Exercise Questions

1) Explain the Basic rules of probability with example.

→ * An experiment is any process that leads to a well-defined outcome. for example, tossing a coin or rolling a die.

* A Sample Space (S) is the set of all possible outcomes of an experiment. for example.
  - Tossing a Coin : S = {Heads, Tails}.
  - Rolling a dice S = {1, 2, 3, 4, 5, 6}.

* An event is any subset of the Sample Space.

example
  - Rolling an even number on a die : E = {2, 4, 6}.
  - Getting heads in a coin toss : E = {Heads}.

* The probability of an event E, denoted by P(E), is the measure of the likelihood that E will occur. it is defined as:

P(E) = Number of favorable outcomes / Total number of outcomes

example,
  - The probability of getting heads when tossing a coin is.

$$P(\text{Heads}) = 1/2$$

* The probability that an event does not occur is the Complement of the probability that the event does occur. This given by :

$$P(\text{not } A) = 1 - P(A)$$

example: if the probability of rolling 3 on die is $P(3) = 1/6$, the probability of not rolling 3 is:

$$P(\text{not } 3) = 1 - \frac{1}{6} = \frac{5}{6}$$

* Addition rule :-

If two events A and B are mutually exclusive (they can happen at the same time), the probability that either event A or event B occurs is.

$$P(A \cup B) = P(A) + P(B).$$

example:- when rolling a die, the probability of rolling a 3 or a 5 is :

$$P(3 \text{ or } 5) = P(3) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

* Multiplication rule :-

If two events A and B are independent (the occurrence of one does not affect the other), that probability that both events occur is .

$$P(A \cap B) = P(A) \times P(B)$$

example:- when flipping a coin and rolling a six-sided die, the probability of getting heads and rolling a 3 is :

$$P(\text{heads and } 3) = P(\text{heads}) \times P(3) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

* Conditional probability : Conditional probability is the probability of one event occuring given that another event has occured . it is denoted by $P(A|B)$ and is calculated as . $P(A|B) = P(A \cap B) / P(B)$.

- Provided $P(B) > 0$.

example:- in a deck of 52 cards, what is the probability of drawing a king given that the card drawn is a face card (king, queen, or Jack)?

→ The event face card includes 12 cards (4 kings, 4 queens, 4 Jacks). The event "king" includes 4 kings .4 kings

Therefore .

$$P(\text{King} | \text{Face card}) = P(\text{King and Face card}) / P(\text{Face card})$$

$$= (4/52) / (12/52) = \frac{4}{12} = \frac{1}{3}$$

* Law of total Probability: This rule helps calculate the total probability of an event by breaking it into several mutually exclusive events.

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \dots + P(A \cap B_n)$$

(or)

$$P(A) = P(B_1)\,P(A|B_1) + P(B_2)\,P(A|B_2) + \dots + P(B_n)\,P(A|B_n)$$

* Baye's Theorem: Baye's theorem allows us to reverse conditional probabilities. It is given by:

$$P(A|B) = P(B|A)\,P(A) / P(B)$$

Where $P(B)$ can be found using the the law of total probability

2) If the probability of drawing a red card from a deck is 0.5, what is the probability of not drawing a red card?

→ using complement rule.

$$P(\text{not red}) = 1 - P(\text{red})$$

Substitute $P(\text{red}) = 0.5$:

$$P(\text{not red}) = 1 - 0.5$$
$$= 0.5$$

So the probability of not drawing red card is 0.5

3) If the probability of ~~drawing~~ a student passing an exam is 0.85, what is the probability that the student fails.

→ The probability of a student passing an exam is $= 0.85$.

$$P(\text{fail}) = 1 - P(\text{Pass})$$

by using complement rule.

$$P(\text{fail}) = 1 - 0.85 = 0.15$$

4) what is the probability of rolling a number less than 3 or an odd number on a six-sided die?

→ . Numbers less than 3 : {1,2} .

. odd Numbers : {1,3,5} .

