Matematična indukcija

- tehnika dokazovanja trditev, formul ipd. kjer nastopa IN število

$T(n)$ za $n \in \mathbb{N}$
(vsaj od nekega zač. št. naprej)

(1.) preverimo $T(1)$ - baza indukcije

(2.) dokažemo $T(n) \Rightarrow T(n+1)$ - korak indukcije

induk. predpostavka

Vaje:

1. a) $1\cdot 2+2\cdot 3+\ldots+n(n+1)=\frac{1}{3}n(n+1)(n+2)$

(1) $n=1$: $1\cdot 2=\frac{1}{3}\cdot 1\cdot 2\cdot 3$

$2=2$ ✓

(2) predpostavimo, da za n velja

$1\cdot 2+2\cdot 3+\ldots+n(n+1)=\frac{1}{3}n(n+1)(n+2)$

hočemo dokazati, da velja tudi

$1\cdot 2+2\cdot 3+\ldots+n(n+1)+(n+1)(n+2)=\frac{1}{3}(n+1)(n+2)(n+3)$

$n+1$: $1\cdot 2+2\cdot 3+\ldots+n(n+1)+(n+1)(n+2)=\frac{1}{3}(n+1)(n+2)(n+3)$

$1\cdot 2+2\cdot 3+\ldots+n(n+1)+(n+1)(n+2)=\underbrace{\frac{1}{3}n(n+1)(n+2)}_{\text{po induk. predp.}}+(n+1)(n+2)=(n+1)(n+2)\left(\frac{n}{3}+1\right)=$

1. IZPOSTAVIMO SKUPNI FAKTOR

R: $=\frac{1}{3}(n+1)(n+2)(n+3)$

2. IZPOSTAVIMO $\frac{1}{3}$

Bistven korak

c) $\underset{1\cdot 2\cdot 3\cdot\ldots\cdot n}{n!}>2^{n-1}$ za $n>2$ ali $n\geq 3$

(1) $n=3$: $3!>2^2$

$6>4$

$n>2$ zato ni 1

(2) i.p.: $n!>2^{n-1}$

dokazujemo: $(n+1)!>2^n$

$(n+1)!=n!\cdot(n+1)$

$n!\cdot(n+1)>2^{n-1}(n+1)$

ko množiš neenačbo pazi da je množitelj pozitiven

po i.p.

$\boxed{2^n=2^{n-1}\cdot 2}$

$n>2$, torej razvidno, da je večje

R: $n!(n+1)>2^{n-1}(n+1)>2^{n-1}\cdot 2$

d) $1\cdot 4+2\cdot 4^2+3\cdot 4^3+\ldots+n\cdot 4^n>\frac{(3n-1)4^{n+1}}{9}$

(1.) $n=1$: $1\cdot 4^1>\frac{2\cdot 4^2}{9}$

$\frac{36}{9}>\frac{32}{9}$ ✓

(2.) i.p.: $1\cdot 4+2\cdot 4^2+3\cdot 4^3+\ldots+n\cdot 4^n>\frac{(3n-1)4^{n+1}}{9}$

$1\cdot 4+2\cdot 4^2+3\cdot 4^3+\ldots+n\cdot 4^n+(n+1)\cdot 4^{n+1}>\frac{(3(n+1))4^{n+2}}{9}$

$\frac{(3n-1)4^{n+1}}{9}+(n+1)\cdot 4^{n+1}=\frac{(3(n+1))4^{n+2}}{9}$

Dodali smo še 0

(4.) $\forall\ n\in\mathbb{N}_0$ $57\mid 7^{n+2}+8^{2n+1}$

(1) $n=0$: $7^2+8^1=57$

$57:57=1$ ✓

(2) predpostavimo $57\mid 7^{n+2}+8^{2n+1}$, $\Leftrightarrow$ $7^{n+2}+8^{2n+1}=57\cdot k_1$

i.p.

hočemo videti, da velja tudi

$57\mid 7^{n+3}+8^{2n+3}$

i.p.: $7^{n+3}+8^{2n+3}=57\cdot k_2$, $k\in\mathbb{Z}$

$7^{n+3}+8^{2n+3}=7\cdot 7^{n+2}+\overset{7+57=64}{64}\cdot 8^{2n+1}=$

$=\underbrace{7\,7^{n+2}+7\cdot 8^{2n+1}}_{\text{i.p.}}+\underline{57\cdot 8^{2n+1}}$

automatsko deljivo s 57

$=7(7^{n+2}+8^{2n+1})+57\cdot 8^{2n+1}=$

$=7\cdot(57\cdot k)+57\cdot 8^{2n+1}$

2.

$\log\left(1-\frac{1}{2^2}\right)+\log\left(1-\frac{1}{3^2}\right)+\ldots+\log\left(1-\frac{1}{n^2}\right)=\log\left(\frac{n+1}{2n}\right)$, $n\geq 2$

① $n=2$: $\log\left(1-\frac{1}{4}\right)=\log\frac{3}{4}$ ✓

② $n+1$: $\underbrace{\log\left(1-\frac{1}{2^2}\right)+\log\left(1-\frac{1}{3^2}\right)+\ldots+\log\left(1-\frac{1}{n^2}\right)}+\log\left(1-\frac{1}{(n+1)^2}\right)=$

hočemo: $\log\left(\frac{n+2}{2(n+1)}\right)$

$\log\left(\frac{n+1}{2n}\right)+\log\left(1-\frac{1}{(n+1)^2}\right)=$

$\log\left(\frac{n+1}{2n}-\frac{n+1}{2n(n+1)^2}\right)=$

$\log\left(\frac{(n+1)^{\cancel{2}2}-n-1}{2n(n+1)^{\cancel{2}}}\right)=\log\left(\frac{n^2+\cancel{2n}+\cancel{1}\cancel{-n}\cancel{-1}}{2n(n+1)}\right)=$

$=\log\left(\frac{\cancel{n}(n+1)}{2\cancel{n}(n+1)}\right)$

5. max sup

min inf

a) $A=\{x\in\mathbb{R}: ||x-1|-2|\geq 1\}$

b) $B=\{x\in A, x\leq 5, x>1\}$

$x\in\mathbb{R}$

$|x|=\begin{cases}x: x\geq 0\\ -x: x<0\end{cases}$

1. način: analitično

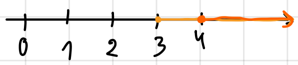
1.1 predpostavimo, da je

$x-1\geq 0$

$\boxed{x\geq 0}$

$x-1-2\geq 0$

$\boxed{x\geq 3}$



$x-3\geq 1$

R: $\boxed{x\geq 4}$

1.2

$x-3<0$

$x<3$

$-(x-3)\geq 1$

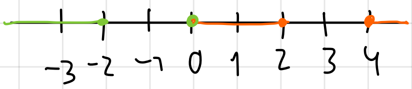
$-x+3\geq 1$

$x\leq 2$

2. Neg.

$x-1<0$

$x<1$



$|-(x-1)-2|\geq 1$

$|-x-1|\geq 1$

$|x+1|\geq 1$

2.1 $x+1\geq 0$

$\boxed{x\geq -1}$

$x+1\geq 1$

$\boxed{x\geq 0}$

2.2. $x+1<0$

$\boxed{x<-1}$

$-(x+1)\geq 1$

$-x-1\geq 1$

$\boxed{x\leq -2}$

A: $(-\infty,-2]\cup[0,2]\cup[4,\infty)$

max \	min \
sup ∞	inf $-\infty$

B: $(1,2]\cup[4,5]$

max 5	min
sup 5	inf 1

DN – 1. teden

$$\frac{n(n+1)(2n+1)}{6}+(n+1)=\frac{(n+1)(n+2)(2n+3)}{6} \quad /\cdot 6$$

$$n(n+1)(2n+1)+6(n+1)=(n+1)(n+2)(2n+3)$$
$$(n^2+n)(2n+1)+6n+6=(n^2+3n+2)(2n+3)$$
$$\cancel{2n^3}+3n^2+7n+\cancel{6}=\cancel{2n^3}+9n^2+13n+\cancel{6}$$
$$-6n^2=6n$$
$$n=-1$$

