (SGD) Cost-function or objective function Likelihood function Log-likelihood function Estimate of $x(t)$ or $x[n]$ Sample mean of $x[n]$ or $x(t)$ Sample mean vector of $x[n]$ or $x(t)$
$\hat{r}_X(au), \hat{R}_X(au)$ $\hat{S}_X(f), \hat{S}_X(j\omega)$	Estimated autocorrelation function of the signal $x(t)$ or $x[n]$ Estimated power spectral density (PSD) of $x(t)$ in linear (f) or angular (ω) frequency

$\hat{\mathbf{R}}_{\mathbf{x}}$	Sample (auto)correlation matrix
$\hat{r}_{x,d}(au), \hat{R}_{x,d}(au)$	Estimated cross-correlation between
	x[n] and $d[n]$ or $x(t)$ and $d(t)$
$\hat{S}_{x,y}(f), \hat{S}_{x,y}(j\omega)$	Estimated cross PSD of $x(t)$ and $y(t)$
	in linear or angular (ω) frequency
$\hat{\mathbf{R}}_{\mathbf{x}\mathbf{y}}$	Sample cross-correlation matrix of
	$\mathbf{R}_{\mathbf{x}\mathbf{y}}$
$\hat{ ho}_{x,y}$	Estimated Pearson correlation coeffi-
•	cient between x and y
$\hat{c}_{x}(au),\hat{C}_{x}(au)$	Estimated autocovariance function of
	the signal $x(t)$ or $x[n]$
$\hat{\mathbf{C}}_{\mathbf{x}},\hat{\mathbf{K}}_{\mathbf{x}},\hat{\mathbf{\Sigma}}_{\mathbf{x}}$	Sample (auto)covariance matrix
$\hat{c}_{xy}(au), \hat{C}_{xy}(au)$	Estimated cross-covariance function
	of the signal $x(t)$ or $x[n]$
$\hat{ extbf{C}}_{ extbf{xy}}, \hat{ extbf{K}}_{ extbf{xy}}, \hat{ extbf{\Sigma}}_{ extbf{xy}}$	Sample cross-covariance matrix
$\mathbf{w}, \mathbf{\theta}$	Parameters, coefficients, or weights
	vector
$\mathbf{w}_{o}, \mathbf{w}^{\star}, \mathbf{\theta}_{o}, \mathbf{\theta}^{\star}$	Optimum value of the parameters,
	coefficients, or weights vector
\mathbf{W}	Matrix of the weights
J	Jacobian matrix
H	Hessian matrix
$\hat{\mathbf{H}}$	Estimate of the Hessian matrix

5 Linear Algebra

5.1 Common matrices and vectors

W, D	Diagonal matrix
P	Projection matrix; Permutation ma-
	trix
J	Jordan matrix
${f L}$	Lower matrix
\mathbf{U}	Upper matrix
\mathbf{C}	Cofactor matrix
$\mathbf{C}_{\mathbf{A}}, \operatorname{cof}\left(\mathbf{A}\right)$	Cofactor matrix of $\bf A$
\mathbf{S}	Symmetric matrix
Q	Orthogonal matrix
\mathbf{I}_N	$N \times N$ -dimensional identity matrix
$0_{M imes N}$	$M \times N$ -dimensional null matrix
0_N	N-dimensional null vector
$1_{M imes N}$	$M \times N$ -dimensional ones matrix
1_N	N-dimensional ones vector

0	Null matrix, vector, or tensor (di-
	mensionality understood by context)
1	Ones matrix, vector, or tensor (di-
	mensionality understood by context)

5.2 Indexing

$x_{i_1,i_2,,i_N}, [\mathcal{X}]_{i_1,i_2,,i_N}$	Element in the position
1, 2, , , 1,	(i_1,i_2,\ldots,i_N) of the tensor $\boldsymbol{\mathcal{X}}$
$\mathcal{X}^{(n)}$	nth tensor of a nontemporal sequence
$\mathbf{x}_n, \mathbf{x}_{:n}$	nth column of the matrix X
\mathbf{x}_{n} :	nth row of the matrix X
$\mathbf{X}_{i_1,,i_{n-1},:,i_{n+1},,i_N}$	Mode- n fiber of the tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{:,i_2,i_3}$	Column fiber (mode-1 fiber) of the
	thrid-order tensor \mathcal{X}
$\mathbf{x}_{i_1,:,i_3}$	Row fiber (mode-2 fiber) of the thrid-
	order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{x}_{i_1,i_2,:}$	Tube fiber (mode-3 fiber) of the
	thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{i_1,:,:}$	Horizontal slice of the thrid-order
	tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{:,i_2,:}$	Lateral slices slice of the thrid-order
	tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{i_3},\mathbf{X}_{:,:,i_3}$	Frontal slices slice of the thrid-order
	tensor \mathcal{X}

