Group-based beliefs and human capital specialization

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September 2, 2020

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Abstract

Although the overall gender gap in postsecondary degree attainment has reversed over the past forty years, significant heterogeneity persists in terms of which fields men and women choose to study. In this paper, I consider the role of group-based beliefs in explaining these differential convergence rates across fields. I assume a student forms their initial belief about their probability of success in a particular field based on past outcomes for their group type. I then incorporate group-based beliefs into the model of gradual human capital specialization from Alon and Fershtman (2019) to show how these differences in priors can drive human capital specialization decisions amongst otherwise similar agents.

1. Introduction

The gender gap in postsecondary degree attainment has reversed in the US over the past fifty years, as seen in figure 1. The overall gender convergence in human capital has been related to a number of economic benefits, including reducing the gender wage gap (Blau and Kahn 2017) and increasing aggregate economic productivity (Hsieh et al. 2019). However, this pattern is not

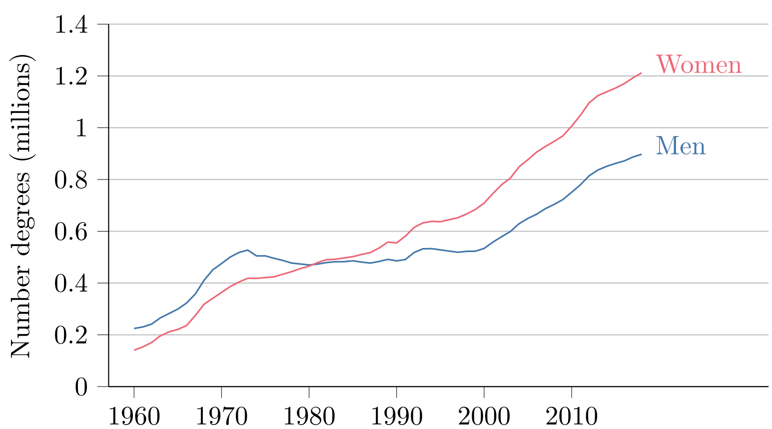

Figure 1. Number of Bachelor's Degrees awarded in US 4-year colleges. Source: IPEDS; Snyder (2013).

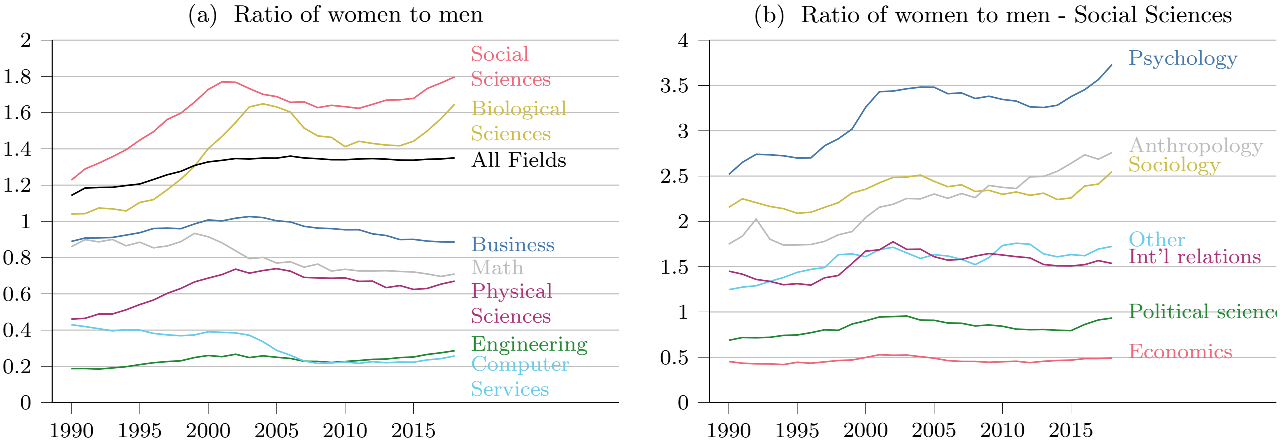

Figure 2. Ratio of women to men completing Bachelor's degrees in U.S. 4-year colleges. Source: IPEDS.

uniformly observed across fields of study. Consider figure 2a, which plots the ratio of women to men completing Bachelor's degrees in historically male-dominated subjects. While the gender ratios of some fields have increased since 1990, others have remained flat or worsened. More generally, aggregations of college majors can easily mask heterogeneity in gender convergence across fields (Black et al. 2008). This can be seen when comparing the overall Social Sciences gender ratio from Figure 2a with the ratios of its subfields in 2b. Although these differences in major choice across genders appear to matter for labor market outcomes (Sloane, Hurst, and Black 2020), the reasons these differences exist is not well understood.

This paper addresses this heterogeneity using a model of group-based belief formation and gradual human capital specialization. Building on Alon and Fershtman (2019), I assume individuals belonging to a particular group choose to work or study in heterogeneous fields. Returns to education are stochastic, and underlying abilities are unknown. Agents form beliefs about their unknown abilities based on existing group outcomes. I use simulations to highlight the different mechanisms of this model, and to specifically show how beliefs can impact decision making. I then use my model to demonstrate how group-based beliefs can lead to a misallocation of talent, and the dynamics of this result.

We can then use this model for counterfactual analysis. First, group-based beliefs may lead to misallocation of talent. I estimate how the misallocation of talent due to group-based beliefs might affect aggregate productivity. To do this, I modify the approach from Hsieh et al. (2019), who estimate the impact of barriers to human capital accumulation on aggregate productivity. Specifically, they build a Roy (1951) model of occupational choice.

There are several key differences between the model outlined below and the one from Hsieh et al. (2019) that warrant attention. First, they explicitly model barriers to field-specific human capital attainment in monetary terms. As such, field specialization in their model is reduced to picking the occupation with the highest indirect expected utility.

What happens if I remove discrimination; how long would it take for women's beliefs to converge to the truth?

Discuss the role of affirmative action. Can affirmative action address these biases?

This paper proceeds as follows. After a brief literature review, I outline the model in section 2. Implications of the model are explored in section 3.

Literature

This paper builds upon the extensive literature on human capital formation (Becker 1962; Ben-Porath 1967; Mincer 1974; Rosen 1983). I expand on Alon and Fershtman's (2019) theoretical model of gradual specialization, a recent contribution to the literature. Their framework closely relates to two classical papers from this field. The first is the seminal Mincer (1974) model of the returns to education. The second is the Roy (1951) model of occupational choice and skill heterogeneity. Alon and Fershtman's (2019) model, and by extension my own, can be viewed as a generalization of the mincerian model of human capital accumulation to include a dynamic Roy model with unknown heterogeneous abilities and sequential learning.

I focus on pre-labor market specialization decisions, specifically college major choice. As such, this project is closely tied to the theoretical and empirical literature on education decisions and college major choice. Altonji's (1993) seminal work on the role of uncertainty in sequential education decisions provides a theoretical antecedent for this paper.

Empirical research on determinants of educational choices and returns to education is too extensive to fully outline here. I will focus on several relevant strands of empirical literature that either motivate my analysis or provide relevant context. Of particular relevance is the empirical literature on the nonpecuniary determinants of college major choice (Arcidiacono 2004; Wiswall and Zafar 2018; Zafar 2013). As noted in Zafar (2013), much of the literature on nonpecuniary determinants focus on the primacy of preferences.

