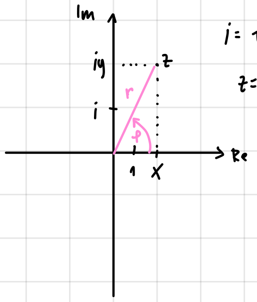
6.10.

Kartezični zapis



$i = \sqrt{-1}$

$z = x + iy$

Polarni zapis

$z = r(\cos\varphi + i\cdot\sin\varphi)$

$\varphi \in \mathbb{R}$ ni enolično določen → do $+k2\pi$ natančno, $k \in \mathbb{Z}$; pri $z=0$ je kot karkoli

$r = |z| \in [0,\infty)$ je enolično določen

argument, polarni kot

$x = r\cdot\cos\varphi$

$y = r\cdot\sin\varphi$

$\tan\varphi = \frac{y}{x} \Rightarrow \varphi = \arctan\frac{y}{x}$

upoštevamo kvadrant!

Primer: Zapiši v polarni obliki

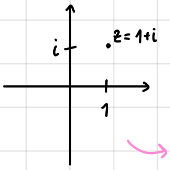
$z = 1+i$

$x = 1$ $\quad r = \sqrt{1^2+1^2} = \sqrt{2}$ (pitagorov)

$y = 1$

$\tan\varphi = \frac{1}{1} = 1$

$\varphi = \frac{\pi}{4} + k\pi, k\in\mathbb{Z}$



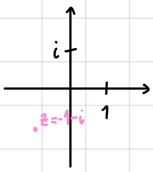
$\varphi \in \left\{\frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4} \ldots \frac{-3\pi}{4}, \frac{-7\pi}{4} \ldots\right\}$

kvadrant → zadosten odg.

$\boxed{\varphi = \frac{\pi}{4}}$

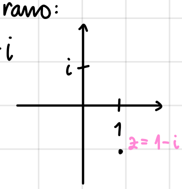
Če bi imeli $z = -1-i$

$\boxed{\varphi = \frac{5\pi}{4}, r = \sqrt{2}}$



Konjugiramo:

$\bar{z} = 1-i$

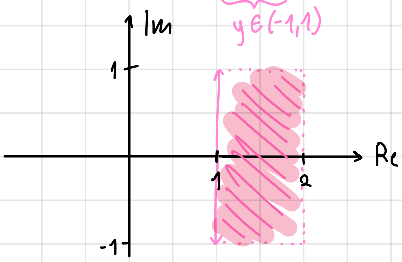


$r = \sqrt{2}$

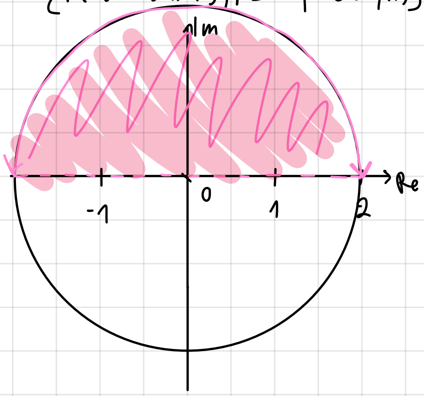
$\varphi = -\frac{\pi}{4}$

Primer: Nariši naslednjo množico

$\{x+iy;\ x\in[1,2), |y|<1\}$ — $y \in (-1,1)$



$\{r(\cos\varphi + i\sin\varphi), r \le 2, \varphi \in (0,\pi)\}$



Množenje v polarnem zapisu

$z_j = |z_j|\cdot(\cos\varphi_j + \sin\varphi_j) \quad j\in\{1,2\}$

$z_1\cdot z_2 = |z_1|\cdot(\cos\varphi_1 + \sin\varphi_1)\cdot|z_2|(\cos\varphi_2 + \sin\varphi_2)$

$= |z_1|\cdot|z_2|\left(\cos\varphi_1\cdot\cos\varphi_2 - \sin\varphi_1\cdot\sin\varphi_2 + i(\cos\varphi_1\cdot\sin\varphi_2 + \sin\varphi_1\cdot\cos\varphi_2)\right)$