$|x+8|+|x+4|<10$

1. $x>0$ 2. $x<0$

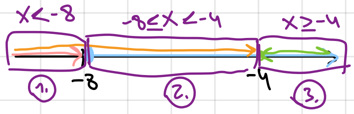
$x+8>0$ $x+8<0$

$\boxed{x\geq -8}$ $\boxed{x<-8}$

$x+4>0$ $x+4<0$

$\boxed{x\geq -4}$ $\boxed{x<-4}$

$x<-8$ $-8\leq x<-4$ $x\geq -4$



1. $x<-8$

$-x-8-x-4<10$

$-2x<22$

$R_1: \boxed{x>-11}$ $x\in(-\infty,-8)\cap(-11,\infty)=(-11,-8)$

2. $-8\leq x<-4$

$x+8-x-4<10$

$R_2: 4<10$ ✓ $x\in[-8,-4)$

3. $x\geq -4$

$x+8+x+4<10$

$2x<-2$

$R_3: \boxed{x<-1}$ ~~$x\in[-4,4)$~~ $x\in[-4,-1)$

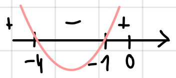
~~$x\in[-8,4)$~~ ~~združim~~

združim

$x\in[-11,-1)$

inf. sup.

$$x^2+5x+4\leq 0$$
$$(x+1)(x+4)\leq 0$$
$$x_1=-1$$
$$x_2=-4$$



$$x\in[-4,-1]$$

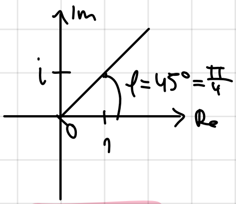
DN – 2. teden

1. $(1+i)^8 \rightarrow z^8$

$\boxed{e^{i\varphi}=\cos\varphi+i\sin\varphi}$

$r^2=(Im_z)^2+(Re_z)^2=1+1$

$r=\sqrt{2}$



$e^{i\frac{\pi}{4}}$

$(re^{i\varphi})^n=r^n e^{in\varphi}$

$(\sqrt{2}\cdot e^{i\frac{\pi}{4}})^8=\sqrt{2}^8\, e^{i\cdot 2\pi}=2^4\cdot e^{i2\pi}=16$ ($2^4 \rightarrow 16$; $e^{i2\pi}$: 0, ker 2π)

2. $\varphi=\frac{315}{7}°$

$r=1+\frac{2}{1000}$

$x=z$

$x^7=?$

$x=r\cdot e^{i\varphi}$

$\boxed{x=1+\frac{2}{1000}\cdot e^{i\frac{315°}{7}}}$

$x^7=\underbrace{(1+\frac{2}{1000})^7}_{r}\cdot\underbrace{e^{i\cdot 315°}}_{\varphi}$

$\varphi'=\varphi,\ -180<\varphi'<180$

$\varphi'=-45°$

ker je 315° manj od 360°

$r=1{,}01408$

3. $z=13-26i$

$w=\overline{z}=13+26i$

$z^2-z=169-676i-676-13+26i=-520-650i$

$r=\sqrt{Im_z^2+Re_z^2}=832{,}40615$

$2w=26+52i$ $Im_z=52$

4. $f(z)=(2z-1-2i)e^{i\frac{\pi}{2}}$

ni tak ko pr normal funk.

Im os

i

$f(z)=\overline{(2z-1-2i)\cdot i}$ prezrcali prek x osi

$f(1+2i)=\overline{(1+2i)\cdot i}=\overline{i-2}=-2-i$

20.10.

7. Nariši naslednjo podmnožico v $\mathbb{C}$:

$$A=\{z\in\mathbb{C}\,;\,|z|\le 1,\ \mathrm{Im}(z)\le\frac{\sqrt{2}}{2},\ 0\le \mathrm{Re}(z)\le\frac{\sqrt{2}}{2}\}$$

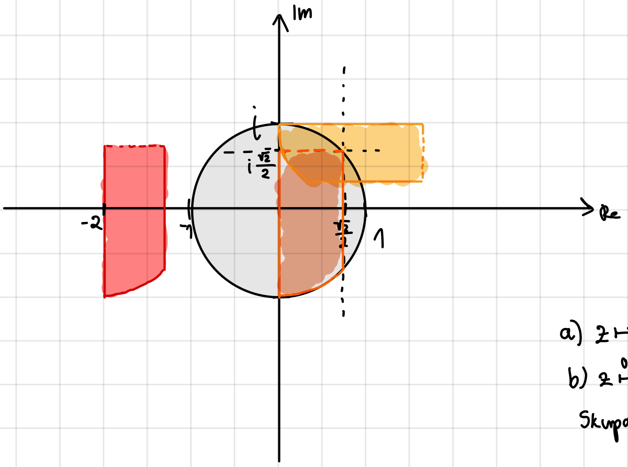
Z območjem A naredimo naslednjo transformacijo:

(a) zavrtimo ga okoli števila 1 za kot $-\pi/2$;

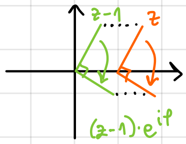
(b) zavrtimo ga okoli števila $-i$ za kot $\pi/2$.

Zapiši predpis $z\mapsto f(z)$, ki opravi to kompleksno transformacijo. Nariši tudi $f(A)$ ter ugotovi, kam se preslika število $1+i\notin A$, če f razširimo na celo kompleksno ravnino.

