

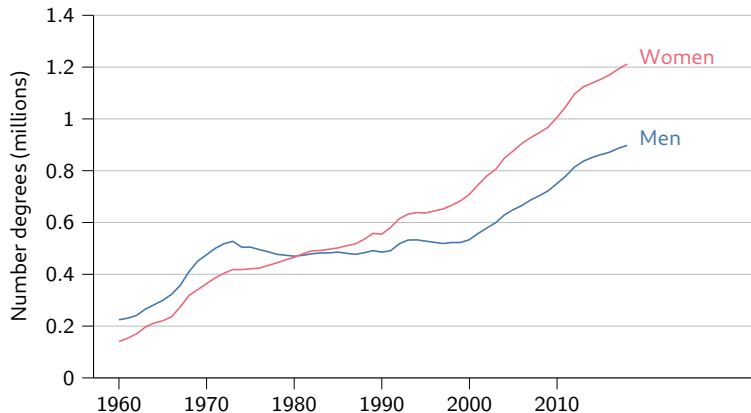
# Group-based beliefs and human capital specialization

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Macro Lunch Presentation  
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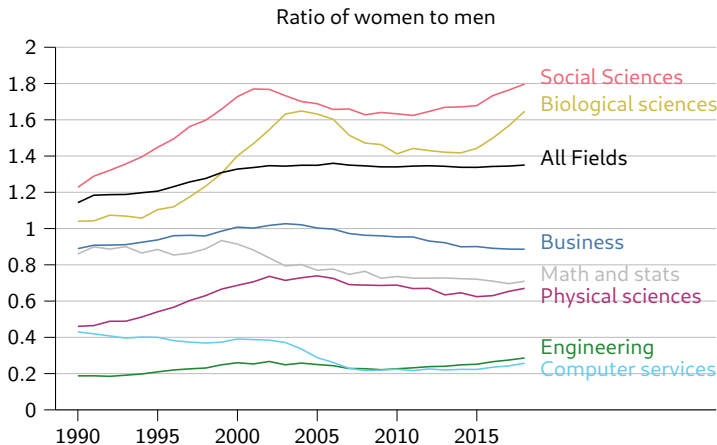
September 12, 2020

## Increased attainment of Bachelor's degrees

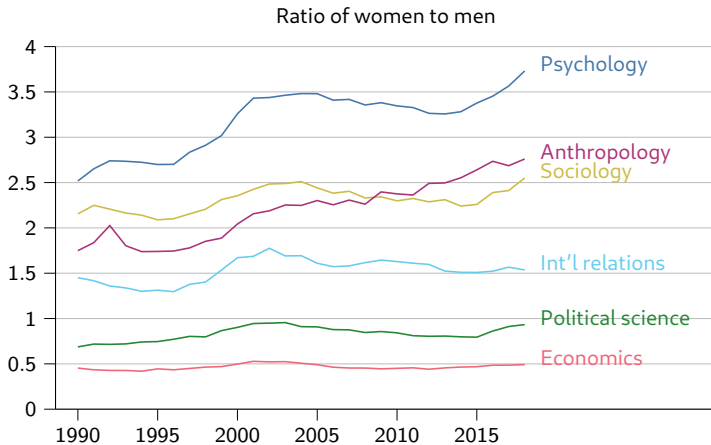


Number of Bachelor's Degrees awarded in US 4-year colleges. Source: IPEDS; Snyder (2013).

# Gender ratio in different fields



# Social Sciences



# Gender convergence across fields

Do differences in gender convergence across fields represent misallocation of talent?

- ▶ If yes, this has macroeconomic consequences

# Gender convergence across fields

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**This paper:** the role of group-based beliefs in human capital specialization decisions

Model of gradual human capital specialization:

- ▶ Unknown heterogeneous abilities
- ▶ Group-based beliefs about abilities
- ▶ Sequential learning and human capital accumulation

⇒ Group-based beliefs play an important role in specialization decisions

What are the productivity costs associated with misallocation of talent implied by the model?

- ▶ Estimate impact of misallocation on aggregate productivity growth in line with Hsieh, Hurst, Jones, and Klenow (2019)

# Limited Literature Review

## 1. Human capital specialization

- ▶ Build on model of gradual specialization from Alon and Fershtman (2019)

## 2. Gender gaps in college choice

- ▶ Empirically motivated by Sloan, Hurst, and Black (2020)

## 3. Determinants of college major choice, in particular the role of beliefs

- ▶ Arcidiacono et al. (2015): model of sequential learning and role of beliefs
- ▶ Subjective expectations literature (Stinebrickner and Stinebrickner, 2014; Wiswall and Zafar, 2019; Zafar, 2013)

## 4. Statistical discrimination literature

- ▶ Lundberg and Startz (1984): efficiency of equal opportunity laws
- ▶ Coate and Loury (1997): permanent affirmative action and patronizing equilibria

# Outline

Model

Model Simulations

Calibration

Conclusion and next steps



## Model preliminaries

Individuals endowed with:

$h_{j0}$ : Field- $j$  specific human capital ( $j = 0, \dots, J$ )