$= |z_1|\cdot|z_2|\left(\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2)\right)$

$\boxed{\text{produkt zmnoži absolutne vrednosti in sešteje kote}}$

Eulerjeva formula

$e^{i\pi}=-1$

$\boxed{e^{i\varphi}=\cos\varphi+i\sin\varphi}$

Polarni zapis

$z=r\cdot e^{i\varphi}$

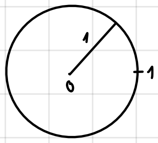
$r_1\cdot e^{i\varphi_1}\cdot r_2\cdot e^{i\varphi_2}=\boxed{r_1\cdot r_2\cdot e^{i(\varphi_1+\varphi_2)}}$

absolutna vrednost

Primer: $\{e^{i\varphi},\varphi\in\mathbb{R}\}$

središče v $(0,0)$

polmer je 1



$\{r\cdot e^{i\frac{\pi}{2}},r\in[0,\infty)\}$

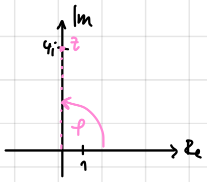
$z=4i=4e^{i\frac{\pi}{2}}$

$x=0$

$y=4$

$r=4$

$\varphi=\frac{\pi}{2}$



Lastnosti polarnega zapisa

1. $x_1+iy_1=x_2+iy_2 \Leftrightarrow x_1=x_2 \wedge y_1=y_2$

$r_1e^{i\varphi_1}=r_2\cdot e^{i\varphi_2}\Leftrightarrow r_1=r_2 \wedge (\varphi_1-\varphi_2)\in k\cdot 2\pi, k\in\mathbb{Z}$

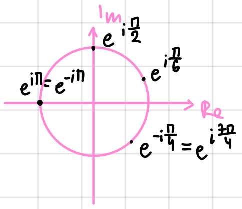
ALI

2. $\overline{r\cdot e^{i\varphi}}=r\cdot e^{-i\varphi}$ $\qquad r_1=r_2=0$

3. $(r\cdot e^{i\varphi})^n=r^n\cdot e^{in\varphi}$ de Moivrejeva formula

4. $(re^{i\varphi})^{-1}=r^{-1}\cdot e^{-i\varphi}$ $\qquad r\neq0$

5. $\frac{r_1 e^{i\varphi_1}}{r_2 e^{i\varphi_2}}=\frac{r_1}{r_2}e^{i(\varphi_1-\varphi_2)}$ $\qquad r_2\neq0$



Primer:

$z=1-i\sqrt{3}$

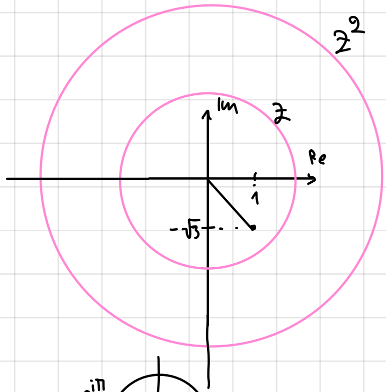
narišemo $z, z^2, z^3\ldots$

$x=1$

$y=-\sqrt{3}$

$r=2=\sqrt{x^2+y^2}$

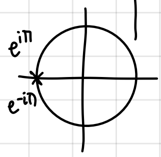
$\varphi=\frac{\pi}{3}$



$z=2\cdot e^{-i\frac{\pi}{3}}$

$z^2=4\cdot e^{-i\frac{2\pi}{3}}$

$z^3=8\ e^{-i\pi}=-8$



Geometrija operacij v ravnini

preslikava	transformacija v $\mathbb{C}$
$z\mapsto\bar{z}$	zrcaljenje preko Re (premica)
$z\mapsto -z$	zrcaljenje preko $z=0$ (točka)
$z\mapsto z+z_0$	premik za z_0
$z\mapsto z\cdot e^{i\varphi}$	rotacija za $+\varphi$
$z\mapsto z\cdot r\cdot e^{i\varphi}$	razteg $\times r$, rotacija $+\varphi$

