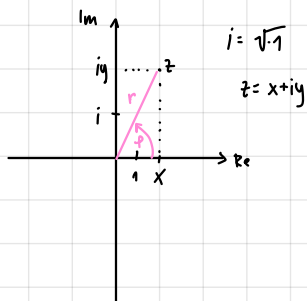


## Kartezični zapis



## Polarni zapis

$$z = r(\cos \varphi + i \sin \varphi)$$

$\varphi \in \mathbb{R}$  ni enolično določen  $\rightarrow$  do  $+k2\pi$  natanko,  $k \in \mathbb{Z}$   
 $r = |z| \in [0, \infty)$  je enolično določen  
 $\rightarrow$  argument, polarni kot  
 pri  $z=0$  je kot karkoli

$$x = r \cdot \cos \varphi$$

$$y = r \cdot \sin \varphi$$

$$\tan \varphi = \frac{y}{x} \Rightarrow \varphi = \arctan \frac{y}{x}$$

upoštevamo kvadrant!

Primer: Zapiši v polarni obliki

$$z = 1 + i$$

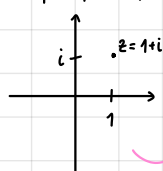
$$x = 1 \quad r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

pitagorov

$$y = 1$$

$$\tan \varphi = \frac{1}{1} = 1$$

$$\varphi = \frac{\pi}{4} + k\pi, k \in \mathbb{Z}$$



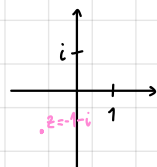
$$\varphi \in \left\{ \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \dots, \frac{-3\pi}{4}, \frac{-7\pi}{4}, \dots \right\}$$

kvadrant

zadosten odg.

če bi imeli  $z = -1 - i$

$$\varphi = \frac{\pi}{4}$$

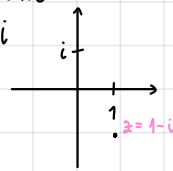


Konjugirano:

$$\bar{z} = 1 - i$$

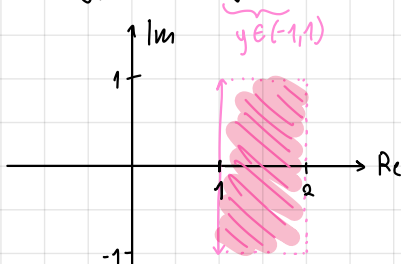
$$r = \sqrt{2}$$

$$\varphi = -\frac{\pi}{4}$$

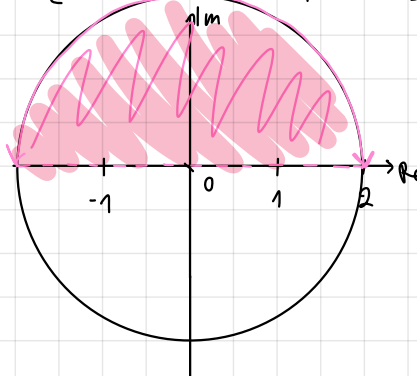


Primer: Nariši naslednjo množico

$$\{x + iy; x \in [1, 2], |y| < 1\}$$



$$\{r(\cos \varphi + i \sin \varphi), r \leq 2, \varphi \in (0, \pi)\}$$



## Množenje v polarnem zapisu

$$z_j = |z_j| \cdot (\cos \varphi_j + i \sin \varphi_j) \quad j \in \{1, 2\}$$

$$z_1 \cdot z_2 = |z_1| \cdot (\cos \varphi_1 + i \sin \varphi_1) \cdot |z_2| \cdot (\cos \varphi_2 + i \sin \varphi_2)$$

$$= |z_1| \cdot |z_2| (\cos \varphi_1 \cos \varphi_2 - \sin \varphi_1 \sin \varphi_2 + i (\cos \varphi_1 \sin \varphi_2 + \sin \varphi_1 \cos \varphi_2))$$

