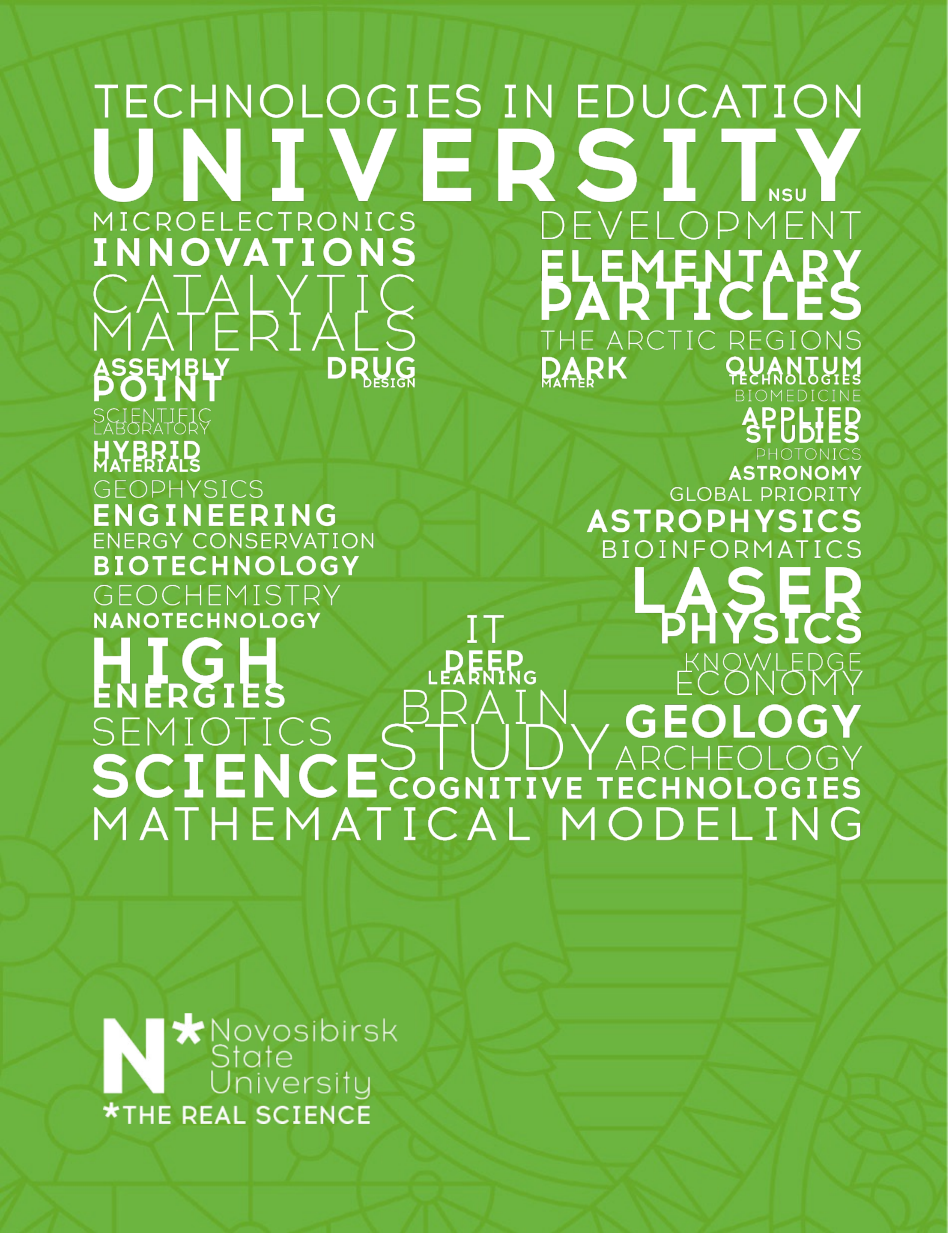

Regression

Luu Minh Sao Khue
Department of Mathematics and Mechanics

Overview

- Introduction to Regression
- Simple Linear Regression
- Practical Implementation
- Model Evaluation and Diagnostics
- Advanced Topics, Applications, and Q&A

Regression

	ENGINESIZE	CYLINDERS	FUELCONSUMPTION_COMB	CO2EMISSIONS
0	2.0	4	8.5	196
1	2.4	4	9.6	221
2	1.5	4	5.9	136
3	3.5	6	11.1	255
4	3.5	6	10.6	244
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7	3.7	6	11.1	255
8	3.7	6	11.6	267
9	2.4	4	9.2	?

