Group-based beliefs and human capital specialization

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Macro Lunch Presentation
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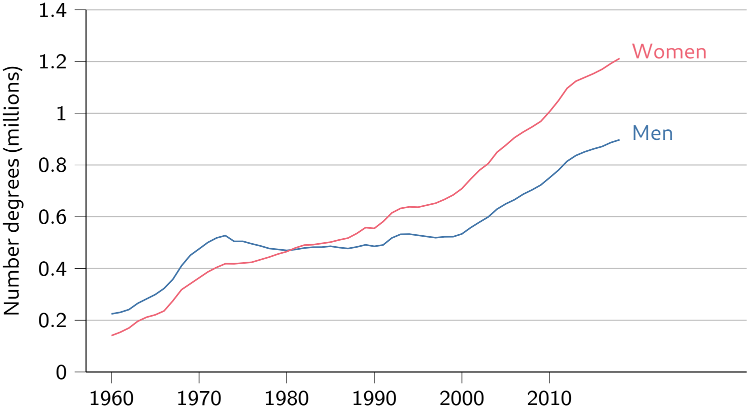
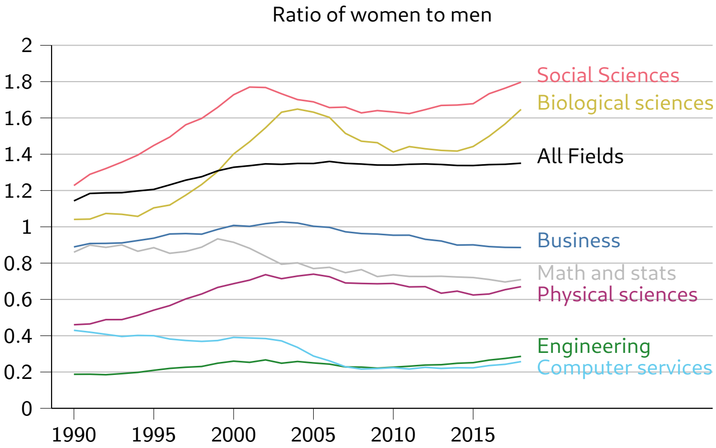
Increased attainment of Bachelor's degrees

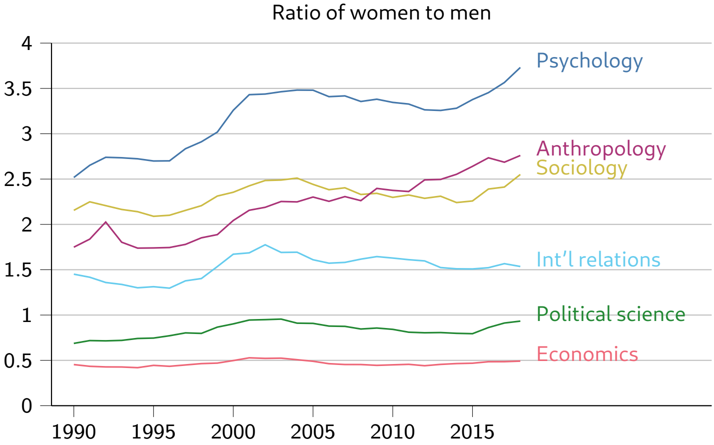


Number of Bachelor's Degrees awarded in US 4-year colleges. Source: IPEDS; Snyder (2013).

Historically male-dominated fields



Social Sciences



Outline

Model preliminaries

Individuals endowed with:

h_{j0}: Skill-j specific human capital ($j = 0, \ldots, J$)

θ_j: Unknown probability of success in j

P_{j0}: Prior beliefs about θ_j

Time constraint: At each t, individuals can choose to either study (m_{jt}) or work (ℓ_{jt}) in one field $j \in \{1, \ldots, J\}$:

$$\sum_{j=0}^{J} (m_{jt} + \ell_{jt}) = 1, \qquad m_{jt}, \ell_{jt} \in \{0, 1\}$$

- Studying may accumulates field-j human capital and reveals information about underlying probability of success in j
- If you work, you receive wage w_j

Enter labor market at time t in skill-j to maximize expected lifetime payoff:

$$\frac{\delta^t}{1-\delta} U_j(w_j, h_{jt}) \ell_{jt} = \frac{\delta^t}{1-\delta} w_j h_{jt} \ell_{jt}$$

Evolution of human capital accumulation and beliefs

Students studying skill-j at time t pass the course with probability θ_j:

$$s_{jt} \sim \text{Bernoulli}(\theta_j)$$

- Accumulate human capital if they pass the course:

$$h_{jt+1} = h_{jt} + \nu_j s_{jt} mjt$$

- Beliefs about θ_j evolve:

$$P_{j,t+1} = \Pi_j(P_{jt}, s_{jt})$$

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Key: How are **priors** formed, and how are they **updated**?

