



## Matematična indukcija

-tehnika dokazovanja  
trditev, formul ipd.  
kjer nastopa  $\mathbb{N}$  število  
 $T(n)$  za  $n \in \mathbb{N}$   
(usaj od nekega zač. št.  
naprej)

① preverimo  $T(1)$  - **baza indukcije**

② dokazemo  $T(n) \Rightarrow T(n+1)$  - **korak indukcije**  
Vaje: **induk. predpostavka**

$$1. a) 1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1) = \frac{1}{3}n(n+1)(n+2)$$

$$\textcircled{1} n=1: 1 \cdot 2 = \frac{1}{3} \cdot 1 \cdot 2 \cdot 3$$

$$2=2 \checkmark$$

② predpostavimo, da za  $n$  velja

$$1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1) = \frac{1}{3}n(n+1)(n+2)$$

hocemo dokazati, da velja tudi

$$1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1) + (n+1)(n+2) = \frac{1}{3}(n+1)(n+2)(n+3)$$

$$n+1: 1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1) + (n+1)(n+2) = \frac{1}{3}(n+1)(n+2)(n+3)$$

$$1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1) + (n+1)(n+2) = \underbrace{\frac{1}{3}n(n+1)(n+2)}_{\text{po induk. predp.}} + (n+1)(n+2) = (n+1)(n+2)\left(\frac{n}{3} + 1\right) = \begin{array}{l} \text{1. izpostavimo skupni faktor} \\ \text{2. izpostavimo } \frac{1}{3} \end{array}$$

$$c) n! > 2^{n-1} \quad \text{za } n > 2$$

$$1 \cdot 2 \cdot 3 \dots n \quad \text{ali} \quad n \geq 3$$

$$\textcircled{1} n=3: 3! > 2^2$$

$$6 > 4$$

$\text{zato in 1}$

$$\textcircled{2} \text{ i.p.: } n! > 2^{n-1}$$

dokazujemo:  $(n+1)! > 2^n$

$$(n+1)! = n! \cdot (n+1)$$

$$n! \cdot (n+1) > 2^{n-1} (n+1) \quad \begin{array}{l} \text{ko množiš neenako} \\ \text{paži da je množično} \\ \text{pozitiven} \end{array}$$

$$2^n = 2^{n-1} \cdot 2$$

$\downarrow$   
 $n \geq 2$ , torej razvidno,  
da je večje

$$\textcircled{R}: n!(n+1) > 2^{n-1}(n+1) > 2^{n-1} \cdot 2$$

dodali smo re 0

$$\textcircled{4} \quad \forall n \in \mathbb{N}_0 \quad 57 | 7^{n+2} + 8^{2n+1}$$

$$\textcircled{1} n=0: 7^2 + 8^1 = 57$$

$$57:57=1 \checkmark$$

$$\textcircled{2} \quad \text{predpostavimo } 57 | 7^{n+2} + 8^{2n+1}, \Leftrightarrow 7^{n+2} + 8^{2n+1} = 57 \cdot k_1$$

hocemo videti, da velja tudi

$$57 | 7^{n+3} + 8^{2n+3}$$

$$\text{i.p.: } 7^{n+3} + 8^{2n+3} = 57 \cdot k_2, \quad k \in \mathbb{Z}$$

$$\begin{aligned} 7^{n+3} + 8^{2n+3} &= 7 \cdot 7^{n+2} + 64 \cdot 8^{2n+1} = \\ &= 7 \cdot 7^{n+2} + 7 \cdot 8^{2n+1} + 57 \cdot 8^{2n+1} \\ &\quad \begin{array}{l} \text{i.p.} \\ \text{ip.} \end{array} \quad \begin{array}{l} \text{64} \\ \text{automatsko} \\ \text{deljujo s 57} \end{array} \\ &= 7(7^{n+2} + 8^{2n+1}) + 57 \cdot 8^{2n+1} = \\ &= 7 \cdot (57 \cdot k_1) + 57 \cdot 8^{2n+1} \end{aligned}$$

2.

$$\log\left(1-\frac{1}{2^2}\right) + \log\left(1-\frac{1}{3^2}\right) + \dots + \log\left(1-\frac{1}{n^2}\right) = \log\left(\frac{n+1}{2n}\right), \quad n \geq 2$$

①  $n=2: \log\left(1-\frac{1}{4}\right) = \log\frac{3}{4}$

②  $n+1: \log\left(1-\frac{1}{2^2}\right) + \log\left(1-\frac{1}{3^2}\right) + \dots + \log\left(1-\frac{1}{n^2}\right) + \log\left(1-\frac{1}{(n+1)^2}\right) =$

nožemo:  $\log\left(\frac{n+2}{2(n+1)}\right)$

$$\left(\log\left(\frac{n+1}{2n}\right) + \log\left(1-\frac{1}{(n+1)^2}\right)\right) =$$

$$\log\left(\frac{n+1}{2n} - \frac{n+1}{2n(n+1)^2}\right) =$$

$$\log\left(\frac{(n+1)^2 - n-1}{2n(n+1)^2}\right) = \log\left(\frac{n^2 + 2n + 1 - n - 1}{2n(n+1)^2}\right) = \\ = \log\left(\frac{n^2 + n}{2n(n+1)}\right)$$

5.  $\max \sup$   
 $\min \inf$

a)  $A = \{x \in \mathbb{R} : |x-1| - 2 \geq 1\}$

b)  $B = \{x \in A, x \leq 5, x > 1\}$

$x \in \mathbb{R}$

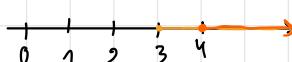
$$|x| = \begin{cases} x: x \geq 0 \\ -x: x < 0 \end{cases}$$

1. način: analitično

1.1 predpostavimo, da je

$$\begin{array}{l} x-1 \geq 0 \\ \boxed{x \geq 0} \end{array}$$

$$\begin{array}{l} x-1-2 \geq 0 \\ \boxed{x \geq 3} \end{array}$$



$x-3 \geq 1$

Bi:  $x \geq 4$

1.2

$$x-3 < 0$$

$$x < 3$$

$$-(x-3) \geq 1$$

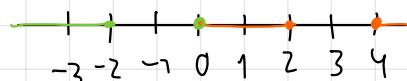
$$-x+3 \geq 1$$

$$x \leq 2$$

2. Neg.

$$x-1 < 0$$

$$x < 1$$



$$|x-1| - 2 \geq 1$$

$$|x-1| \geq 1$$

$$|x+1| \geq 1$$

2.1  $x+1 \geq 0$

$$\boxed{x \geq -1}$$

2.2.  $x+1 \leq 0$

$$\boxed{x < -1}$$

$$x+1 \geq 1$$

$$\boxed{x \geq 0}$$

$$-(x+1) \geq 1$$

$$\begin{array}{l} -x-1 \geq 1 \\ \boxed{x \leq -2} \end{array}$$

$$A: (-\infty, -2] \cup [0, 2] \cup [4, \infty)$$

$\max$	$\min$
$\sup \infty$	$\inf -\infty$

$$B: (1, 2] \cup [4, 5]$$

$\max 5$	$\min 1$
$\sup 5$	$\inf 1$

## DN-1. teden

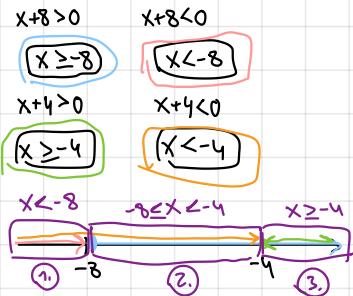
$$\frac{n(n+1)(2n+1)}{6} + (n+1) = \frac{(n+1)(n+2)(2n+3)}{6} \quad | \cdot 6$$

$$\begin{aligned} n(n+1)(2n+1) + 6(n+1) &= (n+1)(n+2)(2n+3) \\ (n^2+n)(2n+1) + 6n+6 &= (n^2+3n+2)(2n+3) \\ 2n^3+3n^2+7n+6 &= 2n^3+9n^2+13n+6 \\ -6n^2-6n &= 0 \end{aligned}$$

