A Dynamic Model of Primaries*

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Abstract

Primary elections are increasingly popular around the world, but historically political parties have typically chosen their candidate selection mechanisms in a decentralized manner. We develop a theory that accounts for variations in the use of primary elections in these settings. In our model, two parties choose candidates for general elections over an infinite horizon. Each party has an elite and a non-elite faction, where the elite faction can choose whether to hold primaries or nominate itself. Primaries produce more electable candidates, but losing a primary also deprives elites of private goods and future elite status. The model predicts that parties adopt primaries under high ideological polarization or when they are electorally disadvantaged. Additionally, we show how rigidities in the ability of winners to change candidate selection mechanisms can increase the universal adoption of primaries in electorally volatile environments.

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1 Introduction

When do political parties choose primary elections over closed methods of candidate selection? Candidate selection mechanisms clearly affect which candidates run for office and are subsequently elected, and academics and political observers have offered numerous rationales for this choice. Primary elections can help parties by improving candidate quality, reducing the power of party bosses, and bringing meaningful competition to areas where one party dominates. However, primaries may also reduce party cohesion, damage the party brand, and direct benefits to "core" rather than swing voters. In assessing this trade-off, parties must consider not only their internal politics, but also how the choices of other parties affect their electoral prospects.

To a significant degree, the question of why parties choose particular selection mechanisms remains largely unaddressed. This is due in part to the context of many studies of primaries: much of the theoretical and empirical scholarship has focused on the United States, where parties have long been legally required to use primaries to select candidates for most important offices (Ware, 2002). Empirically, however, centrally imposed primaries remain the exception rather than the rule across democracies globally (Hazan and Rahat, 2010). In many present-day democracies, primaries are not mandated, and political parties voluntarily adopt or forego the use of primaries in a decentralized manner. As Figure 1 illustrates, the lack of constitutional or legal mandates has produced a diverse set of outcomes. Examining all presidential elections in Spanish- and Portuguese-speaking Latin America since each country's most recent democratization, *every* country has varied in the use of primaries across major parties and election cycles.

We develop a simple theory of party governance that accounts for variations in candidate selection mechanisms under decentralized adoption. Its key feature is that competing parties symmetrically make independent decisions over their mechanisms. In choosing between elite selection and primaries, each party's elite faces a tradeoff between ensuring that its candidate represents the party and improving the party's electability through candidate competition.

These decisions additionally take into account possible effects on future elections.