- Regression is a process of predicting a continuous value

Regression

X: Independent variable

Y: Dependent variable

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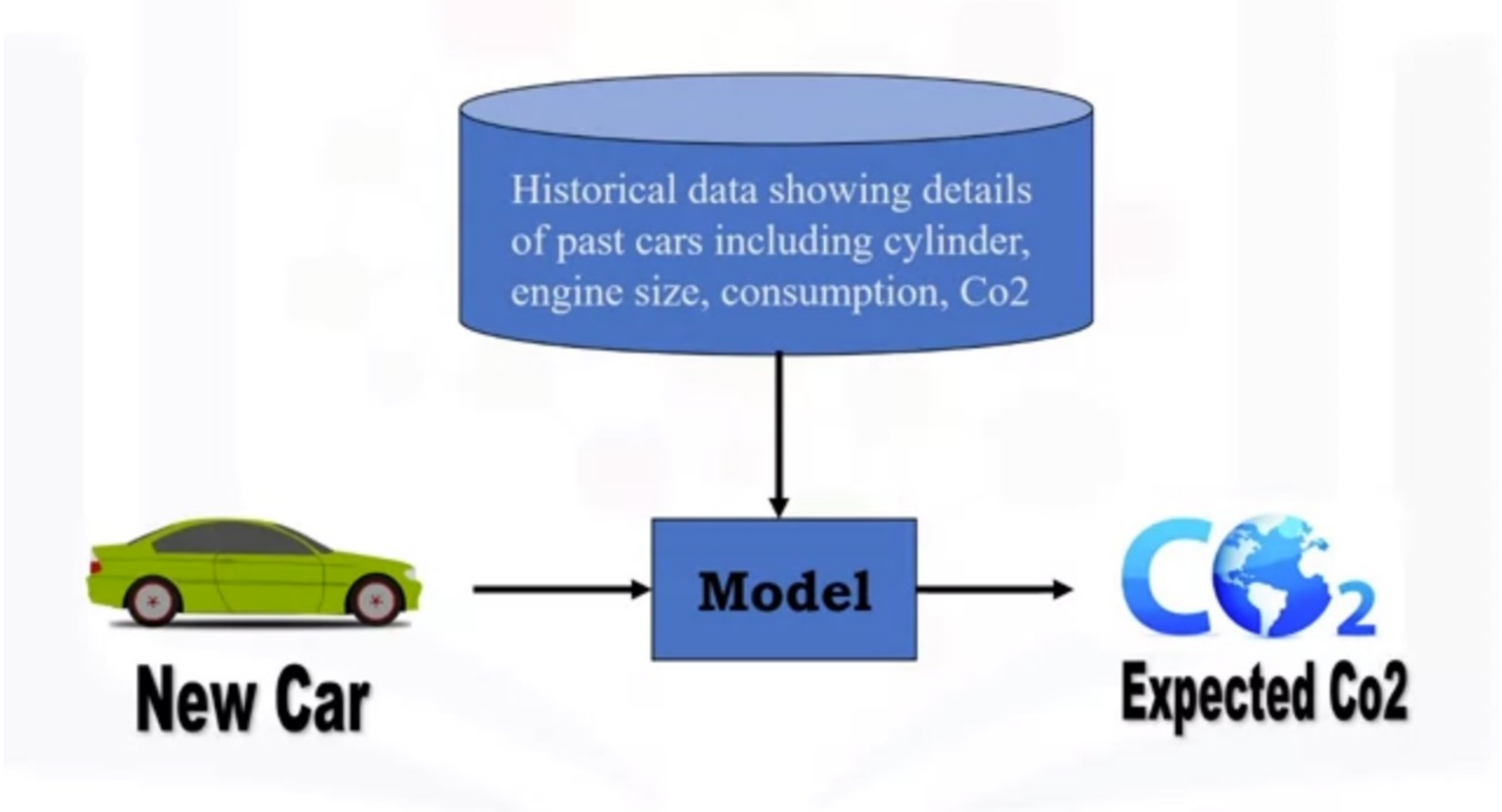
- Dependent variable: the value we want to predict
- Independent variables: the values that explain or cause the value of the dependent variable

Regression



Historical data showing details of past cars including cylinder, engine size, consumption, Co2
Model
New Car
CO2
Expected Co2

Types of regression

- Simple Regression
  - Using one independent variable to predict the dependent variable
  - Ex: predict CO_2 emssion using engine size.
  - Simple Linear Regression, Simple Non-Linear Regresison
- Multiple Regression
  - Using more than one independent variables to predict the dependent variable
  - Ex: predict CO_2 emssion using engine size and number of cylinders.
  - Mulitple Linear Regression, Multiple Non-Linear Regresison

Applications of regression

Sales forcasting

Predict a yearly sale of a person based on Age, Years of Experience, etc.