This paper builds on recent literature in human capital specialization. In particular, I draw on work about college major choice and occupational choice (Altonji, Blom, and Meghir 2012; Altonji, Arcidiacono, and Maurel 2016). Of particular interest is research on the role of beliefs in human capital specialization decisions (Arcidiacono et al., forthcoming). This paper shares several theoretical commonalities with Arcidiacono et al. (2016), who build a dynamic model of school and work decisions, though they focus on attrition; as such, major choice is broadly characterized as a choice between STEM fields and non-STEM fields.

This paper is motivated by empirical work on the relationship between gender and college major choice, and how that relationship has changed over time. Differences in major choice by gender has been documented using both administrative data (Dickson 2010) and survey results

(Zafar 2013). It is worth noting that there has been significant improvement in gender ratios across fields compared to the mid-twentieth century. However, most of this gender convergence ended by the 1980s, well before parity was achieved (Sloane, Hurst, and Black 2020; England and Li 2006).

The literature suggests several reasons for gender differences in college major choice. A number of studies estimate the impact of preferences (Zafar 2013; Wiswall and Zafar 2015), including preferences over lifetime temporal flexibility (Bronson 2015; Wiswall and Zafar 2018). Though my model can accommodate field-specific preferences, my paper primarily focuses on the role of beliefs and belief formation. This approach is consistent with a number of determinants of gender major choice, including the presence of same-gender role models (Porter and Serra 2020; Lim and Meer 2020), and the role of negative feedback (Kugler, Tinsley, and Ukhaneva 2017).

Finally, the results of this paper will be closely tied to statistical discrimination literature. The theory of statistical discrimination, as posited by Arrow (1972) and Phelps (1972), and then formalized in Aigner and Cain (1977), assumes discrimination arises out of imperfect information. For example, the aforementioned papers consider the case where employer's use a job applicant's race as one variable when inferring potential productivity. The A key concern associated with statistical discrimination in labor markets are self-fulfilling prophecies (Lundberg and Startz 1983). As discussed in section TBD, this model in this paper is closely tied to the theory of statistical discrimination. From statistical discrimination literature, know that self-fulfilling prophecies matter:

- Coate and Loury (1993)

Matters from an affirmative action point of view. Estimate inefficiencies across communities (compare to Arcidiacono and Lovenheim (2016))

Statistical discrimination runs into conflicts with legal definitions of discrimination, a point noted in Lundberg and Startz (1983).

2. Model of human capital specialization

This section outlines my theory of group-based beliefs and human capital specialization. Please note that the agent's specialization problem and subsequent decision rule follows Alon and Fershtman's (2019) model of gradual human capital specialization; as such, I refer to the reader to their original paper for details.

2.1. Specialization decision

Assume infinitely lived agents in discrete time choose to work or study among J fields. This means that at each time period t, individuals decide to either study a field j at a postsecondary institution, or to work in a field j, where $j \in \{1, \ldots, J\}$. The variable m_{jt} is equal to one if an agent matriculates and studies field j at time t; otherwise, m_{jt} equals zero. Likewise, ℓ_{jt} indicates whether an agent works in field j at time t. An individual therefore faces the following time constraint in each period t:

$$\sum_{j=1}^{J}(m_{jt}+\ell_{jt})=1.$$

Individuals aim to maximize their expected lifetime utility. Let U_j denote the expected per-period utility associated with working in field j, where U_j is a bounded function that is non-decreasing in field-specific wages and in field-specific human capital (w_j and h_{jt}, respectively). The agent's expected lifetime payoff can be written as:

$$\sum_{t=0}^{\infty}\delta^t\sum_{j=1}^{J}U_j(w_j,h_{jt})\ell_{jt}, \tag{1}$$

where $\delta\in(0,1)$ is the discount rate.

Agents are initially endowed with some level of field-j human capital, h_{j0}. They can stochastically accumulate more field-j human capital by studying that field. Recall that if $m_{jt}=1$, an agent matriculates in time period t to take a course in field j. Let s_{jt} indicate whether an agent studying j at time t succeeds and passes that course; if the student succeeds, then $s_{jt}=1$, otherwise $s_{jt}=0$. I assume that whether a student passes or fails a particular field-j course is stochastic. Specifically, each student is endowed with some immutable probability of success in field-j courses, denoted θ_j. A student then passes any field-j course with probability θ_j, implying that s_{jt} is a Bernoulli random variable with parameter θ_j.[1] Agents only accumulates human capital when they pass courses. Therefore, an agent's field-specific human capital evolves according to:

$$h_{j,t+1}=h_{jt}+\nu_j s_{jt}m_{jt}, \qquad s_{jt}\sim \text{Bernoulli}(\theta_j), \tag{2}$$

where $\nu_j\geq 0$ is the human capital gain associated with passing the course.[2]

A student's probability of success in a field j course, θ_j, is an ability parameter; students with high values of θ_j are more likely to pass any given class in field j, whereas students with low values of θ_j are more likely to fail. Students do not know their personal value of θ_j, but they have beliefs about what their value of θ_j might be. Their beliefs about their ability is initially given by the distribution P_{j0}. As students take courses in field-j, they update their belief about what their value of θ_j may be, according to some updating rule Π_j:

$$P_{j,t+1}=\Pi_j(P_{jt},s_{jt})$$

It is clear from (1) that agents will only want to work in the field that yields the highest

1. The variable s_{jt} is assumed to be independent and stationary over time.

2. The per-period expected accumulation of human capital additionally must be non-negative and bounded. Regularity conditions imposed in section 2.3 ensure that is the case.

expected lifetime utility. The choice of which field to work in is an individual's *specialization decision*. Agents that plan to specialize in field j will study j to accumulate j-specific human capital, and will endogenously enter the labor market as a field-j specialist. The decision of when to stop studying j and enter the labor market is an agent's *stopping decision*. Overall, students have two incentives for studying a field j: first, they can potentially accumulate j-specific human capital, which directly increases lifetime utility if they specialize in j. Second, studying j reveals information about their ability in that field, which is central to the specialization decision.

2.2. Group-based beliefs

Assume each student has a group type, g. To simplify the exposition, consider two groups, men and women ($g \in \{m, f\}$). The distributions of underlying abilities, θ_j, are the same for men and women. However, initial beliefs about underlying abilities, P^g_{j0}, are different for the two groups.

To make this explicit, let's fully parameterize the belief distribution for a type g student. Assume the initial beliefs about θ_j follow a beta distribution with parameters $(\alpha^g_{j0}, \beta^g_{j0})$, implying $P^g_{j0} = \mathcal{B}(\alpha^g_{j0}, \beta^g_{j0})$. To understand why this is a reasonable assumption, recall that our unknown ability parameter, θ_j, is the probability that a student succeeds ($s^g_{jt} = 1$) or fails ($s^g_{jt} = 0$) a field j course at time t. Because the realizations of s_{jt} are independent over time, the sequence of successes and failures for some number of total field j courses taken is a binomial random variable. The beta distribution is a conjugate prior for the binomial distribution, and is thus a natural and tractable choice for modeling beliefs about θ_j (Casella and Berger 2002, pg. 325). Therefore, if we assume that a student updates their beliefs about θ_j using Bayes' rule, their posterior is also a Beta distribution:

$$P^g_{j,t+1} = \mathcal{B}(\alpha^g_{j,t+1}, \beta^g_{j,t+1}), \qquad (\alpha_{j,t+1}, \beta_{j,t+1}) = \begin{cases} (\alpha^g_{jt} + 1, \beta^g_{jt}) & \text{if } s^g_{jt} = 1 \\ (\alpha^g_{jt}, \beta^g_{jt} + 1) & \text{if } s^g_{jt} = 0 \end{cases}$$

To develop intuition about how this assumption influences specialization decisions, it's helpful to proceed with an illustrative, albeit somewhat contrived, parameterization. Let α^g_{j0} and β^g_{j0} denote the number of type g students who have succeeded and failed in field j at time 0, respectively. As an example, suppose a type g student is forming their initial beliefs about their probability of success in field j, and therefore asks five type g upperclassmen about their experiences in field j. If three of those type g upperclassmen passed the introductory course in field j, while two failed, then the student's initial belief parameters $(\alpha^g_{j0}, \beta^g_{j0})$ would equal $(3, 2)$.