$$|x+8| + |x+4| < 10$$

$$n = -1$$

①  $x > 0$     ②  $x < 0$

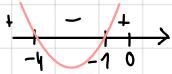


$$x^2 + 5x + 4 \leq 0$$

$$(x+1)(x+4) \leq 0$$

$$x_1 = -1$$

$$x_2 = -4$$



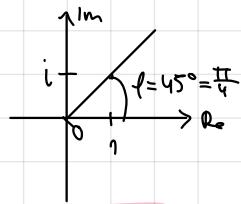
$$x \in [-4, -1]$$

## DN-2. teden

$$\textcircled{1} \quad (1+i)^{\frac{2\pi}{3}} \quad e^{i\varphi} = \cos \varphi + i \sin \varphi$$

$$r^2 = (Im z)^2 + (Re z)^2 = 1+1$$

$$r = \sqrt{2}$$



$$(re^{i\varphi})^n = r^n e^{in\varphi}$$

$$(1\sqrt{2} e^{i\frac{\pi}{4}})^8 = \sqrt{2}^8 e^{i \cdot 2\pi} = 2^8 e^{i \cdot 2\pi} = 16$$

$$\textcircled{2} \quad \varphi = \frac{315}{7}^\circ$$

$$r = 1 + \frac{2}{1000}$$

$$\frac{x}{r} = z$$

$$x^7 = ?$$

$$x = r \cdot e^{i\varphi}$$

$$x = 1 + \frac{2}{1000} \cdot e^{i \cdot \frac{315}{7}^\circ}$$

$$x^7 = \underbrace{\left(1 + \frac{2}{1000}\right)^7}_{r} \cdot \underbrace{e^{i \cdot \frac{315}{7}^\circ}}_{\varphi}$$

$$\varphi' = \varphi, \quad -180^\circ < \varphi' < 180^\circ$$

$$\varphi' = -45^\circ$$

ker je  $315^\circ$  manj od  $360^\circ$

$$r = 1,01408$$

$$z = 13 - 26i$$

$$w = \overline{z} = 13 + 26i$$

$$z^2 - z = 169 - 676i - 676 - 13 + 26i = -520 - 650i$$

$$r = \sqrt{Im z^2 + Re z^2} = 832,40615$$

$$2w = 26 + 52i \quad Im z = 52$$

\textcircled{3}

$$f(z) = (2z - 1 - 2i) e^{i\frac{\pi}{4}}$$

$$f(z) = \overbrace{(2z - 1 - 2i) \cdot i}^{\text{preverili prek } \times \text{ osi}}$$

$$f(1+2i) = \overline{(1+2i) \cdot i} = \overline{i-2} = -2-i$$

## 20.10.

7. Nariši naslednjo podmnožico v  $\mathbb{C}$ :

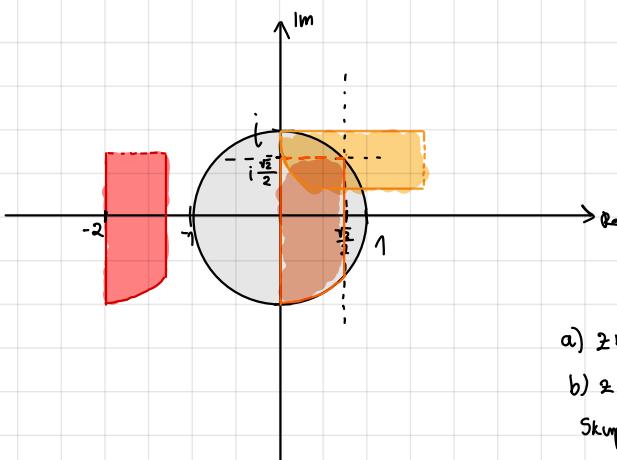
$$A = \{z \in \mathbb{C} : |z| \leq 1, \operatorname{Im}(z) < \frac{\sqrt{2}}{2}, 0 \leq \operatorname{Re}(z) \leq \frac{\sqrt{2}}{2}\}$$

Z območjem  $A$  naredimo naslednjo transformacijo:

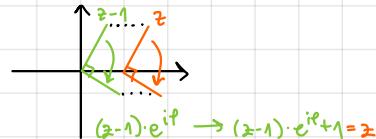
- (a) zavrtimo ga okoli števila 1 za kot  $-\pi/2$ ;
- (b) zavrtimo ga okoli števila  $-i$  za kot  $\pi/2$ .

Zapiši predpis  $z \mapsto f(z)$ , ki opravi to kompleksno transformacijo. Nariši tudi  $f(A)$  ter ugotovi, kam se preslika število  $1+i \notin A$ , če  $f$  razširimo na celo kompleksno ravnino.

Rešitev:  $z \mapsto z - 2, 1 + i \mapsto -1 + i$ .



rotacija okrog 0 za  $\varphi$ :  
 $z \mapsto z \cdot e^{i\varphi}$

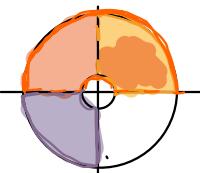


OKROG 1  
a)  $z \mapsto (z-1)(-i) + 1 = -iz + i + 1$   
b)  $z \mapsto \overset{\text{OKROG } -i}{(z+i)} - i = iz - 1 - i$   
Skupaj:  $z \mapsto -iz + i + 1 \mapsto i(-iz + i + 1) - 1 - i$   
 $= 2 - 1 + i - 1 - i =$   
 $= \underline{\underline{z - 2}}$

$$f(z) = z - 2$$

Dodatevne točke za sliko  
 $f(1+i) = 1+i-2 = -1+i$   
 $f(0) = -2$   
 $f(i) = i-2$

8. Nariši naslednje podmnožice v  $\mathbb{C}$ :



$$A = \{z \in \mathbb{C} : 1 \leq |z| \leq 3, 0 \leq \arg(z) < \frac{\pi}{2}\},$$

$$B = \{z \in \mathbb{C} : \frac{1}{3} \leq |z| \leq 3, 0 \leq \arg(z) < \pi\},$$

$$C = \{z \in \mathbb{C} : 1 \leq |z| \leq 3, \pi < \arg(z) \leq \frac{3\pi}{2}\}.$$

Nato poišči kompleksne transformacije, ki transformirajo  $A$  v  $B$ ,  $A$  v  $C$  in  $C$  v  $A$ .

a) b) c)

Rešitev:  $f_{AB} : z \mapsto \frac{1}{3}z^2, f_{CA} : z \mapsto -i\bar{z}, f_{AC} : z \mapsto -i\bar{z}$ .

a)  $f_{AB} : A \mapsto B$

$$z \mapsto z^2 \mapsto \frac{1}{3}z^2$$

$$\pi \in [1, 3] \quad \pi \in [1, 2] \quad \pi \in [\frac{1}{3}, 3]$$

$$f(z) = \frac{1}{3}z^2$$

b)  $f_{AC} : A \mapsto C$

$$z \mapsto \bar{z} \mapsto \bar{z} e^{i(-\frac{\pi}{2})} = -\bar{z}i$$

zrcaljenje  
iz re os  
rotacija za  
 $-\frac{\pi}{2}$

$$f_{AC}(z) = -i\bar{z}$$

c)  $f_{CA} : C \mapsto A$

$$f_{CA} = f_{AC} = -\bar{z}i$$

3. b)  $z^3 = -\frac{27}{\sqrt{2}} + \frac{27}{\sqrt{2}}i$

$$\begin{cases} z = r e^{i\varphi} \\ z^3 = r^3 e^{i3\varphi} \end{cases}$$

$$R = \sqrt{\left(\frac{27}{\sqrt{2}}\right)^2 + \left(\frac{27}{\sqrt{2}}\right)^2} = \boxed{27}$$

