

1. vaja

Booleva algebra

X ... operandi $\{0,1\}$

\circ ... operatorji $\{\vee, \wedge, \neg\}$

P ... postulati

↳ zaprtost

$$P1: x, y \in X; x \vee y \in X$$

$$P1^*: x, y \in X; x \wedge y \in X$$

↳ nevtralni element

$$P2: x, 0 \in X; x \vee 0 = x$$

$$P2^*: x, 1 \in X; x \wedge 1 = x$$

↳ komutativnost

$$P3: x, y \in X; x \vee y = y \vee x$$

$$P3^*: x, y \in X; x \wedge y = y \wedge x$$

↳ distributivnost

$$P4: x, y, z \in X; x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$$

$$P4^*: x, y, z \in X; x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$$

↳ inverzni element

$$P5: \forall x \in X, \exists \bar{x} \in X; x \vee \bar{x} = 1$$

$$P5^*: \forall x \in X, \exists \bar{x} \in X; x \wedge \bar{x} = 0$$

↳ število elementov

$$P6: \exists x, y \in X; x \neq y$$

LASTNOSTI

↳ asociativnost

$$x \vee (y \vee z) = (x \vee y) \vee z = x \vee y \vee z$$

$$x \wedge (y \wedge z) = (x \wedge y) \wedge z = x \wedge y \wedge z$$

↳ idempotenca

$$x \vee x \vee x \dots \vee x = x$$

$$x \wedge x \wedge x \dots \wedge x = x$$

↳ de Morganov izrek

$$\overline{(x_1 \vee x_2 \vee \dots \vee x_n)} = \bar{x}_1 \cdot \bar{x}_2 \cdot \dots \cdot \bar{x}_n$$

$$\overline{(\bar{x}_1 \cdot \bar{x}_2 \cdot \dots \cdot \bar{x}_n)} = \bar{x}_1 \vee \bar{x}_2 \vee \dots \vee \bar{x}_n$$

↳ absorbcija

$$x \vee x \cdot y = x$$

$$x \cdot (x \vee y) = x$$

LOGIČNI OPERATORJI

↳ disjunkcija (or)

$$y = x_1 \vee x_2$$

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

↳ negacija (not)

$$y = \neg x = \bar{x}$$

x	y
0	1
1	0

↳ konjunkcija (and)

$$y = x_1 \wedge x_2$$

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

↳ Peirceov operator (nor)

$$y = \overline{x_1 \vee x_2} = x_1 \downarrow x_2$$

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	0

$$x_1 \downarrow (x_2 \downarrow x_3) \neq (x_1 \downarrow x_2) \downarrow x_3 \neq$$

$$x_1 \downarrow x_2 \downarrow x_3$$

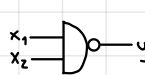
↳ Ekskluzivni ali (xor) \oplus

$$y = x_1 \oplus x_2 = \bar{x}_1 x_2 \vee x_1 \bar{x}_2$$



↳ Shefferjev operator (nand)

$$y = \overline{x_1 \cdot x_2} = x_1 \uparrow x_2$$



x_1	x_2	y
0	0	1
0	1	1
1	0	1
1	1	0

5 postulati dokaži $x \vee x = x$

$$x \vee x = (x \vee x) \cdot 1 \quad (P2^*)$$

$$= (x \vee x) \cdot (x \vee \bar{x}) \quad (P5)$$

$$= x \vee (x \cdot \bar{x}) \quad (P4)$$

$$= x \vee 0 \quad (P5^*)$$

$$= x$$

Poenostavi izraz $x_2 \rightarrow ((x_1 \vee \bar{x}_3) \cdot (\bar{x}_1 \vee \bar{x}_3))$

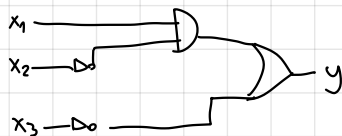
$$= \bar{x}_2 \vee ((x_1 \vee \bar{x}_3) \cdot (\bar{x}_1 \vee \bar{x}_3))$$

$$= \bar{x}_2 \vee ((x_1 \vee \bar{x}_3) \vee (\bar{x}_1 \vee \bar{x}_3))$$

$$= \bar{x}_2 \vee ((\bar{x}_1 \cdot x_3) \vee (x_1 \cdot x_3))$$

$$= \bar{x}_2 \vee x_3 \cdot (\bar{x}_1 \vee x_1)$$

$$= \bar{x}_2 \vee x_3$$



Naloga

$$f(x_1, x_2, x_3) = \bar{x}_1 x_2 \vee x_3$$



x_1	x_2	x_3	$\bar{x}_1 x_2 \vee x_3$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

DN 01 - 4.11.