$$= |z_1| \cdot |z_2| (\cos(\varphi_1 + \varphi_2) + i \sin(\varphi_1 + \varphi_2))$$

produkt množi absolutne vrednosti in sešteja kote

## Eulerjeva formula

$$e^{i\varphi} = \cos \varphi + i \sin \varphi$$

Polarni zapis

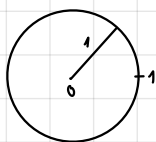
$$z = r \cdot e^{i\varphi}$$

$$r_1 \cdot e^{i\varphi_1} \cdot r_2 \cdot e^{i\varphi_2} = r_1 \cdot r_2 \cdot e^{i(\varphi_1 + \varphi_2)}$$

absolutna vrednost

$$e^{i\pi} = -1$$

Primer:  $\{e^{i\varphi}, \varphi \in \mathbb{R}\}$   
središče v  $(0,0)$   
polmer je 1



$$\{r \cdot e^{i\frac{\pi}{2}}, r \in [0, \infty)\}$$

0

$$z = 4i = 4e^{i\frac{\pi}{2}}$$

$$x = 0$$

$$y = 4$$

$$r = 4$$

$$\varphi = \frac{\pi}{2}$$



## Lastnosti polarne zapisa

1.  $x_1 + iy_1 = x_2 + iy_2 \Leftrightarrow x_1 = x_2 \wedge y_1 = y_2$

$$r_1 e^{i\varphi_1} = r_2 e^{i\varphi_2} \Leftrightarrow r_1 = r_2 \wedge (\varphi_1 - \varphi_2) \in k \cdot 2\pi, k \in \mathbb{Z}$$

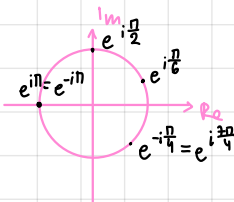
ALI

2.  $\overline{r \cdot e^{i\varphi}} = r \cdot e^{-i\varphi}$   $r_1 = r_2 = 0$

3.  $(r \cdot e^{i\varphi})^n = r^n \cdot e^{in\varphi}$  de Moivrejeva formula

4.  $(r e^{i\varphi})^{-1} = r^{-1} \cdot e^{-i\varphi}$   $r \neq 0$

5.  $\frac{r_1 e^{i\varphi_1}}{r_2 e^{i\varphi_2}} = \frac{r_1}{r_2} e^{i(\varphi_1 - \varphi_2)}$   $r_2 \neq 0$



Primer:

$$z = 1 - i\sqrt{3}$$

navarimo  $z, z^2, z^3 \dots$

$$x = 1$$

$$y = -\sqrt{3}$$

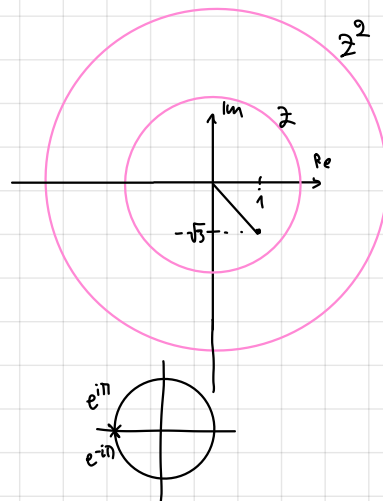
$$r = 2 = \sqrt{x^2 + y^2}$$

$$\varphi = \frac{\pi}{3}$$

$$z = 2 \cdot e^{-i\frac{\pi}{3}}$$

$$z^2 = 4 \cdot e^{-i\frac{2\pi}{3}}$$

$$z^3 = 8 \cdot e^{-i\pi} = -8$$



## Geometrija operacij v ravnini

### preslikava

$$z \mapsto \bar{z}$$

$$z \mapsto -z$$

$$z \mapsto z + z_0$$

$$z \mapsto z \cdot e^{i\varphi}$$

$$z \mapsto z \cdot r \cdot e^{i\varphi}$$

### transformacija v C

zrcaljenje preko Re (pravica)