Belief distribution

Initial prior drawn from Beta distribution

$$P_{j0} = \mathcal{B}(\alpha_{j0}, \beta_{j0})$$

Update according to Bayes Rule $\implies$ posterior drawn from Beta distribution:

$$P_{j,t+1} = \mathcal{B}(\alpha_{j,t+1}, \beta_{j,t+1}), \qquad (\alpha_{j,t+1}, \beta_{j,t+1}) = \begin{cases} (\alpha_{jt} + 1, \beta_{jt}) & \text{if } a_{jt} = 1 \\ (\alpha_{jt}, \beta_{jt} + 1) & \text{if } a_{jt} = 0 \end{cases}$$

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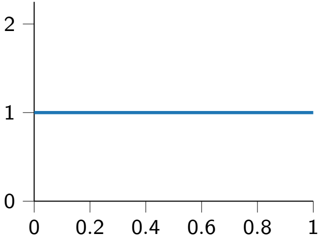
Example: $\alpha_0 = 1$, $\beta_0 = 1$

Beliefs $p(\theta|\alpha, \beta)$



Return: simulation set-up
Return: baseline simulation

Belief distribution

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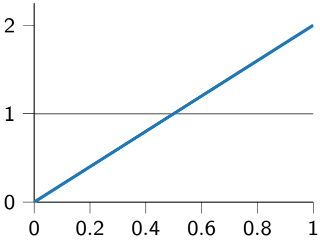
Example: $\alpha_0 = 1$, $\beta_0 = 1$

- success at $t = 1 \implies \alpha_1 = 2$, $\beta_1 = 1$

Beliefs $p(\theta|\alpha, \beta)$



Belief distribution

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Update according to Bayes Rule $\implies$ posterior drawn from Beta distribution:

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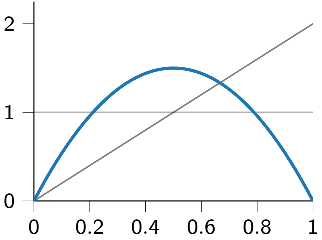
Example: $\alpha_0 = 1$, $\beta_0 = 1$

- success at $t = 1 \implies \alpha_1 = 2$, $\beta_1 = 1$
- failure at $t = 2 \implies \alpha_1 = 2$, $\beta_1 = 2$

Beliefs $p(\theta|\alpha, \beta)$



Return: simulation

Belief distribution

Initial prior drawn from Beta distribution

$$P_{j0} = \mathcal{B}(\alpha_{j0}, \beta_{j0})$$

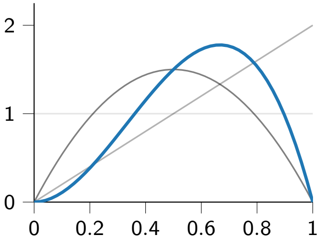
Update according to Bayes Rule $\Longrightarrow$ posterior drawn from Beta distribution:

$$P_{j,t+1} = \mathcal{B}(\alpha_{j,t+1}, \beta_{j,t+1}), \qquad (\alpha_{j,t+1}, \beta_{j,t+1}) = \begin{cases} (\alpha_{jt}+1, \beta_{jt}) & \text{if } a_{jt} = 1 \\ (\alpha_{jt}, \beta_{jt}+1) & \text{if } a_{jt} = 0 \end{cases}$$

Example: $\alpha_0 = 1$, $\beta_0 = 1$

- success at $t = 1 \Longrightarrow \alpha_1 = 2$, $\beta_1 = 1$
- failure at $t = 2 \Longrightarrow \alpha_1 = 2$, $\beta_1 = 2$
- success at $t = 3 \Longrightarrow \alpha_1 = 3$, $\beta_1 = 2$

Beliefs $p(\theta|\alpha, \beta)$



Group-based parametrization

Consider group-based beliefs about abilities:

- Each individual has a group type: $g \in \{m, f\}$
- Students form beliefs, P_{j0}, based on previously observed group successes

Simple parameterization:

α^g_{j0}: Number of type-g students who have succeeded in j

β^g_{j0}: Number of type-g students who have failed in j

$\Longrightarrow$ Observed success rate:

$$\mu^g_{j0} = \frac{\alpha^g_{j0}}{\alpha^g_{j0} + \beta^g_{j0}}.$$

This average is based on a sample size of type g students:

$$n^g_{j0} = \alpha^g_{j0} + \beta^g_{j0}$$

Group-based prior beliefs about probability of success in skill-j courses, θ_j:

$$\mathcal{B}\left(\alpha^g_{j0}, \beta^g_{j0}\right) \quad \Longrightarrow \quad \mathcal{B}\left(\mu^g_{j0} n^g_{j0}, (1 - \mu^g_{j0}) n^g_{j0}\right)$$

