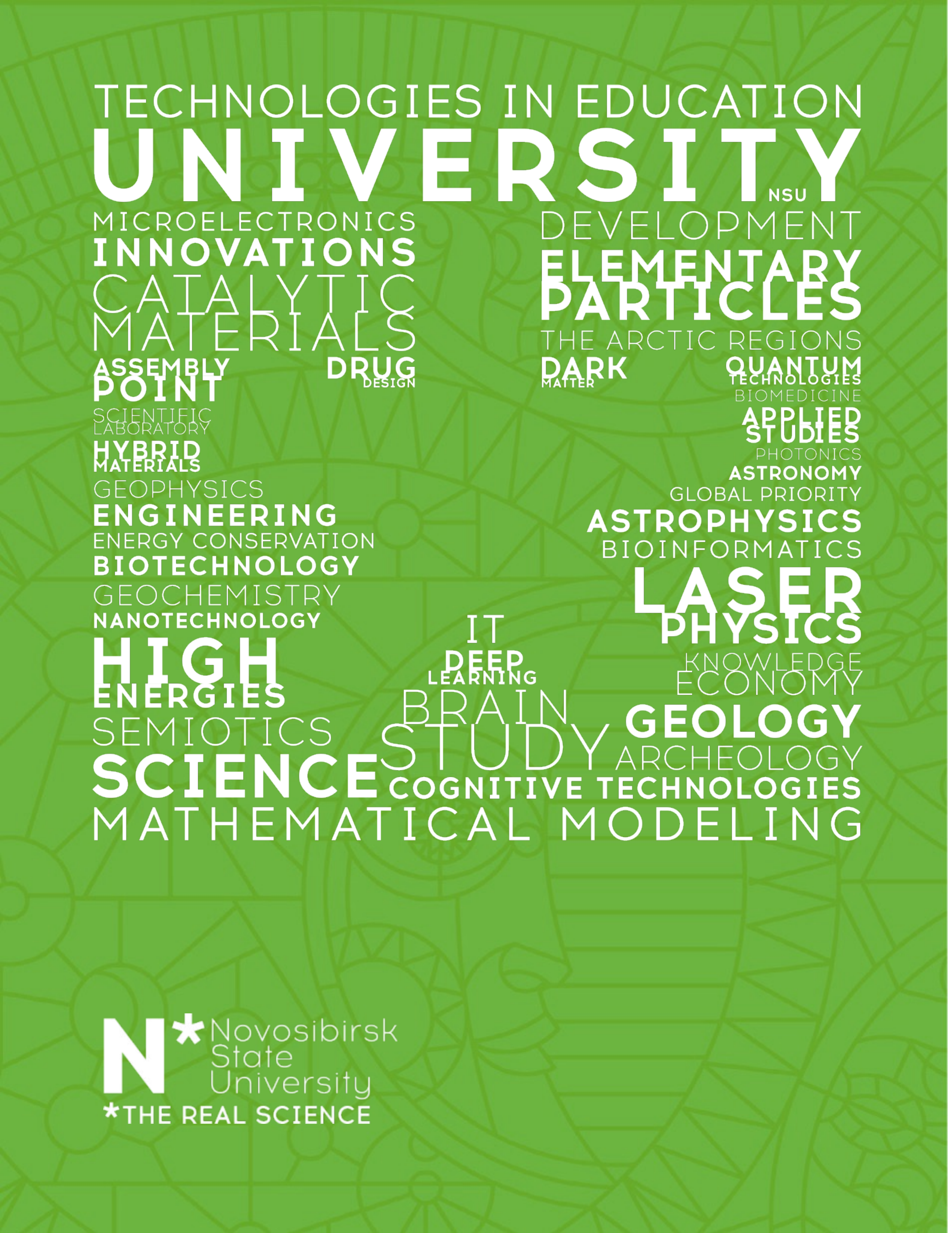

Non-linear Regression

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Department of Mathematics and Mechanics

Overview

- Non-Linear Regression
- Bias vs Variance
- Ridge Regression
- Lasso Regression
- Data Visualization