$\theta_j$ : Unknown probability of success in  $j$

$P_{j0}$ : Prior beliefs about  $\theta_j$

At each time  $t$ , agents can either study one field  $j$  ( $m_{jt}$ ) or work in one field  $j$  ( $\ell_{jt}$ )

$$\sum_{j=0}^J (m_{jt} + \ell_{jt}) = 1, \quad m_{jt}, \ell_{jt} \in \{0, 1\}$$

If an agent chooses to study field  $j$  ( $m_{jt} = 1$ ):

- ▶ Stochastically accumulate field- $j$  human capital
- ▶ Reveal information about  $\theta_j$

If an agent chooses to work in field  $j$  ( $\ell_{jt} = 1$ ):

- ▶ Earn wage  $w_j$

Enter labor market at time  $t$  in skill- $j$  to maximize expected lifetime payoff:

$$\frac{\delta^t}{1 - \delta} U_j(w_j, h_{jt}) \ell_{jt} = \frac{\delta^t}{1 - \delta} w_j h_{jt} \ell_{jt}$$

# Evolution of human capital accumulation and beliefs

Students studying field- $j$  at time  $t$  pass the course with probability  $\theta_j$ :

$$s_{jt} \sim \text{Bernoulli}(\theta_j)$$

- Accumulate human capital if they pass the course:

$$h_{jt+1} = h_{jt} + \nu_j s_{jt} m_j t$$

- Beliefs about  $\theta_j$  evolve:

$$P_{j,t+1} = \Pi_j(P_{jt}, s_{jt})$$

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**Key:** How are **priors** formed, and how are they **updated**?

# Belief distribution

Initial prior drawn from Beta distribution

$$P_{j0} = \mathcal{B}(\alpha_{j0}, \beta_{j0})$$

Update according to Bayes Rule  $\implies$  posterior drawn from Beta distribution:

$$P_{j,t+1} = \mathcal{B}(\alpha_{j,t+1}, \beta_{j,t+1}), \quad (\alpha_{j,t+1}, \beta_{j,t+1}) = \begin{cases} (\alpha_{jt} + 1, \beta_{jt}) & \text{if } a_{jt} = 1 \\ (\alpha_{jt}, \beta_{jt} + 1) & \text{if } a_{jt} = 0 \end{cases}$$

# Belief distribution

Initial prior drawn from Beta distribution

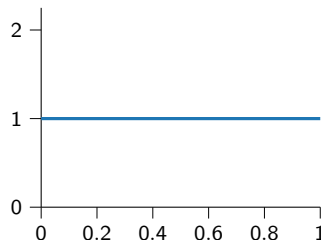
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Example:  $\alpha_0 = 1, \beta_0 = 1$

Beliefs  $p(\theta|\alpha, \beta)$



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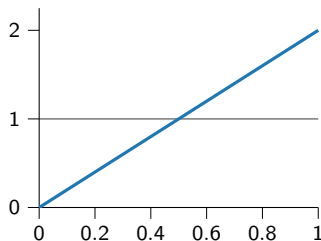
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Example:  $\alpha_0 = 1, \beta_0 = 1$

- success at  $t = 1 \implies \alpha_1 = 2, \beta_1 = 1$

Beliefs  $p(\theta|\alpha, \beta)$



# Belief distribution

Initial prior drawn from Beta distribution

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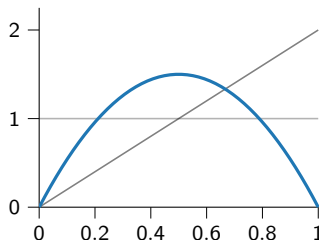
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Example:  $\alpha_0 = 1, \beta_0 = 1$

- ▶ success at  $t = 1 \implies \alpha_1 = 2, \beta_1 = 1$
- ▶ failure at  $t = 2 \implies \alpha_1 = 2, \beta_1 = 2$

Beliefs  $p(\theta|\alpha, \beta)$



# Belief distribution

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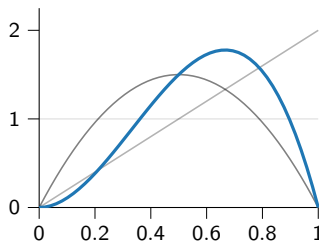
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Example:  $\alpha_0 = 1, \beta_0 = 1$

- ▶ success at  $t = 1 \implies \alpha_1 = 2, \beta_1 = 1$
- ▶ failure at  $t = 2 \implies \alpha_1 = 2, \beta_1 = 2$
- ▶ success at  $t = 3 \implies \alpha_1 = 3, \beta_1 = 2$

Beliefs  $p(\theta|\alpha, \beta)$





# Group-based parametrization

Consider group-based beliefs about abilities:

- ▶ Each individual has a group type:  $g \in \{m, f\}$
- ▶ Students form beliefs,  $P_{j0}$ , based on previously observed group successes

Simple parameterization:

$\alpha_{j0}^g$ : Number of type- $g$  students who have succeeded in  $j$

$\beta_{j0}^g$ : Number of type- $g$  students who have failed in  $j$

$\Rightarrow$  Observed success rate:

$$\mu_{j0}^g = \frac{\alpha_{j0}^g}{\alpha_{j0}^g + \beta_{j0}^g}.$$

This average is based on a sample size of type  $g$  students:

$$n_{j0}^g = \alpha_{j0}^g + \beta_{j0}^g$$

Group-based prior beliefs about probability of success in skill- $j$  courses,  $\theta_j$ :

$$\mathcal{B}(\alpha_{j0}^g, \beta_{j0}^g) \quad \Rightarrow \quad \mathcal{B}(\mu_{j0}^g n_{j0}^g, (1 - \mu_{j0}^g) n_{j0}^g)$$