zrcaljenje preko  $z=0$  (točka)

premik za  $z_0$

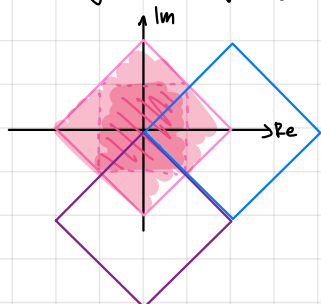
rotacija za  $\varphi$

razteg  $\times r$ , rotacija  $+\varphi$

Primer: V kaj se s preslikavo  $z \mapsto (z(1+i) - 2i) \cdot i$  preslika:  $\{x+iy, |x| < 1, |y| < 1\}$

menja se Re in Im  $\rightarrow$  obrne za  $90^\circ$   
polarna vrednost je  $\sqrt{2}$ , kot pa  $\frac{\pi}{2}$

$$2 \cdot \sqrt{2} \cdot e^{i\frac{\pi}{2}}$$



## Koreni enote

Primer:  $z^5 = 1 = 1 \cdot e^{i0} = 1 \cdot e^{i \cdot \frac{0}{5}}$   
 $z = r \cdot e^{i\varphi}$

$$\underbrace{r^5}_{=1} \cdot \underbrace{e^{i5\varphi}}_{=1} = 1 \cdot e^{i0} = e^{i2\pi} = e^{i4\pi} = e^{i6\pi} = \dots$$

$$r^5 = 1 \quad e^{i5\varphi} = e^{i0}$$

$$r = 1$$

$$① \quad 5\varphi = 0 + \frac{2\pi}{5}$$

$$\boxed{\varphi = \frac{0}{5} + \frac{2\pi}{5}}$$

$$② \quad 5\varphi = 2\pi$$

$$\boxed{\varphi = \frac{2\pi}{5} + \frac{2\pi}{5}}$$

$$③ \quad 5\varphi = 4\pi$$

$$\boxed{\varphi = \frac{4\pi}{5} + \frac{2\pi}{5}}$$

5 različnih  
rešitev

$$z_1 = 1 \cdot e^{i0} = 1$$

$$z_2 = e^{i\frac{2\pi}{5}}$$

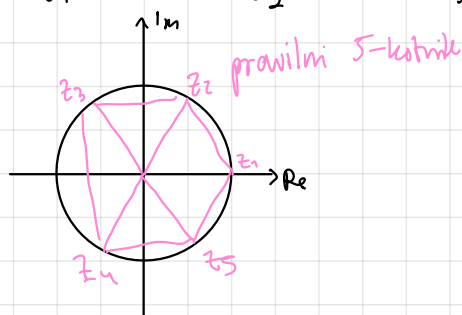
$$z_3 = e^{i\frac{4\pi}{5}}$$

$$z_4 = e^{i\frac{6\pi}{5}}$$

$$z_5 = e^{i\frac{8\pi}{5}}$$

$$z_6 = e^{i\frac{10\pi}{5}} = z_1$$

začnejo se  
ponavljati



## FORMULE ZA KORENJE

$$n \in \mathbb{N}, z^n = r \cdot e^{i\varphi} \quad k = 0, \dots, n-1$$

$$z_k = \sqrt[n]{r} \cdot e^{i(\frac{\varphi}{n} + \frac{2k\pi}{n})}$$

$$= \sqrt[n]{r} \cdot e^{i(\frac{\varphi + 2k\pi}{n})}$$

Pravilni n-kotnik

n različnih,  
razen pri r=0

Primer:  $(z^3 - 2) \cdot (z^2 + i) = 0$

$$① \quad z^3 = 2 \quad \varphi = 0$$

$$z_1 = \sqrt[3]{2} \cdot e^{i\frac{0}{3}}$$

$$z_2 = \sqrt[3]{2} \cdot e^{i\frac{0+2\pi}{3}}$$

$$z_3 = \sqrt[3]{2} \cdot e^{i\frac{0+4\pi}{3}}$$

popolni  $\Delta$

→ realna rešitev  
(ena more bit)