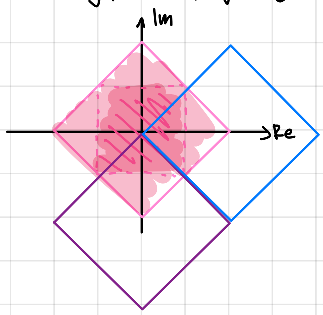
Primer: V kaj se s preslikavo $z\mapsto (z(1+i)-2i)\cdot i$ preslika:

$\{x+iy, |x|<1, |y|<1\}$

menja se Re in Im → obrne za 90°

polarna vrednost je $\sqrt{2}$, kot pa $\frac{\pi}{2}$

$z\cdot\sqrt{2}\cdot e^{i\frac{\pi}{2}}$



Koreni enote

Primer: $z^5 = 1 = 1 \cdot e^{i \cdot 0}$ $= 1 \cdot e^{i \cdot \frac{\pi}{4}}$

$z = r \cdot e^{i\varphi}$

$$r^5 \cdot e^{i 5 \cdot \varphi} = 1 \cdot e^{i0} = e^{i 2\pi + \frac{\pi}{4}} = e^{i 4\pi + \frac{\pi}{4}}$$

$r^5 = 1$ $\qquad e^{i5\varphi} = e^{i \cdot 0}$

$r = 1$

① $5\varphi = 0 + \frac{\pi}{4}$ $\boxed{\varphi = 0 \, \frac{\pi}{20}}$

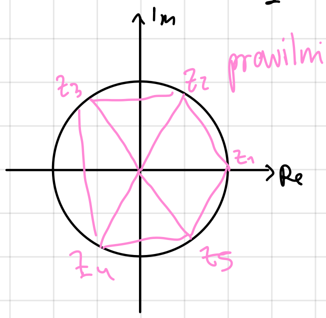
② $5\varphi = 2\pi$ $\boxed{\varphi = \frac{2\pi}{5}} + \frac{\pi}{20}$

③ $5\varphi = 4\pi$ $\boxed{\varphi = \frac{4\pi}{5}} + \frac{\pi}{20}$

$\boxed{\text{5 različnih rešitev}}$

$z_1 = 1 \cdot e^{i \cdot 0} = 1$ $\quad z_2 = e^{i \frac{2\pi}{5}}$ $\quad z_3 = e^{i \frac{4\pi}{5}}$ $\quad z_4 = e^{i \frac{6\pi}{5}}$ $\quad z_5 = e^{i \frac{8\pi}{5}}$ $\quad z_6 = e^{i \frac{10\pi}{5}} = z_1$

začnejo se ponavljat



pravilni 5-kotnik

FORMULE ZA KORENJENJE

$n \in \mathbb{N}$, $z^n = r \cdot e^{i\varphi}$ $\qquad k = 0, \ldots, n-1$

$$z_k = \sqrt[n]{r} \cdot e^{i\left(\frac{\varphi}{n} + \frac{2k\pi}{n}\right)} = \sqrt[n]{r} \cdot e^{i\left(\frac{\varphi + 2k\pi}{n}\right)}$$

$\boxed{\text{n različnih, razen pri } r = 0}$

Pravilni n-kotnik

Primer: $(z^3 - 2) \cdot (z^2 + i) = 0$

① $z^3 = 2$ $\qquad \varphi = 0$

$z_1 = \sqrt[3]{2} \cdot e^{i \frac{0}{3}}$ → realna rešitev (ena more bit)