5.3 General operations

$\langle \cdot, \cdot angle$	Inner product, e.g., $\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^{T} \mathbf{b}$
0	Outer product, e.g., $\mathbf{a} \circ \mathbf{b} = \mathbf{a} \mathbf{b}^{T}$
\otimes	Kronecker product
⊙	Hadamard (or Schur) (elementwise)
	product
$.\odot n$	nth-order Hadamard power
$\cdot \circ \frac{1}{n}$	nth-order Hadamard root
\oslash	Hadamard (or Schur) (elementwise)
	division
♦	Khatri-Rao product
\otimes	Kronecker Product
\times_n	<i>n</i> -mode product

5.4 Operations with matrices and tensors

 \mathbf{A}^{-1} Inverse matrix A^+, A^\dagger Moore-Penrose left pseudoinverse $\mathbf{A}^{ op}$ Transpose $\mathbf{A}^{-\top}$ Transpose of the inverse \mathbf{A}^* Complex conjugate \mathbf{A}^H Hermitian $\|\mathbf{A}\|_{\mathrm{F}}$ Frobenius norm $\|\mathbf{A}\|$ Matrix norm $|\mathbf{A}|, \det(\mathbf{A})$ Determinant $\operatorname{diag}\left(\mathbf{A}\right)$ The elements in the diagonal of A vec(A)Vectorization: stacks the columns of the matrix A into a long column vec $vec_d(\mathbf{A})$ Extracts the diagonal elements of a square matrix and returns them in a column vector $\text{vec}_{l}\left(\mathbf{A}\right)$ Extracts the elements strictly below the main diagonal of a square matrix in a column-wise manner and returns them into a column vector $vec_u(\mathbf{A})$ Extracts the elements strictly above the main diagonal of a square matrix in a column-wise manner and returns them into a column vector $\text{vec}_{\text{b}}\left(\mathbf{A}\right)$ Block vectorization operator: stacks square block matrices of the input into a long block column matrix unvec (A) Reshapes a column vector into a matrix $\mathrm{tr}\left(\mathbf{A}\right)$ trace n-mode matricization of the tensor ${\cal X}$ $\mathbf{X}_{(n)}$

5.5 Operations with vectors

$\ \mathbf{a}\ $	l_1 norm, 1-norm, or Manhatan norm
$\ \mathbf{a}\ , \ \mathbf{a}\ _2$	l_2 norm, 2-norm, or Euclidean norm
$\ \mathbf{a}\ _p$	l_p norm, p -norm, or Minkowski norm
$\ \mathbf{a}\ _{\infty}^{r}$	l_{∞} norm, ∞ -norm, or Chebyshev
	norm
$\operatorname{diag}\left(\mathbf{a}\right)$	Diagonalization: a square, diagonal matrix with entries given by the vec-
	tor a

5.6 Decompositions

$rac{oldsymbol{\Lambda}}{oldsymbol{Q}}$	Eigenvalue matrix Eigenvectors matrix; Orthogonal ma-
R	trix of the QR decomposition Upper triangular matrix of the QR decomposition
\mathbf{U}	Left singular vectors
_	e e e e e e e e e e e e e e e e e e e
$egin{array}{c} \mathbf{U}_r \ \mathbf{\Sigma} \end{array}$	Left singular nondegenerated vectors
	Singular value matrix
Σ_r	Singular value matrix with nonzero
Σ^+	singular values in the main diagonal
L .	Singular value matrix of the pseu-
V +	doinverse
Σ_r^+	Singular value matrix of the pseu-
	doinverse with nonzero singular val-
	ues in the main diagonal
V	Right singular vectors
\mathbf{V}_r	Right singular nondegenerated vec-
	tors
$\operatorname{eig}\left(\mathbf{A} ight)$	Set of the eigenvalues of A
$[\![\mathbf{A},\mathbf{B},\mathbf{C},\ldots]\!]$	CANDECOMP/PARAFAC (CP) de-
	composition of the tensor $\boldsymbol{\mathcal{X}}$ from the
	outer product of column vectors of \mathbf{A} ,
	B, C,
$[\![\boldsymbol{\lambda};\mathbf{A},\mathbf{B},\mathbf{C},\ldots]\!]$	Normalized CANDE-
	COMP/PARAFAC (CP) decom-
	position of the tensor \mathcal{X} from the
	outer product of column vectors of
	A, B, C, \dots
	,,,,