$$\varphi = \arctan \left( \frac{\frac{27}{\sqrt{2}}}{-\frac{27}{\sqrt{2}}} \right) = \arctan(-1) = -\frac{\pi}{4} + \pi = \boxed{\frac{3\pi}{4}}$$

$$z^3 = r^3 e^{i3\varphi} = 27 \cdot e^{i\frac{3\pi}{4}}$$

$$r^3 = 27$$

$$3\varphi = \frac{3\pi}{4}$$

$$\varphi = \frac{\pi}{4} + \frac{2\pi k}{3}$$

$$\begin{cases} k \in \mathbb{Z} \\ (k=0, 1, 2) \end{cases}$$

dobiši  $z_1, z_2, z_3$  = graf roštive je  $\Delta$

d)  $\begin{aligned} z^4 &= -1 + i\sqrt{3} \\ z &= \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2 \\ z &= r \cdot e^{i\varphi} \quad \varphi = \arctan\left(\frac{\sqrt{3}}{-1}\right) = \arctan\left(\frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}}\right) = 120^\circ = \frac{2\pi}{3} \\ r^4 &= R \quad 4\varphi = \varphi' \\ (r = \sqrt[4]{2}) \quad 4\varphi &= \frac{\pi}{6} + \frac{\pi k}{2} \quad (k=0,1,2,3) \end{aligned}$

↳ rešitev je  $\square$

4.  $ax^3 + bx^2 + cx + d = a(x-x_1)(x-x_2)(x-x_3)$

$$x_1 + x_3 \cdot x_2 = -\frac{b}{a}$$

$$x_1 x_2 + x_2 x_3 + x_1 x_3 = -\frac{c}{a}$$

$$x_1 x_2 x_3 = -\frac{d}{a}$$

$$\begin{aligned} ax^3 + bx^2 + cx + d &= a(x-(1+i))(x-(1-i))(x-2) = \\ &= a(x^2 - 2x + 2)(x-2) = \\ &= a(x^3 - 2x^2 + 2x - 2x^2 + 4x - 4) = \\ &= a(x^3 - 4x^2 + 6x - 4) \end{aligned}$$

8. Za vsako izmed spodnjih zaporedij razišči monotonoost, omejenost in konvergenco. Če imajo limito določi tudi prvi člen zaporedja, ki je od nje oddaljen za manj kot  $\frac{1}{10}$ .

- (a) \*  $a_n = \frac{n}{3n-1}$
- (b)  $a_n = \frac{2^n-4}{2^n+4}$
- (c)  $a_n = -\frac{n^2+n}{n^2+1}$

Rešitve:

(a) padajoče od  $n=1$  naprej, navzdol omejeno z 0, limita je  $\frac{1}{3}$ ,  $n_0 = 2$ ,

(b) naraščajoče, navzgor omejeno z 1, limita je 1,  $n_0 = 7$ ,

(c) naraščajoče od  $n=2$  naprej, navzgor omejeno z 0, limita je -1,  $n_0 = 1$ .

a)  $\begin{aligned} a_0 &= 0 \\ a_1 &= \frac{1}{2} \\ a_2 &= \frac{2}{5} \\ a_3 &= \frac{3}{8} \\ a_4 &= \frac{4}{11} \end{aligned}$

↓ pada

$$\begin{aligned} a_{n+1} - a_n &\geq 0 \\ \frac{n}{3n-1} - \frac{n+1}{3n+2} &\geq 0 \\ \frac{3n^2+2n-3n^2-3n-1}{(3n-1)(3n+2)} &\geq 0 \\ \frac{1}{(3n-1)(3n+2)} &\geq 0 \\ 1 &\geq 0 \quad \checkmark \end{aligned}$$

### DN - 3. teden

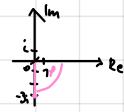
$$\textcircled{1} \quad z = -3i$$

$$2\operatorname{Re}z = 0$$

$$\operatorname{Im}z = -3$$

$$r = |z| = 3$$

$$\varphi = -\frac{\pi}{2}$$



$$z = r(\cos \varphi + i \sin \varphi)$$

sedmi koren:  $n=7$

$$z_k = \sqrt[7]{r} \cdot e^{i\left(\frac{\varphi}{n} + \frac{2k\pi}{n}\right)}$$

$$z_4 = \sqrt[7]{3} \cdot e^{i\left(-\frac{\pi}{14} + \frac{2\pi}{7}\right)}$$

$$\operatorname{Re}z = -1,14060$$

$$\operatorname{Im}z = 0,260033$$

\textcircled{2} pravilni 10-kotnik

$$A(3, 0^\circ) \Rightarrow z = 7 + \frac{1}{2}i$$

$$r = |z| = 7,0783$$

$$\varphi = \arctan \frac{\operatorname{Im}z}{\operatorname{Re}z}$$

$$\varphi = 4,09^\circ$$

kot med naslednjima angleščina:  $\frac{360^\circ}{10} = 36^\circ$

$$(x_k, y_k) = (r \cdot \cos(\varphi_0 + k \cdot 36^\circ), r \cdot \sin(\varphi_0 + k \cdot 36^\circ)), \quad k = 0, 1, 2, \dots, 9$$

največji y je pri  $k=2$   
 $\downarrow$   
 kot =  $76,09^\circ$

$$(x_2, y_2) = (4^{\circ}637771, 6^{\circ}80743)$$

$1^{\circ}63206, 6^{\circ}81434$

$$\textcircled{3} \quad a_{24} = ?$$

$$\textcircled{4} \quad a_n = \frac{15n^2 + 3n - 1}{5n^2 + 2}$$

$$a_{n+1} = a_n + 13$$

$$a_1 = 10$$

aritmetična zaporedje

$$a_n = a_1 + (n-1) \cdot d$$

$$a_{24} = 10 + 23 \cdot 13$$

$$a_{24} = 309$$

$$\lim_{n \rightarrow \infty} \frac{15n^2 + 3n - 1}{5n^2 + 2} = \frac{\frac{15}{n^2} + \frac{3}{n} - \frac{1}{n^2}}{\frac{5}{n^2} + \frac{2}{n^2}} \xrightarrow{n \rightarrow \infty} \frac{15}{5} = 3$$

$$\begin{aligned} & \text{limita } \varepsilon \\ & \uparrow \quad \uparrow \\ & |a_{k+3}| < \frac{1}{30} \\ & \left| \frac{15k^2 + 3k - 1}{5k^2 + 2} - 3 \right| < \frac{1}{30} \\ & \frac{30k^2 + 3k - 1 - 15k^2 - 2}{5k^2 + 2} < \frac{1}{30} \\ & 15k^2 + 3k - 1 < \frac{1}{30} \\ & 450k^2 + 90k - 30 < 1 \\ & 450k^2 + 90k < 30 \\ & 50k^2 + 10k < 3 \\ & 5k^2 + 10k < 3 \\ & 5k(k+2) < 3 \end{aligned}$$

$$\begin{aligned} & 3k - 7 > 0 \\ & 3k > 7 \\ & k > \frac{7}{3} \\ & \downarrow \\ & \boxed{k > 3} \end{aligned}$$

$$90k - 210 < 5k^2 + 2$$

$$5k^2 - 90k + 212 > 0$$

$$D = b^2 - 4ac = 3660$$

$$\sqrt{D} = 62,429$$

$$k_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-15 \pm 21}{2 \cdot 5} \Rightarrow k = 1,6$$

$$z = 2,8$$

## 3.11.

1. \* Podani sta vrsti

$$a) \sum_{n=1}^{\infty} \frac{2}{n(n+2)} \quad \text{in} \quad b) \sum_{n=0}^{\infty} (-1)^n n.$$

Za vsako izmed omenjenih vrst ugani in dokazi formula za  $n$ -to delno vsoto ter po definiciji izracunaj vsoto vrste, če obstaja.Rešitev: Prva vrsta konvergira k  $\frac{2}{3}$ , delne vsote so  $s_n = \frac{2}{3} - \frac{1}{n+1} - \frac{1}{n+2}$ , divergira, ker je alternirajoča, členi pa ne konvergirajo k 0.