Using this parameterization, the observed group-g success rate, μ^g_{j0}, is given by:

$$\mu^g_{j0} = \frac{\alpha^g_{j0}}{\alpha^g_{j0} + \beta^g_{j0}}.$$

This average is based on a sample of $n^g_{j0} = \alpha^g_{j0} + \beta^g_{j0}$ type-g students. Note that the beta distribution

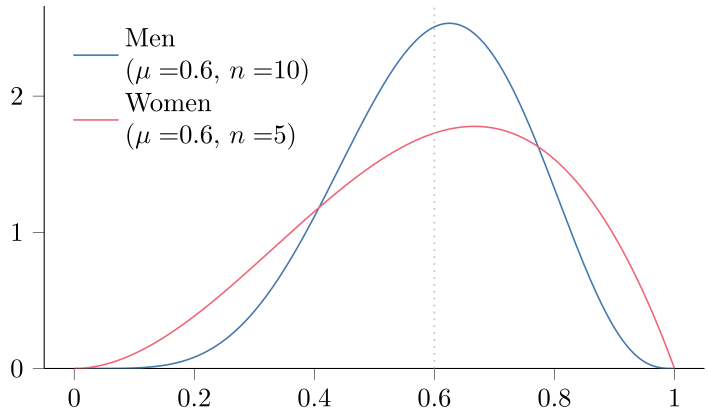

Figure 3. PDF of Beta distribution

parameters, α^g_{j0} and β^g_{j0}, can be expressed using the average success rate, μ^g_{j0}, and the sample size, n^g_{j0}:

$$\alpha^g_{j0} = \mu^g_{j0} n^g_{j0}, \qquad \beta^g_{j0} = (1 - \mu^g_{j0}) n^g_{j0}.$$

Therefore, an alternative parameterization of prior is given by $\mathcal{B}\left(\alpha^g_{j0}, \beta^g_{j0}\right) = \mathcal{B}\left(\mu^g_{j0} n^g_{j0}, (1 - \mu^g_{j0}) n^g_{j0}\right)$.

Assume the sample size of men is larger than that of women, but that the observed success rate is the same for the two genders:

$$n^m_{j0} > n^f_{j0}, \qquad \mu_{j0} = \mu^m_{j0} = \mu^w_{j0}.$$

Figure 3 provides a numerical example to illustrate how these assumptions affect the priors of men and women. Although women and men have the same probability of success in expectation, women have more initial uncertainty regarding their underlying abilities.

In section 3, I discuss the implications of differential initial beliefs. Before moving forward, it is helpful to highlight some shortcomings of the above illustrative example. First, I am being purposefully vague about what "success" means when agents form their initial priors, α^g_{j0} and β^g_{j0}. The example above, in which students solicit feedback from upperclassmen, is helpful for building intuition, and is consistent with the literature highlighting the importance of role-models in specialization decisions; see Porter and Serra (2020) for directly relevant empirical evidence. However, success could mean many things in this model. It could be the number of type g students who graduate into field j, the number of type g professors, the number of students attaining graduate degrees in field j, etc. I will discuss this matter further when calibrating the model. Additionally, while it is illustrative to use the parameters α^g_{j0} and β^g_{j0} to tally the total number of type g successes and failures, it is by no means necessary.

2.3. Optimal policy

To summarize the individual's problem, let h_t^g, P_t^g, m_t^g, ℓ_t^g denote the $J \times 1$ vectors of field-specific human capital, beliefs, study decisions, and labor decisions, respectively. A policy $\pi : (h_t^g, P_t^g) \to (m_t^g, \ell_t^g)$ is optimal if it maximizes lifetime expected utility:

$$\mathbb{E}^{\pi}\left[\sum_{t=0}^{\infty} \delta^t \left(\sum_{j=1}^{J} U_j(h_{jt}^g, w_j)\ell_{jt}^g\right) \middle| h_0^g, P_0^g\right], \tag{3}$$

given the following time constraint:

$$\sum_{j=1}^{J}(m_{jt}^g + \ell_{jt}^g) = 1, \qquad m_{jt}^g, \ell_{jt}^g \in \{0, 1\},$$

subject to the human capital accumulation and belief transition laws:

$$\begin{aligned} h_{jt+1}^g =& h_{jt}^g + \nu_j s_{jt}^g m_{jt}^g, \qquad s_{jt}^g \sim \text{Bernoulli}(\theta_j), \qquad \theta_j \sim P_{j0}^g \equiv \mathcal{B}(\alpha_{j0}^g, \beta_{j0}^g), \\ P_{j,t+1}^g =& \mathcal{B}(\alpha_{j,t+1}^g, \beta_{j,t+1}^g), \qquad (\alpha_{j,t+1}^g, \beta_{j,t+1}^g) = \begin{cases} (\alpha_{jt}^g + 1, \beta_{jt}^g) & \text{if } m_{jt}^g = 1 \text{ and } s_{jt}^g = 1 \\ (\alpha_{jt}^g, \beta_{jt}^g + 1) & \text{if } m_{jt}^g = 1 \text{ and } s_{jt}^g = 0 \\ (\alpha_{jt}^g, \beta_{jt}^g) & \text{if } m_{jt}^g = 0 \end{cases}. \end{aligned}$$

Alon and Fershtman (2019) characterize the optimal policy to the above problem. To apply their results, first assume the following initial condition to ensure optimality:

$$h_{j0} \le \alpha_{j0}^g \nu_j. \tag{4}$$

Let τ denote the optimal stopping rule defined over $\{s_{j1}^g, s_{j2}^g, \dots\}$. Define the field-$j$ index as the expected lifetime payoff an agent would receive if they commit to studying field-j given their state (h_{jt}^g, P_{jt}^g):

$$\mathcal{I}_{jt}(h_j^g, P_j^g) = \sup_{\tau \ge 0} \mathbb{E}^{\tau}\left[\sum_{t=0}^{\infty} \delta^t U_j(h_{jt}^g, w_j)\ell_{jt}^g \middle| (h_{j0}^g, P_{j0}^g) = (h_j^g, P_j^g)\right] \tag{5}$$

Define the graduation region of field j as the states where an agent committed to studying field j would choose to stop studying and enter the labor market:

$$\mathcal{G}_j(h_j^g, P_j^g) = \left\{(h_j^g, P_j^g) \middle| \arg\max_{\tau \ge 0} \mathbb{E}^{\tau}\left[\sum_{t=0}^{\infty} \delta^t U_j(h_{jt}^g, w_j)\ell_{jt}^g \middle| (h_j, P_j^g)\right] = 0\right\} \tag{6}$$

Then the following policy $\pi : (h_t^g, P_t^g) \to (m_t^g, \ell_t^g)$ is optimal:

1. At each $t \ge 0$, choose skill $j^* = \arg\max_{j \in J} \mathcal{I}_j$, breaking ties according to any rule
2. If $(h_{j^*}, P_{j^*}^g) \in \mathcal{G}_j$, then enter the labor market as a j^* specialist. Otherwise, study j^* for an

additional period.

