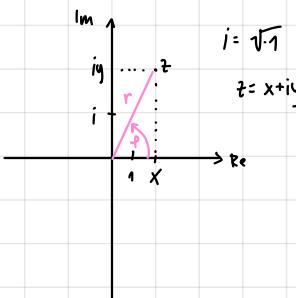


## Kartezični zapis



## Polarni zapis

$z = r(\cos \varphi + i \sin \varphi)$   
 $\varphi \in \mathbb{R}$  ni enotično določen  
 $\Rightarrow$  do  $+k2\pi$  natančno,  $k \in \mathbb{Z}$   
 $r = |z| \in [0, \infty)$  je enotično določen  
 $\Rightarrow$  argument, polarni kot  
 $x = r \cdot \cos \varphi$   
 $y = r \cdot \sin \varphi$   
 $\tan \varphi = \frac{y}{x} \Rightarrow \varphi = \arctan \frac{y}{x}$   
 (upoštevamo kvadrant)

Primer: Zapiši v polarni obliko

$$z = 1+i$$

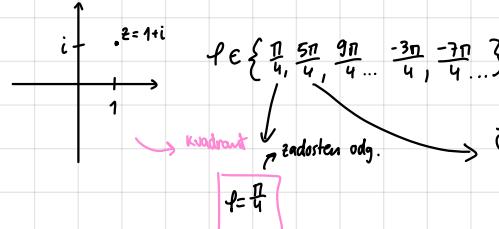
pitagorov

$$x = 1 \quad r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$y = 1$$

$$\tan \varphi = \frac{1}{1} = 1$$

$$\varphi = \frac{\pi}{4} + k\pi, k \in \mathbb{Z}$$



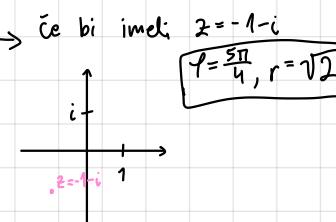
Konjugiramo:

$$z = 1-i$$

z = 1-i

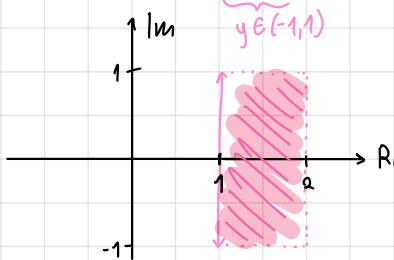
$$r = \sqrt{2}$$

$$\varphi = -\frac{\pi}{4}$$

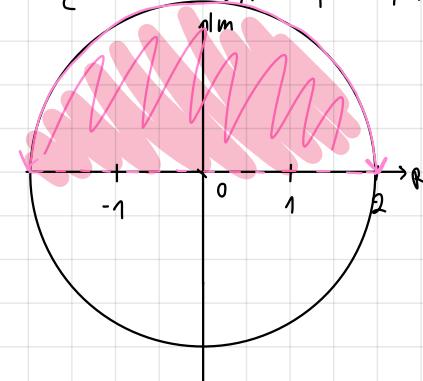


Primer: Nariši naslednjo množico

$$\{x+iy; x \in [1,2], |y| < 1\}$$



$$\{r(\cos \varphi + i \sin \varphi), r \leq 2, \varphi \in (0, \pi)\}$$



## Množenje v polarnem zapisu

$$z_j = |z_j| \cdot (\cos \varphi_j + i \sin \varphi_j) \quad j \in \{1, 2\}$$

$$z_1 \cdot z_2 = |z_1| \cdot (\cos \varphi_1 + i \sin \varphi_1) \cdot |z_2| \cdot (\cos \varphi_2 + i \sin \varphi_2)$$

$$= |z_1| \cdot |z_2| \cdot (\cos \varphi_1 \cdot \cos \varphi_2 - \sin \varphi_1 \cdot \sin \varphi_2 + i(\cos \varphi_1 \cdot \sin \varphi_2 + \sin \varphi_1 \cdot \cos \varphi_2))$$

$$= |z_1| \cdot |z_2| \cdot (\cos(\varphi_1 + \varphi_2) + i \sin(\varphi_1 + \varphi_2))$$