$$\begin{aligned} a) \quad a_1 &= \frac{2}{3} \\ a_2 &= \frac{4}{12} \\ a_3 &= \frac{2}{18} \\ a_4 &= \frac{1}{72} \\ S_1 &= a_1 = \frac{2}{3} \\ S_2 &= a_1 + a_2 = \frac{14}{12} \\ S_3 &= S_2 + a_3 = \frac{63}{60} \\ S_4 &= S_3 + a_4 = \frac{63}{60} \end{aligned}$$

 $S_n = ??$ 

$$a_n = \frac{2+0 \cdot n}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2} = \frac{(n+2)A+nB}{n(n+2)} = \frac{n(A+B)+2A}{n(n+2)}$$

$$\begin{array}{c} A+B=0 \\ 2A=2 \\ \hline A=1 \quad B=-1 \end{array}$$

$$a_n = \frac{2}{n(n+2)} = \frac{1}{n} - \frac{1}{n+2}$$

$$\begin{aligned} S_n &= a_1 + a_2 + a_3 + \dots + a_n = \frac{1}{1} - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \dots + \frac{1}{n-1} - \frac{1}{n+1} + \frac{1}{n} - \frac{1}{n+2} = \\ &= \frac{3}{2} - \frac{1}{n-1} - \frac{1}{n+2} \\ \lim_{n \rightarrow \infty} S_n &= \lim_{n \rightarrow \infty} \left( \frac{3}{2} - \frac{1}{n-1} - \frac{1}{n+2} \right) = \frac{3}{2} \end{aligned}$$

↳ vsota vrste

2. Izračunaj vsoto naslednjih geometrijskih vrst.

(a) \*  $\sum_{n=1}^{\infty} \frac{10}{3^n}$

(b)  $\sum_{n=2}^{\infty} \frac{2^n}{3^{2n-1}}$

(c) \*  $3/2 + 1/2 + 2/3 + 4/9 + 8/27 + \dots$

(d)  $\sum_{n=1}^{\infty} \frac{(-2)^n}{3 \cdot 2^{3n-2}}$

(e) \*  $\sum_{n=1}^{\infty} \left(\frac{x}{2}\right)^{3n}$ , za tiste  $x \in \mathbb{R}$ , za katere vrsta konvergira.

Rešitev: (a) 5, (b)  $\frac{4}{21}$ , (c)  $\frac{9}{2}$ , (d)  $-\frac{4}{15}$ , (e)  $\frac{x^3}{8-x^3}$ .

$$a) \sum_{n=1}^{\infty} \frac{10}{3^n} = 10 \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = 10 \cdot \left(\frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \dots + \left(\frac{1}{3}\right)^n\right) = \boxed{10} \cdot \left(1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \dots + \left(\frac{1}{3}\right)^n\right)$$

$$k = \frac{a_2}{a_1} = \frac{1}{3}$$

$$S_n = \frac{a_1}{1-k} = \frac{10}{3} \cdot \frac{1}{1-\frac{1}{3}} = \frac{10}{3} \cdot \frac{1}{\frac{2}{3}} = 5$$

$$c) \frac{3}{2} + 1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \dots = \frac{3}{2} + \sum_{n=1}^{\infty} k^n = \frac{3}{2} + \frac{1}{1-k} = \frac{3}{2} + \frac{1}{1-\frac{1}{3}} = \frac{9}{2} = S_n$$

$\sum_{n=1}^{\infty} k^n$

$$\text{če } a_n \not\rightarrow 0 \Rightarrow \sum a_n = \infty$$

(divergira)

če  $a_n \rightarrow 0 \Rightarrow ??$ 

$$\sum \frac{1}{n^2} < \infty \quad \sum \frac{1}{n} = \infty$$

$$\lim_{n \rightarrow \infty} \frac{1}{n^2} = 0 \quad \lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$$

#### DN-4. teden

④

$$a_n = \frac{a_{n-1} + \frac{7}{a_{n-1}}}{2}$$

$$a_1 = 2$$


---


$$a_2 = ?$$

$$\lim_{n \rightarrow \infty} a_n = ?$$

$$a_2 = \frac{2 + \frac{7}{2}}{2} = \frac{\frac{11}{2}}{2} = \frac{11}{4} = 2,75$$

$$a_3 = \frac{\frac{11}{4} + \frac{7}{\frac{11}{4}}}{2} = \frac{\frac{11}{4} + \frac{28}{11}}{2} = \frac{\frac{121+112}{44}}{2} = \frac{\frac{233}{44}}{2} = \frac{233}{8} \approx 2,6477$$

$$a_4 \approx 2,6463$$

NEWTON-RAPHSONOVA METODA

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f(x) = x^2 - 7 \quad \rightarrow \quad x_{n+1} = x_n - \frac{x_n^2 - 7}{2x_n} = \frac{x_n + \frac{7}{x_n}}{2}$$

$$\lim_{n \rightarrow \infty} a_n = -\sqrt{7}$$

$$a_0 \approx \sqrt{7} \approx 2,64575$$

②

$$\sum_{n=2}^{\infty} \frac{24}{4^n}$$

$$k = \frac{a_3}{a_2} = \frac{1}{4}$$

$$a_2 = \frac{3}{2}$$

$$S_n = \frac{a_2}{1-k} = \frac{\frac{3}{2}}{1-\frac{1}{4}} = \frac{\frac{3}{2}}{\frac{3}{4}} = \frac{12}{6} = 2$$

### 10.11.

1. \* Določi definicijsko območje naslednjih funkcij in ugotovi, ali so injektivne. Če niso, definicijsko območje smiselnega zmanjšaj in nato določi inverzno funkcijo.

- (a)  $f(x) = \sqrt{e^{2x} - 3e^x + 2}$ ,
- (b)  $f(x) = \log(2\sin x - \sqrt{3})$ .

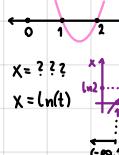
Bilježi:  
(a)  $D_f = (-\infty, 0] \cup [\log 2, \infty)$ ,  $D_g = [0, \infty)$ , sarti tih niti neda, ali injektivna na  $D_f$ , je injektivna na  $[0, \infty)$ .  
(b)  $D_f = (-\pi, -\frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi)$ ,  $D_g = (-\frac{\pi}{2}, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, \pi)$ , sarti tih niti neda, ali injektivna na  $D_f$ , je injektivna, če se omrežimo sipe, na  $(\frac{\pi}{2}, \pi)$ .

a)  $\underbrace{\sqrt{e^{2x} - 3e^x + 2}}_{\geq 0}$

$\sqrt{x}$   
 $D_f = [0, \infty)$

$e^{2x} - 3e^x + 2 \geq 0$   
 $t = e^x$   
 $t^2 - 3t + 2 \geq 0$

$(t-2)(t-1) \geq 0$   
 $t \in (-\infty, 1] \cup [2, \infty)$

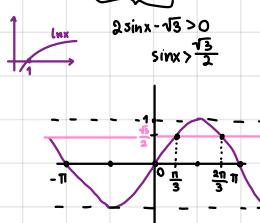


①  $\left. \begin{array}{l} t \in (-\infty, 1] \\ t = e^x > 0 \end{array} \right\} \quad \left. \begin{array}{l} t \in (0, 1] \rightarrow x \in (-\infty, 0) \\ t \in [2, \infty) \rightarrow x \in [0, \infty) \end{array} \right.$