The union of these sets is : {1,2,3,5} .

There are 4 favorable outcomes : 1, 2, 3 and 5 .

$$P = \frac{\text{Number of favorable outcomes}}{\text{Total Number of outcomes}} = \frac{4}{6} = \frac{2}{3}$$

5) What is the probability of drawing a red card or a face Card from a standard deck of 52 Cards.

→ Red Cards → 26 red Cards in a deck (13 hearts and 13 diamonds) .

Face Cards → Jack, Queen, and King .

3 face Cards per suit × 4 suits = 12 face cards in total.

The Red face Cards = 6 .

Red Cards = 26 .

Face Cards = 12 .

Total no. of favorable outcomes is,

Red face Cards + Red Cards + Face Cards .

~~26 + 12 + 6~~ = 32

26 + 12 - 6 = 32

$$P = \frac{\text{total no. of favorable outcomes}}{\text{total no of outcomes}} = \frac{32}{52} = \frac{8}{13}$$

6)

→

7)

-

6) What is the probability of rolling a prime number or a multiple of 2 on a six-sided die?

→ Prime number = {3, 5}

Multiple of 2 = {2, 4, 6}.

union = {2, 3, 4, 5, 6}.

$$= \frac{\text{total no/. favorable outcomes}}{\text{total no/. outcomes}} = \frac{5}{6} = 1$$

7) What is the probability of drawing two kings in succession from a standard deck of 52 cards without replacement?

→ 4 kings in 52 cards.

* 1st king.

$$P(\text{first king}) = \frac{4}{52} = \frac{1}{13}.$$

* 2nd king.

$$P(\text{second king}) = \frac{3}{52} = \frac{1}{17}.$$

* 3rd king.

$$P(\text{Third king}) = \frac{2}{52} = \frac{1}{26}$$

* 4th king $= \frac{1}{52} = \frac{1}{52}.$

Calculating the total probability.

$$P(\text{two kings}) = P(\text{first king}) \times P(\text{second king})$$

$$= \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$$

8) What is the probability of drawing a spade (or) a Number card (2-10) from a standard deck of 52 cards?

8)

→ There are 13 Spades in 1 deck.

total no/. of Cards (2-10) = 9.

$9 \times 4 = 36$ Cards.

Total Favorable outcomes.

- total no/. of spades = 13.
- total no/. from (2-10) = 36.
- Subtract the overlap Card = 9.

So $13 + 36 - 9 = 40$

So the probability is $= \frac{40}{52} = \frac{10}{13}$

9) what is the probability of rolling a number greater than 2 or a Multiple of 3 on a Six sided die

→ greater than 2 = {3, 4, 5, 6}.

Multiple of 3 = {3, 6}.

Union = {3, 4, 5, 6}.

~~total no/. of~~ Probability $= \frac{4}{6} = \frac{2}{3}$

10) we roll a six-sided die. what is the probability that the roll is a 4, given that the outcome is greater than 2?

→ Number greater than 2 = {3, 4, 5, 6}.

$\frac{\text{P (favorable outcomes)}}{\text{total no/. of outcomes}} = \frac{1}{4}, = \frac{1}{4}.$

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11) A card is drawn from a standard deck of 52 cards. what is the probability that it is a heart, Given that it is a red card?

→ The total number of red cards = 26.
total number of heart = 13.

$$P = \frac{\text{(total no. of favorable outcome)}}{\text{total no. of outcome}} = \frac{13}{26} = \frac{1}{3}$$

12) A bag contains 3 red, 5 blue, and 2 green balls. if a randomly selected ball is not red, what is the probability that it is blue?

→ Red balls = 3.
Blue Balls = 5
Green Balls = 2.
total no. of balls = 3+5+2 = 10 balls.
Non-Red balls = Blue balls + Green balls = 5+2 = 7 balls

$$\text{Probability} = \frac{\text{Number of blue balls}}{\text{No. of non-red balls}} = 5/7$$

13) We roll two 6-sided dice. what is the probability that the sum is 10. given that at least one die shows a 5?

