List of Symbols

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1 Font notation

$a, b, c, \ldots, A, B, C, \ldots$	Scalars
$\mathbf{a}, \mathbf{b}, \mathbf{c}, \ldots$	Vectors
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$	Matrices
$\boldsymbol{\mathcal{A}}, \boldsymbol{\mathcal{B}}, \boldsymbol{\mathcal{C}}, \ldots$	Tensors
$A, B, C, \ldots, \mathcal{A}, \mathcal{B}, \mathcal{C}, \ldots$	Sets

2 Common symbols

$\boldsymbol{\nabla} f, \mathbf{g}$	Gradient vector
$\boldsymbol{\nabla}_x f, \mathbf{g}_x$	Gradient vector with respect x
$\mathbf{g}$ (or $\hat{\mathbf{g}}$ if the gradient vector is $\mathbf{g}$)	Stochastic approximation of the gradient vector
$J(\cdot)$	Cost-function or objective function
$\Lambda(\cdot)$	Likelihood function
$\Lambda_l(\cdot)$	Log-likelihood function
$\mathcal{O}(\cdot), O(\cdot)$	big-O notation
$\boldsymbol{\mu}_x, \mathbf{m}_x$	Mean vector
$\hat{\boldsymbol{\mu}}_x, \hat{\mathbf{m}}_x$	Sample mean vector
$r_x(\tau), R_x(\tau)$	Autocorrelation function of the signal $x(t)$ or $x[n]$
$\hat{r}_x(\tau), \hat{R}_x(\tau)$	Estimated autocorrelation function of the signal $x(t)$ or $x[n]$
$\mathbf{R}_\mathbf{x}$	(Auto)correlation matrix of $\mathbf{x}$
$\hat{\mathbf{R}}_\mathbf{x}$	Sample (auto)correlation matrix
$r_{x,d}(\tau), R_{x,d}(\tau)$	Cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$
$\hat{r}_{x,d}(\tau), \hat{R}_{x,d}(\tau)$	Estimated cross-correlation between $x[n]$ and $d[n]$ or $x(t)$ and $d(t)$
$\mathbf{R}_\mathbf{xy}$	Cross-correlation matrix of $\mathbf{x}$ and $\mathbf{y}$

$\hat{\mathbf{R}}_{\mathbf{xy}}$	Sample cross-correlation matrix of $\mathbf{R}_{\mathbf{xy}}$
$\mathbf{p}_{\mathbf{x}d}$	Cross-correlation vector
$\rho_{x,y}$	Pearson correlation coefficient between x and y
$\hat{\rho}_{x,y}$	Estimated Pearson correlation coefficient between x and y
$c_x(\tau), C_x(\tau)$	Autocovariance function of the signal $x(t)$ or $x[n]$
$\hat{c}_x(\tau), \hat{C}_x(\tau)$	Estimated autocovariance function of the signal $x(t)$ or $x[n]$
$\mathbf{C}_{\mathbf{x}}, \mathbf{K}_{\mathbf{x}}, \mathbf{\Sigma}_{\mathbf{x}}$	(Auto)covariance matrix of $\mathbf{x}$
$\hat{\mathbf{C}}_{\mathbf{x}}, \hat{\mathbf{K}}_{\mathbf{x}}, \hat{\mathbf{\Sigma}}_{\mathbf{x}}$	Sample (auto)covariance matrix
$c_{xy}(\tau), C_{xy}(\tau)$	Cross-covariance function of the signal $x(t)$ or $x[n]$
$\hat{c}_{xy}(\tau), \hat{C}_{xy}(\tau)$	Estimated cross-covariance function of the signal $x(t)$ or $x[n]$
$\mathbf{C}_{\mathbf{xy}}, \mathbf{K}_{\mathbf{xy}}, \mathbf{\Sigma}_{\mathbf{xy}}$	Cross-covariance matrix of $\mathbf{x}$
$\hat{\mathbf{C}}_{\mathbf{xy}}, \hat{\mathbf{K}}_{\mathbf{xy}}, \hat{\mathbf{\Sigma}}_{\mathbf{xy}}$	Sample cross-covariance matrix
$\delta(t)$	Delta function
$\delta[n]$	Kronecker function
$h(t), h[n]$	Impulse response (continuous and discrete time)
$\mathbf{C}$	Cofactor matrix
$\mathbf{W}, \mathbf{D}$	Diagonal matrix
$\mathbf{w}, \boldsymbol{\theta}$	Parameters, coefficients, or weights vector
$\mathbf{w}_o, \mathbf{w}^\star, \boldsymbol{\theta}_o, \boldsymbol{\theta}^\star$	Optimum value of the parameters, coefficients, or weights vector
$\mathbf{W}$	Matrix of the weights
$\mathbf{P}$	Projection matrix; Permutation matrix
$\mathbf{\Lambda}$	Eigenvalue matrix
$\mathbf{\Sigma}$	Singular value matrix
$\mathbf{U}$	Upper matrix; Left singular vectors
$\mathbf{L}$	Lower matrix
$\mathbf{V}$	Right singular vectors
$\mathbf{J}$	Jordan matrix; Jacobian matrix
$\mathbf{S}$	Symmetric matrix
$\mathbf{Q}$	Orthogonal matrix
$\mathbf{I}_N$	$N \times N$-dimensional identity matrix
$\mathbf{0}_{M\times N}$	$M \times N$-dimensional null matrix
$\mathbf{0}_N$	N-dimensional null vector
$\mathbf{0}$	Null matrix, vector, or tensor (dimensionality understood by context)

