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An Experiment about Factors Involved in a Catapult Design
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An Experiment about Factors Involved in a Catapult Design

Abstract

The present study experimented with physics and math involved in catapults by building and analyzing a Mangonel Catapult of volume 21000cm3. The Literature Review discusses different designs that may qualify as a Mangonel Catapult. Also, the energy transformed during a projectile motion and the tension applied by rubber bands were investigated so that the projectile achieves a long distance after launch. Moreover, different attempts were made to analyze the most efficient launching angle, which led to the conclusion that when the launch is made from 70°, the distance traveled can be maximized.
Furthermore, the projectile's initial velocity, which is the same as the arm's final velocity, must be high, considering inertia, so that the objectives regarding distance traveled can be achieved. This requirement could be met through high tension applied by rubber bands and an optimal draw-back angle of the arm. Overall, this process was crucial for the authors to reflect on topics such as circular motion, projectile motion, and energy transformation in reallife situations.

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## Introduction

The usage of catapults has had a significant change in the past centuries. Catapults were first used by the Greek people during the 3rd century before Christ to fire missiles at castles and conquer land (Chakraborti and Chakrabarti, 2022). Since then, catapults have been replaced by modern technology, but they are still used by engineers to study forces, energy transformation, and projectile motion (Güner, T., 2013). This report aims to analyze the physics and math involved in a catapult and translate these aspects into a popsicle stick model. Through this analysis, the chosen design of the catapult will allow the Malteser to travel an average horizontal distance of 4.95m by maximizing the initial velocity. This will be possible by applying a force of 8.87N on the arm, launching at an angle of 70°, and drawing back the arm by 125°. In summary, the force applied to the catapult, the drawback and launching angle, and the initial velocity will directly impact the total distance traveled by the projectile.

## Literature Review

In general, the engineering involved in catapults includes tension, torsion, and gravity, which are used differently to launch projectiles. Some types of catapults are Trebuchet, Ballista, and Mangonel (Chakraborti and Chakrabarti, 2022). Concerning Mangonel catapults, there are conflicts between researchers regarding what qualifies a catapult as a Mangonel. Hassaan's (2014) study claims that a Mangonel is an Arabic Trebuchet catapult, which has a system that consists of ropes, a pulley, and a gear train. Contrastingly, the research done by Neel and Hardin (2015) emphasizes that it is a siege catapult from medieval times, which relies on torque applied by twisted ropes. Oxford Science's (2016) study expands on this idea and explains that tension is applied to the arm through a pulling force in the opposite direction of motion, which is caused by a stretched



rope. Additionally, Cove Editions (n.d.) states that the launch angle of the projectile could be changed by moving the cross beam nearer to or further away from the launching arm. In general, the second design is more commonly accepted as a Mangonel catapult.

Methodology

The requirements for the construction of the catapult were that it must fit in the dimensions 30cm x 30cm x 30cm, and it should be able to launch the projectile more than once without collapsing. Also, it needed an attached trigger that did not increase the force applied to the projectile. Finally, it had to launch a Malteser as far as possible and the spendings should be low.

In light of this, specific materials were chosen to meet these requirements. First, to build the catapult shown in Figure 1 and Figure 2, popsicle sticks were picked, as they are sturdier than cardboard and easier to manage, increasing the overall support strength of the body. Furthermore, elastic bands were used, as they have a high elastic potential energy, which can make the net force added to the projectile larger. These help achieve longer distances, which is the project's main objective. Whereas, for the trigger, the best choice acknowledged was strings, as they are easy to manipulate and do not add any force to the projectile. Also, hot glue was chosen to join the whole structure as it could withstand the applied pressure and made the process easier regarding aligning the joints of the frame.

Lastly, a small bottle lid was selected as the catapult bucket, because it would minimalize air resistance, thus reducing opposing forces. At last, the lid’s walls would prevent the Malteser from falling out before the launch.