proizvod zimnoži absolute vrednosti in sešteja kote

## Eulerjeva formula

$$e^{i\varphi} = \cos \varphi + i \sin \varphi$$

Polarni zapis

$$z = r \cdot e^{i\varphi}$$

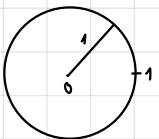
$$r_1 \cdot e^{i\varphi_1} \cdot r_2 \cdot e^{i\varphi_2} = r_1 \cdot r_2 \cdot e^{i(\varphi_1 + \varphi_2)}$$

absolutna vrednost

$$e^{i\pi} = -1$$

$$\text{Primer: } \{e^{i\varphi}, \varphi \in \mathbb{R}\}$$

središče v  $(0,0)$   
polmer je 1



$$\{r \cdot e^{i\frac{\pi}{2}}, r \in [0, \infty)\}$$



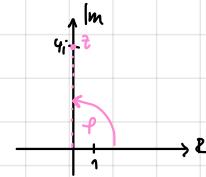
$$z = 4i = 4e^{i\frac{\pi}{2}}$$

$$x = 0$$

$$y = 4$$

$$r = 4$$

$$\varphi = \frac{\pi}{2}$$



## Lastnosti polarnega zapisa

$$1. x_1 + iy_1 = x_2 + iy_2 \Leftrightarrow x_1 = x_2 \wedge y_1 = y_2$$

$$r_1 e^{i\varphi_1} = r_2 e^{i\varphi_2} \Leftrightarrow r_1 = r_2 \wedge (\varphi_1 - \varphi_2) \in k \cdot 2\pi, k \in \mathbb{Z}$$

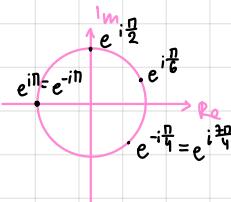
Ali

$$2. \overline{r_1 e^{i\varphi}} = r_1 e^{-i\varphi} \quad r_1 = r_2 = 0$$

$$3. (r \cdot e^{i\varphi})^n = r^n \cdot e^{in\varphi} \text{ de Moivrejeva formula}$$

$$4. (r e^{i\varphi})^{-1} = r^{-1} \cdot e^{-i\varphi} \quad r \neq 0$$

$$5. \frac{r_1 e^{i\varphi_1}}{r_2 e^{i\varphi_2}} = \frac{r_1}{r_2} e^{i(\varphi_1 - \varphi_2)} \quad r_2 \neq 0$$



## Primer:

$$z = 1 - i\sqrt{3}$$

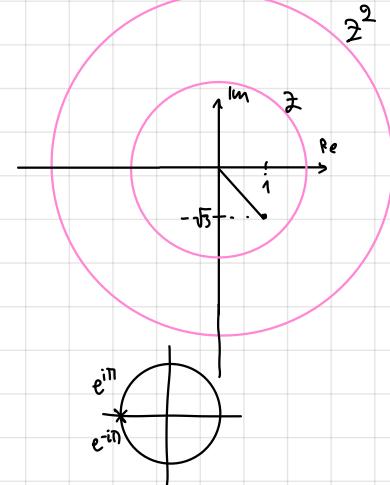
navisemo  $z, z^2, z^3, \dots$

$$x = 1$$

$$y = -\sqrt{3}$$

$$r = 2 = \sqrt{x^2 + y^2}$$

$$\varphi = \frac{\pi}{3}$$



## Geometrijske operacije v ravnini

### preslikava

$$z \mapsto \bar{z}$$

$$z \mapsto -z$$

$$z \mapsto z + z_0$$

$$z \mapsto z \cdot e^{i\varphi}$$

$$z \mapsto z \cdot r \cdot e^{i\varphi}$$

### transformacija v $\mathbb{C}$

zrcaljenje preko  $\text{Re}$  (premica)

zrcaljenje preko  $z=0$  (točka)

premik za  $z_0$

rotacija za  $\varphi$

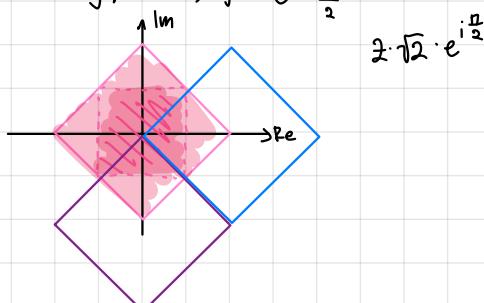
razteg  $\times r$ , rotacija  $+\varphi$