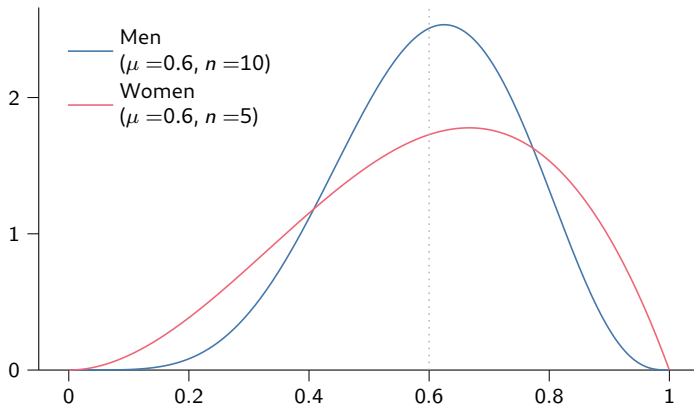
## Group-based belief distribution

Suppose there are more men than women in field  $j$ :

$$n_{j0}^m > n_{j0}^f$$

But the observed success rate is the same for the two groups:

$$\mu_{j0} = \mu_{j0}^m = \mu_{j0}^w$$



# Individual problem

A policy  $\pi : (h_t, P_t^g) \rightarrow (s_t, \ell_t)$  is optimal if it maximizes:

$$\mathbb{E}^{\pi} \left[ \sum_{t=0}^{\infty} \delta^t \left( \sum_{j=1}^J h_{jt} w_j \ell_{jt} \right) \middle| ((h_{10}, P_{10}^g), \dots, (h_{J0}, P_{J0}^g)) \right]$$

Subject to the human capital accumulation and belief transition laws:

$$h_{jt+1} = h_{jt} + \nu_j s_{jt} m_{jt}, \quad s_{jt} \sim \text{Bernoulli}(\theta_j), \quad \theta_j \sim P_{j0}^g \equiv \mathcal{B}(\alpha_{j0}^g, \beta_{j0}^g),$$

$$P_{j,t+1}^g = \mathcal{B}(\alpha_{j,t+1}^g, \beta_{j,t+1}^g), \quad (\alpha_{j,t+1}^g, \beta_{j,t+1}^g) = \begin{cases} (\alpha_{jt}^g + 1, \beta_{jt}^g) & \text{if } m_{jt}^g = 1 \text{ and } s_{jt}^g = 1 \\ (\alpha_{jt}^g, \beta_{jt}^g + 1) & \text{if } m_{jt}^g = 1 \text{ and } s_{jt}^g = 0 \\ (\alpha_{jt}^g, \beta_{jt}^g) & \text{if } m_{jt}^g = 0 \end{cases}.$$

And subject to the constraints:

$$\sum_{j=1}^J (m_{jt} + \ell_{jt}) = 1, \quad m_{jt}, \ell_{jt} \in \{0, 1\}$$

$$h_{j0} \leq \nu_j \alpha_{j0}^g$$

# Optimal policy rule

Define the field- $j$  index as the expected payoff if you committed to studying  $j$ :

$$\mathcal{I}_{jt}(h_j^g, P_j^g) = \sup_{\tau \geq 0} \mathbb{E}^{\tau} \left[ \sum_{t=0}^{\infty} \delta^t U_j(h_{jt}^g, w_j) \ell_{jt}^g \middle| (h_{j0}^g, P_{j0}^g) = (h_j^g, P_j^g) \right]$$

Define the graduation region of field  $j$  as:

$$\mathcal{G}_j(h_j^g, P_j^g) = \left\{ (h_j^g, P_j^g) \middle| \arg \max_{\tau \geq 0} \mathbb{E}^{\tau} \left[ \sum_{t=0}^{\infty} \delta^t U_j(h_{jt}^g, w_j) \ell_{jt}^g \middle| (h_j, P_j^g) \right] = 0 \right\}$$

The following policy  $\pi : (h_t, P_t^g) \rightarrow (s_t, \ell_t)$  is optimal:

1. At each  $t \geq 0$ , choose field  $j^* = \arg \max_{j \in J} \mathcal{I}_j$ , breaking ties according to any rule
2. If  $(h_{j^*}, P_{j^*}^g) \in \mathcal{G}_{j^*}$ , then enter the labor market as a  $j^*$  specialist. Otherwise, study  $j^*$  for an additional period.

Return: Identification

Model

Model Simulations

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Conclusion and next steps

# Simulate agent behavior

How can the model explain different specialization outcomes?