②  $t \in [2, \infty) \rightarrow x \in [\ln 2, \infty)$

$x \in (-\infty, 0] \cup [\ln 2, \infty)$   
 $D_f$

b)  $f(x) = \log(2\sin x - \sqrt{3})$



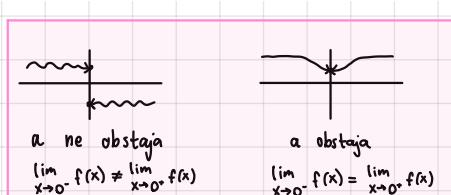
$D_f = \dots \cup (-\frac{5\pi}{3}, -\frac{4\pi}{3}) \cup (\frac{\pi}{3}, \frac{2\pi}{3}) \cup (\frac{3\pi}{3}, \frac{5\pi}{3}) \cup \dots = \bigcup_{k \in \mathbb{Z}} (\frac{\pi}{3} + 2k\pi, \frac{2\pi}{3} + 2k\pi)$   
 $+ \frac{2\pi}{3}$   
 $+ \frac{5\pi}{3}$

$f$  ni injektivna:  $\left. \begin{array}{l} x_1 = \frac{\pi}{3} \\ x_2 = \frac{5\pi}{3} \end{array} \right\} f(x) = \log 2 - \sqrt{3} = f(x_1) = f(x_2)$

3. \* Določi konstanto  $a$  (če obstaja), da bo  $f$  zvezna funkcija.

$$f(x) = \begin{cases} \arctan\left(1 + \frac{1}{x}\right), & x \neq 0, \\ a, & x = 0. \end{cases}$$

Rešitev: Tak  $a$  ne obstaja, ker sta leva in desna limita v točki  $x = 0$  različni.



$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \arctan\left(1 + \frac{1}{x}\right) = -\frac{\pi}{2}$

$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \arctan\left(1 + \frac{1}{x}\right) = \frac{\pi}{2}$

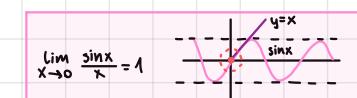
Limita  $a$  ne obstaja, ker  $\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$

4. \* Določi konstante  $a$  in  $b$ , da bo  $f$  zvezna funkcija.

$$f(x) = \begin{cases} \frac{\sin(3x)(x-2)}{x}, & x < 0, \\ ax+b, & 0 \leq x \leq 1, \\ 2e^{x-1} - \cos(\pi x), & x > 1. \end{cases}$$

Rešitev:  $a = 9$ ,  $b = -6$ .

④  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{\sin(3x)(x-2)}{x} \stackrel{x \rightarrow 0}{=} -2 \lim_{x \rightarrow 0^-} \frac{\sin(3x)}{x} = -6 \lim_{x \rightarrow 0^-} \frac{3\sin(3x)}{3x} = -6$   
 $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (ax+b) = b$



②  $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (ax+b) = a+b = a-b$   
 $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{(2e^{x-1} - \cos(\pi x))}{2} = 3$

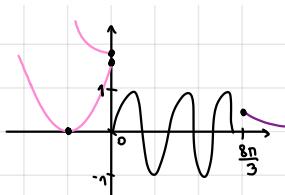
$\frac{2e^{x-1} - \cos(\pi x)}{2} \underset{\cos \pi = -1}{=} 3$

$\frac{a-b}{a+b} = 3$

5. Skiciraj graf funkcije  $f$  za  $a = -1$  in  $b = 1$ . Nato določi taki konstanti  $a$  in  $b$  (če obstajata), da bo  $f$  zvezna funkcija.

$$f(x) = \begin{cases} (x-a)^2, & x \leq 0, \\ \sin(4x), & 0 < x \leq \frac{8\pi}{3}, \\ \frac{b\sqrt{3}}{x}, & x > \frac{8\pi}{3}. \end{cases}$$

Rešitev:  $a = 0, b = \frac{4\pi}{3}$ .



$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= f(0) = (0-a)^2 = a^2 \quad \rightarrow a^2 = 0 \\ \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} \sin(4x) = 0 \quad \boxed{a=0} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow \frac{8\pi}{3}^+} f(x) &= \lim_{x \rightarrow \frac{8\pi}{3}^+} \frac{b\sqrt{3}}{x} = \frac{b\sqrt{3}}{\frac{8\pi}{3}} = \frac{3\sqrt{3}}{8\pi} b \quad \rightarrow \frac{3\sqrt{3}}{8\pi} b = \frac{\sqrt{3}}{2} \\ \lim_{x \rightarrow \frac{8\pi}{3}^+} f(x) &= f\left(\frac{8\pi}{3}\right) = \sin\left(4 \cdot \frac{8\pi}{3}\right) = \sin\left(\frac{32\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \boxed{b = \frac{4\pi}{3}} \end{aligned}$$

2. Izračunaj

$$\lim_{x \nearrow 0} (1+x)^{\frac{1}{x}} \quad \text{in} \quad \lim_{x \searrow 0} (1+x)^{\frac{1}{x}}.$$

Rešitev: Obe limiti sta enaki  $e$ .

$$\begin{aligned} \lim_{x \rightarrow 0^-} (1+x)^{\frac{1}{x}} &= \\ &= \left(1 + \frac{1}{x}\right)^{\frac{1}{x}} = e \end{aligned}$$

$$\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}} = e$$

GCC gem

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n+1}$  v števku naj bo 1  
 $\left(1 + \frac{1}{n}\right)^{2n+1}$  naj bo večes +  
 $\left(1 + \frac{1}{n}\right)^{2n+1}$  naj bo eksponent enak imenovalec  
 $\left(\left(1 + \frac{1}{n}\right)^{\frac{n}{n}}\right)^{2n+1}$  posledji eksponent  
 $\downarrow$   
 $\lim_{n \rightarrow \infty} \frac{2n+1}{n} = \frac{(2n+1) \cdot 1}{n} = \frac{2n+1}{n} = 12 + \frac{1}{n} \rightarrow 0$

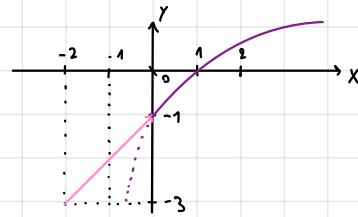
$\lim_{n \rightarrow \infty} e^{12 + \frac{1}{n}} = e$

6. \* Prepričaj se, da je funkcija

$$f(x) = \begin{cases} \frac{x-1}{x+1}, & x \geq 0, \\ x-1, & x < 0. \end{cases}$$

zvezna na intervalu  $[-2, 2]$ . Poisci največjo vrednost  $M$  in najmanjšo vrednost  $m$ , ki ju zavzame na tem intervalu. Ali ima enačba  $f(x) = 0$  rešitev na tem intervalu? Kaj pa enačba  $f(x) = 3$ ? Poisci vse rešitve, če obstajajo!

Rešitev:  $m = -3, M = \frac{1}{3}$ . Einačba  $f(x) = 0$  ima rešitev  $x = 1$ . Einačba  $f(x) = 3$  ni rešljiva, ker  $3 \notin [m, M]$ .



$f$  je zvezna na  $[-2, 2]$

1. problem: pol rac. funkcije  $\frac{x-1}{x+1}$  je v  $-1$   
 predpis uporabimo za  $x \geq 0 \checkmark$  OK

2. problem:  $x=0$

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^+} f(x) = f(0) = -1 \\ \lim_{x \rightarrow 0^+} (x-1) &= -1 \quad \checkmark \text{OK} \end{aligned}$$

Če je  $f$  zvezna na  $[a, b]$ , potem  $f$  doseže

$\min m$  in  $\max M$  in  $Z_f = [m, M]$

Kandidati za  $m, M$ :

- točke kjer je  $f'$  = 0 (tangenta vodoravna)
- krajišča intervala ( $x=a, x=b$ )
- točke kjer  $f'$  ni definiran

↳ odvod

$$f'(x) = \left( \frac{x-1}{x+1} \right)' = \frac{(x+1)-(x-1) \cdot 1}{(x+1)^2} = \frac{2}{(x+1)^2}$$

•  $f'(x) = \begin{cases} \frac{2}{(x+1)^2}; & x > 0 \\ 1; & x < 0 \end{cases} \neq 0$

$$\bullet f(2) = \frac{2-1}{2+1} = \frac{1}{3} \quad f(-2) = -2-1=-3$$

$$\bullet f(0) = -1$$

$$\begin{cases} M = \frac{1}{3} \text{ pri } x=2 \\ m = -3 \text{ pri } x=-2 \end{cases} \quad Z_f = [-3, \frac{1}{3}]$$

