POZNAŃ UNIVERSITY
OF ECONOMICS
AND BUSINESS

Taras Kryvyy

Credit Valuation Adjustments for Derivative Contracts

Korekty Wyceny Kredytowej dla Kontraktów Pochodnych

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Thesis Supervisor: dr Marcin Bartkowiak

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Abstract

Credit crunch of late 2000th and European sovereign debt crisis of early 2010th plainly showed significance of counterparty credit risk. Credit valuation adjustment (CVA) is a price of losses arising from such risk. Nowadays, every major bank has a desk dedicated to trading and hedging CVA and similar valuation adjustments (so-called xVA). As financial institutions get more and more regulated, a need for a systematized quantification of these adjustments emerged.

This thesis covers an entire process of CVA calculation from estimation of default probabilities of a counterparty, through generation of exposures, to analysis of netting effect on CVA of the derivative instruments portfolio. Each step is illustrated by sample calculations based on anonymized market data provided during Financial Engineering Project held in summer semester 2019 at Poznan University of Economics and Business. Provided data includes smoothed zero-coupon bond price curves of EUR and PLN, EUR\PLN FX rate and several tenors of CDS spreads. The calculations assume that the dealer is a Poland-based financial institution while the counterparty is an entity operating in Eurozone.

The portfolio of analyzed derivatives are FX forward, interest rate swap, cross-currency interest rate swap and interest rate cap. The thesis demonstrates that (i) unilateral CVA is linearly dependent on CDS spreads used for extraction of default probabilities; (ii) growth of assumed recovery rate has a limited effect on unilateral CVA, as long as the recovery rate is within reasonable boundaries; (iii) netting significantly reduces counterparty credit risk.

Contents

Preface

The main goal of this thesis is to develop an end-to-end model to price a unilateral Credit Valuation Adjustment of a portfolio of derivative instruments traded with the same counterparty using the real market data.

The thesis is testing a hypothesis that netting exposures of the trades before scenario aggregation lowers counterparty credit risk.

The first chapter explores the historical background of the counterparty credit risk by describing main credit events of the last several decades and giving some theoretical reasons for the adjustments of the risk-neutral prices. The counterparty credit risk is formally defined and a high-level overview of CVA is given.

Chapter 2 briefly goes through the main methods of default probability estimation and then dives into mechanics & pricing of a CDS, and a CDS bootstrapping algorithm, implementation of which is available in the Appendix.

The third chapter introduces the analyzed derivative instruments: their mechanics, payoffs and pricing formulas. Those derivatives are FX forward rate, interest rate swap, cross-currency interest rate swap and interest rate cap.

Chapter 4 defines major risk factors of the derivatives in question and proposes a calibration methodology. Since the biggest risk factor is the interest rate, main interest rate models are overviewed.

Chapter 5 is devoted to exposure generation and aggregation. Approaches to exposure calculation are established, focusing on Monte Carlo simulation method. Analysis of exposure profiles is presented.

The final, sixth chapter delves into the unilateral CVA. It formally introduces the adjustment and then, imposing some assumptions, displays a simplified CVA formula. Using the latter, CVA of each separate instrument as well as the netted portfolio is given. The chapter ends with sensitivity analysis of the netted portfolio CVA.

1. Introduction

1.1. Historical perspective

Failure of the Long-Term Capital Management (LTCM) (1998) brought a lot of attention to counterparty risk. LTCM was a significant counterparty to all major banks, and Federal Reserve had to organize a bailout due to fear of chain reaction. As a result, Basel 2 (2004) had clearly laid out counterparty risk capital requirements within one of its “pillars”.

At the same time, with development of derivatives markets and defaults of such major clients as Enron (2001) and WorldCom (2002), large banks came to realize the importance of quantifying and allocating credit related losses. That is how the first CVA (Credit Valuation Adjustment) desks emerged (Gregory, 2015). The idea was to calculate historical default probability of a counterparty and then to incorporate it in a profit of a transaction. With passage of time, accounting standards started requiring “fair value” of a derivative that should have included the party’s own default risk as well. CVA’ mirror twin DVA (Debt Valuation Adjustment) came to existence.

In 2007, a global financial crisis started. Beginning as a US subprime mortgage market collapse, it had suddenly grown into the international banking crisis involving bankruptcy of a major investment bank Lehman Brothers (2008). Prior to the crisis, many financial institutions were known to take excessive risks that were insufficiently backed by the capital. As a result, governments attempted to preserve the financial system from failing, and massive bailouts took place around the world. Hence, the responsibility for the extra risk appetite effectively transferred to the taxpayers. Resulting economic recession that affected almost the whole world lasted for years.

Shortly after, a multi-year European debt crisis (2009) with Greece, Portugal, Ireland, Spain and Cyprus not being able to repay or refinance their government debt took place. That situation clearly strengthened an argument against assumptions of a risk-free counterparty even if it is a sovereign one.

In the aftermath of the global financial crisis, it became obvious that banks should be subject to much stricter regulation. There was little doubt that those “too big to fail” should not be allowed to fail in the first place and should therefore be even more closely controlled. In order to implement such attitude, a new regulatory framework Basel III emerged. It mostly focused on over-the-counter (OTC) derivatives market, which is highly exposed to counterparty and liquidity risk. In order to tackle counterparty risk, mandatory central clearing, bilateral rules towards collateral posting and stricter capital charges appeared. Net stable funding ratio and leverage ratio were aiming at liquidity risk (Gregory, 2015).

Alongside the regulatory shift, banks recognized importance of valuation of the credit, funding and regulatory capital requirements set in derivative transactions. Consequently, they introduced CVA and DVA to quantify credit risk, FVA (Funding Valuation Adjustment) to account for funding costs that consist of MVA (Margin Valuation Adjustment) to account for initial margin funding costs and KVA (Capital Valuation Adjustment) for Regulatory Capital funding costs, and TVA (Tax Valuation Adjustment) to assess taxation impact on P&L. In fact, those adjustments turned out to have a significant effect on value of a derivatives portfolio.

One can interpret these adjustments as partial side effects of numerous simplifying assumptions embedding Black-Scholes-Merton risk-neutral valuation framework. Authors have been gradually relaxing those assumptions (Green, 2015):

- Constant interest rates – Merton (1973)
- No dividends – Merton (1973)
- No transaction costs – Ingersoll (1976)
- Jumps – Merton (1976), Cox and Ross (1976)

The original Nobel award winning partial differential equation eventually became a basis of a fundamental theorem of asset pricing by Harrison and Pliska (1981) which assumes that arbitrage opportunities do not exist and that market is complete. Market completeness holds true when every contingent claim (e.g. option) can be replicated with another financial instrument with the same characteristics. In other words, the survival of this approach depends on law of one price, which states that if there is no arbitrage opportunity, the same asset should have the same price on every market.

However, according to Green (2015), the law of one price is not applicable to over-the-counter (OTC) derivatives with following arguments:

- ISDA (International Swaps and Derivatives Association) and CSA (Credit Support Annex) legal rules considerably vary, and that effectively makes each OTC trade unique.
- Even the strongest CSA terms are not eliminating counterparty risk entirely, which means that each derivative with a different counterparty is uniquely valued
- Asymmetric access to markets of counterparties
- Different accounting approaches lead to different valuation

In fact, only liquid exchange-traded derivatives such as futures and exchange-traded options can fulfill law of one price criteria.

On top of that, post-crisis world turned out to be even more hostile to risk-neutral valuation framework and its assumptions. OIS (Overnight Index Swap) replaced LIBOR as a hypothetical “risk free” interest rate, and banks could no longer borrow money unsecured at that rate as their funding costs soared up. Regulatory capital requirements made cost of trading derivatives material. (Green, 2015)

As a result, post-crisis construction of derivative prices underwent significant changes compared to the pre-crisis one. Before 2007, the price typically consisted of a LIBOR-discounted risk-neutral price, hedging costs, mostly unilateral CVA and a profit itself. However, after 2009 the derivative price included OIS discounted risk-neutral price, hedging costs, accountants’ suggested bilateral CVA&DVA and profit. In addition to that, the post-crisis prices contained FVA (encompassed by liquidity buffer cost), KVA, which was capital’s lifetime cost, MVA/CCP costs and TVA defined as tax on profits/losses. (Green, 2015)

1.2. Counterparty risk

Basel is a set of recommendation on banking regulation issued by the Basel Committee on Banking Supervision. Basel II and Basel III are mostly concerned with capital requirements for banks. In fact, banks tend to set aside as little capital as possible in order to use the remaining funds for their activity and for liquidity as well. Three main types of risk are taken into account: credit (or counterparty) risk, market risk and operational risk. Counterparty risk can be measured under three levels of increasing complexity: standardized approach, foundation of internal rating-based approach (IRBA) and advanced IRBA. As standardized approach to calculation tends to return the highest capital requirements, banks have an incentive to develop their own models that are more sophisticated. (Crépey, Bielecki, & Brigo, 2014)

The counterparty credit risk is defined as the risk that the counterparty to a transaction could default before the final settlement of the transaction's cash flows. An economic loss would occur if the transactions or portfolio of transactions with the counterparty has a positive economic value at the time of default. (Basel II, Annex 4, 2/A, 2006)

Counterparty risk can be viewed from two different perspectives:

- From capital requirements measurement perspective (Basel II and credit Value-at-Risk) which is calculated within $\mathbb{P}$ probability measure

- From pricing perspective. Adjustment of a risk-neutral price of a derivative instrument charged to a specific counterparty resulting from the counterparty's default risk. It is called credit valuation adjustment or CVA and is calculated within $\mathbb{Q}$ probability measure.

As CVA is calculated solely within $\mathbb{Q}$ probability measure and not within $\mathbb{P}$ one, the final price does not include risk statistics. Further information can be found in Duffie & Huang (1996), Bielecki & Rutkowski (2001), Brigo & Masetti (2005). (Crépey, Bielecki, & Brigo, 2014)

1.3. Introduction of CVA

CVA is a measure of the market's counterparty risk embedded in bilateral transactions. In terms of pricing, CVA is a market value of counterparty risk. One can see in Equation 19 that a CVA of a trade is a difference between Risk-neutral price and Price with counterparty risk, where condition "$including\ default$" constitutes possible default events.

$$\begin{aligned} CVA(trade) = {} & Price(trade|no\ default) \\ & - Price\ (trade|including\ default) \end{aligned} \tag{1}$$

CVA formula bases on three key components:

1. P*robabilities of counterparty's default.* A term structure of market-implied Probability of Default (PD) must be estimated from credit spreads observed in the market. It is generally given by a survival probability curve $\left(P(t)\right)_{t\in[0,T]}: P(t) = P(\tau \geq t), \tau$ is a random default time. There is also an associated curve $\left(\bar{P}(t)\right)_{t\in[0,T]}: \bar{P}(t) = 1 - P(t)$ with probability of default prior to time t.

2. *Exposure at default.* Future exposure paths produced by pricing all trade in the portfolio as a stochastic process $(E_t)_{t\in[0,T]}$. E_t is non-negative amount of loss resulting from counterparty default. E_t can potentially include mitigation mechanisms (e.g. netting, collateral and margin).
3. *Recovery rate.* Percentage number (R) denoting the share of loss recovered at default generally based on trader marks. R takes values from zero to 100 percent, typically in range between 20 and 50 percent.

One can separate CVA into unilateral and bilateral cases:

Unilateral Case. In a situation when both sides agree that one of the sides is default-free, unilateral valuation takes place. If risky counterparty defaults, the default-free institution will bear losses because the counterparty would not fulfill its payment obligations. So, from the default-free institution's perspective, a positive value is being subtracted from its payment obligations as a premium for dealing with a risky counterparty. It is known as unilateral CVA (UCVA). On the other hand, if risky counterparty defaults, they would no longer have to fulfill their payments obligations and hence would gain. From the risky counterparty's perspective this value is being added to its payment obligations. This is known as unilateral DVA (UDVA). As a result, $UCVA(institution) = UDVA(counterparty)$ since the amount of adjustment is the same for both parties and the difference is in the sign. Because the institution is assumed not to bear risk, $UDVA(institution) = UCVA(counterparty) = 0$.

Bilateral Case. In case the parties mutually do not recognize each other as default free, the only way to agree on price is to systematically introduce their default probabilities into the valuation. As previously, $CVA(institution) = DVA(counterparty)$, however $DVA(institution) = CVA(counterparty)$ becomes a positive amount. Let us denote MtM as a default risk-free price of the deal. The final price will be computed as following:

$Final\ Price = MtM - CVA + DVA$, and hence

$Price\ To\ The\ Institution = MtM(Institution) - CVA(Institution) + DVA(institution)$ and similarly

$Price\ To\ The\ Counterparty = MtM(Counterparty) - CVA(Counterparty) + DVA(Counterparty)$. Obviously,

$MtM(Institution) = -MtM(Counterparty)$, and according to previous statement about CVA-DVA parity we are getting

$Price\ To\ The\ Institution = -Price\ To\ The\ Counterparty$, which leads to consensus over the price. (Crépey, Bielecki, & Brigo, 2014)

2. Implied default probability

Estimation of default probabilities of the counterparty throughout the derivative transaction lifetime is an integral part of CVA calculation. One can obtain them from rating agencies, bond prices, Credit Default Swaps (CDS) and other instruments/methodologies.

There are three major rating agencies: Standard and Poor's Agency, Fitch Rating and Moody's Investors Services. They independently assess creditworthiness of debt instruments originated by sovereigns and big corporations. In case of government debt, the agencies consider factors such as political risk, economic indicators, fiscal policy etc. Evaluation of corporates involves analysis of financial statements, growth potential, and exposure to risk factors as well as gathering information privately from the managers. As a result, those entities receive ratings (a combination of letters A, B, C and some sign) that indicate their creditworthiness. Using the ratings and historical data, gathered by the agencies, about average cumulative default rates, it is possible to estimate probability of a particular entity to receive the lowest credit rating (i.e. to default) within some time. However, the agencies have failed to predict major credit events connected with Bear Stearns or Enron, which turned attention to the counterparty risk in the first place. Hence, this thesis does not focus on this method.

