

1. vaja

Bolova algebra

X ... operandi $\{0, 1\}$

O ... operatorji $\{\vee, \wedge, \neg\}$

P ... postulati

↳ zapisi:

$$P1: x, y \in X; x \vee y \in X$$

$$P1^*: x, y \in X; x \wedge y \in X$$

↳ neutralni element

$$P2: x, 0 \in X; x \vee 0 = x$$

$$P2^*: x, 1 \in X; x \wedge 1 = x$$

↳ komutativnost

$$P3: x, y \in X; x \vee y = y \vee x$$

$$P3^*: x, y \in X; x \wedge y = y \wedge x$$

↳ distributivnost

$$P4: x, y, z \in X; x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$$

$$P4^*: x, y, z \in X; x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$$

↳ inverzni element

$$P5: \forall x \in X, \exists \bar{x} \in X; x \vee \bar{x} = 1$$

$$P5^*: \forall x \in X, \exists \bar{x} \in X; x \cdot \bar{x} = 0$$

↳ številočni elementar

$$P6: \exists x, y \in X; x \neq y$$

LASTNOSTI

↳ asociativnost

$$x \vee (y \vee z) = (x \vee y) \vee z = x \vee y \vee z$$

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z = x \cdot y \cdot z$$

↳ de Morganov izrek

$$(x_1 \vee x_2 \vee \dots \vee x_n) = \bar{x}_1 \cdot \bar{x}_2 \cdot \dots \cdot \bar{x}_n$$

$$(x_1 \cdot x_2 \cdot \dots \cdot x_n) = \bar{x}_1 \vee \bar{x}_2 \vee \dots \vee \bar{x}_n$$

↳ idempotencija

$$x \vee x \vee x \dots \vee x = x$$

$$x \cdot x \cdot x \dots \cdot x = x$$

↳ absorbacija

$$x \vee x \cdot y = x$$

$$x \cdot (x \vee y) = x$$

LOGIČNI OPERATORJI

↳ disjunkcija (or)

$y = x_1 \vee x_2$	x_1	x_2	y
	0	0	0
	0	1	1
	1	0	1
	1	1	1

↳ negacija (not)

$y = \neg x = \bar{x}$	x	y
	0	1
	1	0

↳ konjunkcija (and)

$y = x_1 \wedge x_2$	x_1	x_2	y
	0	0	0
	0	1	0
	1	0	0
	1	1	1

↳ Peirceov operator (nor)

$y = \overline{x_1 \vee x_2} = x_1 \downarrow x_2$	x_1	x_2	y
	0	0	1
	0	1	0
	1	0	0
	1	1	0

$$x_1 \downarrow (x_2 \downarrow x_3) \neq (x_1 \downarrow x_2) \downarrow x_3 \neq$$

$$\neq x_1 \downarrow x_2 \downarrow x_3$$

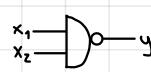
↳ ekskluzivni ali (xor) \oplus

$$y = x_1 \Delta x_2 = \overline{x_1} \cdot x_2 \vee x_1 \cdot \overline{x_2}$$



↳ Shefferjev operator (nand)

$$y = \overline{x_1 \cdot x_2} = x_1 \uparrow x_2$$



S postulati dokazi $x \vee x = x$

$$x \vee x = (x \vee x) \cdot 1 \quad (P2^*)$$

$$= (x \vee x) \cdot (x \vee \bar{x}) \quad (P5)$$

$$= x \vee (x \cdot \bar{x}) \quad (P4)$$

$$= x \vee 0 \quad (P5^*)$$

$$= x$$

Pozornost na iteraz $x_2 \rightarrow (\overline{(x_1 \vee \bar{x}_3)} \cdot \overline{(x_1 \vee x_3)})$