→ 1st die 5 = {(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)}.
2nd die 5 = {(1,5), (2,5), (3,5), (4,5), (5,5), (6,5)}.

total no. of outcomes = {(5,1), (5,2) (5,3) (5,4) (5,5), (5,6), (1,5) (2,5) (3,5) (4,5) (6,5)}
= 11

$$\frac{\text{total no. of favorable outcomes}}{\text{total outcome}} = \frac{1}{11}$$

14) Explain the Components of Bayesian Statistics Compon

→ • Bayesian statistics is a probabilistic framework, that blends prior beliefs with observed data to update and refine our understanding of uncertainty.

• This approach is particularly powerful in situations with limited data, or when incorporating existing knowledge is essential, offering a flexible and continuous learning model that empowers decision-making in diverse fields like medicine, finance and Machine learning.

* Prior Distribution.
* Likelihood distribution
* posterior distribution.

* Prior distribution :- These represent your initial beliefs about a parameter or variable before observing any d... They can be subjective or objective. Choosing an appropriate prior is crucial, as it influences the final outcome, but the beauty of Bayesian method l... in their flexibility to handle even vague or subjecti... priors.

* Likelihood distribution :- This quantifies the probabili... of observing the actual data given a specific value of parameter you're interested in. It acts as a bridge betw... your beliefs and the observed reality, telling you how w... your hypothesis explains the data.

* Posterior distribution :- The culmination of the Bayesian. the posterior distribution reflects your updated belief after considering both prior knowledge and observed evidence. It's a powerful tool for summarizing uncertainty, providing not just a point estimate for the parameter but also a range of plausible values with their associated probabilities.

15) State and Prove Baye's Theorem.

→ if $A_1, A_2 \ldots A_n$ are mutually disjoint events with $P(A_i) \neq 0$. then for any orbitory event E which is subset of $\cup_i A_i$ such that $P(E) > 0$,

we have $$P(A_i/E) = \frac{P(A_i)\, P(E/A_i)}{\Sigma P(A_i)\, P(E/A_i)}$$

Proof:- $A_1 \cap A_2 \cap \ldots \cap A_n = \phi$.

$$E \subset \bigcup_{i=1}^{n} A_i.$$

To prove :- $$P(A_i/E) = \frac{P(A_i)\, P(E/A_i)}{\Sigma P(A_i)\, P(E/A_i)}.$$

$$E = E \cap \cup A_i = \cup (E \cap A_i)$$

$$P(E) = \sum_{i=1}^{n} P(E \cap A_i)$$

$$= \Sigma P(A_i) P(E/A_i) \rightarrow ①.$$

By Conditional probability.

$$P(A_i/E) = \frac{P(A_i \cap E)}{P(E)}$$

$$P(A_i/E) = \frac{P(A_i)\, P(E/A_i)}{\Sigma P(A_i)\, P(E/A_i)} \quad (\because \text{By } ①)$$

Hence proved.

16) A Coin is tossed 5 times. what is the probability of getting atleast 3 heads?

→ A Coin has 2 outcomes Head or Tail.

Possible outcomes:- each coin has 2 possible outcomes (H or T). Since the coin is tossed 5 times

$$2 \times 2 \times 2 \times 2 \times 2 = 32.$$

+ 3 heads. $\frac{5}{3} = 10$ ways.

+ 4 heads $\frac{5}{4} = 5$ ways

+ 5 heads $\frac{5}{5} = 1$ way.

total no/- of favorable outcomes are $= 16$

$$\frac{16}{32} = \frac{1}{2} = 50\%$$

17) A biased coin, where the probability of getting heads is 0.6, is flipped 4 times. what is the probability of getting ~~i) no success~~ ii) ~~exactly 3 success~~ ~~iii) at most 4~~ exactly 2 heads.