5.7 Spaces

$\mathrm{span}\left(\mathbf{a}_{1},\mathbf{a}_{2},\ldots,\mathbf{a}_{n}\right)$	Vector space spanned by the argument vectors
C(A), columnspace(A), range(A), span(A), image(A)	Columnspace, range or image, i.e., the space span $(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$, where \mathbf{a}_i is the ith column vector of the ma-
	trix A
$C(\mathbf{A}^{H})$	Row space
$N(\mathbf{A})$, nullspace(\mathbf{A}), kernel(\mathbf{A})	Nullspace (or kernel space)
$N(\mathbf{A}^{H})$	Left nullspace
$\operatorname{rank}(\mathbf{A})$	Rank, that is, $\dim(\operatorname{span}(\mathbf{A})) =$
` '	$\dim (C(\mathbf{A}))$
nullity (A)	Nullity of \mathbf{A} , i.e., dim (N (\mathbf{A}))
$\mathbf{a} \perp \mathbf{b}$	a is orthogonal to b
a ≠ b	${f a}$ is not orthogonal to ${f b}$

5.8 Inequalities

\mathcal{X}	\leq	0
a :	≤ _K	b

 $\mathbf{a} \prec_K \mathbf{b}$

 $\mathbf{a} \leq \mathbf{b}$

 $\mathbf{a} \prec \mathbf{b}$

 $\mathbf{A} \leq_K \mathbf{B}$

 $\mathbf{A} \prec_K \mathbf{B}$

 $A \leq B$

A < B

Nonnegative tensor

Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the conic subset K in the space \mathbb{R}^n

Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the interior of the conic subset K in the space \mathbb{R}^n Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the nonnegative orthant conic subset, \mathbb{R}^n_+ , in the space \mathbb{R}^n .

Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the positive orthant conic subset, \mathbb{R}^n_{++} , in the space \mathbb{R}^n

Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the conic subset K in the space \mathbb{S}^n

Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the interior of the conic subset K in the space \mathbb{S}^n Generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive semidefinite conic subset, \mathbb{S}^n_+ , in the space \mathbb{S}^n Strict generalized inequality meaning that $\mathbf{B} - \mathbf{A}$ belongs to the positive orthant conic subset, \mathbb{S}^n_{++} , in the space \mathbb{S}^n

6 Sets

A + B
A - B
$A \setminus B, A - B$

 $A \cup B$ $A \cap B$ $A \times B$ A^n

Set addition (Minkowski sum)

Minkowski difference

Set difference or set subtraction, i.e., the set containing the elements

of A that are not in B

Set of union Set of intersection Cartesian product $A \times A \times \cdots \times A$

n times

A^{\perp}	Orthogonal complement of A , e.g., $N(\mathbf{A}) = C(\mathbf{A}^{\top})^{\perp}$
$A \oplus B$	Direct sum, e.g., $C(\mathbf{A}^{T}) \oplus C(\mathbf{A}^{T})^{\perp} =$
	\mathbb{R}^n
A^c, \bar{A}	Complement set (given U)
#A, A	Cardinality
$a \in A$	a is element of A
$a \notin A$	a is not element of A
$\{1,2,\ldots,n\}$	Discrete set containing the integer el-
	ements $1, 2, \ldots, n$
U	Universe
2^A	Power set of A
\mathbb{R}	Set of real numbers
\mathbb{C}	Set of complex numbers
$\mathbb Z$	Set of integer number
$\mathbb{B} = \{0, 1\}$	Boolean set
Ø	Empty set
N	Set of natural numbers
$\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$	Real or complex space (field)
$\mathbb{K}^{I_1 \times I_2 \times \cdots \times I_N}$	$I_1 \times I_2 \times \cdots \times I_N$ -dimensional real (or
	complex) space
\mathbb{K}_{+}	Nonnegative real (or complex) space
K++	Positive real (or complex) space, i.e., $\mathbb{K}_{++} = \mathbb{K}_{+} \setminus \{0\}$
$\mathbb{S}^n, \mathcal{S}^n$	Conic set of the symmetric matrices
, , ,	in $\mathbb{R}^{n \times n}$
$\mathbb{S}^n_+, \mathcal{S}^n_+$	Conic set of the symmetric positive
	semidefinite matrices in $\mathbb{R}^{n \times n}$
$\mathbb{S}^n_{++}, \mathcal{S}^n_{++}$	Conic set of the symmetric positive
	definite matrices in $\mathbb{R}^{n\times n}$, i.e., \mathbb{S}^n_{++}
	$\mathbb{S}^n_+\setminus\{0\}$
\mathbb{H}^n	Set of all hermitian matrices in $\mathbb{C}^{n\times n}$
[a,b]	Closed interval of a real set from a to
	b
(a,b)	Opened interval of a real set from a
	to b
[a,b),(a,b]	Half-opened intervals of a real set
	from a to b