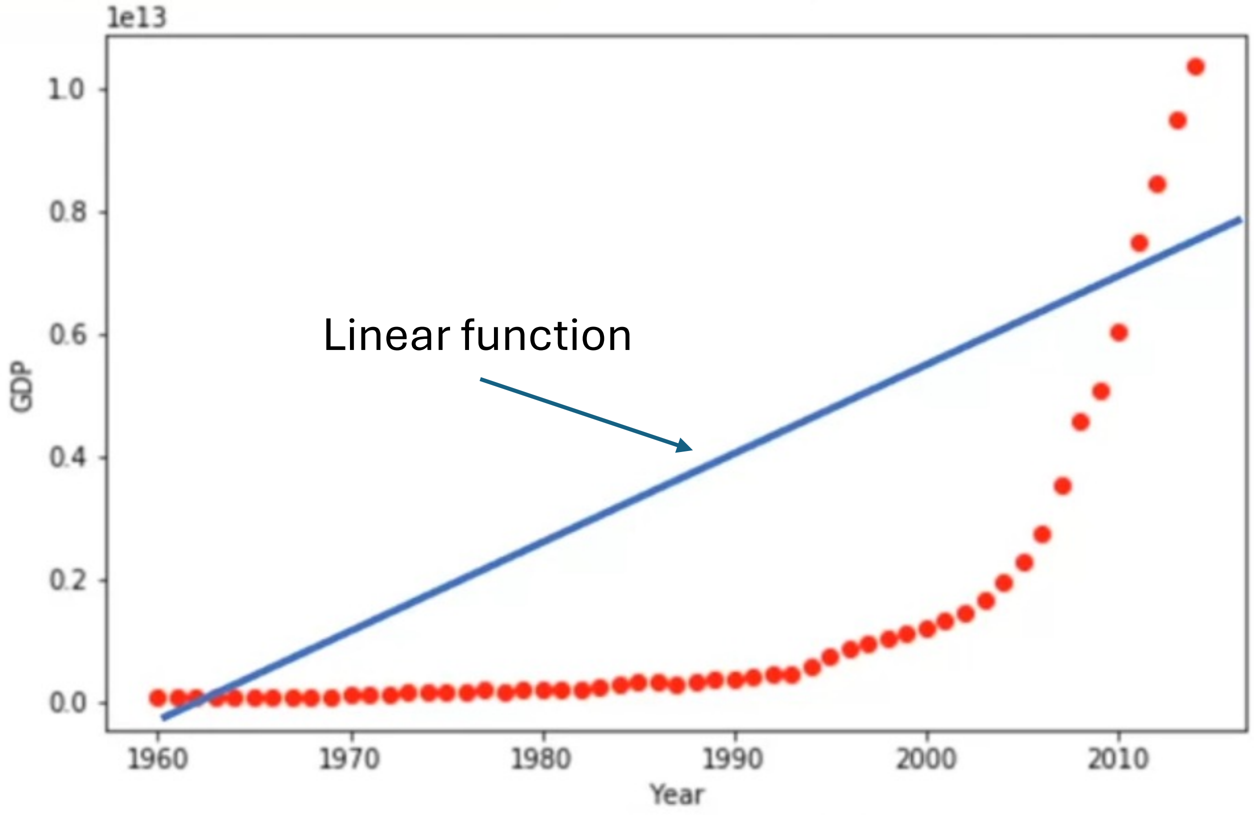
Non-Linear Regression

Non-Linear Regression

- **Non-linear regression** is a type of regression analysis in machine learning where the relationship between the independent variable(s) and the dependent variable cannot be modeled by a straight line (linear equation). Instead, it involves a **non-linear function** of the parameters to model more complex relationships.