Consider a world with two fields, X and Y

- ▶ Wages are equal:  $w_X = w_Y$
- ▶ The agent's probabilities of success are equal:  $\theta_X = \theta_Y$
- ▶ Initial beliefs are equal to the uniform prior: [PDF of beliefs](#)

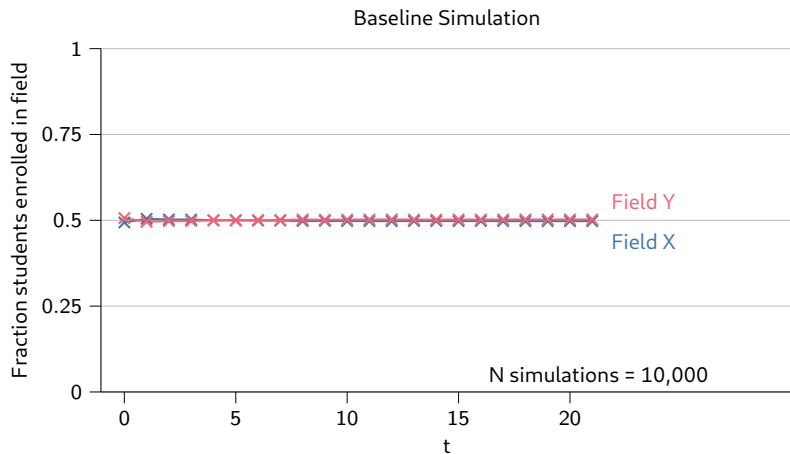
$$(\alpha_{X0}, \beta_{X0}) = (\alpha_{Y0}, \beta_{Y0}) = (1, 1)$$

- ▶ Assume  $h_{j0} = \nu \alpha_{j0}$  [Details](#)

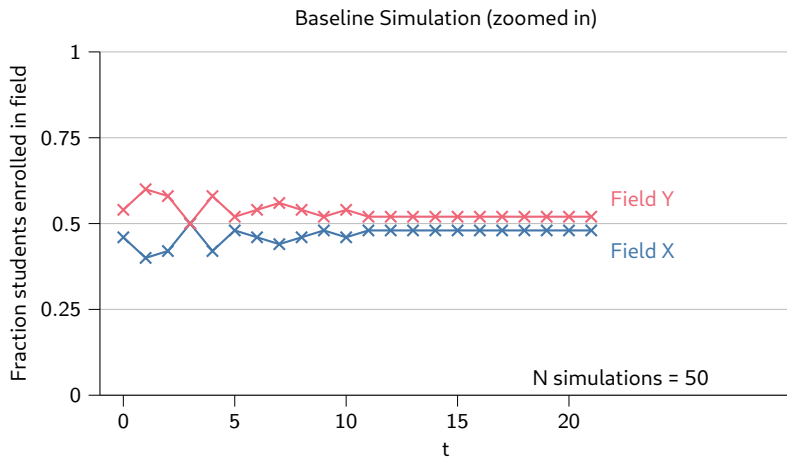
Simulate agent's specialization decisions when choosing between X and Y