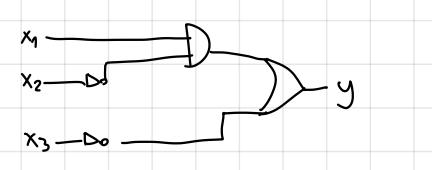
$$= \overline{x_2} \vee (\overline{(x_1 \vee \bar{x}_3)} \cdot \overline{(x_1 \vee x_3)})$$

$$= \overline{x_2} \vee ((\overline{x_1} \cdot \bar{x}_3) \vee (\overline{x_1} \cdot x_3))$$

$$= \overline{x_2} \vee ((\bar{x}_1 \cdot x_3) \vee (x_1 \cdot \bar{x}_3))$$

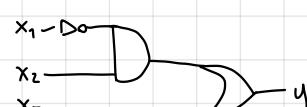
$$= \overline{x_2} \vee x_3$$

$$= \overline{x_2} \vee x_3$$



Naloga

$$f(x_1, x_2, x_3) = \overline{x_1} \cdot x_2 \vee x_3$$

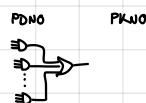


x_1	x_2	x_3	$\overline{x_1} \cdot x_2 \vee x_3$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

2. vaja

Oblike zapisa preklopnih funk.

- POPOLNE
- NORMALNE
- MINIMALNA



PDNO

$$f(x_1, x_2, \dots, x_n) = \bigvee_{i=0}^{2^n-1} m_i f(\vec{w}_i)$$

$$m_i = x_1^{w_{1,i}} \cdot x_2^{w_{2,i}} \cdots x_n^{w_{n,i}} ; i = 0, 1, 2, \dots, 2^n - 1$$

$$x^w = \begin{cases} x, w=1 \\ \bar{x}, w=0 \end{cases}$$

Zgled:

$n=4$

$$i = g_{[00]} = 1001_{[2]}$$

$$m_9 = x_1^0 x_2^0 x_3^0 x_4^1 = x_1 \bar{x}_2 \bar{x}_3 x_4$$

x_1	x_2	x_3	$f(x_1, x_2, x_3)$	minterm	f
0	0	0	$f(\vec{w}_0)$	m_0	1
0	0	1	$f(\vec{w}_1)$	m_1	0
0	1	0	$f(\vec{w}_2)$	m_2	0
0	1	1	$f(\vec{w}_3)$	m_3	0
1	0	0	$f(\vec{w}_4)$	m_4	1
1	0	1	$f(\vec{w}_5)$	m_5	1
1	1	0	$f(\vec{w}_6)$	m_6	1
1	1	1	$f(\vec{w}_7)$	m_7	1

Krajši način zapisa PDNO: $f(x_1, x_2, \dots, x_n) = V^w [i_1, i_2, \dots, i_n]$ indeksi mintermor

↓ Primer za to tabelo:

$$f(x_1, x_2, x_3) = \bar{x}_1 \bar{x}_2 \bar{x}_3 \vee x_1 \bar{x}_2 \bar{x}_3 \vee x_1 \bar{x}_2 x_3 \vee x_1 x_2 \bar{x}_3 \vee x_1 x_2 x_3 = V^3 (0, 4, 5, 6, 7)$$

ANALITIČNO:

$$\begin{aligned} &x_1 \vee x_1 \bar{x}_2 \vee \bar{x}_2 \bar{x}_3 \\ &= x_1 (x_2 \vee \bar{x}_2) (x_3 \vee \bar{x}_3) \vee x_1 \bar{x}_2 (x_3 \vee \bar{x}_3) \vee (x_1 \vee \bar{x}_1) \bar{x}_2 \bar{x}_3 \\ &= x_1 x_2 x_3 \vee x_1 x_2 \bar{x}_3 \vee x_1 \bar{x}_2 x_3 \vee x_1 \bar{x}_2 \bar{x}_3 \vee x_1 \bar{x}_2 x_3 \vee x_1 \bar{x}_2 \bar{x}_3 \\ &= V^3 (7, 6, 5, 4, 0) \\ &= V^3 (0, 4, 5, 6, 7) \end{aligned}$$

PKNO

$$f(x_1, x_2, \dots, x_n) = \&_{i=0}^{2^n-1} (M_{2^n-1-i} \vee f(\vec{w}_i))$$

$$M_i = x_1^{\bar{w}_{1,i}} \vee x_2^{\bar{w}_{2,i}} \vee \dots \vee x_n^{\bar{w}_{n,i}} ; i = 0, 1, \dots, 2^n - 1$$

x_1	x_2	x_3	$f(x_1, x_2, x_3)$	maksterm	f
0	0	0	$f(\vec{w}_0)$	M_7	1
0	0	1	$f(\vec{w}_1)$	M_6	0
0	1	0	$f(\vec{w}_2)$	M_5	0
0	1	1	$f(\vec{w}_3)$	M_4	0
1	0	0	$f(\vec{w}_4)$	M_3	1
1	0	1	$f(\vec{w}_5)$	M_2	1
1	1	0	$f(\vec{w}_6)$	M_1	1
1	1	1	$f(\vec{w}_7)$	M_0	1