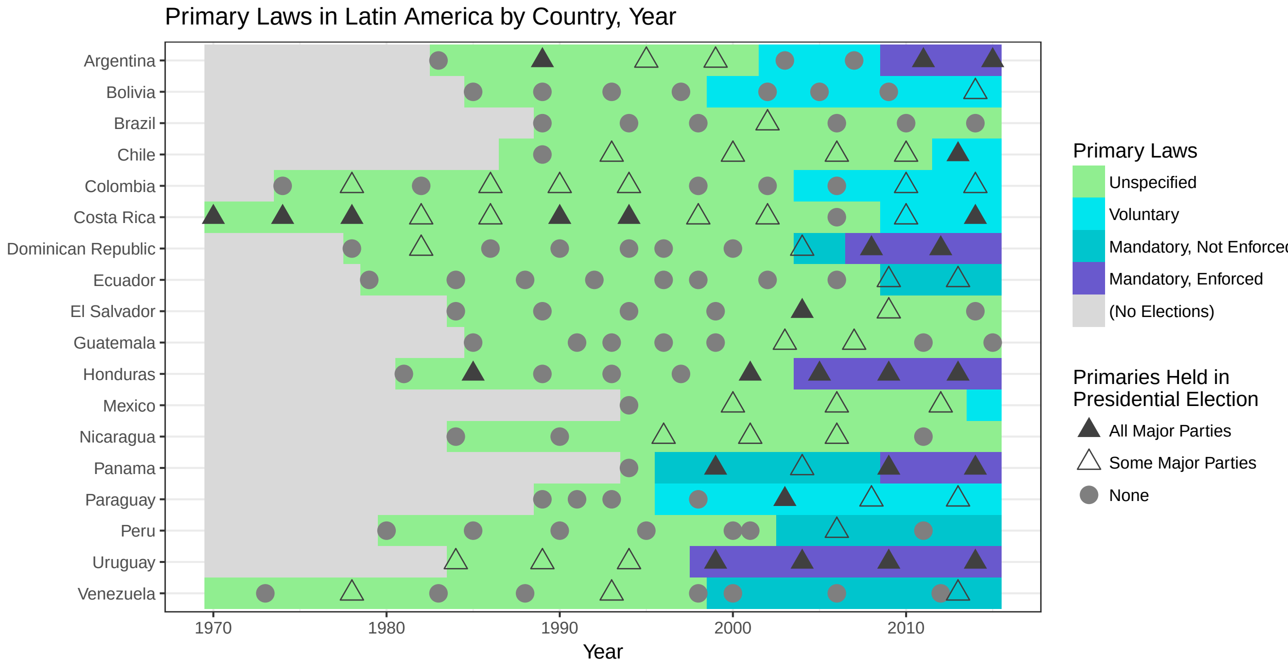

Figure 1: Primary election laws and the employment of primaries in presidential elections in eighteen Latin American countries from 1970-2015. The light gray segments precede a country's most recent democratic transition. The background colors correspond to the legal requirements for primaries as specified in national constitutions and electoral law. The points indicate each presidential election in the subsequent to these transitions.

Our model features two party electoral competition within a single constituency over an infinite horizon. Parties are ideologically homogeneous but consist of two symmetrical factions, elite and non-elite. The elite faction leads the party and chooses between two methods of candidate selection: elite selection and closed primaries. Under the former, the party elite simply nominates itself as the party's general election candidate. Under the latter, the two factions compete for the support of a decisive party voter. If eligible to compete, candidates choose costly platforms that specify the level of public goods they will provide if elected. After candidates are determined, the constituency chooses a general election winner. In addition to public goods and ideology, voters also care about an electability or valence bonus that one faction in each party realizes immediately before the general election.

Elite status within a party matters for two reasons. First, the faction that wins the general election receives faction-specific private goods, which might correspond to patronage benefits. As such, in choosing to hold primaries, elites must weigh the possibility of losing the primary (and thus losing private goods with certainty) against the possible general election benefit of increased electability. Studies of factionalism in Latin American parties emphasize the high value of party control ("elite status") in terms of patronage or rents (Benton, 2007; Taylor, 1996).

Second, elite status can shift across periods. One potential cost of holding primaries is that the winning faction becomes the next period's elite, and can thereby shut the original elites out of private goods. Case studies of factionalism within Latin American political parties from Honduras to Uruguay have suggested that the prospect of losing influence within or control of the party may deter primary adoption (Coppedge, 1994; Martz, 1999; Morgenstern, 2001; Taylor, 1996). The first factor imposes a static trade-off in candidate selection, while the second imposes a dynamic trade-off.

In equilibrium, primary adoption depends on the location of the median voter and ideological polarization among parties. Our first main prediction is that primaries become more appealing as the ideological distance between parties increases—in other words, as the ideological cost of an election loss grows. At low levels of ideological polarization, the electoral stakes are lower and elites can more safely maintain control. The second prediction is that the electorally disadvantaged party will tend to be the sole adopter of primaries; the party that is electorally favored will employ elite selection, since it has less need for the electoral benefit provided by primaries. These two results imply that the combination of high party polarization and low partisan imbalance encourages universal adoption of primaries.

These results are driven in large part by the model's static incentives, as they hold even when factions do not care about the future. However, dynamic considerations matter greatly for the incentive to hold primaries. The possible loss of elite status discourages primaries in the sense of expanding the set of parameters supporting an equilibrium with no primaries or only one primary. Yet there is no reduction in the set of parameters supporting two primaries, and two primaries remain the unique prediction for centrist electorates with highly polarized parties. Thus, even far-sighted politicians can arrive at system-wide adoption of primaries in highly polarized political systems.

Our results are broadly consistent with patterns of primary adoption in Latin America, which we describe in the next section. In particular, they strongly suggest that decentralized choice plays a crucial role in determining a party system's configuration of candidate selection mechanisms. Consistent with our theoretical predictions, electorally dominant Latin American parties have not generally adopted primaries for themselves. By contrast, Snyder and Ting (2011) present evidence the least competitive U.S. states were generally the earliest to impose primaries statewide.