Rešitev: $z\mapsto z-2$, $1+i\mapsto -1+i$.



rotacija okrog 0 za φ:
$z\mapsto z\cdot e^{i\varphi}$



$(z-1)\cdot e^{i\varphi} \longrightarrow (z-1)\cdot e^{i\varphi}+1 = z$

a) $z \xmapsto{\text{okrog } 1} (z-1)(-i)+1=-iz+i+1$

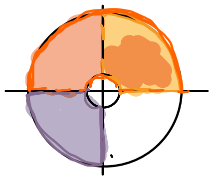
b) $z \xmapsto{\text{okrog } -i} (z+i)i-i = iz-1-i$

Skupaj: $z\mapsto -iz+i+1 \mapsto i(-iz+i+1)-1-i$
$= z-1+i-1-i =$
$= \underline{\underline{z-2}}$

$\boxed{f(z)=z-2}$

Dodatne točke za sliko:
$f(1+i)=1+i-2=-1+i$
$f(0)=-2$
$f(i)=i-2$

8. Nariši naslednje podmnožice v $\mathbb{C}$:



$A=\{z\in\mathbb{C}\,;\,1\le|z|\le3,\ 0\le\arg(z)<\frac{\pi}{2}\}$,

$B=\{z\in\mathbb{C}\,;\,\frac{1}{3}\le|z|\le3,\ 0\le\arg(z)<\pi\}$,

$C=\{z\in\mathbb{C}\,;\,1\le|z|\le3,\ \pi<\arg(z)\le\frac{3\pi}{2}\}$.

Nato poišči kompleksne transformacije, ki transformirajo A v B, A v C in C v A.
a) b) c)

Rešitev: $f_{AB}: z\mapsto \frac{1}{3}z^2$, $f_{CA}: z\mapsto -i\bar{z}$, $f_{AC}: z\mapsto -i\bar{z}$.

a) $f_{AB}: A\mapsto B$

$z\mapsto z^2\mapsto \frac{1}{3}z^2$

$r\in[1,3]$ $r\in[1,9]$ $r[\frac{1}{3},3]$

$f(z)=\frac{1}{3}z^2$

b) $f_{AC}: A\mapsto C$

$z \xmapsto{\text{zrcaljenje čez Re os}} \bar{z} \xmapsto{\text{rotacija za } -\frac{\pi}{2}} \bar{z}e^{i(-\frac{\pi}{2})} = -\bar{z}i$

$f_{AC}(z)=-i\bar{z}$

c) $f_{CA}: C\mapsto A$

$f_{CA}=f_{AC}=-\bar{z}i$

3. b) $z^3=-\frac{27}{\sqrt{2}}+\frac{27}{\sqrt{2}}i$

$\boxed{\begin{array}{l} z=re^{i\varphi} \\ z^3=r^3e^{i\cdot3\varphi} \end{array}}$

$R=\sqrt{\left(\frac{27}{\sqrt{2}}\right)^2+\left(\frac{27}{\sqrt{2}}\right)^2}=\boxed{27}$

$\varphi' = \arctan\left(\frac{\frac{27}{\sqrt{2}}}{-\frac{27}{\sqrt{2}}}\right)=\arctan(-1)=-\frac{\pi}{4}+\pi=\boxed{\frac{3\pi}{4}}$

$z^3=r^3\cdot e^{i\cdot3\varphi}=27\cdot e^{i\frac{3\pi}{4}}$

$r^3=27$ $\quad 3\varphi=\frac{3\pi}{4}$ $\quad k\in\mathbb{Z}$

$\boxed{r=3}$ $\quad \boxed{\varphi=\frac{\pi}{4}+\frac{2\pi k}{3}}$ $\quad (k=0,1,2)$

dobiš z_1,z_2,z_3 = graf rešitve je $\triangle$

d) $z^4 = -1 + i\sqrt{3}$

$z = r \cdot e^{i\varphi}$

$z = r^4 \cdot e^{i \cdot 4\varphi}$

$R = \sqrt{(-1)^2 + \sqrt{3}^2} = \sqrt{4} = 2$

$\varphi' = \arctan\left(\frac{\sqrt{3} \cdot \frac{1}{2}}{-1 \cdot \frac{1}{2}}\right) = \arctan\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right) = 120° = \boxed{\frac{2\pi}{3}}$

sin

cos

$r^4 = R$

$r = \sqrt[4]{2}$

$4\varphi = \varphi'$

$\varphi = \frac{\pi}{6} + \frac{\pi k}{2} \quad (k = 0, 1, 2, 3)$

rešitev je $\square$

4. $ax^3 + bx^2 + cx + d = a(x - x_1)(x - x_2)(x - x_3)$

$x_1 + x_3 x_2 = -\frac{b}{a}$

$x_1 x_2 + x_2 x_3 + x_1 x_3 = \frac{c}{a}$

$x_1 x_2 x_3 = -\frac{d}{a}$

$$ax^3 + bx^2 + cx + d = a(x - (1+i))(x - (1-i))(x - 2) =$$
$$= a(x^2 - 2x + 2)(x - 2) =$$
$$= a(x^3 - 2x^2 + 2x - 2x^2 + 4x - 4) =$$
$$= a(x^3 - 4x^2 + 6x - 4)$$

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8. Za vsako izmed spodnjih zaporedij razišči monotonost, omejenost in konvergenco. Če imajo limito določi tudi prvi člen zaporedja, ki je od nje oddaljen za manj kot $\frac{1}{10}$.

(a) * $a_n = \frac{n}{3n-1}$

(b) $a_n = \frac{2^n - 4}{2^n + 4}$

(c) $a_n = -\frac{n^2 + n}{n^2 + 1}$

Rešitve:

(a) padajoče od $n = 1$ naprej, navzdol omejeno z 0, limita je $\frac{1}{3}$, $n_0 = 2$,

(b) naraščajoče, navzgor omejeno z 1, limita je 1, $n_0 = 7$,

(c) naraščajoče od $n = 2$ naprej, navzgor omejeno z 0, limita je -1, $n_0 = 1$.

a) $a_0 = 0$

$a_1 = \frac{1}{2}$

$a_2 = \frac{2}{5}$

$a_3 = \frac{3}{8}$

$a_4 = \frac{4}{11}$

$\downarrow$ pada

$a_n - a_{n+1} \geq 0$

$\frac{n}{3n-1} - \frac{n+1}{3n+2} \geq 0$

$\frac{\cancel{3n^2} + \cancel{2n} - \cancel{3n^2} - \cancel{3n} + \cancel{n} + 1}{(3n-1)(3n+2)} \geq 0$

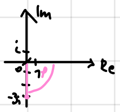
$\frac{1}{(3n-1)(3n+2)} \geq 0$

$1 \geq 0$ ✓✓

DN – 3. teden

① $z=-3i$
$\mathrm{Re}\,z=0$
$\mathrm{Im}\,z=-3$

$r=|z|=3$
$\varphi=-\frac{\pi}{2}$



$z=r(\cos\varphi+i\sin\varphi)$

sedmi koren: $n=7$

$$z_k=\sqrt[n]{r}\cdot e^{i\left(\frac{\varphi}{n}+\frac{2k\pi}{n}\right)}=\sqrt[n]{r}\cdot e^{i\left(\frac{\varphi+2k\pi}{n}\right)}$$