$\mathbf{1}_{M \times N}$	$M \times N$-dimensional ones matrix
$\mathbf{1}_N$	N-dimensional ones vector
$\mathbf{1}$	Ones matrix, vector, or tensor (dimensionality understood by context)
j	$\sqrt{-1}$

3 Linear Algebra operations

$\mathbf{A}^{-1}$	Inverse matrix
$\mathbf{A}^{+}, \mathbf{A}^{\dagger}$	Moore-Penrose pseudoinverse
$\mathbf{A}^{\top}$	Transpose
$\mathbf{A}^{*}$	Conjugate
$\mathbf{A}^{\mathsf{H}}$	Hermitian
$\|\mathbf{A}\|_{\mathrm{F}}$	Frobenius norm
$\|\mathbf{A}\|$	Matrix norm
$\|\mathbf{a}\|$	l_1 norm, 1-norm, or Manhatan norm
$\|\mathbf{a}\|, \|\mathbf{a}\|_2$	l_2 norm, 2-norm, or Euclidean norm
$\|\mathbf{a}\|_p$	l_p norm, p-norm, or Minkowski norm
$\|\mathbf{a}\|_\infty$	l_∞ norm, ∞-norm, or Chebyshev norm
$\lvert\mathbf{A}\rvert, \det(\mathbf{A})$	Determinant
$\operatorname{diag}(\mathbf{a}), \operatorname{diag}(\mathbf{A})$	Diagonalization: a square, diagonal matrix with entries given by the vector **a** or the elements in the diagonal of **A**
$\operatorname{vec}(\mathbf{A})$	Vectorization: stacks the columns of the matrix **A** into a long column vector
$\operatorname{vec}_{\mathrm{d}}(\mathbf{A})$	Extracts the diagonal elements of a square matrix and returns them in a column vector
$\operatorname{vec}_{\mathrm{l}}(\mathbf{A})$	Extracts the elements strictly below the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\operatorname{vec}_{\mathrm{u}}(\mathbf{A})$	Extracts the elements strictly above the main diagonal of a square matrix in a column-wise manner and returns them into a column vector
$\operatorname{vec}_{\mathrm{b}}(\mathbf{A})$	Block vectorization operator: stacks square block matrices of the input into a long block column matrix
$\operatorname{unvec}(\mathbf{A})$	Reshapes a column vector into a matrix