We next push the model further to explore the effects of an electoral volatility and factional entrenchment. Electoral conditions might favor the previously elected party, and a losing party might be more easily able to adjust its candidate selection mechanism than the winner. We therefore consider a setting in which the previous period's winning party is constrained to hold its candidate selection mechanism fixed and general election voter preferences are stochastic but correlated over time. To simplify the analysis we assume myopic factions, and even then only numerical results are possible. Our results show that in the long run, both mutual adoption and mutual non-adoption of primaries can become more prevalent as the constituency becomes more ideologically volatile. Heightened volatility corresponds to a reduction in the incumbency advantage, and so these observations imply that a low incumbency advantage will be associated with a reduced the likelihood of elections with partial use of primaries. The framework also presents some promising opportunities for further examining dynamic electoral competition.

The mechanisms identified in this paper are novel contributions to the literature on primaries (Serra, 2018). Most theoretical models of primaries address consequences such as candidate attributes, effort, ideological extremity, or public goods provision, but most do not address endogenous primary adoption (Meirowitz, 2005; Owen and Grofman, 2006;

Jackson, Mathevet, and Mattes, 2007; Hirano, Snyder, and Ting, 2009; Hummel, 2010; Casas, 2016; Serra, 2011; Agranov, 2016; Hummel, 2013). Several models have endogenous primary adoption, but do so in a limited fashion, where primary adoption is centralized (sometimes determined by the favored party) or considered for only one party (Aragón, 2014; Adams and Merrill, 2008; Snyder and Ting, 2011; Buisseret and Van Weelden, 2018).

Two notable exceptions to this literature are Crutzen, Castanheira, and Sahuguet (2010) and Crutzen (2013), which allow competing parties to choose candidate selection mechanisms in a decentralized manner. These papers focus mainly on features such as voter information, office rents, and electoral systems. By contrast, this paper focuses on polarization, electorate ideology and the dynamics of elite status, and provides a complete characterization of the ideological conditions favoring different configurations of candidate selection mechanisms. Thus, to our knowledge, our model is the first that can address the empirically observed cross-national variation in primary adoption.

The paper proceeds as follows. In the next section, we describe the model's empirical motivation. Section 3 describes the model, and section 4 presents its results. Section 5 then develops the alternative infinite horizon model with stochastic voter ideal points and factional entrenchment. The final section concludes.

2 Background

Drawing from data on presidential candidate selection mechanisms from the eighteen Spanish- and Portuguese-speaking countries in Latin America, three sets of observations motivate our theoretical model. First, as suggested in the introduction, electoral laws in the region generally provide for the adoption of primaries as a party-level (decentralized) decision. Second, on the basis of subsequent election results and characteristics of the party system, we provide suggestive evidence that primary adoption is non-random; primaries appear to be implemented as a strategic response of parties to electoral conditions. Finally, a more detailed examination of candidate selection mechanisms over time within parties provides motivation for a dynamic model of decentralized primary adoption.

The decentralized adoption of primary elections requires that parties have the right to choose their method of candidate selection, namely elite selection or primary elections. In settings where primary elections are mandated or prohibited, this choice does not exist. The presidential democracies of Latin America provide fertile grounds for examining (a) the frequency with which electoral law allows for the possibility of decentralized adoption of primaries; and (b) the degree to which parties' methods of candidate selection vary in such settings.[1] Figure 1 depicts the electoral setting in Latin America since 1970 or a country's most recent democratization.[2] Background colors indicate the state of electoral laws regarding primaries. We code whether or not primaries are specified in national constitutions or electoral law over each country's recent democratic history. This coding draws upon the analysis of Estaun (2015) as well as supplementary archival research.

In all countries in Figure 1, national constitutions or electoral law did not mandate or ban the use of primaries as a means of candidate selection in the years immediately following democratization. More recently, fourteen of the eighteen countries have adopted some language specifying the voluntary or mandatory use of primaries for candidate selection. However, the formal or "parchment" laws mandating the use of primaries are not enforced in several of the countries that have adopted them, consistent with broader arguments about political institutions in the region (Levitsky and Murillo, 2009; Carey, 2000). Where such laws are absent, or under voluntary or unenforced mandatory primary laws, parties have a wide scope to select candidate selection mechanisms. For much of these nations' recent democratic histories, this strategic choice has been available to party leaders. Moreover, this choice remains available to parties in thirteen of eighteen countries.

Given this freedom of choice, to what extent do we observe variation in the candidate

[1]Decentralized primary adoption has also occurred in democracies outside of the region, including but not limited to France, Spain, Italy, Armenia, Portugal, Finland, the UK, Greece, Japan, Israel, South Korea, Taiwan, Ghana, and Botswana.