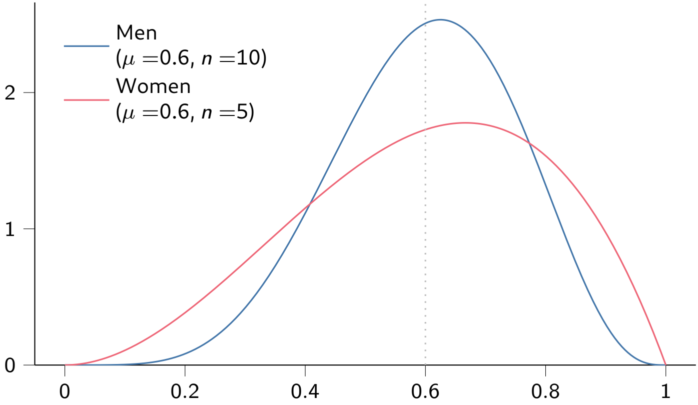
Group-based belief distribution

Suppose there are more men then women in field j:

$$n_{j0}^{m} > n_{j0}^{f}$$

But the observed success rate is the same for the two groups:

$$\mu_{j0} = \mu_{j0}^{m} = \mu_{j0}^{w}$$



Individual problem

A policy $\pi : (h_t, P_t^g) \to (s_t, \ell_t)$ is optimal if it maximizes:

$$\mathbb{E}^\pi \left[\sum_{t=0}^{\infty} \delta^t \left(\sum_{j=1}^{J} h_{jt} w_j \ell_{jt} \right) \middle| \left((h_{10}, P_{10}^g), \ldots, (h_{J0}, P_{J0}^g) \right) \right]$$

Subject to the human capital accumulation and belief transition laws:

$$h_{jt+1} = h_{jt} + \nu_j s_{jt} m_{jt}, \qquad s_{jt} \sim \text{Bernoulli}(\theta_j), \qquad \theta_j \sim P_{j0}^g \equiv \mathcal{B}(\alpha_{j0}^g, \beta_{j0}^g),$$

$$P_{j,t+1}^g = \mathcal{B}(\alpha_{j,t+1}^g, \beta_{j,t+1}^g), \qquad (\alpha_{j,t+1}^g, \beta_{j,t+1}^g) = \begin{cases} (\alpha_{jt}^g + 1, \beta_{jt}^g) & \text{if } m_{jt}^g = 1 \text{ and } s_{jt}^g = 1 \\ (\alpha_{jt}^g, \beta_{jt}^g + 1) & \text{if } m_{jt}^g = 1 \text{ and } s_{jt}^g = 0 \\ (\alpha_{jt}^g, \beta_{jt}^g) & \text{if } m_{jt}^g = 0 \end{cases}.$$

And subject to the constraints:

$$\sum_{j=1}^{J} (m_{jt} + \ell_{jt}) = 1, \qquad m_{jt}, \ell_{jt} \in \{0, 1\}$$

$$h_{j0} \leq \nu_j \alpha_{j0}^g$$

Optimal policy rule

Define the skill j index as the expected payoff if you committed to studying j:

$$\mathcal{I}_{jt}(h_j^g, P_j^g) = \sup_{\tau \geq 0} \mathbb{E}^\tau \left[\sum_{t=0}^{\infty} \delta^t U_j(h_{jt}^g, w_j)\ell_{jt}^g \middle| (h_{j0}^g, P_{j0}^g) = (h_j^g, P_j^g) \right]$$

Define the graduation region of skill j as:

$$\mathcal{G}_j(h_j^g, P_j^g) = \left\{ (h_j^g, P_j^g) \middle| \arg\max_{\tau \geq 0} \mathbb{E}^\tau \left[\sum_{t=0}^{\infty} \delta^t U_j(h_{jt}^g, w_j)\ell_{jt}^g \middle| (h_j, P_j^g) \right] = 0 \right\}$$

The following policy $\pi : (h_t, P_t^g) \rightarrow (s_t, \ell_t)$ is optimal:

1. At each $t \geq 0$, choose skill $j^* = \arg\max_{j \in J} \mathcal{I}_j$, breaking ties according to any rule
2. If $(h_{j^*}, P_{j^*}^g) \in \mathcal{G}_j$, then enter the labor market as a j^* specialist. Otherwise, study j^* for an additional period.

Model

Model Simulations

Simulate agent behavior

How can the model explain different specialization outcomes?

