Group-based beliefs and human capital specialization

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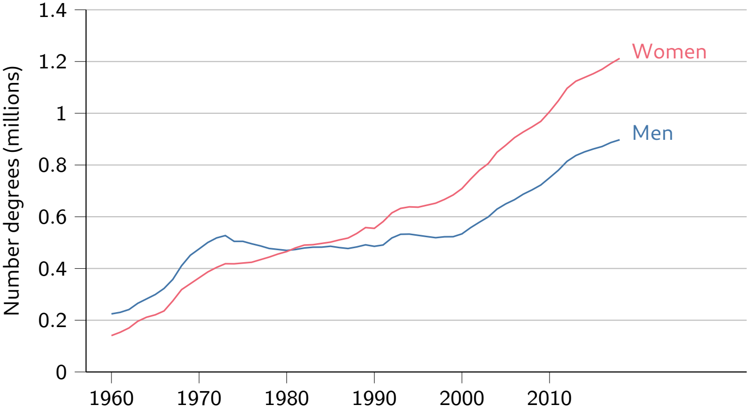
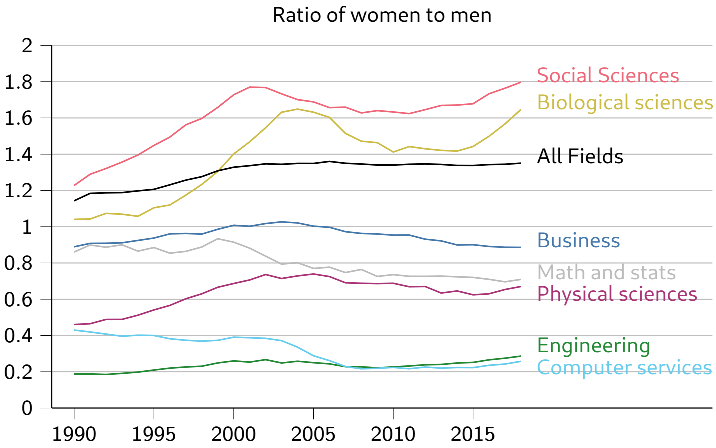
Increased attainment of Bachelor’s degrees



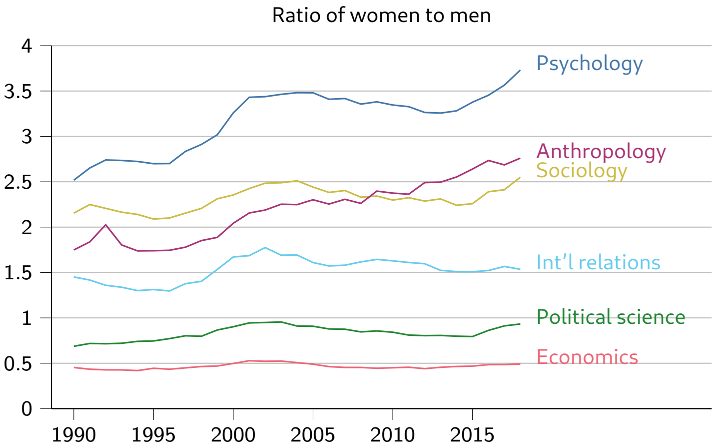
Number of Bachelor’s Degrees awarded in US 4-year colleges. Source: IPEDS; Snyder (2013).

Gender ratio in different fields



Return: Next steps

Social Sciences



Number degrees completed | Social Science CIP | Engineering | Business | Computer Science | Education | Science and math

What drives differences in gender convergence across fields?

Why might we see these differences?

- Differences in wages
- Differences in preferences or norms
- Social networks and role models
- Uncertainty about abilities

Problem: how do these things interact?

- How can your network influence your beliefs about your abilities?
- How can shared uncertainty influence norms?

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My research: the role of group-based beliefs in human capital specialization decisions

Model of gradual human capital specialization:

- Unknown heterogeneous abilities
- Group-based beliefs about abilities
- Sequential learning and human capital accumulation

$\Longrightarrow$ Group-based beliefs play an important role in specialization decisions

Dissertation road map

Chapter 1: Introduction to model

- Incorporate group-base beliefs into Alon and Fershtman (2019)
- Simulate decisions given different beliefs
- Connection to statistical discrimination

Chapter 2: Re-create dynamics of major choice across gender

- Incorporate intergenerational learning into model
- Identify model parameters

Chapter 3: Quantitative exercises

- Implications of group-based beliefs for aggregate productivity
- Policy analysis: affirmative action

Limited Literature Review

1. Human capital specialization
   - Build on model of gradual specialization from Alon and Fershtman (2019)

2. Gender gaps in college choice
   - Empirically motivated by Sloan, Hurst, and Black (2020)

3. Determinants of college major choice, in particular the role of beliefs
   - Arcidiacono et al. (2015): model of sequential learning and role of beliefs
   - Subjective expectations literature (Stinebrickner and Stinebrickner, 2014; Wiswall and Zafar, 2019; Zafar, 2013)

4. Statistical discrimination literature
   - Lundberg and Startz (1984): efficiency of equal opportunity laws
   - Coate and Loury (1997): permanent affirmative action and patronizing equilibria

Outline

Model preliminaries

Individuals endowed with:

h_{j0}: Field-j specific human capital ($j = 0, \ldots, J$)