Primer: V kaj se s preslikavo  $z \mapsto (z(1+i) - 2i)$  menjata Re in Im  $\Rightarrow$  obrne za  $90^\circ$

preslikava:

$$\{x+iy, |x| < 1, |y| < 1\}$$

polarna vrednost  $\sqrt{2}$ , kot pa  $\frac{\pi}{2}$



$$2\sqrt{2} \cdot e^{i\frac{\pi}{2}}$$

## Korenji enote

Primer:  $z^5 = 1 = 1 \cdot e^{i \cdot 0} = 1 \cdot e^{i \cdot \frac{\pi}{4}}$   
 $z = r \cdot e^{i\varphi}$

$$\frac{r^5 \cdot e^{i5\varphi}}{r^5} = \frac{1 \cdot e^{i0}}{1} = e^{i2\pi} = e^{i4\pi}$$

$$r^5 = 1 \quad e^{i5\varphi} = e^{i0}$$

$$r = 1$$

$$\textcircled{1} 5\varphi = 0 + \frac{\pi}{4} \quad \textcircled{2} 5\varphi = 2\pi$$

$$\varphi = 0 \frac{\pi}{20}$$

$$\varphi = \frac{2\pi}{5} + \frac{\pi}{20}$$

$$\textcircled{3} 5\varphi = 4\pi$$

$$\varphi = \frac{4\pi}{5} + \frac{\pi}{20}$$

5 različnih rešitev

$$z_1 = 1 \cdot e^{i0} = 1$$

$$z_2 = e^{i\frac{2\pi}{5}}$$

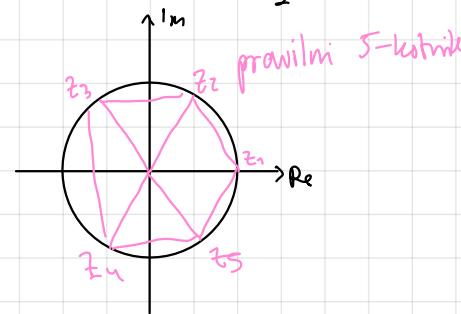
$$z_3 = e^{i\frac{4\pi}{5}}$$

$$z_4 = e^{i\frac{6\pi}{5}}$$

$$z_5 = e^{i\frac{8\pi}{5}}$$

$$z_6 = e^{i\frac{10\pi}{5}}$$

začnejo se ponavljati



## FORMULE ZA KORENJENJE

$$n \in \mathbb{N}, z^n = r \cdot e^{i\varphi} \quad k = 0, \dots, n-1$$

$$z_k = \sqrt[n]{r} \cdot e^{i\left(\frac{\varphi}{n} + \frac{2k\pi}{n}\right)}$$

$n$  različnih, razen pri  $r=0$

Pravilni  $n$ -kotnik

Primer:  $(z^3 - 2) \cdot (z^2 + i) = 0$

$$\textcircled{1} z^3 = 2 \quad \varphi = 0$$

$$z_1 = \sqrt[3]{2} \cdot e^{i\frac{0}{3}} \quad \text{realna rešitev (črna morebit)}$$