Non-Linear Regression

	Year	Value
0	1960	5.918412e+10
1	1961	4.955705e+10
2	1962	4.668518e+10
3	1963	5.009730e+10
4	1964	5.906225e+10
5	1965	6.970915e+10
6	1966	7.587943e+10
7	1967	7.205703e+10
8	1968	6.999350e+10
9	1969	7.871882e+10
...	.....	....



Non-Linear Regression

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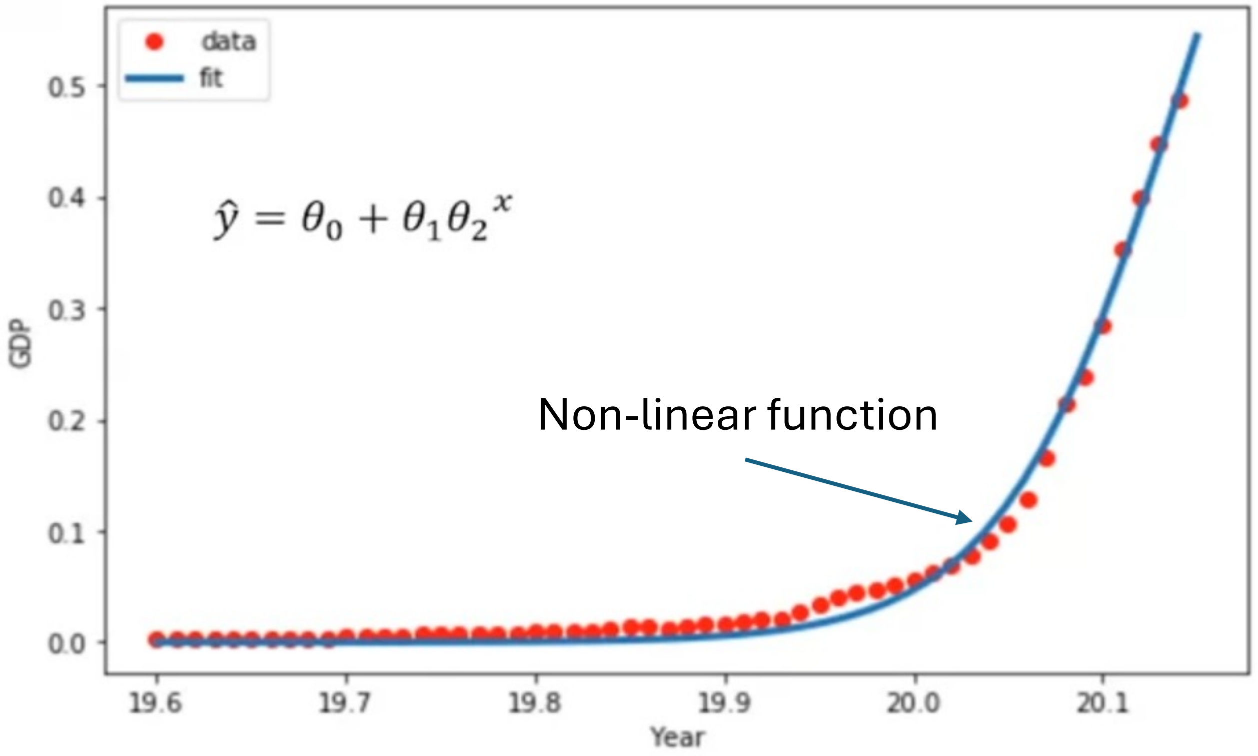
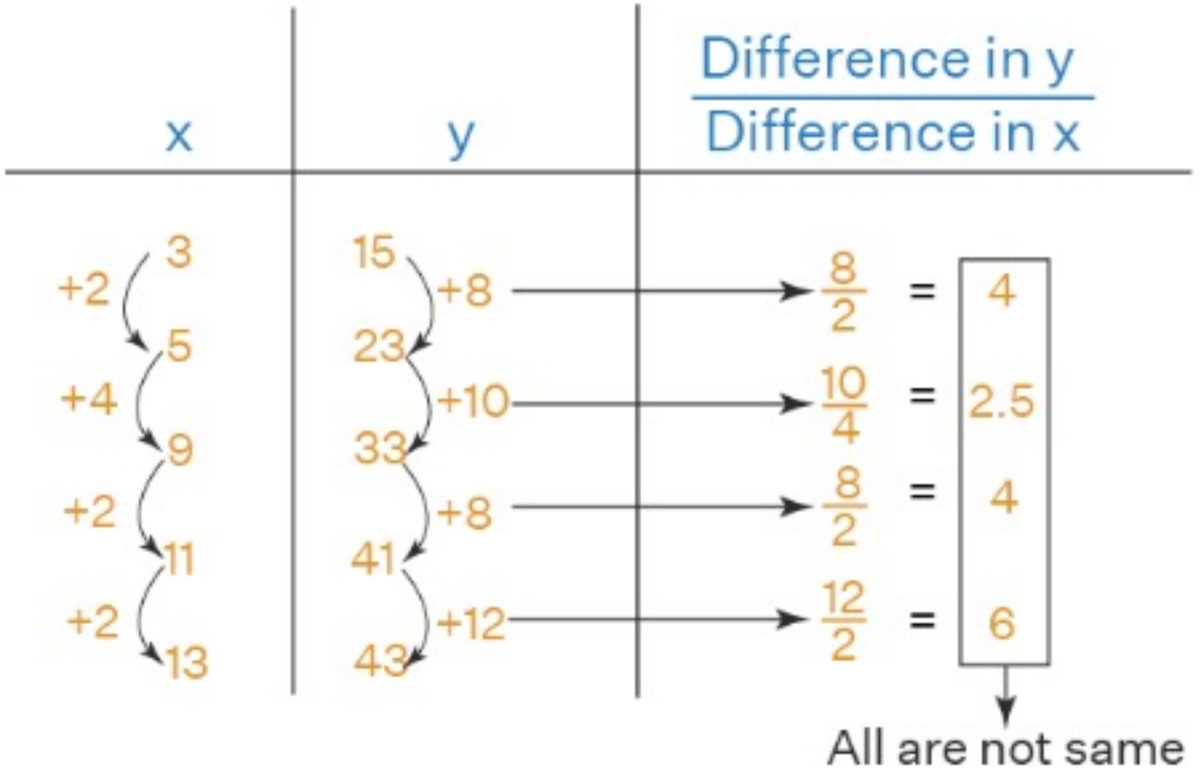
...	.....	....