$[\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots]$	CANDECOMP/PARAFAC (CP) decomposition of the tensor $\boldsymbol{\mathcal{X}}$ from the outer product of column vectors of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}, \dots$ (TODO: change the square brackets to the double one by using the commented commands)
$[\boldsymbol{\lambda}; \mathbf{A}, \mathbf{B}, \mathbf{C}, \dots]$	Normalized CANDECOMP/PARAFAC (CP) decomposition of the tensor $\boldsymbol{\mathcal{X}}$ from the outer product of column vectors of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}, \dots$ (TODO: change the square brackets to the double one by using the commented commands)
$\mathrm{N}(\mathbf{A}), \mathrm{nullspace}(\mathbf{A}), \mathrm{kernel}(\mathbf{A})$	Nullspace (or kernel)
$\mathrm{C}(\mathbf{A}), \mathrm{columnspace}(\mathbf{A}), \mathrm{range}(\mathbf{A})$	Columnspace (or range), i.e., the space $\mathrm{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$, where $\mathbf{a}_i$ is the ith column vector of the matrix $\mathbf{A}$
$\mathrm{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$	Space spanned by the argument vectors
$\mathrm{span}(\mathbf{A})$	Space spanned by the column vectors of $\mathbf{A}$
$\mathrm{rank}(\mathbf{A})$	Rank, that is, $\dim(\mathrm{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)) = \dim(\mathrm{C}(\mathbf{A}))$, where $\mathbf{a}_i$ is the ith column vector of the matrix $\mathbf{A}$
$\mathrm{nullity}(\mathbf{A})$	Nullity of $\mathbf{A}$, i.e., $\dim(\mathrm{N}(\mathbf{A}))$
$\mathrm{tr}(\mathbf{A})$	trace
$\mathbf{a} \perp \mathbf{b}$	$\mathbf{a}$ is orthogonal to $\mathbf{b}$
$\mathbf{a} \not\perp \mathbf{b}$	$\mathbf{a}$ is not orthogonal to $\mathbf{b}$
$\langle \mathbf{a}, \mathbf{b} \rangle$	Inner product, i.e., $\mathbf{a}^\top \mathbf{b}$
$\mathbf{a} \circ \mathbf{b}$	Outer product, i.e., $\mathbf{a}\mathbf{b}^\top$
$\otimes$	Kronecker product
$\odot$	Hadamard (elementwise) product
$\diamond$	Khatri-Rao product
$\otimes$	Kronecker Product
$\times_n$	n-mode product
$\mathbf{X}_{(n)}$	n-mode matricization of the tensor $\boldsymbol{\mathcal{X}}$
$\boldsymbol{\mathcal{X}} \leq 0$	Nonnegative tensor
$\mathbf{a} \preceq_K \mathbf{b}$	Generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the conic subset K in the space $\mathbb{R}^n$
$\mathbf{a} \prec_K \mathbf{b}$	Strict generalized inequality meaning that $\mathbf{b} - \mathbf{a}$ belongs to the interior of the conic subset K in the space $\mathbb{R}^n$

$\mathbf{a} \preceq \mathbf{b}$	Generalized inequality meaning that $\mathbf{b}-\mathbf{a}$ belongs to the nonnegative orthant conic subset, $\mathbb{R}^n_+$, in the space $\mathbb{R}^n$
$\mathbf{a} \prec \mathbf{b}$	Strict generalized inequality meaning that $\mathbf{b}-\mathbf{a}$ belongs to the positive orthant conic subset, $\mathbb{R}^n_{++}$, in the space $\mathbb{R}^n$
$\mathbf{A} \preceq_K \mathbf{B}$	Generalized inequality meaning that $\mathbf{B}-\mathbf{A}$ belongs to the conic subset K in the space $\mathcal{S}^n$
$\mathbf{A} \prec_K \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B}-\mathbf{A}$ belongs to the interior of the conic subset K in the space $\mathcal{S}^n$
$\mathbf{A} \preceq \mathbf{B}$	Generalized inequality meaning that $\mathbf{B}-\mathbf{A}$ belongs to the positive semidefinite conic subset, $\mathcal{S}^n_+$, in the space $\mathcal{S}^n$
$\mathbf{A} \prec \mathbf{B}$	Strict generalized inequality meaning that $\mathbf{B}-\mathbf{A}$ belongs to the positive orthant conic subset, $\mathcal{S}^n_{++}$, in the space $\mathcal{S}^n$