Liquidly traded bonds are another important source of extracting default probabilities as they reflect market expectations about the credit risk. This information is contained in a credit spread – a difference between the bond yield and a risk-free rate. Using a simple formula and assuming a recovery rate, one can estimate a default probability of a counterparty within maturity of the bond. In order to estimate a term structure of the default probabilities, one needs to find credit spreads of the bonds issued by the same counterparty maturing at different moments, which is not a trivial task.

CDS is the most popular credit derivative instrument. Traded with different maturities, it allows managing credit risk, acting as an insurance against default of some reference instrument. Due to availability of data and wide acceptance among market practitioners, this thesis focuses on extraction of default probabilities from CDS contracts using bootstrap method. Next subchapter dives deeper into the topic.

It is worth to mention some other methods of estimating default probabilities. Empirically estimated internal credit ratings such as Altman's Z-score that uses data from financial statements to assess probability of default of smaller firms within some time. Another method is using equity prices. Proposed by Robert Merton in 1974, it relies on option pricing theory. For a more detailed treatment of the topic, one can refer to (Jorion, 2011).

2.1. Implying probability of default via CDS bootstrapping

CDS is an instrument that transfers credit risk of debt of a reference entity (reference asset) from a debt holder (protection buyer) to a CDS originator (protection seller). CDS involves an exchange of two streams of cash flows that are contingent on a credit event. A definition of a credit event usually refers to an International Swaps and Derivatives Association (ISDA) master agreement executed during CDS contract negotiation. Protection buyer regularly pays a premium called a CDS spread for a right to claim not recovered fraction of value of the debt in case a reference entity defaults. Figure 1, summarizes the mechanics of CDS.

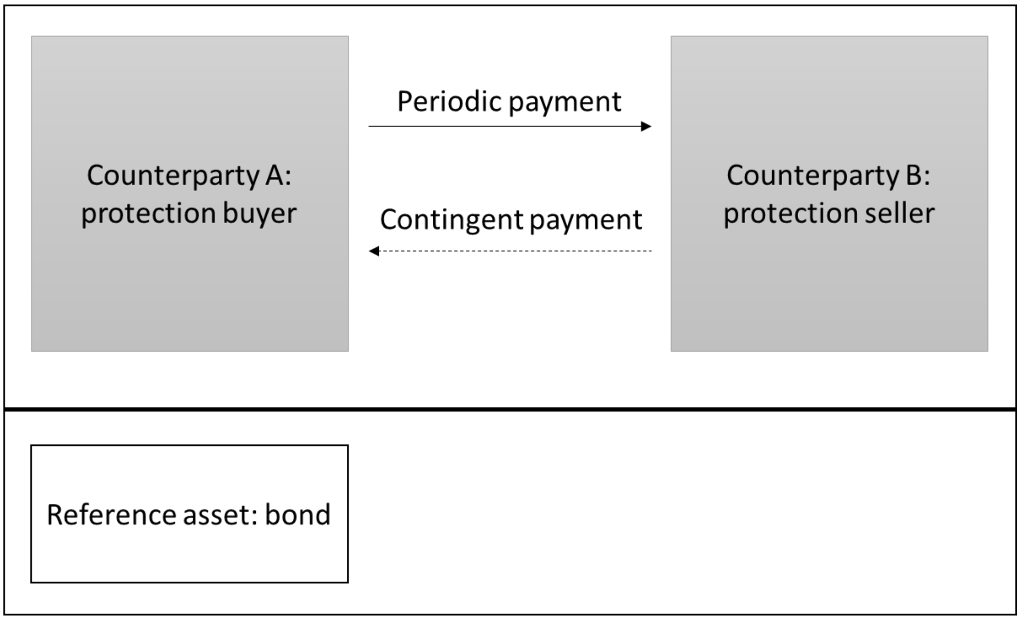

Figure 1: Mechanics of CDS

Source: (Jorion, 2011)

In order price a CDS, one has to calculate a difference between present values of the two legs of a swap. Those legs correspond to two mutually exclusive streams of cash flows: a premium leg and a default leg – when the latter starts, the former ends.

Present value formulas of premium and default legs are present in Equation 2 and 3 respectively. For purposes of this thesis, the situation when default happens is a subject of the analysis.

$$PL_N = S_N \cdot \sum_{n=1}^{N} d(T_n) \cdot PS(T_n) \cdot \Delta t_n + S_n \cdot \frac{\sum_{n=1}^{N} d(T_n) \cdot [PS(T_{n-1}) - PS(T_n)] \cdot \Delta t_n}{q} \tag{2}$$

$$DL_N = LGD \cdot \sum_{n=1}^{N} d(T_n) \cdot [PS(T_{n-1}) - PS(T_n)] \tag{3}$$

- S_N – CDS spread with tenor N
- $d(T_n)$ – risk-free discount factor from time 0 to time T_n
- $PS(T_n)$ – probability of survival from time 0 to time T_n
- Δt_n – annualized difference between T_n and T_{n-1}
- LGD – loss given default
- $\frac{\Delta t_n}{q}$ – annualized difference between T_{n-1} and time of default

Premium leg payments are happening with some frequency specified in the contract, however, a credit event can happen at any moment. Hence, in order to compensate for cost of insurance time between the last premium payment and the moment of default, a fraction of the premium is still to be paid at the next payment date.

Like in vanilla Interest Rate Swap, at moment zero, present value of both legs should be the same so that value of the swap is zero. Hence, after equalizing Equations 2 and 3 and solving for CDS spread one gets:

$$S_N = \frac{LGD \cdot \sum_{n=1}^{N} d(T_n) \cdot [PS(T_{n-1}) - PS(T_n)]}{\sum_{n=1}^{N} d(T_n) \cdot \Delta t_n + S_n \cdot \frac{\sum_{n=1}^{N} d(T_n) \cdot [PS(T_{n-1}) - PS(T_n)] \cdot \Delta t_n}{q}} \tag{4}$$

Solving Equation 4 for the cumulative probability of survival:

$$PS(T_N) = \frac{\sum_{n=1}^{N-1} d(T_n)[LGD - PS(T_{n-1}) - PS(T_n)(LGD + S_n\Delta t_n)]}{d(T_n)(LGD + S_N\Delta t_N)} + \frac{PS(T_{N-1})LGD}{LGD + S_N\Delta t_N} \tag{5}$$

Almost every element of RHS of Equation 5 is known at moment zero, except cumulative survival probability at within T_{N-1} time. Nevertheless, assuming that the reference entity has not yet defaulted, Equation 6 holds true.

This allows bootstrapping the CDS, which means one can extract survival probabilities using recursive formulas. (Cesari, et al., 2009)

$$PS(T_0) = 1 \tag{6}$$

$$PS(T_1) = \frac{LGD}{S_1\Delta t_1 + LGD}$$

$$\ldots$$

$$PS(T_N) = \frac{\sum_{n=1}^{N-1} d(T_n)[LGD - PS(T_{n-1}) - PS(T_n)(LGD + S_n\Delta t_n)]}{d(T_n)(LGD + S_N\Delta t_N)} + \frac{PS(T_{N-1})LGD}{LGD + S_N\Delta t_N}$$

Implementation of the algorithm is available in Appendix. The implementation breaks the RHS of Equation 5 into three parts: numerator of the first element, denominator of the first element and the second element.

This method allows finding marginal default probability from CDS maturities t_{j-1} to t_j, where $j = 1,2,\ldots,N$ and $S_{t_N}$is a CDS spread with the longest maturity. However, for the purposes of this thesis, one is interested in marginal default probability over some granular time-grid. Hence, before bootstrapping, one has to interpolate the CDS spreads. Figure 2 shows the CDS spread of a European counterparty. Equation 7 presents linear interpolation formula of CDS spreads S_{t_i}, where $t_{j-1} < t_i < t_j$. Figure 3 displays discount factors applicable for the currency of the European counterparty. After applying the bootstrapping algorithm, one gets probabilities that the counterparty will default in the middle of the time-step of the time-grid that are shown in Figure 4.

$$S_{t_i} = S_{t_{j-1}} + \frac{\left(S_{t_j} - S_{t_{j-1}}\right) \cdot \left(t_i - t_{j-1}\right)}{t_j - t_{j-1}} \tag{7}$$

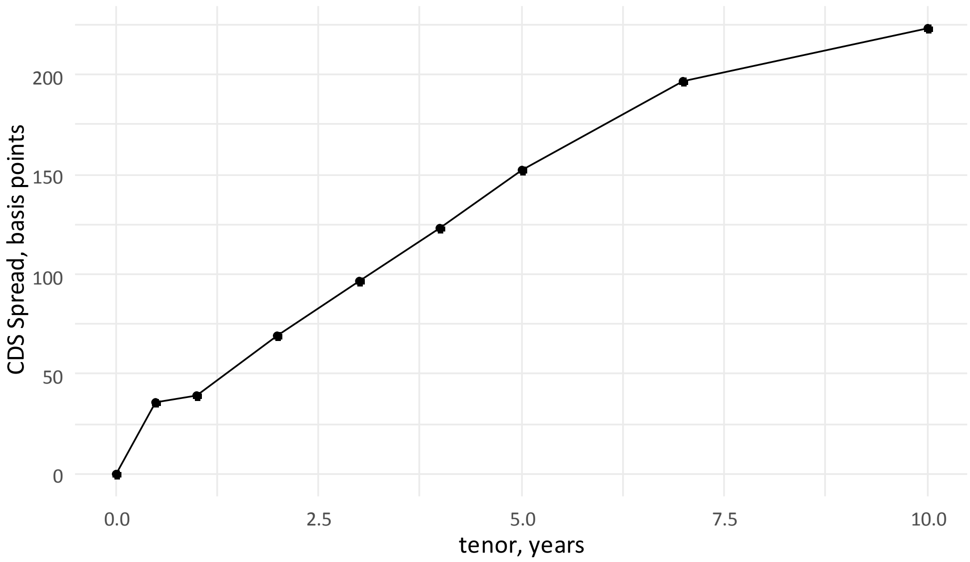

Figure 2: CDS Spreads of the Counterparty

Source: Financial Engineering project data

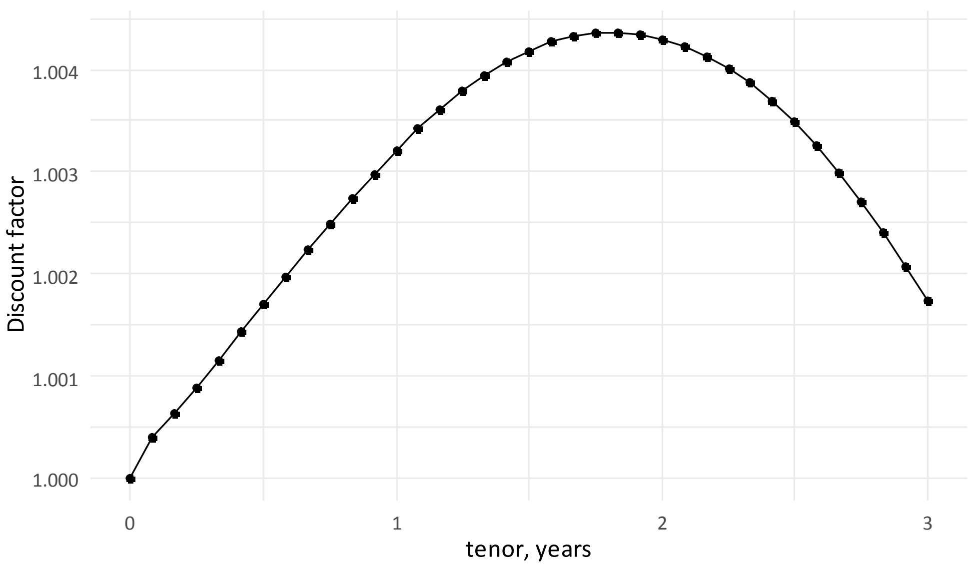

Figure 3: EUR Discount Factors

Source: Financial Engineering project data

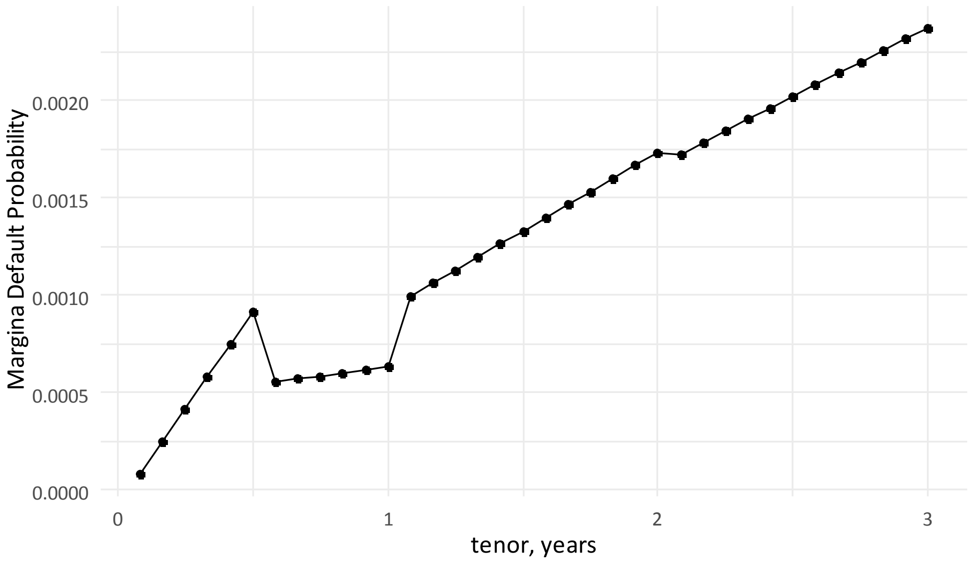

Figure 4: Marginal Default Probabilities of the Counterparty

Source: own research

3. Analysis of derivative instruments in question

Interest rate derivatives play a crucial role in today's economy. Institutions widely use them to hedge against adverse market movements as well as to enhance risk profile.