$$z_4=\sqrt[7]{3}\cdot e^{i\left(\frac{-\frac{\pi}{2}+2k\pi}{7}\right)}$$

$$z_4=\sqrt[7]{3}\left(\cos\frac{6.5\pi}{7}+i\sin\frac{6.5\pi}{7}\right)$$

$\mathrm{Re}\,z: -1,14060$

$\mathrm{Im}\,z: -0,260033$

② pravilni 10-kotnik

$A(7, 0'5) \Rightarrow z=7+\frac{1}{2}i$

$r=|z|=7,01783$

$\varphi=\arctan\frac{\mathrm{Im}\,z}{\mathrm{Re}\,z}$

$\varphi=4,09°$

kot med sosednjima oglišiema: $\frac{360°}{10}=36°$

$$(x_k, y_k)=(r\cdot\cos(\varphi_0+k\cdot 36°),\ r\cdot\sin(\varphi_0+k\cdot 36°)),\quad k=0,1,2,\dots,9$$

največji y je pri $k=2$
$\downarrow$
kot $=76,09°$

$(x_2, y_2)=(\cancel{1'69771, 6'80713})$
$1'67206,\ 6'81434$

③ $a_{24}=?$

$a_{n+1}=a_n+13$
$a_1=10$

aritmetično zaporedje

$a_n=a_1+(n-1)\cdot d$

$a_{24}=10+23\cdot 13$

$a_{24}=309$

④ $a_n=\frac{15n^2+3n-1}{5n^2+2}$

$$\lim_{n\to\infty}\frac{15n^2+3n-1}{5n^2+2}\ \overset{:n^2}{\underset{:n^2}{=}}\ \frac{15+\frac{3}{n}-\frac{1}{n^2}}{5+\frac{2}{n^2}}\xrightarrow{n\to\infty}\frac{15}{5}=3$$

limita $\qquad \varepsilon$

$$|a_k-3|<\frac{1}{30}$$

$$\left|\frac{15k^2+3k-1}{5k^2+2}-3\right|<\frac{1}{30}$$

$$\frac{\cancel{15k^2}+3k-1-\cancel{15k^2}-6}{5k^2+2}<\frac{1}{30}$$

$$\boxed{\frac{3k-7}{5k^2+2}}<\frac{1}{30}$$

$3k-7>0$
$3k>7$
$k>\frac{7}{3}$
$\Downarrow$
$\boxed{k>3}$

$90k-210<5k^2+2$

$5k^2-90k+212>0$

$D=b^2-4ac=3860$

$\sqrt{D}=62,129$

$$k_{1,2}=\frac{-b\pm\sqrt{D}}{2a}=\begin{array}{l}1=15,21\ \checkmark\\ 2=2,8\end{array}\Rightarrow k=16$$

3.11.

1. * Podani sta vrsti

$$a)\ \sum_{n=1}^{\infty}\frac{2}{n(n+2)} \quad \text{in} \quad b)\ \sum_{n=0}^{\infty}(-1)^n n.$$

Za vsako izmed omenjenih vrst ugani in dokaži formulo za n-to delno vsoto ter po definiciji izračunaj vsoto vrste, če obstaja.

Rešitve: Prva vrsta konvergira k $\frac{3}{2}$, delne vsote so $s_n = \frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}$, divergira, ker je alternirajoča, členi pa ne konvergirajo k 0.

VRSTA

$$\sum_{n=k}^{\infty} a_n = a_k + a_{k+1} + \ldots$$

$$\sum_{n=0}^{\infty} a_n \qquad S_0 = a_0$$

$S_1 = a_0 + a_1$

$S_2 = a_0 + a_1 + a_2$

$\vdots$

$\boxed{S_n = S_{n-1} + a_n}$

$$\sum a_n = \lim_{n\to\infty} S_n$$

a) $a_1 = \frac{2}{3}$ $\quad S_1 = a_1 = \frac{2}{3}$

$a_2 = \frac{1}{4}$ $\quad S_2 = a_1 + a_2 = \frac{11}{12}$

$a_3 = \frac{2}{15}$ $\quad S_3 = S_2 + a_3 = \frac{63}{60}$

$a_4 = \frac{1}{12}$ $\quad S_4 = S_3 + a_4 = \frac{68}{60}$

$S_n = ??$

$$a_n = \frac{2 + 0\cdot n}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2} = \frac{(n+2)A + nB}{n(n+2)} = \frac{n(A+B) + 2A}{n(n+2)}$$

$A + B = 0$

$2A = 2$

$\boxed{A = 1 \quad B = -1}$

$$a_n = \frac{2}{n(n+2)} = \frac{1}{n} - \frac{1}{n+2}$$

$$S_n = a_1 + a_2 + a_3 + \ldots + a_n = \frac{1}{1} - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \ldots + \frac{1}{n-1} - \frac{1}{n+1} + \frac{1}{n} - \frac{1}{n+2} =$$

$$= \frac{3}{2} - \frac{1}{n-1} - \frac{1}{n-2}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{3}{2} - \overbrace{\frac{1}{n-1}}^{=0} - \overbrace{\frac{1}{n-2}}^{=0}\right) = \frac{3}{2}$$

↳ vsota vrste

2. Izračunaj vsoto naslednjih geometrijskih vrst.

(a) * $\sum_{n=1}^{\infty} \frac{10}{3^n}$

(b) $\sum_{n=2}^{\infty} \frac{2^n}{3^{2n-1}}$

(c) * $3/2 + 1 + 2/3 + 4/9 + 8/27 + \ldots$

(d) $\sum_{n=1}^{\infty} \frac{(-2)^n}{3\cdot 2^{3n-2}}$

(e) * $\sum_{n=1}^{\infty} (\frac{x}{2})^{3n}$, za tiste $x \in \mathbb{R}$, za katere vrsta konvergira.

Rešitve: (a) 5, (b) $\frac{4}{21}$, (c) $\frac{9}{2}$, (d) $-\frac{4}{15}$, (e) $\frac{x^3}{8-x^3}$.

$$a)\ \sum_{n=1}^{\infty}\frac{10}{3^n} = 10\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n = 10\cdot\left(\frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \ldots + \left(\frac{1}{3}\right)^n\right) = \boxed{\frac{10}{3}}\cdot\left(1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \ldots + \left(\frac{1}{3}\right)^n\right)$$

$\boxed{k = \frac{a_2}{a_1} = \frac{1}{3}}$

$$S_n = \frac{a_1}{1-k} = \frac{10}{3}\cdot\frac{1}{1-\frac{1}{3}} = \frac{10}{3}\cdot\frac{1}{\frac{2}{3}} = 5$$

$$c)\ \frac{3}{2} + \underbrace{1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots}_{\sum_{n=1}^{\infty} k^n} = \frac{3}{2} + \sum_{n=1}^{\infty} k^n = \frac{3}{2} + \frac{1}{1-k} = \frac{9}{2} = S_n$$

$k = \frac{2}{3}$

če $a_n \not\to 0 \Rightarrow \sum a_n = \infty$ (divergira)

če $a_n \to 0 \Rightarrow ??$

$\sum \frac{1}{n^2} < \infty \qquad \sum \frac{1}{n} = \infty$

$\lim_{n\to\infty} \frac{1}{n^2} = 0 \qquad \lim_{n\to\infty} \frac{1}{n} = 0$

$$\lim_{n\to\infty} \sqrt[n]{n} = 1$$

DN - 4. teden

1.