①. $(x \vee \bar{y}) \cdot y = x \cdot y$

$(x \cdot y) \vee (\bar{y} \cdot y) \sim$ P4* ✓ P3*

$(x \cdot y) \vee 0 \sim$ P5*

$\sim x \cdot y$ P2

②. $f(x, y, z) = \overline{(\bar{x}\bar{y} \vee yz)} \vee (x \vee z) \sim$

$\overline{(\bar{x}\bar{y} \vee yz)} \cdot \overline{(x \vee z)} \sim$ De Morgan

$\overline{(\bar{x}\bar{y})} \cdot \overline{(yz)} \cdot (\bar{x} \cdot \bar{z}) \sim$ De Morgan

$\sim(x \vee y) \cdot (\bar{y} \vee \bar{z}) \cdot (\bar{x} \cdot \bar{z})$ De Morgan

$\sim(\bar{x}\bar{y} \vee x\bar{z} \vee \bar{y}y \vee y\bar{z}) \cdot (\bar{x} \cdot \bar{z})$ P4*

$\sim(x\bar{y} \vee x\bar{z} \vee y\bar{z}) \cdot \bar{x}\bar{z} \sim$ P5*

$\sim \underbrace{x\bar{y}\bar{x}\bar{z}}_0 \vee \underbrace{x\bar{z}\bar{x}\bar{z}}_0 \vee y\bar{z} \cdot \bar{x}\bar{z} \sim$ P5*

$\sim y\bar{z}\bar{x}\bar{z} \sim \bar{x}y\bar{z}$

③. $f(x_1, x_2, x_3, x_4) = \mathbb{Z}^1(14, 12, 11, 8, 7, 3, 2, 0)$

maximajouit termi: $M_{15}, M_{13}, M_{10}, M_7, M_6, M_5, M_4, M_1$

preluoriti: $\bar{M}_i = m_{2^n-1-i}$

$f(x_1, x_2, x_3, x_4) = \mathbb{V}^3(0, 2, 5, 6, 9, 10, 11, 14)$

3. vaja

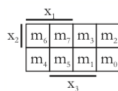
Veitchevi diagrami



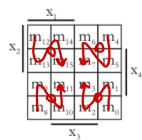
(a)



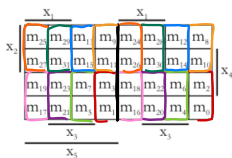
(b)



(c)



(d)

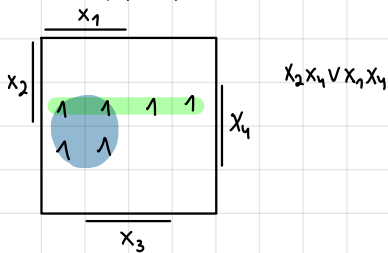


(e)

$$d) m_{13} = x_1 x_2 \bar{x}_3 x_4$$

PRIMER:

$$F = V^4(5, 7, 9, 11, 13, 15)$$



FUNKCIJSKO POLN SISTEM

$$\bullet \{V, 1, 7\}$$

$$\bullet \{V, 7\}$$

$$\bullet \{1, 7\}$$

$$\bullet \{1\} ??$$

$$\hookrightarrow 1: \bar{x} = \bar{x} \cdot \bar{x} = x \uparrow x$$

$$\wedge: x_1 \wedge x_2 = \overline{\overline{x_1 \wedge x_2}} = \overline{x_1 \uparrow x_2} = (x_1 \uparrow x_2) \uparrow (x_1 \uparrow x_2)$$

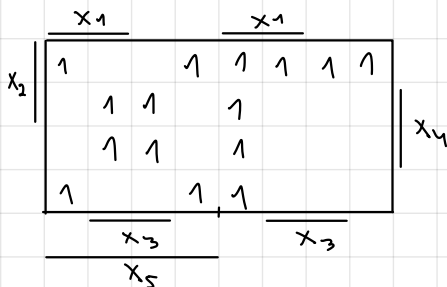
$$\vee: x_1 \vee x_2 = \overline{\overline{x_1 \vee x_2}} = \overline{\overline{x_1} \cdot \overline{x_2}} = \overline{\overline{x_1} \uparrow \overline{x_2}} = (x_1 \uparrow x_2) \uparrow (x_2 \uparrow x_2)$$

Naloga

manjkajoči: 30, 24, 23, 22, 19, 16, 15, 14, 13, 8, 7, 6, 5, 3, 0

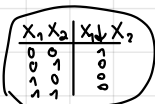
$$\textcircled{1} f(x_1, x_2, x_3, x_4, x_5) = \mathbb{B}^5(31, 29, 28, 27, 26, 25, 21, 20, 18, 17, 12, 11, 10, 9, 4, 2, 1)$$

$$f(x_1, x_2, x_3, x_4, x_5) = V^5(1, 7, 8, 9, 12, 15, 16, 17, 18, 23, 24, 25, 26, 28, 31)$$