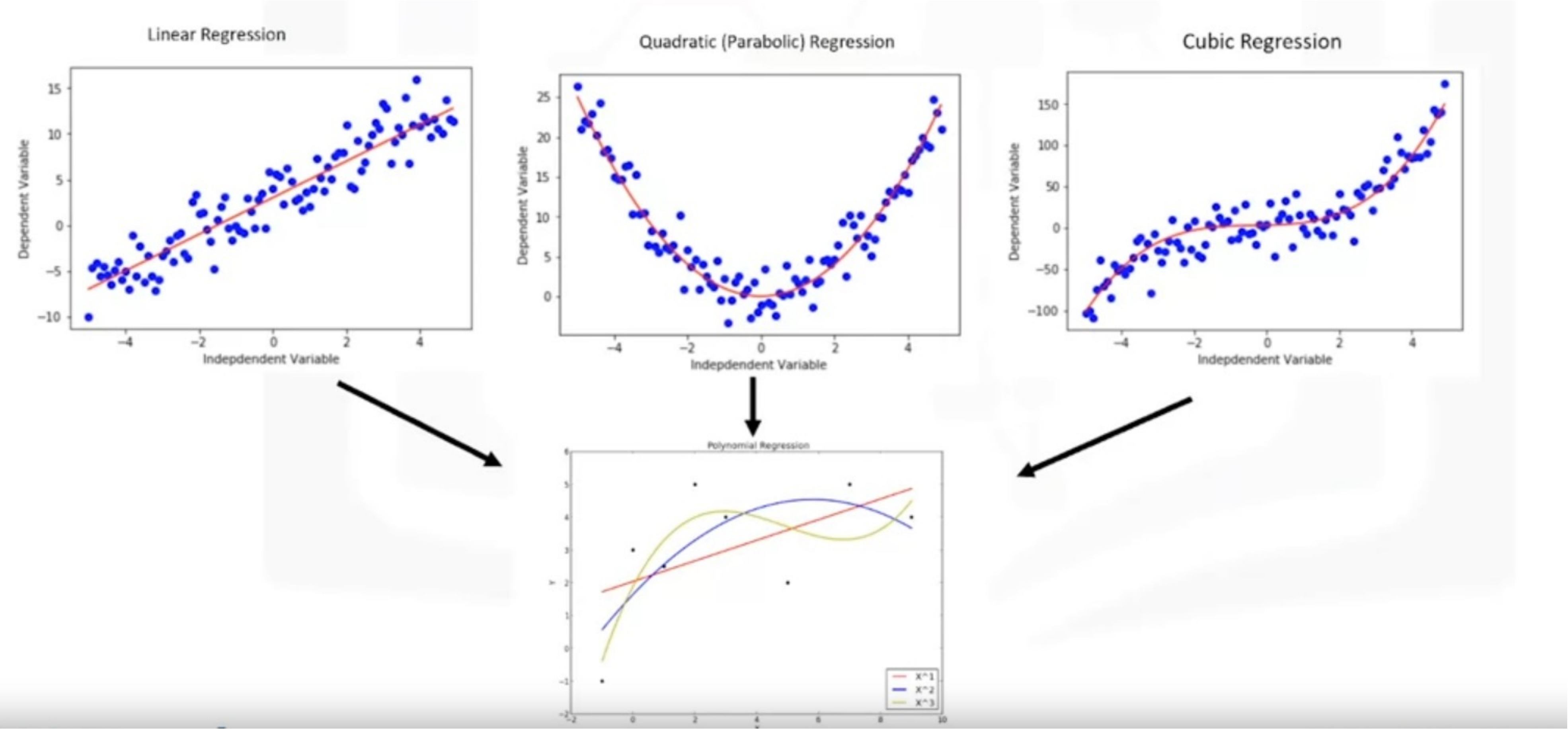
How to identify non-linear relationship



Visualize

Source: https://www.cuemath.com/calculus/nonlinear-functions/

Polinomial Regression



Limitations of Polynomial Regression

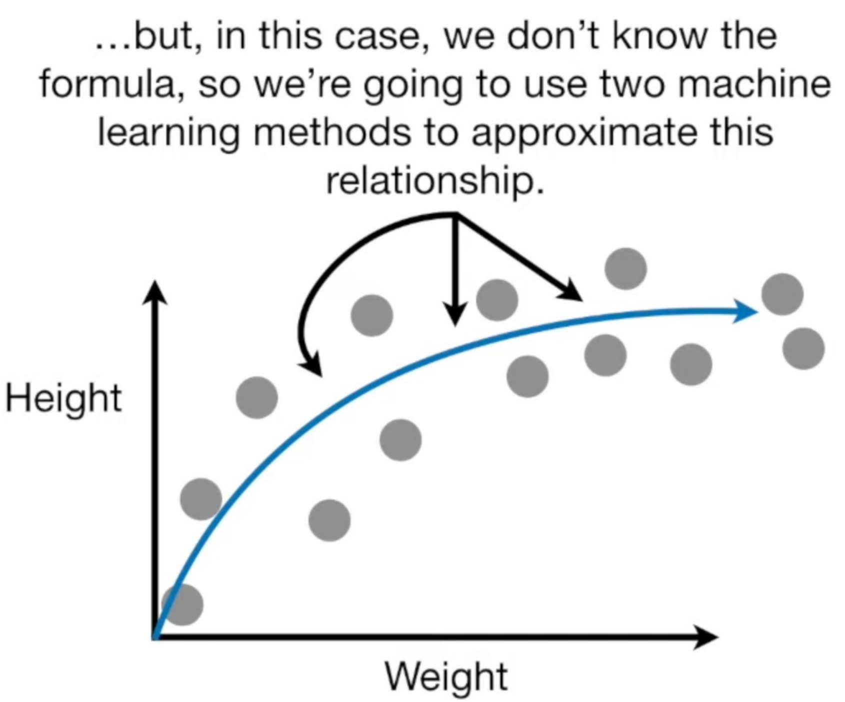
- **Overfitting**:
- Increasing the degree of a polynomial improves the fit on training data but worsens performance on new data.
- The model becomes too specific to the training set and captures noise.
- **Unpredictable Behavior**:
- High-degree polynomials diverge rapidly outside the training data range:
  - **Odd degrees**: Shoot off in opposite directions.
  - **Even degrees**: Shoot off in the same direction.
- **When to Use Polynomial Regression**:
- Only when the underlying relationship is **known to follow a polynomial pattern** (e.g., natural laws like projectile motion).

Linear vs Non-Linear Regression

	Linear Regression	Non-Linear Regression
Model Form	Linear equation	Non-linear equation
Complexity	Simple, easy to interpret	Flexible, hard to interpret
Parameters Estimation	Closed-form solution	Iterative optimization
Computation	Low	High

Bias vs Variance

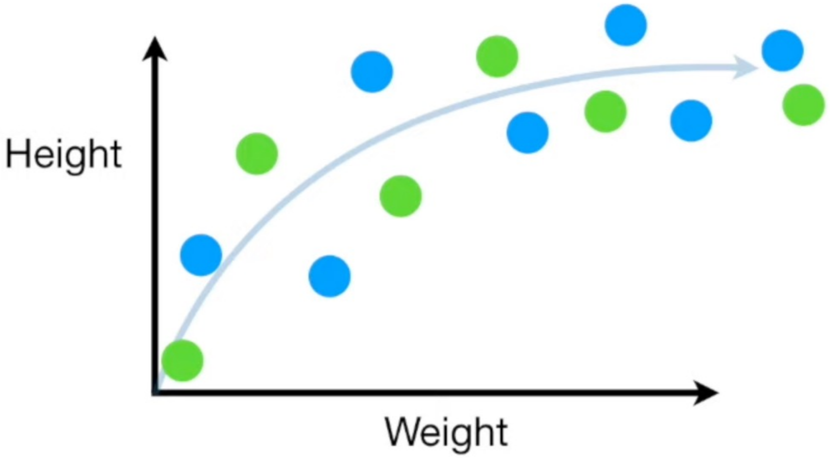
Bias vs Variance



Source: https://www.youtube.com/watch?v=EuBBz3bI-aA

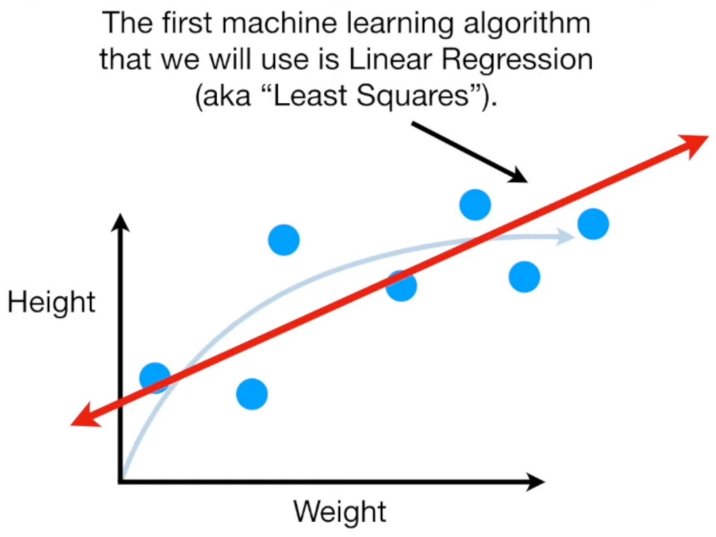
Bias vs Variance

The first thing we do is split the data into two sets, one for training the machine learning algorithms and one for testing them.