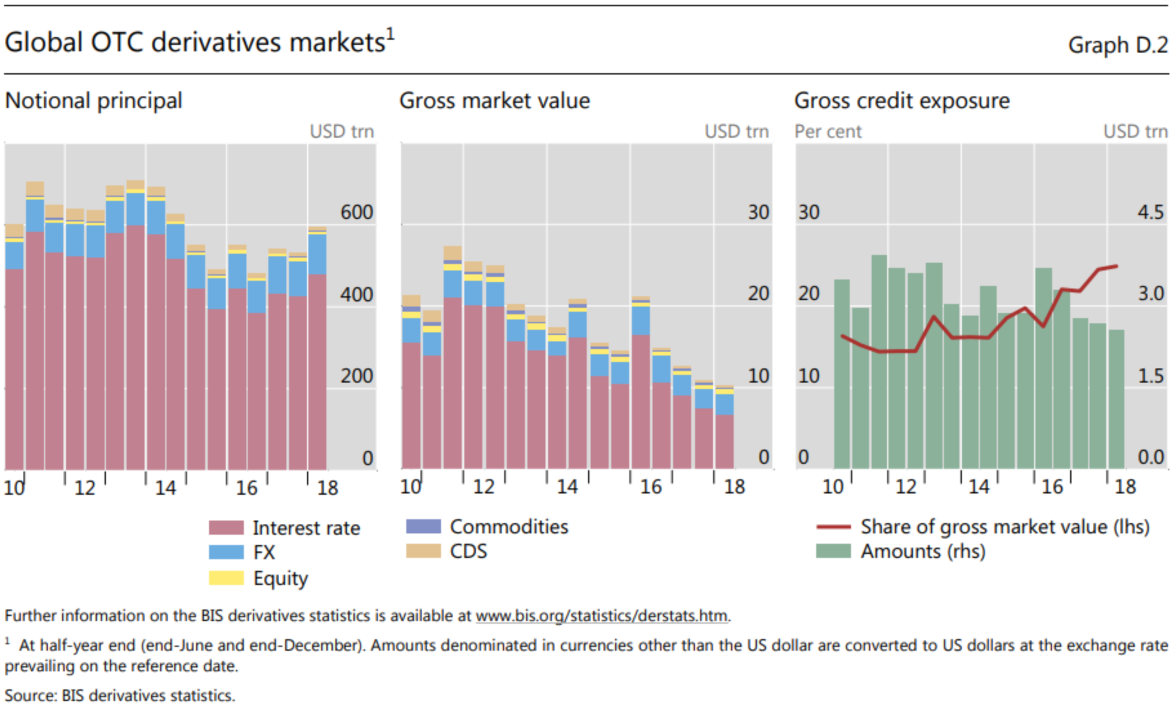

Figure 5: The importance of the interest rate market

Source: BIS Quarterly Review 2018

Nowadays, according to the most recent quarterly review prepared by Bank of International Settlements, interest rate market represents a vast majority of the global OTC derivatives market according to both a notional principal and gross market value.

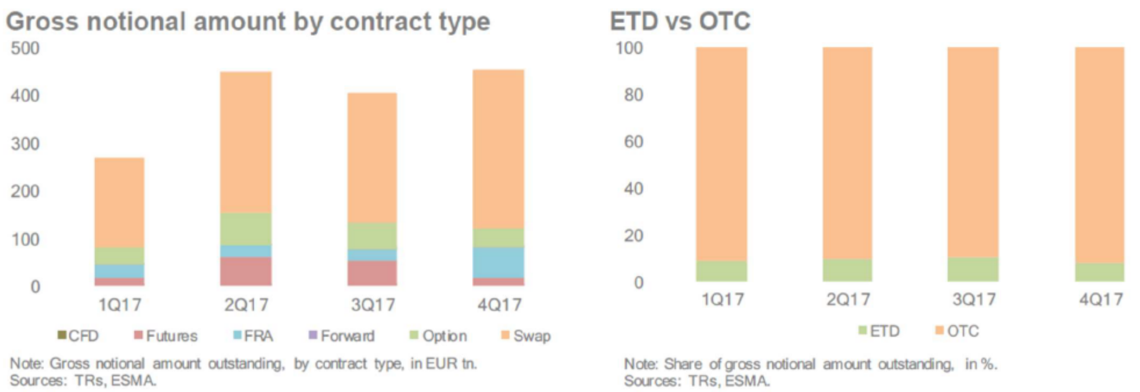

Figure 6: Key features of European Interest Rate Derivatives market

Source: ESMA Annual Statistical Report EU Derivatives Markets 2018

As one can see from the Figure 6, throughout 2017, OTC instruments were prevailing in the European Interest Rates sector, namely swaps, options, and forwards. This is why this thesis focuses on the derivatives such as FX Forward, IRS (interest rate swap), CIRS (cross-currency interest rate swap) and interest rate cap.

This chapter aims to briefly introduce mechanics, payoff and valuation of the analyzed derivatives, which are mentioned above.

3.1. FX Forward

Generic Forward contract is one of the simplest OTC-traded derivatives. One counterparty, after negotiation, is obliged to buy a particular asset at a particular moment at agreed in advance (forward) price from the second counterparty. Individually negotiated, they are flexible in terms of the size of the contract, settlement date, subject of the contract and method of settlement ideally suiting the needs of investors. Some of the things to agree on are:

- Underlying asset (stock, FX rate, interest rate, etc.)
- Notional
- Price
- Date of settlement
- Type of delivery (physical delivery or cash settled)

Forward contract is unconditional, which means that each party is obliged to fulfill the co ntract conditions, even if it is unprofitable. This instrument is symmetric, which means that gains of one counterparty are equal to losses of the other counterparty. Payoff of such contract is defined by the difference between forward price and market (spot) price of the underlying asset at maturity of the contract. Equation 8 depicts the payoff of a long position in a FX Forward contract. (Bartkowiak & Echaust, 2014)

$$V_T = S_T - F_0 \tag{8}$$

- S_T – the value of the FX spot rate at maturity
- F_0 – agreed exchange rate
- $-V_T$ – for short position

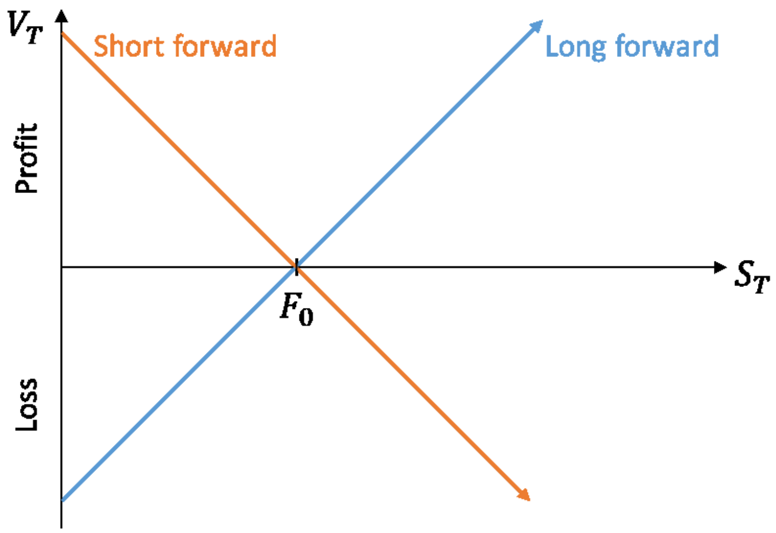

Figure 7: Payoff profile of a Forward contract

Source: Own research

Figure 7 displays payoff of a Forward contract as a function of the value of the instrument at maturity. One can observe that Forward contract has a linear payoff.

At moment of initiation ($t = 0$) of the Forward contract, its value is supposed to be equal to zero so that neither party is at loss. At moment $t > 0$, the value can take non-zero values. At maturity ($t = T$), the value is equal to the payoff introduced in Equation 8.

In order to find price of Generic Forward, one has to assume that arbitrage opportunities do not exist and then to calculate difference between current price of underlying asset and discounted forward price. In case of FX Forward, presence of an additional currency complicates the analysis by introducing foreign interest rate. One discounts FX forward rate using domestic interest rate and FX spot rate using foreign interest rate. Equation 9 summarizes pricing formula using continuous compounding. (Bartkowiak & Echaust, 2014)

$$V_t^{long} = S_t e^{-\tilde{r}_F \tau} - F_0 e^{-\tilde{r}\tau} \tag{9}$$

$$V_t^{short} = -V_t^{long}$$

- S_t – FX spot rate
- F_0 – FX forward rate
- $\tilde{r}_F$ – foreign continuously-compounded interest rate
- $\tilde{r}$ – domestic continuously-compounded interest rate
- τ – time remaining to maturity, $\tau = T - t$

- t – current moment, $t \in [0, T]$
- T – moment of contract settlement

3.2. Interest Rate Swap

Interest Rate Swap (IRS) is an agreement between two counterparties to exchange two streams of interest rate payments based on some principal. These streams constitute a floating leg and a fixed leg. The floating leg is usually a market reference rate + spread, e.g. LIBOR + 50 basis points. The fixed leg represents a fixed interest rate. The counterparty that pays fixed interest rate enters payer swap while the counterparty that pays floating interest rate enters receiver swap. Assuming that both legs have the same payment frequency, one can represent an IRS as a portfolio of Forward Rate Agreement (FRA) contracts with the same fixed rate. (Flavell, 2010)

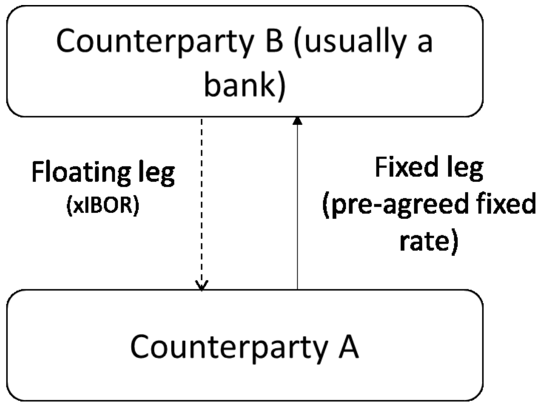

Figure 8: Mechanics of IRS

Source: Own research

Broadly speaking, one can define a payoff of the IRS as a difference between its legs. From the payer's perspective, floating interest payments exceeding fixed interest payments imply a positive payoff and vice versa from the receiver's perspective. Figure 9 summarizes the intuition behind IRS payoff.

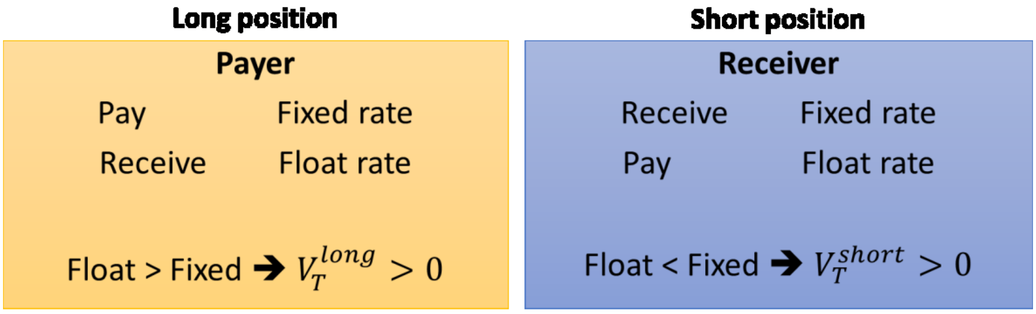

Figure 9: Intuition behind IRS payoff

Source: Own research

Equation 10 displays an IRS valuation formula at any moment based on forward rates method. (Bartkowiak & Echaust, 2014) It assumes that payment frequency of both legs are the equal to each other. Essentially, it is a difference between present values of fixed and floating leg cash flows. Forward rates are playing a role of proxies for the future spot floating rates.

$$V_t^{long} = N\left(\frac{c}{m}e^{-\tilde{s}_{t_1}t_1} + \frac{f_{t_1,t_2}}{m}e^{-\tilde{s}_{t_2}t_2} + \cdots + \frac{f_{t_{k-1},t_k}}{m}e^{-\tilde{s}_{t_k}t_k}\right) - N\left(\frac{r_{IRS}}{m}e^{-\tilde{s}_{t_1}t_1} + \frac{r_{IRS}}{m}e^{-\tilde{s}_{t_2}t_2} + \cdots + \frac{r_{IRS}}{m}e^{-\tilde{s}_{t_k}t_k}\right) \quad (10)$$

$$V_t^{short} = -V_t^{long}$$

- N – notional amount
- c – floating interest rate that is known at moment t
- m – number of payments per year
- r_{IRS} – fixed rate
- $\tilde{s}_{t_k}$ – continuously compounded spot rate at time t for the k^{th} (last) cash flow
- $f_{t_{k-1},t_k} = \frac{\left(\frac{d(t_{k-1})}{d(t_k)}-1\right)}{t_k - t_{k-1}}$ – forward rate starting from the moment t_{k-1} to the moment t_k
- $d(t_k)$ – price of a zero-coupon bond maturing at moment t_k

At moment zero, value of the IRS is always equal to zero. It is achieved by adjusting fixed interest rate r_{IRS} such that value of the RHS of Equation 10 is set to zero.