$z_2 = \sqrt[3]{2} \cdot e^{i \frac{0 + 2\pi}{3}}$

$z_3 = \sqrt[3]{2} \cdot e^{i \frac{0 + 4\pi}{3}}$

popolni △

② $z^2 = -i = 1 \cdot e^{-i\frac{\pi}{2}}$ $\qquad \varphi = -\frac{\pi}{2}$

$z_4 = 1 \cdot e^{i\left(\frac{-\frac{\pi}{2}}{2}\right)}$

$z_5 = 1 \cdot e^{i\left(\frac{-\frac{\pi}{2} + 2\pi}{2}\right)} = -z_4$

popolna —

OSNOVNI IZREK ALGEBRE

Vsaka enačba $\qquad a_k \in \mathbb{C}, a_n \neq 0$

$$a_n \cdot z^n + a_{n-1} \cdot z^{n-1} + \ldots + a_1 \cdot z + a_0 = 0$$

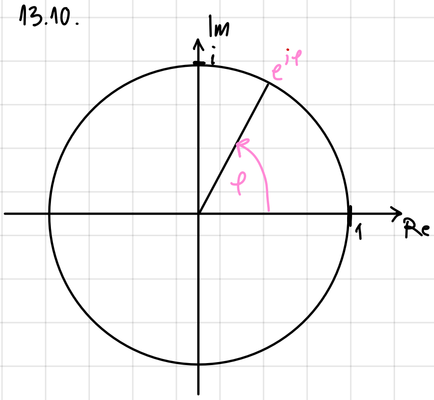
ima natanko n rešitev

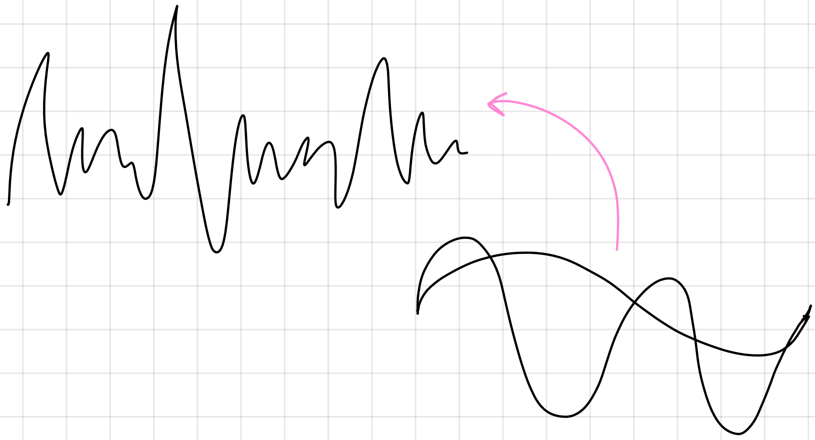
$$p(z) = a_n (z - z_1)^{v_1} \cdot (z - z_2)^{v_2} \cdot \ldots \cdot (z - z_n)^{v_m}$$

$$v_1 + \ldots + v_m = n$$

Če so $a_k \in \mathbb{R}, \forall k$
potem ne-realne ničle
nastopajo v konjugiranih parih

13.10.





Diskretna Fourierjeva transformacija – DFT

"nek signal" → $(x_0, x_1, \dots, x_N)$ ⇝ $(y_0, y_1, \dots, y_N)$ ← zaporedje frekvenc

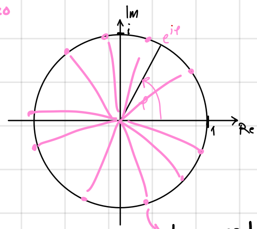
zaporedje jakosti

kuk visok smo pri

kolikokrat se valj s frekvenco 1

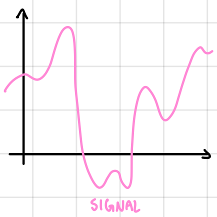
$$y_K = \sum_{n=0}^{N-1} x_n \cdot e^{-i\frac{2\pi}{N} \cdot k \cdot n}$$

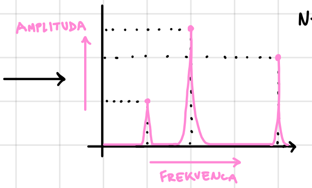
kulkrat gre gor pa dol



velikost vala – abs. vred.
phase – kot

krog razdelimo na N-delov





ZAPOREDJE

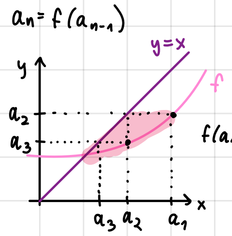
Def: preslikava $\mathbb{N} \to \mathbb{R}$, $n \to a_n$ $\quad (a_n) = (a_1, a_2 \dots)$

Podajanje zaporedja:

- EKSPLICITNO: $a_n = f(n)$
- REKURZIVNO: $a_n = f(a_{n-1}, a_{n-2})$ 2-člena + ZAČETNI ČLEN
  $a_n = f(a_{n-1})$ 1-člen

GRAFIČNI PRIKAZ REKURZIVNEGA ZAP.