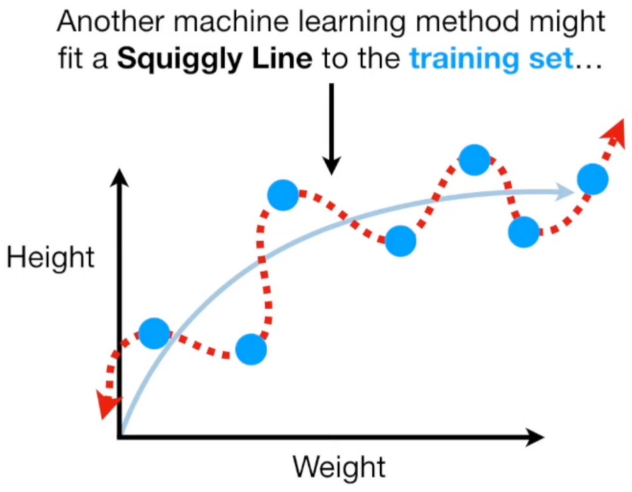


Source: https://www.youtube.com/watch?v=EuBBz3bI-aA

Bias vs Variance



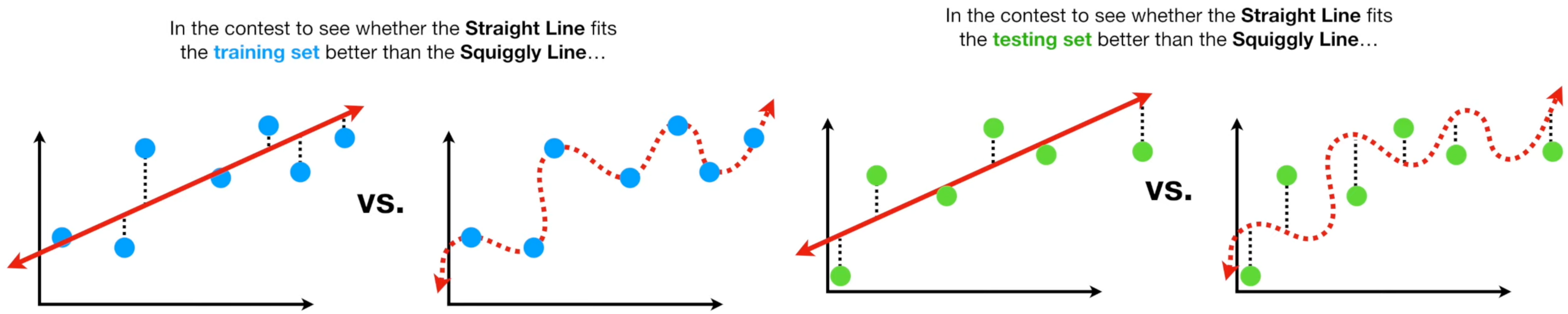
High Bias



High Variance

Source: https://www.youtube.com/watch?v=EuBBz3bI-aA

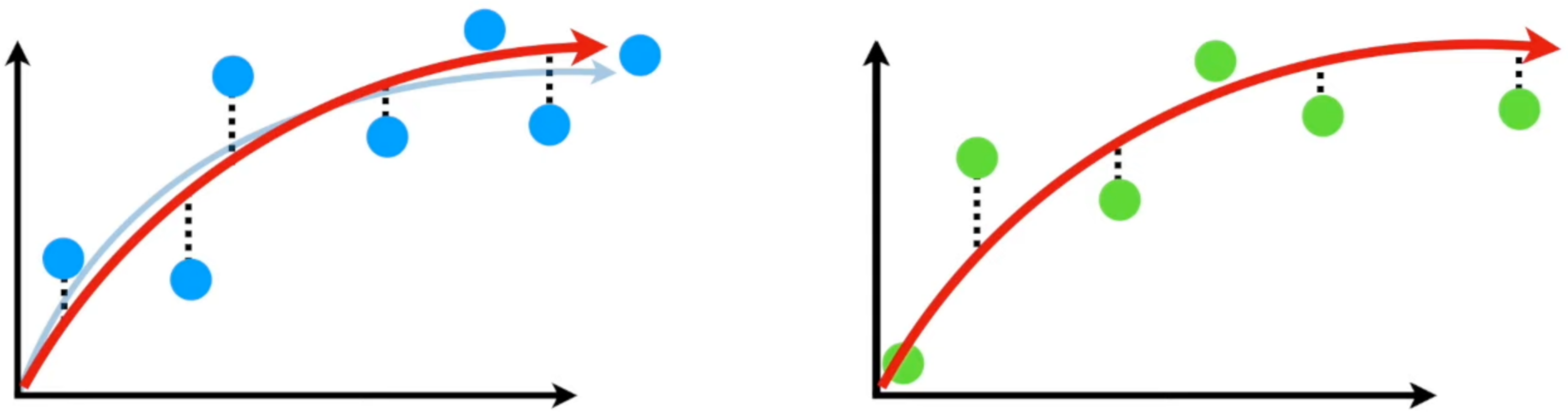
Bias vs Variance



Source: https://www.youtube.com/watch?v=EuBBz3bI-aA

Bias vs Variance

Low Bias Low Variance



Source: https://www.youtube.com/watch?v=EuBBz3bI-aA

Bias vs Variance

- Regularization -> add penalties to the loss function
- Bagging -> reduce variance
- Boosting -> reduce bias