$f(x)=0$  rešljiva?

$$\text{Einačba } f(x)=k \text{ je rešljiva} \Leftrightarrow k \in Z_f$$

Ja, ker je  $0 \in Z_f = [-3, \frac{1}{3}]$

Npr.  $x=1$  (ne vem, če je edina)

$f(x)=3$  ni rešljiva ker  $3 \notin Z_f = [-3, \frac{1}{3}]$

### DN - 5. teden

①  $v_1 = v_2 = 10 \frac{\text{km}}{\text{h}}$



$m = 160 \frac{\text{km}}{\text{h}}$

$s = 160 \text{ km}$

$x = ?$

$v_1 = 140$

$$s_n = \frac{a_1}{1-k} = \frac{140}{\frac{1}{2}} = \boxed{280}$$

$\frac{140}{160} = t = 52,5 \text{ min} = 0,875 \text{ h}$

↓

v tem čas u

vlak naredi

$v \cdot t = s = 75 \text{ km}$

$$140 - 2 \cdot 35 = 70 = a_2$$

$k = \frac{a_2}{a_1} = \frac{1}{2}$

$$\sum_{n=12}^{\infty} \frac{24}{n^2-1} = 24 \left( \frac{1+0 \cdot n}{(n-1)(n+1)} \right) = 24 \sum_{n=12}^{\infty} \left( \frac{\frac{1}{2}}{n-1} + \frac{-\frac{1}{2}}{n+1} \right) = 24 \sum_{n=12}^{\infty} \left( \frac{\frac{1}{2}}{n-1} - \frac{\frac{1}{2}}{n+1} \right) = 24 \cdot \left( \frac{\frac{1}{2}}{11} - \frac{\frac{1}{2}}{13} \right) + \left( \frac{\frac{1}{2}}{12} - \frac{\frac{1}{2}}{14} \right) + \left( \frac{\frac{1}{2}}{13} - \frac{\frac{1}{2}}{15} \right) + \dots + \left( \frac{\frac{1}{2}}{n-1} - \frac{\frac{1}{2}}{n+1} \right)$$

konvergira

1.  $A+B=0$   
2.  $A-B=1$   
 $A=1+B$

$$\begin{cases} 1+B+B=0 \\ B=-\frac{1}{2} \\ A=\frac{1}{2} \end{cases}$$

$s_n = 24 \cdot (a_1 + a_2 + \left( \frac{\frac{1}{2}}{n-1} - \frac{\frac{1}{2}}{n+1} \right)) = \frac{23}{71} = 2,09091$

✓

1. \* Določi definicijsko območje funkcije

$$f(x, y) = x^2 + \frac{y^2}{4}$$

in skiciraj nivojnico.

Rešitev:  $D_f = \mathbb{R} \times \mathbb{R}$ , nivojnice so elipse s središčem v izhodišču in polosoma  $\sqrt{a}, 2\sqrt{a}$ .

$$D_f = \mathbb{R} \times \mathbb{R}$$

nivojnice je krivulja  $f(x, y) = c$ za  $c \in \mathbb{R}$ ,

$$f(x, y) = c$$

$$x^2 + \frac{y^2}{4} = c \quad | :c \quad \text{je } c \neq 0$$

$$\frac{x^2}{c} + \frac{y^2}{4c} = 1 \Rightarrow \text{elipsa } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{(c)^2} + \frac{y^2}{(2c)^2} = 1$$

kaj pa če  $c = 0$ ?

$$x^2 + \frac{y^2}{4} = 0$$

$$x = y = 0$$

$$(0, 0)$$

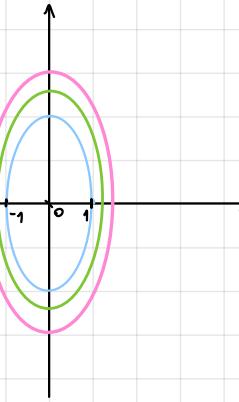
skicirai tako, da računaš c

$$c=1 \quad a=1, b=2$$

$$c=2 \quad a=\sqrt{2}, b=2\sqrt{2}$$

$$c=3 \quad a=\sqrt{3}, b=2\sqrt{3}$$

Nivojnice:



2. \* Skiciraj definicijska območja in nivojnice danih funkcij.

$$(a) f(x, y) = \log(y - x^2 + 2), y > 0$$

$$(b) f(x, y) = e^{y/x}$$

$$(c) f(x, y) = \sqrt{x^2 - 2}$$

Rešitev:

$$(a) D_f = \{(x, y) \mid y > x^2 - 2\}, \text{ nivojnice so parabole } y = x^2 + b, b \in (-2, \infty)$$

$$(b) D_f = \{(x, y) \mid y \neq 0\}, \text{ nivojnice so premice skroz izhodišča brez izhodišča (razen y=0)}$$

$$(c) D_f = \{(x, y) \mid x \neq 0, x \cdot \sin y > 0\}, \text{ nivojnice so krivulje } \sin y = a^2 x$$

$$b) f(x, y) = e^{\frac{y}{x}}$$

$$D_f = \{(x, y) \in \mathbb{R}^2; x \neq 0\}$$

$$\mathbb{R}^2 \setminus \{(0, 0)\}; y \in \mathbb{R}\}$$

$$\mathbb{R}^2 \setminus y=0$$

$$f(x, y) = c$$

$$e^{\frac{y}{x}} = c \quad | \ln$$

$$\frac{y}{x} = \ln c$$

$$y = (\ln c)x \rightarrow \text{premica skozi } (0, 0) \text{ (brez } 0, 0)$$

konst.

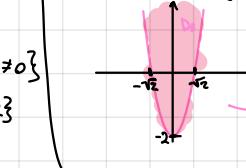
Pogoj za logaritem

$$a) y > x^2 - 2$$

$$y = p x^2 \quad \text{PARABOLA}$$

teme v  $(0, -2)$ 

$$D_f = \{(x, y) \in \mathbb{R}^2; y > x^2 - 2\}$$



$$f(x, y) = c$$

$$\log(y - x^2 + 2) = c$$

uporabimo definicijo  $\log_a x = y \Leftrightarrow a^y = x$ 

$$y - x^2 + 2 = c^e$$

konst. parabola

$$c \in \mathbb{Z}_e \quad e^c - 2 \in (-2, \infty)$$

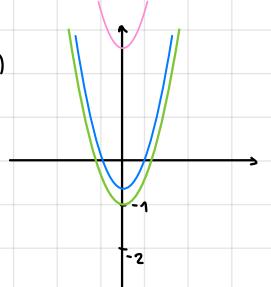
$$y = x^2 + e^{-2}$$

$$c=0 \quad y = x^2 - 1$$

$$c=1 \quad y = x^2 + e^{-2} \quad 0.718\dots$$

$$c=2 \quad y = x^2 + e^{-2} \quad 1\dots$$

$$c=-1 \quad y = x^2 + \frac{1}{e^2} - 2 \quad 0.59\dots$$



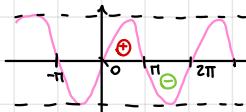
$$c) f(x, y) = \sqrt{\frac{\sin y}{x}}, x \neq 0$$