$$z_2 = \sqrt[3]{2} \cdot e^{i\frac{2\pi}{3}}$$

$$z_3 = \sqrt[3]{2} \cdot e^{i\frac{4\pi}{3}}$$

popolni  $\Delta$

$$\varphi = -\frac{\pi}{2}$$

$$\textcircled{2} z^2 = -i = 1 \cdot e^{-i\frac{\pi}{2}}$$

$$z_4 = 1 \cdot e^{i\left(\frac{\pi}{2}\right)}$$

$$z_5 = 1 \cdot e^{i\left(\frac{\pi}{2} + 2\pi\right)} = -z_4$$

popolna  $\Delta$

## OSNOVNI IZREK ALGEBRE

Vsiaka enačba  $a_k \in \mathbb{C}, a_n \neq 0$

$$a_n \cdot z^n + a_{n-1} \cdot z^{n-1} + \dots + a_1 \cdot z + a_0 = 0$$

ima natanko  $n$  rešitev

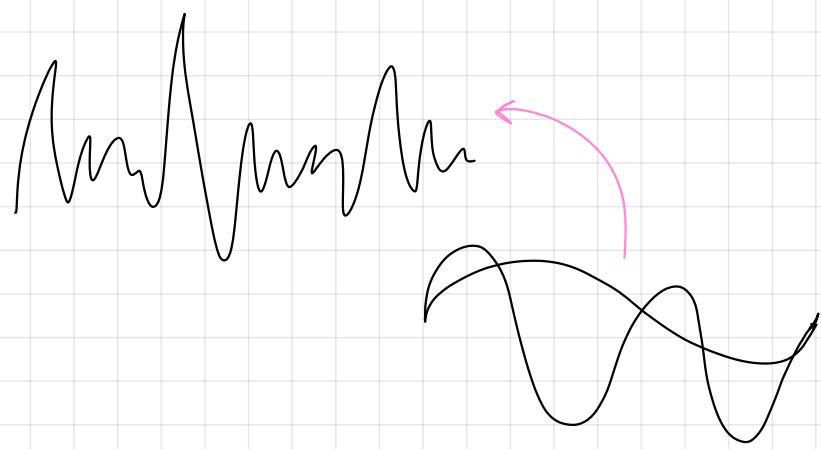
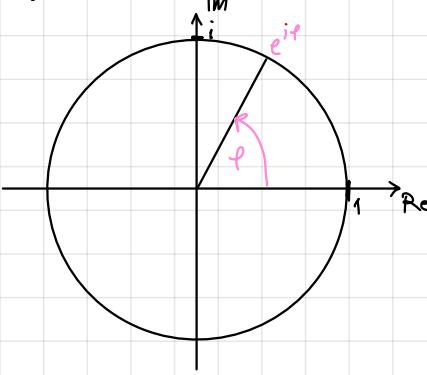
$$\varphi(z) = a_n (z - z_n)^{\frac{1}{n}} (z - z_2)^{\frac{1}{n}} \dots (z - z_n)^{\frac{1}{n}}$$

$$v_1 + \dots + v_m = r$$

Če so  $a_k \in \mathbb{R}, \forall k$

potem ne-realne nizke

nastopajo v konjugiranih parih



## Diskretna Fourierjeva transformacija - DFT

"nek signal"  $\rightarrow$  kaj visok smo pri  $\omega$

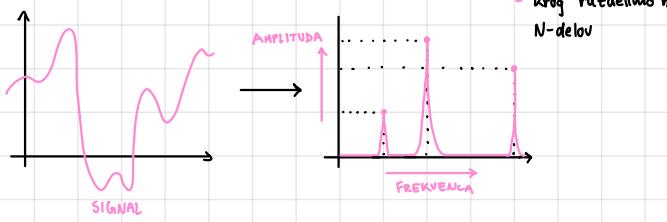
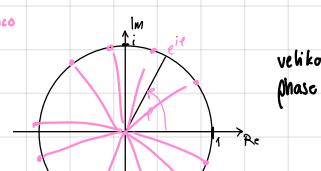
$(x_0, x_1, \dots, x_N) \rightsquigarrow (y_0, y_1, \dots, y_N)$   $\leftarrow$  zaporedje frekvenc  
čaporedje jakosti

$$Y_k = \sum_{n=0}^{N-1} x_n e^{-i \frac{2\pi}{N} k \cdot n}$$

kolikorat se valj i frekvenco

klikorat gre po gor pa dol

velikost vala - abs. vred.  
faz - kot



## ZAPOREDJE

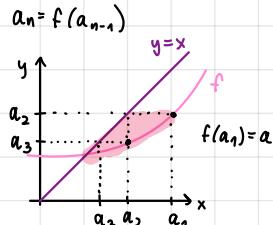
Def: preslikava  $\mathbb{N} \rightarrow \mathbb{R}$   $(a_n) = (a_1, a_2, \dots)$

Podajanje zaporedja:

• EKSPlicitno:  $a_n = f(n)$

• REKURZIVNO:  $a_n = f(a_{n-1}, a_{n-2})$  2-člen  
+ začetni člen  
 $a_n = f(a_{n-1})$  1-člen

GRAFICKI PRIKAZ REKURZIVNEGA ZAP.