KRAJŠI NAČIN ZAPISA

$$f(x_1, x_2, \dots, x_n) = \&^n (i_n, i_{n-1}, \dots, i_1)$$

Zgled:

$n=4$

$$2^{n-1}-i = 15 - 9 = 6$$

$$2^{n-1}-i = 6_{[10]} = 0110_{[2]}$$

$$M_9 = x_1^0 x_2^1 x_3^1 x_4^0 = x_1 \bar{x}_2 \bar{x}_3 x_4$$

Pretvorba
PDNO \Leftrightarrow PKNO

① Izpišemo manjkajoče terme

$$f(x_1, x_2, x_3) = V^3 (0, 4, 5, 6, 7)$$

manjkojo: m_1, m_2, m_3

$$f(x_1, x_2, x_3) = V^3 (1, 2, 3)$$

② Funkcijo drugič negiramo

pretvorimo v terme

[7-1, 7-2, 7-3]

$$f_{\text{PKNO}}(x_1, x_2, x_3) = \&^3 (6, 5, 4)$$

2. NALOGA: Podana je preklopna funkcija:

$$f(x_1, x_2, x_3, x_4) = \&^4 (14, 13, 10, 9, 6, 5, 2, 1)$$

ZAPIŠI

a) PKNO v eksplicitni obliki

b) PDNO v eksplicitni in skrajšani obliki

c) pravilnostno tabelo

① izpisemo manjkajoče terme

manjkojo: $M_{15}, M_{12}, M_{11}, M_8, M_7, M_4, M_3, M_0$

② Funkcijo drugič negiramo

$$f_{\text{PDNO}}(x_1, x_2, x_3, x_4) = V^4 (0, 3, 4, 7, 8, 11, 12, 15)$$

$$\begin{aligned} &f(x_1, x_2, x_3, x_4) = \bar{x}_1 \bar{x}_2 \bar{x}_3 \bar{x}_4 \vee \bar{x}_1 \bar{x}_2 x_3 \bar{x}_4 \vee \bar{x}_1 x_2 \bar{x}_3 \bar{x}_4 \vee \bar{x}_1 x_2 x_3 \bar{x}_4 \vee x_1 \bar{x}_2 \bar{x}_3 \bar{x}_4 \vee x_1 \bar{x}_2 x_3 \bar{x}_4 \vee x_1 x_2 \bar{x}_3 \bar{x}_4 \vee x_1 x_2 x_3 \bar{x}_4 = \\ &= \bar{x}_1 (\bar{x}_2 \bar{x}_3 \bar{x}_4 \vee \bar{x}_2 x_3 \bar{x}_4 \vee x_2 \bar{x}_3 \bar{x}_4 \vee x_2 x_3 \bar{x}_4) \vee x_1 (\bar{x}_2 \bar{x}_3 \bar{x}_4 \vee \bar{x}_2 x_3 \bar{x}_4 \vee x_2 \bar{x}_3 \bar{x}_4 \vee x_2 x_3 \bar{x}_4) = \\ &= (\bar{x}_1 \vee x_1) (\bar{x}_2 \bar{x}_3 \bar{x}_4 \vee \bar{x}_2 x_3 \bar{x}_4 \vee x_2 \bar{x}_3 \bar{x}_4 \vee x_2 x_3 \bar{x}_4) = \\ &= \bar{x}_2 \bar{x}_3 \bar{x}_4 \vee x_2 x_3 \bar{x}_4 \vee x_2 \bar{x}_3 \bar{x}_4 \vee x_2 x_3 \bar{x}_4 = \\ &= (\bar{x}_2 \vee x_2) (\bar{x}_3 \bar{x}_4 \vee x_3 x_4) = \\ &= \bar{x}_3 \bar{x}_4 \vee x_3 x_4 = x_3 \Leftrightarrow x_4 \end{aligned}$$