θ_j: Unknown probability of success in j

P_{j0}: Prior beliefs about θ_j

At each time t, agents can either study one field j (m_{jt}) or work in one field j (ℓ_{jt})

$$\sum_{j=0}^{J}(m_{jt} + \ell_{jt}) = 1, \qquad m_{jt}, \ell_{jt} \in \{0, 1\}$$

If an agent chooses to study field j ($m_{jt} = 1$):

- Stochastically accumulate field-j human capital
- Reveal information about θ_j

If an agent chooses to work in field j ($\ell_{jt} = 1$):

- Earn $w_j h_{jt}$

Enter labor market at time t in skill-j to maximize expected lifetime payoff:

$$\frac{\delta^t}{1-\delta} U_j(w_j, h_{jt})\ell_{jt} = \frac{\delta^t}{1-\delta} w_j h_{jt} \ell_{jt}$$

Evolution of human capital accumulation and beliefs

Students studying field-j at time t pass the course with probability θ_j:

$$s_{jt} \sim \text{Bernoulli}(\theta_j)$$

- Accumulate human capital if they pass the course:

$$h_{jt+1} = h_{jt} + \nu_j s_{jt} mjt$$

- Beliefs about θ_j evolve:

$$P_{j,t+1} = \Pi_j(P_{jt}, s_{jt})$$

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Key: How are **priors** formed, and how are they **updated**?

Belief distribution

Initial prior drawn from Beta distribution

$$P_{j0} = \mathcal{B}(\alpha_{j0}, \beta_{j0})$$

Update according to Bayes Rule $\implies$ posterior drawn from Beta distribution:

$$P_{j,t+1} = \mathcal{B}(\alpha_{j,t+1}, \beta_{j,t+1}), \qquad (\alpha_{j,t+1}, \beta_{j,t+1}) = \begin{cases} (\alpha_{jt} + 1, \beta_{jt}) & \text{if } s_{jt} = 1 \\ (\alpha_{jt}, \beta_{jt} + 1) & \text{if } s_{jt} = 0 \end{cases}$$

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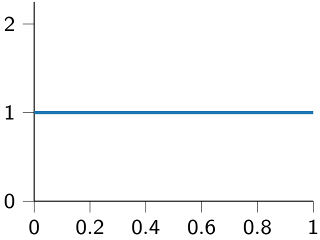
Example: $\alpha_0 = 1$, $\beta_0 = 1$

Beliefs $p(\theta|\alpha, \beta)$



Return: simulation set-up Return: baseline simulation

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- success at $t = 1 \implies \alpha_1 = 2$, $\beta_1 = 1$

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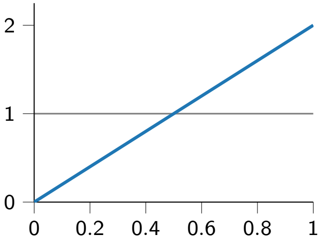

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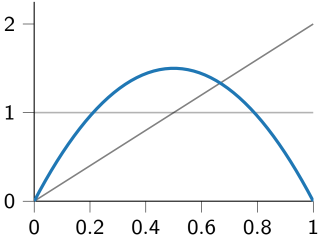
Example: $\alpha_0 = 1$, $\beta_0 = 1$

- success at $t = 1 \implies \alpha_1 = 2$, $\beta_1 = 1$
- failure at $t = 2 \implies \alpha_1 = 2$, $\beta_1 = 2$

Beliefs $p(\theta|\alpha, \beta)$



Return: simulation

Belief distribution

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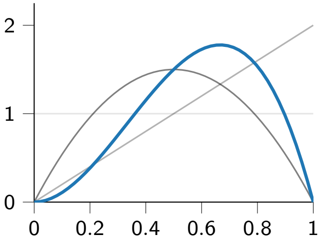
Example: $\alpha_0 = 1$, $\beta_0 = 1$

- success at $t = 1 \implies \alpha_1 = 2$, $\beta_1 = 1$
- failure at $t = 2 \implies \alpha_1 = 2$, $\beta_1 = 2$
- success at $t = 3 \implies \alpha_1 = 3$, $\beta_1 = 2$

Beliefs $p(\theta|\alpha, \beta)$



Group-based parametrization

Consider group-based beliefs about abilities:

- Each individual has a group type: $g \in \{m, f\}$
- Students form beliefs, P_{j0}, based on previously observed group successes

Simple parameterization:

α^g_{j0}: Number of type-g students who have succeeded in j

β^g_{j0}: Number of type-g students who have failed in j

$\Longrightarrow$ Observed success rate:

$$\mu^g_{j0} = \frac{\alpha^g_{j0}}{\alpha^g_{j0} + \beta^g_{j0}}.$$

This average is based on a sample size of type g students:

$$n^g_{j0} = \alpha^g_{j0} + \beta^g_{j0}$$

Group-based prior beliefs about probability of success in skill-j courses, θ_j:

$$\mathcal{B}\left(\alpha^g_{j0}, \beta^g_{j0}\right) \quad \Longrightarrow \quad \mathcal{B}\left(\mu^g_{j0} n^g_{j0}, (1 - \mu^g_{j0}) n^g_{j0}\right)$$