$$② \quad z^2 = -i = 1 \cdot e^{-i\frac{\pi}{2}}$$

$$z_4 = 1 \cdot e^{i(\frac{-\pi}{2})}$$

$$z_5 = 1 \cdot e^{i(\frac{-\pi+2\pi}{2})} = -z_4$$

popolna

## OSNOVNI IZREK ALGEBRE

Vsaka enačba  $a_n \in \mathbb{C}, a_n \neq 0$

$$a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0 = 0$$

ima natanko n rešitev

$$p(z) = a_n(z-z_1)^{v_1} \cdot (z-z_2)^{v_2} \cdot \dots \cdot (z-z_n)^{v_n}$$

$$v_1 + \dots + v_n = n$$

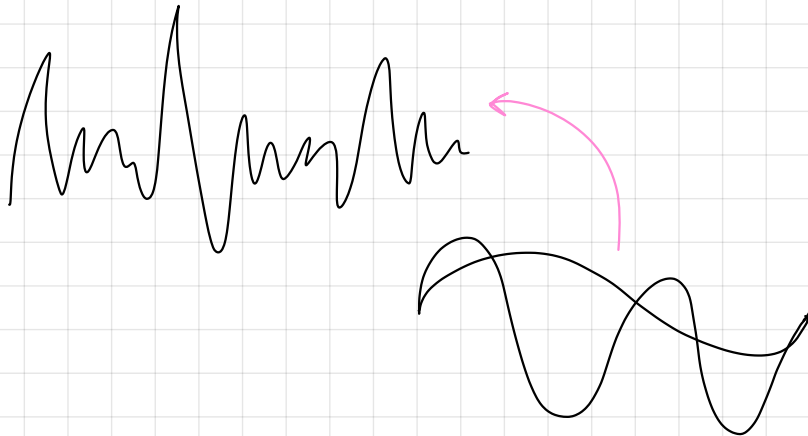
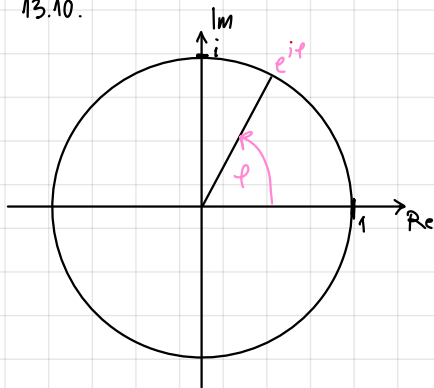
Če so  $a_k \in \mathbb{R}, \forall k$

potem ne-realne ničle

nastopajo v konjugiranih

parih

13.10.



## Diskretna Fourierjeva transformacija - DFT

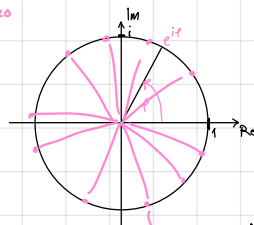
"nek signal" → kuk visok smo pri  $N/M/N$

$(x_0, x_1, \dots, x_N) \rightsquigarrow (y_0, y_1, \dots, y_N) \leftarrow$  zaporedje frekvenc  
 ↳ zaporedje jakosti

$$y_k = \sum_{n=0}^{N-1} x_n e^{-i \frac{2\pi}{N} k \cdot n}$$

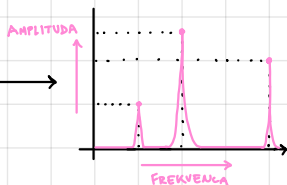
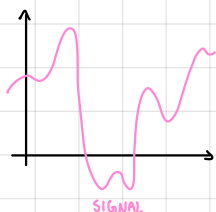
kolkokrat gre gor pa dol

kolikokrat se valj s frekvenco 1



velikost vala - abs. vred.  
 faza - kot

krog razdelimo na N-delov



## ČAPOREDJE

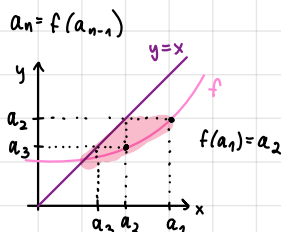
Def: preslikava  $\mathbb{N} \rightarrow \mathbb{R}$   
 $n \rightarrow a_n$   $(a_n) = (a_1, a_2, \dots)$