$a_n = f(a_{n-1})$



Primeri: $a_n = (-1)^n$

- Aritmetično zap.
  $a_n = a_0 + n \cdot d$
  ali
  $a_n = a_{n-1} + d$
- Geometrijsko zap.
  $a_n = a_0 \cdot q^n$
  ali
  $a_n = q \cdot a_{n-1}$
- Fibonaccijevo zap.
  $a_1 = 1 = a_2$
  $a_n = a_{n-1} + a_{n-2}$
- Collatzova domneva (1937)
  $$a_{n+1} = \begin{cases} \frac{a_n}{2}; & a_n \text{ sodo} \\ 3 \cdot a_n + 1; & a_n \text{ liha} \end{cases}$$
  Ali se za $\forall a_0 \in \mathbb{N}$ to zap. vrne v 1?
  ↳ Preverjeno do 2^{71}

Lastnosti zaporedja: $(a_n)_n$ je

- **Navzgor omejeno**, če obstaja zgornja meja $M \in \mathbb{R}: \forall n \in \mathbb{N} \wedge a_n \leq M$
  Supremum $\sup_{n \in \mathbb{N}} a_n$ = najmanjša zgornja meja
- **Navzdol omejeno**, če obstaja spodnja meja $m \in \mathbb{R}: \forall n \in \mathbb{N} \wedge a_n \geq m$
  Infimum $\inf_{n \in \mathbb{N}} a_n$ = najvišja spodnja meja
- **Omejeno**, če je navzgor in navzdol omejeno
- **Naraščajoče**, če $\forall n: a_{n+1} \geq a_n$
  strogo naraščajoče, če $\forall n$ $a_{n+1} > a_n$
- **Padajoče**, če $\forall n: a_{n+1} \leq a_n$
  strogo padajoče, če $\forall n$ $a_{n+1} < a_n$
- **Monotono**, če je povsod ali padajoče ali naraščajoče

Primeri: $n = 1, 2, 3 \dots$

1. $\frac{n^2-1}{n} = n - \frac{1}{n}$ ↓ omejeno, ↑ neomejeno, ↑ strogo
2. $b_n = \frac{b_{n-1}}{2}$
   2.1 $b_1 > 0$ strogo padajoče, $\inf_n b_n = 0$
   2.2 $b_1 < 0$ strogo naraščajoče, $\sup_n = 0$
   2.3 $b_1 = 0$ konst. v 0, OBOJE DRŽI

LIMITA ZAPOREDJA

Def: $a \in \mathbb{R}$ je limita zap. $(a_n)_n$, če
$\forall \varepsilon > 0 \quad \exists N \in \mathbb{N}$
$\forall n > N \quad |a_n - a| < \varepsilon$

Zaporedje je konvergentno, če ima limito
Zaporedje je divergentno, če nima limite

Zaporedje $(a_n)_n$

- Narašča preko vsake meje $[\lim_{n\to\infty} a_n = \infty]$ ni konv. zap.!
  $\forall M \in \mathbb{R} \;\exists N \in \mathbb{N}: a_n > M, \;\forall n \geq N$
- Pada pod vsako mejo $[\lim_{n\to\infty} a_n = -\infty]$
  $\forall m \in \mathbb{R} \;\exists N \in \mathbb{N}: a_n < m, \;\forall n \geq N$

Pravila za rač. limit

Naj bo $a = \lim_{n\to\infty} a_n$, $b = \lim_{n\to\infty} b_n$

1 $\lim_{n\to\infty} (a_n + b_n) = \quad a_n + \quad b_n$

2 $(a_n \cdot b_n) = \quad a_n \cdot \quad b_n$

3 V smislu naraščanja in padanja preko vseh mej:
$\infty + \infty = \infty$
$5 \cdot \infty = \infty$
$\infty \cdot \infty = \infty$
$-\infty \cdot \infty = -\infty$
$\frac{1}{\infty} = 0$

Primeri:

1. $a_n = \frac{1}{n^2}$

Dokažimo, da je $\lim_{n\to\infty} a_n = 0$

$n^2, \varepsilon > 0$

Izberemo $\varepsilon > 0$ po def.