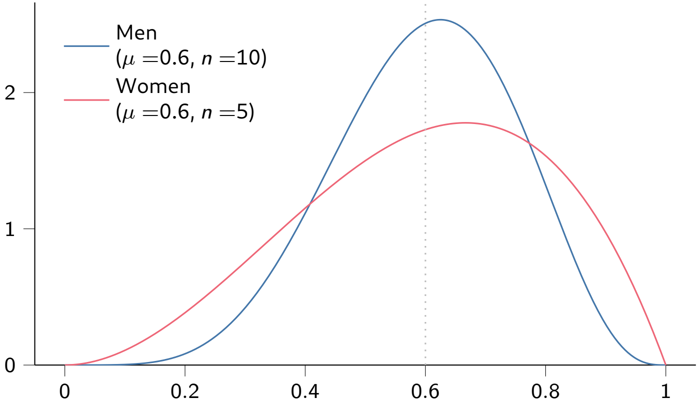
Group-based belief distribution

Suppose there are more men then women in field j:

$$n_{j0}^{m} > n_{j0}^{f}$$

But the observed success rate is the same for the two groups:

$$\mu_{j0} = \mu_{j0}^{m} = \mu_{j0}^{w}$$



Individual problem

A policy $\pi : (h_t, P_t^g) \to (s_t, \ell_t)$ is optimal if it maximizes:

$$\mathbb{E}^\pi \left[\sum_{t=0}^{\infty} \delta^t \left(\sum_{j=1}^{J} h_{jt} w_j \ell_{jt} \right) \middle| ((h_{10}, P_{10}^g), \ldots, (h_{J0}, P_{J0}^g)) \right]$$

Subject to the human capital accumulation and belief transition laws:

$$h_{jt+1} = h_{jt} + \nu_j s_{jt} m_{jt}, \qquad s_{jt} \sim \text{Bernoulli}(\theta_j), \qquad \theta_j \sim P_{j0}^g \equiv \mathcal{B}(\alpha_{j0}^g, \beta_{j0}^g),$$

$$P_{j,t+1}^g = \mathcal{B}(\alpha_{j,t+1}^g, \beta_{j,t+1}^g), \qquad (\alpha_{j,t+1}^g, \beta_{j,t+1}^g) = \begin{cases} (\alpha_{jt}^g + 1, \beta_{jt}^g) & \text{if } m_{jt}^g = 1 \text{ and } s_{jt}^g = 1 \\ (\alpha_{jt}^g, \beta_{jt}^g + 1) & \text{if } m_{jt}^g = 1 \text{ and } s_{jt}^g = 0 \\ (\alpha_{jt}^g, \beta_{jt}^g) & \text{if } m_{jt}^g = 0 \end{cases}.$$

And subject to the constraints:

$$\sum_{j=1}^{J} (m_{jt} + \ell_{jt}) = 1, \qquad m_{jt}, \ell_{jt} \in \{0, 1\}$$

$$h_{j0} \leq \nu_j \alpha_{j0}^g$$

Optimal policy rule

Define the field-j index as the expected payoff if you committed to studying j:

$$\mathcal{I}_{jt}(h_j^g, P_j^g) = \sup_{\tau \geq 0} \mathbb{E}^\tau \left[\sum_{t=0}^{\infty} \delta^t U_j(h_{jt}^g, w_j) \ell_{jt}^g \middle| (h_{j0}^g, P_{j0}^g) = (h_j^g, P_j^g) \right]$$

Define the graduation region of field j as:

$$\mathcal{G}_j(h_j^g, P_j^g) = \left\{ (h_j^g, P_j^g) \middle| \arg\max_{\tau \geq 0} \mathbb{E}^\tau \left[\sum_{t=0}^{\infty} \delta^t U_j(h_{jt}^g, w_j) \ell_{jt}^g \middle| (h_j, P_j^g) \right] = 0 \right\}$$

The following policy $\pi : (h_t, P_t^g) \to (s_t, \ell_t)$ is optimal:

1. At each $t \geq 0$, choose field $j^* = \arg\max_{j \in J} \mathcal{I}_j$, breaking ties according to any rule
2. If $(h_{j^*}, P_{j^*}^g) \in \mathcal{G}_j$, then enter the labor market as a j^* specialist. Otherwise, study j^* for an additional period.

Return: Likelihood

Simulate agent behavior

How can the model explain different specialization outcomes?

Consider a world with two fields, X and Y

- Wages are equal: $w_X = w_Y$
- The agent's probabilities of success are equal: $\theta_X = \theta_Y$
- Initial beliefs are equal to the uniform prior: PDF of beliefs

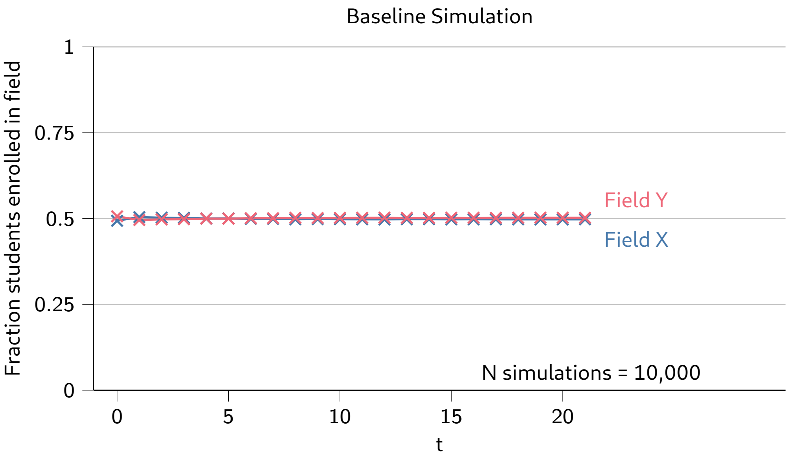
$$(\alpha_{X0}, \beta_{X0}) = (\alpha_{Y0}, \beta_{Y0}) = (1, 1)$$