Satisfaction analysis

Detemine individual satisfaction based on demographic and psychological factors.

Price estimation

Predict a price of a house based on its size, number of rooms, etc.

Employment income

to predict employment income for independent variables such as hours of work, education, occupation, sex, age, years of experience

Quiz

- Which one is a sample application of regression?
  - Predicting whether a patient has cancer or not.
  - Grouping of similar houses in an area.
  - Forecasting rainfall amount for next day.
  - Predicting if a team will win or not.

Some regresison algorithms

- Ordinal Regression
- Poisson Regression
- **Linear Regression**
- Polinomial Regression
- Lasso Regression
- Ridge Regression
- Decision Forest Regression
- Boosted Decision Tree Regression

Simple Linear Regresison

Simple Linear Regresison

- Linear regression is the approximation of a linear model used to describe the relationship between two or more variables

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Simple Linear Regression

Changes in 1 variable explain changes in the other

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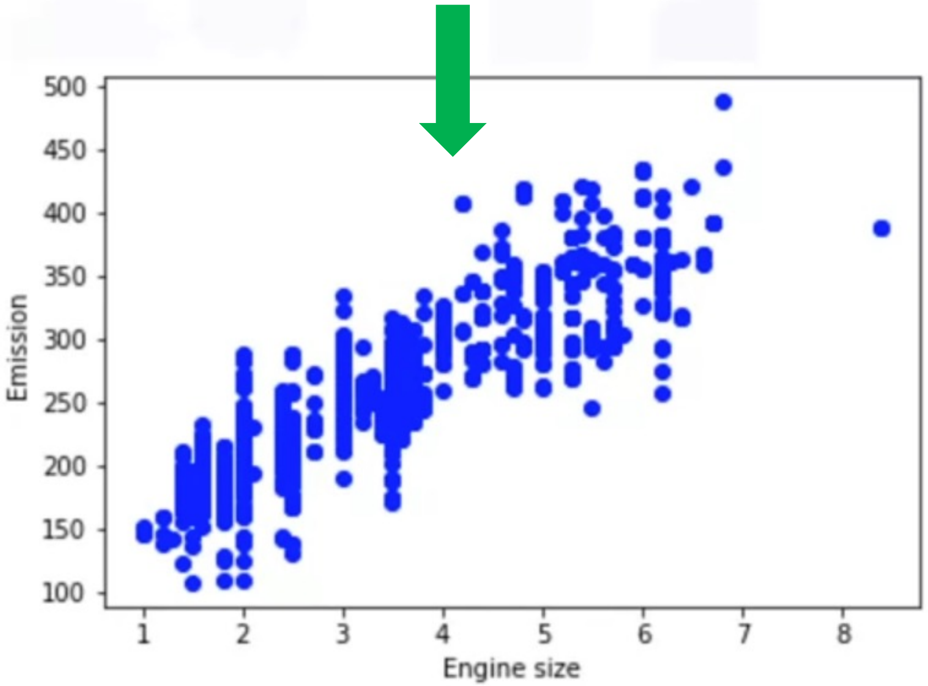

Simple Linear Regression

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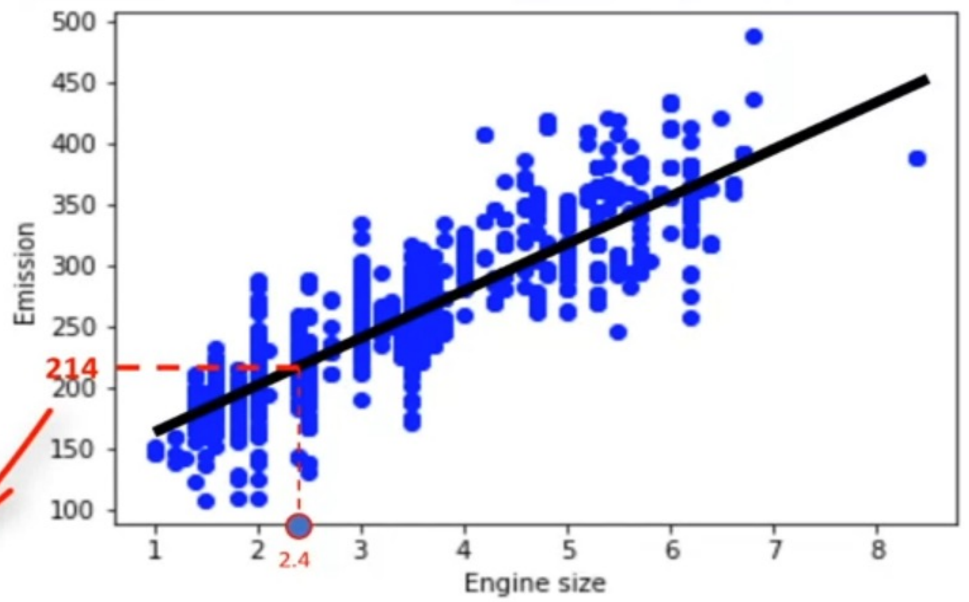