Source: https://www.youtube.com/watch?v=EuBBz3bI-aA

Ridge Regression

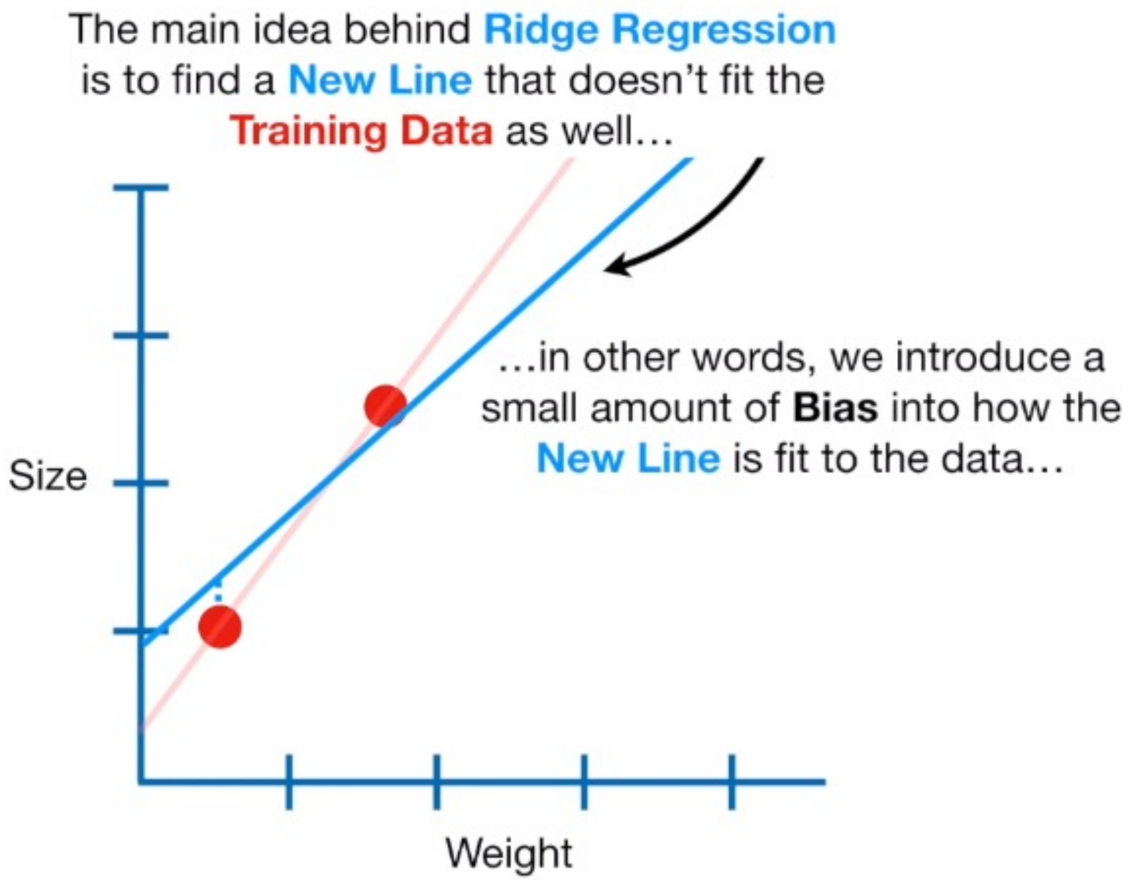
Regularization

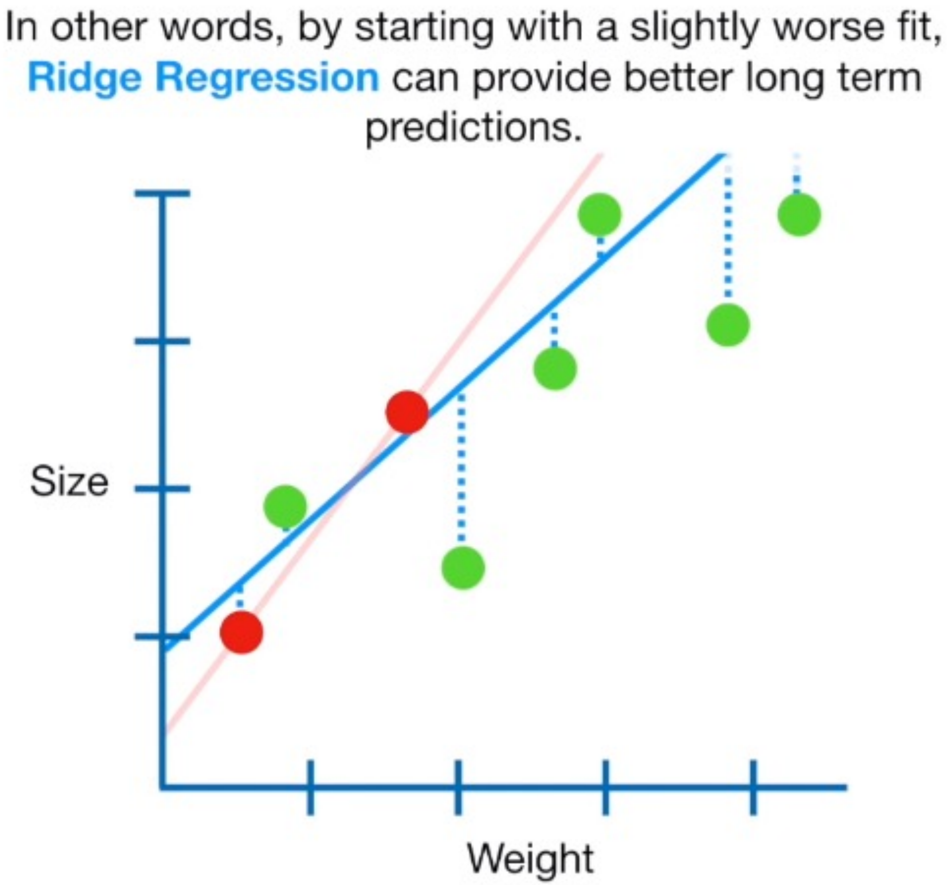
- Regularization is a technique used in machine learning and statistics to improve the performance and generalization of a model by adding a penalty to the loss function. This penalty discourages the model from becoming too complex or overly reliant on specific features, which can lead to **overfitting**.

Ridge Regression

- **Ridge regression**, also known as **L2 Regularization**, is a linear regression technique that introduces a penalty term to the loss function. It is used to address the problem of **multicollinearity** (high correlation among predictors) and to prevent **overfitting** in models with a large number of features or small datasets.
- Formula: https://github.com/luumsk/NSU_ML/blob/main/Lectures/Day4_RidgeRegression.pdf