[2]While Colombia was ostensibly democratic prior to 1974, the National Front arrangement (1958-1974) precluded multi-party competition during this period.

selection methods employed in these contexts? The points in Figure 1 represent each presidential election in each country. Their shapes and shading correspond to the candidate selection mechanisms adopted by major parties in each election.[3] Three distinct candidate selection arrangements emerge: (1) no major party holds primaries; (2) at least one but not all major parties hold primaries; or (3) all major parties hold primaries. Taken together with the chronology of primary laws from the region, the decentralized adoption of primaries has induced wide variation in the profile of candidate selection mechanisms in most Latin American countries.

We next consider whether the use of primaries corresponds to electoral conditions. For the same set of countries, Figure 2 depicts the distribution of parties' general election presidential vote margins subsequent to primaries and elite selection.[4] This graph suggests a possible relationship between the prevalence of primaries as a selection method and a party's electoral strength: primaries are more common among parties with smaller vote margins. They are also distributed more broadly among losing parties (where vote margin is negative).

Ideological polarization may also play a role. Figure 3 depicts the adoption of primaries in elections between 2005 and 2009 as a function of vote share and ideological distance between parties.[5] This measure is cross sectional, and scores were not available for all parties within our dataset, so the sample shown in Figure 3 is reduced. Despite this, a few notable trends emerge. First, within this recent sample, there is a visible relationship between polarization and primary adoption: primaries are more common in scenarios where there is more ideological variation among competing parties. In addition, primary adoption

[3]A major party is defined as one that achieves a vote share of at least 20% in the general election (first-round where applicable) or one that advances to a runoff with less than 20% of the first-round vote. All electoral data comes from Nohlen (2005) supplemented by subsequent electoral results from each nation's electoral body. We draw upon a dataset by Carey and Polga-Hecimovich (2006) and amended by Kemahlioglu, Weitz-Shapiro, and Hirano (2009) as well as original data collection to extend the time series through 2015.

[4]Vote margin is calculated as a party's vote share relative to that of its strongest opponent. In two round (runoff) election systems, the vote margin graphed is from the first round.

[5]Our measure of ideological distance comes from the Kitschelt (2013) expert survey on political parties: parties were rated on a common 10-point left-right scale, and an environmental polarization measure was constructed by taking the standard deviation of this measure within a country-election period.