$$a_n = \frac{a_{n-1} + \frac{7}{a_{n-1}}}{2}$$

$a_1 = 2$

$a_9 = ?$

$\lim_{n\to\infty} a_n = ?$

$$a_2 = \frac{2+\frac{7}{2}}{2} = \frac{\frac{11}{2}}{2} = \frac{11}{4} = 2{,}75$$

$$a_3 = \frac{\frac{11}{4} + \frac{7}{\frac{11}{4}}}{2} = \frac{\frac{11}{4} + \frac{28}{11}}{2} = \frac{\frac{121+112}{44}}{2} = \frac{\frac{233}{4}}{2} = \frac{233}{8} \simeq 2{,}6477$$

$a_4 \simeq 2{,}6463$

NEWTON-RAPHSONOVA METODA

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\left.\begin{aligned} f(x) &= x^2 - 7 \\ f'(x) &= 2x \end{aligned}\right\} \rightarrow x_{n+1} = x_n - \frac{x_n^2 - 7}{2x_n} = \frac{x_n + \frac{7}{x_n}}{2}$$

$\lim_{n\to\infty} a_n = \sqrt{7}$

$a_9 \simeq \sqrt{7} \simeq 2{,}64575$

2. $\sum_{n=2}^{\infty} \frac{2^4}{4^n}$ $\quad k = \frac{a_3}{a_2} = \frac{1}{4}$

$a_2 = \frac{3}{2}$

$$S_n = \frac{a_2}{1-k} = \frac{\frac{3}{2}}{1-\frac{1}{4}} = \frac{\frac{3}{2}}{\frac{3}{4}} = \frac{12}{6} = 2$$

10. 11.

1. * Določi definicijsko območje naslednjih funkcij in ugotovi, ali so injektivne. Če niso, definicijsko območje smiselno zmanjšaj in nato določi inverzno funkcijo.

   (a) $f(x) = \sqrt{e^{2x} - 3e^{x} + 2}$,

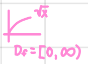
   (b) $f(x) = \log(2\sin x - \sqrt{3})$.

Rešitve:

(a) $\mathcal{D}_f = (-\infty, 0] \cup [\log 2, \infty)$, $\mathcal{Z}_f = [0, \infty)$, niti liha niti soda, ni injektivna na $\mathcal{D}_f$, je injektivna na $[\log 2, \infty)$,

(b) $\mathcal{D}_f = \cdots \cup \left(-\frac{5\pi}{3}, -\frac{4\pi}{3}\right) \cup \left(\frac{\pi}{3}, \frac{2\pi}{3}\right) \cup \left(\frac{7\pi}{3}, \frac{8\pi}{3}\right) \cup \cdots$, $\mathcal{Z}_f = \left(-\infty, \log(2-\sqrt{3})\right]$, niti liha niti soda, ni injektivna na $\mathcal{D}_f$, je injektivna, če se omejimo npr. na $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

a) $\underbrace{\sqrt{e^{2x} - 3e^{x} + 2}}_{\geq 0}$



$D_f = [0, \infty)$

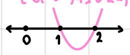
$e^{2x} - 3e^{x} + 2 \geq 0$

$t = e^{x}$

$t^2 - 3t + 2 \geq 0$

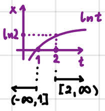
$(t-2)(t-1) \geq 0$

$t \in (-\infty, 1] \cup [2, \infty)$



$x = ???$

$x = \ln(t)$



① $\left.\begin{array}{l} t \in (-\infty, 1] \\ t = e^{x} > 0 \end{array}\right\} t \in (0, 1] \longrightarrow x \in (-\infty, 0]$

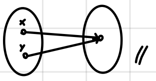
② $t \in [2, \infty) \longrightarrow x \in [\ln 2, \infty)$

$\underbrace{x \in (-\overset{①}{\infty}, 0] \cup [\ln \overset{②}{2}, \infty)}_{D_f}$

f je injektivna $\Leftrightarrow \forall x, y: f(x) = f(y) \Rightarrow x = y$

$\Leftrightarrow \forall x, y: x \neq y \Rightarrow f(x) \neq f(y)$

$\Leftrightarrow$ vsaka vodoravna premica seka graf največ enkrat



$t_1 = 1$ $\quad$ $t_2 = 2$

$x_1 = \ln 1$ $\quad$ $x_2 = \ln 2$

$x_1 = 0$

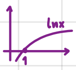
$\boxed{f(x_1) = 0 \quad f(x_2) = 0}$

Za x_1 in x_2 funkcija ni injektivna.

$f|_{[\ln 2, \infty)}$ je injektivna (ker je $t^2 - 3t + 2$ injektivna na $[2, \infty)$)

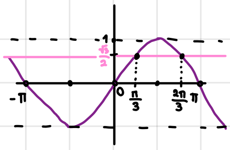
$f|_{(-\infty, 0]}$ je injektivna

b) $f(x) = \log(\underbrace{2\sin x - \sqrt{3}}_{})$



$2\sin x - \sqrt{3} > 0$

$\sin x > \frac{\sqrt{3}}{2}$



$D_f = \dots \cup \left(-\frac{5\pi}{3}, -\frac{4\pi}{3}\right) \cup \left(\frac{\pi}{3}, \frac{2\pi}{3}\right) \cup \left(\frac{7\pi}{3}, \frac{8\pi}{3}\right) \cup \dots = \bigcup_{k \in \mathbb{Z}} \left(\frac{\pi}{3} + 2k\pi, \frac{2\pi}{3} + 2k\pi\right)$

$+2\pi$

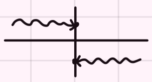
$+\frac{6\pi}{3}$

f ni injektivna: $\left.\begin{array}{l} x_1 = \frac{\pi}{2} \\ x_2 = \frac{5\pi}{2} \end{array}\right\} f(x) = \log 2 - \sqrt{3} = f(x_1) = f(x_2)$

3. * Določi konstanto a (če obstaja), da bo f zvezna funkcija.

$$f(x) = \begin{cases} \arctan\left(1 + \frac{1}{x}\right), & x \neq 0, \\ a, & x = 0. \end{cases}$$

Rešitev: Tak a ne obstaja, ker sta leva in desna limita v točki $x = 0$ različni.



a ne obstaja

$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

a obstaja

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \overbrace{\arctan\left(1 + \tfrac{1}{x}\right)}^{-\frac{\pi}{2}} = -\frac{\pi}{2}$ $\quad$ $\left(\tfrac{1}{x} \to -\infty\right)$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \overbrace{\arctan\left(1 + \tfrac{1}{x}\right)}^{\frac{\pi}{2}} = \frac{\pi}{2}$ $\quad$ $\left(\tfrac{1}{x} \to \infty\right)$

Limita a ne obstaja, ker $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

4. * Določi konstanti a in b, da bo f zvezna funkcija.

$$f(x) = \begin{cases} \frac{\sin(3x)(x-2)}{x}, & x < 0, \\ ax + b, & 0 \leq x \leq 1, \\ 2e^{x-1} - \cos(\pi x), & x > 1. \end{cases}$$

Rešitev: $a = 9$, $b = -6$.