$\frac{1}{n^2} < \varepsilon \;/ \cdot \frac{n^2}{\varepsilon}$

$\frac{1}{\varepsilon} < n^2$

R: $n > \left\lceil \sqrt{\frac{1}{\varepsilon}} \right\rceil \in \mathbb{N}$

$x > 0$
$\lceil x \rceil = \min \{ n \in \mathbb{N}_0;\ n \geq x \}$
$\lfloor x \rfloor = \max \{ n \in \mathbb{N};\ n \leq x \}$

2 $b_n = (-1)^n$

Ali je 0 limita?

Izberemo $\varepsilon > 0$

$|(-1)^n - 0| < \varepsilon$

$1 < \varepsilon$

Ni okej za $\varepsilon < 1$!

R: $\boxed{\text{NI LIMITA}}$

MANJKA: 20.-26.10.

Predavanja: Pravila za računanje z limitami, izrek o sendviču, izrek o monotoni konvergenci, e kot limita zaporedja $a_n = (1 + \frac{1}{n})^n$. Vrste - delne vsote, konvergenca, geometrijska vrsta.

27.10.

Pravila za računanje vrst

Naj bosta $\sum_{n=1}^{\infty} a_n, \sum_{n=1}^{\infty} b_n$ konvergentni

Tedaj konvergirata:

$$\sum_{n=1}^{\infty}(a_n+b_n)=\sum_{n=1}^{\infty} a_n+\sum_{n=1}^{\infty} b_n$$

$$c\cdot\sum_{n=1}^{\infty} a_n=\sum_{n=1}^{\infty} c\, a_n$$

Primer:

a) $$\sum_{n=1}^{\infty}\left(\left(\tfrac{1}{2}\right)^n-\left(\tfrac{1}{3}\right)^n\right)=\sum_{n=1}^{\infty}\frac{1}{2^n}-\sum_{n=1}^{\infty}\frac{1}{3^n}=$$

$$=\frac{\frac{1}{2}}{1-\frac{1}{2}}-\frac{\frac{1}{3}}{1-\frac{1}{3}}= \dots$$

b) $$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)=\sum_{n=1}^{\infty}\frac{1}{n}-\sum_{n=1}^{\infty}\frac{1}{n+1}=\cancel{\infty-\infty}\,?=1$$

Dominiranje vrst $a_n, b_n>0$

$\sum_{n=1}^{\infty} a_n$ dominira $\sum_{n=1}^{\infty} b_n$, če $a_n\geq b_n, \forall n \Rightarrow S_n^a\geq S_n^b$

V tem primeru velja:

Če $\sum_{n=1}^{\infty} b_n$ div., potem tudi $\sum_{n=1}^{\infty} a_n$ divergira

$\sum_{n=1}^{\infty} a_n$ konvergira $\Rightarrow \sum_{n=1}^{\infty} b_n$ konv.

Primer:

a) $$\sum_{n=1}^{\infty}\frac{(\sin(e^{6x^3}-8\pi^2))^6}{2^n}$$

$-1<\sin(\ )<1$

$$\underbrace{\sum_{n=1}^{\infty}\frac{(\sin(e^{6x^3}-8\pi^2))^6}{2^n}}_{\Downarrow \text{ konvergira}}\leq\underbrace{\frac{1}{2^n}}_{\text{konvergira}}$$

$$\underbrace{\sum_{n=1}^{\infty}\frac{\ln(n)}{n}}_{\text{divergira}}\geq\underbrace{\sum_{n=1}^{\infty}\frac{1}{n}}_{\text{divergira}}$$

večja $\to$ konv.

manjša $\to$ div.