Podajanje zaporedja:

• **Eksplicitno:**  $a_n = f(n)$

• **Rekurzivno:**  $a_n = f(a_{n-1}, a_{n-2})$  2-člena  
 + začetni člen  
 $a_n = f(a_{n-1})$  1-člen

GRAFIČNI PRIKAZ REKURZIVNEGA ZAP.



Primeri:  $a_n = (-1)^n$

• **Aritmetično zap.**

$$a_n = a_0 + n \cdot d$$

ali

$$a_n = a_{n-1} + d$$

• **Geometrijsko zap.**

$$a_n = a_0 \cdot q^n$$

ali

$$a_n = q \cdot a_{n-1}$$

• **Fibonaccijevo zap.**

$$a_1 = 1 = a_2$$

$$a_n = a_{n-1} + a_{n-2}$$

• **Collatzova domneva** (1937)

$$a_{n+1} = \begin{cases} \frac{a_n}{2} & \text{an sodo} \\ 3 \cdot a_n + 1 & \text{an liha} \end{cases}$$

Ali se za  $\forall a_0 \in \mathbb{N}$  to zap. vrne v 1?

↳ Preverjeno do  $2^{71}$

Lastnosti zaporedja:  $(a_n)_n$  je

• **Navzgor omejeno**, če obstaja zgornja meja  $M \in \mathbb{R}: \forall n \in \mathbb{N} \wedge a_n \leq M$

Supremum  $\sup_{n \in \mathbb{N}} a_n =$  najmanjša zgornja meja

• **Navzdol omejeno**, če obstaja spodnja meja  $m \in \mathbb{R}: \forall n \in \mathbb{N} \wedge a_n \geq m$

Infimum  $\inf_{n \in \mathbb{N}} a_n =$  najvišja spodnja meja

• **Omejeno**, če je navzgor in navzdol omejeno

• **Naraščajoče**, če  $\forall n: a_{n+1} \geq a_n$

strogo naraščajoče, če  $\forall n: a_{n+1} > a_n$

• **Padajoče**, če  $\forall n: a_{n+1} \leq a_n$

strogo padajoče, če  $\forall n: a_{n+1} < a_n$

• **Monotono**, če je povsod ali padajoče ali naraščajoče

Primeri:  $n = 1, 2, 3, \dots$

1.  $\frac{n^2-1}{n} = n - \frac{1}{n}$  ↓ omejeno, ↑ neomejeno, ↑ strogo

2.  $b_n = \frac{b_{n-1}}{2}$

2.1  $b_1 > 0$   
 strogo padajoče  
 $\inf b_n = 0$

2.2  $b_1 < 0$   
 strogo naraščajoče  
 $\sup b_n = 0$

2.3  $b_1 = 0$   
 konst. v 0  
 OBOJE DRŽI

## LIMITA ZAPOREDJA

Def:  $a \in \mathbb{R}$  je limita zap.  $(a_n)_n$ , če

$$\forall \varepsilon > 0 \exists N \in \mathbb{N}$$

$$\forall n > N \quad |a_n - a| < \varepsilon$$

Zaporedje je konvergentno, če ima limito

Zaporedje je divergentno, če nima limite

Primeri:

1.  $a_n = \frac{1}{n^2}$

Dokažimo, da je  $\lim_{n \rightarrow \infty} a_n = 0$

$$n^2, \varepsilon > 0$$

Izberemo  $\varepsilon > 0$  po def.