Consider a world with two fields, X and Y

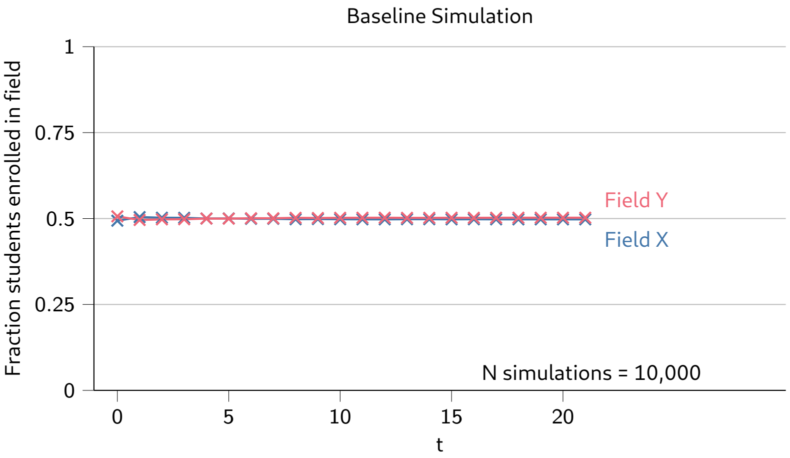
- Wages are equal: $w_X = w_Y$
- The agent's probabilities of success are equal: $\theta_X = \theta_Y$
- Initial beliefs are equal to the uninformative prior: PDF of beliefs

$$(\alpha_{X0}, \beta_{X0}) = (\alpha_{Y0}, \beta_{Y0}) = (1, 1)$$

Simulate agent's specialization decisions when choosing between X and Y

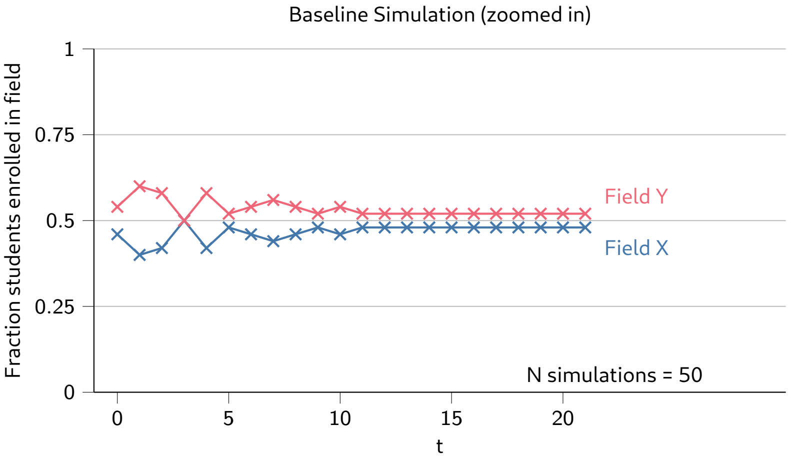
- Model fraction of simulated agents choosing X or Y at time t
- Assume $h_{j0} = \nu\alpha_{j0}$ Details