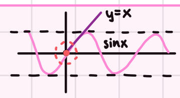
① $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\overset{\to 0}{\sin(3x)}\overset{\to -2}{(x-2)}}{x} = -2 \lim_{x \to 0^-} \frac{\sin(3x)}{x} = -6 \lim_{x \to 0^-} \frac{3\sin(3x)}{3x} = -6$ $\quad$ $\left(\to \frac{0}{0} = 0\right)$

$\parallel$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (ax + b) = b$

$\boxed{b = -6}$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$



② $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (ax + b) = a + b = a - b$

$\parallel$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2e^{x-1} - \cos(\pi x)) = 3$ $\quad$ $(2e^{x-1} \to 2$, $\cos(\pi x) \to \cos\pi = -1)$

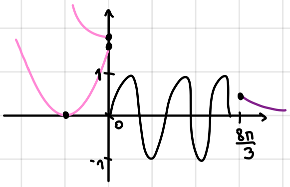
$a - 6 = 3$

$\boxed{a = 9}$

5. Skiciraj graf funkcije f za $a = -1$ in $b = 1$. Nato določi taki konstanti a in b (če obstajata), da bo f zvezna funkcija.

$$f(x) = \begin{cases} (x-a)^2, & x \leq 0, \\ \sin(4x), & 0 < x \leq \frac{8\pi}{3}, \\ \frac{b\sqrt{3}}{x}, & x > \frac{8\pi}{3}. \end{cases}$$

Rešitev: $a = 0$, $b = \frac{4\pi}{3}$.



(1) $\lim_{x\to 0^-} f(x) = f(0) = (0-a)^2 = a^2$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \sin(4x) = 0$

$\to a^2 = 0$, $\boxed{a = 0}$

(2) $\lim_{x\to \frac{8\pi}{3}^+} f(x) = \lim_{x\to \frac{8\pi}{3}^+} \frac{b\sqrt{3}}{x} = \frac{b\sqrt{3}}{\frac{8\pi}{3}} = \frac{3\sqrt{3}}{8\pi} b$

$\|$

$\lim_{x\to \frac{8\pi}{3}^+} f(x) = f\left(\frac{8\pi}{3}\right) = \sin\left(4 \cdot \frac{8\pi}{3}\right) = \sin\left(\frac{32\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\to \frac{3\sqrt{3}}{8\pi} b = \frac{\sqrt{3}}{2}$, $\boxed{b = \frac{4\pi}{3}}$

2. Izračunaj

$$lim_{x\nearrow 0}(1+x)^{\frac{1}{x}} \quad \text{in} \quad lim_{x\searrow 0}(1+x)^{\frac{1}{x}}.$$

Rešitev: Obe limiti sta enaki e.

$\lim_{x\to 0^-} (1+x)^{\frac{1}{x}} = \underbrace{\left(1+\frac{1}{\frac{1}{x}}\right)^{\frac{1}{x}}}_{e} = e$

$\lim_{x\to 0^+} (1+x)^{\frac{1}{x}} = e$

GCC gem

$\boxed{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e}$

$\lim_{n\to\infty}\left(1-\frac{6}{n}\right)^{2n+1}$ v števcu naj bo 1

$\left(1-\frac{1}{\frac{n}{6}}\right)^{2n+1}$ naj bo vmes +

$\left(1+\frac{1}{-\frac{n}{6}}\right)^{2n+1}$ naj bo eksponent enak imenovalcu

$\left(\left(1+\frac{1}{-\frac{n}{6}}\right)^{-\frac{n}{6}}\right)^{\frac{2n+1}{-\frac{n}{6}}}$ poišči limito eksponenta

$\Downarrow$ e

$\lim_{n\to\infty} \frac{\frac{2n+1}{1}}{-\frac{n}{6}} = \frac{(2n+1)\cdot 6}{n} = \frac{12n+6}{n} = 12 + \frac{6}{n} \to 0$

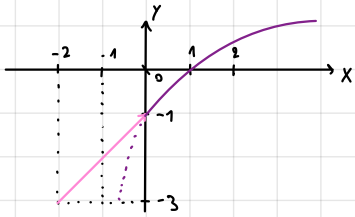
$\lim_{n\to\infty} e^{12}$

6. * Prepričaj se, da je funkcija

$$f(x) = \begin{cases} \frac{x-1}{x+1}, & x \geq 0, \\ x-1, & x < 0. \end{cases}$$

zvezna na intervalu $[-2, 2]$. Poišči največjo vrednost M in najmanjšo vrednost m, ki ju zavzame na tem intervalu. Ali ima enačba $f(x) = 0$ rešitev na tem intervalu? Kaj pa enačba $f(x) = 3$? Poišči vse rešitve, če obstajajo!

Rešitev: $m = -3$, $M = \frac{1}{3}$. Enačba $f(x) = 0$ ima rešitev $x = 1$. Enačba $f(x) = 3$ ni rešljiva, ker $3 \notin [m, M]$.



f je zvezna na $[-2,2]$

1. problem: pol rac. funkcije $\frac{x-1}{x+1}$ je v -1

predpis uporabimo za $x \geq 0$ ✓ ok

2. problem: $x = 0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0) = -1$

$\|$

$\lim_{x\to 0^-} (x-1) = -1$ ✓ ok

Če je f zvezna na $[a,b]$, potem f doseže

min m in max M in $Z_f = [m, M]$

Kandidati za m, M:

- točke kjer je $f' = 0$ (tangenta vodoravna)
- krajišča intervala ($x = a$, $x = b$)
- točke kjer f' ni definiran

↳ odvod

$f'(x) = \left(\frac{x-1}{x+1}\right)' = \frac{(x+1)-(x-1)\cdot 1}{(x+1)^2} = \frac{2}{(x+1)^2}$

- $f'(x) = \begin{cases} \frac{2}{(x+1)^2}; & x > 0 \\ 1; & x < 0 \end{cases} \neq 0$

- $f(2) = \frac{2-1}{2+1} = \frac{1}{3}$ $\quad f(-2) = -2-1 = -3$
- $f(0) = -1$

$M = \frac{1}{3}$ pri $x = 2$

$m = -3$ pri $x = -2$

$\Big\} Z_f = [-3, \frac{1}{3}]$

$f(x) = 0$ rešljiva?