$$\frac{1}{n^2} < \varepsilon \Leftrightarrow \frac{n^2}{1} > \frac{1}{\varepsilon}$$

$$\frac{1}{\varepsilon} < n^2$$

$$R: n > \sqrt{\frac{1}{\varepsilon}} \in \mathbb{N}$$

$$\begin{aligned} x > 0 \\ \lceil x \rceil &= \min \{ n \in \mathbb{N}_0; n \geq x \} \\ \lfloor x \rfloor &= \max \{ n \in \mathbb{N}; n \leq x \} \end{aligned}$$

2.  $b_n = (-1)^n$

Ali je 0 limita?

Izberemo  $\varepsilon > 0$

$$|(-1)^n - 0| < \varepsilon$$

$$1 < \varepsilon$$

Ni okej za  $\varepsilon < 1$ !

R: NI LIMITA

Zaporedje  $(a_n)_n$

• Narašča preko vsake meje  $\left[ \lim_{n \rightarrow \infty} a_n = \infty \right]$  ni konv. zap.

$$\forall M \in \mathbb{R} \exists N \in \mathbb{N}: a_n > M, \forall n \geq N$$

• Pada pod vsako mejo  $\left[ \lim_{n \rightarrow \infty} a_n = -\infty \right]$

$$\forall m \in \mathbb{R} \exists N \in \mathbb{N}: a_n < m, \forall n \geq N$$



Pravila za rač. limit

Naj bo  $a = \lim_{n \rightarrow \infty} a_n, b = \lim_{n \rightarrow \infty} b_n$

1.  $\lim_{n \rightarrow \infty} (a_n + b_n) = a + b$

2.  $\lim_{n \rightarrow \infty} (a_n \cdot b_n) = a \cdot b$

3. V smislu naraščanja in padanja preko vseh mej:

$$\infty + \infty = \infty$$

$$\infty \cdot \infty = \infty$$

$$0 \cdot \infty = 0$$

$$-\infty \cdot \infty = -\infty$$

$$\frac{1}{\infty} = 0$$

## MANJKA: 20.-26.10.

Predavanja: Pravila za računanje z limitami, izrek o sendviču, izrek o monotoni konvergenci, e kot limita zaporedja  $a_n = \left(1 + \frac{1}{n}\right)^n$ . Vrste - delne vsote, konvergenca, geometrijska vrsta.

## Pravila za računanje vrst

Naj bosta  $\sum_{n=1}^{\infty} a_n, \sum_{n=1}^{\infty} b_n$  konvergentni

Tedaj konvergirata:

$$\sum_{n=1}^{\infty} (a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$$

$$c \cdot \sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} c a_n$$

Primer:

$$a) \sum_{n=1}^{\infty} \left( \left( \frac{1}{2} \right)^n - \left( \frac{1}{3} \right)^n \right) = \sum_{n=1}^{\infty} \frac{1}{2^n} - \sum_{n=1}^{\infty} \frac{1}{3^n} =$$

$$= \frac{1}{1 - \frac{1}{2}} - \frac{1}{1 - \frac{1}{3}} = \dots$$

$$b) \sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{n+1} \right) = \sum_{n=1}^{\infty} \frac{1}{n} - \sum_{n=1}^{\infty} \frac{1}{n+1} = \infty - \infty = ? = 1$$

Dominiranje vrst  $a_n, b_n > 0$ 

$\sum_{n=1}^{\infty} a_n$  dominira  $\sum_{n=1}^{\infty} b_n$ , če  $a_n \geq b_n, \forall n \Rightarrow S_n^a \geq S_n^b$

V tem primeru velja:

Če  $\sum_{n=1}^{\infty} b_n$  div., potem tudi  $\sum_{n=1}^{\infty} a_n$  divergira

$\sum_{n=1}^{\infty} a_n$  konvergira  $\Rightarrow \sum_{n=1}^{\infty} b_n$  konv.