Ridge Regression





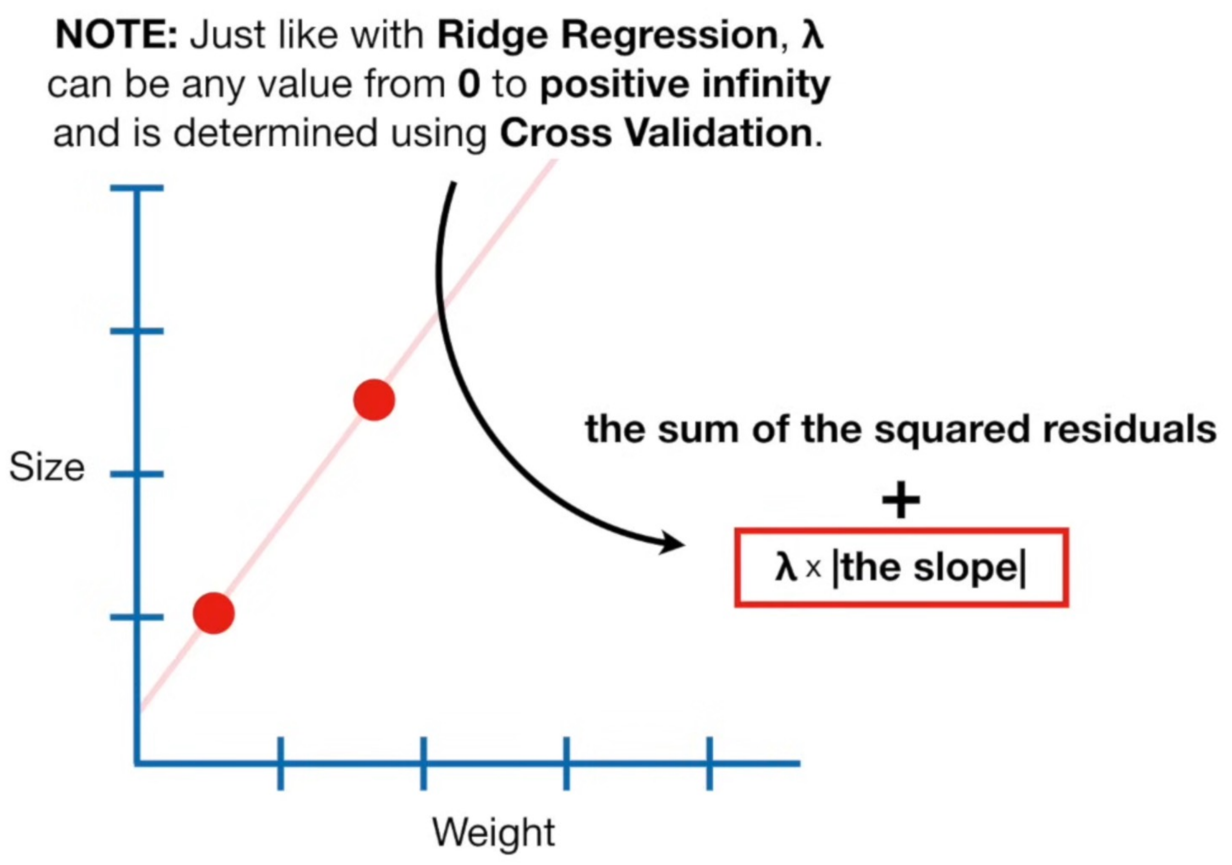
When to use Ridge Regression

- You have **many correlated predictors**.
- You want to improve **generalization** without removing features.
- You are dealing with **small datasets** or **high-dimensional data**.

Lasso Regression

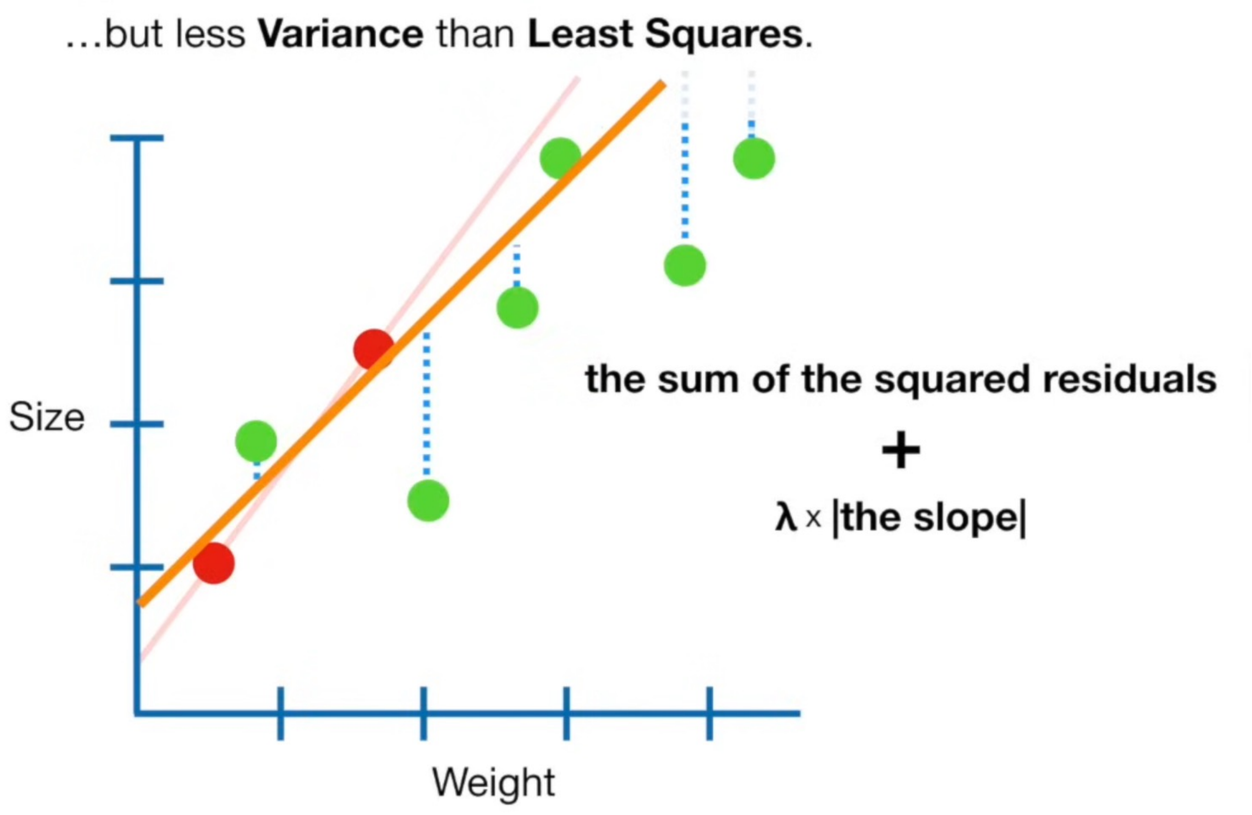
Lasso Regression

Least Absolute Shrinkage
and Selection Operator



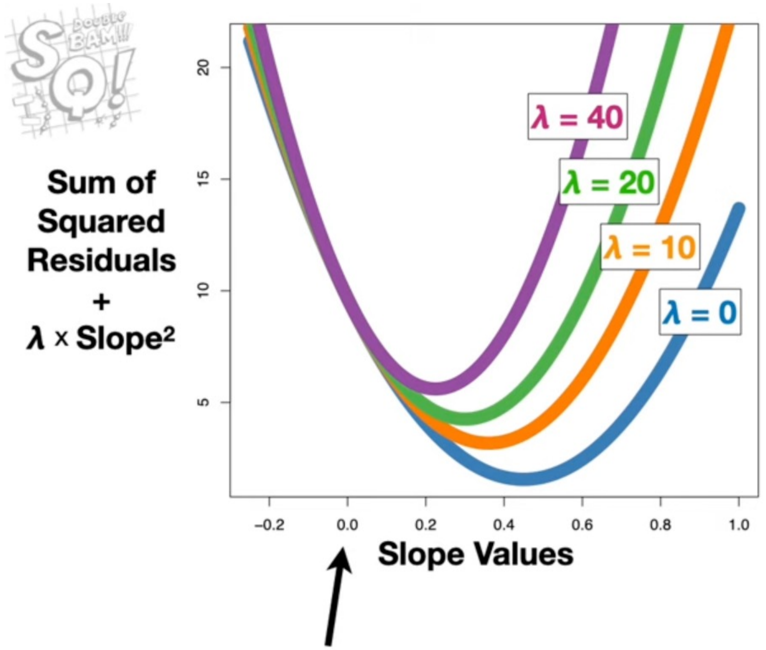
Source: https://www.youtube.com/watch?v=Xm2C_gTAl8c