Default parameterization

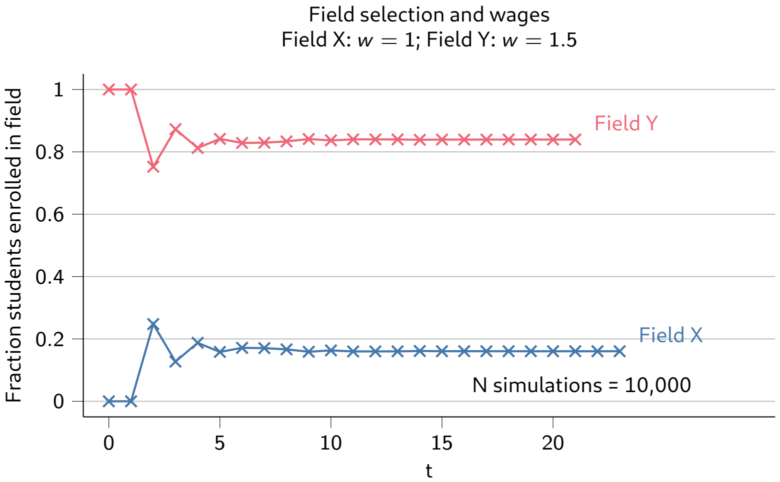


PDF of beliefs

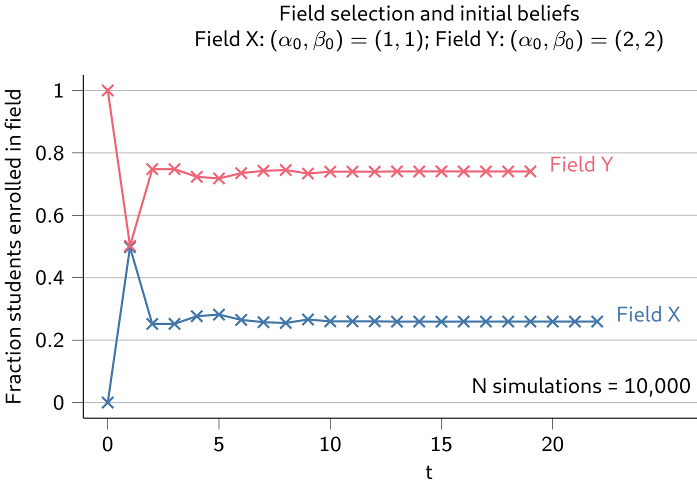
Zooming in


Baseline Simulation (zoomed in)
Fraction students enrolled in field
1
0.75
0.5
0.25
0
0
5
10
15
20
t
Field Y
Field X
N simulations = 50

Wage effects

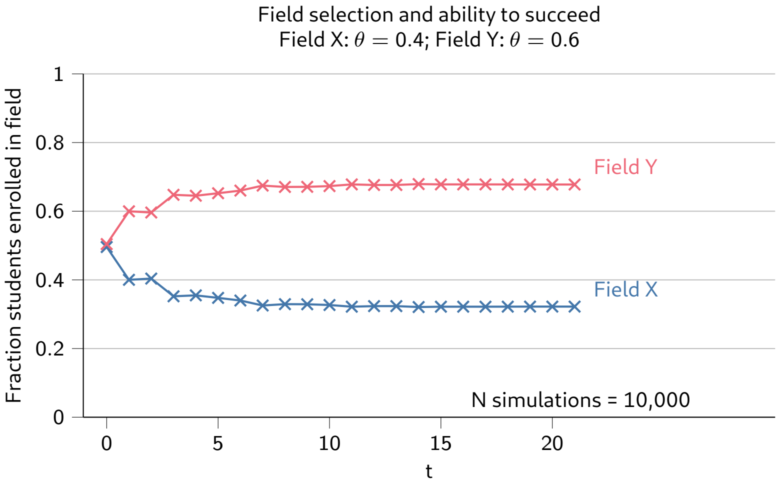


Belief effects



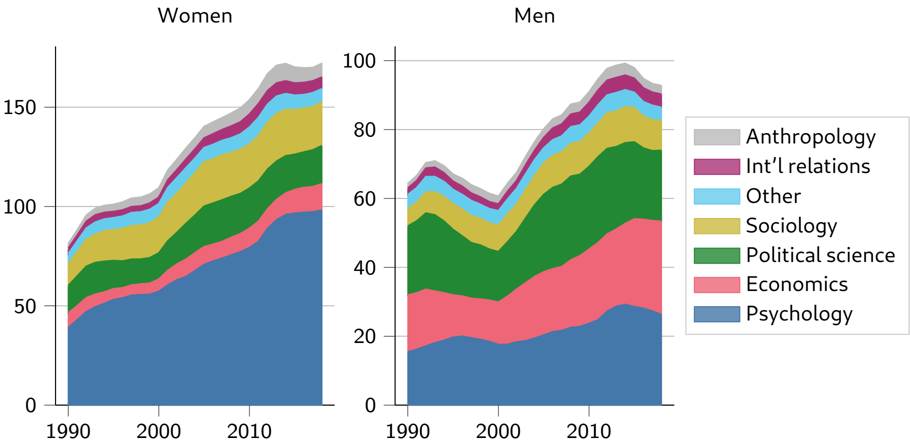
PDF of beliefs | Parameterization | Let $\alpha_{X0}\nu_X = \alpha_{Y0}\nu_Y$

Ability to succeed



ν simulation

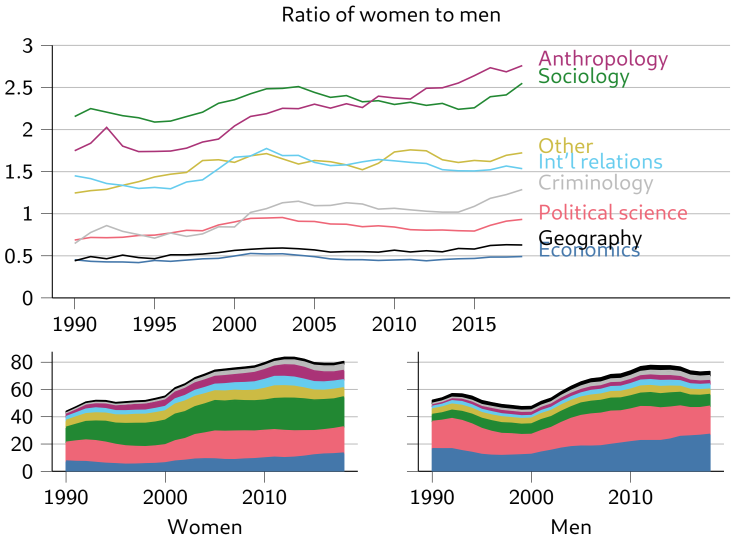
Appendix



Social Science - number Bachelor's degrees awarded (thousands). Source: IPEDS.