- ▶ Model fraction of simulated agents choosing X or Y at time  $t$

# Default parameterization

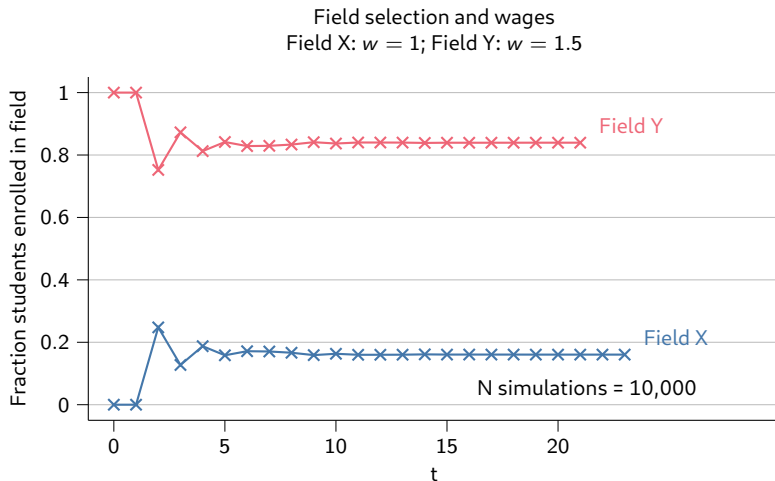


# Zooming in



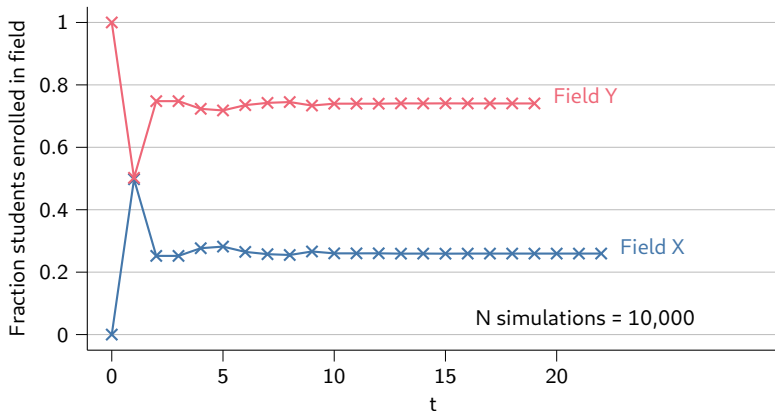


# Wage effects



# Belief effects

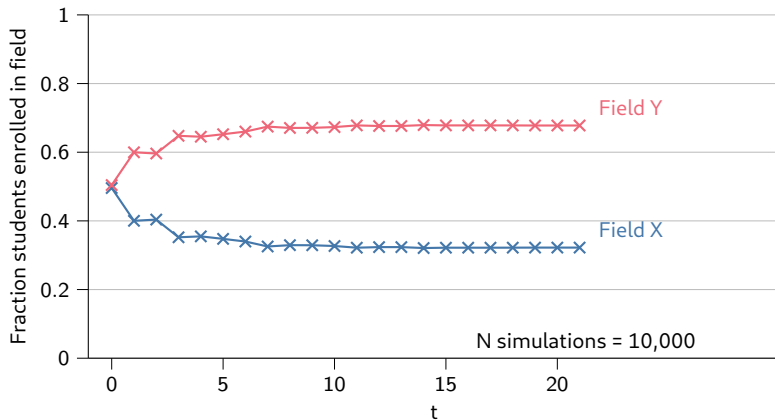
Field selection and initial beliefs  
Field X:  $(\alpha_0, \beta_0) = (1, 1)$ ; Field Y:  $(\alpha_0, \beta_0) = (2, 2)$



# Ability to succeed

Field selection and ability to succeed

Field X:  $\theta = 0.4$ ; Field Y:  $\theta = 0.6$



N simulations = 10,000

Model

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# Identification problem

Identification problem: how to identify  $(\alpha_{j0}^g, \beta_{j0}^g)$ ?

- Parameters that characterize prior beliefs about probability of success ( $\theta_j$ )

The following state variables are observable using transcript data (BPS or NLSY97):

$$\bar{m}_{jt} = \sum_{r=0}^{t-1} m_{jr}, \quad \bar{s}_{jt} = \sum_{r=0}^{t-1} s_{jr}$$

Goal: find likelihood function to estimate  $(\alpha_{j0}^g, \beta_{j0}^g)$ .

- Determine the best way to incorporate population heterogeneity (random utility? Semi-nonparametric approaches?)
- Balance endogeneity concerns with taking advantage of the recursive structure

Possible example: conditional logit approach:

$$\log \mathcal{L} = \log \sum_{i=1}^n \sum_{j=1}^J m_{ijt} \log P(m_{ijt} = 1 | \bar{m}_{ijt}, \bar{s}_{ijt}, \alpha_{j0}, \beta_{j0}, \theta_{ji})$$

## Useful model notes

Under the parametric assumption, agents study a field for a deterministic number of periods: [Details](#)

$$m_j^* = \left\lceil \frac{\delta}{1-\delta} \right\rceil - \alpha_{j0} - \beta_{j0}$$

Can analytically characterize Index in field  $j$ :

$$\mathcal{I}_{jt}(\alpha_{j0}, \beta_{j0}, \bar{m}_{jt}, \bar{s}_{jt}) = \begin{cases} \frac{w_j h_{jt}}{1-\delta} \\ \frac{w_j}{1-\delta} \delta^{m_j^* - \bar{m}_{jt}} \mathbb{E}_t \left[ h_{j, t+m_j^* - \bar{m}_{jt}} \right] \end{cases} \quad \begin{array}{l} \text{if } (\alpha_{j0}, \beta_{j0}) \in \mathcal{G}_j \\ \text{otherwise.} \end{array}$$

Probability an agent chooses  $j$  at time  $t$  depends on the index:

$$G_j(x) = P(\mathcal{I}_{jt} < x | \cdot) = P\left(\delta^{-(\alpha_{j0} + \beta_{j0})} \frac{\alpha_{j0} + \bar{s}_{jt}}{\alpha_{j0} + \beta_{j0} + \bar{m}_{jt}} < c_j x \mid \bar{m}_{jt}, \bar{s}_{jt}, \alpha_{j0}, \beta_{j0}\right)$$

Under conditional independence given all states:

$$\begin{aligned} P(m_{jt} = 1 | \cdot) &= P(\mathcal{I}_{jt} > \mathcal{I}_{kt} \forall k \neq j | \cdot) \\ &= \int \prod_{k \neq j} G_k(x) dG_j(x) \end{aligned}$$

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## Conclusion and next steps

Model suggests that group-based beliefs play an important role in specialization decisions

Next steps:

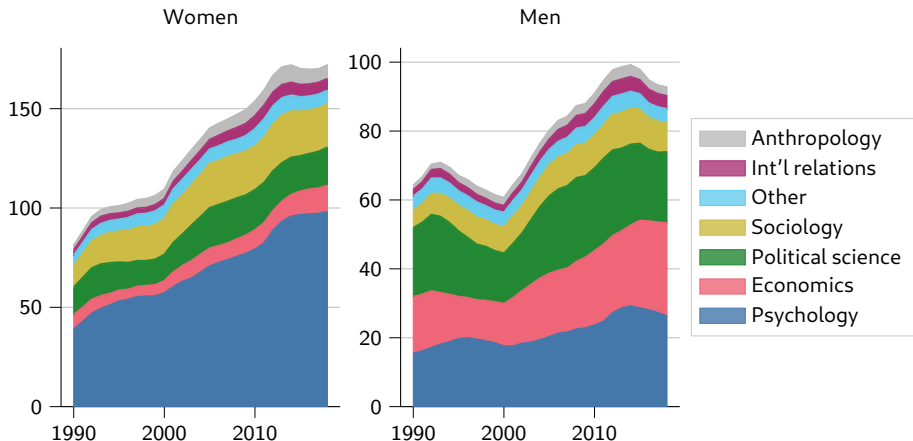
- ▶ Identification of model parameters
- ▶ Explore counterfactuals: productivity differences if we remove misallocation of talent?