x_1	x_2	x_3	x_4	m	M	f
0	0	0	0	m_0	M_{15}	1
0	0	0	1	m_1	M_{14}	0
0	0	1	0	m_2	M_{13}	0
0	0	1	1	m_3	M_{12}	1
0	1	0	0	m_4	M_{11}	1
0	1	0	1	m_5	M_{10}	1
0	1	1	0	m_6	M_9	0
0	1	1	1	m_7	M_8	0
1	0	0	0	m_8	M_7	1
1	0	0	1	m_9	M_6	0
1	0	1	0	m_{10}	M_5	0
1	0	1	1	m_{11}	M_4	1
1	1	0	0	m_{12}	M_3	1
1	1	0	1	m_{13}	M_2	1
1	1	1	0	m_{14}	M_1	1
1	1	1	1	m_{15}	M_0	1

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① $(x \vee \bar{y}) \cdot y = x \cdot y$ ✓ P₃*

$(x \cdot y) \vee (\bar{y} \cdot y) \sim$ P_{4*} ✗ P₃*

$(x \cdot y) \vee 0 \sim$ P_{5*}

$\sim x \cdot y$ P₂

② $f(x, y, z) = \overline{(\bar{x} \bar{y} \vee y z) \vee (x \vee z)} \sim$

$\overline{(\bar{x} \bar{y} \vee y z) \cdot (\bar{x} \vee z)} \sim$ DeMorgan

$\overline{(\bar{x} \bar{y}) \cdot (\bar{y} z) \cdot (\bar{x} \cdot \bar{z})} \sim$ De Morgan

$\sim (x y) \cdot (\bar{y} \vee \bar{z}) \cdot (\bar{x} \cdot \bar{z})$ De Morgan

$\sim (\bar{x} \bar{y} \vee x \bar{z} \vee \underbrace{y \bar{y}}_{0} \vee y \bar{z}) \cdot (\bar{x} \bar{z})$ P_{4*}

$\sim (\bar{x} \bar{y} \vee x \bar{z} \vee y \bar{z}) \cdot \bar{x} \bar{z} \sim$ P_{5*}

$\sim \underbrace{\bar{x} \bar{y} \bar{x} \bar{z}}_0 \vee \underbrace{x \bar{z} \cdot \bar{x} \bar{z}}_0 \vee y \bar{z} \cdot \bar{x} \bar{z} \sim$ P_{5*}

$\sim y \bar{z} \bar{x} \bar{z} \sim \bar{x} y \bar{z}$

③ $f(x_1, x_2, x_3, x_4) = \&^4(14, 12, 11, 8, 7, 3, 2, 0)$

manykajou termi: M₁₅, M₁₃, M₁₀, M₉, M₆, M₅, M₄, M₁

pretransformi: $\bar{m}_i = m_{2^n - i}$

$f(x_1, x_2, x_3, x_4) = V^4(0, 2, 5, 6, 9, 10, 11, 14)$

3. vaja

Veitševi diagrami

$$\begin{array}{|c|c|} \hline x & \\ \hline m_1 & m_0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|} \hline x_1 & & & \\ \hline m_1 & m_1 & m_1 & \\ \hline m_1 & m_0 & m_0 & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|c|c|} \hline x_1 & & & & & \\ \hline m_1 & m_1 & m_1 & m_1 & m_1 & m_0 \\ \hline m_1 & m_1 & m_1 & m_0 & m_1 & m_0 \\ \hline m_1 & m_1 & m_0 & m_1 & m_1 & m_0 \\ \hline m_1 & m_0 & m_1 & m_1 & m_1 & m_1 \\ \hline \end{array}$$

(a)

(b)

(c)

$$\begin{array}{|c|c|c|c|c|c|} \hline x_1 & & & & & \\ \hline m_1 & m_1 & m_1 & m_1 & m_1 & m_1 \\ \hline m_1 & m_1 & m_1 & m_1 & m_1 & m_1 \\ \hline m_1 & m_1 & m_1 & m_1 & m_1 & m_1 \\ \hline m_1 & m_1 & m_1 & m_1 & m_1 & m_1 \\ \hline m_1 & m_1 & m_1 & m_1 & m_1 & m_1 \\ \hline \end{array}$$