Return: Social science ratio

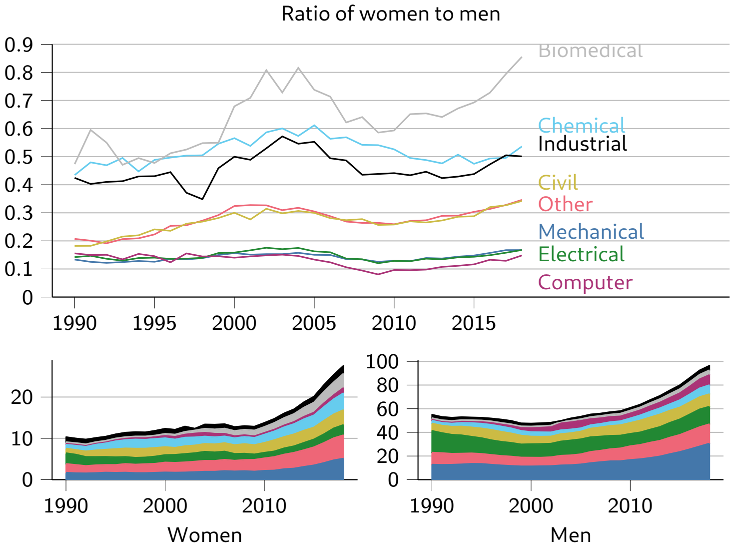
Social Sciences



Number Bachelor's degrees awarded (thousands). Source: IPEDS.

Return: Social science ratio

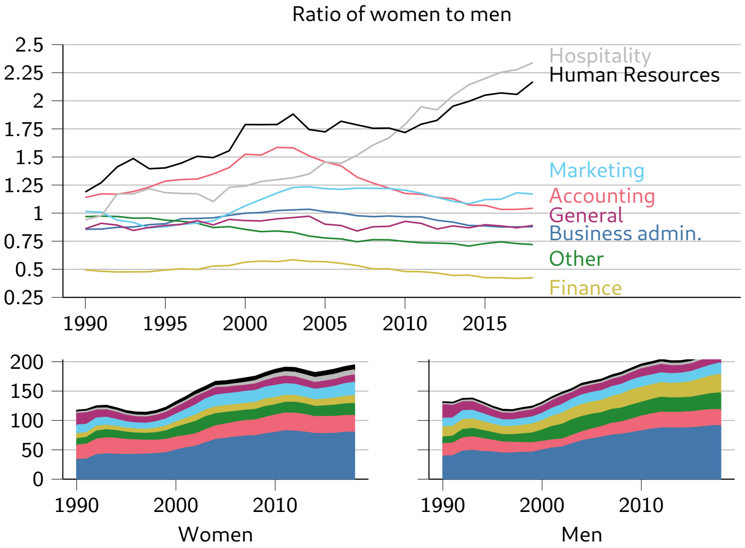
Engineering



Number Bachelor's degrees awarded (thousands). Source: IPEDS.

Return: Social science ratio

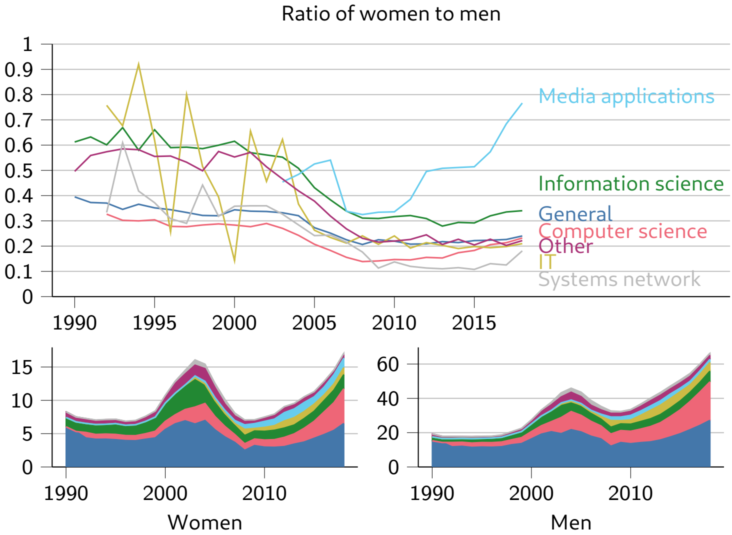
Business



Number Bachelor’s degrees awarded (thousands). Source: IPEDS.