3.3. Cross-currency Interest Rate Swap

Cross-currency Interest Rate Swap (CIRS) is) is a contract in which two counterparties agree to exchange/swap two sets of cash flows based on different fixed or floating rates in two different currencies. Usually, there is also an exchange of notional at the beginning and at the end of the CIRS lifetime. Figure 10 depicts mechanics of such CIRS. At stage 1, counterparties exchange notional in two different currencies according to spot FX rate. At stage 2, the counterparties swap interest payments for the nominal received at stage 1. At stage 3, the notional is exchanged back according to the spot FX rate at the swap initiation moment. (Bartkowiak & Echaust, 2014)

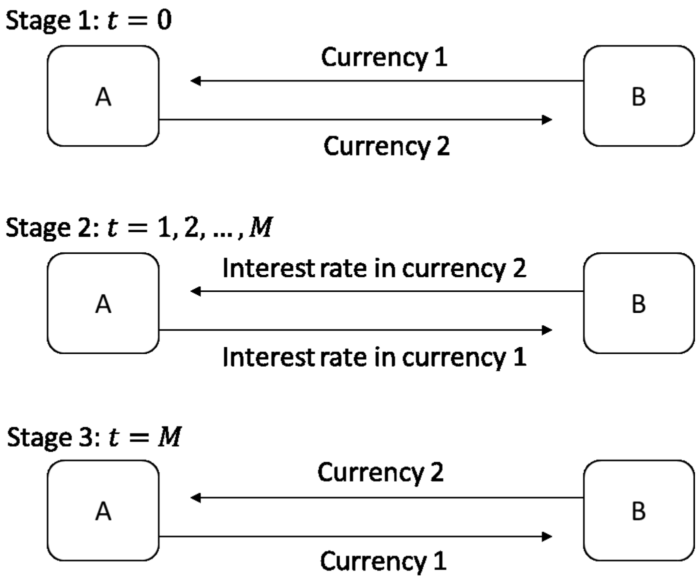

Figure 10: Mechanics of CIRS

Source: (Bartkowiak & Echaust, 2014)

Since CIRS is a swap, the same payoff intuition applies as in case of IRS. However, the analysis is complicated by the exchange rate:

For party A:

$$CF_A - CF_B \cdot F_A > 0, \quad V_A > 0$$

For party B:

$$CF_B - CF_A \cdot F_B > 0, \; V_B > 0$$

- CF_A – cash flows to party A
- $F_A = 1/F_B$ – current exchange rate for A

Valuation of CIRS depends on types and frequencies of the cash flows in it. An example of CIRS can be a EUR/PLN payer CIRS, with fixed leg in EUR paid annually, WIBOR 3M as a floating leg, maturity of 3 years and notional exchange at the beginning as well as end of the contract. Equation 11 presents valuation formula of such instrument.

$$\begin{aligned} V^{payer} = N\left(-1 + \frac{f_{0,0.25}}{4} d(0.25) + \cdots + \left(1 + \frac{f_{2.75,3}}{4}\right) d(3)\right) \\ - X^{EUR\backslash PLN} N_f\big(-1 + r_{CIRS} d_f(1) + r_{CIRS} d_f(2) \\ + (1 + r_{CIRS}) d_f(3)\big) \end{aligned} \tag{11}$$

- $d(0.25)$ – present value of 1 PLN in 3 months (i.e. discount factor)
- $d_f(1)$ – present value of 1 EUR in 1 year (i.e. discount factor)
- N – contract notional in domestic currency
- N_f – contract notional in foreign currency
- $f_{0.25,0.5} = \frac{\left(\frac{d(0.25)}{d(0.5)} - 1\right)}{0.5 - 0.25}$ – current forward rate that starts in 3 months and matures in 6 months
- r_{CIRS} – fixed interest rate
- $X^{EUR\backslash PLN}$ – current exchange rate

3.4. Interest Rate Cap

Interest rate cap (floor) is a financial instrument that grants the buyer with a right to obtain interest on the agreed notional based on the excess of underlying floating rate above (below) some rate. This rate is called cap (floor) rate. One can see interest rate cap (floor) as a portfolio of call (put) options on a reference rate, which are called caplets (floorlets). Caplet (floorlet) payoffs occur on reset days with a frequency defined by a tenor of the reference rate. Since the reference rates are usually annualized, one has to adjust them to the compounding frequency of the tenor.

Assuming that the entire lifetime of an interest rate cap is T, the reset dates are $t_1, t_2, \ldots, t_n$, and defining T as t_{n+1}, the payoff of the caplet at moment $t_{i+1}, (i = 1, 2, \ldots, n)$ is present in Equation 12. (Hull, 2018)

$$Payoff_{i+1}^{Caplet} = N \cdot \Delta t_i (s_i - s^{Cap})^+ \tag{12}$$

where:

- N – contract notional

- Δt_i – time step between t_{i+1} and t_i
- s_i – reference rate
- s^{Cap} – cap rate

Black model defines value of a caplet in Equation 13. (Hull, 2018)

$$V^{Caplet} = N \cdot \Delta t_i \cdot d(t_{i+1}) \cdot \left(f_{t_i,t_{i+1}} \mathcal{N}(d_1) - s^{Cap} \mathcal{N}(d_2) \right) \tag{13}$$

$$d_1 = \frac{\ln\left(\frac{f_{t_i,t_{i+1}}}{s^{Cap}}\right) + \frac{\sigma_i^2 t_i}{2}}{\sigma_i \sqrt{t_i}}$$

$$d_2 = \frac{\ln\left(\frac{f_{t_i,t_{i+1}}}{s^{Cap}}\right) - \frac{\sigma_i^2 t_i}{2}}{\sigma_i \sqrt{t_i}} = d_1 - \sigma_i \sqrt{t_i}$$

- $f_{t_i,t_{i+1}}$ – forward rate from moment t_i to t_{i+1}
- $d(t_{i+1})$ – price of a zero-coupon bond maturing at moment t_{i+1}
- $\mathcal{N}$ – cumulative standard normal distribution function
- σ_i – volatility of the forward rate $f_{t_i,t_{i+1}}$

4. Risk factors simulation

Valuation of the analyzed derivative instruments requires knowledge of all the variables used in the formulas. Those variables are available at current moment, which allows finding current value of the transaction only. However, CVA requires knowledge of transaction value at each moment of the transaction's lifetime. In order to achieve that, one has to define risk factors (i.e. variables one cannot assume to be constant over time) that affect value of the derivative and to simulate them. Next subchapters focus on a simulation methodology and calibration to market data.

4.1. Interest rate modeling techniques

Valuation of almost any derivative instrument depends on the interest rate, and estimation of exposure profile depends on evolution of those interest rates either directly (instrument payoff formula includes interest rates, e.g. IRS) or indirectly (as an amortization effect, e.g. FX forward). This is why it is so crucial to establish the model and to use it consistently over the entire portfolio of the instruments. This subchapter attempts to summarize a broad range of interest rate models based on (Lukasewich, 2016).

Studies of relationship between interest rates maturing at different moments are of prominent importance in the domain of finance and investments. The most important research stream is analysis of so-called yield curve. Such curve is a plot that reflects how prices and yields of respective debt instruments (e.g. government or corporate bonds) change depending on their maturity. Figure 11 displays an example of a smoothed yield curve. Essentially, it is a snapshot of interest rate term structure on a particular market at a particular moment reflecting current and maybe future conjuncture of that debt market. It affects activities of financial markets, shapes prices of credit resources, gives expectations about inflation and sometimes even predicts economic recessions.

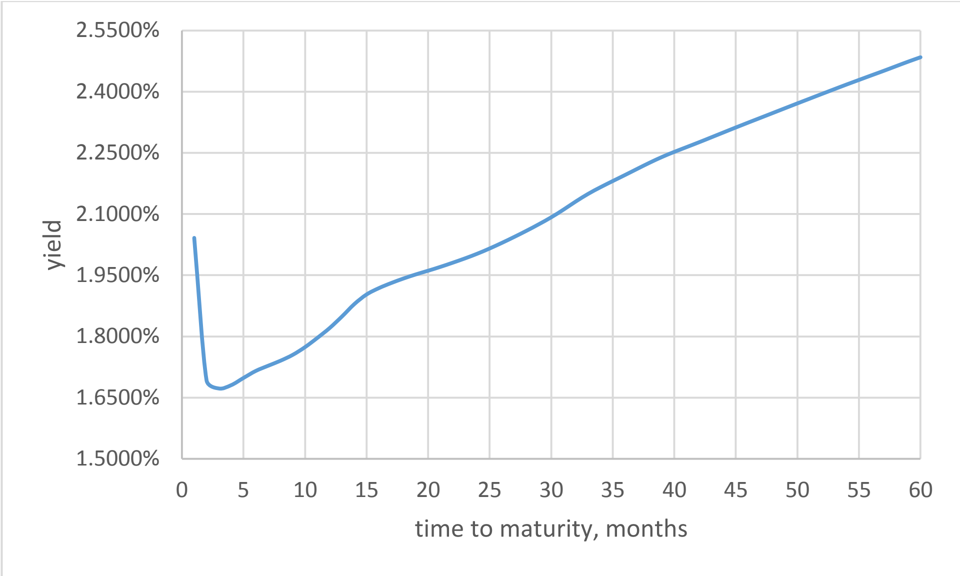

Figure 11: Yield curve of Polish government zero-coupon bonds

Source: Financial Engineering Project

There are three main streams in term structure theory: liquidity preference theory, expectations theory and segmentation theory. Liquidity preference theory is the most empirically confirmed theory; it implies that investors prefer more liquid instruments, and hence on average expected yield should go up as maturity rises due to growth in liquidity premium. Expectations theory assumes that forward rates approximate spot rates, where precision of the approximation is inversely related to uncertainty on the market. Market segmentation theory divides yield term structure into separate markets (e.g. short- and long-term debt), each market governed by its own demand-supply forces. All three streams can explain the term structure of government debt.

When it comes to development of the interest rate models, one can build them according to three approaches:

- statistical (time-series models), based on classical methods of time-series analysis (Keynes, models with regressive expectations, combined model of Modigliani, McCulloch spline);
- econometric (volatility models), accounting for changes in volatility and covariance (ARCH, GARCH, log-normal stochastic volatility)

- stochastic (diffusion models), building on some stochastic process (Bachilier, Black-Scholes-Merton, Vasicek, Cox-Ross-Ingersoll, Hull-White, Heath-Jarrow-Morton)

This thesis applies stochastic approach. Initially author was considering Vasicek model based on Ornstein-Uhlenbeck process. Equation 14 depicts the model.

$$dr_t = \alpha(\bar{r} - r_t)dt + \sigma dW_t \tag{14}$$

Where $\bar{r}$ and α play role of equilibrium interest rate and speed of convergence of r_t towards $\bar{r}$, respectively. This model requires calibration using econometric methods. After discretizing Equation 14 and making some algebraic transformations and substitutions one gets regression depicted in Equation 15.

$$\Delta r_t = \alpha(\bar{r} - r_t)\Delta t + \sigma \epsilon_t$$

$$\epsilon \sim N(0, \Delta t)$$

$$\Delta r_t = \alpha \bar{r} - \alpha \cdot \Delta t \cdot r_t + \sigma \epsilon_t$$

$$\Delta r_t = \beta_0 + \beta_1 \cdot r_t + \xi \tag{15}$$

However, in case of WIBOR 3M (reference rate analyzed in the thesis); calibration to market data fails due to statistical insignificance of both parameters estimated. One can explain it by lack of variability (or liquidity) of WIBOR 3M. One can see a time-series plot of the reference rate in Figure 12.

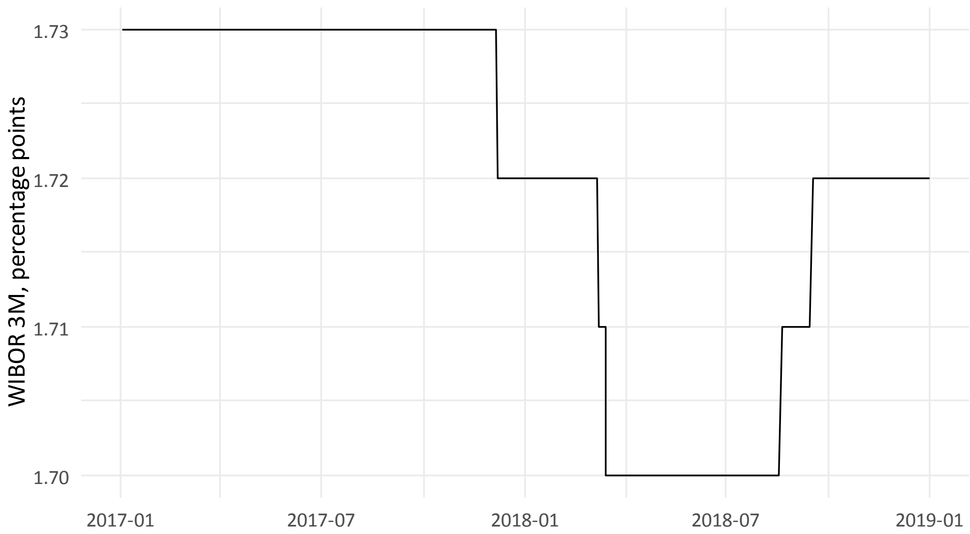

Figure 12: WIBOR 3M historical data

Source: stooq.pl

This is why, the author decided to use Arithmetic Brownian Motion (ABM) to describe interest rate movement. Moreover, nowadays, negative interest rates are present in many countries, so the fact that ABM permits negative values only advocates for this model. Equation 16 shows how changes in a spot rate across time can be explained by some constant drift and volatility parameters along with changes of Wiener process. 16

$$dS_t = \mu_S \cdot dt + \sigma_S \cdot dW_t \qquad (16)$$

- μ_S – historically calibrated drift parameter
- σ_S – historically calibrated volatility parameter
- W_t – standard Wiener process

4.2. Assumption about term structure of spot rates

The entire spot rate curve is shifting with simulated xIBOR 3M. PLN example is given in Figure 13. It illustrates three steps: (i) extract spot rates from given discount factors, (ii) **simulate the spot** rates, (iii) transform them back into the discount factors.