Primeri:  $a_n = (-1)^n$

• Aritmetično zap.

$$a_n = a_0 + n \cdot d$$

$$a_n = a_0 + n \cdot d$$

• Geometrično zap.

$$a_n = a_0 \cdot q^n$$

$$a_n = q \cdot a_{n-1}$$

• Fibonaccijevo zap.

$$a_1 = 1 = a_2$$

$$a_n = a_{n-1} + a_{n-2}$$

• Collatzova domneva (1937)

$$a_{n+1} = \begin{cases} \frac{a_n}{2}; & a_n \text{ sodo} \\ 3a_n + 1; & a_n \text{ loba} \end{cases}$$

Ali se za  $\forall a_0 \in \mathbb{N}$  to zap. vrne v 1?

↳ Preverjeno do  $2^{34}$

Lastnosti zaporedja:  $(a_n)_n$  je

- Navzgor omejena, če obstaja zgornja meja  $M \in \mathbb{R}$ :  $\forall n \in \mathbb{N} \wedge a_n \leq M$   
Supremum  $\sup_{n \in \mathbb{N}} a_n =$  najmanjša zgornja meja
- Navzdol omejeno, če obstaja spodnja meja  $m \in \mathbb{R}$ :  $\forall n \in \mathbb{N} \wedge a_n \geq m$   
Infimum  $\inf_{n \in \mathbb{N}} a_n =$  najvišja spodnja meja
- Omejeno, če je navzgor in navzdol omejeno
- Naraščajoče, če  $\forall n: a_{n+1} \geq a_n$   
strago naraščajoče, če  $\forall n: a_{n+1} > a_n$
- Padajoče, če  $\forall n: a_{n+1} \leq a_n$   
strago padajoče, če  $\forall n: a_{n+1} < a_n$
- Monoton, če je povsod ali padajoče ali naraščajoče

Primeri:  $n = 1, 2, 3, \dots$

1.  $\frac{n^2-1}{n} = n - \frac{1}{n} \downarrow$  omejeno,  $\uparrow$  neomejeno,  $\uparrow$  strago

2.  $b_n = \frac{b_{n-1}}{2}$  2.1  $b_1 > 0$  2.2  $b_1 < 0$  2.3  $b_1 = 0$   
strago padajoče strago naraščajoče konst. v 0  
 $\inf_n b_n = 0$   $\sup_n b_n = 0$  OBOJE DRŽI

## LIMITA ZAPOREDJA

Def:  $a \in \mathbb{R}$  je limita zap.  $(a_n)_n$ , če

$\forall \varepsilon > 0 \exists N \in \mathbb{N}$

$\forall n > N \quad |a_n - a| < \varepsilon$

Zaporedje je konvergentno, če ima limito

Zaporedje je divergentno, če nima limite

### Zaporedje $(a_n)_n$

- Narašča preko vseh meje  $[\lim_{n \rightarrow \infty} a_n = \infty] \leftarrow$  ni konv.
- $\forall M \in \mathbb{R} \exists N \in \mathbb{N}: a_n > M, \forall n \geq N$
- Pada pod vseh mejo  $[\lim_{n \rightarrow \infty} a_n = -\infty] \leftarrow$
- $\forall m \in \mathbb{R} \exists N \in \mathbb{N}: a_n < m, \forall n \geq N$

Primeri: 1.  $a_n = \frac{1}{n^2}$

Dokažimo, da je  $\lim_{n \rightarrow \infty} a_n = 0$

$n^2, \varepsilon > 0$

Izberemo  $\varepsilon > 0$  po def.

$$\frac{1}{n^2} < \varepsilon / \frac{n^2}{\varepsilon}$$

$$n > \sqrt{\frac{1}{\varepsilon}} \in \mathbb{N}$$

$x > 0$

$$\lceil x \rceil = \min \{ n \in \mathbb{N}_0; n \geq x \}$$

$$\lfloor x \rceil = \max \{ n \in \mathbb{N}; n \leq x \}$$

2.  $b_n = (-1)^n$

Ali je 0 limita?