3. Implications of model

An agent's specialization decision is affected by field, individual, and group characteristics. This section illustrates how these factors motivate an individual's behavior using a simplified version of the model. Section 3.1 develops a version of the model where an agent chooses between two completely symmetric fields. Simulations are explored in section 3.2 to illustrate how different factors influence decision making. In particular, I emphasize the role that beliefs play in an agent's specialization decision.

3.1. Choice between symmetric fields

Assume a student can choose to work or study in one of two fields, field X or field Y. Utility in field $j \in \{X, Y\}$ at time t is equal to income:

$$U_j(w_j, h_{jt}^g)\ell_{jt}^g = w_j h_{jt}^g \ell_{jt}^g \tag{7}$$

Wages in fields X and Y are equal and will be normalized to 1 ($w_X = w_Y = 1$), as are returns to successfully studying human capital ($\nu_X = \nu_Y = 1$). The student's underlying abilities in the two fields, θ_X and θ_Y, are both equal to 0.5. Therefore, the student has a 50% chance of passing any given field X or field Y course. Finally, I assume the student's beliefs about their own abilities in field X and Y are equal to the uniform prior:[3]

$$P_{X,0} = \mathcal{B}(\alpha_{X,0}, \beta_{X,0}) = \mathcal{B}(1,1), \qquad P_{Y,0} = \mathcal{B}(\alpha_{Y,0}, \beta_{Y,0}) = \mathcal{B}(1,1),$$

For tractability, I modify the assumption in equation (4) as follows:

$$h_{j0}^g = \nu_j \alpha_{j0}^g. \tag{8}$$

Equation (8) is consistent with the human capital accumulation function in equation (2), and allows for an analytical solution to the index and graduation region. The analytical results associated with (8) and its weaker form, (4), warrant discussion, and thus the remainder of this section will briefly discuss these derivations. For additional details, see Alon and Fershtman (2019).

Recall that the graduation index, (6), characterizes the states (α_j, β_j) where an individual would stop studying field-j and enter the labor market as a field-j specialist, ignoring all other fields. As noted in section 2.1, this is the field-specific stopping problem facing an individual agent. Under the weaker assumption $h_{j0} \leq \alpha_{j0}^g \nu_j$, the stopping problem for a given field is monotone.

3. Note that if $(\alpha, \beta) = (1, 1)$, the beta distribution $\mathcal{B}(\alpha, \beta)$ equals the uniform distribution over $[0, 1]$. This distribution can be seen graphically in figure 5a.

Intuitively, monotonicity means that an agent who wants to stop studying j at time t would still want to stop studying j at time $t+1$ if they continued on, independent of stochastic outcomes. Therefore, when an agent at time t chooses whether to stop studying j and enter the labor market as a field-j specialist, they compare their current expected lifetime payoff in j with the expected payoff in $t+1$. In other words, monotonicity implies the optimality of a one-step-look-ahead comparison for the field-specific stopping problem. An agent evaluating field j at time t will stop studying if their expected lifetime payoff in the current period exceeds their expected lifetime payoff in the $t+1$:

$$\frac{1}{1-\delta}w_j h_{jt} \geq \frac{1}{1-\delta}w_j \mathrm{E}[h_{j,t+1}]$$

Assuming (4) implies this stopping condition simplifies to:

$$\frac{1-\delta}{\delta} \geq \frac{\nu_j \alpha_{j0}^g + h_{jt} - h_{j0}}{h_{jt}(\alpha_{jt}^g + \beta_{jt}^g)}$$

Assuming (8) further simplifies the stopping condition:

$$\frac{1-\delta}{\delta} \geq \frac{1}{h_{jt}(\alpha_{jt}^g + \beta_{jt}^g)}. \tag{9}$$

Let c_{jt} denote the number of field-j courses a student has taken at time t.[4] Then the stopping condition (9) can be written as:

$$\frac{1-\delta}{\delta} \geq \frac{1}{h_{jt}(c_{jt} + \alpha_{j0}^g + \beta_{j0}^g)}.$$

Now, the optimal number of field-j courses is a deterministic function of the agent's initial beliefs:

$$c_j^* = \left\lceil \frac{\delta}{1-\delta} \right\rceil - (\alpha_{j0} + \beta_{j0})$$

This means that an agent who specializes in field j will take exactly c_j^* courses in field j, where c_j^* is a function of an agent's initial field-j beliefs.[5]

Therefore, assuming (8) and linear utility (7) implies the optimal field-j graduation region is given by:

$$\mathcal{G}_j(\alpha_{jt}, \beta_{jt}) = \left\{ \alpha_{jt}, \beta_{jt} \,\middle|\, \frac{\delta}{1-\delta} \leq \alpha_{jt} + \beta_{jt} \right\}$$

4. Specifically, c_{jt} equals the total number of successes, $\alpha_{jt}^g - \alpha_{j0}^g$, plus the total number of failures equals $\beta_{jt}^g - \beta_{j0}^g$.
5. Additionally, this assumption, and to a lesser extent the relaxed version in equation (4), ties together beliefs and initial levels of human capital. This may have relevant implications when applying this model to specific groups. Specifically, if we make assumptions about initial group-based beliefs in line with section 2.2, equation (8) implies that (1) women begin with lower levels of initial human capital than men, and (2) women study for more periods than men in the fields in which they are more uncertain. The first implication is not out of line with the literature, and the second implication here has some empirical support. It is worth being aware of these implications moving forward.

In this example, note that $\mathcal{G}_Y = \mathcal{G}_X$. Index in the graduation region given by $\frac{h_{jt}}{1-\delta}$. Index when not in graduation region given by Binomial distribution with parameters $\left(c_j^* - c_j, \frac{h_{jt}}{\nu(c_{jt}+\alpha_{j0}+\beta j0)}\right)$. The index from equation (5) can be simplified to:

$$\mathcal{I}_{jt}(h_{jt}, \alpha_{jt}, \beta_{jt}) = \begin{cases} \frac{w_{jt}h_{jt}}{1-\delta} & \text{if } \{\alpha_{jt}, \beta_{jt}\} \in \mathcal{G}_j, \\ \frac{w_{jt}h_{jt}}{1-\delta}\left\lceil \frac{\lceil \frac{\delta}{1-\delta} \rceil \delta^{\lceil \frac{\delta}{1-\delta} \rceil - c_{jt} - \alpha_{j0} - \beta_{j0}}}{c_{jt}+\alpha_{j0}+\beta_{j0}} \right\rceil & \text{if } \{\alpha_{jt}, \beta_{jt}\} \notin \mathcal{G}_j \end{cases}$$

3.2. Simulations

Each subplot in figure 4 plots the fraction of simulated agents choosing to study field X or field Y at each time period t. Recall that agents studying field j at time t will either pass and successfully accumulate human capital ($s_{jt}^g = 1$) or they will fail ($s_{jt}^g = 0$), where $s_{jt}^g \sim \text{Bernoulli}(\theta_j)$. The student then updates their beliefs about their own underlying ability, θ_j. Line movements in figure 4 are caused by agents switching fields in response to updated beliefs. Eventually, students will specialize in one field and enter the labor market as a field-X or field-Y specialist. Specialization in figure 4 is represented by a flattening of the curve; once a student has made their specialization decision, they no longer switch fields. For clarity, the line for any field j ends once any agent specializing in j stops studying and enters the labor market. Therefore, the length of the lines in figure 4 denote the minimum amount of time an agents spends studying before becoming a field-j specialist.