- Assume $h_{j0} = \nu\alpha_{j0}$ Details

Simulate agent's specialization decisions when choosing between X and Y

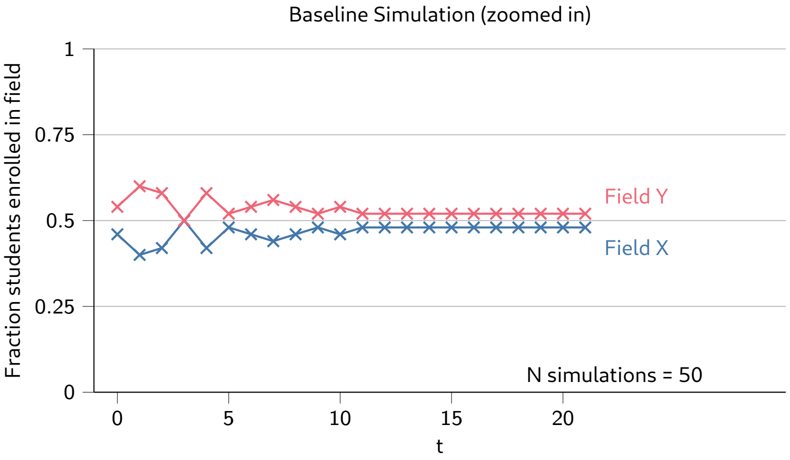
- Model fraction of simulated agents choosing X or Y at time t

Default parameterization

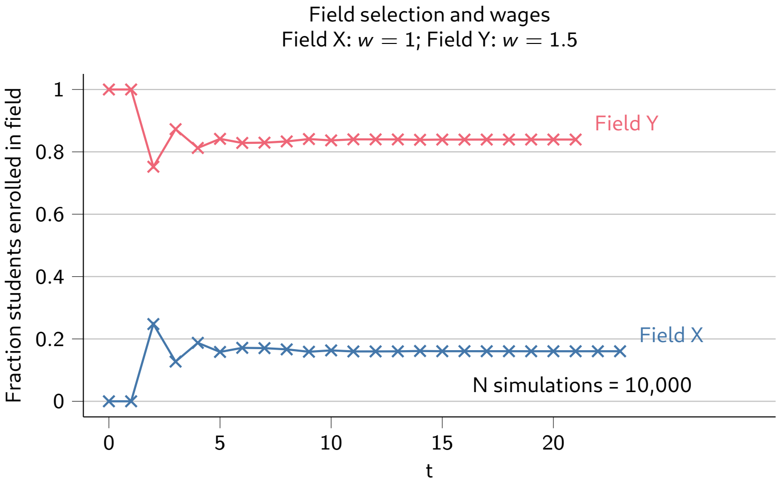


PDF of beliefs

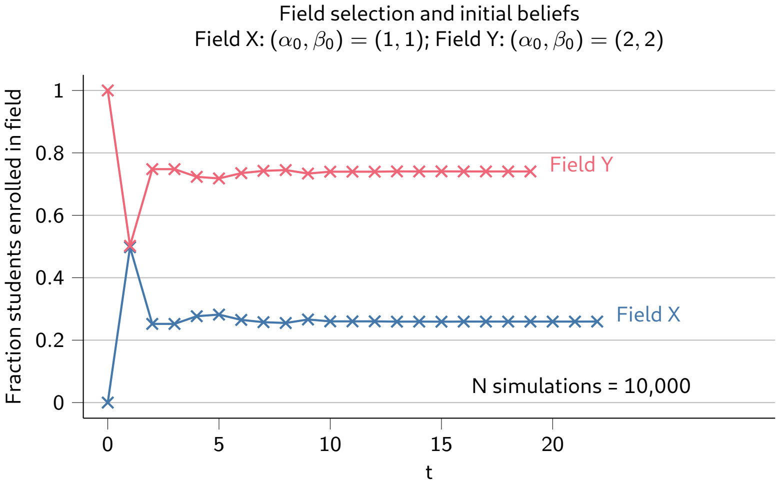
Zooming in


Baseline Simulation (zoomed in)
Fraction students enrolled in field
1
0.75
0.5
0.25
0
0
5
10
15
20
t
Field Y
Field X
N simulations = 50

Wage effects

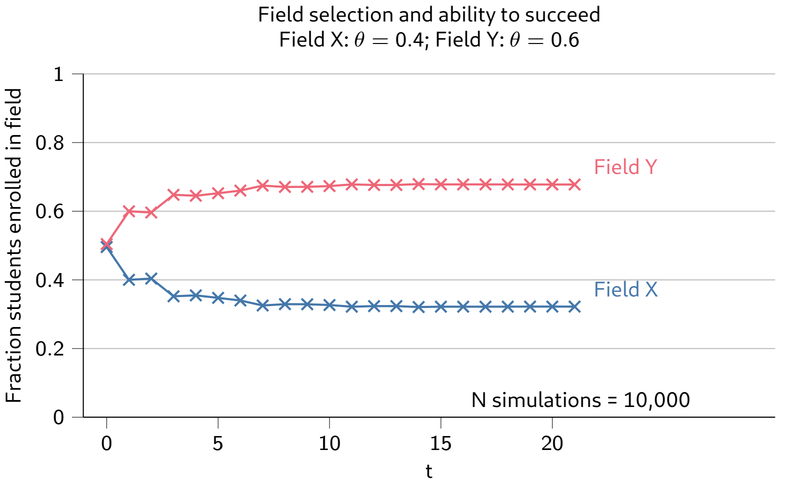


Belief effects



PDF of beliefs | Parameterization | Let $\alpha_{X0}\nu_X = \alpha_{Y0}\nu_Y$

Ability to succeed



ν simulation

Discrimination in model

Definition of statistical discrimination (Lundberg and Startz, 1983)

Economic discrimination exists when groups with equal average initial endowments of productive ability do not receive equal average compensation in equilibrium.