Apart from the materials used, the design chosen is intended to meet the requirements and achieve long distances. Firstly, the triangular shape facilitates better stability across the prototype and distributes the forces evenly. Secondly, the original launch angle was 45°, as



this is an optimal launch angle for a greater distance to be traveled in a perfect system (Tsokos, 2014, pg. 50). However, through trial and error, it was found that 70° is the best launch angle. Additionally, using circular motion, it was deduced that having a long arm drawn back by a larger angle would lead to a maximized tangential velocity (Tsokos, 2014, pg. 249). Finally, three rubber bands were added to the structure as a form of adding force in the direction of the launch. Two were twisted and stretched between the posts of the frame to serve as a pivot, while the third band was stretched between the main frame and the launch arm, thus drawing the arm towards the crossbeam.

This design exploits the transformation of energy involved in the launch of a projectile. Firstly, when the arm is cocked, there is elastic potential energy stored in the twisted and stretched rubber bands. After the trigger is pulled, there will be potential and kinetic energy in the arm and the projectile. When the arm hits the crossbeam, some energy is converted into sound and heat, while the projectile maintains its energy. During the launch, the projectile experiences potential and kinetic energy, until it hits the ground. This is considered the end point of the motion observed, however, it is important to emphasize that when the projectile hits the ground, it will still have kinetic energy, as it carries on moving.

All in all, elastic energy was transformed into kinetic energy to allow the projectile to move and achieve maximum distances.

F i g u r e 1

Catapult in its relaxed position

F i g u r e 2

Cocked catapult



# Results

The catapult was tested three times, and by filming and observing it, the results presented in Table 1 could be found. The average distance traveled by the projectile was 4.59 m.

Moreover, the projectile motion was depicted in Figure 3 by using the values from the table mentioned. Also, Figure 4, Figure 5, Figure 6, and Figure 7 present free-body diagrams of the forces acting on the projectile during different stages of the motion.

Table 1

Results of the launch of the catapult

| Launch number | Max height (m) | Distance travelled |
| --- | --- | --- |

(m)

Time spent in air
(s)

Initial velocity
(m/s)

1

1.68

4.93

1.00

7.21

2

1.79

4.56

1.26

6.55

3

0.980

4.27

0.700

7.10

Average

1.48

4.59

1.03

6.95

F i g u r e 3
Graph representing the projectiles motion

Change in vertical distance (m)

y

f(x)
x
Change in horizontal distance (m)

8

F i g u r e 4

R

R = Normal force
W = weight

W
45

F i g u r e 5
Free body diagram of the forces acting on the Malteser just before the launch arm hits the
crossbeam

R
Fa = Force of arm on the Malteser
Ff

W

Ff = Air friction force
W = weight

70

9

F i g u r e 6
Free body diagram of the forces acting on the Malteser when it reaches maximum height
Ff

W

Ff = Air friction
W = weight

F i g u r e 7
Free body diagram of the forces acting on the Malteser once it has landed
R = Normal force
R
W= weight
W

Discussion

Before the catapult was built, the launch results were predicted. These results were acquired by assuming that the time it takes the launch arm to complete a full rotation is 0.2s and that the length of a popsicle stick is around 11.4cm. For the projectile's motion, the height of the arm was ignored, therefore the projectile's vertical displacement is zero and it moves in a perfect parabolic arc. In addition, air friction (Ff) was ignored. This is because Ff = kv, where "k" stands for a constant determined by the surface area in contact with the air and "v" is the



velocity, and in this case both k and v are very small. All formulas are from Binas (2016) in sections 35A 1, 2, and 3. The directions of the vectors are indicated in Figure 8.

The initial velocity of the projectile:

$$\boldsymbol{v}_t = \frac{2\pi r}{T}$$

$\boldsymbol{v}_t$ = 3.581415625 m.s−1

Due to inertia $\boldsymbol{v}_t = \boldsymbol{v}_i$ of the projectile so $\boldsymbol{v}_i$ = 3.581415625 m.s−1

Maximum height reached by the projectile:

At max height $\boldsymbol{v}_f = 0$

$\boldsymbol{v}_f^2 = \boldsymbol{v}_{iy}^2 + 2\boldsymbol{a}\Delta\boldsymbol{y}$ (Tsokos, 2014, p. 40)

$\Delta\boldsymbol{y}$ = 0.326874054 m

Distance travelled by projectile:

Since projectiles move in a parabolic motion with a starting height of 0m, the time it takes the projectile to land (t2) is double the amount of time it takes to reach maximum height (t1).