(d)

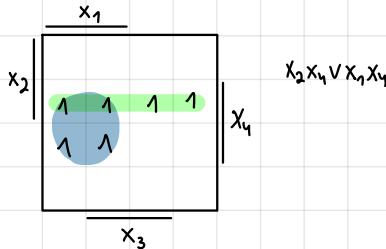
$$\begin{array}{|c|c|c|c|c|c|} \hline x_1 & & & & & \\ \hline m_1 & m_1 & m_1 & m_1 & m_1 & m_1 \\ \hline m_1 & m_1 & m_1 & m_1 & m_1 & m_1 \\ \hline m_1 & m_1 & m_1 & m_1 & m_1 & m_1 \\ \hline m_1 & m_1 & m_1 & m_1 & m_1 & m_1 \\ \hline m_1 & m_1 & m_1 & m_1 & m_1 & m_1 \\ \hline \end{array}$$

(e)

$$d) m_{13} = x_1 x_2 \bar{x}_3 x_4$$

PRIMER:

$$f = V^U(5, 7, 9, 11, 13, 15)$$



FUNKCIJSKO POLN SISTEM

- {V, A, 7}
- {V, 7}
- {A, 7}

- {↑} ??

$$\hookrightarrow 1: \bar{x} = \overline{x \cdot x} = x \uparrow x$$

$$\wedge: x_1 \wedge x_2 = \overline{\overline{x_1} \wedge x_2} = \overline{x_1 \uparrow x_2} = (x_1 \uparrow x_2) \uparrow (x_1 \uparrow x_2)$$

$$V: x_1 V x_2 = \overline{\overline{x_1} V x_2} = \overline{x_1 \cdot x_2} = \overline{x_1} \uparrow \overline{x_2} = (x_1 \uparrow x_1) \uparrow (x_2 \uparrow x_2)$$

Naloga

mangajumi: 30, 24, 23, 22, 19, 16, 15, 14, 13, 9, 7, 6, 5, 3, 0

$$① f(x_1, x_2, x_3, x_4, x_5) = \&^5(31, 23, 28, 27, 26, 25, 21, 20, 18, 17, 12, 11, 10, 9, 6, 2, 1)$$

$$f(x_1, x_2, x_3, x_4, x_5) = V^5(1, 7, 8, 9, 12, 15, 16, 17, 18, 23, 24, 25, 26, 28, 31)$$

$$\begin{array}{|c|c|c|c|c|c|c|c|} \hline x_1 & & & & & & & \\ \hline 1 & & 1 & 1 & 1 & 1 & 1 & \\ \hline & 1 & 1 & & 1 & & & \\ \hline & 1 & 1 & 1 & & & & \\ \hline & 1 & & 1 & 1 & & & \\ \hline \hline x_2 & & & & & & & \\ \hline & & & & & & & \\ \hline \end{array}$$

$$\left\{ \begin{array}{l} x_1 x_2 \\ x_1 \downarrow x_2 \\ \hline 0 0 \\ 0 1 \\ 1 0 \\ 1 1 \end{array} \right.$$

②

- {↓}

$$\hookrightarrow 1: \bar{x} = x \downarrow x$$

$$\wedge: x_1 \wedge x_2 = \overline{\overline{x_1} \wedge x_2} = \overline{x_1 \downarrow x_2} = (x_1 \downarrow x_2) \downarrow (x_1 \downarrow x_2)$$

$$V: x_1 V x_2 = \overline{\overline{x_1} V x_2} = \overline{x_1 \downarrow x_2} = (x_1 \downarrow x_1) \downarrow (x_2 \downarrow x_2)$$

4. vaja

Zaprti razredi (Postov teorem)

$$T_0 \dots f(0,0,\dots,0) = 0$$

$$T_1 \dots f(1,1,\dots,1) = 1$$

$$S \dots \bar{f}(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) = f(x_1, x_2, \dots, x_n)$$

$$L \dots f(x_1, x_2, \dots, x_n) = a_0 \nabla a_1 x_1 \nabla \dots \nabla a_n x_n, f \in L$$

$$M \dots f \in M, \forall i; \vec{w}_i < \vec{w}_j \rightarrow f(\vec{w}_i) \leq f(\vec{w}_j)$$

Primer:

Ali je $\sum V_i \rightarrow \Lambda^3$ poln?