Enačba $f(x) = k$ je rešljiva $\Leftrightarrow k \in Z_f$

Ja, ker je $0 \in Z_f = [-3, \frac{1}{3}]$

Npr. $x = 1$ (ne vemo, če je edina)

$f(x) = 3$ ni rešljiva ker $3 \notin Z_f = [-3, \frac{1}{3}]$

DN – 5. teden

1. $v_1 = v_2 = 40\,\frac{km}{h}$

$m = 160\,\frac{km}{h}$

$\frac{140}{160} = t = 52{,}5\,min = 0{,}875\,h$

$\downarrow$

v tem času vlak naredi

$v \cdot t = s$ 35 km

$s = 140\,km$

$x = ?$

$a_1 = 140$

$140 - 2\cdot 35 = 70 = a_2$

$k = \frac{a_2}{a_1} = \frac{1}{2}$

$$S_n = \frac{a_1}{1-k} = \frac{140}{\frac{1}{2}} = \boxed{280}$$

2. $$\sum_{n=12}^{\infty} \frac{24}{n^2-1} = 24\left(\frac{1 + 0\cdot n}{(n-1)(n+1)}\right) = 24\sum_{n=12}^{\infty}\left(\frac{\frac{1}{2}}{n-1} + \frac{-\frac{1}{2}}{n+1}\right) = 24\sum_{n=12}^{\infty}\left(\frac{\frac{1}{2}}{n-1} - \frac{\frac{1}{2}}{n+1}\right) = 24\cdot\left(\frac{\frac{1}{2}}{11} - \frac{\frac{1}{2}}{13}\right) + \left(\frac{\frac{1}{2}}{12} - \frac{\frac{1}{2}}{14}\right) + \left(\frac{\frac{1}{2}}{13} - \frac{\frac{1}{2}}{15}\right) + \left(\frac{\frac{1}{2}}{14} - \frac{\frac{1}{2}}{16}\right) + \ldots + \underbrace{\left(\frac{\frac{1}{2}}{n-1} - \frac{\frac{1}{2}}{n+1}\right)}_{0}$$

konvergira

$$\frac{A}{n-1} + \frac{B}{n+1} = \frac{(n+1)A + (n-1)B}{(n-1)(n+1)} = \frac{\overbrace{n(A+B)}^{1.} + \overbrace{1(A-B)}^{2.}}{n^2-1}$$

1. $A + B = 0$
2. $A - B = 1$

$A = 1 + B$

$1 + B + B = 0$

$\boxed{B = -\frac{1}{2}}$

$\boxed{A = \frac{1}{2}}$

$$S_n = 24\cdot\left(a_1 + a_2 + \left(\frac{\frac{1}{2}}{n-1} - \frac{\frac{1}{2}}{n+1}\right)\right) = \frac{23}{11} = 2{,}09091$$

17. 11.

1. * Določi definicijsko območje funkcije

$$f(x,y) = x^2 + \frac{y^2}{4}$$

in skiciraj nivojnice.

Rešitev: $\mathcal{D}_f = \mathbb{R} \times \mathbb{R}$, nivojnice so elipse s središčem v izhodišču in polosema $\sqrt{a}$, $2\sqrt{a}$.

$D_f = \mathbb{R} \times \mathbb{R}$

$\boxed{\text{nivojnica je krivulja } f(x,y)=c \text{ za } c \in Z_f}$

$f(x,y) = c$

$x^2 + \frac{y^2}{4} = c \quad / : c$ če $c \neq 0$

$\frac{x^2}{c} + \frac{y^2}{4c} = 1 \Rightarrow$ elipsa $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ polmer

$\boxed{\frac{x^2}{(\sqrt{c})^2} + \frac{y^2}{(2\sqrt{c})^2} = 1}$

kaj pa če $c = 0$?

$x^2 + \frac{y^2}{4} = 0$

$x = y = 0$

$(0,0)$

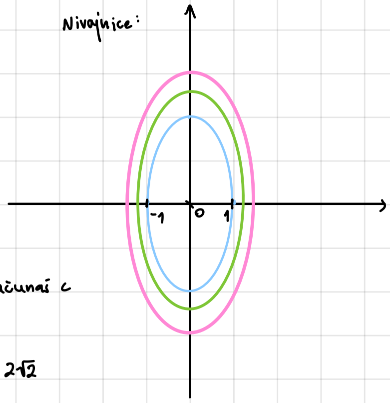
Skiciraj tako, da računaš c

$c=1$ $a=1, b=2$

$c=2$ $a=\sqrt{2}, b=2\sqrt{2}$

$c=3$ $a=\sqrt{3}, b=2\sqrt{3}$

Nivojnice:



2. * Skiciraj definicijska območja in nivojnice danih funkcij.

(a) $f(x,y) = \log(y - x^2 + 2)$, ln >0

(b) $f(x,y) = e^{y/x}$,

(c) $f(x,y) = \sqrt{x^{-1}\sin(y)}$.

Rešitve:

(a) $\mathcal{D}_f = \{(x,y) \mid y > x^2 - 2\}$, nivojnice so parabole $y = x^2 + b$, $b \in (-2, \infty)$,

(b) $\mathcal{D}_f = \{(x,y) \mid x \neq 0\}$, nivojnice so premice skozi izhodišče brez izhodišča (razen y-osi),

(c) $\mathcal{D}_f = \{(x,y) \mid x \neq 0, x \cdot \sin y > 0\}$, nivojnice so krivulje $\sin y = a^2 x$.

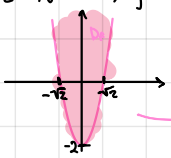
Pogoj za logaritem

a) $y - x^2 + 2 > 0$

$\boxed{y > x^2 - 2}$

$y = px^2$ PARABOLA teme $(0,-2)$

$D_f = \{(x,y) \in \mathbb{R}^2 ; y > x^2 - 2\}$



$f(x,y) = c$

ln $\log(y - x^2 + 2) = c$

uporabimo definicijo $\log_a x = y \Leftrightarrow a^y = x$

$y - x^2 + 2 = e^c$ konst. parabola

$c \in Z_f = \mathbb{R}$ $e^c - 2 \in (-2, \infty)$

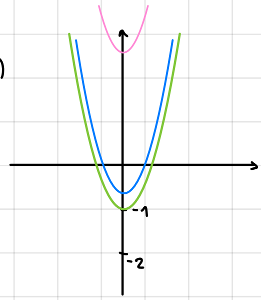
$y = x^2 + e^c - 2$

$c=0$ $y = x^2 - 1$

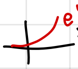
$c=1$ $y = x^2 + e - 2$ 0.718...

$c=2$ $y = x^2 + e^2 - 2$ 6,...

$c=-1$ $y = x^2 + \frac{1}{e} - 2$ 0.37...



b) $f(x,y) = e^{\frac{y}{x}}$ $Z_f = \mathbb{R}^+$



$x \neq 0$

$D_f = \{(x,y) \in \mathbb{R}^2 ; x \neq 0\}$

$\mathbb{R}^2 \setminus \{(0,y) ; y \in \mathbb{R}\}$

$\mathbb{R}^2 \setminus y\text{-os}$

$f(x,y) = c$

$e^{\frac{y}{x}} = c \quad / \ln$

$\frac{y}{x} = \ln c$

$y = \underbrace{(\ln c)}_{\text{konst.}} x \rightarrow$ premica skozi $(0,0)$ (brez $0,0$)

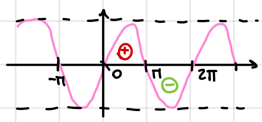
c) $f(x,y) = \sqrt{\frac{\sin y}{x}}$ $x \neq 0$

Pogoj za koren

$\frac{\sin y}{x} \geq 0 \, / \cdot x \neq 0$

$\sin y \geq 0$ $\boxed{x > 0}$

$\sin y \leq 0$ $\boxed{x < 0}$



$\oplus$ $[0, \pi] \rightarrow \bigcup_{k \in \mathbb{Z}} [0 + 2\pi k, \pi + 2\pi k] = \ldots \cup [0,\pi] \cup [2\pi, 3\pi] \cup \ldots$