Konvergenčni kriteriji: $a_n>0, \forall n$

① Kvocientni kriterij

$a_n=q^n$

$$\boxed{\sum_{n=1}^{\infty} q^n\begin{cases}\text{konv.} & |q|<1\\ \text{div.} & |q|>1\end{cases}}$$

$q=\frac{a_{n+1}}{a_n}$

$D_n=\frac{a_{n+1}}{a_n}$, naj obstaja $\lim_{n=\infty} D_n\in\mathbb{R}$ // :D

$D>1\Rightarrow$ vrsta divergira $\left(\sum_{n=1}^{\infty} a_n=\infty\right)$

$D<1\Rightarrow$ vrsta konvergira

$D=1\Rightarrow$ ¯\\(ツ)/¯

Primer: a) Za katere $x>0$ vrsta konv. $\sum_{n=1}^{\infty} n\cdot x^n$?

$a_n=n\cdot x^n$

$$D_n=\frac{(n+1)x^{n+1}}{n\cdot x^n}=\underbrace{\frac{n+1}{n}}_{\lim_{n\to\infty}\frac{n+1}{n}}\cdot x\xrightarrow{n\to\infty}x$$

konv. za $x<1$

div. za $x>1$

b) $\sum_{n=1}^{\infty}\frac{x^n}{n!}$

$a_n=\frac{x^n}{n!}$

$$\frac{a_{n+1}}{a_n}=\frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}}=\frac{x}{n+1}\xrightarrow{n\to\infty}0$$

konv. za vse $x>0$

② Korenski kriterij

$D_n=\sqrt[n]{a_n}$, naj obstaja $\lim_{n=\infty} D_n\in\mathbb{R}$ // :D

$D>1\Rightarrow$ vrsta divergira $\left(\sum_{n=1}^{\infty} a_n=\infty\right)$

$D<1\Rightarrow$ vrsta konvergira

$D=1\Rightarrow$ ¯\\(ツ)/¯

Primer:

$\sum_{n=1}^{\infty}\left(\frac{x}{n}\right)^n$

$a_n=\left(\frac{x}{n}\right)^n$

$\sqrt[n]{a_n}=\frac{x}{n}\xrightarrow{n\to\infty}0$

konv. za $x>0$

③ Leibnitzov kriterij

$a_n\searrow^{n\to\infty} 0$ monotono (tj.: $a_{n+1}\leq a_n, \forall n$)

$\Rightarrow \sum_{n=1}^{\infty}(-1)^n a_n$ konvergira

Primer:

$\sum_{n=1}^{\infty}\frac{1}{n}$ div. $\quad \sum_{n=1}^{\infty}\frac{1}{2n}$ div.

$\sum_{n=1}^{\infty}\frac{1}{2n-1}$ div.

$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\dots=\infty$

$-1-\frac{1}{3}-\frac{1}{5}-\frac{1}{7}\dots=-\infty$

! VRSTNI RED SEŠT. JE POMEMBEN!

$\sum_{n=1}^{\infty}(-1)^n\cdot\frac{1}{n}$ konv.

3.11.

Limita funkcije.

1. $\lim\limits_{x\to a} f(x) = b$

   $\forall \varepsilon > 0 \quad \exists \delta > 0$

   $|x-a| < \delta \Rightarrow |f(x)-b| < \varepsilon$

   $x \neq a$

   $a-\delta \quad a \quad a+\delta \qquad b-\varepsilon \quad b \quad b+\varepsilon$