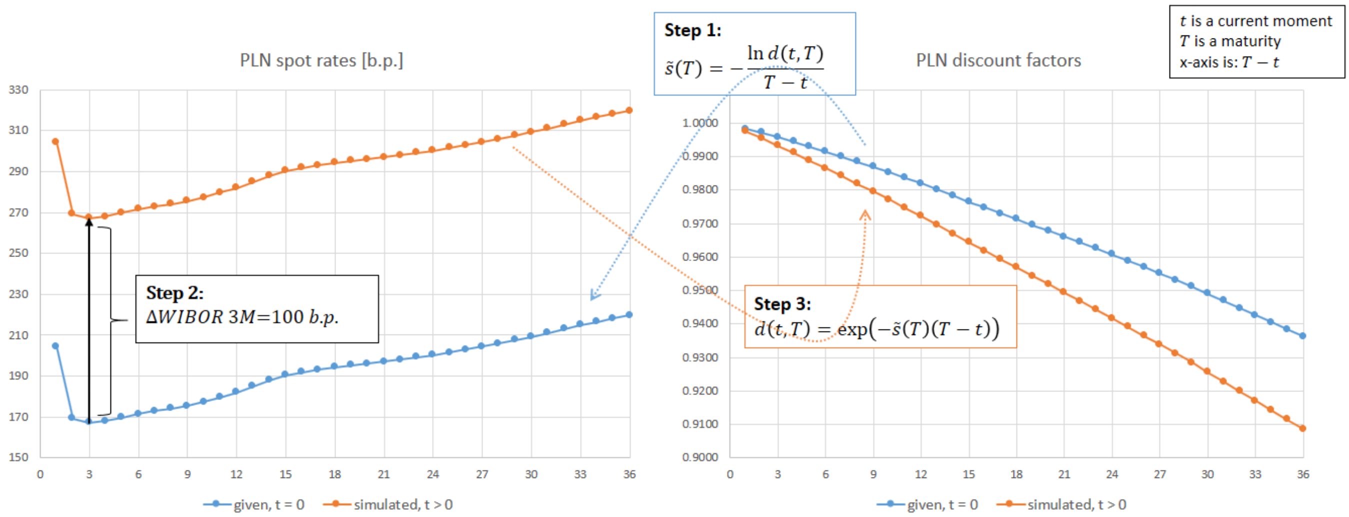

Figure 13: Quasi-dynamic term structure of spot rates

Source: own research

- $d(t,T)$ – price of a zero-coupon bond with maturity *T* at moment *t*.
- $\tilde{s}(T)$ – continuously compounded spot rate with maturity *T*.

4.3. FX forward

The main risk factor in an FX forward is the value of the FX spot rate at maturity.

FX spot rates cannot be negative. Therefore, a standard model for historically calibrated FX spot rates is a lognormal process; Geometric Brownian Motion is applied:

$$dS_t = \mu S_t dt + \sigma_S S_t dW_t$$

- S_t – FX spot rate at time t
- μ – drift parameter
- σ_S – historically-calibrated volatility parameter
- W_t – standard Wiener process

Source: (Lalley, 2001)

Calibrating drift to market data, one assumes no arbitrage opportunity.

$$\mu = r_d - r_f$$

- $r_d = -\frac{\ln(d_{PLN}(0,0.25))}{0.25}$ – domestic risk-free interest rate extracted from PLN 3M zero-coupon bond price
- $r_f = -\frac{\ln(d_{EUR}(0,0.25))}{0.25}$ – foreign risk-free interest rate extracted from EUR 3M zero-coupon bond price

Source: (Lalley, 2001)

Due to nature of the given data, the real date of a derivative transaction is unknown, so it is better to extract the spot rates from the given data, i.e. zero-coupon bond price maturing in 3 months, instead of assuming the date and taking it from the market data. 3M tenor is taken, as it is the most liquid one. No mean-reversion needed because of short maturity of the instrument

We calibrate volatility to market data as historical volatility:

$$\sigma_S = \sqrt{Var\left(\log\left(\frac{S_{i+1}}{S_i}\right)\right) \times N_{trading}}$$

- $N_{trading}$ – number of trading periods per year, e.g. 252 for daily frequency of data

Then, after Euler discretization, one can simulate FX spot rates using the formula:

$$S_t = S_{t-1} \cdot e^{\left(r_d - r_f - \frac{\sigma_S^2}{2}\right)\Delta t + \sigma_S \sqrt{\Delta t} * \epsilon_t}$$

- ϵ_t $\sim \mathcal{N}(0,1)$

- S_t – FX spot rate at moment t
- Δt – simulation time-step

Source: (Lalley, 2001)

4.4. Interest Rate Swap

The main risk factor in IRS is WIBOR 3M. Since interest rates can take negative values, Arithmetic Brownian Motion can describe spot interest rate process:

$$dS_t = \mu_s \cdot dt + \sigma_S \cdot dW_t$$

- μ_s – historically calibrated drift parameter
- σ_s – historically calibrated volatility parameter
- W_t – standard Wiener process

Drift and volatility calibration to market data is happening as follows:

$$\mu = \mathbb{E}[\Delta S_i] \times N_{trading}$$

$$\sigma_S = \sqrt{Var(\Delta S_i) \times N_{trading}}$$

Source: (Green, 2015)

Then, after Euler discretization, one simulates WIBOR 3M spot rates using formula:

$$S_t = S_{t-1} + (r_d - r_f)\Delta t + \sigma_S * \sqrt{\Delta t} * \epsilon_t$$

- ϵ_t $\sim \mathcal{N}(0,1)$
- S_t – WIBOR 3M spot rate at time t
- Δt – simulation time-step

4.5. Cross-currency Interest Rate Swap

In CIRS, there are two correlated risk factors, PLN spot interest rate and spot FX rate.

$$dS_t^{FX} = \mu S_t^{FX} dt + \sigma_{S^{FX}} S_t^{FX} dW_t$$

$$dS_t = \mu_s \cdot dt + \sigma_S \cdot dW_t$$

In order to generate correlated random variables, one uses Cholesky Factorization:

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix}$$

- ρ – correlation between risk factors
- ϵ – i.i.d. random variable
- z – correlated random variable

Source: (Jorion, 2011)

One calibrates drift and volatility to market data as follows:

- For the spot FX rate: $\mu = r_d - r_f$, $\sigma_S = \sqrt{Var(\Delta S_i) \times N_{trading}}$

- For the WIBOR 3M rate: $\mu = \mathbb{E}[\Delta S_i] \times N_{trading}$, $\sigma_S = \sqrt{Var(\Delta S_i) \times N_{trading}}$

Then, after Euler discretization, one simulates both spot rates using formulas:

- For the spot FX rate:
    - $S_t = S_{t-1} \cdot e^{\left(r_d - r_f - \frac{\sigma_S^2}{2}\right)\Delta t + \sigma_S \cdot z_t^{FX}}$
    - $z_t^{FX} = 1 \cdot \epsilon_t^{FX} + 0 \cdot \epsilon_t^{WIBOR} \sim \mathcal{N}(0,1)$
    - S_t – FX spot rate at moment t
    - Δt – simulation time-step
- For the WIBOR 3M rate:
    - $S_t = S_{t-1} + \left(r_d - r_f\right)\Delta t + \sigma_S \cdot \sqrt{\Delta t} \cdot z_t^{WIBOR}$
    - $z_t^{WIBOR} = \rho \cdot \epsilon_t^{FX} + \sqrt{1 - \rho^2} \cdot \epsilon_t^{WIBOR} \sim \mathcal{N}(0,1)$
    - S_t – WIBOR 3M spot rate at time t
    - ρ – correlation between the risk factors
    - Δt – simulation time-step

Source: (Jorion, 2011), (Lalley, 2001)

4.6. Interest Rate Cap

The main risk factor of an Interest Rate Cap (in addition to the reference rate) is volatility of the underlying forward interest rate. Usually, separate implied volatilities are extracted from the market-traded instruments (e.g. interest rate options, swaptions) and then are used for each separate caplet. An alternative approach is to use the same volatility for each caplet. Due to lack of data, this thesis uses an alternative approach. 3M forward rate volatility is proxied by annualized standard deviation of daily historical data of WIBOR 3M.

In addition, dynamics of discount factors and forward rates have been taken from the IRS model.

5. Expected positive exposure (EPE) for the analyzed derivatives

Generation of counterparty credit exposures is one of the key functions of risk management departments in financial institutions. The exposure profiles are necessary to set limits on the trades, to calculate economic and regulatory capital and to price counterparty credit risk.

Counterparty credit exposure describes potential losses (or gains) due to default of one of two counterparties. Hence, in order to quantify the credit exposure, one has to somehow 'predict' the way an instrument will behave in the future and apply some transformation. For example, in order to calculate a unilateral CVA, one requires Expected Negative Exposure (ENE). This is because, CVA is a price of losses due default of the counterparty.

Two effects accompany quantification of the exposures across time. On the one hand, the further one looks in time, the more uncertain the value of the transaction becomes, and hence the risk goes up alongside with the exposure. On the other hand, the further in time some cash flow happens, the more amortized it gets, which means that exposure is decreases. Those two effects are responsible for the shape of an exposure profile.

One can construct exposure using techniques of various complexity, such as MtM + add-ons, semi-analytical methods or Monte Carlo simulation.

Mark-to-market + add-ons method approximates future potential exposure by taking a positive MtM value and adding some element reflecting characteristics of the trade. Those characteristics are the (i) how far in time the exposure is, (ii) how uncertain the underlying asset is and (iii) some transaction-specific characteristics. Those add-ons usually take form of a notional value percentage. This approach is the least complicated both conceptually and computationally.

Semi-analytical methods are more advanced and involve some simplifying assumptions about stochastic nature of the transaction's risk factors. Using those assumptions, one can derive a distribution of the future exposure and hence approximate it with some level of confidence. The biggest strength of this approach is its ease of computation. It can handle linear instruments such as forwards well. However, this method is not flexible enough for more complex instruments such as American-style options. In addition, it is not easy to incorporate collateral and netting effects.

The most versatile and currently most used method is Monte Carlo Simulation. Despite being the most computation-heavy approach, it allows to generate exposure of required granularity and with instrument-specific features.

The key component of the simulation is a time-grid. It is important to pick a right density of the grid. If it is too sparse, the simulation might not catch main features of the exposure. For example, if an IRS contains quarterly cash flows, a grid with annual frequency will not suffice. On the other hand, too many time-steps might lead to computational issues.

The algorithm to calculate Expected Positive Exposure (EPE) of the derivatives contains six steps:

1. Pick variables that determine present value of future cash flows (i.e. risk factors)

2. Find out in which direction risk factors move and how unstable they are across time (i.e. calibrate drift and volatility parameters)

3. Simulate risk factors (i.e. generate scenarios)

4. Price a derivative at each time step using simulated risk factors

5. Include portfolio effects such as netting or collateral

6. Apply necessary transformations to calculate EPE

One can use following definition of EPE (Gregory, 2010):

$$NettedPositiveExposure_{t,k} = \left(\sum_{i=1}^{m} V_{i,t,k}\right)^{+}$$

$$EPE_t = \frac{\sum_{k=1}^{K} NettedPositiveExposure_{t,k}}{K}$$

- $V_{i,t,k}$ – value of derivative i (out of m instruments in portfolio) at moment t in scenario k (out of K scenarios)

5.1. Key features of EPE for the analyzed derivatives

Using pricing formulas and simulated risk factors introduced in previous chapters, 10 000 Positive Exposure trajectories were generated for each instrument under consideration. This subchapter introduces main characteristics of the exposure profiles.

5.1.1. FX Forward

Exposure profile of EUR\PLN FX Forward with maturity of 1 year, notional of 100 000 EUR and strike FX rate of 4.3930 has been evaluated. Figure 14 presents 100 sample Positive Exposure trajectories generated for this instrument. One can see that in separate scenarios, exposure reaches extreme values right before maturities, but at the beginning of all scenarios exposure does not vary significantly. It is an illustration of a diffusion effect.

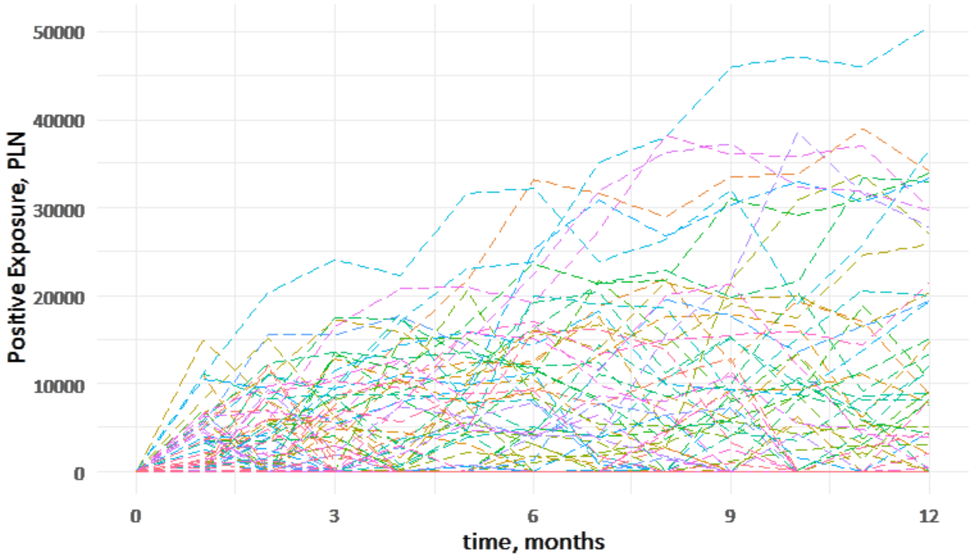

Figure 14: Sample trajectories of FX Forward Positive Exposure

Source: own research

Figure 15 crystalizes the pattern behind different scenarios. Exposure grows with progression of time. Despite linearity of the instrument's payoff, the EPE is non-linear. It is a consequence of discounting and of the transformation of exposure intro positive exposure.