$\ominus$ $[\pi, 2\pi] \rightarrow \bigcup_{k \in \mathbb{Z}} [\pi + 2\pi k, 2\pi + 2\pi k] = \ldots \cup [\pi, 2\pi] \cup [3\pi, 4\pi] \cup \ldots$

$f(x,y) = c$

$\sqrt{\frac{\sin y}{x}} = c$

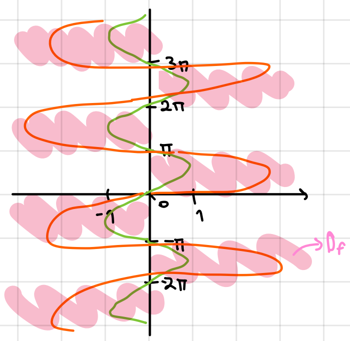
$\sin y = x c^2$

$\boxed{x = \frac{1}{c^2} \cdot \sin y}$ $c \neq 0$

$c=1$ $x = \sin y$

$c=2$ $x = \frac{1}{4} \sin y$

$c = \frac{1}{\sqrt{5}}$ $x = 5 \sin y$



3. * Izračunaj odvode naslednjih funkcij:

(a) $f(x) = \sqrt{x}(2 - x^2)$,

(b) $f(x) = \frac{x}{1+x^2}$, $\to \ln$

(c) $f(x) = \log \frac{1+x}{1-x}$,

(d) $f(x) = \arcsin(\sqrt{1 - x^2})$.

Rešitve: (a) $\frac{1}{2\sqrt{x}}(2 - 5x^2)$, (b) $\frac{1-x^2}{(1+x^2)^2}$, (c) $\frac{2}{1-x^2}$, (d) $-\frac{x}{|x|} \cdot \frac{1}{\sqrt{1-x^2}}$.

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$ $\quad$ $\log = \ln$

$(x^n)' = n x^{n-1}$ $\quad$ $(\ln x)' = \frac{1}{x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

$(\arctan x)' = \frac{1}{1+x^2}$

a) $f(x) = \sqrt{x}(2 - x^2)$

$f'(x) = \frac{1}{2\sqrt{x}}(2 - x^2) + \sqrt{x}(-2x) = \frac{1}{\sqrt{x}} - \frac{x^2}{2\sqrt{x}} - 2x\sqrt{x} =$

b) $\left(\frac{x}{1+x^2}\right)' = \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$

c) $\left(\log \frac{1+x}{1-x}\right)' = \frac{1}{\frac{1+x}{1-x}} \cdot \frac{1\cdot(1-x) - (1+x)(-1)}{(1-x)^2} = \frac{\cancel{1-x}}{1+x} \cdot \frac{2}{(1-x)^{\cancel{2}}} = \frac{2}{1-x^2}$

d) $(\arcsin(\sqrt{1-x^2}))' = \frac{1}{\sqrt{1-(\sqrt{1-x^2})^2}} \cdot \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) = \frac{1}{|x|} \cdot \frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}} = -\frac{x}{|x|} \cdot \frac{1}{\sqrt{1-x^2}}$

$$\boxed{\begin{aligned}(\sqrt{x})^2 &= x\\ \sqrt{x^2} &= |x|\end{aligned}}$$

5. * Poišči tisto normalo na krivuljo $y = x\log(x)$, ki je pravokotna na premico z enačbo $2x - 2y - 3 = 0$.

Rešitev: Enačba normale je $y = -x + 1$.

funk: $\quad$ premica:

$f(x) = x \ln x$ $\quad$ $2x - 2y - 3 = 0$

$f'(x) = \ln x + x \cdot \frac{1}{x} = \boxed{\ln x + 1} = k_t$ $\quad$ $y = x - \frac{3}{2}$

$\ln x = 0$

$\boxed{x_0 = 1}$

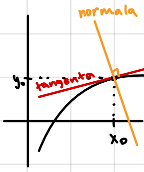
$k_t = 1$

$k_N = -1$

GCC gem

TANGENTA:

1. $T(x_0, y_0)$
2. $f'(x)$
3. $f'(x_0) = k$
4. tangenta: $y - y_0 = k(x - x_0)$



NORMALA

1. $T(x_0, y_0)$
2. $f'(x)$
3. $f'(x_0) = k \to$ tangente
4. $k_N = -\frac{1}{k}$ $\to$ normale
5. normala: $y - y_0 = k_N(x - x_0)$

kot med $f(x)$ in x:

1. $T(x_0, 0)$
2. $\tan\varphi = f'(x_0)$

kot med $f(x)$ in y:

1. $T(0, y_0)$
2. $\tan\varphi' = f'(0)$
3. $\varphi = 90° - \varphi'$

kot med $f(x)$ in $g(x)$

1. Presečišče (x_0, y_0)
2. $f'(x_0) = k_1$, $g'(x_0) = k_2$
3. $\tan\varphi = \left|\frac{k_2 - k_1}{1 + k_1 k_2}\right|$

6. * Naj bo z enačbo $5x^2 - 8xy + 5y^2 = 1$ implicitno podana funkcija $y(x)$.

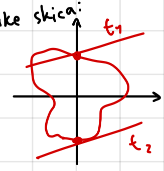
(a) Zapiši enačbi tangent na graf krivulje v obeh točkah, kjer je $x = 0$.

(b) Za katere vrednosti x je naklon tangente enak $k = \frac{1}{2}$?

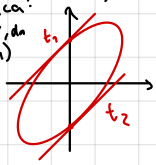
Rešitvi: (a) $y = \frac{4}{5}x + \frac{1}{\sqrt{5}}$ in $y = \frac{4}{5}x - \frac{1}{\sqrt{5}}$, (b) $x = \pm\frac{1}{3}$.

$ax^2 + by^2 + cxy + dx + f = 0$ STOŽNICA

Fake skica:



Real skica:
(ko ugotoviš, da je elipsa)



a) 1. Točka v $x = 0$

$y_1 = \frac{1}{\sqrt{5}}$

$y_2 = -\frac{1}{\sqrt{5}}$

$(xy)' = x' \cdot y + x \cdot y' = y + xy'$

$(y^2)' = 2y \cdot y'$

2. $f'(x,y) =$

$= 10x - 8(xy' + y) + 10yy'$

$10x - 8y - 8xy' + 10yy' = 0$ $\quad$ $| : 2$

$y'(-4x + 5y) = 4y - 5x$

$y' = \frac{4y - 5x}{-4x + 5y}$ $\to k_t$

3. $f'(0, \frac{1}{\sqrt{5}}) = \frac{4}{5} = k_t$

$f'(0, -\frac{1}{\sqrt{5}}) = \frac{4}{5} = k_t$

4. t_1: $y = \frac{4}{5}x + \frac{1}{\sqrt{5}}$

t_2: $y = \frac{4}{5}x - \frac{1}{\sqrt{5}}$

b) $5x^2 - 16x^2 + 20x^2 = 1$

$x = \frac{\pm 1}{3}$

$x = \frac{3}{4\sqrt{5}} = \frac{1}{2\sqrt{5}}$ X