

Functions			
Function	Convex?	Proof	
$\mathbf{y} = \max(f_1, f_2)$	Yes, if $f_1$ and $f_2$ are convex functions		
$\mathbf{y} = \min(f_1, f_2)$	Not always		
$C = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{Ax} = \mathbf{b}, \mathbf{A} \in \mathbb{R}^{m \times n}, \mathbf{b} \in \mathbb{R}^m\}$	It is an affine set (all affine set is a convex set)		
$y = \mathbf{c}^\top \mathbf{x}$ (linear function)	Yes (but not strictly convex)		
$y = \ \mathbf{x}\ _p$ (p-norm)	Yes (for any $p \in \mathbb{N}_+$ )	$\ \theta \mathbf{x} + (1 - \theta)\mathbf{y}\  \leq \theta \ \mathbf{x}\  + (1 - \theta) \ \mathbf{y}\ $ (triangular inequality)	
$f(g(\mathbf{x}))$	Yes, if $f, g$ are convex		
Function	Domain	Codomain	Comments
System of linear equation: $\mathbf{b} = f(\mathbf{x}) = \mathbf{Ax}$	$D = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{Ax} = \mathbf{b} \in C, \mathbf{A} \in \mathbb{R}^{m \times n}\}$	$C = \{\mathbf{b} \in \mathbb{R}^m \mid \mathbf{b} = \mathbf{Ax}, \forall \mathbf{x} \in D\}$	If $D$ is an affine set, so $C$ is also affine set which, in turn, is a convex set.

Sets		
Set	Convex?	Commens
Convex hull: $\text{conv } C = \left\{ \sum_{i=1}^k \theta_i \mathbf{x}_i \mid \mathbf{x}_i \in C, 0 \leq \theta_i \leq 1, i = 1, \dots, k, \sum_{i=1}^k \theta_i = 1 \right\}$	Yes.	<ul style="list-style-type: none"> <li><math>A</math> will be the smallest convex set that contains <math>C</math>.</li> <li><math>\text{conv } C</math> will be a finite set as long as <math>C</math> is also finite.</li> </ul>
Affine hull: $\text{aff } C = \left\{ \sum_{i=1}^k \theta_i \mathbf{x}_i \mid \mathbf{x}_i \in C, i = 1, \dots, k, \sum_{i=1}^k \theta_i = 1 \right\}$	Yes.	<ul style="list-style-type: none"> <li><math>A</math> will be the smallest affine set that contains <math>C</math>.</li> <li>Different from the convex set, <math>\theta_i</math> is not restricted between 0 and 1</li> <li><math>\text{aff } C</math> will always be an infinite set. If <math>\text{aff } C</math> contains the origin, it is also a subspace.</li> </ul>
Conic hull: $A = \left\{ \sum_{i=1}^k \theta_i \mathbf{x}_i \mid \mathbf{x}_i \in C, \theta_i > 0, i = 1, \dots, k \right\}$	Yes.	<ul style="list-style-type: none"> <li><math>A</math> will be the smallest convex conic that contains <math>C</math>.</li> <li>Different from the convex and affine sets, <math>\theta_i</math> does not need to sum up 1.</li> </ul>
Euclidean ball: $B(\mathbf{x}_c, r) = \{\mathbf{x} \mid \ \mathbf{x} - \mathbf{x}_c\ _2 \leq r\}$ $B(\mathbf{x}_c, r) = \left\{ \mathbf{x} \mid (\mathbf{x} - \mathbf{x}_c)^\top (\mathbf{x} - \mathbf{x}_c) \leq r^2 \right\}$ $B(\mathbf{x}_c, r) = \{\mathbf{x}_c + r \ \mathbf{u}\  \mid \ \mathbf{u}\  \leq 1\}$	Yes	<ul style="list-style-type: none"> <li><math>B(\mathbf{x}_c, r)</math> as long as <math>r &lt; \infty</math>.</li> <li><math>\mathbf{x}_c</math> is the center of the ball.</li> <li><math>r</math> is its radius.</li> </ul>
Ellipsoid: $\mathcal{E} = \{\mathbf{x} \mid (\mathbf{x} - \mathbf{x}_c)^\top \mathbf{P}^{-1} (\mathbf{x} - \mathbf{x}_c) \leq 1\}$ $\mathcal{E} = \{\mathbf{x}_c + \mathbf{A}\mathbf{u} \mid \ \mathbf{u}\  \leq 1\}$	Yes	<ul style="list-style-type: none"> <li><math>\mathcal{E}</math> is a finite set as long as <math>\mathbf{P}</math> is a finite matrix.</li> <li><math>\mathbf{P}</math> is symmetric and positive definite, that is, <math>\mathbf{P} = \mathbf{P}^\top \succ 0</math>.</li> <li><math>\mathbf{x}_c</math> is the center of the ellipsoid.</li> <li>The lengths of the semi-axes are given by <math>\sqrt{\lambda_i}</math>.</li> <li><math>\mathbf{A} = \mathbf{P}^{1/2}</math>.</li> <li><math>\mathbf{A}</math> is invertible. When it is not, we say that <math>\mathcal{E}</math> is a degenerated ellipsoid (degenerated ellipsoids are also convex).</li> </ul>
$C = A \cup B$	Not always.	
$C = A \cap B$	Yes, if $A$ and $B$ are convex sets.	