→ Binomial probability.

$$P(\text{exactly } k \text{ heads}) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$$

$$P(2 \text{ heads}) = \binom{4}{2} \cdot (0.6)^2 \cdot (0.4)^2$$

Combinations $\binom{4}{2} = 6$

Probability of heads $(0.6)^2 = 0.36$.

Probability of tails $(0.4)^2 = 0.16$.

$$P(2 \text{ heads}) = 6 \times 0.36 \times 0.16 = 0.3456$$

or 34.56%

18) A pair of dice is rolled 8 times, and rolling a sum of 7 is considered a success. what is the probability of getting i) no success ii) exactly 3 success iii) at most 4 success.

→ Probability of success (sum of 7)

36 possible ways.

To get sum as 7, $= \{(1,6)\ (2,5)\ (3,4)\ (4,3)\ (5,2)\ (6,1)$

$= 6$ outcomes.

$$P(\text{success}) = \frac{6}{36} = \frac{1}{6}$$

$$P(\text{failure}) = 1 - P(\text{success}) = 1 - \frac{1}{6} = \frac{5}{6}$$

i) $k=0$.

$$P(0\text{ success}) = \binom{8}{0} \cdot \left(\frac{1}{6}\right)^0 \cdot \left(\frac{5}{6}\right)^8 = 0.0232$$

ii) exactly 3 success $k=3$.

$$P(3\text{ success}) = \binom{8}{3} \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right)^5 = 0.104$$

iii) 4 success, $K = 0, 1, 2, 3, 4$

P (at most 4 success) = P(0 success) + P(1 success) + P(2 success) + P(3 success) + P(4 success)

$P(x \geq 4) = P(X=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$P(x=0) = 0.0232$

$P(x=1) = {}^8C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{8-1} = 8 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^7 = \frac{8}{6} \times \frac{78125}{279932}$

$= 625000/1679592 = 0.372$

$P(x=2) = {}^8C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{8-2} = 28 \times \frac{1}{36} \times \left(\frac{5}{6}\right)^6 = \frac{28}{36} \times \frac{5625}{46652}$

$= 0.7778 \times 0.3349 = 0.2604$

$P(x=3) = 0.104$

$P(x=4) = {}^8C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^{8-4} = 70 \times \frac{1}{1296} \times \frac{625}{1296} = \frac{43750}{1679616} = 0.026$

$P(x \geq 4) = 0.0232 + 0.372 + 0.2604 + 0.104 + 0.026$

$= 0.994$

19) A basketball player has a 70% chance of making a free throw. if they take 5 shots, what is the probability that that the make at least 3 shots.

→ $p = 0.7$, $n = 5$, $1 - p = 1 - 0.7 = 0.3$,

P (at least 3 shots) ⇒ $P(x \geq 3) = P(x=3) + P(x=4) + P(x=5)$

$P(x=3) = {}^5C_3 (0.7)^3 (0.3)^{5-3} = 10 \times 0.343 \times (0.3)^2 = 10 \times 0.343 \times 0.09 = 0.3087$

$P(x=4) = {}^5C_4 (0.7)^4 (0.3)^1 = 10 \times 0.2401 \times 0.3 = 0.36015$

$P(x=5) = {}^5C_5 (0.7)^5 (0.3)^0 = 1 \times 0.16807 \times 1 = 0.16807$

$P(x \geq 3) = 0.3087 + 0.36015 + 0.16807$

$= 0.83692$

20) A Factory produces 10% defective items. if 6 items are randomly selected, what is the probability that
i) none are defective. ii) exactly 2 are defective
iii) atmost 3 are defective

→ $P = 0.1, \quad n = 6, \quad 1-P = 1 - 0.1 = 0.9$

i) $P(x=0) = {}^6C_0 (0.1)^0 (0.9)^6 = 1 \times 1 \times 0.531441 = 0.531441$

ii) $P(x=2) = {}^6C_2 (0.1)^2 (0.9)^{6-2} = 15 \times 0.01 \times (0.9)^4 = 15 \times 0.01 \times 0.6561 = 0.098415$

iii) $P(x \geq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$P(x=0) = 0.531441$

