Initial Conditions

- **Dry Mass** (M_f): 10.65 kg
- **Target Apogee**: 1050 m
- **Burn Time** ($t_{burnout}$): 3.5 s
- **Specific Impulse** (I_{sp}): $I_{sp} \leq 2800\ Ns$

Variables and Their Definitions

- g: Acceleration due to gravity (*9.81 m/s*$^{2)}$
- M_f : Final/Dry mass of the rocket (dry mass)
- M_i : Initial mass of the rocket (dry mass + propellant mass)
- M_p: Mass of the propellant
- *T*: Thrust
- $v_{burnout}$: Burnout velocity
- Δv : Change in velocity required to reach target apogee

Step 1: Estimating Δv Using Energy Equation

Using the formula:

$$\Delta v = \sqrt{(2gh)}$$

We can estimate Δv :

$$\Delta v = \sqrt{(2 \cdot 9.81 \cdot 1050)}$$

$$\Delta v = 143.53$$

Step 2: Tsiolkovsky Equation

Using the Tsiolkovsky rocket equation:

$$\Delta v = ve \cdot \log\left(\frac{m_i}{m_f}\right)$$

Specific Impulse Formulas:

$$Isp = \frac{ve}{g} = \frac{T}{\frac{dm}{dt} \cdot g}$$

Thrust Definition:

$$\Delta v = \frac{T}{\frac{dm}{dt}} \cdot \log\left(\frac{m_i}{m_f}\right)$$

Step 3: Finding Burnout Velocity

Total apogee reached is the sum of height during burnout and height of coasting phase:

$$H_{total} = H_{burnout} + H_{coasting}$$

Where:

$$H_{burnout} = \frac{1}{2} a t_{burnout}^2$$

$$H_{coasting} = \frac{v_{burnout}^2}{2 \cdot g}$$

Combining these gives:

$$H_{total} = \frac{1}{2} a t_{burnout}^2 + \frac{v_{burnout}^2}{2 \cdot g} = \frac{1}{2} v_{burnout} . t_{burnout} + \frac{(v_{burnout})^2}{2 \cdot g}$$

Setting *H=1050 m, and t=3.5 s and* :

$$1050 = \frac{3.5}{2} v_{burnout \cdot} + \frac{(v_{burnout})^2}{2 \cdot (9.81)}$$

Solving this equation yields:

$$v_{burnout} = 127.39 \frac{m}{s}$$

Step 4: Finding Propellant Weight Needed

Using the effective velocity equation:

$$v_{burnout} = \left(\frac{T_{avg}\ \cos(\propto)}{m_{avg}} - g\right) \cdot t_{burnout}$$

Importing thrust from Tsiolkovsky Equation,

$$v_{burnout} = \left(\frac{\Delta v \cdot \frac{dm}{dt} \cdot \cos(\propto)}{m_{avg} \cdot \log \left(\frac{m_i}{m_f}\right)} - g\right) \cdot t_{burnout}$$

As $m_p = dm \cdot t_{burnout}$ and $m_{avg} = \frac{(m_i + m_f)}{2} = m_f + \frac{m_p}{2}$

$$v_{burnout} = \left(\frac{\Delta v \cdot m_p \cos(\propto)}{\left(m_i + \frac{m_p}{2}\right) \cdot \log \left(\frac{m_i}{m_f}\right)} - g \cdot t_{burnout}\right)$$

$$\frac{v_{burnout}}{\Delta v \cdot \cos(\propto)} = \left(\frac{m_p}{\left(m_i + \frac{m_p}{2}\right) \cdot \log \left(\frac{m_i}{m_f}\right)} - g \cdot t_{burnout}\right)$$

On applying the values:

$$\frac{1.12674}{\cos(\propto)} = \left(\frac{m_p}{\left(10.5 + \frac{m_p}{2}\right) \cdot \log \left(1 + \frac{m_p}{10.5}\right)}\right)$$

Step 5: Values at different angles of attack

a) Angle: $90°$

$$\left(\frac{m_p}{\left(10.5 + \frac{m_p}{2}\right) \cdot \log \left(1 + \frac{m_p}{10.5}\right)}\right) = 1.12674$$

On doing iterations we get
m_p = 1.2626658982 kg
Error = -4.771516515234e-12

b) Angle: $85°$

$$\left(\frac{m_p}{\left(10.5 + \frac{m_p}{2}\right) \cdot \log \quad \left(1 + \frac{m_p}{10.5}\right)} \right) = 1.16647$$

On doing iterations we get

M_p = 1.6318742415 Kg
Error = -2.02469152554841e-11

c) Angle: $80°$

$$\left(\frac{m_p}{\left(10.5 + \frac{m_p}{2}\right) \cdot \log \quad \left(1 + \frac{m_p}{10.5}\right)} \right) = 1.1990$$

On doing iterations we get

M_p = 1.925382576 Kg
Error = -3.22228910221156e-11

Step 6: Evaluation of thrust and specific impulse

a) Angle: 90°

Total Mass $= 10.5 + 1.262 = 11.762\, kg$

Thrust, $T = \frac{(v_{burnout} + g \cdot t) \cdot m_{avg}}{t} = \frac{(127.39 + 34.34) \cdot (10.5 + 0.631)}{3.5} = 514.319\, N$

Total Impulse, $I_t = T_{avg} \cdot t_{burnout} = 514.319 \cdot 3.5 = 1800.117\, Ns$

b) Angle: 85

Total Mass $= 10.5 + 1.665 = 12.165\, kg$

Thrust,

$$T = \frac{(v_{burnout} + g \cdot t) \cdot m_{avg}}{t} = \frac{(127.39 + 34.34) \cdot (10.5 + 0.8325)}{3.5}$$
$$= 523.63\,N$$

Total Impulse, $I_t = T_{avg} \cdot t_{burnout} = 523.63 \cdot 3.5 = 1832.70\,Ns$

c) Angle: 80°

Total Mass $= 10.5 + 1.925 = 12.425\,kg$

Thrust,

$$T = \frac{(v_{burnout} + g \cdot t) \cdot m_{avg}}{t} = \frac{(127.39 + 34.34) \cdot (10.5 + 0.9625)}{3.5}$$
$$= 529.636\,N$$

Total Impulse,

$$I_t = T_{avg} \cdot t_{burnout} = 529.63 \cdot 3.5 = 1853.7273\,Ns$$