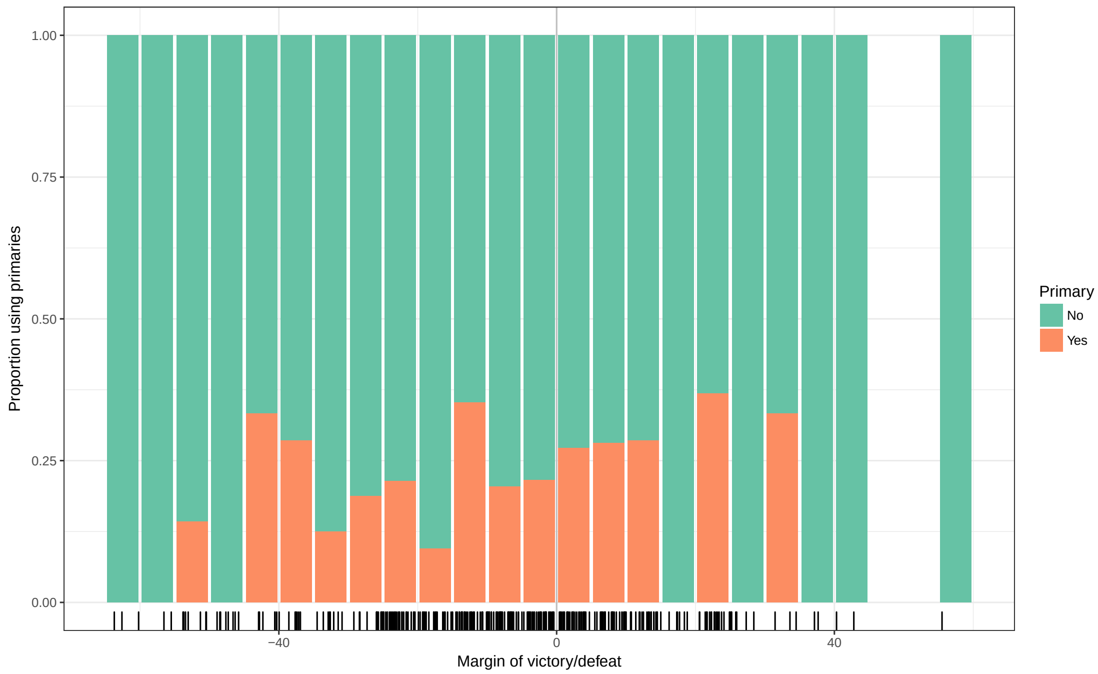

Figure 2: Proportion of parties using primaries as a function of vote margin. Margin for each party is calculated relative to the strongest opponent; margins are grouped into bins of five percentage points. Tick marks at the bottom of the plot indicate the density distribution of data.

is more common among losing parties; only two out of twelve winners (17%) held primaries, while eleven out of thirty-one losers (35%) did so. Collectively, this data suggests that the adoption of primaries is far from random: electoral conditions, especially vote share and polarization, seem to shape parties' choices.

Finally, beyond the choice of whether or not to hold a primary in a given election, examining the adoption (or lack thereof) of primaries over time reveals several distinctive patterns. Figure 4 shows several of these patterns, by party, in four Latin American countries over time. Each party that has held a primary has later returned to elite selection in at least one election, suggesting that decentralized decisions to hold primaries are not particularly "sticky" from one election to the next. Additionally, the frequency of primaries varies substantially from country to country: Costa Rica and Brazil represent the countries with the most and least frequent adoption of primaries, respectively. There also exists substantial variation in the degree to which major parties mirror competitors' behavior with respect

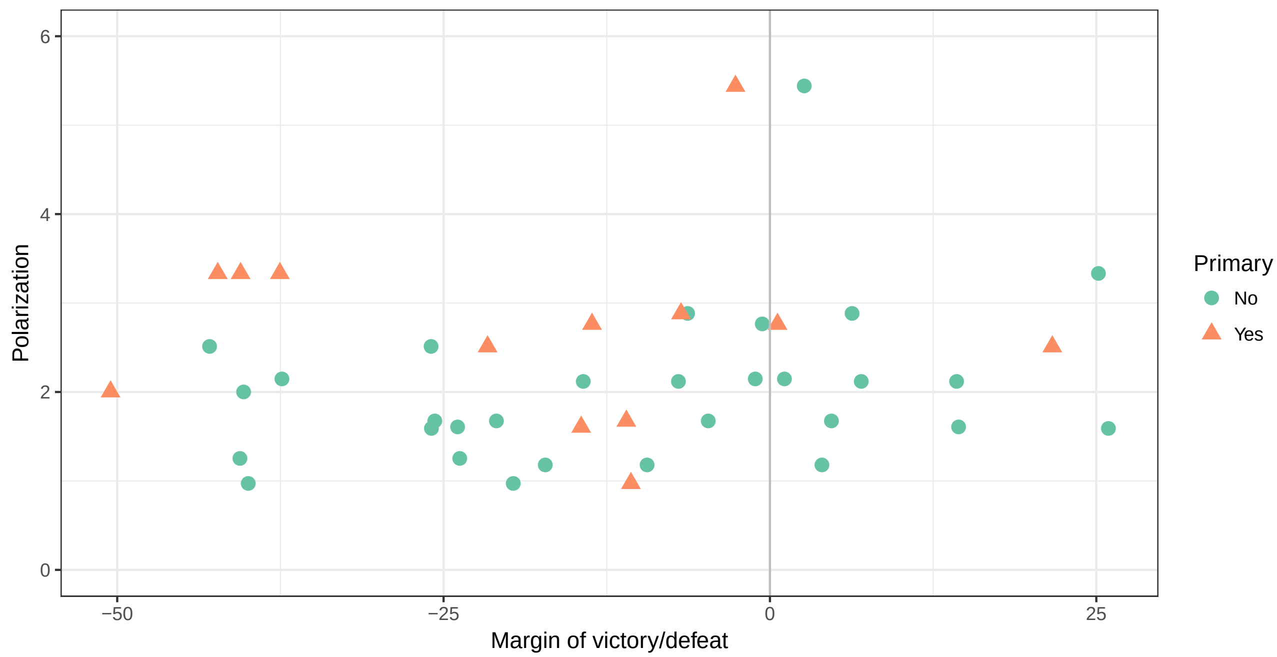

Figure 3: Primary adoption in presidential elections in relation to ideological polarization and vote margin. Positive vote margin indicates victory. Polarization is measured as the standard deviation of the ideological positions of all parties competing within a country election period.

to holding primaries. Here, Honduras and Colombia represent two distinctive patterns. In Honduras, prior to the implementation of mandatory primaries before the 2005 election, Partido Nacional and Partido Liberal adopted primaries in parallel. In Colombia, at most one (major) party has held primaries in a given presidential election.