$\textcircled{2}$

$$\{ \downarrow \}$$



$$1: \bar{x} = x \downarrow x$$

$$\vee: x_1 \vee x_2 = \overline{\overline{x_1 \vee x_2}} = \overline{x_1 \downarrow x_2} = (x_1 \downarrow x_2) \downarrow (x_1 \downarrow x_2)$$

$$\wedge: x_1 \wedge x_2 = \overline{\overline{x_1 \wedge x_2}} = \overline{\overline{x_1} \downarrow \overline{x_2}} = (x_1 \downarrow x_2) \downarrow (x_2 \downarrow x_2)$$

4. vaja

žepni razredi (Postov teorem)

$T_0 \dots f(0,0,\dots,0) = 0$

$T_1 \dots f(1,1,\dots,1) = 1$

$S \dots \overline{f(\overline{x}_1, \overline{x}_2, \dots, \overline{x}_n)} = f(x_1, x_2, \dots, x_n)$

$L \dots f(x_1, x_2, \dots, x_n) = a_0 \nabla a_1 x_1 \nabla \dots \nabla a_n x_n, f \in L$

$M \dots f \in M, \forall i, j; \vec{w}_i < \vec{w}_j \rightarrow f(\vec{w}_i) \leq f(\vec{w}_j)$

Primer:

Ali je $\{V, \rightarrow, \wedge\}$ poln?

T_0 :

$V: 0 \vee 0 = 0; \forall \in T_0$

$\rightarrow: 0 \rightarrow 0 = 1; \rightarrow \notin T_0$

T_1 :

$V: 1 \vee 1 = 1; \forall \in T_1$

$\rightarrow: 1 \rightarrow 1 = 1; \rightarrow \in T_1$

$\wedge: 1 \wedge 1 = 1; \wedge \in T_1$

..... tukaj se že lahko ustavi

L :

V : analitično

$f(x_1, x_2) = f_L(x_1, x_2) = a_0 \nabla a_1 x_1 \nabla a_2 x_2$

$f_L(0,0) = a_0 \nabla a_1 0 \nabla a_2 0 = a_0 = 0$

$f_L(0,1) = a_0 \nabla a_1 0 \nabla a_2 1 = 0_2 = 1$

$f_L(1,0) = a_0 \nabla a_1 1 \nabla a_2 0 = a_1 = 1$

$f_L(x_1, x_2) = 0 \nabla 1 x_1 \nabla 1 x_2 = x_1 \nabla x_2$

Preverimo $f = f_L$ pri ostalih vnosih

$f_L(1,1) = 1 \nabla 1 = 0$

$f_L(1,1) \neq f(1,1); \forall \notin L$

$f(1,1) = 1 \vee 1 = 1$

S :

V : analitično

$\overline{x_1 \vee x_2} = \overline{x_1} \wedge \overline{x_2}$

$x_1 x_2 \neq x_1 \vee x_2; \forall \notin S$

$\vee^2(3) \neq \vee^2(1,2,3)$

prav. tabela

x_1	x_2	\vee
0	0	0
0	1	1
1	0	1
1	1	1

vse preizkušnje morajo biti različne

M :

\rightarrow :

$\vec{w}_i < \vec{w}_j; w_{ki} \leq w_{kj}$

$[1,0,1,0] < [1,1,1,0]$

Dovolj, da preverimo sosedo, ki se razlik. po enem bitu

x_1	x_2	\rightarrow
0	0	1
0	1	1
1	0	0
1	1	1

$\vec{w}_0 < \vec{w}_1, f(\vec{w}_0) \leq f(\vec{w}_1)$

$\vec{w}_0 < \vec{w}_2, f(\vec{w}_0) \neq f(\vec{w}_2); \rightarrow \notin M$

Primer: Preveri pripadnost $f = \mathcal{L}^1(0,1,6,8,9,14)$ razreda L

$x_1 x_2 x_3 x_4$	f
0000	1
0001	0
0010	0
0011	1
0100	1
0101	0
0110	0
0111	1
1000	1
1001	0
1010	1
1011	0
1100	1
1101	0
1110	0
1111	0

$f_L(0,0,0,0) = a_0 \nabla a_1 0 \nabla a_2 0 \nabla a_3 0 \nabla a_4 0 = a_0 = 1$

istega ko izideš daš na 1

$f_L(0,0,0,1) = a_0 \nabla a_1 0 \nabla a_2 0 \nabla a_3 0 \nabla a_4 1 = 1 \nabla a_4 = 0$