Additional counterfactual exercise: affirmative action

- ▶ If we remove discrimination, how long would it take for women's beliefs to converge?
- ▶ Can affirmative action address these biases?



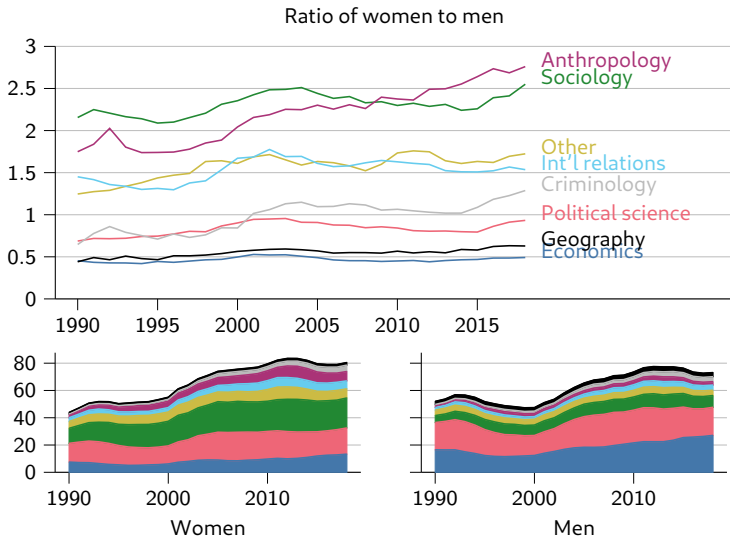
## Appendix



Social Science - number Bachelor's degrees awarded (thousands). Source: IPEDS.

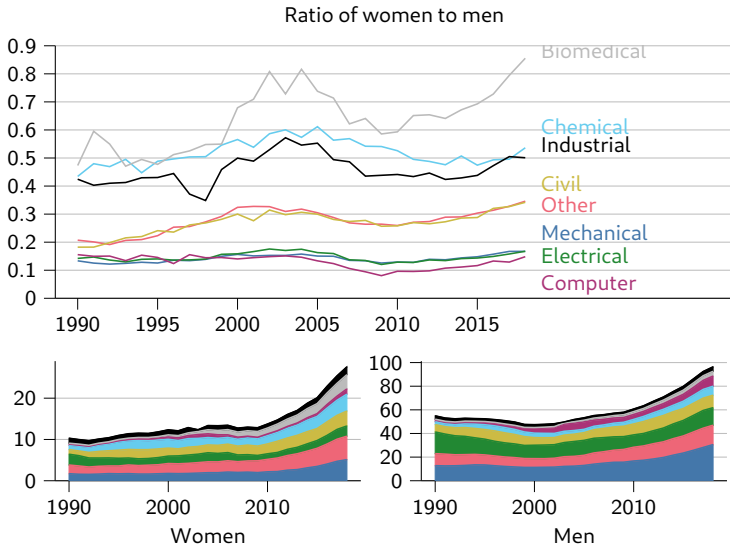
[Return: Social science ratio](#)

# Social Sciences



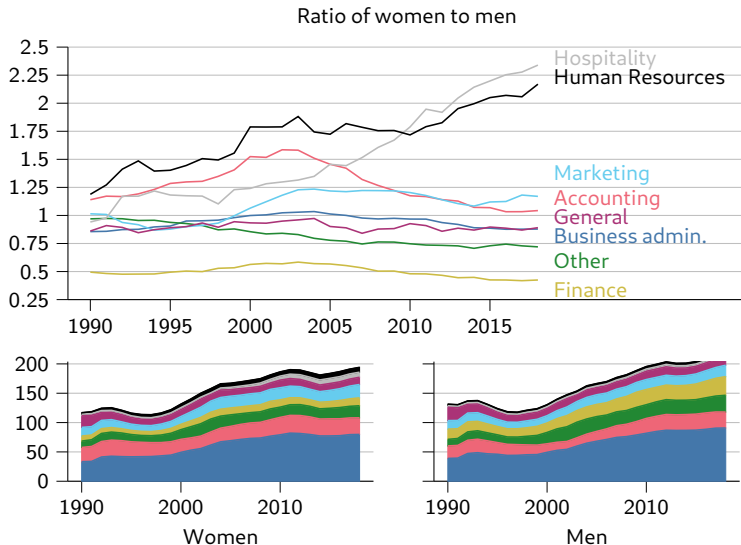
Number Bachelor's degrees awarded (thousands). Source: IPEDS.