3.1 Indexing

$x_{i_1,i_2,\ldots,i_N}$	Element in the position $(i_1, i_2, \ldots, i_N)$ of the tensor $\boldsymbol{\mathcal{X}}$
$\boldsymbol{\mathcal{X}}^{(n)}$	nth tensor in a nontemporal sequence
$[\boldsymbol{\mathcal{X}}]_{i_1,i_2,\ldots,i_N}$	Element $x_{i_1,i_2,\ldots,i_N}$
$\mathbf{x}_j, \mathbf{x}_{:j}$	jth column of the matrix X
$\mathbf{x}_{j:}$	jth row of the matrix X
$\mathbf{x}_{i_1,\ldots,i_{j-1},:,i_{j+1},\ldots,i_N}$	Mode-j fiber of the tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{x}_{:,i_2,i_3}$	Column fiber (mode-1 fiber) of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{x}_{i_1,:,i_3}$	Row fiber (mode-2 fiber) of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{x}_{i_1,i_2,:}$	Tube fiber (mode-3 fiber) of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{i_1,:,:}$	Horizontal slice of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{:,i_2,:}$	Lateral slices slice of the thrid-order tensor $\boldsymbol{\mathcal{X}}$
$\mathbf{X}_{i_3}, \mathbf{X}_{:,:,i_3}$	Frontal slices slice of the thrid-order tensor $\boldsymbol{\mathcal{X}}$

4 Sets

$A \setminus B$	Set subtraction, i.e., the set containing the elements of A that are not in B
$A \cup B$	Set of union
$A \cap B$	Set of intersection
$A \times B$	Cartesian product
$A \oplus B$	Direct sum, e.g., $\mathrm{C}(A^\top) \oplus \mathrm{C}(A^\top)^\perp = \mathbb{R}^n$
$A^\perp$	Orthogonal complement
A^c	Complement
$\#A$	Cardinality
$a \in A$	a is element of A
$a \notin A$	a is not element of A
$\{1, 2, \ldots, n\}$	Discrete set containing the integer elements $1, 2, \ldots, n$
$\mathbb{R}$	Set of real numbers
$\mathbb{C}$	Set of complex numbers
$\mathbb{Z}$	Set of integer number
$\mathbb{B} = \{0, 1\}$	Boolean set
$\varnothing$	Empty set
$\mathbb{N}$	Set of natural numbers
$\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$	???
$\mathbb{K}^{I_1 \times I_2 \times \cdots \times I_N}$	$I_1 \times I_2 \times \cdots \times I_N$-dimensional real (or complex) space
$\mathbb{K}_+$	Nonnegative real (or complex) space
$\mathbb{K}_{++}$	Positive real (or complex) space, i.e., $\mathbb{K}_{++} = \mathbb{K}_+ \setminus \{\mathbf{0}\}$
$\mathbb{S}^n, \mathcal{S}^n$	Conic set of the symmetric matrices in $\mathbb{R}^{n\times n}$
$\mathbb{S}^n_+, \mathcal{S}^n_+$	Conic set of the symmetric positive semidefinite matrices in $\mathbb{R}^{n\times n}$
$\mathbb{S}^n_{++}, \mathcal{S}^n_{++}$	Conic set of the symmetric positive definite matrices in $\mathbb{R}^{n\times n}$, i.e., $\mathbb{S}^n_{++} = \mathbb{S}^n_+ \setminus \{\mathbf{0}\}$
$\mathbb{H}^n$	Set of all hermitian matrices in $\mathbb{C}^{n\times n}$
$[a, b]$	Closed interval of a real set from a to b
(a, b)	Opened interval of a real set from a to b
$[a, b), (a, b]$	Half-opened intervals of a real set from a to b