Primer:

$$a) \sum_{n=1}^{\infty} \left( \sin(e^{\frac{6x^3}{2^n}} - 8\pi^2) \right)^6$$

$$\sum_{n=1}^{\infty} \frac{(\sin(e^{\frac{6x^3}{2^n}} - 8\pi^2))^6}{2^n} \leq \frac{1}{2^n}$$

konvergira

konvergira

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n} \geq \sum_{n=1}^{\infty} \frac{1}{n}$$

divergira

večja  $\rightarrow$  konv.

manjša  $\rightarrow$  div.

Konvergenčni kriteriji:  $a_n > 0, \forall n$ 

## ① Kvocienčni kriterij

$$a_n = q^n$$

$$\sum_{n=1}^{\infty} q^n \begin{cases} \text{konv. } |q| < 1 \\ \text{div. } |q| > 1 \end{cases}$$

$$q = \frac{a_{n+1}}{a_n}$$

$$D_n = \frac{a_{n+1}}{a_n}, \text{ naj obstaja } \lim_{n \rightarrow \infty} D_n \in \mathbb{R}$$

$D > 1 \Rightarrow$  vrsta divergira ( $\sum_{n=1}^{\infty} a_n = \infty$ )

$D < 1 \Rightarrow$  vrsta konvergira

$D = 1 \Rightarrow \neg(\text{"/})$

Primer: a) Za katere  $x > 0$  vrsta konv.

$$\sum_{n=1}^{\infty} n \cdot x^n ?$$

$$a_n = n \cdot x^n$$

$$D_n = \frac{(n+1)x^{n+1}}{n \cdot x^n} = \frac{n+1}{n} \cdot x \xrightarrow{n \rightarrow \infty} x$$

konv. za  $x < 1$

div. za  $x > 1$

$$b) \sum_{n=1}^{\infty} \frac{x^n}{n!}$$

$$a_n = \frac{x^n}{n!}$$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}} = \frac{x}{n+1} \xrightarrow{n \rightarrow \infty} 0$$

konv. za vse  $x > 0$

## ② Korenski kriterij

$$D_n = \sqrt[n]{a_n}, \text{ naj obstaja } \lim_{n \rightarrow \infty} D_n \in \mathbb{R}$$

$D > 1 \Rightarrow$  vrsta divergira ( $\sum_{n=1}^{\infty} a_n = \infty$ )

$D < 1 \Rightarrow$  vrsta konvergira

$D = 1 \Rightarrow \neg(\text{"/})$

Primer:

$$\sum_{n=1}^{\infty} \left( \frac{x}{n} \right)^n$$

$$a_n = \left( \frac{x}{n} \right)^n$$

$$\sqrt[n]{a_n} = \frac{x}{n} \xrightarrow{n \rightarrow \infty} 0$$

konv. za  $x > 0$

## ③ Leibnitzov kriterij

$$a_n \xrightarrow{n \rightarrow \infty} 0 \text{ monotono (t.j.: } a_{n+1} \leq a_n, \forall n)$$

$$\Rightarrow \sum_{n=1}^{\infty} (-1)^n a_n \text{ konvergen}$$

Primer:

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ div. } \sum_{n=1}^{\infty} \frac{1}{2^n} \text{ div.}$$

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} \text{ div.}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \dots = \infty$$

$$-1 - \frac{1}{3} - \frac{1}{5} - \frac{1}{7} \dots = -\infty$$

! VESTNI RGD SEŠT: JE POMENBEJ!