The baseline scenarios in figures 4a and 4b illustrate these dynamics. Figure 4a plots the baseline scenario for 10,000 simulations; figure 4b plots the first 50 of these simulations. Our first takeaway is that the agent's specialization decision in the baseline is effectively a coin flip. This is most clearly seen in figure 4a; at all time periods, approximately 50% of the agents are studying field X and and 50% are studying field Y. This should be expected, as fields X and Y are completely symmetric.

Some of the more subtle decision dynamics can only be seen with fewer observations. Therefore, 4b zooms in on the first 50 of these simulations. Note that the fraction of students studying field X or field Y moves in early periods, but flattens out in later periods. This is because students at the beginning of their education will update their beliefs in response to course outcomes. These updated beliefs may cause students to switch fields, shifting the composition of simulated agents studying X or Y. In later periods, simulated agents have made their specialization decision and no longer switch fields. This specialization is represented by the flattening of the lines in figure 4b. As in 4a, approximately 50% of agents specialize in field X, and 50% specialize in field Y.

The remainder of figure 4 plots variations of the baseline for $N = 10,000$ simulations. In figure 4c, wages in field Y are 50% higher than wages in field X. All other variables are identical to the baseline scenario. Unsurprisingly, higher wages in Y drive specialization into that field. Because

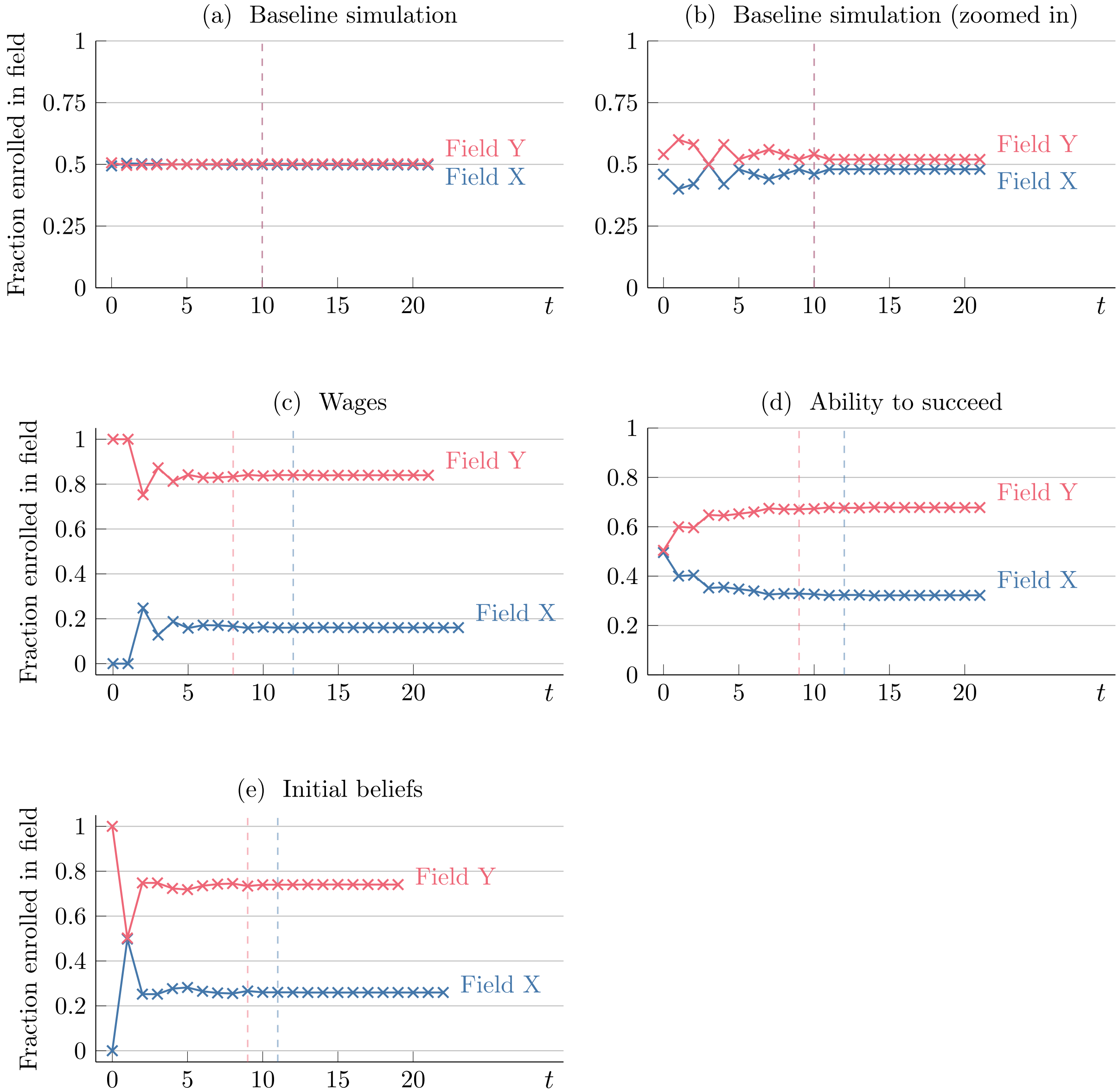

Figure 4. Simulations of simple version of model. Figure (a) presents the baseline for $N = 10,000$ simulations; figure (b) does the same for the first 50 simulations. The remaining figures have $N = 10,000$ simulations. Figure (c) repeats the simulations for $w_X = 1$ and $w_Y = 1.5$. Figure (d) repeats the simulations when $\theta_X = 0.4$ and $\theta_Y = 0.6$. Figure (e) repeats the simulations when $(\alpha_{X0}, \beta_{X0}) = (1, 1)$ and $(\alpha_{Y0}, \beta_{Y0}) = (2, 2)$.

the expected lifetime payoff is so much higher, approximately 80% of agents choose to specialize in Y.

The field X line in figure 4c is longer than the field Y line, implying that agents who specialize in X spend more time in school. To understand why this happens, first note that all agents begin their education studying Y because of the higher relative wages. However, after two periods, a large

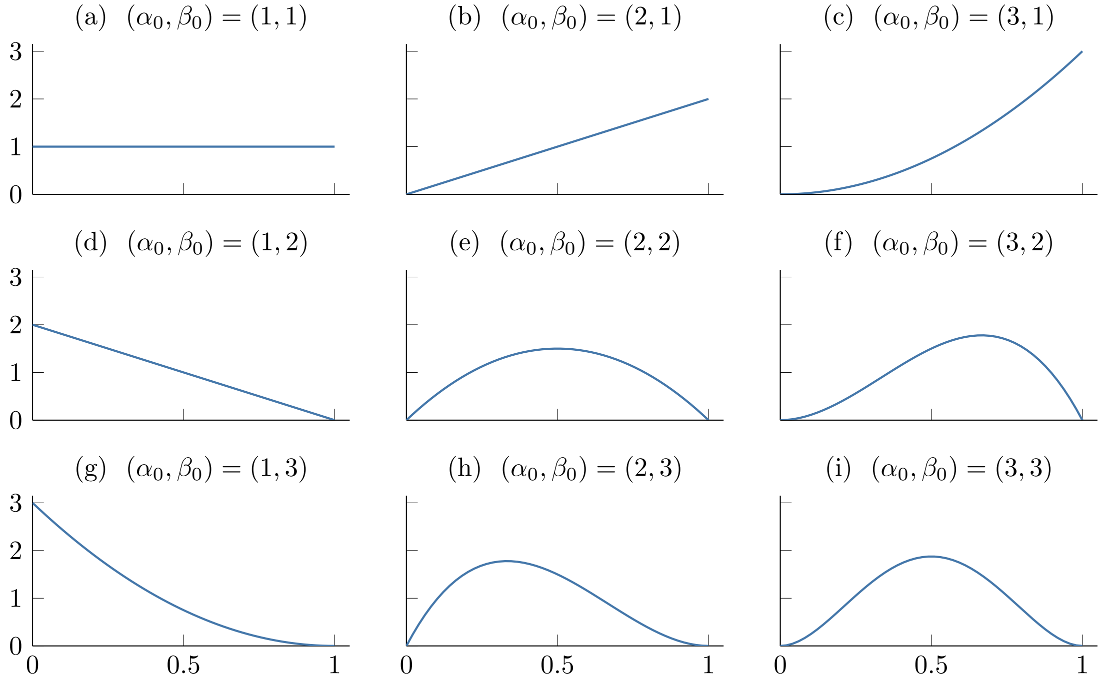

Figure 5. Evolution of the Beta distribution $\mathcal{B}(\alpha_0, \beta_0)$ for different values of (α_0, β_0).