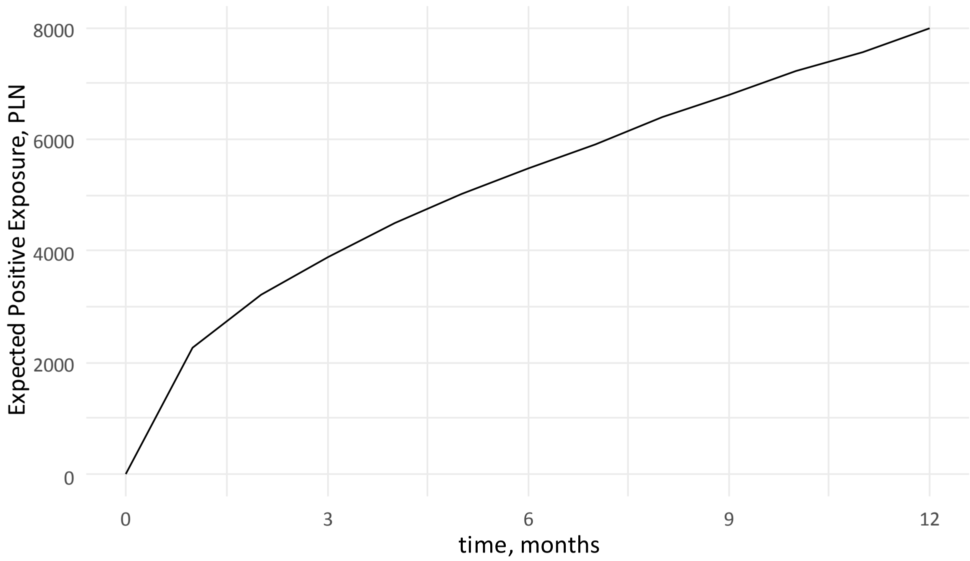

Figure 15: Expected Positive Exposure of FX Forward

Source: own research

5.1.2. Interest Rate Swap

Receiver PLN WIBOR 3M IRS with maturity of 3 years, notional of 500 000 PLN and annually paid fixed rate of 2.2144% has been under consideration. Figure 16 shows 10 sample trajectories of IRS positive exposure. Due to extremely small historically calibrated volatility of WIBOR 3M, there is almost no noise in the exposure. However, one can notice the variation of the exposures goes down as time passes (and consequently number of remaining cash flows decreases).

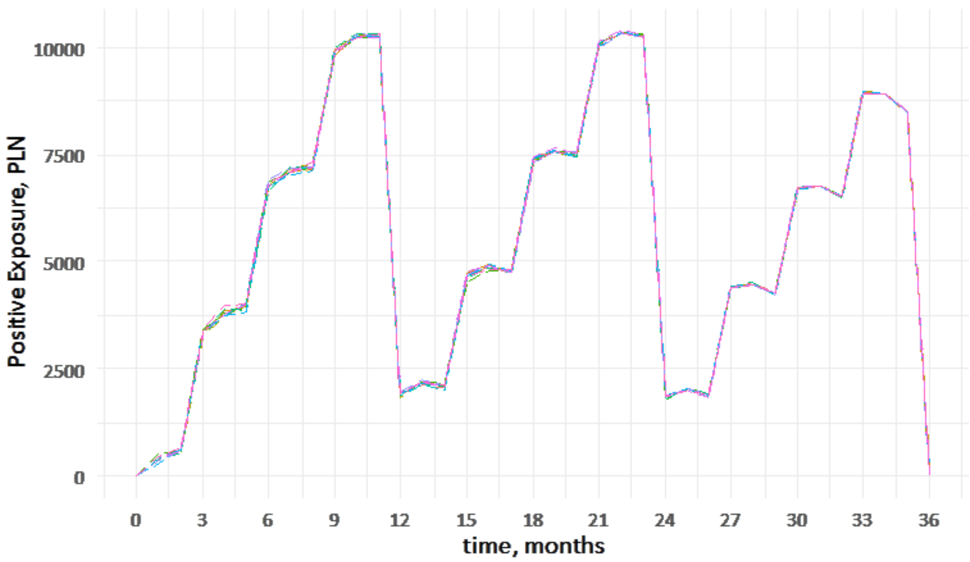

Figure 16: Sample trajectories of IRS Positive Exposure

Source: own research

Figure 17 confirms that 10 sample trajectories from Figure 16 are approximating EPE quite well. Due to different payment frequencies, this IRS exhibits an unusual exposure profile. One can see that the peak exposures are occurring right before the annual fixed leg payments. Prior to those peaks are quarterly spikes before floating leg payments and after those peaks, a drastic decrease in exposure happens. Despite insignificant volatility of the WIBOR 3M and different leg payment schedules, the maximum exposure is still happening approximately at 1/3 of the instrument's maturity, which is a classical feature of IRS exposure profile. It happens because of combination of amortization and diffusion effects in addition to effect of reducing number of cash flows remaining.

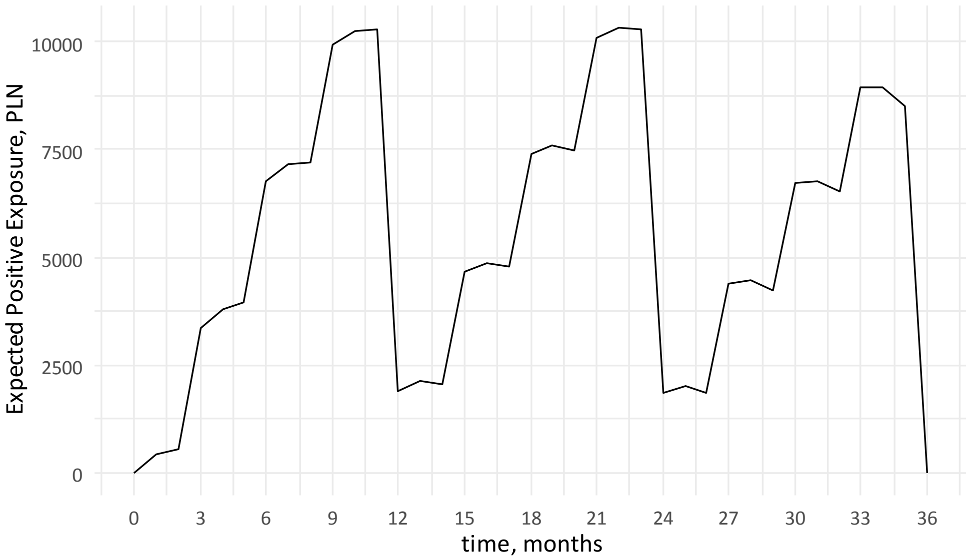

Figure 17: Expected Positive Exposure of IRS

Source: own research

5.1.3. Cross-currency Interest Rate Swap

The next instrument is Payer EUR/PLN CIRS. It has following specifications: fixed rate of -0.0575% in EUR, floating rate is WIBOR 3M, notional exchange at the beginning as well as end of the contract, maturity is 3 years, notional is 100 000 EUR/ 430 000 PLN. The main feature that defines exposure profile of the instrument is notional exchange prior to maturity. It means that entire value of the notional amount can be subject to FX rate fluctuation. Figure 18Figure 27 illustrates that with 100 scenarios of positive exposure of CIRS. Right before the maturity, value of the transaction takes most extreme values.

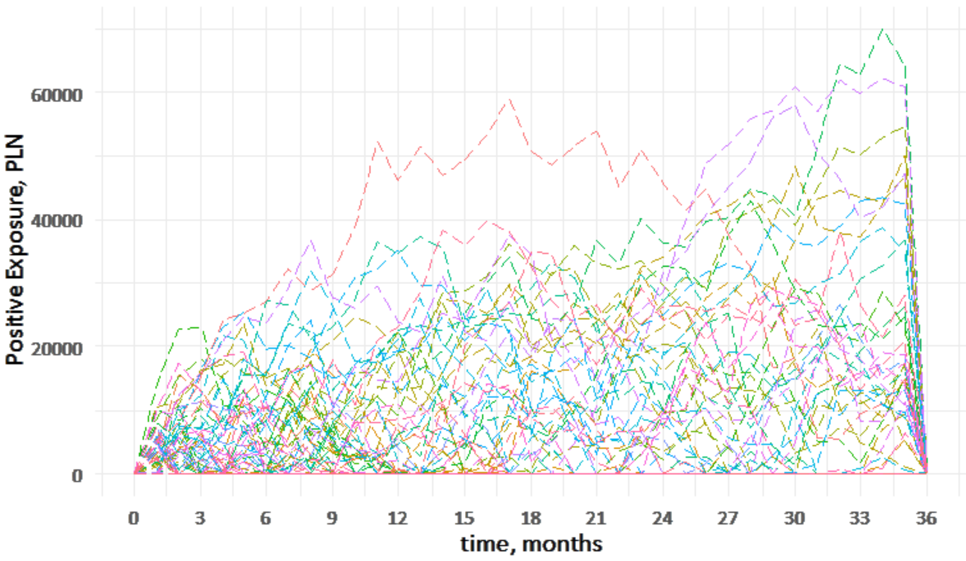

Figure 18: Sample trajectories of CIRS Positive Exposure

Source: own research

EPE of CIRS is depicted in Figure 19. The profile plainly displays quarterly spikes of exposure caused by 3 months frequency of a floating leg. The plot does not reveal annual patterns of fixed leg due to negative (payer is receiving positive cash flows on both legs) and extremely small in magnitude fixed CIRS rate. In addition, unexpected result is that EPE around end of maturity is not well emphasized.

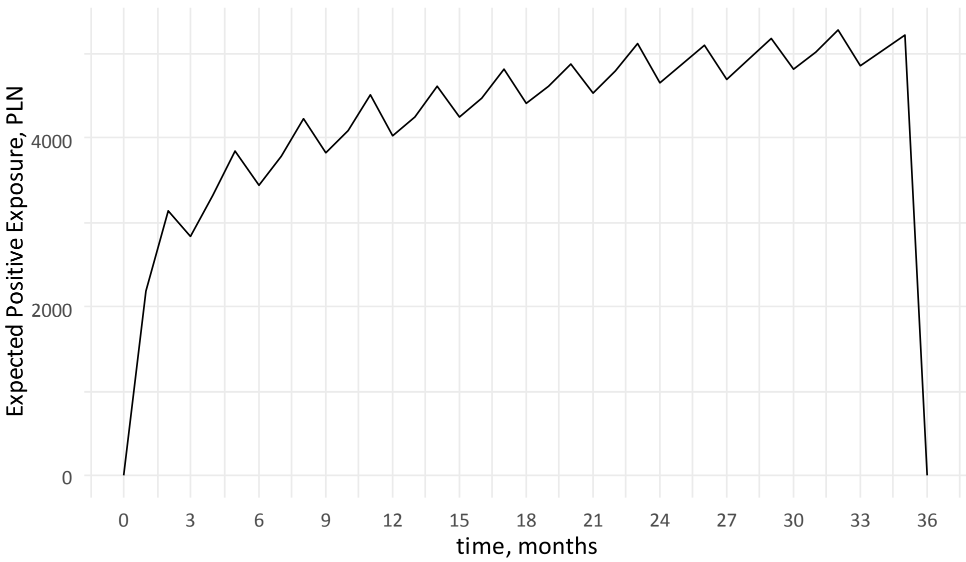

Figure 19: Expected Positive Exposure of CIRS

Source: own research

In order to explain this finding, simple exposures (before transformation into positive exposures) were considered. Sample of 100 exposure trajectories depicted in Figure 20 show that in majority of the scenarios, MtM valuations were becoming more negative as time passed. Positive sign of drift parameter calibrated for the FX rate can explain this. From the payer's perspective, growing FX rate means that notional exchange at the end of the contract incurs losses for the payer (they receive 100 000 EUR according to EUR\PLN = 4.3, whereas EUR\PLN is higher than 4.3 at moment of maturity).

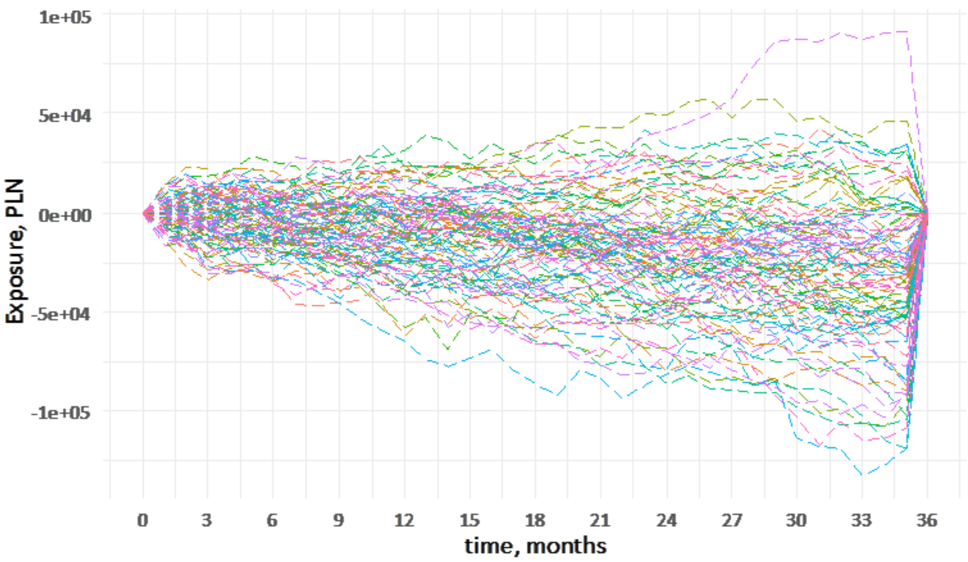

Figure 20: Sample trajectories of CIRS Exposure

Source: own research

Figure 21 confirms this explanation. After just 6 months, magnitude of the expected exposure reaches maximum magnitude of EPE depicted in Figure 19 and peaks with five times bigger magnitude at the end of the contract. It means that this CIRS should have even bigger ENE (expected negative exposure) and should yield a significant DVA and FVA in bilateral xVA setting where both parties are assumed to bear risk of default.

