Written HW 1

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1.

a. the program ends when $i \geq n$

let i_x be the value of i after x iterations

$i_0 = 2 \quad i_1 = 2^2 \quad i_2 = 2^4 \quad i_3 = 2^8 \quad i_4 = 2^{16}$

these can be rewritten as

$i_0 = 2^{2^0} \quad i_1 = 2^{2^1} \quad i_2 = 2^{2^2} \quad i_3 = 2^{2^3} \quad i_4 = 2^{2^4}$

$\therefore \quad i_x = 2^{2^x}$

since the loop stops when $i_x \geq n$

$2^{2^x} \geq n$

$\Rightarrow 2^x \geq \log_2 n$

$\Rightarrow x \geq \log_2 \log_2 n \quad \therefore x = \boxed{\Theta(\log(\log n))}$

b. the if statement activates when i is a multiple of $\sqrt{n}$

this means the inner loop will run $\sqrt{n}$ times at maximum until $i = n$

let $t = \sqrt{n}$

$\therefore$ $i = k\sqrt{n}$ where $k = 1, 2, 3, \ldots t$

$\therefore$ each loop does $(k\sqrt{n})^3$ work inside

$$\therefore = \sum_{k=1}^{t} (k\sqrt{n})^3 = \sqrt{n}^3 \sum_{k=1}^{t} k^3 \qquad \sum_{k=1}^{t} k^3 = \left(\frac{t(t+1)}{2}\right)^2$$

$$= \left(\frac{t^2+t}{2}\right)^2$$

$$= \Theta(t^4)$$

only relevant term

$$\therefore = (\sqrt{n})^3 \cdot \Theta(t^4)$$

since $t = \sqrt{n}$

$$= (\sqrt{n})^7 = (n^{\frac{1}{2}})^7$$

$$= n^{\frac{7}{2}}$$

$$\boxed{\Theta(n^{\frac{7}{2}})}$$

C. each outer for loop runs n times so together cost $O(n^2)$

for each loop the number of times the if statement activates is once for each i $\therefore$ it runs at most n times

the loop within that runs log n times everytime the if statement activates as the increment is m+m or 2m

$\therefore$ the inner loop is $O(n \log n)$

$\therefore O(n^2) + O(n \log n)$

for Θ only n^2 is relevant

$\therefore \boxed{\Theta(n^2)}$

d. the outer loop iterates n times making that O(n)

when size is hit the new array is created since this only occurs sometimes copying the new array takes O(n)

$$\therefore = O(n) + O(n)$$

$$= O(2n) \Rightarrow O(n)$$

$$\boxed{= \Theta(n)}$$

2. a. llrec (1, 5)

then 1->next = llrec (5, 2)

which sets 5->next = llrec(2, 6)
which sets 2->next = llrec(6, 3)
which sets 6->next = llrec(3, nullptr)
b/c there is no node linked after to 6
which returns
3 as 6->next
& 4 is already linked to 3
then llrec(6, 3) returns 6 as 2->next
then llrec(2, 6) returns 2 as 5->next
and llrec(5, 2) returns 5 as 1->next
∴ the final list is ∴ $\boxed{1, 5, 2, 6, 3, 4}$

b. with in1 = nullptr
and in2 = 2

the function would just return in2

in this case in2 is a single node

$\boxed{2}$