Izberemo  $\varepsilon > 0$

$$|(-1)^n - 0| < \varepsilon$$

$1 < \varepsilon$

Ni okej za  $\varepsilon < 1$

R: NI LIMITA

### Pravila za računski limiti

Naj bo  $a = \lim_{n \rightarrow \infty} a_n, b = \lim_{n \rightarrow \infty} b_n$

$$1. \lim_{n \rightarrow \infty} (a_n + b_n) = a + b$$

$$2. (a_n \cdot b_n) = a \cdot b$$

3. V smislu naraščanja in padačja preko vseh mej:

$$\infty + \infty = \infty$$

$$5 \cdot \infty = \infty$$

$$\infty \cdot \infty = \infty$$

$$-\infty \cdot \infty = -\infty$$

$$\frac{1}{\infty} = 0$$

**MANJKA:** 20.-26.10.

Predavanja: Pravila za računanje z limitami, izrek o sendviču, izrek o monotoni konvergenci, e kot limita zaporedja  $a_n = (1 + \frac{1}{n})^n$ . Vrste - delne vsote, konvergencia, geometrijska vrsta.

## Pravila za računanje vrst

Naj bosta  $\sum_{n=1}^{\infty} a_n, \sum_{n=1}^{\infty} b_n$  konvergentni

Tedaj konvergirata:

$$\sum_{n=1}^{\infty} (a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$$

$$c \cdot \sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} c a_n$$

Primer:

$$a) \sum_{n=1}^{\infty} \left( \left( \frac{1}{2} \right)^n - \left( \frac{1}{3} \right)^n \right) = \sum_{n=1}^{\infty} \frac{1}{2^n} - \sum_{n=1}^{\infty} \frac{1}{3^n} =$$

$$= \frac{\frac{1}{2}}{1 - \frac{1}{2}} - \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \dots$$

$$b) \sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{n+1} \right) = \sum_{n=1}^{\infty} \frac{1}{n} - \sum_{n=1}^{\infty} \frac{1}{n+1} = \infty - \infty ? = 1$$

Dominiranje vrst  $a_n, b_n > 0$ 

$$\sum_{n=1}^{\infty} a_n \text{ dominira} \sum_{n=1}^{\infty} b_n, \text{ če } a_n \geq b_n, \forall n \Rightarrow S_n^a \geq S_n^b$$

V tem primeru velja:

Če  $\sum_{n=1}^{\infty} b_n$  div., potem tudi  $\sum_{n=1}^{\infty} a_n$  divergira

$\sum_{n=1}^{\infty} a_n$  konvergira  $\Rightarrow \sum_{n=1}^{\infty} b_n$  konv.

Primer:

$$a) \sum_{n=1}^{\infty} \frac{(\sin(e^{6x^2} - 8\pi^2))^6}{2^n}$$

$$\sum_{n=1}^{\infty} \frac{(\sin(e^{6x^2} - 8\pi^2))^6}{2^n} \leq \frac{1}{2^n}$$

konvergira

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n} \geq \sum_{n=1}^{\infty} \frac{1}{n}$$

divergira

vezja  $\rightarrow$  konv.  
mangja  $\rightarrow$  div.

Konvergenčni kriterij:  $a_n > 0, \forall n$

① Kvocientni kriterij

$$a_n = g^n$$

$$\sum_{n=1}^{\infty} g^n \begin{cases} \text{konv. } |g| < 1 \\ \text{div. } |g| > 1 \end{cases}$$

$$g = \frac{a_{n+1}}{a_n}$$

$$D_n = \frac{a_{n+1}}{a_n}, \text{ naj obstaja } \lim_{n \rightarrow \infty} D_n \in \mathbb{R}$$

$$D > 1 \Rightarrow \text{vrsta divergira } (\sum_{n=1}^{\infty} a_n = \infty)$$

$$D < 1 \Rightarrow \text{vrsta konvergira}$$

$$D = 1 \Rightarrow \neg(\text{konv.}) \neg(\text{div.})$$

Primer: a) 2a. katero  $X > 0$  vrsta konv.