The three main stylized facts developed in this section motivate our theoretical model. First, electoral laws allowing for the decentralized adoption of primaries are relatively widespread in Latin America, demonstrating wide applicability for this model. Second, the decision to adopt primaries appears to be systematically related to electoral conditions, specifically polarization and competitiveness of presidential elections. Finally, the variations in the use of primaries within parties across elections suggest a role for dynamic considerations.

3 Model

Our basic model is an infinitely repeated game of electoral competition between two parties, labeled L and R, in a single constituency. In every period there is a general election between

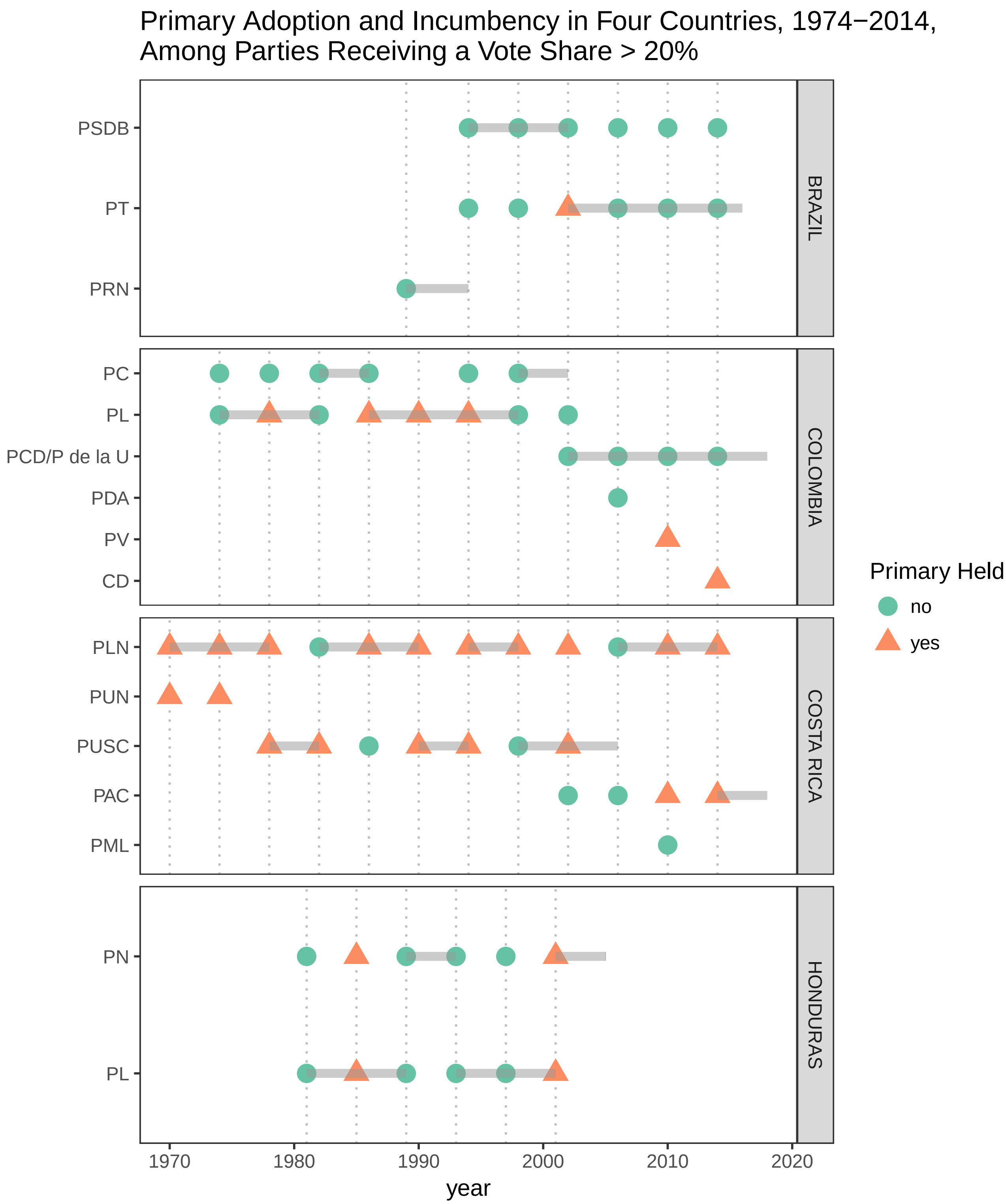

Figure 4: Party-level patterns of primary adoption in four Latin American countries. Gray segments correspond to the president's party. The vertical lines correspond to presidential election years. Blank years in the left portion of the panels are prior to the most recent democratization. Blank years in the right portion of the Honduras panel indicate that Honduras mandated (and enforced) primary elections starting in 2005.

the parties and possibly primary elections within each party. The elections determine both ideological and public goods outcomes. We omit notation for time periods throughout, as these are unnecessary for describing the model.

Players in the game are either party factions or voters. Each party has two factions, labeled *elite* (E) and *non-elite* (N), that can serve as the party's general election candidate. Party i factions share a common ideological ideal point $x_i \in \mathbb{R}$, where $x_L < 0 < x_R$ and $-x_L = x_R$. Let $\Delta = x_R - x_L$ denote the ideological distance between the parties. A continuum of citizens vote in the general election. The ideological median among these has an ideal point $x_m \in (x_L, x_R)$ (equivalently, $x_m \in (-\Delta/2, \Delta/2)$). For each party i, a voter with ideal point x_i additionally serves as the representative voter in its primary elections. All players are infinite-lived and discount future periods by a common factor $\delta < 1$.

Elite status corresponds to current leadership of the party, and confers control over candidate selection procedures. In every period, faction E in party i chooses selection method $c_i \in \{elite, primary\}$. Under *elite* selection, faction E simply nominates herself and excludes the faction N candidate. Under *primary* selection, a primary election determines the nominee. An important feature of the model is that elite status is endogenous. To capture the possibility that losing a primary may require the elites to cede leadership, we assume the winner of a primary election adopts the role of the party elite in the subsequent period. It will be convenient to use $c \in \{\emptyset, L, R, LR\}$ to identify the set of parties holding primaries.