Pogoj za koren

$$\frac{\sin y}{x} \geq 0 \quad | \quad x \neq 0$$

$$\sin y \geq 0 \quad (x > 0)$$

$$\sin y \leq 0 \quad (x < 0)$$



$$\oplus [0, \pi] \rightarrow \bigcup_{k \in \mathbb{Z}} [0 + 2\pi k, \pi + 2\pi k] = \dots \cup [0, \pi] \cup [2\pi, 3\pi] \cup \dots$$

$$\ominus [\pi, 2\pi] \rightarrow \bigcup_{k \in \mathbb{Z}} [\pi + 2\pi k, 2\pi + 2\pi k] = \dots \cup [\pi, 2\pi] \cup [3\pi, 4\pi] \cup \dots$$

$$f(x, y) = c$$

$$\sqrt{\frac{\sin y}{x}} = c$$

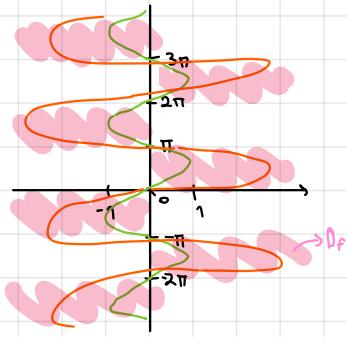
$$\sin y = x c^2$$

$$x = \frac{1}{c^2} \sin y \quad c \neq 0$$

$$c=1 \quad x = \sin y$$

$$c=2 \quad x = \frac{1}{4} \sin y$$

$$c=\frac{1}{4\pi} \quad x = 5 \sin y$$



3. \* Izračunaj odvode naslednjih funkcij:

(a)  $f(x) = \sqrt{x}(2 - x^2)$ ,

(b)  $f(x) = \frac{x}{1+x^2}$ ,

(c)  $f(x) = \ln \frac{1+x}{1-x}$ ,

(d)  $f(x) = \arcsin(\sqrt{1 - x^2})$ .

Rešitev: (a)  $\frac{1}{2\sqrt{x}}(2 - 5x^2)$ , (b)  $\frac{1-x^2}{(1+x^2)^2}$ , (c)  $\frac{2}{1-x^2}$ , (d)  $-\frac{x}{|x|} \cdot \frac{1}{\sqrt{1-x^2}}$ .

a)  $f(x) = \sqrt{x}(2 - x^2)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$f'(x) = \frac{1}{2\sqrt{x}}(2 - x^2) + \sqrt{x}(-2x) = \frac{1}{2\sqrt{x}} - \frac{x^2}{2\sqrt{x}} - 2x\sqrt{x} =$

b)  $\left(\frac{x}{1+x^2}\right)' = \frac{1+x^2 - 2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$

c)  $(\ln \frac{1+x}{1-x})' = \frac{1}{1+x} \cdot \frac{1 \cdot (1-x) - (1+x)(-1)}{(1-x)^2} = \frac{1-x}{1+x} \cdot \frac{2}{(1-x)^2} = \frac{2}{1-x^2}$

d)  $(\arcsin(\sqrt{1-x^2}))' = \frac{1}{\sqrt{1-(1-x^2)^2}} \cdot \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) = \frac{1}{|x|} \cdot \frac{-2x}{2\sqrt{1-x^2}} = -\frac{x}{|x|} \cdot \frac{1}{\sqrt{1-x^2}}$

$\log = \ln$

$(\ln x)' = \frac{1}{x}$

$(\arcsin \sin x)' = \frac{1}{\sqrt{1-x^2}}$

$(\arcsin \tan x)' = \frac{1}{1+x^2}$

$(\sqrt{x})^2 = x$   
 $\sqrt{x^2} = |x|$

5. \* Poisci tisto normalo na krivuljo  $y = x \ln x$ , ki je pravokotna na premico z enačbo  $2x - 2y - 3 = 0$ .

Rešitev: Enačba normal je  $y = -x + 1$ .

funk:

$f(x) = x \ln x$

premica:

$2x - 2y - 3 = 0$

$f'(x) = \ln x + x \cdot \frac{1}{x} = \ln x + 1 = k_t$

$\begin{cases} \ln x = 0 \\ x = 1 \end{cases}$   
 $k_t = 1$   
 $k_t = -1$

$y = x - \frac{3}{2}$

GCC gem

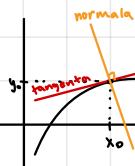
TANGENTA:

1.  $T(x_0, y_0)$

2.  $f'(x)$

3.  $f'(x_0) = k$

4. tangenta:  $y - y_0 = k(x - x_0)$



NORMALA

1.  $T(x_0, y_0)$

2.  $f'(x)$

3.  $f'(x_0) = k \rightarrow$  tangente

4.  $k_N = -\frac{1}{k} \rightarrow$  normala

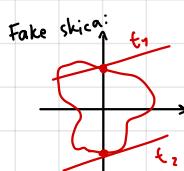
5. normala:  $y - y_0 = k_N(x - x_0)$

6. \* Naj bo z enačbo  $5x^2 - 8xy + 5y^2 = 1$  implicitno podana funkcija  $y(x)$ .

(a) Zapiši enačbi tangent na graf krivulje v obeh točkah, kjer je  $x = 0$ .

(b) Za katere vrednosti  $x$  je naklon tangenten enak  $k = \frac{1}{2}$ ?

Rešitev: (a)  $y = \frac{3}{5}x + \frac{1}{\sqrt{5}}$  in  $y = \frac{3}{5}x - \frac{1}{\sqrt{5}}$ , (b)  $x = \pm \frac{1}{3}$ .



$\text{STOŽNICA}$

$ax^2 + by^2 + cxy + dx + f = 0$

a) 1. Točka  $\vee x=0$

$y_1 = \frac{1}{\sqrt{5}}$   
 $y_2 = -\frac{1}{\sqrt{5}}$

$(xy)' = x \cdot y + x \cdot y' = y + xy'$

$(y^2)' = 2y \cdot y'$

$= 10x - 8(xy' + y) + 10yy'$

$10x - 8y - 8xy' + 10yy' = 0 \quad | :2$

$y'(-4x + 5y) = 4y - 5x$

$y' = \frac{4y - 5x}{-4x + 5y}$

$k_t$

$x = \frac{3}{5\sqrt{5}} = \frac{1}{2\sqrt{5}} x$

3.  $f'(0, \frac{1}{\sqrt{5}}) = \frac{4}{5} = k_t$

$f'(0, -\frac{1}{\sqrt{5}}) = -\frac{4}{5} = k_t$

4.  $t_1: y = \frac{4}{5}x + \frac{1}{\sqrt{5}}$

$t_2: y = -\frac{4}{5}x - \frac{1}{\sqrt{5}}$

kot med  $f(x)$  in  $x$ :

1.  $T(x_0, 0)$

2.  $\tan \varphi = f'(x_0)$

kot med  $f(x)$  in  $y$ :

1.  $T(0, y_0)$

2.  $\tan \varphi = f'(0)$

3.  $\varphi = 90^\circ - \varphi'$

kot med  $f(x)$  in  $g(x)$ :

1. Presecice  $(x_0, y_0)$

2.  $f'(x_0) = k_1, g'(x_0) = k_2$

3.  $\tan \varphi = \left| \frac{k_2 - k_1}{1 + k_1 k_2} \right|$

## DN- 7. teden

① Funkcija  $f$  je dana s predpisom

$$f(x) = \begin{cases} -a + 2x - 2 & : x < -1 \\ x^2 - b & : -1 \leq x \leq 1 \\ ax - 16 & : x > 1 \end{cases}$$

Določi vrednosti za  $a$  in  $b$ , da bo funkcija  $f$  zvezna.

$$a = \boxed{\quad}, b = \boxed{\quad}$$

Preveri

$$\begin{aligned} \text{④} \lim_{x \rightarrow -1^-} -a + 2x - 2 &= f(-1) = -a - 4 & -a - 4 = a - 16 \\ \lim_{x \rightarrow -1^+} x^2 - b &= f(-1) = 1 - b & 2a = 12 \\ && a = 6 \\ \lim_{x \rightarrow 1^+} x^2 - b &= f(1) = 1 - b & -6 - 4 = 1 - b \\ \lim_{x \rightarrow 1^+} ax - 16 &= f(1) = a - 16 & b = 11 \end{aligned}$$

② Za funkcijo  $f(x) = \begin{cases} -2 + 2x + x^2 & x \leq 3 \\ ax + 1 & x \geq 3 \end{cases}$  določi  $a$  tako, da bo funkcija zvezna.

$$\text{Vpiši } a: \boxed{\quad}$$

Preveri

$$\begin{aligned} \lim_{x \rightarrow 3^-} -2 + 2x + x^2 &= f(3) = 13 & 3a + 1 = 13 \\ \lim_{x \rightarrow 3^+} ax + 1 &= 3a + 1 & 3a = 12 \\ && a = 4 \end{aligned}$$

## DN- 6. teden

Pošči definicijsko območje funkcije  $\sqrt{\log(-x^2 - 2x + 1)}$ . Krajišči definicijskega območja:  $\boxed{\quad}, \boxed{\quad}$ .