fraction of agents switch from studying Y to studying X. This is due to agents (randomly) failing their first two courses in field Y, and switching into field X. The reason agents switch fields can be seen by the evolution of their belief distributions, shown in figure 5. The student's initial belief distribution is plotted in figure 5a. A student that fails their first course in field Y updates their beliefs about their underlying ability in Y to the distribution plotted in figure 5d; if they fail their second course in Y, they update their beliefs to 5g. As we can see from figure 5g, a student that fails their first two classes in Y will believe they likely have a lower ability in that field. As such, if they choose to specialize in Y, they would not expect to successfully accumulate much human capital over the course of their studies, implying a lower expected lifetime payoff. As a result, these agents switch to studying field X, in spite of the lower wages. This switching leads to more overall time in school; as mentioned above, equation (8) implies that the number of periods an agents spends studying field X or Y before becoming a specialist is a deterministic function of initial beliefs. Agents' initial beliefs about their abilities in X and Y are the same, and as such, agents specializing in either X or Y will study their chosen discipline for the same number of periods. However, because all agents spend their first two periods studying Field Y, those who specialize in Field X will study for a minimum of two more periods.

Figure 4d augments the baseline scenario so agents have a higher ability in field Y. Specifically, probability of success in any given field X course, θ_X, equals 0.4, whereas the probability of success in field Y is given by $\theta_Y = 0.6$. Unsurprisingly, a higher ability in field Y drives specialization into that field.

We now turn to the impact of differential priors on specialization dynamics, plotted in Figure 4e. I assume simulated agents are initially more certain about their abilities in field Y relative to field X. Specifically, I assume a student's initial prior about their ability in Y is given by $P_{Y0} = \mathcal{B}(2, 2)$; this corresponds to the distribution plotted in figure 5e. Their initial prior about their ability in X continues to equal the uniform distribution, $P_{X0} = \mathcal{B}(1, 1)$. Note that agents have the same belief about their probability of success in X and Y in expectation. However, the variances of the initial distributions suggest that agents have more certainty about their underlying ability in field Y than in field X.

The first consequence of this assumption is that agents specializing in field X study for more periods than those specializing in field Y, as shown in figure 4e. As mentioned above, equation (8) implies that the number of periods an agent spends studying j before specializing in that field is a deterministic function of the agent's initial beliefs. Agents have more initial uncertainty about their abilities in X than in Y, and therefore they will study X for more periods before specializing in that field. The second takeaway from 4e is that all agents begin their education studying field Y. Agents know that if they become field Y specialists, they will finish their education earlier, and begin earning an income sooner. The prospect of ending their education earlier drives agents to initially study field Y.

The key takeaway from figure 4e is that increased initial certainty about field Y abilities causes more agents to specialize in field Y. [6] Although agents are equally likely to succeed in fields X and Y, and although the payoffs for specializing in these fields are the same, differential initial beliefs about underlying abilities drives the majority of simulated agents to specialize in field Y. Thus, initial beliefs play a key role in specialization decisions. I now consider how those beliefs are formed, and the consequences of forming those beliefs based on existing group outcomes.

4. Estimation

Data sources

Characteristics of all U.S. postsecondary institutions are reported in the Integrated Postsecondary Education Data System (IPEDS) data. These data are collected annually by the National Center for Educational Statistics and describe the universe of institutions that participate in federal student financial aid programs. The empirical motivation for this analysis relies on the IPEDS Completion Surveys, which describe all degrees and certificates awarded at postsecondary institutions by field of study, gender, and race.[7] University-level graduation rates are estimated using the IPEDS

6. It's worth emphasizing that this is not driven by risk aversion across fields; assuming linear utility in (7) ensures that agents are risk neutral across fields. Rather, concavity due to discounting ensures that agents are risk averse across time.

7. For more details on the IPEDS series, please visit `https://nces.ed.gov/ipeds/`. IPEDS data are available from 1986 until the present, though I begin empirical analysis in 1990 due to changes in how fields of study are classified. For earlier data, such as those used in Figure 1, I supplement the IPEDS series with data from Snyder (1993). However, it is worth noting that the predecessor to IPEDS series is the Higher Education General Information

graduation surveys. Details on IPEDS graduation surveys are in the Appendix.

Data on field-switching are from the 2012/17 Beginning Postsecondary Students Longitudinal Study, conducted by the National Center for Education Statistics. This longitudinal study collects education and employment data from a nationally representative sample of first time beginning postsecondary students. Respondents are initially surveyed in 2011-2012, the beginning of their postsecondary studies. Follow-up surveys were conducted three and six years after they began their studies.[8]

Individual-level data on degree completions are from the American Community Survey (ACS), as accessed using Ruggles et al. (2020). Beginning in 2009, the ACS began recording up to two undergraduate field of study. I follow the data selection procedure outlined in Appendix A of Sloane, Hurst, and Black (2020).

4.1. Identification

The first key parameter for this analysis are the number of fields of study, J. I utilize both IPEDS and ACS data for this analysis, and therefore wanted a classification system that applies across both fields.

Appendix

Table A1. ACS degree fields

Broad classification	Detailed classification
Agriculture	General Agriculture
Agriculture	Agriculture Production and Management
Agriculture	Agricultural Economics
Agriculture	Animal Sciences
Agriculture	Food Science
Agriculture	Plant Science and Agronomy
Agriculture	Soil Science
Agriculture	Miscellaneous Agriculture
Environment and Natural Resources	Environmental Science
Environment and Natural Resources	Forestry

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Survey (HEGIS), available through the International Archive of Education Data at University of Michigan (https://www.icpsr.umich.edu/web/ICPSR/series/00030). As such, I refer the reader to the HEGIS series for a more detailed portrait of postsecondary education statistics than available in Snyder (1993).

8. For additional information, see https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2020504. Restricted-use licenses are required for access to BPS microdata. However, data aggregates and simple regression analysis are available through the NCES PowerStats DataLab.