Each non-excluded faction $j \in \{E, N\}$ of party $i \in \{L, R\}$ competes by offering a platform $q_i^j \geq 0$ of public goods. This platform benefits the entire electorate if she wins the election, and imposes a cost $(q_i^j)^2$ on the offering faction. The cost might represent the difficulty of developing credible policies, or simply the opportunity cost of foregone favors to factional allies. We denote the set of platforms chosen by non-excluded factions $\mathbf{q}$, and the platform of the party i general election candidate q_i.

Following the choice of candidate selection method but before the primary and general elections, all factions receive a candidate-specific electability shock, $b_i^j \in \{0, b\}$, where $b > 0$.

Voters value this shock in the same way as public goods, while factions do not receive utility directly from it. Within each party, faction E receives the favorable b shock with probability 1/2, and faction N receives it otherwise. Note that faction N can receive the b shock regardless of whether there is a primary. Shocks are i.i.d. across parties and periods. Let b_i denote the shock of the party i general election candidate.

Primary elections in party i are determined by its decisive primary voter. This voter is fully strategic and anticipates the results of the general election. The general election matches the two parties' nominees, and is decided by the entire population of voters. Immediately prior to this election, voters receive an election-specific utility shock $\phi \sim U[-\alpha, \alpha]$ to the party R candidate, where α is a measure of electoral volatility. The general election winner receives a private benefit v from the victory. This might be interpreted as patronage or office-holding utility.

A voter with ideal point x receives the following utility from electing the faction j candidate from each party:

$$
\begin{aligned}
U(x_R, q_R^j, b_R^j; x) &= -|x - x_R| + q_R^j + b_R^j + \phi && (1)\\
U(x_L, q_L^j, b_L^j; x) &= -|x - x_L| + q_L^j + b_L^j. && (2)
\end{aligned}
$$

Factions care about ideology, public goods, and private benefits. They care about the electability shock only insofar as it affects the probability of winning, and their utility from the other party's candidate is based on her public goods offer and the ideological distance between parties. Faction j in party i thus receives the following utility from each outcome:

$$
U_i^j(\mathbf{q}) = \begin{cases} q_i^j + v - (q_i^j)^2 & \text{if she wins the general election} \\ q_i^{\neg j} - (q_i^j)^2 & \text{if the opposing faction wins general election} \\ q_{\neg i}^k - \Delta - (q_i^j)^2 & \text{if faction } k \text{ of the opposing party wins general election} \end{cases} \quad (3)
$$

Each period of the game proceeds as follows:

1. Party elites simultaneously choose candidate selection methods c_i.

2. Eligible factions simultaneously choose public goods platforms q_i^j.

3. The valence shock b_i^j for each faction is realized.

4. For each party:

   - Under *primary*, the party decisive voter chooses the primary winner.