Preveri

$$\begin{aligned} \log x &\dots x > 0 \\ -x^2 - 2x + 1 &> 0 \\ x^2 + 2x - 1 &< 0 \\ D = 4 + 4 & \quad x_1, 2 = \frac{-2 \pm \sqrt{16}}{2} = -1 \pm \sqrt{2} \\ \sqrt{D} = \sqrt{16} & \quad x \in (-1 - \sqrt{2}, -1 + \sqrt{2}) \end{aligned}$$

$$\begin{aligned} \sqrt{x} &\dots x \geq 0 \\ \log(-x^2 - 2x + 1) &\geq 0 \\ e^0 &\leq -x^2 - 2x + 1 \\ x^2 + 2x &\leq 0 \\ x(x+2) &\leq 0 \\ x_1 = 0, x_2 = -2 & \quad x \in [0, -2] \end{aligned}$$

Pošči interval, ki predstavlja definicijsko območje funkcije  $f(x) = \log(-(x^2 - 8x + 15))$ .  $(\boxed{\quad}, \boxed{\quad})$ .

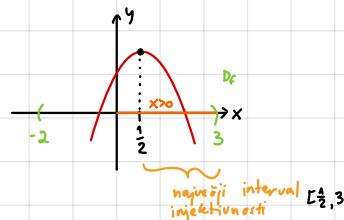
Preveri

$$\begin{aligned} \log x &\dots x > 0 \\ -x^2 + 8x - 15 &> 0 \\ x^2 - 8x + 15 &< 0 \\ (x-3)(x-5) &< 0 \\ x_1 = 3, x_2 = 5 & \\ D_x = (3, 5) & \end{aligned}$$

$$\begin{aligned} D_f: \log x &\dots x > 0 \\ -x^2 + x + 6 &> 0 \\ x^2 - x - 6 &< 0 \\ (x-3)(x+2) &< 0 \\ x_1 = 3, x_2 = -2 & \\ D_f = [-2, 3] & \end{aligned}$$

$$\begin{aligned} z_f: f(x) &= -x^2 + x + 6 \\ p = -\frac{b}{2a} &= \frac{1}{2} \\ \left(\frac{1}{2}\right)^2 &= \frac{1}{4} \\ f_{\max} &= \log\left(\frac{25}{4}\right) \\ z_f = (-\infty, \log\frac{25}{4}] & \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{2} \\ \log(-x^2 + x + 6) &= \frac{1}{2} \\ e^{\frac{1}{2}} &= (-x+3)(x+2) \\ x_1 &= \frac{1-\sqrt{25-4\cdot 6}}{2} \\ x_2 &= \frac{1+\sqrt{25-4\cdot 6}}{2} \end{aligned}$$



Dana je funkcija  $f(x) = \log(-x^2 + x + 6)$ . Določi definicijsko območje in zalogu vrednosti funkcije  $f$ . Poišči vse vrednosti  $x$ , za katere je vrednost funkcije enaka  $a = 1/2$ . Ali je funkcija injektivna?

Vpiši krajišča intervalov, ki sestavljajo definicijsko območje:

$$\boxed{\quad}, \boxed{\quad}$$

Vpiši največji element v zalogi vrednosti:  $\boxed{\quad}$ .

Vrednosti  $x$ , pri katerih je  $f(x) = a$ :

$$x > 0$$

Določi največji interval, ki vsebuje le pozitivna števila in na katerem je funkcija  $f(x)$  injektivna. Vpiši krajišča intervala:  $\boxed{\quad}, \boxed{\quad}$ .

24.11.

$$\lim_{x \rightarrow \infty} \sqrt[n]{x} = 1$$

Andrejeva razlaganje:

$$\textcircled{1} \quad \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x^2+x}\right)^{x^2} = \left(\left(1 + \frac{1}{x^2+x}\right)^{x^2+x}\right)^{\frac{x^2}{x^2+x}} = e$$

$$\textcircled{2} \quad \lim_{x \rightarrow \infty} x^{\frac{1}{x}} = e^{\ln(\lim(x^{\frac{1}{x}}))} = e^0 = 1$$

$\lim \ln(x^{\frac{1}{x}}) = \lim \frac{1}{x} \ln x = \lim \frac{\ln x}{x} = \frac{\infty}{\infty} \stackrel{\text{L'H}}{=} \frac{\frac{1}{x}}{1} = \frac{0}{1} = 0$

$\lim_{x \rightarrow \infty} a^x = e^{\lim_{x \rightarrow \infty} (x \cdot \ln a)}$

Approx.

1.  $\sin\left(\frac{2\pi}{3}\right)$

$60^\circ = x_0 = \frac{\pi}{3}$

$45^\circ = x_0 = \frac{\pi}{4}$

$0^\circ = x_0$

$h = -\frac{2}{3}$

$60^\circ: T_r(x) = \sin \frac{\pi}{3} - \frac{1}{3} \cdot \cos \frac{\pi}{3} = \frac{\sqrt{3}}{2} - \frac{1}{3} \cdot \frac{1}{2} \quad \textcircled{3}$

$45^\circ: T_r(x) = \sin \frac{\pi}{4} - \frac{1}{3} \cdot \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} - \frac{1}{3} \cdot \frac{\sqrt{2}}{2} \quad \textcircled{4}$

$0^\circ: T_r(x) = \sin 0 + \frac{2}{3} \cdot \cos 0 = 0 + \frac{2}{3} \cdot 1 = \frac{2}{3} \quad \textcircled{5}$

$\sin\left(\frac{2\pi}{3}\right) \approx \frac{2}{3}$

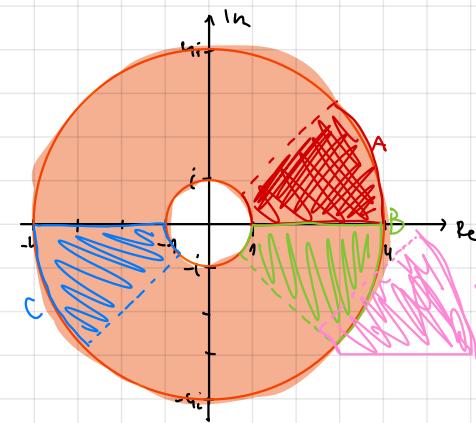
## Vaje 2. - teden

6. \* Nariši naslednjo podmnožico v  $\mathbb{C}$ :

$$A = \{z \in \mathbb{C} ; 1 \leq |z| \leq 4, 0 \leq \arg(z) < \pi/4\}$$

Z območjem  $A$  naredimo naslednjo transformacijo:

- (a) prezrcalimo ga preko osi  $\operatorname{Re}(z)$  (dobimo območje  $B$ ),



- (b) zavrtimo ga okoli števila 0 za kot  $\pi$  (dobimo območje  $C$ )

- (c) premaknemo ga za 2 v desno in 3 navzdol (dobimo območje  $D$ ).

Zapiši predpis  $z \mapsto f(z)$ , ki opravi kompleksno transformacijo območja  $A$  v območje  $D$ .

Nariši tudi  $f(A) = D$  in ugotovi, kam se s predpisom  $f$  preslika število  $1 + i$ .

Rešitev:  $z \mapsto -\bar{z} + 2 - 3i$ ;  $1 + i \mapsto 1 - 2i$ .