$P(x=1) = {}^6C_1 (0.1)^1 (0.9)^{6-1} = 6 \times 0.1 \times (0.9)^5 = 0.6 \times 0.59049 = 0.354294$

$P(x=2) = 0.098415$

$P(x=3) = {}^6C_3 (0.1)^3 (0.9)^{6-3} = 20 \times 0.001 \times (0.9)^3 = 0.02 \times 0.729 = 0.01458$

$P(x \geq 3) = 0.531441 + 0.354294 + 0.098415 + 0.01458 = 0.998$

21) Calculate the mean of the poisson distribution, given that the no of trails is 25 and the probability of success is 0.4.

→ $n = 25, \; P = 0.4$

Mean $(\lambda) = n.p = 25 \times 0.4 = 10$

22) A factory produces light bulbs, and the probability of a defective bulb is 0.05. If 50 bulbs are tested, what is the mean number of defective bulbs.

→ $n = 50, \; P = 0.05, \; \lambda = n.p = 50 \times 0.05 = 2.5$

23) On average, a call center recieves calls per hour what is the probability that they recieve exactly 10 calls in an hour?

→ $\lambda = 12, \; P(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}, \quad e = 2.7182$

$P(x=10) = \dfrac{(12)^{10} e^{-(12)}}{10!}$

$= \dfrac{61917364224}{10!} \times \dfrac{1}{e^{12}} = 0.104$

24) A traffic light on a busy road recieves an average of 8 cars per minute. what is the mean number of cars passing in a 5-minute interval?

$\rightarrow \lambda = 8.$ λ 5 minutes $= 8 \times 5 = 40$

25) The no of typing errors a typist makes per page follows a poisson distribution with a mean of 2. what is the probability that a randomly selected page has exactly 3 errors.

$\rightarrow \lambda = 2, \quad P(x=3) = \frac{2^3 e^{-2}}{3!} = \frac{8}{6} \times \frac{1}{e^2} = \frac{8}{6 \times (2.7182)^2}$

$\frac{8}{44.33166744} = 0.180$

26) A Random Variable x follows a uniform distribution over the interval $[-3, 5]$

a) find k for which $P(x > k) = 1/3$

b) Evalute $P(x < 2)$

c) Find $P(|x - 0.5| < 1.5)$

$\rightarrow f(x) = \frac{1}{b-a} = \frac{1}{5-(-3)} = \frac{1}{8}$ $(a, b) = [-3, 5]$

$(c, d) = (k, 5)$

a) $P(x > k) + P(x \leq k) = 1$

$P(x > k) = 1 - P(x \leq k)$

$= \int_k^5 f(x) dx = \frac{1}{8} \int_k^5 dx \Rightarrow \frac{1}{8} (x)_k^5$

$= \frac{1}{8}(5-k) = \frac{1}{3}$

$= \frac{1}{8}(5-k) = 1/3$

$= \frac{1}{8}(5-k) = \frac{1}{3} \Rightarrow 5-k = \frac{8}{3} \Rightarrow k = 5 - \frac{8}{3}$

$= k = 15/3 - 8/3 \Rightarrow \boxed{k = 7/3}$

b) evalute $P(x < 2)$

$P(x < 2) = \frac{2-(-3)}{5-(-3)} = \frac{2+3}{5+3} \Rightarrow \frac{5}{8} = 0.625$

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c) $P(|x-0.5| < 1.5)$

$$|x-0.5| < 1.5$$

$$-1.5 < x-0.5 < 1.5$$

$$-1.5+0.5 < x < 1.5+0.5$$

$$-1 < x < 2$$

Calculate the proportion of the interval from -1 to 2

$$P(-1<x<2) = \frac{2-(-1)}{5-(-3)} \Rightarrow \frac{2+1}{5+3} = \frac{3}{8} = 0.375$$

27) A random variable x follows a uniform distribution over the interval [1,9] find the probability that x lies between 3 and 7.