Simple Linear Regression

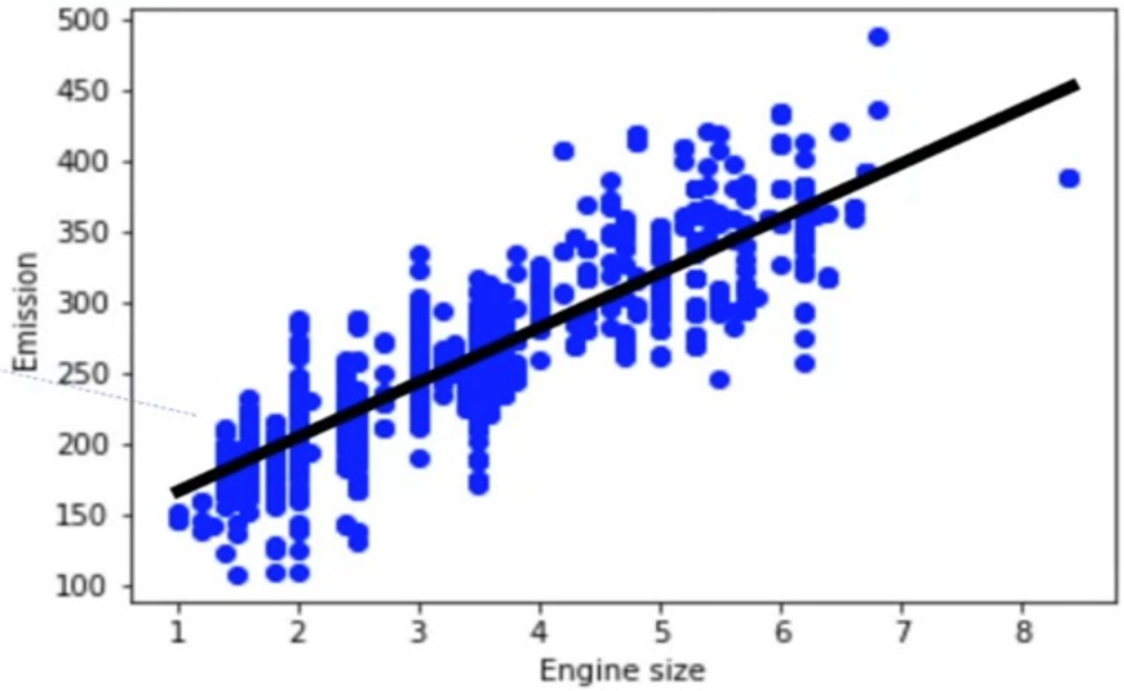
Parameters we need to find

$$\hat{y} = \theta_0 + \theta_1 x_1$$

Independent variable

Independent variable

y



x_1

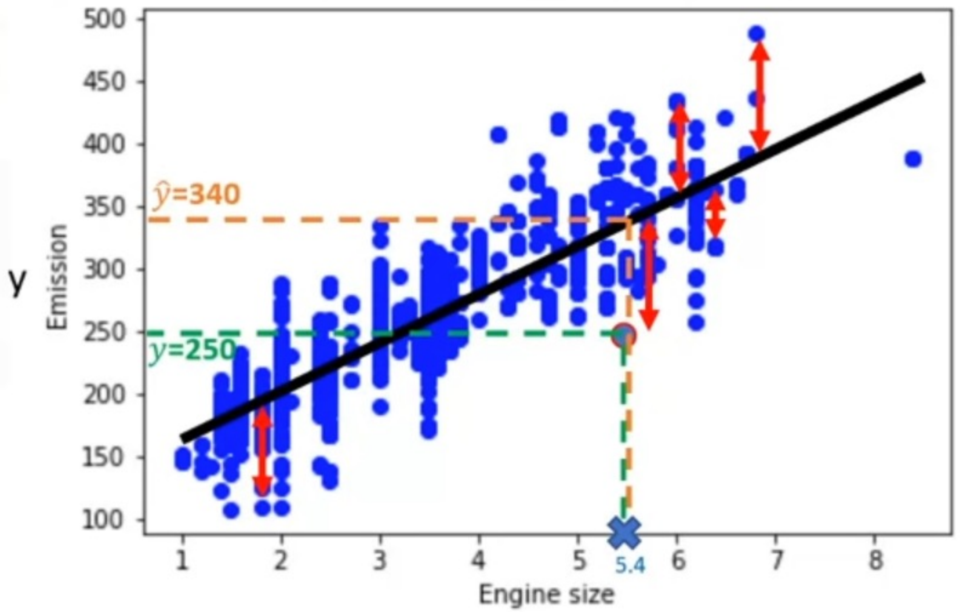
How to find the best fit?