Return: Social science ratio

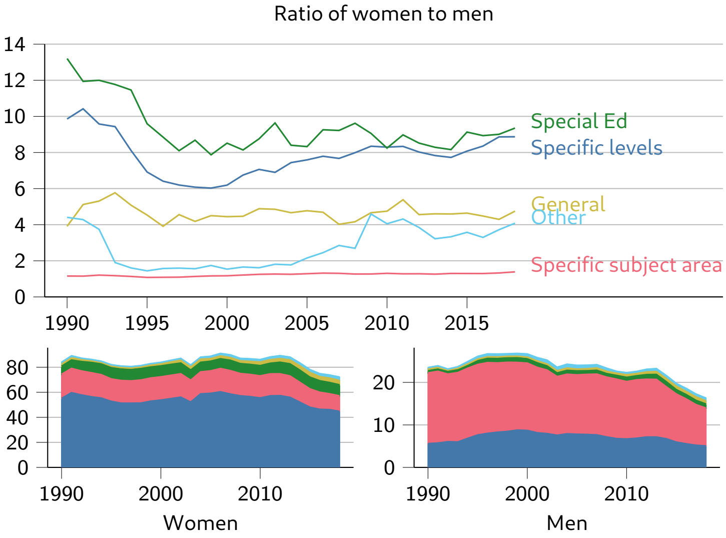
Computer Science



Number Bachelor's degrees awarded (thousands). Source: IPEDS.

Return: Social science ratio

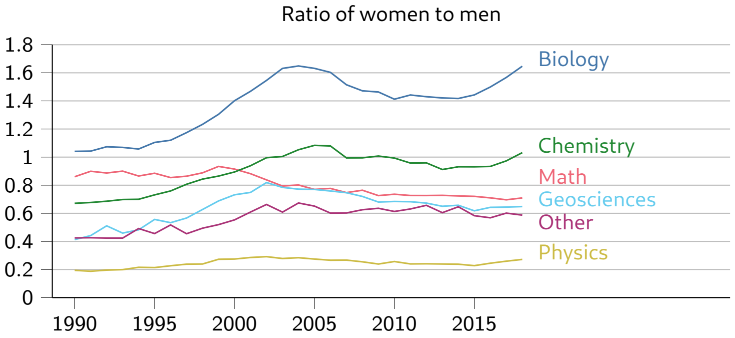
Education

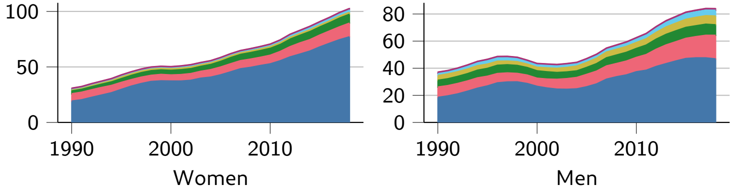


Number Bachelor's degrees awarded (thousands). Source: IPEDS.

Return: Social science ratio

Biological and Physical Sciences and Mathematics

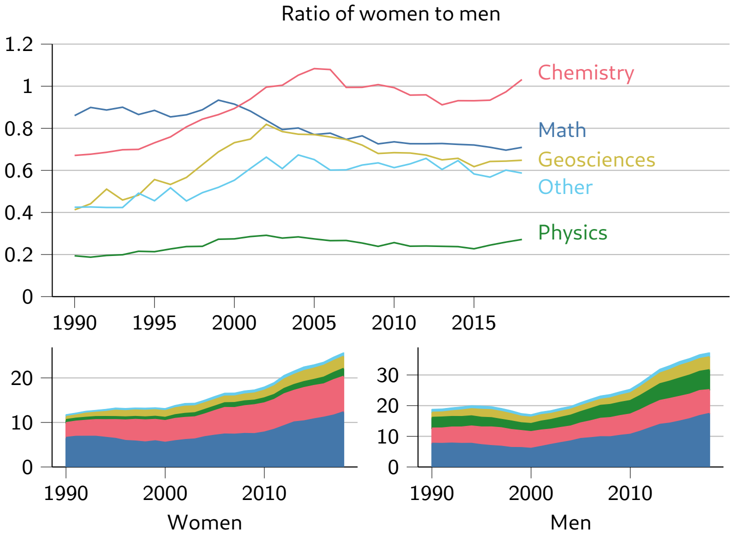




Number Bachelor's degrees awarded (thousands). Source: IPEDS.

Return: Social science ratio
No biology

Physical Sciences and math



Number Bachelor's degrees awarded (thousands). Source: IPEDS.