5 Signals and functions operations and indexing

$f : A \rightarrow B$	A function f whose domain is A and codomain is B
$f^{(n)}$	nth derivative of the function f
$f \circ g$	Composition of the functions f and g
$\inf_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Infimum
$\sup_{\mathbf{y} \in \mathcal{A}} g(\mathbf{x}, \mathbf{y})$	Supremum
$*$	Convolution
$x(t)$	Continuous-time t
$x[n], x[k], x[m], x[i], \ldots$	Discrete-time $n, k, m, i, \ldots$
$x(n), x(k), x(m), x(i), \ldots$	Discrete-time $n, k, m, i, \ldots$ (it should be used only if there are no continuous-time signals in the context to avoid ambiguity)
$\tilde{x}(t)$ or $\tilde{x}[n]$	Estimate of $x(t)$ or $x[n]$; the Hilbert transform of $x(t)$ or $x[n]$
$x_I(t)$ or $x_I[n]$	Real or in-phase part of $x(t)$ or $x[n]$
$x_Q(t)$ or $x_Q[n]$	Imaginary or quadrature part of $x(t)$ or $x[n]$
$X(s)$	Laplace transform of $x(t)$
$X(f)$	Fourier transform (in linear frequency, Hz) of $x(t)$
$X(j\omega)$	Fourier transform (in angular frequency, rad/sec) of $x(t)$
$S_x(f)$	Power spectral density of $x(t)$ in linear frequency
$S_x(j\omega)$	Power spectral density of $x(t)$ in angular frequency
$X(z)$	Z-transform of $x[n]$

6 Probability and stochastic processes

$E[\cdot]$	Statistical expectation
$E_u[\cdot]$	Statistical expectation with respect to u
$\mathrm{var}(x)$	Variance of the random variable x
$\mathrm{erfc}(\cdot)$	Complementary error function
$P(A)$	Probability of the event or set A
$p(\cdot)$	Probability density function

$p(x \mid A)$	Conditional probability density function
$a \sim P$	Random variable a with distribution P
$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution of a random variable with mean μ and variance σ^2
$\mathcal{CN}(\mu, \sigma^2)$	Complex Gaussian distribution of a random variable with mean μ and variance σ^2
$\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	Gaussian distribution of a vector random variable with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$
$\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	Complex Gaussian distribution of a vector random variable with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$
$\mathcal{U}(a, b)$	Uniform distribution from a to b

7 General notations

$a \wedge b$	Logical AND of a and b
$a \vee b$	Logical OR of a and b
$\neg a$	Logical negation of a
$\exists$	There exists
$\nexists$	There does not exist
$\exists!$	There exist an unique
$\forall$	For all
$\mid$	Such that
$\therefore$	Therefore
$\iff$	Logical equivalence
$\triangleq$	Equal by definition
$\neq$	Not equal
∞	Infinity
$\lvert a \rvert$	Absolute value of a
$\log$	Base-10 logarithm or decimal logarithm
$\ln$	Natual logarithm
$\operatorname{Re}\{x\}$	Real part of x
$\operatorname{Im}\{x\}$	Imaginary part of x
$\lceil \cdot \rceil$	Ceiling operation
$\lfloor \cdot \rfloor$	Floor operation
$\angle \cdot$	phase (complex argument)
$x \bmod y$	Remainder, i.e., $x - y\lfloor x/y \rfloor$
$\operatorname{frac}(x)$	Fractional part, i.e., $x \bmod 1$

8 Abbreviations

wrt.	With respect to
st.	Subject to
iff.	If and only if
EVD	Eigenvalue decomposition, or eigen-decomposition
SVD	Singular value decomposition
CP	CANDECOMP/PARAFAC