$x_1 = 5.4$ independent variable
y= 250 actual Co2 emission of x1

$\hat{y} = \theta_0 + \theta_1 x_1$
$\hat{y}$ = 340 the predicted emission of x1

Error = y- $\hat{y}$
= 250 – 340
= -90

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$



How to find the best fit? (Mathematic approach)

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6	3.5	6	10.1	232
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$$\hat{y} = \theta_0 + \theta_1 x_1$$

$$\theta_1 = \frac{\sum_{i=1}^{s}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{s}(x_i - \bar{x})^2}$$

$\bar{x} = (2.0 + 2.4 + 1.5 + \dots)/9 = 3.03$

$\bar{y} = (196 + 221 + 136 + \dots)/9 = 226.22$

$$\theta_0 = \bar{y} - \theta_1 \bar{x}$$

Minimizing the sum of squared errors (SSE)

Prediction with simple linear regression

	ENGINESIZE	CYLINDERS	FUELCONSUMPTION_COMB	CO2EMISSIONS
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$$\hat{y} = \theta_0 + \theta_1 x_1$$

$$Co2Emission = \theta_0 + \theta_1\ EngineSize$$

$$Co2Emission = 125 + 39\ EngineSize$$

$$Co2Emission = 125 + 39 \times 2.4$$

$$Co2Emission = 218.6$$

Advantages

- Very fast
- No hyperparameters tuning
- Intepretable

Disadvantages

- Sensitive to outliers
- Cannot handle non-linear relationships

How to test the accuracy of the model

Train all data

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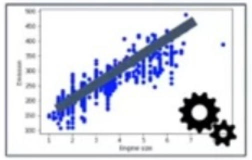
8	3.7	6	11.6	267
9	2.4	4	9.2	212

Train

Test

Actual values



	Prediction
6	234
7	256
8	267
9	210

Predicted values

How to test the accuracy of the model

Train/Test split

	ENGINESIZE	CYLINDERS	FUELCONSUMPTION_COMB	CO2EMISSIONS
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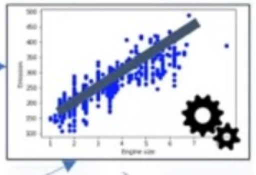
Train