$$\sum_{n=1}^{\infty} n \cdot X^n ?$$

$$\lim_{n \rightarrow \infty} \frac{n+1}{n}$$

$$a_n = n \cdot X^n$$

$$D_n = \frac{(n+1)X^{n+1}}{n \cdot X^n} = \frac{n+1}{n} \cdot X \xrightarrow{n \rightarrow \infty} X$$

konv. za  $X < 1$

div. za  $X > 1$

$$b) \sum_{n=1}^{\infty} \frac{X^n}{n!}$$

$$a_n = \frac{X^n}{n!}$$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{X^{n+1}}{(n+1)!}}{\frac{X^n}{n!}} = \frac{X}{n+1} \xrightarrow{n \rightarrow \infty} 0$$

konv za  $X > 0$

② Korenski kriterij

$$D_n = \sqrt[n]{a_n}, \text{ naj obstaja } \lim_{n \rightarrow \infty} D_n \in \mathbb{R}$$

$$D > 1 \Rightarrow \text{vrsta divergira } (\sum_{n=1}^{\infty} a_n = \infty)$$

$$D < 1 \Rightarrow \text{vrsta konvergira}$$

$$D = 1 \Rightarrow \neg(\text{konv.}) \neg(\text{div.})$$

Primer:

$$\sum_{n=1}^{\infty} \left( \frac{X}{n} \right)^n$$

$$a_n = \left( \frac{X}{n} \right)^n$$

$$\sqrt[n]{a_n} = \frac{X}{n} \xrightarrow{n \rightarrow \infty} 0$$

konv. za  $X > 0$

③ Leibnitzov kriterij

$$a_n \xrightarrow{n \rightarrow \infty} 0 \text{ monotono (tj.: } a_{n+1} \leq a_n, \forall n)$$

$$\Rightarrow \sum_{n=1}^{\infty} (-1)^n a_n \text{ konvergira}$$

Primer:

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ div.}$$

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} \text{ div.}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \dots = \infty$$

$$-1 - \frac{1}{3} - \frac{1}{5} - \frac{1}{7} - \dots = -\infty$$

! VESTNI RED SEŠT. JE TOHOBEN!

$$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n} \text{ konv.}$$

**Limita funkcije:**

$$\textcircled{1} \lim_{x \rightarrow a} f(x) = b$$

$$\forall \varepsilon > 0 \exists \delta > 0$$

$$|x-a| < \delta \Rightarrow |f(x)-b| < \varepsilon$$

$$x \neq a$$

$$a-\delta \ x \ a+\delta \quad b-\varepsilon \ x \ b+\varepsilon$$

$$\text{Leva limita} \quad \textcircled{2} \lim_{\substack{x \rightarrow a \\ x < a}} f(x) = b$$

$$\forall \varepsilon > 0 \exists \delta > 0$$

$$x \in (a-\delta, a) \Rightarrow |f(x)-b| < \varepsilon$$

$$\text{Desna limita} \quad \textcircled{3} \lim_{\substack{x \rightarrow a \\ x > a}} f(x) = b$$

$$\lim_{x \rightarrow a^+} f(x) = b$$

$$\forall \varepsilon > 0 \exists \delta > 0$$

$$x \in (a, a+\delta) \Rightarrow |f(x)-b| < \varepsilon$$

$$\textcircled{4} \lim_{x \rightarrow \infty} f(x) = b$$

$$\forall \varepsilon > 0 \exists M > 0$$

$$x > M \Rightarrow |f(x)-b| < \varepsilon$$

$$\textcircled{5} \lim_{x \rightarrow -\infty} f(x) = b$$

$$\forall \varepsilon > 0 \exists m \in \mathbb{R}$$

$$x < m \Rightarrow |f(x)-b| < \varepsilon$$

$$\textcircled{6} \lim_{x \rightarrow a} f(x) = \infty$$

$$\forall N > 0 \exists \delta > 0$$

$$x \in (a-\delta, a+\delta) \setminus \{a\} \Rightarrow f(x) > N$$

$$\textcircled{7} \lim_{x \rightarrow a^-} f(x) = \infty$$

$$\textcircled{8} \lim_{x \rightarrow a^+} f(x) = \infty$$

$$\textcircled{9} \lim_{x \rightarrow \infty} f(x) = \infty$$

$$\forall N \exists M$$

$$x > M \Rightarrow f(x) > N$$

$$\textcircled{10} \lim_{x \rightarrow -\infty} f(x) = \infty$$

$$\textcircled{11} \lim_{x \rightarrow \infty} f(x) = -\infty$$

$$\textcircled{12} \lim_{x \rightarrow -\infty} f(x) = -\infty$$

Trditve:

Limita  $f$  v  $a$  obstaja, natančno takrat

ko obstajata ujemajoči se leva in desna limita.