# Engineering



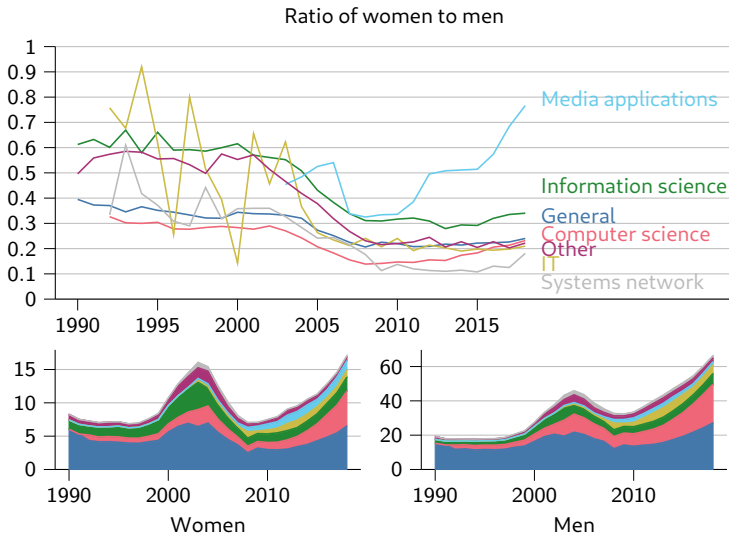
Number Bachelor's degrees awarded (thousands). Source: IPEDS.

# Business

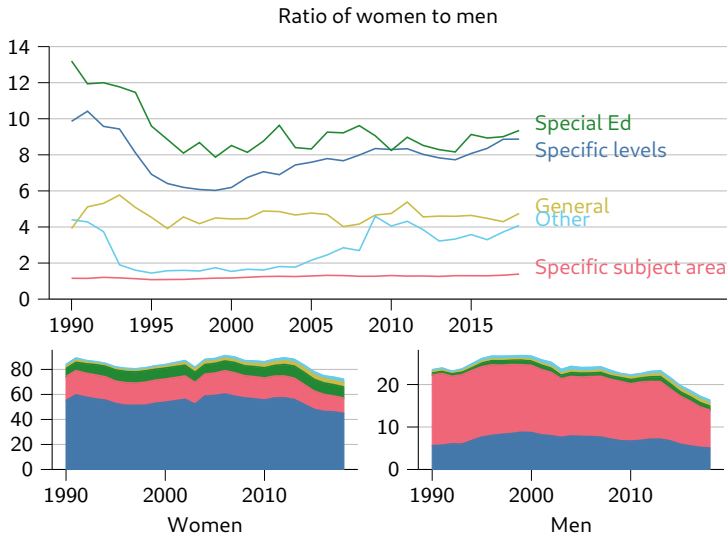


Number Bachelor's degrees awarded (thousands). Source: IPEDS.

# Computer Science

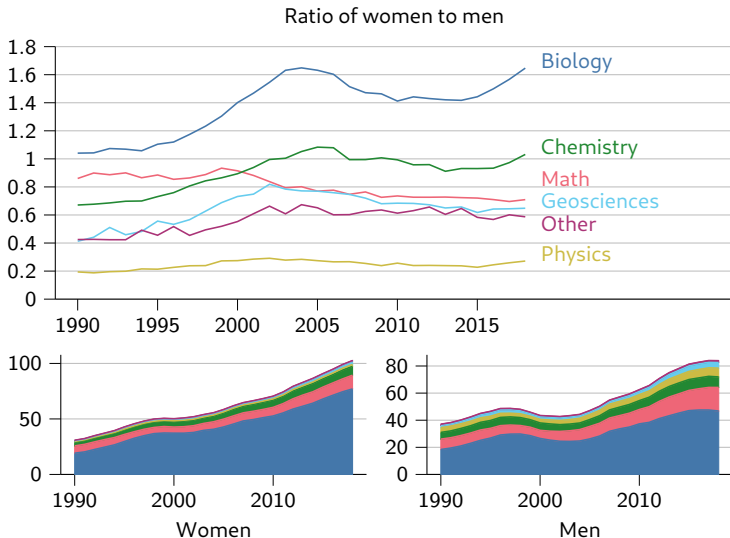


# Education



Number Bachelor's degrees awarded (thousands). Source: IPEDS.

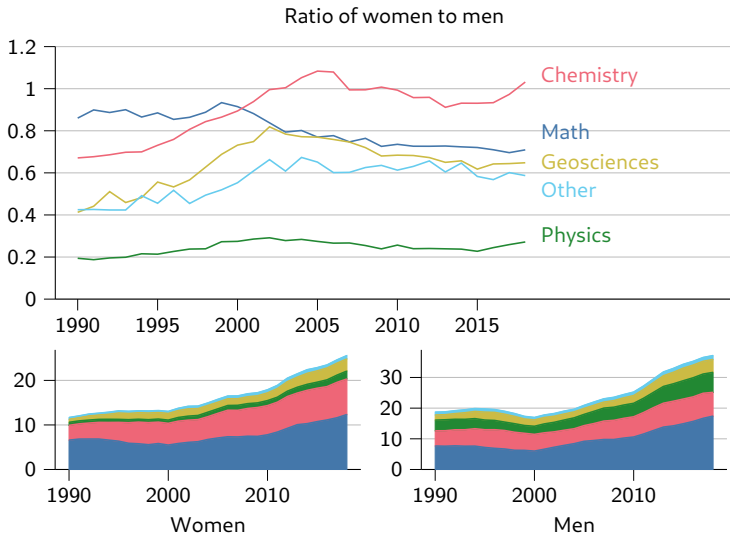
# Biological and Physical Sciences and Mathematics



Number Bachelor's degrees awarded (thousands). Source: IPEDS.



