1A:- given,

$a \in \mathbb{Z}_p$

$(a+p)^n \pmod p = a^n \pmod p$

$$\left({}^nC_0\, a^0 p^n + {}^nC_1\, a^1 p^{n-1} + {}^nC_2\, a^2 p^{n-2} \ldots \ldots + {}^nC_n\, a^n p^0 \right) \bmod p$$

$$= (0 + 0 + \ldots + 0 + a^n) \bmod p$$

$$= a^n \bmod p$$

2A:-

$\mathbb{Z}_5$:- {

$a = \{1, 2, 3, 4\}$

$a^{-1} = \{1, 3, 2, 4\}$

$\mathbb{Z}_{11}$:-

$a = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$a^{-1} = \{1, 6, 4, 3, 9, 2, 8, 7, 5, 10\}$

3A:- euclidean algorithm to find gcd:

gcd(56245, 43159) = ?

$56245 = 1 \times 43159 + 13086$

$43159 = 3 \times 13086 + 3901$

$13086 = 3 \times 3901 + 1383$

$3901 = 2 \times 1383 + 1135$

$1383 = 1 \times 1135 + 248$

$1135 = 4 \times 248 + 143$

$248 = 1 \times 143 + 105$

$143 = 1 \times 105 + 38$

$105 = 2 \times 38 + 29$

$38 = 1 \times 29 + 9$

$29 = 3 \times 9 + 2$

$9 = 4 \times 2 + 1$

$2 = 2 \times 1 + 0$

$\boxed{\therefore \text{gcd} = 1}$

$\boxed{\therefore gcd = 1}$

4A:- $\phi(3^4)$

$\because$ 3 is a prime, w.k.t $\phi(p^e) = p^e - p^{e-1}$

$\Rightarrow \phi(3^4) = 3^4 - 3^{4-1}$

$= 3^4 - 3^3$

$= 3^3(3-1)$

$= 27 \times 2 = \boxed{54}$

$\phi(2^{10}) = 2^{10} - 2^9$

$= 1024 - 512$

$= 512$

5A:-

$3^{100} \bmod (31319)$

$100 = 1100100$

$= 2^6 + 2^5 + 2^2$

$(3)^{100} = (3)^{2^6 + 2^5 + 2^2}$

$= (3)^{2^6} \times (3)^{2^5} \times (3)^{2^2}$

$3^{100} (\bmod 31319) = \left((3)^{2^6} \times (3)^{2^5} \times (3)^{2^2}\right) (\bmod 3131$

$(3)^{2^0} \pmod{31319} = 3$

$(3)^{2^1} \equiv (3^{2^0})^2$

$= 9$

$\equiv 9 \pmod{31319}$

$(3)^{2^2} = (3^{2^1})^2$

$\equiv 9^2 \pmod{31319}$

$\equiv 81 \bmod (31319)$

$(3)^{2^3} = (3^{2^2})^2$

$= (81)^2 \pmod{31319}$

$= 6561 \pmod{31319}$

$\cancel{\equiv 85}$

$(3)^{2^4} = (3^{2^3})^2$

$= (6561)^2 \pmod{31319}$

$= 14415$

$(3)^{2^5} = (3^{2^4})^2 = (14415)^2 \pmod{31319}$

$= 207792225 \pmod{31319}$

$= 21979$