$[a,b] = [1,9]$

$[c,d] = [3,7]$

$$\rightarrow P(3<x<7) = \frac{d-c}{b-a}$$

$$= \frac{7-3}{9-1} = \frac{4}{8} = \frac{1}{2} = 0.5$$

28) A Random Variable x follows a uniform distribution over the interval [5,15]. Determine the probability that x is between 6 and 12.

$[a,b] = [5,15]$

$[c,d] = [6,12]$

$$\rightarrow P(6<x<7) = \frac{d-c}{b-a}$$

$$= \frac{12-6}{15-5}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

$$= 0.6$$

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29) Standard Normal distribution.

a) what is the probability that z lies between 0 and 1.85?.

b) what is the probability that z is greater than 1.60?.

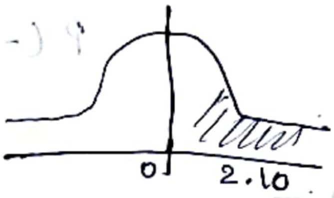
c) what is the probability that z is greater than −1.10?.

d) what is the probability that z is less than −0.80?.

e) what is the probability that z is less than 2.10?.

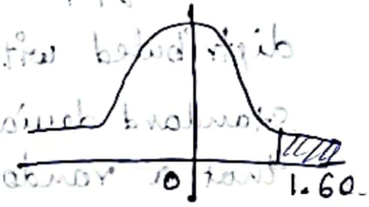
→

a) $P(0 < Z < 1.85) = 0.4678$



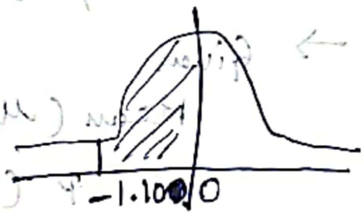
b) $P(Z > 1.60) = 1 - P(Z \le 1.60)$

$= 1 - 0.9452 = 0.0548$

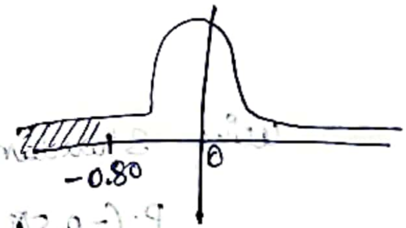


c) $P(Z > -1.10) = 1 - P(Z \le -1.10)$

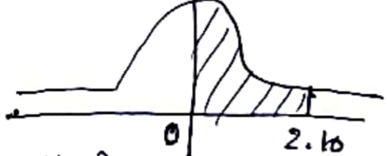
$= 1 - 0.1357 = 0.8643$



d) $P(Z < -0.80) = 0.2119$



e) $P(Z < 2.10) = 0.9821$



30) The heights of students in a college are normally distributed with a mean height of 170 cm and a Student deviation of 10cm. what is the probability that a randomly selected student has a height between 165cm and 190cm?

→

→ Given.

mean (μ) = 170 cm; SD = 10 cm

$$P(165 < x\ 190) = P\left(\frac{165-170}{10} < z < \frac{190-170}{10}\right)$$

$$= P(-0.5 < z < 2)$$

using standard Normal distribution:-

$$P(-0.5 < z < 2) = 0.9772 - 0.3085$$

$$= 0.6687$$

31) The lifespan of mobile phone batteries are normally distributed with a mean lifespan of 500 hours and a standard deviation of 50 hours. What is the probability that a randomly selected battery lasts between 475 hours and 540 hours?

→ Given.

Mean (μ) = 500 hours, SD = 50 hours.

$$P(475 < x < 540) = \left(\frac{475-500}{50} < z < \frac{540-500}{50}\right)$$

$$= P(-0.5 < z < 0.8)$$

using Standard Normal distribution.

$$P(-0.5 < z < 0.8) = 0.7881 - 0.3085$$

$$= 0.4796$$