# Physical Sciences and math



Number Bachelor's degrees awarded (thousands). Source: IPEDS.

## Parametric example

Assuming  $h_{j0} = \nu\alpha_{j0}$  and letting  $c_{jt}$  be time spent studying  $j$ :

⇒ Deterministic stopping function

$$\frac{1-\delta}{\delta} \geq \frac{1}{c_{jt} + \alpha_{j0} + \beta_{j0}} \implies c_j^* = \left\lceil \frac{\delta}{1-\delta} \right\rceil - (\alpha_{j0} + \beta_{j0})$$

Graduation regions given by:

$$\mathcal{G}_j(\alpha_{jt}, \beta_{jt}) = \left\{ \alpha_{jt}, \beta_{jt} \mid \frac{\delta}{1-\delta} \leq \alpha_{jt} + \beta_{jt} \right\}$$

In this example, note that  $\mathcal{G}_Y = \mathcal{G}_X$ . Index in the graduation region given by  $\frac{h_{jt}}{1-\delta}$ . Index when not in graduation region given by Binomial distribution with parameters  $(c_j^* - c_j, \frac{h_{jt}}{\nu(c_{jt} + \alpha_{j0} + \beta_{j0})})$ :

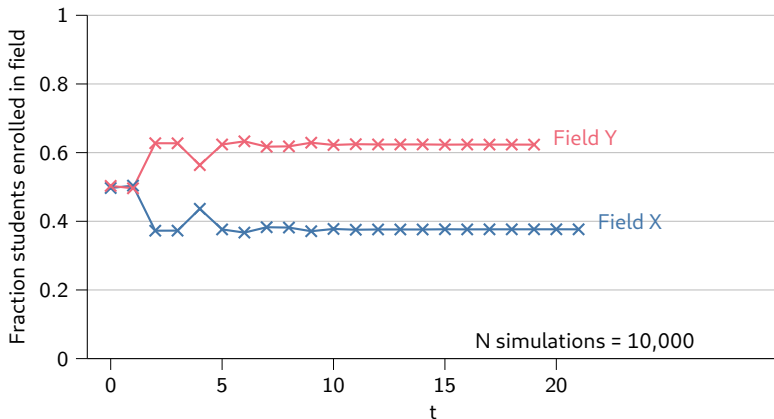
$$\mathcal{I}_{jt}(h_{jt}, \alpha_{jt}, \beta_{jt}) = \begin{cases} \frac{w_{jt} h_{jt}}{1-\delta} & \text{if } \{\alpha_{jt}, \beta_{jt}\} \in \mathcal{G}_j, \\ \frac{w_{jt} h_{jt}}{1-\delta} \left[ \frac{\left\lceil \frac{\delta}{1-\delta} \right\rceil \delta \left\lceil \frac{\delta}{1-\delta} \right\rceil - c_{jt} - \alpha_{j0} - \beta_{j0}}{c_{jt} + \alpha_{j0} + \beta_{j0}} \right] & \text{if } \{\alpha_{jt}, \beta_{jt}\} \notin \mathcal{G}_j \end{cases}$$

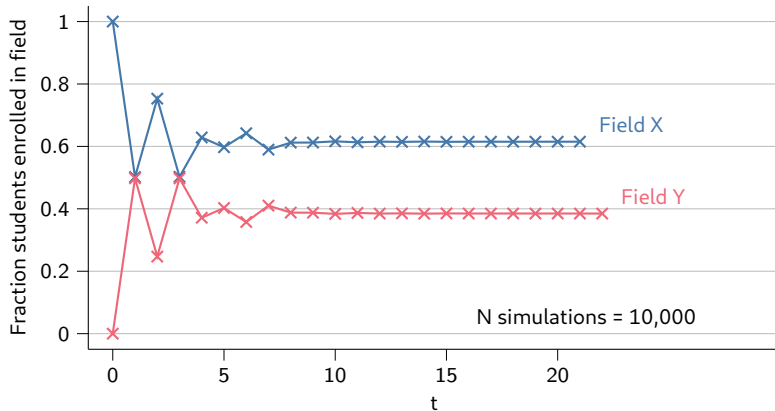
If  $\nu_X = \frac{\alpha_{X0} + \beta_{X0}}{\alpha_{Y0} + \beta_{Y0}} \cdot \frac{\alpha_{Y0}}{\alpha_{X0}} \cdot \delta^{\alpha_{X0} + \beta_{X0} - \alpha_{Y0} - \beta_{Y0}}$ , then:

- ▶  $h_{X0} = h_{Y0}$ , and
- ▶ Agents randomly choose between fields  $X$  and  $Y$  at  $t = 0$

Field X:  $\nu = 1.09$ ; Field Y:  $\nu = 1$

Field X:  $(\alpha_0, \beta_0) = (1, 1)$ ; Field Y:  $(\alpha_0, \beta_0) = (2, 2)$



$\nu$  effectsReturn:  $\alpha_{X0} \nu_X = \alpha_{Y0} \nu_Y$ 

Return: belief simulation

Return: ability simulation