Leva limita

2. $\lim\limits_{x\nearrow a} f(x) = b$ ALI $x \to a^-$

   $\forall \varepsilon > 0 \quad \exists \delta > 0$

   $x \in (a-\delta, a) \Rightarrow |f(x)-b| < \varepsilon$

Desna limita

3. $\lim\limits_{x\searrow a} f(x) = b$ ALI $x \to a^+$

   $\forall \varepsilon > 0 \quad \exists \delta > 0$

   $x \in (a, a+\delta) \Rightarrow |f(x)-b| < \varepsilon$

4. $\lim\limits_{x\to\infty} f(x) = b$

   $\forall \varepsilon > 0 \quad \exists M > 0$

   $x > M \Rightarrow |f(x)-b| < \varepsilon$

5. $\lim\limits_{x\to-\infty} f(x) = b$

   $\forall \varepsilon > 0 \quad \exists m, m \in \mathbb{R}$

   $x < m \Rightarrow |f(x)-b| < \varepsilon$

6. $\lim\limits_{x\to a} f(x) = \infty$

   $\forall N > 0 \quad \exists \delta > 0$

   $x \in (a-\delta, a+\delta) \setminus \{a\} \Rightarrow f(x) > N$

7. $\lim\limits_{x\to a^-} f(x) = \infty$

8. $\lim\limits_{x\to a^+} f(x) = \infty$

9. $\lim\limits_{x\to\infty} f(x) = \infty$

   $\forall N \quad \exists M$

   $x > M \Rightarrow f(x) > N$

10. $\lim\limits_{x\to-\infty} f(x) = \infty$

11. $\lim\limits_{x\to\infty} f(x) = -\infty$

12. $\lim\limits_{x\to-\infty} f(x) = -\infty$

Trditev:

Limita f v a obstaja natanko takrat ko obstajata ujemajoči se leva in desna limita.

Primer: 1. $f(x) - \operatorname{sgn}(x)$

2. $f(x) = \dfrac{1}{1+e^{\frac{1}{x}}}$

$$\lim_{x\to 0^-} \frac{1}{1+e^{\frac{1}{x}}}$$

$$\lim_{x\to 0^+} \frac{1}{1+e^{\frac{1}{x}}} = 0$$

$$\lim_{x\to 0^-} \frac{1}{x} = -\infty \qquad \lim_{t\to\infty} e^{-t} = 1 \qquad \lim_{s\to 1} \frac{1}{s} = 1$$

$$\lim_{x\to 0^+} \frac{1}{x} = \infty \qquad \lim_{t\to\infty} e^{t} = \infty \qquad \lim_{s\to\infty} \frac{1}{s} = 0$$

$$\lim_{x\nearrow 0} \frac{1}{1+e^{\frac{1}{x}}}$$

$\frac{1}{1} = 1$

$-\infty$

0

$1+0=1$

$e^{-100} = \frac{1}{e^{100}}$

$e^{-1000} = \frac{1}{e^{1000}}$ } → bliža se 0

Lastnosti limit funkcij

LASTNOSTI SO ISTE KOT PRI ZAPOREDJIH

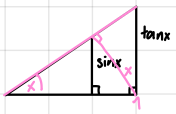
$\alpha, \beta \in \mathbb{R} \cup \{\pm\infty\}$; tedaj velja

$$\lim_{x\to\alpha} f(x) = \beta$$

<u>Primer:</u>

1. $\lim\limits_{x\to\infty} \sin\frac{1}{x} = \lim\limits_{t\to 0} \sin t = 0$
2. $\lim\limits_{x\to 0} \sin\frac{1}{x} = /$
3. $\lim\limits_{x\to\frac{2}{\pi}} \sin\frac{1}{x} = 1$

Dokaz za $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$ $\boxed{x>0}$



$\sin x < x < \tan x$

$\frac{1}{\sin x} > \frac{1}{x} > \frac{\cos x}{\sin x}$

$1 > \frac{\sin x}{x} > \cos x$

$\downarrow \quad \Downarrow \quad \downarrow$

$1 \quad 1 \quad 1$

sklep

Zveznost funkcije

Funkcija f je zvezna

v $a \in D_f$, če $\lim\limits_{x\to a} f(x) = f(a)$

LASTNOSTI ZVEZNOSTI

1. Vse elementarne funkcije so zvezne
2. Če sta f in g zvezni v a:
   - → grafa sta v a neprekinjeni krivulji
   - → so zvezne $f+\lambda\cdot g$, $\sin(g)$ v a
   - → lahko zamenjamo vrstni red funk. in limite v a
3. f zvezna v a, g zvezna v $f(a)$ $\Rightarrow$ $g\circ f$ zvezna v a
4. Če $\lim\limits_{x\to b} f(x)$ obstaja, $b \notin D_f$, lahko f s predpisom
   $$f(b) = \lim_{x\to b} f(x)$$
   zvezno razširimo na b
5. Če je f zvezna na intervalu $[a,b]$

MANJKA:

ničle zveznih funk.

slika omejenega zaprt. intervala