T_0 :

$$V: 0 \vee 0 = 0; V \in T_0$$

$$\rightarrow: 0 \rightarrow 0 = 1; \rightarrow \notin T_0$$

T_1 :

$$V: 1 \vee 1 = 1; V \in T_1$$

$$\begin{aligned} \rightarrow: 1 \rightarrow 1 &= 1; \rightarrow \in T_1 \\ 1: 1 &= 1; 1 \in T_1 \\ \dots \dots \dots \text{tukaj se že lahko ustavi} \end{aligned}$$

$S:$

V : analitično

$$\bar{x}_1 \bar{x}_2 = x_1 \vee x_2$$

$$x_1 x_2 \neq x_1 \vee x_2; V \notin S$$

$$V^2(3) \neq V^2(1,2,3)$$

prav. tabela

x_1	x_2	V
0	0	0
0	1	1
1	0	1
1	1	1

vs. preizpoljene
morajo biti
različne

L :

V : analitično

$$f(x_1, x_2) = f_L(x_1, x_2) = a_0 \nabla a_1 x_1 \nabla a_2 x_2$$

$$f_L(0,0) = a_0 \nabla a_1 0 \nabla a_2 0 = a_0 = 0$$

$$f_L(0,1) = a_0 \nabla a_1 0 \nabla a_2 \cdot 1 = a_2 = 1$$

$$f_L(1,0) = a_0 \nabla a_1 \cdot 1 \nabla a_2 0 = a_1 = 1$$

$$f_L(1,1) = 0 \nabla 1 x_1 \nabla 1 x_2 = x_1 \nabla x_2$$

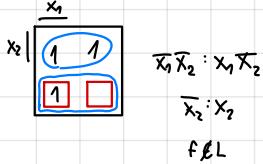
Preverimo $f = f_L$ pri ostalih vhodih

$$f_L(1,1) = 0$$

$$f_L(1,1) \neq f(1,1); V \notin L$$

$$f(1,1) = 1 \vee 1 = 1$$

Veitchev diagram



M :

$\rightarrow:$

$$\vec{w}_i < \vec{w}_j: w_{k_i} \leq w_{k_j}$$

$$[1,0,1,0] < [1,1,1,0]$$

Dovolj, da preverimo
sosedje, ki se razlikuj.
po enem bitu

x_1	x_2	\rightarrow
0	0	1
0	1	1
1	0	0
1	1	1

$$\vec{w}_0 < \vec{w}_1, f(\vec{w}_0) \leq f(\vec{w}_1)$$

$$\vec{w}_0 < \vec{w}_2, f(\vec{w}_0) \neq f(\vec{w}_2); \rightarrow \notin M$$

Primer: Preveri pripadnost $f = \delta^4(0,1,6,8,9,14)$ razreda L

x_1	x_2	x_3	x_4	f	f_L
0	0	0	0	1	
0	0	0	1	1	
0	0	1	0	1	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	1	<small>tisteža bo ites' dat na 1</small>
1	0	0	1	0	<small>$f_L(0,0,0,1) = a_0 \nabla a_1 0 \nabla a_2 0 \nabla a_3 0 \nabla a_4 1 = 1 \nabla a_4 = 0$</small>
1	0	1	0	1	<small>$f_L(0,0,1,0) = 1 \nabla a_3 = 1$</small>
1	0	1	1	0	<small>$a_3 = 0$</small>
1	1	0	0	1	<small>$a_2 = 0$</small>
1	1	0	1	0	<small>$a_1 = 0$</small>
1	1	1	0	0	<small>$a_0 = 0$</small>
1	1	1	1	1	

$$f_L(x_1, x_2, x_3, x_4) = 1 \nabla 0 x_1 \nabla 0 x_2 \nabla 0 x_3 \nabla 1 \cdot x_4 = \underline{\underline{1 \nabla x_1}} = \bar{x}_4 \quad x_4 = 0 \text{ je} \\ f \neq f_L; f \notin L \quad \text{ponosod kjen je} \\ \text{funkcija } f_L = 1$$