Test



	Prediction
6	234
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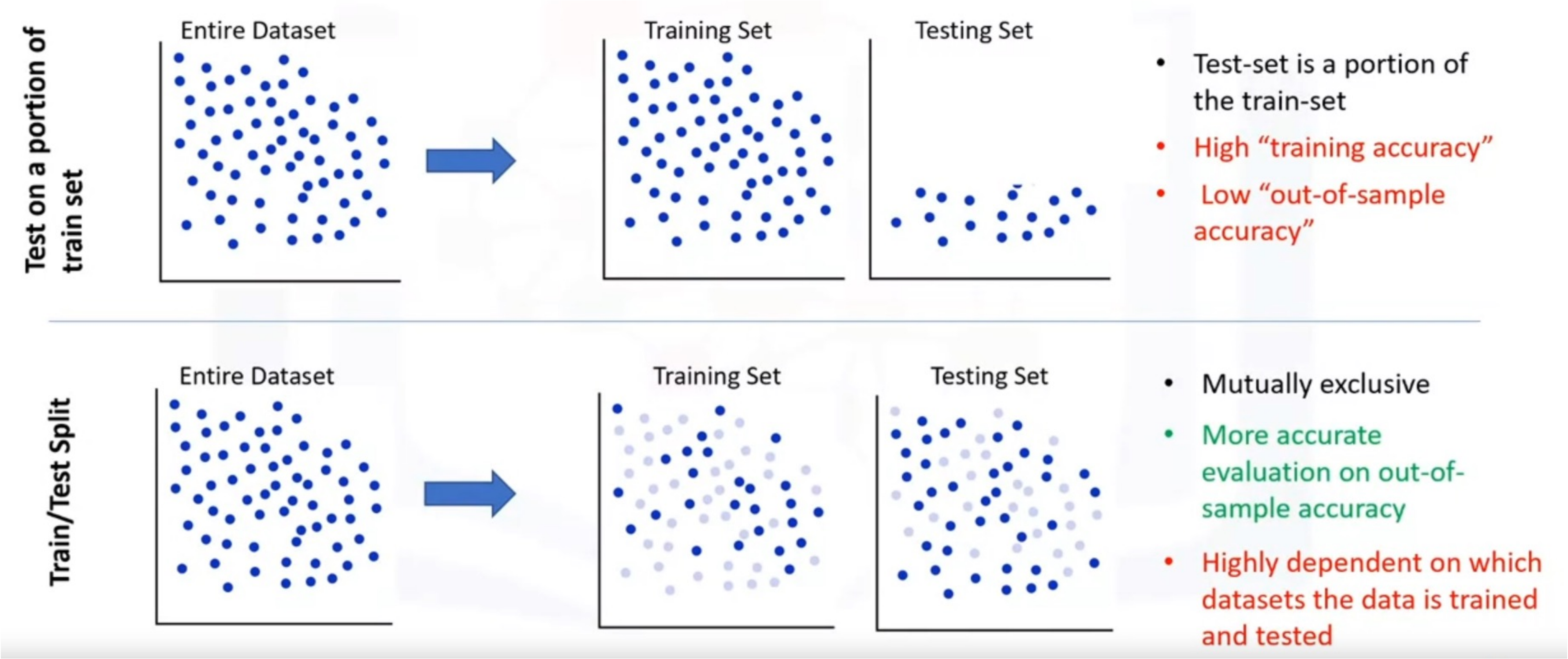
8	267
9	210