Consider if agents begin with

- The same levels of initial human capital, h_{j0}, and abilities, θ_j
- Different levels of beliefs, $(\alpha_{j0}, \beta_{j0})$

Simulations above show this leads to different specialization decisions

$\Longrightarrow$ This is connected to statistical discrimination

Large statistical discrimination literature to draw on:

- Inaccurate statistical discrimination
- Dynamic discrimination

Next steps

Chapter 2: Re-create dynamics of major choice across gender Dynamics

- Incorporate intergenerational learning into model
- Identify model parameters

Chapter 3: Additional quantitative exercises

- Implications of group-based beliefs for aggregate productivity
- Policy analysis: affirmative action

Identification problem for a single cohort

Suppose I can access:

- Data on college major choice by gender at different points in time
- Transcript data
- Estimates of w_j, δ, and v_j

$\Longrightarrow$ **Question:** How to recover estimates of $\left(h_{ij0}, \alpha_{j0}^{g(i)}, \beta_{j0}^{g(i)}\right)$?

Problem #1: Endogeneity of major choice

- Solution: Focus on estimating parameters at the beginning of college

Likelihood for a single cohort

Problem #2: Defining likelihood function for agent i at time $t = 0$

$$\mathcal{L} = \sum_{j=1}^{J} m_{ijt} \mathbb{P}\left(m_{ijt} = 1 \middle| h_{j0}, \alpha_{j0}^{g(i)}, \beta_{j0}^{g(i)}\right)$$
$$= \sum_{j=1}^{J} m_{ijt} \mathbb{P}\left(\mathcal{I}_{j0} > \mathcal{I}_{k0} \; \forall k \neq j \middle| h_{j0}, \alpha_{j0}^{g(i)}, \beta_{j0}^{g(i)}\right),$$

where $\mathcal{I}_{jt}$ is the agent's expected payoff if they graduate in j after studying N_j periods:

Optimal Policy

$$\mathcal{I}_{jt} = \frac{1}{1-\delta} \delta^{\mathbb{E}_0\left[N_j^{g(i)} | \cdot\right]} w_j \left(h_{ij0} + \nu_j \mathbb{E}_0\left[N_j^{g(i)} \theta_j^{g(i)} | \cdot\right]\right)$$

- How can I recover $(\alpha_{j0}, \beta_{j0}, h_{j0})$?
- What assumptions do I need on individual heterogeneity h_{j0}?

Question: If I could run an experiment to recover my parameters for a single cohort, what would that experiment look like?

- Do I need to use a different belief structure? Or human capital accumulation function?

Beliefs over time

Question: How are beliefs $(\alpha^g_{j0}, \beta^g_{j0})$ changing over time?

- Answering this depends on how I develop a dynamic version of the model

Goal: Build dynamic version of the model

- Replicate dynamics of major choice Dynamics
- Possible resources: Fernandez (2013) model of cultural learning and labor force participation

Cultural norms and female labor force participation (Fernandez 2013)

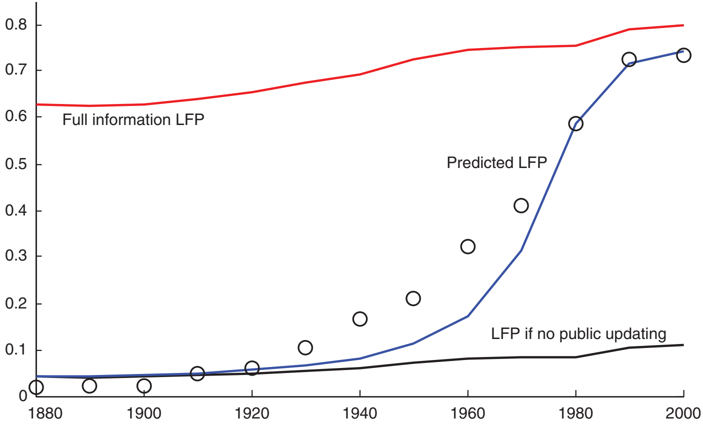


Figure 7. Solution Parameters from Calibrated Model Without Public Learning

Source: Fernandez (2013)

Fernandez (2013)

Women making labor decisions:

- Have unknown disutility form working
- Have beliefs about their disutility

Beliefs are updated according to:

- A private signal inherited from their mother
- The previous generation's participation in the labor force

$\Longrightarrow$ Re-creates S-shaped participation rate

- Theoretical concept: information cascade

Conclusion

Presented a model of how group-based beliefs can influence specialization

- How realistic is this model?