7 Communication systems

One-sided bandwidth of the transmitted signal, in ${\rm Hz}$

WOne-sided bandwidth of the transmitted signal, in rad/s Real or in-phase part of x x_i Imaginary or quadrature part of x x_q f_c, f_{RF} Carrier frequency f_{IF} Intermediate frequency Sampling frequency or sampling rate f_s T_s Sampling period R Bit rate TBit period T_{sy} Symbol period Transmitted signal in RF SRFTransmitted signal in FI SFILow-pass equivalent signal or enve- S, S_1 lope complex of transmitted signal Received signal in RF r_{RF} Received signal in FI r_{FI} Low-pass equivalent signal or enve r, r_l lope complex of received signal φ Signal phase Initial phase ϕ_0 Noise in RF η_{RF}, w_{RF} Noise in FI η_{FI}, w_{FI} Noise in baseband η, w Timing delay τ Timing error (delay - estimated) $\Delta \tau$ Phase offset $\Delta \varphi$ Phase error (offset - estimated) Linear Doppler frequency f_d Frequency error (Doppler frequency - Δf_d estimated) ν Angular Doppler frequency Frequency error (Doppler frequency - $\Delta \nu$ estimated) γ, A Transmitted signal amplitude Combined effect of the path loss and γ_0, A_0 antenna gain

8 Other notations

8.1 Mathematical symbols

∃ There exists
∄ There does not exist

∃!	There exist an unique
€	Belongs to
∉	Does not belong to
	Q.E.D.
\therefore	Therefore
:	Because
\forall	For all
ļ,:	Such that
\iff	Logical equivalence
≜,:=	Equal by definition
≠	Not equal
∞	Infinity
j	$\sqrt{-1}$
W_N	Twiddle factor, $e^{-j\frac{2\pi}{N}}$