Lasso Regression



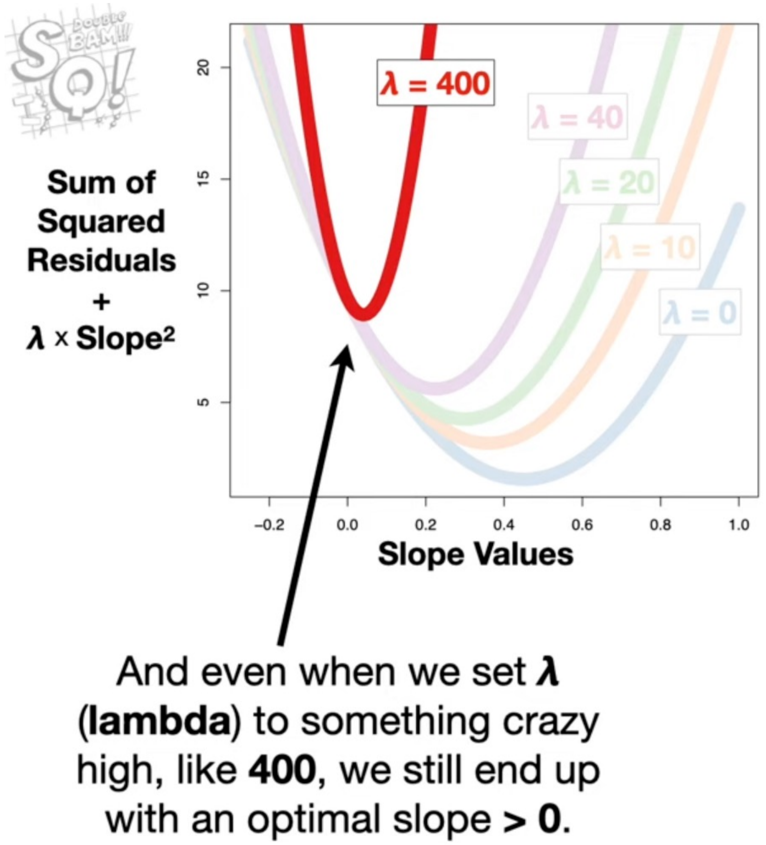
Source: https://www.youtube.com/watch?v=Xm2C_gTAl8c

Ridge vs Lasso Regression



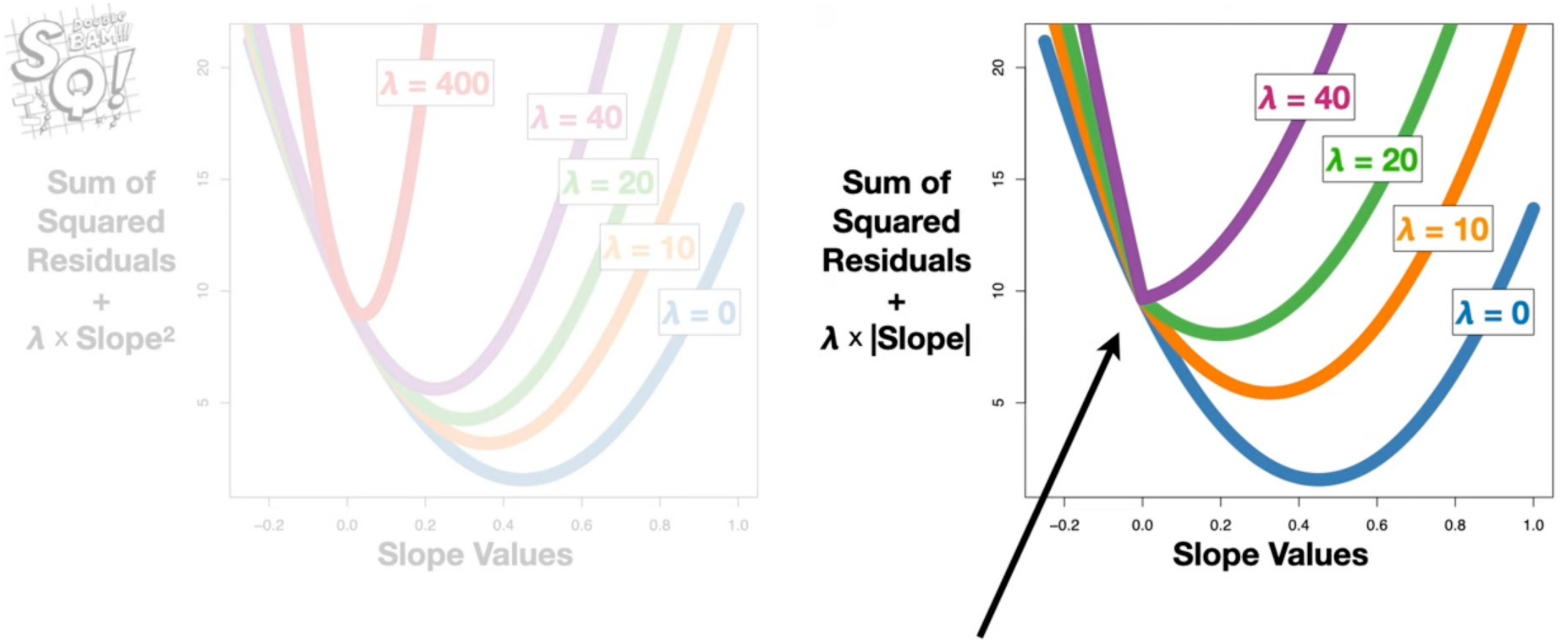
When we increase the **Ridge Regression Penalty,** aka the **L2 Penalty**, aka the **Square Penalty**…

Ridge vs Lasso Regression



Source: https://www.youtube.com/watch?v=Xm2C_gTAl8c

Ridge vs Lasso Regression



…the optimal value shifts towards **0**, but, since we have a kink at **0**, **0** ends up being the optimal slope.

Source: https://www.youtube.com/watch?v=Xm2C_gTAl8c

Data Visualization

Data Visualization

- Lab: https://www.kaggle.com/code/khueluu/data-visualization

Graded Assignment

- TBA
- Deadline: **Monday, 02.12.2024**

References

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- https://tahera-firdose.medium.com/understanding-polynomial-regression-603eb25501d
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