Next goal: re-create dynamics of major choice over time

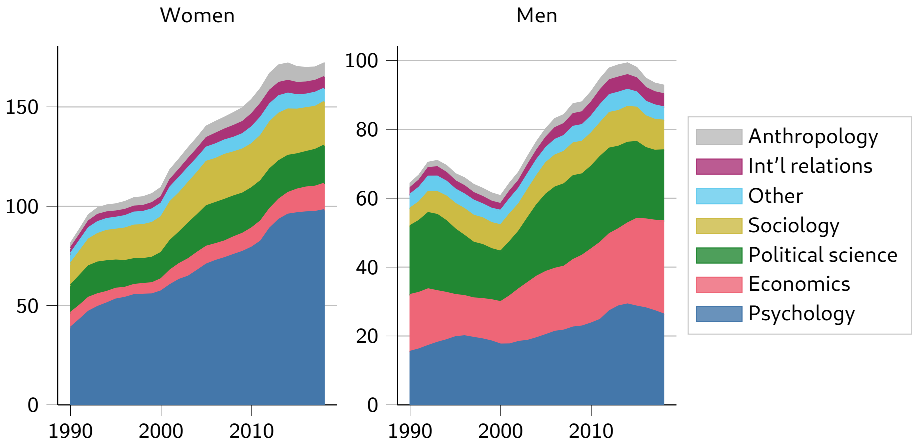
- Building a dynamic model of major choice
- Identifying model parameters

Plan to do this:

- Determine the diffusion process first to build the dynamic model
- Think about identification in terms of an experiment
  - What experiment could I run to recover my parameters of interest?
  - Should I change some model fundamentals?

Feedback on the best way to proceed more than welcome!

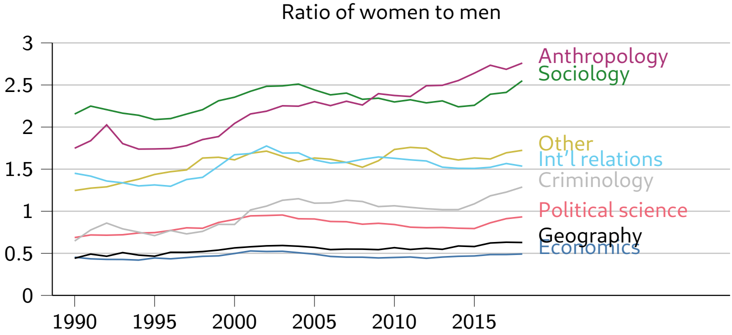
Appendix

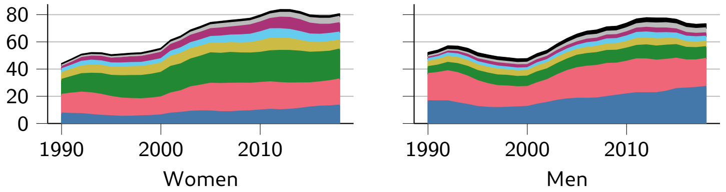


Social Science - number Bachelor's degrees awarded (thousands). Source: IPEDS.

Return: Social science ratio

Social Sciences





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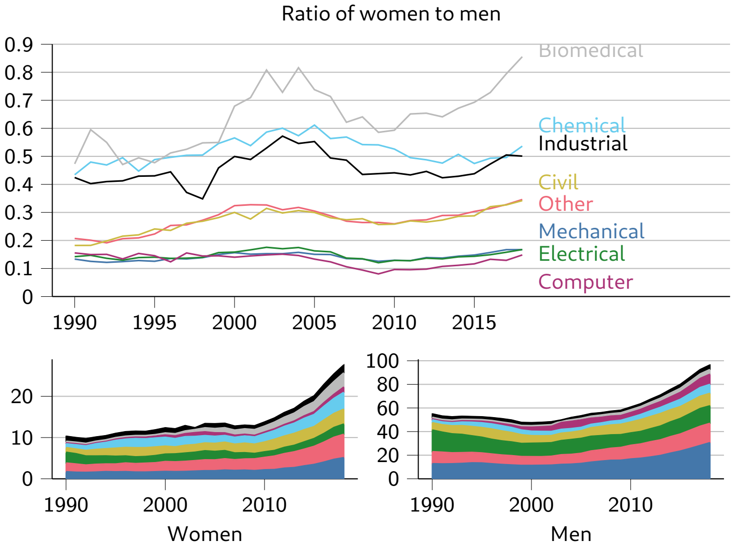
Return: Social science ratio

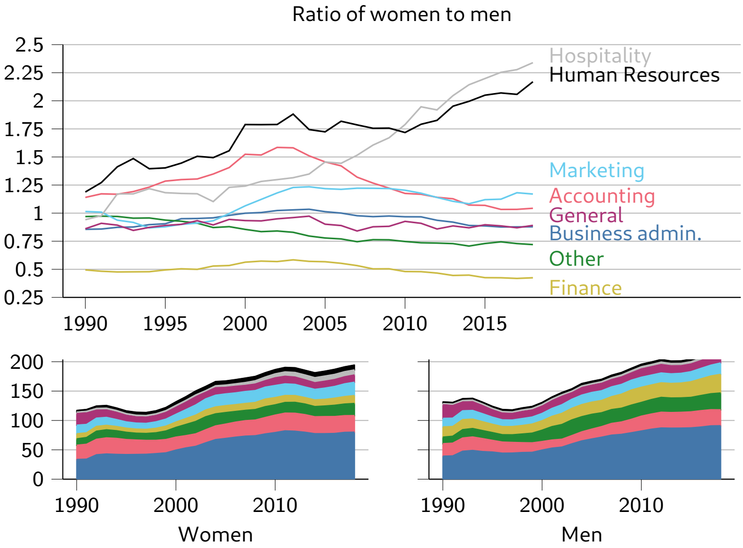
Engineering



Number Bachelor's degrees awarded (thousands). Source: IPEDS.