8.2 Operations

arg min $f(x)$ $x \in A$ inf $g(\mathbf{x}, \mathbf{y})$ $y \in A$ Sup $g(\mathbf{x}, \mathbf{y})$ $y \in A$ log log Base-10 logarithm or decimal logarithm Re $\{x\}$ Real part of x Imaginary part of x Imaginary part of x $x \mod y$ Remainder, i.e., $x - y \lfloor x/y \rfloor$ frac (x) $x \wedge b$ Logical AND of a and b $a \vee b$ Logical negation [·] Ceiling operation Floor operation	$ \operatorname{argmax} f(x) $	Value of x that minimizes x
$\inf_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$ $\sup_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$ $\lim_{\mathbf{x} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y}$		Value of x that minimizes x
$\sup g(\mathbf{x}, \mathbf{y})$ Supremum $y \in \mathcal{A}$ Absolute value of a \log Base-10 logarithm or decimal logarithm $\lim \{x\}$ Real part of x $\lim \{x\}$ Imaginary part of x \mathcal{A} Phase (complex argument) \mathcal{A} Remainder, i.e., $x - y \lfloor x/y \rfloor$ \mathcal{A} Fractional part, i.e., $x \mod 1$ $\mathcal{A} \wedge \mathcal{B}$ Logical AND of a and a $\mathcal{A} \vee \mathcal{B}$ Logical OR of a and a \mathcal{A} Logical negation of a \mathcal{A} Ceiling operation	$\inf g(\mathbf{x}, \mathbf{y})$	Infimum
$ a \qquad \qquad \text{Absolute value of } a \\ \log \qquad \qquad \text{Base-10 logarithm or decimal logarithm} \\ \ln \qquad \qquad \text{Natual logarithm} \\ \text{Re } \{x\} \qquad \qquad \text{Real part of } x \\ \text{Im } \{x\} \qquad \qquad \text{Imaginary part of } x \\ \angle \cdot \qquad \qquad \text{phase (complex argument)} \\ x \bmod y \qquad \qquad \text{Remainder, i.e., } x - y \lfloor x/y \rfloor \\ \text{frac } (x) \qquad \qquad \text{Fractional part, i.e., } x \bmod 1 \\ a \wedge b \qquad \qquad \text{Logical AND of } a \text{ and } b \\ a \vee b \qquad \qquad \text{Logical OR of } a \text{ and } b \\ -a \qquad \qquad \text{Logical negation of } a \\ \lceil \cdot \rceil \qquad \qquad \text{Ceiling operation} \\ \end{cases}$	$\sup g(\mathbf{x},\mathbf{y})$	Supremum
rithm In Natual logarithm Re $\{x\}$ Real part of x Im $\{x\}$ Imaginary part of x \emptyset \emptyset Phase (complex argument) \emptyset Remainder, i.e., \emptyset \emptyset Fractional part, i.e., \emptyset mod 1 \emptyset A b Logical AND of \emptyset and \emptyset Logical OR of \emptyset and \emptyset Logical negation of \emptyset \emptyset Ceiling operation	· .	Absolute value of a
InNatual logarithmRe $\{x\}$ Real part of x Im $\{x\}$ Imaginary part of x \angle phase (complex argument) $x \mod y$ Remainder, i.e., $x - y \lfloor x/y \rfloor$ frac (x) Fractional part, i.e., $x \mod 1$ $a \wedge b$ Logical AND of a and b $a \vee b$ Logical OR of a and b $\neg a$ Logical negation of a $\lceil \cdot \rceil$ Ceiling operation	log	Base-10 logarithm or decimal loga-
Re $\{x\}$ Real part of x Im $\{x\}$ Imaginary part of x $ \angle \cdot$ phase (complex argument) $ x \mod y $ Remainder, i.e., $ x - y \lfloor x/y \rfloor $ frac $ x \mod 1 $ Expectational part, i.e., $ x \mod 1 $ and $ x \mod 1 $ Expectational part, i.e., $ x \mod 1 $ Expectational part, i.e., $ x \mod 1 $ and $ x \mod 1 $ Expectational part, i.e., $ x $		rithm
Im $\{x\}$ Imaginary part of x \angle phase (complex argument) $x \mod y$ Remainder, i.e., $x - y \lfloor x/y \rfloor$ frac (x) Fractional part, i.e., $x \mod 1$ $a \wedge b$ Logical AND of a and b $a \vee b$ Logical OR of a and b $\neg a$ Logical negation of a $\lceil \cdot \rceil$ Ceiling operation	ln	Natual logarithm
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\operatorname{Re}\left\{ x\right\}$	Real part of x
$x \mod y$ Remainder, i.e., $x - y \lfloor x/y \rfloor$ $frac(x)$ Fractional part, i.e., $x \mod 1$ $a \wedge b$ Logical AND of a and b $a \vee b$ Logical OR of a and b $\neg a$ Logical negation of a $[\cdot]$ Ceiling operation	$\operatorname{Im}\left\{ x\right\}$	Imaginary part of x
frac (x) Fractional part, i.e., $x \mod 1$ $a \wedge b$ Logical AND of a and b $a \vee b$ Logical OR of a and b Logical negation of a [\cdot] Ceiling operation	∠.	phase (complex argument)
$a \wedge b$ Logical AND of a and b and b Logical OR of a and b Logical negation of a [.]	$x \mod y$	Remainder, i.e., $x - y \lfloor x/y \rfloor$
$a \lor b$ Logical OR of a and b Logical negation of a [·] Ceiling operation	$\operatorname{frac}(x)$	Fractional part, i.e., $x \mod 1$
$\neg a$ Logical negation of a [·] Ceiling operation	$a \wedge b$	Logical AND of a and b
[·] Ceiling operation	$a \lor b$	Logical OR of a and b
	$\neg a$	Logical negation of a
$\lfloor \cdot \rfloor$ Floor operation	[·]	Ceiling operation
	[·]	Floor operation

8.3 Functions

$\mathcal{O}(\cdot), O(\cdot)$	Big-O notation
$\Gamma(\cdot)$	Gamma function

9 Abbreviations

wrt. With respect to st. Subject to iff. If and only if

EVD Eigenvalue decomposition, or eigen-

decomposition

SVD Singular value decomposition CP CANDECOMP/PARAFAC