Primer: 1.  $f(x) = \text{sgn}(x)$

$$2. f(x) = \frac{1}{1+e^{\frac{1}{x}}}$$

$$\lim_{x \rightarrow 0^-} \frac{1}{1+e^{\frac{1}{x}}} = 0$$

$$\lim_{x \rightarrow 0^+} \frac{1}{1+e^{\frac{1}{x}}} = 0$$

$$\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$$

$$\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$$

$$\lim_{t \rightarrow \infty} e^{-t} = 0$$

$$\lim_{t \rightarrow \infty} e^t = \infty$$

$$\lim_{s \rightarrow 1} \frac{1}{s} = 1$$

$$\lim_{s \rightarrow \infty} \frac{1}{s} = 0$$

$$\lim_{x \rightarrow 0} \frac{1}{1+e^{\frac{1}{x}}} = 1$$

Diagram illustrating the behavior of the function  $f(x) = \frac{1}{1+e^{\frac{1}{x}}}$  as  $x \rightarrow 0$ . The function is shown as a blue curve that approaches the x-axis ( $y=0$ ) from above as  $x \rightarrow 0^-$  and the y-axis ( $x=0$ ) from below as  $x \rightarrow 0^+$ . The point  $(0, 1)$  is marked on the curve. The region near the origin is shaded in light blue. The labels  $1+0=1$  and  $0+0=0$  are shown near the x-axis, and  $e^{+\infty} = \infty$  and  $e^{-\infty} = 0$  are shown near the y-axis. The text "blizu se 0" is written at the bottom right.

## Lastnosti limit funkcij

LASTNOSTI SO ISTE KOT  
PRI ŽAPOREDJIH

$x, b \in \mathbb{R} \cup \{\pm \infty\}$ ; tedaj velja

$$\lim_{x \rightarrow a} f(x) = f(a)$$

Primer:

$$1. \lim_{x \rightarrow \infty} \sin \frac{1}{x} = \lim_{t \rightarrow 0} \sin t = 0$$

$$2. \lim_{x \rightarrow 0} \sin \frac{1}{x} = \infty$$

$$3. \lim_{x \rightarrow \frac{\pi}{2}} \sin \frac{1}{x} = 1$$

Dokaz za  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad (x > 0)$



$$\begin{aligned} \sin x &< x < \tan x \\ \frac{1}{\sin x} &> \frac{1}{x} > \frac{\cos x}{\sin x} \\ 1 &> \frac{\sin x}{x} > \cos x \\ \downarrow & \quad \downarrow & \quad \downarrow \\ 1 & > 1 & = 1 \end{aligned}$$

sklep

## Zveznost funkcije

Funkcija  $f$  je zvezna.

$$\forall a \in D_f, \text{ če } \lim_{x \rightarrow a} f(x) = f(a)$$

LASTNOSTI ZVEZNOSTI

1. Vse elementarne funkcije so zvezne.

2. Če sta  $f$  in  $g$  zvezni v  $a$ :

→ grafa sta v  $a$  neprekiniti krivulji

→ so zvezne  $f + g, g, \sin(g)$  in  $a$

→ lahko zamenjamo vrstni red funk. in limite v  $a$

3.  $f$  zvezna v  $a$ ,  $g$  zvezna v  $f(a)$   $\Rightarrow g \circ f$  zvezna v  $a$

4. Če  $\lim_{x \rightarrow b} f(x)$  obstaja,  $b \in D_f$ , lahko  $f$  s predpisom

$$f(b) = \lim_{x \rightarrow b} f(x)$$

zvezno razširimo na  $b$

5. Če je  $f$  zvezna na intervalu  $[a, b]$

## MANJKA:

ničle zveznih funk.

slika omejjenega zaprt. intervala