Return: Social science ratio

Business



Number Bachelor's degrees awarded (thousands). Source: IPEDS.

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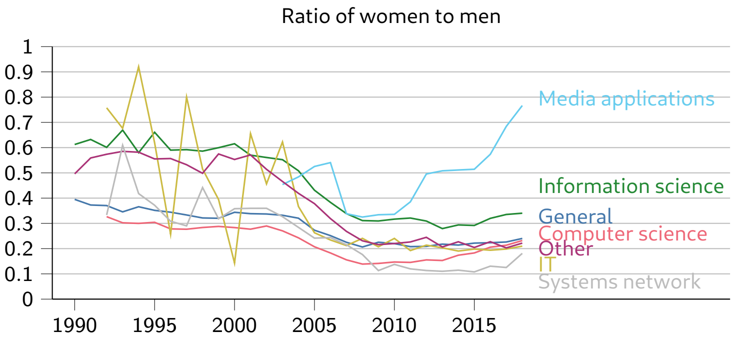
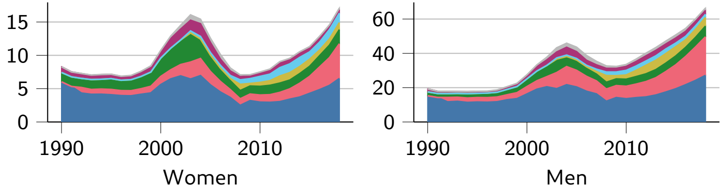
Computer Science

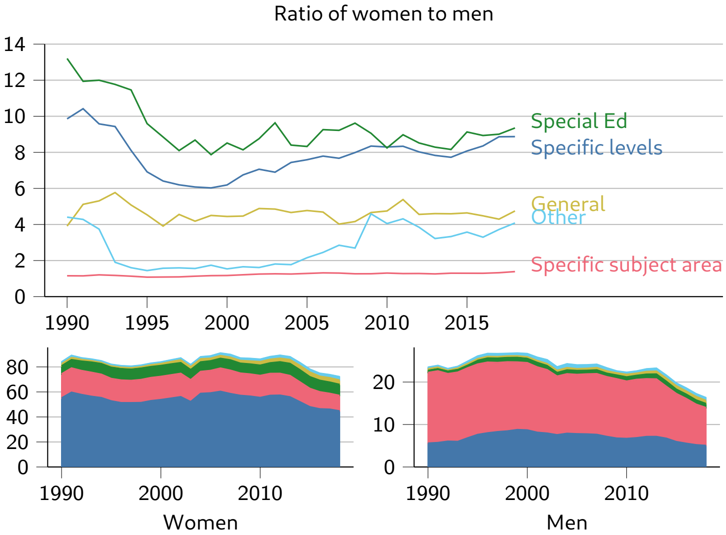




Number Bachelor's degrees awarded (thousands). Source: IPEDS.

Return: Social science ratio

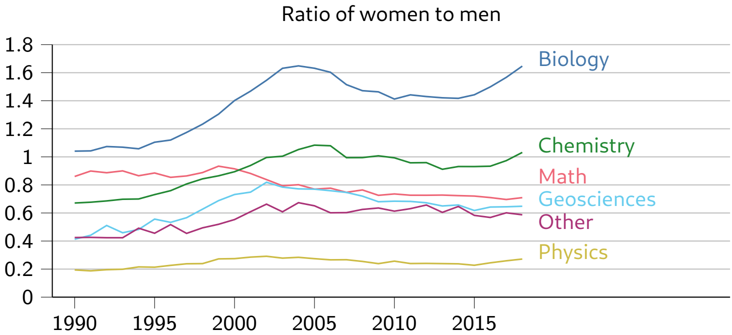
Education

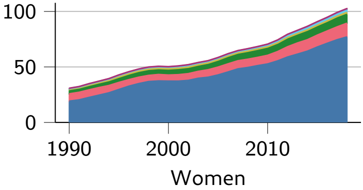


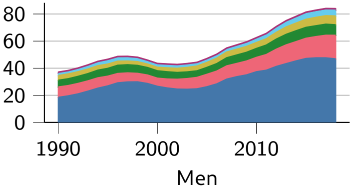
Number Bachelor's degrees awarded (thousands). Source: IPEDS.

Return: Social science ratio

Biological and Physical Sciences and Mathematics



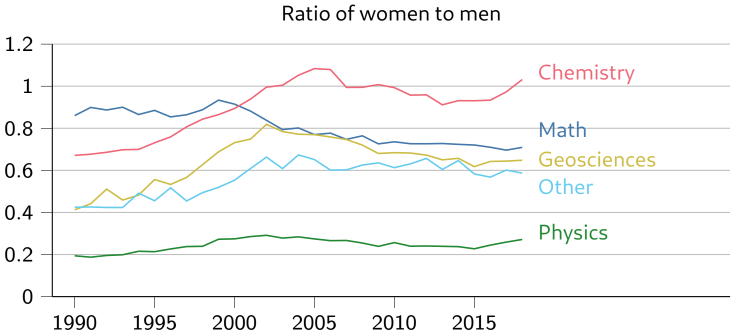


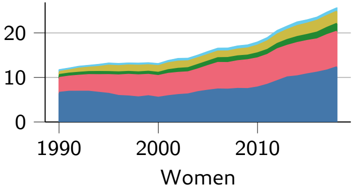


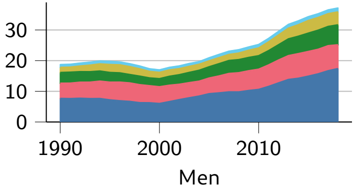
Number Bachelor's degrees awarded (thousands). Source: IPEDS.