$$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n} \text{ konv.}$$

②  $\lim_{x \nearrow a} f(x) = b \iff \forall \epsilon > 0 \exists \delta > 0 \forall x \in D_f \cap (a-\delta, a) : |f(x) - b| < \epsilon$

$$x \in (a-\delta, a) \Rightarrow |f(x) - b| < \epsilon$$

③  $\lim_{x \rightarrow a} f(x) = b$   $\Leftrightarrow \forall \epsilon > 0 \exists \delta > 0$   $x \rightarrow a^+$

$$x \in (a, a + \delta) \Rightarrow |f(x) - b| < \epsilon$$



$$\forall \varepsilon > 0 \quad \exists M > 0$$

$$\forall \varepsilon > 0 \quad \exists m, m \in \mathbb{R}$$

$$\forall N > 0 \quad \exists \delta > 0$$

8.  $\lim_{x \rightarrow a^+} f(x) = \infty$

ME NA

⑩  $\lim_{x \rightarrow -\infty} f(x) = \infty$

12.  $\lim_{x \rightarrow -\infty} f(x) = -\infty$

$$2. f(x) = \frac{1}{1+e^x}$$

$$\lim_{x \rightarrow 0^-} \frac{1}{1+e^{\frac{1}{x}}}$$

$$\lim_{x \rightarrow 0^+} \frac{1}{1 + e^{\frac{1}{x}}} = 0$$

$$\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$$

$$\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$$

$$\lim_{t \rightarrow \infty} e^{-t} = 1$$

$$\lim_{t \rightarrow \infty} e^t = \infty$$

$$\lim_{s \rightarrow 1} \frac{1}{s} = 1$$

$$\lim_{s \rightarrow \infty} \frac{1}{s} = 0$$

$\lim_{x \rightarrow 0} \frac{1}{1 + e^{\frac{1}{x}}}$

$$\begin{aligned} e^{-1000} &= \frac{1}{e^{1000}} \\ e^{-10000} &= \frac{1}{e^{10000}} \end{aligned} \quad \left. \vphantom{\begin{aligned} e^{-1000} &= \frac{1}{e^{1000}} \\ e^{-10000} &= \frac{1}{e^{10000}} \end{aligned}} \right\} \text{bliža se } 0$$

## Lastnosti limit funkcij

$\alpha, \beta \in \mathbb{R}$  v  $\{\pm \infty\}$ ; tedaj velja

$$\lim_{x \rightarrow \alpha} f(x) = \beta$$

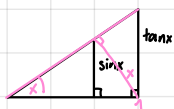
Primer:

$$1. \lim_{x \rightarrow \infty} \sin \frac{1}{x} = \lim_{t \rightarrow 0} \sin t = 0$$

$$2. \lim_{x \rightarrow 0} \sin \frac{1}{x} = /$$

$$3. \lim_{x \rightarrow \frac{\pi}{2}} \sin \frac{1}{x} = 1$$

Dokaz da  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$   $(x > 0)$



$$\sin x < x < \tan x$$

$$\frac{1}{\sin x} > \frac{1}{x} > \frac{\cos x}{\sin x}$$

$$1 > \frac{\sin x}{x} > \cos x$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$1 \quad 1 \quad 1$$

sklep

## Žveznost funkcije

Funkcija  $f$  je žvezna

$$\text{N} \ a \in D_f, \text{ če } \lim_{x \rightarrow a} f(x) = f(a)$$

### LASTNOSTI ŽVEZNOSTI

1. Vse elementarne funkcije so žvezne

2. Če sta  $f$  in  $g$  žvezni v  $a$ :

→ grafa sta v  $a$  neprekinjeni krivulji

→ so žvezne  $f+g$ ,  $\sin(g)$  in  $a$

→ lahko zamenjamo vrstni red funk. in limite v  $a$

3.  $f$  žvezna v  $a$ ,  $g$  žvezna v  $f(a) \Rightarrow g \circ f$  žvezna v  $a$

4. Če  $\lim_{x \rightarrow b} f(x)$  obstaja,  $b \in D_f$ , lahko  $f$  s predpisom

$$f(b) = \lim_{x \rightarrow b} f(x)$$

žvezno razširimo na  $b$

5. Če je  $f$  žvezna na intervalu  $[a, b]$

# MANJKA:

niče žveznih funk.

slika omejenega zaprt. intervala