$f_L(0,0,1,0) = 1 \nabla a_3 = 1$

$a_3 = 0$

$f_L(0,1,0,0) = 1 \nabla a_2 = 1$

$a_2 = 0$

$f_L(1,0,0,0) = 1 \nabla a_1 = 1$

$a_1 = 0$

$f_L(1,0,0,0) = 1 \nabla a_1 = 1$

$a_1 = 0$

$f_L(x_1, x_2, x_3, x_4) = 1 \nabla 0 x_1 \nabla 0 x_2 \nabla 0 x_3 \nabla 1 x_4 = 1 \nabla x_4 = \overline{x_4}$

$f \neq f_L; f \notin L$

ponovljen je

$x_4 = 0$ je

funkcija $f_L = 1$

x_1	x_2	x_3	x_4
1	1		
1	1		
		1	1
		1	1

$$f(x_1, x_2, x_3, x_4) = (x_1 \rightarrow x_2) \vee (x_3 \equiv x_4) = \vee^4(1, 2, 5, 6, 8, 11, 13, 14)$$

• preveri pripadnost zaprtim razredom, L na oba načina

• po potrebi dopolni do funkc. polnosti

\vec{w}_3	\vec{w}_1	x_1	x_2	x_3	x_4	f	f_i
		0	0	0	0	0	
		0	0	0	1	1	
		0	0	1	0	0	
		0	0	1	1	1	
		0	1	0	0	0	
		0	1	0	1	1	
		0	1	1	0	0	
		0	1	1	1	1	
		1	0	0	0	1	
		1	0	0	1	0	
		1	0	1	0	0	
		1	0	1	1	1	
		1	1	0	0	0	
		1	1	0	1	1	
		1	1	1	0	1	
		1	1	1	1	0	

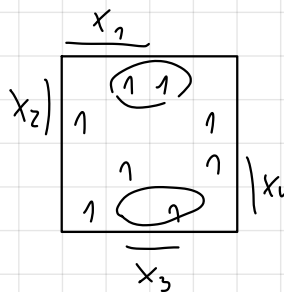
$$L: f_i(0,0,0,0) = a_0 \vee a_1 \vee a_2 \vee a_3 \vee a_4 = a_0 = 0 \quad a_0 = 0$$

$$f_i(0,0,0,1) = 0 \vee a_1 \vee a_2 \vee a_3 \vee a_4 = 0 \vee a_4 = 1 \quad a_4 = 1$$

$$f_i(0,0,1,0) = 0 \vee a_3 = 1 \quad a_3 = 1$$

$$f_i(0,1,0,0) = 0 \vee a_2 = 0 \quad a_2 = 0$$

$$f_i(1,0,0,0) = 0 \vee a_1 = 1 \quad a_1 = 1$$



$$x_1 \bar{x}_2 x_3 \bar{x}_4 : \bar{x}_1 \bar{x}_2 x_3 \bar{x}_4$$

$$f_L(x_1, x_2, x_3, x_4) = 0 \vee 1 \cdot x_1 \vee 0 \cdot x_2 \vee 1 \cdot x_3 \vee 1 \cdot x_4 = x_1 \vee x_3 \vee x_4$$

	T_0	T_1	S	L	M
f	0	1	0	1	1

Protiprimer:

$$f_i(0,1,1,0) = 0 \vee 0 \vee 0 \vee 1 \vee 0 = 1$$

$$f_i(0,1,1,1) = 0 \vee 0 \vee 0 \vee 1 \vee 1 = 0$$

$$f_i(0,0,1,1) = 0 \vee 0 \vee 0 \vee 1 \vee 1 = 0$$

$$f_i(1,1,0,0) = 0 \vee 1 \vee 0 \vee 0 \vee 0 = 1 \quad \text{Protiprimer}$$

S:

$$f(x_1, x_2, x_3, x_4) = \bar{f}(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4)$$

$$f(0,0,1,1) \neq \bar{f}(\bar{0}, \bar{0}, \bar{1}, \bar{1})$$

T_0 :

$$0 \rightarrow 0 = 1$$

$$0 \equiv 0 = 1$$

$$\rightarrow 1 \wedge 1 = 0$$

$$M: [0, 0, 0, 1] < [0, 0, 1, 1]; \quad f(0, 0, 0, 1) \neq f(0, 0, 1, 1)$$

$$f \notin M$$

$$1$$

$$0$$

T_1 :

$$1 \rightarrow 1 = 1$$

$$1 \equiv 1 = 1$$

$$\rightarrow 1 \vee 1 = 0$$

Funkcija je polna pri $\{f, 1\}$, da je $f \in T_0$