F i g u r e 8

Figure depicting the direction and extra information of the vectors

Maximum height ($\Delta\boldsymbol{y}$) with a final velocity ($\boldsymbol{v}_f$ ) of

0m.s−1

y

+

Gravitational acceleration ($\boldsymbol{a}$)

Initial velocity ($\boldsymbol{v}_i$ )

−

45°

⋯⋯⋯

Final position ($\Delta x$)

$t1$

x

$t2$

+

11

$v_f = v_{iy} + a t_1$

$t_1 =$

$v_i \sin 45°$

9.81

$t_2 = 2t_1$

$t_2 = 0.5162960706$ s

$\Delta x = v_{ix}\ t_2$

$\Delta x = v_i \cos 45°\ t_2$

$\Delta x = 1.307490512$ m ≈ 1.31 m.

After analyzing three different launches, it was concluded that the average distance travelled by the Malteser was 4.95m, which exceeded the expectations from the thesis. It is important to emphasize that because of the different conditions between reality and a perfect system, the mathematical hypothesis differed from the actual results. Additionally, the launch angle and initial height of the actual project differed from the hypothetical model. The launch arm was drawn-back to −55°, which is the maximum angle it could be pulled back without the projectile falling. Even though hypothetically 45° is the best launching angle, as stated before, launching it from 70° allowed a greater rotation of the arm from the initial position, which increases the initial velocity. Human error should also be considered since it affects the exact values of the results.

Overall, all the design factors allowed an increase in force and initial velocity, which impacted the total distance travelled, making the projectile go further. Through analysis of the launch, it was concluded that the draw-back angle is the most crucial point to attain the maximum distances the projectile will reach on the x-axis and y-axis. According to an experiment made by Science Oxford (2016), the arm’s draw-back angle is the main aspect that influences the distance travelled and the height reached by the projectile.



These results could be even greater; however, some requirements and limitations had to be met while the catapult was built. One of the limitations was the tension acting in the catapult. In other words, as the catapult should be able to launch the projectile more than once without collapsing, the force applied was limited. Moreover, the dimensions of the system are restricted, which affects the initial height. Also, with constrained time, few attempts could be made, limiting the improvement of the project by trial and error. Another limitation is that there is minimal research surrounding catapults built with popsicle sticks, which restricts the authors’ knowledge about the advantages and disadvantages of this design.

## Conclusion

Summarily, the design of the catapult manipulated force, energy conserved, and launch angle to maximize the horizontal distance traveled by the projectile. This distance was 4.95 m. Additionally, it was concluded that the drawback angle of the arm plays a vital role in the total distance that the projectile can reach by increasing the tension applied by the rubber bands, since they are being more stretched, and increasing the tangential velocity. It is recommended, for future analysis, to reconsider building a popsicle stick catapult, since there is little academic research about it. Furthermore, rubber bands could be added at the top of the arm to increase the traveled distance. Overall, building and analyzing a catapult leads to an enriched understanding of engineering studies by applying physics and math formulas to real-life situations and manipulating numbers to have positive outcomes.

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Appendix

Full mathematical hypothesis working out

Assume that spring constant of a band is 38.8 N.m-1 (Sunday Academy, 2018, pg3), that it takes 0.05 seconds for the launch arm to hit the cross-beam (travel 90°) once it is released, therefore taking 0.2 seconds to complete 1 rotation, the length of a popsicle stick is around 11.4cm, and that the change in distance for both the twisted and the stretched rubber band is 22.85cm. For the projectile motion, ignore the height of the

arm. Therefore, the
projectile's vertical displacement is zero and it moves in a perfect parabolic arc. All formulas
are taken from the Binas (2016) in sections 35A 1, 2 and 3. The directions of the vectors are
indicated in Figure 6

Force of the rubber bands:

$\boldsymbol{F} = k\boldsymbol{\Delta x}$

$\boldsymbol{F} = 38.8 \times 22.85 \times 10^{-2}$

$\boldsymbol{F} = 8.8658$ N towards the crossbeam

There are two bands therefore the net force is 17.7316 N towards the crossbeam:

$\boldsymbol{v}t = \dfrac{2\pi r}{T}$

$\boldsymbol{v}t = \dfrac{2\pi(11.4 \times 10^{-2})}{0.2}$

$\boldsymbol{v}t = 3.581415625 \text{ m.s}^{-1}$

Due to inertia $\boldsymbol{v}t = \boldsymbol{v}i$ of the projectile so $\boldsymbol{v}i = 3.581415625 \text{ m.s}^{-1}$

Maximum height reached by the projectile:

At max height $\boldsymbol{v}f = 0$

$\boldsymbol{v}f^2 = \boldsymbol{v}iy^2 + 2\boldsymbol{a}\Delta\boldsymbol{y}$

$0 = (\boldsymbol{v}i \sin 45°)^2 + 2\boldsymbol{a}\Delta\boldsymbol{y}$

$0 = (3.581415625 \sin 45°)^2 + 2(-9.81)\Delta\boldsymbol{y}$

$\Delta\boldsymbol{y} = 0.326874054$ m

(Tsokos, 2014, p. 40)

Distance travelled by projectile:

Since projectiles move in a parabolic motion, the time it takes the projectile to land (t2) is
double the amount of time it takes to reach maximum height (t1)

$\boldsymbol{v}f = \boldsymbol{v}iy + \boldsymbol{a}t$

$0 = \boldsymbol{v}iy - 9.81t1$



$t1 = \dfrac{\boldsymbol{v}i \sin 45°}{9.81}$

$t2 = 2t1$

$t2 = \dfrac{2(\boldsymbol{v}i \sin 45°)}{}$

9.81

$t2$ =

2(3.581415625 sin 45°)
9.81

$t2 = 0.5162960706$ s

$\Delta x = v_{ix}\ t_2$

$\Delta x = v_i \cos 45°\ t_2$

$\Delta x = 3.581415625 \cos 45° \times 0.5162960706$

$\Delta x = 1.307490512$ m ≈ 1.31 m

F i g u r e 9
Graph representing the relationship between the vertical distance travelled by the projectile
and time for multiple launches
x

Horizontal distance travelled (m)

g(x) is the red graph and represents the
first launch results
h(x) is the blue graph and represents
the second launch results
i(x) is the green graph and represent
the third launch results
j(x) is the purple graph and it represents
the average result of all the launches
y
Time (s)

16

F i g u r e 10
Graph representing the relationship between the vertical distance travelled by the projectile
and time

k(x) is the red graph and represents
the first launch results

Vertical distance (m)

l(x) is the blue graph and represents
the second launch results
m(x) is the green graph and
represent the third launch results
n(x) is the purple graph and it
represents the average result of all
the launches

Time (s)

F i g u r e 11
Graph comparing the hypothetical distances travelled to the actual distances travelled
5
4,5

Distance travelled (m)

4
3,5
3
2,5
2
1,5
1
0,5
0
Vertical plane
Hypothetical results

Horizontal plane
Actual results



Full calculation of the equation of the projectile
By plotting values from the Graph 2 and Graph 3, both based on time, the following
results can be observed.
Table 2
The results of specific points of Figure 9 and Figure 10
Time (s)
0.000
0.200
0.400
0.600
0.800
1.00

Distance in X- axis (m)
0.000
0.878
1.76
2.63
3.51
4.40

Distance in Y-axis (m)
0.275
1.05
1.43

1.43
1.03
0.230

For the projectile motion, the equation considered must be $y = ax$ 2 + $bx$ + $c$, since it represents a parabola.

By applying values from the chart:

For x=0.000: y = c = 0.275

For t = 1.000 s:

0.230 = $a(4.400)2 + 4.400b + 0.275$

For t = 0.2000 s:

1.051 = $a(0.878)2 + 0.878b + 0.275$

Through a piecewise function of both equations, it can be found that a= −0.254, b= 1.106 and, as stated before, c= 0.275

In the light of these, the final equation of the projectile motion is $y = 0.254x$ 2 + $1.106x$ + 0.275

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F i g u r e 12
Image representing of the launch of the projectile and the position of the projectile every 0.2 seconds