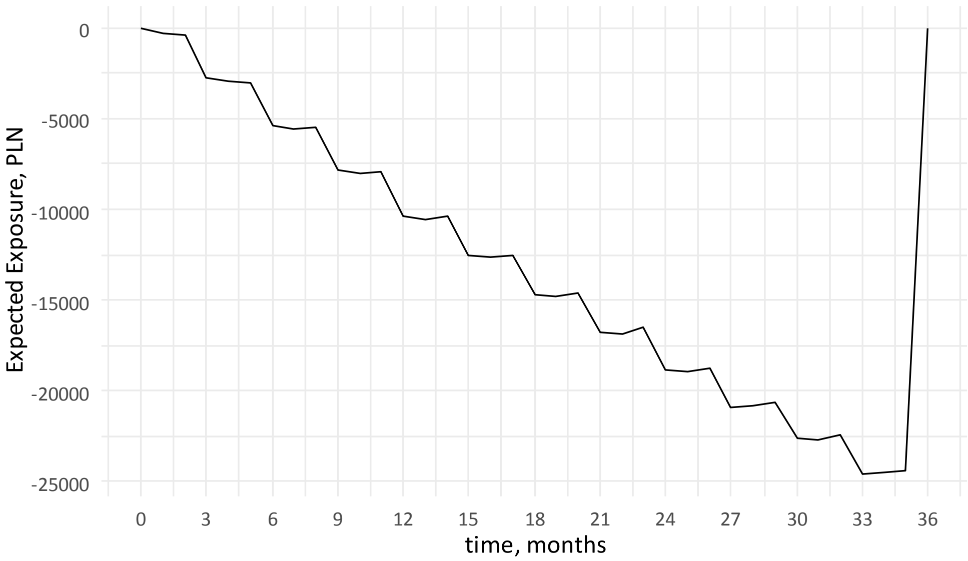

Figure 21: Expected Exposure of CIRS

Source: own research

5.1.4. Interest Rate Cap

The last instrument in a hypothetical portfolio is in-the-money Interest Rate Cap on WIBOR 3M with cap rate of 1.1558% and maturity of 3 years. Figure 22 shows 10 sample trajectories of the positive exposure. Given extremely small volatility of the WIBOR 3M, there is almost no noise on the plot.

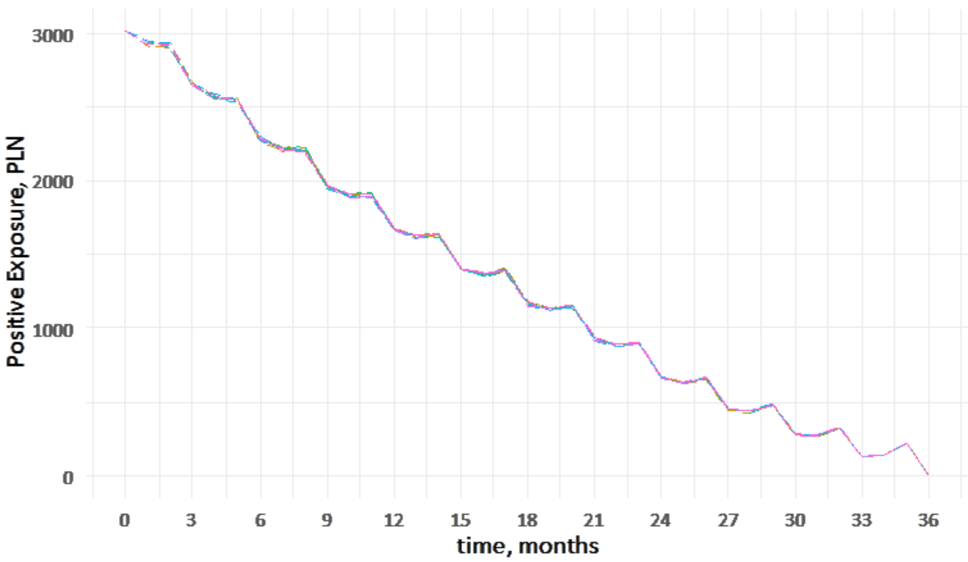

Figure 22: Sample trajectories of Interest Rate Cap Positive Exposure

Source: own research

Figure 23 depicts EPE of the instrument. Quarterly reset frequency is visible on the plot.

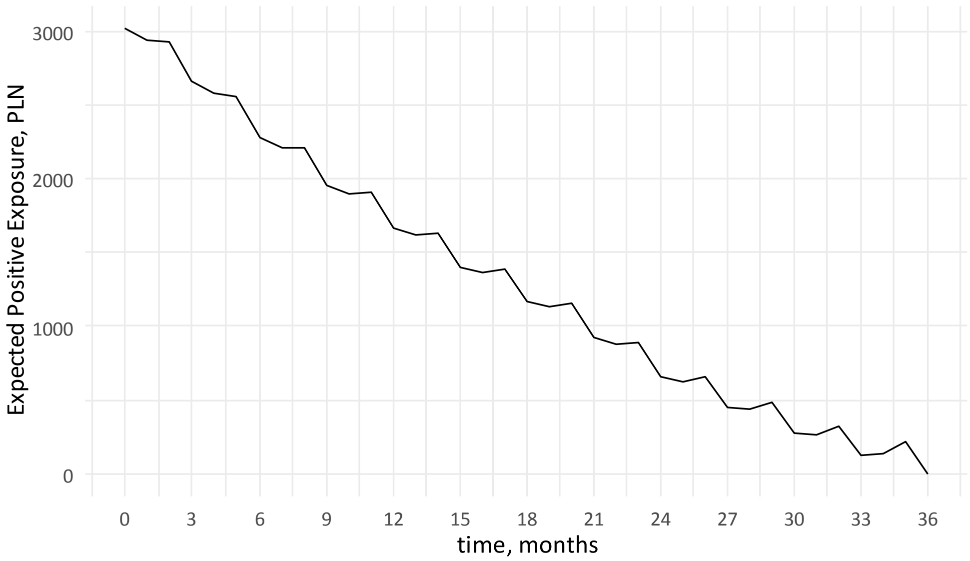

Figure 23: Expected Positive Exposure of Interest Rate Cap

Source: own research

5.1.5. Netted Portfolio

A case when the portfolio of the above-mentioned instruments is 100% netted has been analyzed. Expected exposure (before transformation into positive exposure) shown in Figure 24 is dominated by negative exposures generated by the CIRS. Meanwhile, EPE depicted on Figure 25 is peaking at the IRS-level of exposure. One can find 100 sample trajectories used to calculate the two above-mentioned metrics on Figure 26 and Figure 27, respectively.

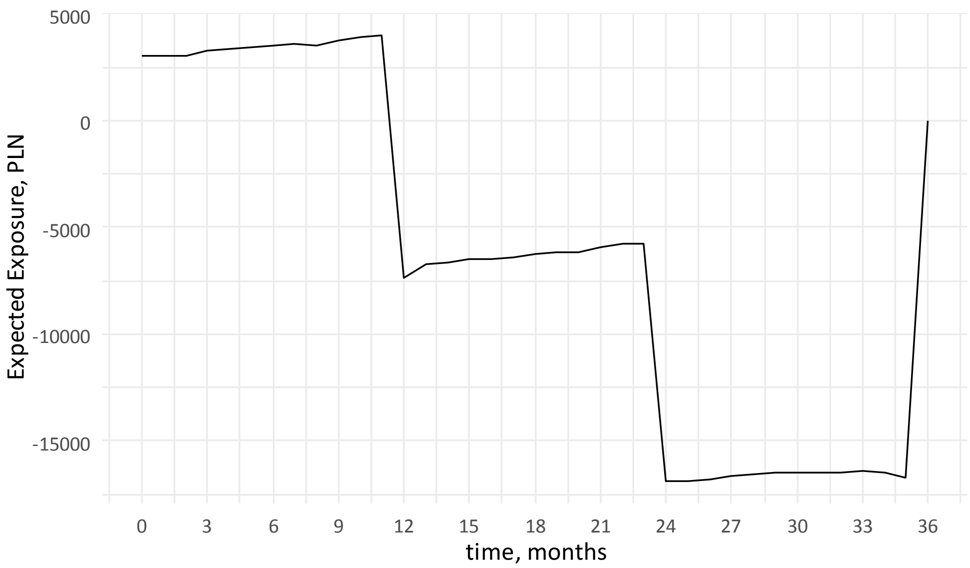

Figure 24: Expected Exposure of Netted Portfolio

Source: own research

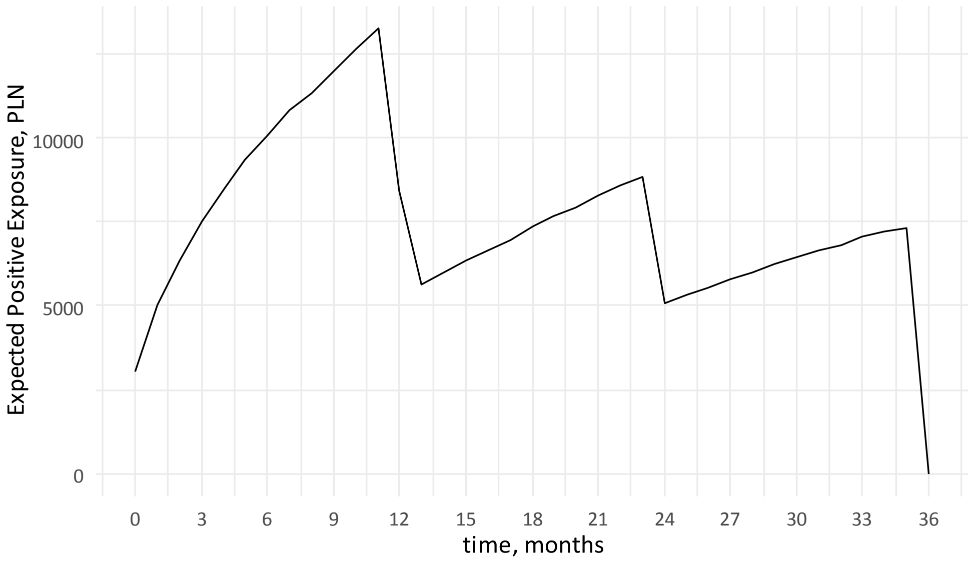

Figure 25: Expected Positive Exposure of Netted Portfolio

Source: own research

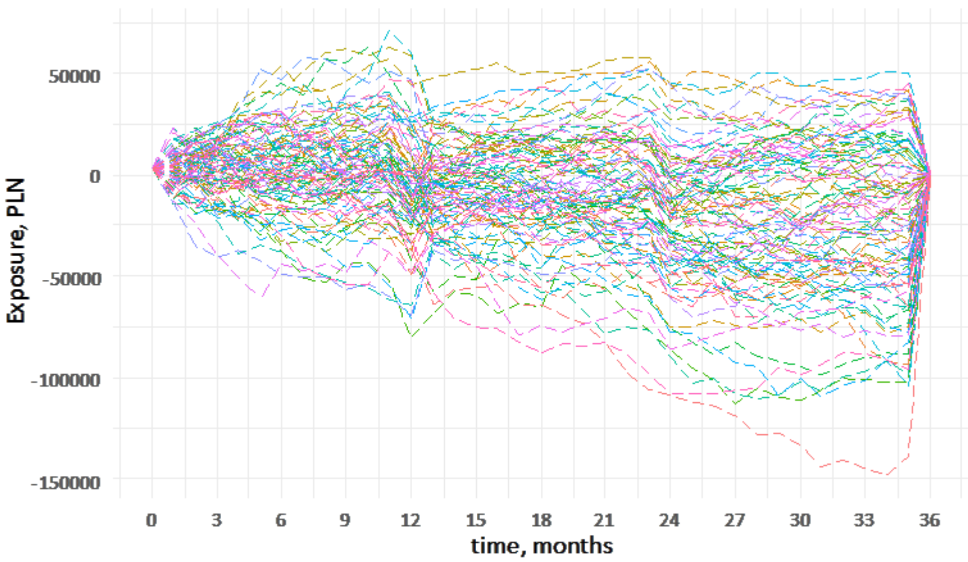

Figure 26: Sample trajectories of Netted Portfolio Exposure

Source: own research

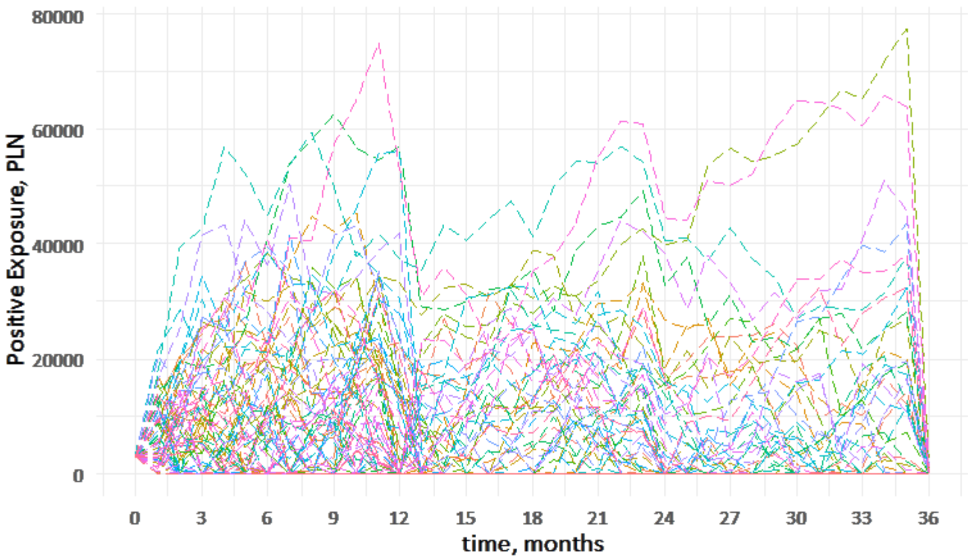

Figure 27: Sample trajectories of Netted Portfolio Positive Exposure

Source: own research

6. CVA calculation for the analyzed derivatives

This chapter briefly introduces notation used to calculate unilateral CVA and presents findings of the thesis.