Table A1. ACS degree fields

Broad classification	Detailed classification
Environment and Natural Resources	Natural Resources Management
Architecture	Architecture
Area, Ethnic, and Civilization Studies	Area, Ethnic, and Civilization Studies
Communications	Communications
Communications	Journalism
Communications	Mass Media
Communications	Advertising and Public Relations
Communication Technologies	Communication Technologies
Computer and Information Sciences	Computer and Information Systems
Computer and Information Sciences	Computer Programming and Data Processing
Computer and Information Sciences	Computer Science
Computer and Information Sciences	Information Sciences
Computer and Information Sciences	Computer Information Management and Security
Computer and Information Sciences	Computer Networking and Telecommunications
Cosmetology Services and Culinary Arts	Cosmetology Services and Culinary Arts
Education Administration and Teaching	General Education
Education Administration and Teaching	Educational Administration and Supervision
Education Administration and Teaching	School Student Counseling
Education Administration and Teaching	Elementary Education
Education Administration and Teaching	Mathematics Teacher Education
Education Administration and Teaching	Physical and Health Education Teaching
Education Administration and Teaching	Early Childhood Education
Education Administration and Teaching	Science and Computer Teacher Education
Education Administration and Teaching	Secondary Teacher Education
Education Administration and Teaching	Special Needs Education
Education Administration and Teaching	Social Science or History Teacher Education
Education Administration and Teaching	Teacher Education: Multiple Levels
Education Administration and Teaching	Language and Drama Education
Education Administration and Teaching	Art and Music Education
Education Administration and Teaching	Miscellaneous Education
Engineering	General Engineering
Engineering	Aerospace Engineering
Engineering	Biological Engineering
Engineering	Architectural Engineering

Continued on next page

Table A1. ACS degree fields

Broad classification	Detailed classification
Engineering	Biomedical Engineering
Engineering	Chemical Engineering
Engineering	Civil Engineering
Engineering	Computer Engineering
Engineering	Electrical Engineering
Engineering	Engineering Mechanics, Physics, and Science
Engineering	Environmental Engineering
Engineering	Geological and Geophysical Engineering
Engineering	Industrial and Manufacturing Engineering
Engineering	Materials Engineering and Materials Science
Engineering	Mechanical Engineering
Engineering	Metallurgical Engineering
Engineering	Mining and Mineral Engineering
Engineering	Naval Architecture and Marine Engineering
Engineering	Nuclear Engineering
Engineering	Petroleum Engineering
Engineering	Miscellaneous Engineering
Engineering Technologies	Engineering Technologies
Engineering Technologies	Engineering and Industrial Management
Engineering Technologies	Electrical Engineering Technology
Engineering Technologies	Industrial Production Technologies
Engineering Technologies	Mechanical Engineering Related Technologies
Engineering Technologies	Miscellaneous Engineering Technologies
Linguistics and Foreign Languages	Linguistics and Comparative Language and Liter...
Linguistics and Foreign Languages	French, German, Latin and Other Common Foreign...
Linguistics and Foreign Languages	Other Foreign Languages
Family and Consumer Sciences	Family and Consumer Sciences
Law	Court Reporting
Law	Pre-Law and Legal Studies
English Language, Literature, and Composition	English Language and Literature
English Language, Literature, and Composition	Composition and Speech
Liberal Arts and Humanities	Liberal Arts
Liberal Arts and Humanities	Humanities
Library Science	Library Science

Continued on next page

Table A1. ACS degree fields

Broad classification	Detailed classification
Biology and Life Sciences	Biology
Biology and Life Sciences	Biochemical Sciences
Biology and Life Sciences	Botany
Biology and Life Sciences	Molecular Biology
Biology and Life Sciences	Ecology
Biology and Life Sciences	Genetics
Biology and Life Sciences	Microbiology
Biology and Life Sciences	Pharmacology
Biology and Life Sciences	Physiology
Biology and Life Sciences	Zoology
Biology and Life Sciences	Neuroscience
Biology and Life Sciences	Miscellaneous Biology
Mathematics and Statistics	Mathematics
Mathematics and Statistics	Applied Mathematics
Mathematics and Statistics	Statistics and Decision Science
Military Technologies	Military Technologies
Interdisciplinary and Multi-Disciplinary Studi...	Interdisciplinary and Multi-Disciplinary Studi...
Interdisciplinary and Multi-Disciplinary Studi...	Intercultural and International Studies
Interdisciplinary and Multi-Disciplinary Studi...	Nutrition Sciences
Interdisciplinary and Multi-Disciplinary Studi...	Mathematics and Computer Science
Interdisciplinary and Multi-Disciplinary Studi...	Cognitive Science and Biopsychology
Interdisciplinary and Multi-Disciplinary Studi...	Interdisciplinary Social Sciences
Physical Fitness, Parks, Recreation, and Leisure	Physical Fitness, Parks, Recreation, and Leisure
Philosophy and Religious Studies	Philosophy and Religious Studies
Theology and Religious Vocations	Theology and Religious Vocations
Physical Sciences	Physical Sciences
Physical Sciences	Astronomy and Astrophysics
Physical Sciences	Atmospheric Sciences and Meteorology
Physical Sciences	Chemistry
Physical Sciences	Geology and Earth Science
Physical Sciences	Geosciences
Physical Sciences	Oceanography
Physical Sciences	Physics
Physical Sciences	Materials Science

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Table A1. ACS degree fields

Broad classification	Detailed classification
Physical Sciences	Multi-disciplinary or General Science
Nuclear, Industrial Radiology, and Biological ...	Nuclear, Industrial Radiology, and Biological ...
Psychology	Psychology
Psychology	Educational Psychology
Psychology	Clinical Psychology
Psychology	Counseling Psychology
Psychology	Industrial and Organizational Psychology
Psychology	Social Psychology
Psychology	Miscellaneous Psychology
Criminal Justice and Fire Protection	Criminal Justice and Fire Protection
Public Affairs, Policy, and Social Work	Public Administration
Public Affairs, Policy, and Social Work	Public Policy
Public Affairs, Policy, and Social Work	Human Services and Community Organization
Public Affairs, Policy, and Social Work	Social Work
Social Sciences	General Social Sciences
Social Sciences	Economics
Social Sciences	Anthropology and Archeology
Social Sciences	Criminology
Social Sciences	Geography
Social Sciences	International Relations
Social Sciences	Political Science and Government
Social Sciences	Sociology
Social Sciences	Miscellaneous Social Sciences
Construction Services	Construction Services
Electrical and Mechanic Repairs and Technologies	Electrical and Mechanic Repairs and Technologies
Transportation Sciences and Technologies	Transportation Sciences and Technologies
Fine Arts	Fine Arts
Fine Arts	Drama and Theater Arts
Fine Arts	Music
Fine Arts	Visual and Performing Arts
Fine Arts	Commercial Art and Graphic Design
Fine Arts	Film, Video and Photographic Arts
Fine Arts	Art History and Criticism
Fine Arts	Studio Arts

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Table A1. ACS degree fields

Broad classification	Detailed classification
Fine Arts	Miscellaneous Fine Arts
Medical and Health Sciences and Services	General Medical and Health Services
Medical and Health Sciences and Services	Communication Disorders Sciences and Services
Medical and Health Sciences and Services	Health and Medical Administrative Services
Medical and Health Sciences and Services	Medical Assisting Services
Medical and Health Sciences and Services	Medical Technologies Technicians
Medical and Health Sciences and Services	Health and Medical Preparatory Programs
Medical and Health Sciences and Services	Nursing
Medical and Health Sciences and Services	Pharmacy, Pharmaceutical Sciences, and Adminis...
Medical and Health Sciences and Services	Treatment Therapy Professions
Medical and Health Sciences and Services	Community and Public Health
Medical and Health Sciences and Services	Miscellaneous Health Medical Professions
Business	General Business
Business	Accounting
Business	Actuarial Science
Business	Business Management and Administration
Business	Operations, Logistics and E-Commerce
Business	Business Economics
Business	Marketing and Marketing Research
Business	Finance
Business	Human Resources and Personnel Management
Business	International Business
Business	Hospitality Management
Business	Management Information Systems and Statistics
Business	Miscellaneous Business and Medical Administration
History	History
History	United States History

References

Aigner, Dennis J., and Glen G. Cain. 1977. "Statistical Theories of Discrimination in Labor Markets." *ILR Review* 30, no. 2 (January): 175–87. https://www.jstor.org/stable/2522871.