Return: Social science ratio | No biology

Physical Sciences and math







Number Bachelor's degrees awarded (thousands). Source: IPEDS.

Return: Science and math
Return: Social science ratio

Parametric example

Assuming $h_{j0} = \nu \alpha_{j0}$ and letting c_{jt} be time spent studying j:

$\implies$ Deterministic stopping function

$$\frac{1-\delta}{\delta} \geq \frac{1}{c_{jt} + \alpha_{j0} + \beta_{j0}} \implies c_j^* = \left\lceil \frac{\delta}{1-\delta} \right\rceil - (\alpha_{j0} + \beta_{j0})$$

Graduation regions given by:

$$\mathcal{G}_j(\alpha_{jt}, \beta_{jt}) = \left\{ \alpha_{jt}, \beta_{jt} \,\middle|\, \frac{\delta}{1-\delta} \leq \alpha_{jt} + \beta_{jt} \right\}$$

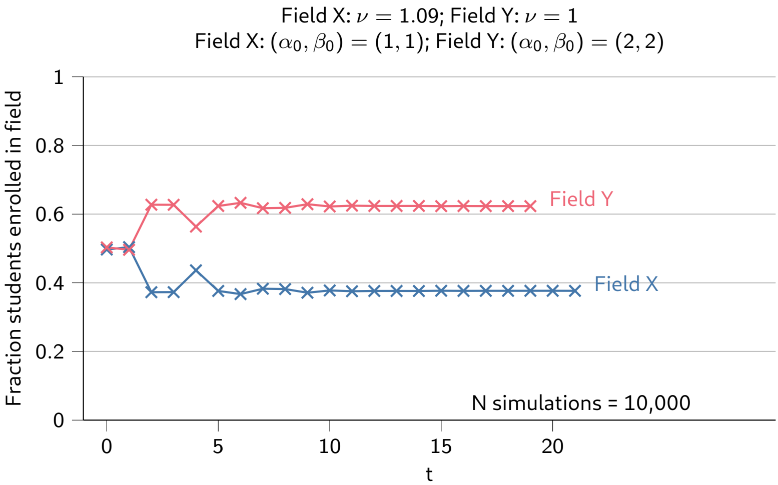
In this example, note that $\mathcal{G}_Y = \mathcal{G}_X$. Index in the graduation region given by $\frac{h_{jt}}{1-\delta}$. Index when not in graduation region given by Binomial distribution with parameters $\left(c_j^* - c_j, \frac{h_{jt}}{\nu(c_{jt}+\alpha_{j0}+\beta j0)}\right)$:

$$\mathcal{I}_{jt}(h_{jt}, \alpha_{jt}, \beta_{jt}) = \begin{cases} \frac{w_{jt} h_{jt}}{1-\delta} & \text{if } \{\alpha_{jt}, \beta_{jt}\} \in \mathcal{G}_j, \\ \frac{w_{jt} h_{jt}}{1-\delta} \left[\frac{\left\lceil \frac{\delta}{1-\delta} \right\rceil \delta^{\left\lceil \frac{\delta}{1-\delta} \right\rceil - c_{jt} - \alpha_{j0} - \beta_{j0}}}{c_{jt}+\alpha_{j0}+\beta_{j0}} \right] & \text{if } \{\alpha_{jt}, \beta_{jt}\} \notin \mathcal{G}_j \end{cases}$$

Return: simulation set-up Return: belief simulation Return: ID notes

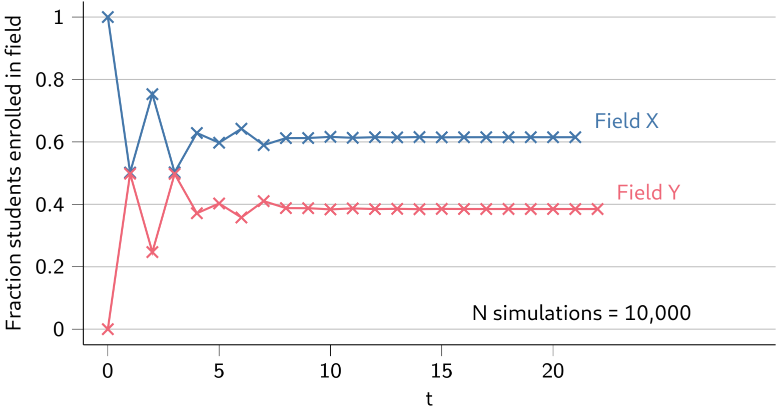
If $\nu_X = \frac{\alpha_{X0}+\beta_{X0}}{\alpha_{Y0}+\beta_{Y0}} \cdot \frac{\alpha_{Y0}}{\alpha_{X0}} \cdot \delta^{\alpha_{X0}+\beta_{X0}-\alpha_{Y0}-\beta_{Y0}}$, then:

- $h_{X0} = h_{Y0}$, and
- Agents randomly choose between fields X and Y at $t = 0$



ν effects Return: belief simulation

ν effects



Return: $\alpha_{X0}\nu_X = \alpha_{Y0}\nu_Y$ Return: belief simulation Return: ability simulation