x_1	x_2	x_3	x_4
1	1		
1	1		
1	1	1	
1	1	1	1

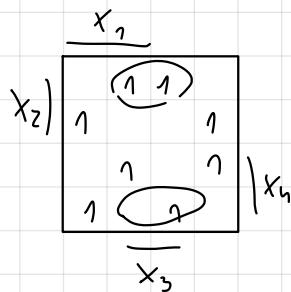
$$f(x_1, x_2, x_3, x_4) = (x_1 \rightarrow x_2) \nabla (x_3 \equiv x_4) = V^4(1, 2, 5, 6, 8, 9, 13, 14)$$

- preveri pripadnost zaprtim razredom, L na obojna načina
- po potrebi dopolni do funk. polnosti

x_1	x_2	x_3	x_4	f	f_L
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	1	0
0	1	0	1	1	1
0	1	1	0	1	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	0	1	1	1
1	0	1	0	1	1
1	0	1	1	1	1
1	1	0	0	1	1
1	1	0	1	1	1
1	1	1	0	1	1
1	1	1	1	1	1

L: $f_L(0,0,0,0) = 0 \nabla a_0, 0 \nabla a_2, 0 \nabla a_3, 0 \nabla a_4, 0 = a_0 = 0 \quad a_0 = 0$
 $f_L(0,0,0,1) = 0 \nabla a_0, 0 \nabla a_2, 0 \nabla a_3, 0 \nabla a_4, 1 = 0 \nabla a_4 = 1 \quad a_4 = 1$
 $f_L(0,0,1,0) = 0 \nabla a_3 = 1 \quad a_3 = 1$
 $f_L(0,1,0,0) = 0 \nabla a_2 = 0 \quad a_2 = 0$
 $f_L(1,0,0,0) = 0 \nabla a_1 = 1 \quad a_1 = 1$

$$f_L(x_1, x_2, x_3, x_4) = 0 \nabla 1 \cdot x_1 \nabla 0 \cdot x_2 \nabla 1 \cdot x_3 \nabla 1 \cdot x_4 = x_1 \nabla x_3 \nabla x_4$$



$$\overline{x_1} \overline{x_2} x_3 \overline{x_4} : \overline{x_1} \overline{x_2} x_3 x_4 \\ \overline{x_2} x_3 \overline{x_4} : x_2 x_3 \overline{x_4}$$

f	T ₀	T ₁	S	L	M
\overline{f}	E	E	X	X	l

S:

$$f(x_1, x_2, x_3, x_4) = \overline{f}(\overline{x}_1, \overline{x}_2, \overline{x}_3, \overline{x}_4)$$

$$f(0,0,1,1) \neq \overline{f}(\overline{0}, \overline{0}, \overline{1}, \overline{1})$$

Protiprimer:

$$f_L(0,1,1,0) = 0 \nabla 0 \nabla 0 \nabla 1 \nabla 0 = 1$$

$$f_L(0,1,1,1) = 0 \nabla 0 \nabla 0 \nabla 1 \nabla 1 = 0$$

$$f_L(0,0,1,1) = 0 \nabla 0 \nabla 0 \nabla 1 \nabla 1 = 0$$

$$f_L(1,1,0,0) = 0 \nabla 1 \nabla 0 \nabla 0 \nabla 0 = 1 \quad \text{Protiprimer}$$

T₀:

$$\begin{aligned} 0 \rightarrow 0 &= 1 \\ 0 \equiv 0 &= 1 \end{aligned} \rightarrow 1 \vee 1 = 0$$

$$M: [0, 0, 0, 1] \subset [0, 0, 1, 1]; \quad f(0, 0, 0, 1) \notin f(0, 0, 1, 1)$$

$\overrightarrow{w_1}$ $\overrightarrow{w_2}$

$f \notin M$ $\begin{matrix} 1 & & 1 \\ & 1 & \\ & & 0 \end{matrix}$

T₁:

$$\begin{aligned} 1 \rightarrow 1 &= 1 \\ 1 \equiv 1 &= 1 \end{aligned} \rightarrow 1 \vee 1 = 0$$

Funkcija je polna pri $\{f, 1\}$, da je $f \in T_0$.