6.1. Practitioner's CVA formula

Credit Valuation Adjustment is a difference between a risk-neutral valuation and a real valuation that includes risk of a counterparty's default. Unilateral CVA is given by the risk-neutral expectation of the discounted loss conditional on counterparty's default. It is formalized in Equation 17, where T is a maturity of the instrument, RR is recovery rate, B_t is future value of a money account at moment t, $E(t)$ is exposure at moment t, τ is a moment of counterparty's default and $dPD(0,t)$ is the first derivative of risk-neutral probability of counterparty's default between moment 0 and moment t.

$$CVA(T) = (1 - RR) \cdot \int_0^T \mathbb{E}^{\mathbb{Q}} \left[\frac{B_0}{B_t} \cdot E(t) \middle| t = \tau \right] \cdot dPD(0,t) \tag{17}$$

Assumption of no correlation between counterparty's exposure and its default probabilities (i.e. no wrong- or wright- way risk) greatly simplifies the analysis, and yields Equation 18.

$$CVA(T) = (1 - RR) \cdot \int_0^T \mathbb{E}^{\mathbb{Q}} \left[\frac{B_0}{B_t} \cdot E(t) \right] \cdot dPD(0,t) \tag{18}$$

Because Monte Carlo simulation yields discrete EPE, this thesis applies discrete approximation formula to calculate CVA, which is presented in Equation 19. 19 (Gregory, 2010)

$$CVA \approx (1 - RR) \cdot \sum_{i=1}^{N} EPE(t_i) \cdot d(t_i) \cdot mPD(t_{i-1}, t_i) \tag{19}$$

- $EPE(t_i)$ – EPE at moment t_i
- $d(t_i)$ – current discount factor for the term t_i
- $mPD(t_{i-1}, t_i) = PS(t_{i-1}) - PS(t_i)$ – marginal probability of counterparty's default for period from t_{i-1} to t_i
- $PS(t_i)$ – survival probability of the counterparty from moment t_0 to t_i
- RR – recovery rate

- N – number of time-steps (grid size)

6.2. Calculated CVA values for the portfolio of the instruments

Based on simulations of 10 000 scenarios, CVA values have been calculated for FX Forward, IRS, CIRS, Interest Rate Cap separately and for the entire portfolio with and without netting. Table 1 summarizes these values.

Table 1: CVA calculated for each instrument separately and for the portfolio

Instrument	*CVA value, PLN*
FX Forward	24.20668
Interest Rate Swap	167.5133
Cross-currency Interest Rate Swap	122.8299
Interest Rate Cap	24.61465
Portfolio (no netting)	**339.1645**
Portfolio (with netting)	**199.2669**

Source: own research

One can see that CVA of the portfolio after netting is significantly smaller than without netting. Netting reduces CVA for the portfolio by 41%.

Figure 28 depicts an effect of bumping CDS spreads (excluding tenor 0) on portfolio CVA after netting. One can see that the effect is linear.

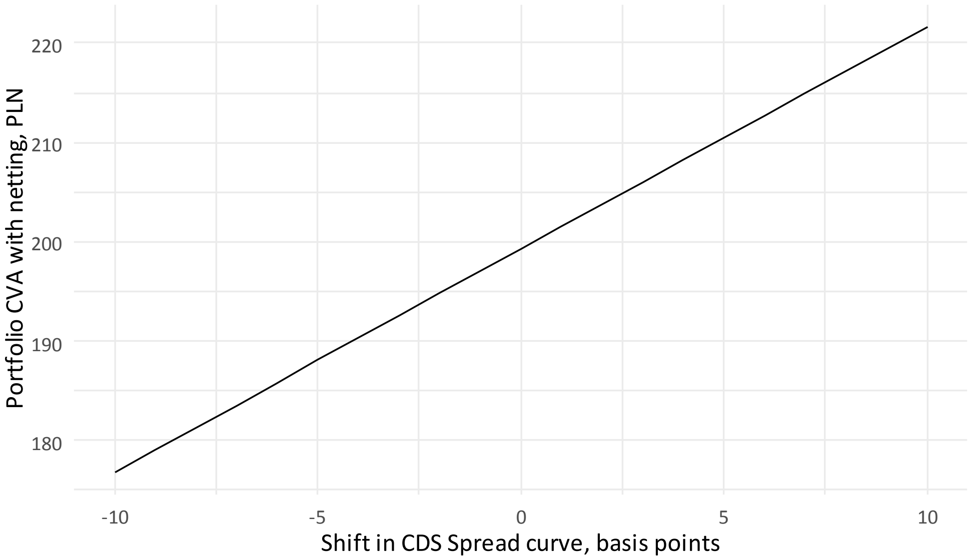

Figure 28: Sensitivity of Portfolio CVA after netting to shifts in CDS Spread curve

Source: own research

Figure 29 shows sensitivity of portfolio CVA after netting to a Recovery Rate assuming that Recovery Rate used in CVA formula and in CDS bootstrapping formula are the same. The plot depicts non-linearity. It is happening due to the above-mentioned assumption. Since there is a positive relation between Recovery Rate and Default Probability, the CDS bootstrapping algorithm returns larger values of marginal default probabilities, which increases CVA. At the same time, high Recovery Rate linearly reduces CVA on a level of the CVA formula. These two opposite effects lead to a stable CVA across majority of the Recovery Rate spectrum.

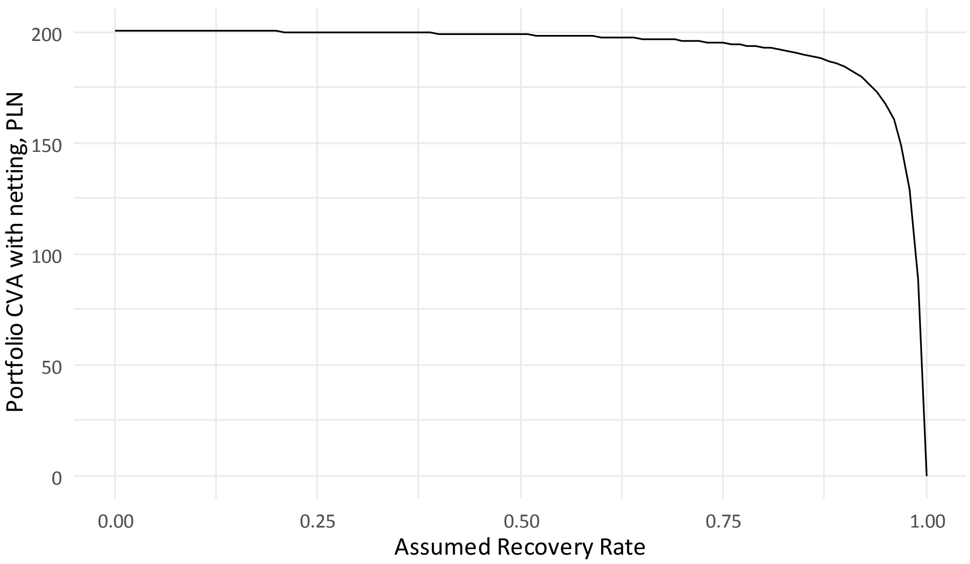

Figure 29: Sensitivity of Portfolio CVA after netting to a Recovery Rate

Source: own research

Conclusion

The aim of this thesis has been to develop a methodology for calculation of unilateral Credit Valuation Adjustment for a portfolio of various derivative instruments that is robust to current financial realities such as negative interest rates in Eurozone and lack of liquidity in Polish interbank borrowing market. Chapter 1 explained motivation behind the family of Valuation Adjustments. Chapter 2 described main ways to estimate default probabilities of a counterparty focusing on CDS bootstrapping method, implementation of which is available in the Appendix. Chapter 3 provided an overview of the derivative instruments in question including their mechanics and valuation. Chapter 4 briefly introduced the main risk factors of the derivatives in question as well as calibration of the parameters. The most notable developments were Arithmetic Brownian Motion model of WIBOR 3M and quasi-dynamic structure of Polish yield curve. Chapter 5 involved construction of exposure profiles using Monte Carlo method accompanied by elaborations about the exposure profiles generated. Chapter 6 revisited CVA more formally and presented the final output of the model alongside with basic sensitivity analysis.

Achievements of this thesis are the developed model itself, implementation of the CDS bootstrapping algorithm and the findings about sensitivity of CVA to CDS spreads shift & to a recovery rate. The thesis confirms a hypothesis that netting reduces the counterparty credit risk, which is quantified in a smaller CVA value.

The main model limitation is computational complexity of Monte Carlo exposure generation engine. Calculation of a numerically stable expected exposure requires 10 000 scenarios, which for the entire portfolio and each time-step, translates into 1 200 000 simulations. This complicates sensitivity analysis of the portfolio CVA to the pre-simulation variables such as volatility parameters of the risk factors.

The major model assumptions are independence between counterparty's credit quality and the exposure, deterministic recovery rate and non-stochastic nature of the default probability process.

Future research direction can be in depth and in breadth. The former one might be gradual relaxation of the model assumptions (such as incorporation of a wrong way risk) and dealing with the model limitations e.g. development of highly efficient valuation algorithms.

The latter one means exploration of bilateral setting, of other members of xVA family, development of more sophisticated interest rate models or including exotic instruments into the portfolio. Such research would heavily rely on availability of market data.

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List of figures

Appendices

CDS bootstrapping for sample market quotes: algorithm implementation in R

```r
implied_probab <- function(CDS_spreads, CDS_tenors, DF, grid, RR) {
  # Loss given default
  LGD <- 1 - RR
  # Emptty vector for interpolation of the CDS spreads
  CDS_spreads_inter <- grid*0

  # Linear interpolation of the CDS spreads
  for (i in 1:(length(grid))) {
    if (any(grid[i]==CDS_tenors)) {
      CDS_spreads_inter[i] <- CDS_spreads[grep(TRUE, grid[i]==CDS_tenors)]
    }
    else {
      j <- grep(TRUE, grid[i]<CDS_tenors)[1]
      CDS_spreads_inter[i] <- CDS_spreads[j-1]+(CDS_spreads[j]-CDS_spreads[j-1])*(grid[i]-C
DS_tenors[j-1])/(CDS_tenors[j] - CDS_tenors[j-1])
    }
  }

  # CDS spreads denominated in percentage points
  S <- CDS_spreads_inter[-1] / 10000

  # Delta t (time step is evenly spaced in our case)
  dt <- diff(grid)
  # Inputs used for bootstrapping
  input <- data.frame(S, DF[-1], dt)

  # Number of tenors to bootstrap
  N <- dim(input)[1]

  # Numerator of the first element of the equation
  num <- matrix(rep(0, N * N), ncol = N)
  # Denominator of the first element of the equation
  den <- numeric(N)
  # First element of the equation
  fst <- numeric(N)
  # Second element of the equation
  snd <- numeric(N)
  # Cumulative survival probability
  PS <- numeric(N)
  # Probability of survival at moment 0. Will be concatenated to the "PS" vector in the end
  PS0 <- 1

  # Calculation of tenor 1 of the second element of the equation
  snd[1] <- PS0 * LGD / (dt[1] * S[1] + LGD)
  # Probability of survival at moment 1
  PS[1] <- snd[1]

  # Calculating the denominator of the first element of equation but leaving tenor 1 to be
zero
  den <- c(0, DF[-(1:2)] * (LGD + S[-1] * dt[-1]))

  # Calculation of probability of survival at tenor 2. Forced to do this in order to stick
to the notation from the formula.
  num[2, 1] <- DF[1] * (LGD * PS0 - PS[1] * (LGD + S[2] * dt[2]))
  fst[2] <- sum(num[2, ]) / den[2]
  snd[2] <- PS[1] * LGD / (dt[2] * S[2] + LGD)
  PS[2] <-  fst[2] + snd[2]

  # Calculation of the general formula for probability of survival from tenor 3 to tenor N
  for (i in 3:N) {
    num[i, 1] <- DF[1] * (LGD * PS0 - PS[1] * (LGD + S[i] * dt[1]))
```

```
    for (n in 2:(i - 1)) {
      num[i, n] <- DF[n] * (LGD * PS[n - 1] - PS[n] * (LGD + S[i] * dt[n]))
    }
    fst[i] <- sum(num[i, ]) / den[i]
    snd[i] <- PS[i - 1] * LGD / (dt[i] * S[i] + LGD)
    PS[i] <- fst[i] + snd[i]
  }

  # Adding almost sure probability of survival at tenor 0
  PS <- c(PS0, PS)

  # Converting cumulative survival probability into marginal default probability
  mPD <- -diff(PS)

  return(mPD)
}
```

Data used

Table 2: EUR and PLN discount factors, EUR/PLN FX Forward Curve

Month	EUR_DF	PLN_DF	EUR/PLN	Month	EUR_DF	PLN_DF	EUR/PLN
0	1.0000	1.0000	4.3000	**19**	1.0043	0.9696	4.4539
1	1.0004	0.9983	4.3090	**20**	1.0043	0.9678	4.4621
2	1.0006	0.9972	4.3149	**21**	1.0044	0.9661	4.4703
3	1.0009	0.9958	4.3218	**22**	1.0044	0.9643	4.4785
4	1.0012	0.9944	4.3292	**23**	1.0043	0.9626	4.4867
5	1.0014	0.9929	4.3367	**24**	1.0043	0.9607	4.4950
6	1.0017	0.9915	4.3444	**25**	1.0042	0.9589	4.5034
7	1.0020	0.9900	4.3521	**26**	1.0041	0.9570	4.5119
8	1.0022	0.9885	4.3599	**27**	1.0040	0.9551	4.5205
9	1.0025	0.9869	4.3678	**28**	1.0039	0.9531	4.5291
10	1.0027	0.9853	4.3760	**29**	1.0037	0.9511	4.5378
11	1.0030	0.9837	4.3844	**30**	1.0035	0.9490	4.5467
12	1.0032	0.9820	4.3930	**31**	1.0033	0.9469	4.5558
13	1.0034	0.9802	4.4020	**32**	1.0030	0.9448	4.5650
14	1.0036	0.9783	4.4112	**33**	1.0027	0.9426	4.5742
15	1.0038	0.9765	4.4202	**34**	1.0024	0.9405	4.5831
16	1.0039	0.9747	4.4288	**35**	1.0021	0.9384	4.5919
17	1.0041	0.9730	4.4373	**36**	1.0017	0.9362	4.6007
18	1.0042	0.9713	4.4456				

Source: Financial Engineering Project

Table 3: CDS Spreads Curve of the counterparty

CDS tenor	CDS spread, b.p.
6M	35.88
1Y	39.40
2Y	69.35
3Y	96.70
4Y	123.33
5Y	152.07
7Y	196.58
10Y	223.30

Source: Financial Engineering Project