PST exercise 3

Part 1

① $$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^3(a)}{3!}(x-a)^3 + \dots$$

a) $f(x_{k+1}) = f(x_k) + f'(x_k)(x_{k+1} - x_k) + \frac{f''(x_k)}{2!}(x_{k+1} - x_k)^2 + \dots$

b) $f(x_{k+1}) = f(x_k) + f'(x_k)(x_{k+1} - x_k) + O((x_{k+1} - x_k)^2)$

c) $f(x_{k+1}) = 0$ and drop residual part

$\Rightarrow 0 = f(x_k) + f'(x_k)(x_{k+1} - x_k)$

d) $\Rightarrow f'(x_k)(x_{k+1} - x_k) = -f(x_k)$

$\Rightarrow x_{k+1} - x_k = -\frac{f(x_k)}{f'(x_k)} \Rightarrow x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

②

a) $y = mx + c \Rightarrow f(x_k) = m\,x_k + c$

$m =$ slope $\Rightarrow f'(x_k)$ $\quad c =$ intercept

$\Rightarrow c = f(x_k) - f'(x_k)x_k$

b) use the slope-point form: $y - y_1 = m(x - x_1)$

$m =$ slope $= f'(x_k))$ $\quad y = f(x_{k+1}) = 0 \quad x = x_{k+1}$

$y_1 = f(x_k) \quad x_1 = x_k$

$\Rightarrow -f(x_k) = f'(x_k)(x_{k+1} - x_k)$

$\Rightarrow x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

Part 2

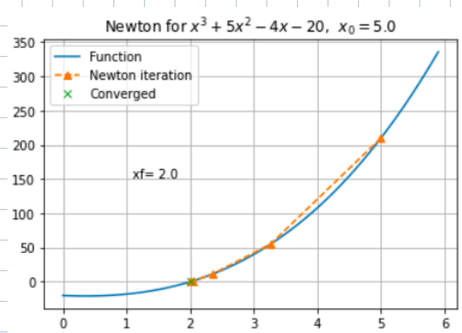
1)

```python
def Newton1d(f, df, x0, tol, maxit=50 ):
    xk=x0  # init
    k=0
    xlist=[x0] # to store all x for plotting
    while k<maxit:
        k += 1  # iteration counter
        xk1 = xk- (f(xk)/df(xk)) #newtons
        print(xk1)

        if abs(xk1-xk)<tol: # check convergance
            break
        xk=xk1 #update values
        xlist.append(xk1)


    if (k>=maxit):
        print("Iterations reached limit") #warn if not converged

    return xk1, xlist
```

given PST3_Part2_def.py

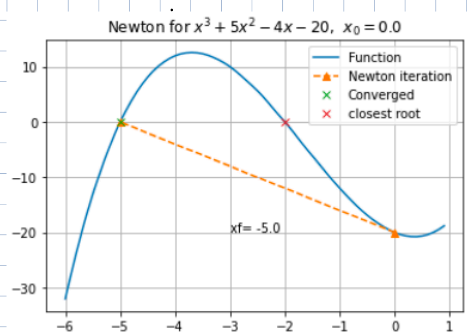
2) all given file PST3_Part2_2.py

a)



converge to $x^* = 2.0$

b)



converges to

$x^* = -5.0$

this is because

$$f(0) = -20$$
$$f'(0) = -4$$

$$\Rightarrow \quad x_{k+1} = -\frac{-20}{-4} = \underline{\underline{-5}}$$

wich in the next iteration will give $x_{k+1} = x_k$

wich is in tolerance and stop

3)

a) it does not converge as each iteration

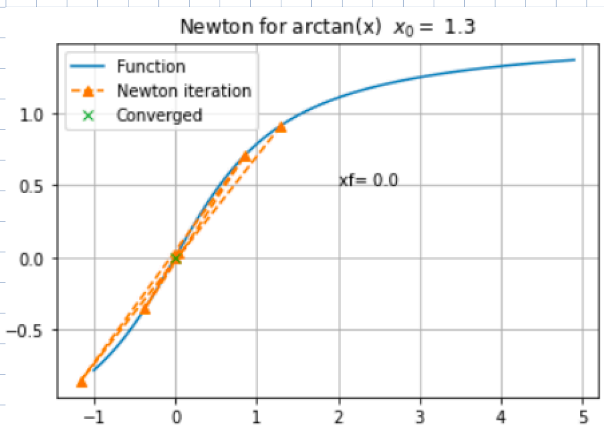
moves the next x further from the

root x in either direction

this is because the x^2 term in the denominator

in the differential term

b) yes $|x_0| < 1.3$ converges



Part 3

1)

$$f(x) = e^{\sin(x)}$$

$$f'(x) = \cos(x)\, e^{\sin(x)} \quad \text{chain rule}$$

$$f''(x) = \cos^2(x)\, e^{\sin(x)} - \sin(x)\, e^{\sin(x)} \quad \text{product rule}$$

2) this code compares the differential given from the forward to the true differentials to compare and plots the error at a given h

the error increases at a point as the precision is high reach the maximum computer error which causes error in rounding

3)

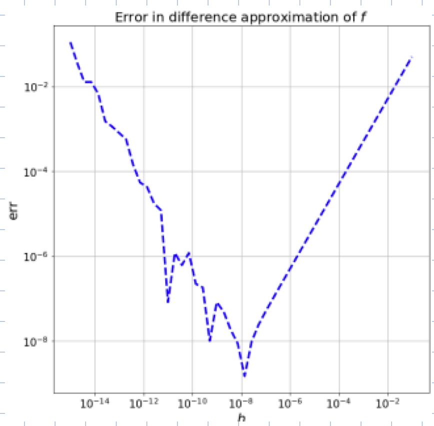
$$h = 2\sqrt{\varepsilon \frac{|f(x)|}{|f''(x)|}}$$

$f(0) = 1$

$f''(0) = 1$

using $\varepsilon = 2.2 \cdot 10^{-16}$

$h \approx 2.97 \cdot 10^{-9}$



looks correct

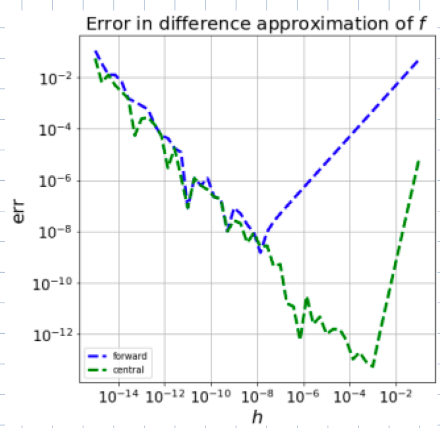
4) finding the minimum in the error array:

```
print(f'minimum: {np.min(err_fd)}')
```

```
minimum: 1.4874659282071434e-09
```

5)

a)



b)

```
minimum: 5.46229728115577e-14
```

c) approx $h = 10^{-3}$