Return: Science and math
Return: Social science ratio

Parametric example

Assuming $h_{j0} = \nu\alpha_{j0}$ and letting c_{jt} be time spent studying j:

$\implies$ Deterministic stopping function

$$\frac{1-\delta}{\delta} \geq \frac{1}{c_{jt} + \alpha_{j0} + \beta_{j0}} \implies c_j^* = \left\lceil \frac{\delta}{1-\delta} \right\rceil - (\alpha_{j0} + \beta_{j0})$$

Graduation regions given by:

$$\mathcal{G}_j(\alpha_{jt}, \beta_{jt}) = \left\{ \alpha_{jt}, \beta_{jt} \,\middle|\, \frac{\delta}{1-\delta} \leq \alpha_{jt} + \beta_{jt} \right\}$$

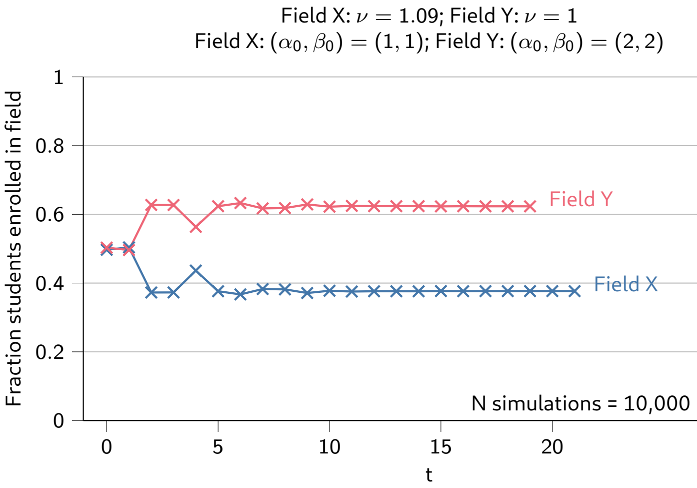
In this example, note that $\mathcal{G}_Y = \mathcal{G}_X$. Index in the graduation region given by $\frac{h_{jt}}{1-\delta}$. Index when not in graduation region given by Binomial distribution with parameters $\left(c_j^* - c_j, \frac{h_{jt}}{\nu(c_{jt}+\alpha_{j0}+\beta j0)}\right)$:

$$\mathcal{I}_{jt}(h_{jt}, \alpha_{jt}, \beta_{jt}) = \begin{cases} \frac{w_{jt} h_{jt}}{1-\delta} & \text{if } \{\alpha_{jt}, \beta_{jt}\} \in \mathcal{G}_j, \\ \frac{w_{jt} h_{jt}}{1-\delta} \left[\frac{\left\lceil \frac{\delta}{1-\delta} \right\rceil \delta^{\left\lceil \frac{\delta}{1-\delta} \right\rceil - c_{jt} - \alpha_{j0} - \beta_{j0}}}{c_{jt}+\alpha_{j0}+\beta_{j0}} \right] & \text{if } \{\alpha_{jt}, \beta_{jt}\} \notin \mathcal{G}_j \end{cases}$$

Return: simulation set-up
Return: belief simulation

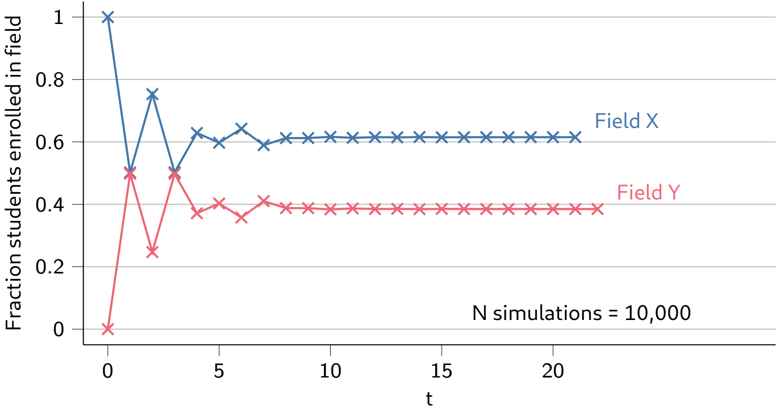
If $\nu_X = \frac{\alpha_{X0}+\beta_{X0}}{\alpha_{Y0}+\beta_{Y0}} \cdot \frac{\alpha_{Y0}}{\alpha_{X0}} \cdot \delta^{\alpha_{X0}+\beta_{X0}-\alpha_{Y0}-\beta_{Y0}}$, then:

- $h_{X0} = h_{Y0}$, and
- Agents randomly choose between fields X and Y at $t = 0$



ν effects
Return: belief simulation

ν effects



Return: $\alpha_{X0}\nu_X = \alpha_{Y0}\nu_Y$
Return: belief simulation
Return: ability simulation