Alon, Titan, and Daniel Fershtman. 2019. "A Theory of Human Capital Specialization." https://drive.google.com/file/d/1LXjGzQSAxl_ijlPMDE11dTPzvRLkV92q/view?usp=sharing.

Altonji, Joseph G. 1993. “The Demand for and Return to Education When Education Outcomes are Uncertain.” *Journal of Labor Economics* 11, no. 1 (January): 48–83. doi:10.1086/298317.

Altonji, Joseph G., Peter Arcidiacono, and Arnaud Maurel. 2016. “The Analysis of Field Choice in College and Graduate School: Determinants and Wage Effects.” Chap. 7 in *Handbook of the Economics of Education,* edited by Eric A. Hanushek, Stephen Machin, and Ludger Woessmann, 5:305–96. doi:10.1016/B978-0-444-63459-7.00007-5.

Altonji, Joseph G., Erica Blom, and Costas Meghir. 2012. “Heterogeneity in Human Capital Investments: High School Curriculum, College Major, and Careers.” *Annual Review of Economics* 4, no. 1 (July): 185–223. doi:10.1146/annurev-economics-080511-110908.

Arcidiacono, Peter. 2004. “Ability sorting and the returns to college major.” *Journal of Econometrics* 121 (2): 343–75. doi:10.1016/j.jeconom.2003.10.010.

Arcidiacono, Peter, Esteban Aucejo, Arnuad Maurel, and Tyler Ransom. 2016. “College Attrition and the Dynamics of Information Revelation.” NBER Working Paper no. 22325. https://www.nber.org/papers/w22325.pdf.

Arcidiacono, Peter, V. Joseph Hotz, Arnaud Maurel, and Teresa Romano. Forthcoming. “Ex-ante Returns and Occupational Choice.” *Journal of Political Economy.* doi:10.1086/710559.

Arcidiacono, Peter, and Michael Lovenheim. 2016. “Affirmative Action and the Quality-Fit Tradeoff.” *Journal of Economic Literature* 54 (1): 3–51. doi:10.1257/jel.54.1.3.

Arrow, Kenneth J. 1972. “Some Mathematical Models of Race in the Labor Market.” In *Racial Discrimination in Economic Life,* Anthony H. Pascal, 187–203. Lexington, MA: D.C. Health.

Becker, Gary S. 1962. “Investment in Human Capital: A Theoretical Analysis.” *Journal of Political Economy* 7, no. 5 (October): 9–49. doi:10.1086/258724.

Ben-Porath, Yoram. 1967. “The Production of Human Capital and the Life Cycle of Earnings.” *Journal of Political Economy* 75, no. 4 (August): 352–65. doi:10.1086/259291.

Black, Dan A., Amelia M. Haviland, Seth G. Sanders, and Lowell J. Taylor. 2008. “Gender Wage Disparities among the Highly Educated.” *Journal of Human Resources* 43 (3): 630–59. doi:10.3368/jhr.43.3.630.

Blau, Francine D., and Lawrence M. Kahn. 2017. “The Gender Wage Gap: Extent, Trends, and Explanations.” *Journal of Economic Literature* 55 (3): 789–865. doi:10.1257/jel.20160995.

Bronson, Mary Ann. 2015. “Degrees are Forever: Marriage, Educational Investment, and Lifecycle Labor Decisions of Men and Women.”

Casella, George, and Roger L. Berger. 2002. *Statistical Inference.* Second Edition. Belmont, CA: Brooks/Cole Cengage Learning.

Coate, Stephen, and Glenn C. Loury. 1993. "Will Affirmative-Action Policies Eliminate Negative Stereotypes?" *The American Economic Review* 83, no. 5 (December): 1220–40. https://www.jstor.org/stable/2117558.

Dickson, Lisa. 2010. "Race and Gender Differences in College Major Choice." *The Annals of the American Academy of Political and Social Science* 627:108–24. doi:10.1177/0002716209348747.

England, Paula, and Sun Li. 2006. "Desegregation Stalled: The Changing Gender Composition of College Majors, 1971-2002." *Gender & Society* 20 (5): 657–77. doi:10.1177/0891243206290753.

Hsieh, Chang-Tai, Erik Hurst, Charles I. Jones, and Peter J. Klenow. 2019. "The Allocation of Talent and U.S. Economic Growth." *Econometrica* 87, no. 5 (September): 1439–74. doi:10.3982/ECTA11427.

Kugler, Adriana D., Catherine H. Tinsley, and Olga Ukhaneva. 2017. "Choice of Majors: Are Women Really Different from Men?" NBER Working Paper No. 23735. https://www.nber.org/papers/w23735.

Lim, Jaegeum, and Jonathan Meer. 2020. "Persistent Effects of Teacher-Student Gender Matches." *Journal of Human Resources* 55 (3): 809–35. doi:10.3368/jhr.55.3.0218-9314R4.

Lundberg, Shelly J., and Richard Startz. 1983. "Private Discrimination and Social Intervention in Competitive Labor Markets." *American Economic Review* 73, no. 3 (June): 340–7. https://www.jstor.org/stable/1808117.

Mincer, Jacob. 1974. *Schooling, Experience, and Earnings.* New York, NY: National Bureau of Economic Research. http://www.nber.org/books/minc74-1.

Phelps, Edmund S. 1972. "The Statistical Theory of Racism and Sexism." *American Economic Review* 62, no. 4 (September): 659–61. https://www.jstor.org/stable/1806107.

Porter, Catherine, and Danila Serra. 2020. "Gender Differences and the Choice of Major: The Importance of Female Role Models." *American Economic Journal - Applied Economics* 12 (3): 226–254. doi:https://doi.org/10.1257/app.20180426.

Rosen, Sherwin. 1983. "Specialization and Human Capital." *Journal of Labor Economics* 1, no. 1 (January): 43–9. doi:10.1086/298003.

Roy, A. D. 1951. "Some Thoughts on the Distribution of Earnings." *Oxford Economic Papers* 3 (2): 135–146. http://www.jstor.org/stable/2662082.

Ruggles, Steven, Sarah Flood, Ronald Goeken, Josiah Grover, Erin Meyer, Jose Pacas, and Matthew Sobek. 2020. *IPUMS USA: Version 10.0.* Dataset. Minneapolis, MN. doi:10.18128/D010.V10.0.

Sloane, Carolyn, Erik Hurst, and Dan Black. 2020. "A Cross-Cohort Analysis of Human Capital Specialization and the College Gender Wage Gap." NBER Working Paper No. 26348. https://www.nber.org/papers/w26348.

Snyder, Thomas D., ed. 1993. *120 Years of American Education: a Statistical Portrait.* National Center for Education Statistics. https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=93442.

Wiswall, Matthew, and Basit Zafar. 2015. "Determinants of College Major Choice: Identification Using an Information Experiment." *The Review of Economic Studies* 82, no. 2 (April): 791–824. doi:10.1093/restud/rdu044.

———. 2018. "Preference for the Workplace, Investment in Human Capital, and Gender." *Quarterly Journal of Economics* 133, no. 1 (February): 457–507. doi:10.1093/qje/qjx035.

Zafar, Basit. 2013. "College Major Choice and the Gender Gap." *Journal of Human Resources* 48 (3): 545–95. doi:10.3368/jhr.48.3.545.