Actual values

Predicted values

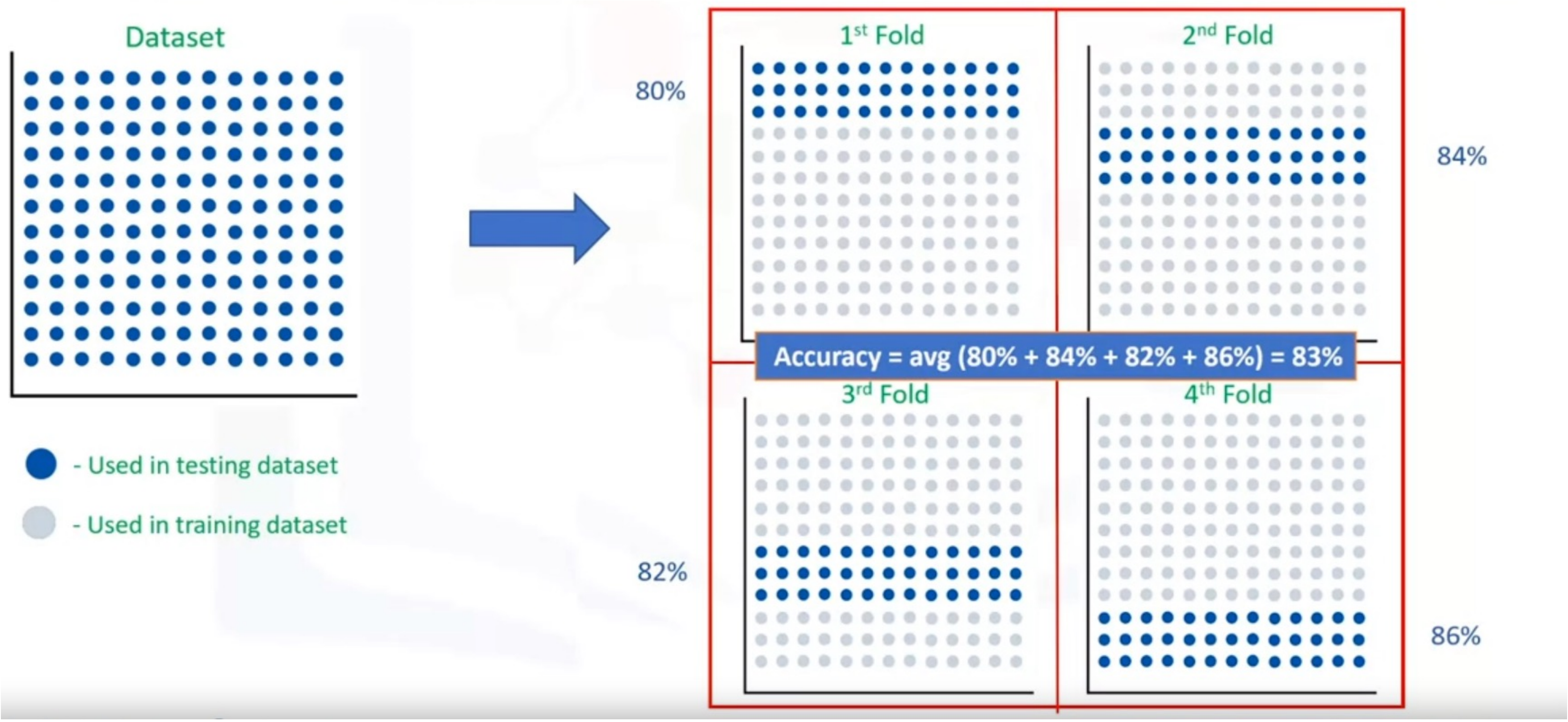
How to test the accuracy of the model

Train/Test split

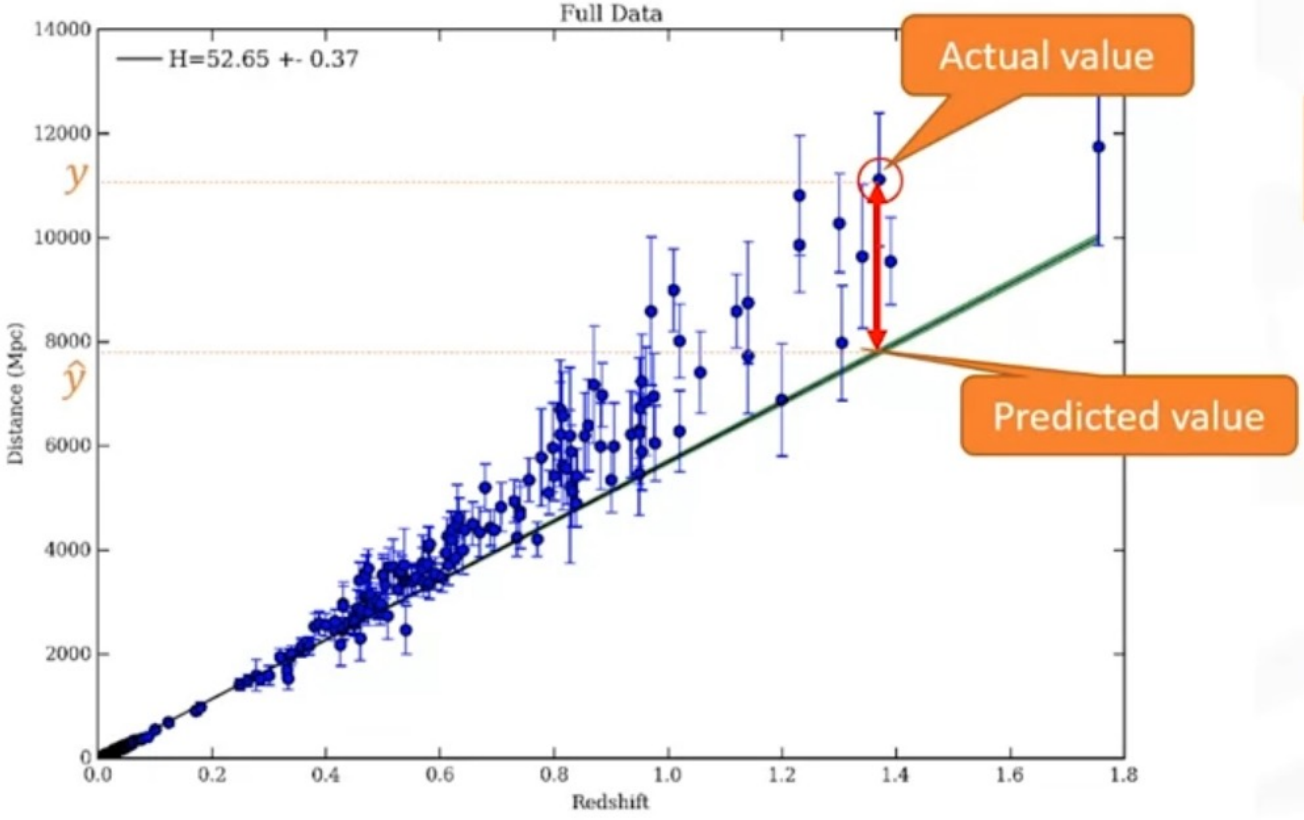


How to test the accuracy of the model

K-fold Cross Validation



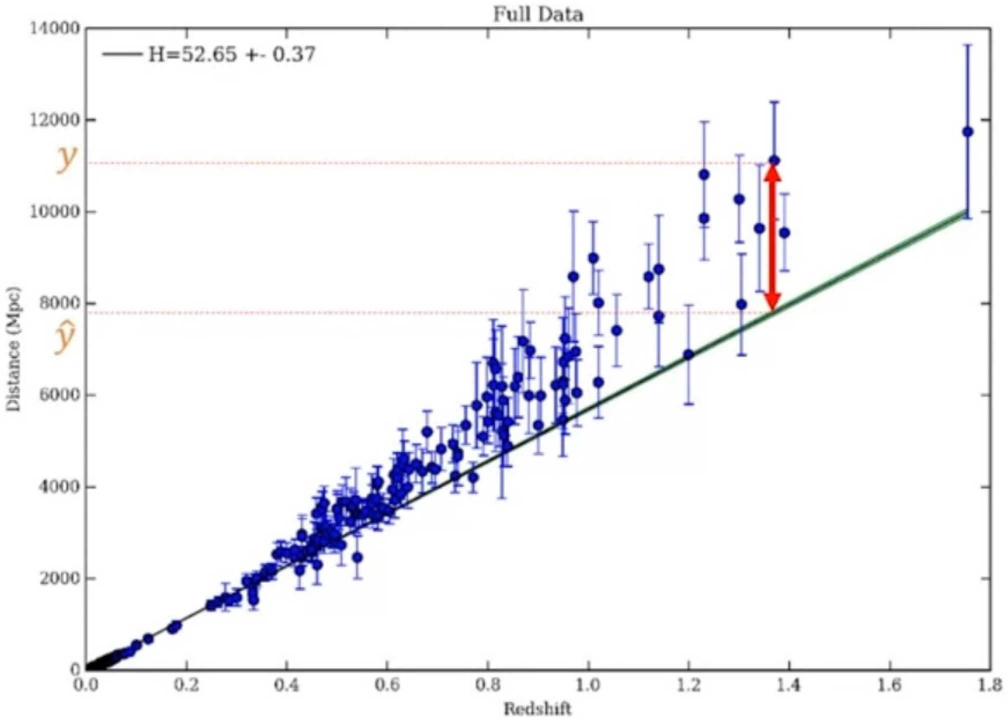
Evaluation Metrics in Regression Models



Error: measure of how far the data is from the fitted regression line.

Error is the difference between the data points and the trend line generated by the algorithm

Evaluation Metrics in Regression Models



$$MAE = \frac{1}{n}\sum_{j=1}^{n}|y_j - \hat{y}_j|$$

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(y_j - \hat{y}_j)^2}$$

$$RAE = \frac{\sum_{j=1}^{n}|y_j - \hat{y}_j|}{\sum_{j=1}^{n}|y_j - \bar{y}|}$$

$$RSE = \frac{\sum_{j=1}^{n}(y_j - \hat{y}_j)^2}{\sum_{j=1}^{n}(y_j - \bar{y})^2}$$

$$R^2 = 1 - RSE$$

Multiple Linear Regression

Multiple Linear Regression

$$Co2\ Em = \theta_0 + \theta_1 Engine\ size + \theta_2 Cylinders + \ \ldots$$

$$\hat{y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \ \ldots + \theta_n x_n$$

$$\hat{y} = \theta^T X$$

$$\theta^T = [\theta_0, \theta_1, \theta_2, \ldots] \qquad X = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ \ldots \end{bmatrix}$$

X: Independent variable

Y: Dependent variable

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9	2.4	4	9.2	?

How to find best parameters for the Multiple Linear Regression?

$\hat{y} = \theta^T X$

$\hat{y}_i = 140$ the predicted emission of x_i

$y_i = 196$ actual value of x_i

$y_i - \hat{y}_i = 196 - 140 = 56$ residual error

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

Minimize

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How to find best parameters for the Multiple Linear Regression?

- Mathematical approach
  - Linear Algebra operation
  - For small dataset
- Optimization approach
  - Gradient Descent
  - For large dataset

Graded Assignment

Using Linear Regression to predict house price for this dataset:

https://www.kaggle.com/code/khueluu/house-price-prediction

- Deadline: **Monday, 25.11.2024**

References

- https://en.wikipedia.org/wiki/Linear_regression
- https://machinelearningmastery.com/regression-metrics-for-machine-learning/
- https://www.coursera.org/learn/machine-learning-with-python?specialization=ibm-data-science