   - Under *elite*, the elite faction is nominated.

5. The preference shock of the median voter, ϕ, is realized.

6. The general election is held.

We adopt three parametric restrictions to simplify the exposition. To keep public goods platforms non-negative, we assume that general election outcomes are sufficiently random: $\alpha > \left(1 + \sqrt{4\Delta + 1}\right)/2$.[6] We also eliminate some uninteresting equilibria by assuming:

$$v > \frac{(3\alpha - 2)b}{4\alpha^2 - 6\alpha + 2}. \tag{4}$$

This ensures that elite status is of sufficient concern to factions. Finally, to ensure interior probabilities of victory, we assume:

$$x_m \in \left[\frac{b}{4} - \frac{\alpha(\alpha-1)}{2\alpha-1}, \frac{\alpha(\alpha-1)}{2\alpha-1} - \frac{b}{4}\right]. \tag{5}$$

We characterize stationary, subgame perfect Nash equilibria in pure strategies. The equilibria are also symmetric with respect to factions within each party. Combined with stationarity, this implies that elite factions make identical choices in each period. Since there is a continuum of voters, we adopt the standard assumption that voters vote as if pivotal. The party i elite faction's candidate selection mechanism strategy is the choice

[6]When this assumption is satisfied, the assumption that $x_m \in (-\Delta/2, \Delta/2)$ is sufficient for interior platforms.

$c_i \in \{elite, primary\}$. Each eligible faction's platform strategy $q_i^j : \{elite, primary\}^2 \to \mathbb{R}_+$ maps the candidate selection mechanisms into a public goods platform (where, of course, faction N's platform is irrelevant under elite selection). A primary voting strategy is a mapping $\{elite, primary\}^2 \times \mathbb{R}_+^4 \times \{E, N\}^2 \to \{E, N\}$ of these choices and the electability shock realizations into a candidate choice. Finally, the general election voters' strategies $\{elite, primary\}^2 \times \mathbb{R}_+^4 \times \{E, N\}^2 \times \{E, N\}^2 \times [-\alpha, \alpha] \to \{L, R\}$ map the preceding choices, the realized candidates, and the utility shock into a vote.

To examine the effects of factional entrenchment and a changing electoral environment, we adapt this game to a somewhat different setting in section 5. In that version, the location of x_m can vary across periods. Additionally, factions live for only a single period but and winning parties are entrenched in the sense that only the previous period's losing party can alter its candidate selection mechanism. The long run patterns of primary adoption in this model require require numerical simulation.

4 Results

Our main results concern the circumstances under which party elites hold primaries. Primaries contrast with elite selection in three important ways. First, they increase the probability of nominating a more electable candidate for the general election. Second, they may deprive status quo elites of private goods if their party wins the general election under the non-elite faction. Third, the loss of elite status may threaten future private goods as well. These produce the tension in the elite's choice of candidate selection mechanism.

4.1 Voting

We begin by deriving voting strategies. An important general observation is that factions are fully symmetric from the perspective of all voters. By assumption, factions have the same ideology, and voters have no intrinsic preferences over which has elite status. Additionally, since factions act symmetrically in equilibrium, the identity of the elite faction has no payoff implications for any voter. This implies that voters can disregard the effect of their votes on internal party governance.

For the general election, the electorate is the entire continuum of voters. The election outcome affects their payoffs only in the current period and does not affect future game play, so voters need only consider the current election.[7] Voting as if pivotal then allows each voter to choose the candidate that maximizes her immediate utility. In standard fashion, this ensures that the constituency-wide median voter is decisive.

Given the uniform distribution of the voters' utility shock ϕ, party R's probability of victory is then:

$$\begin{aligned}\pi(q_L + b_L, q_R + b_R) &= \Pr\{\phi > x_L - x_m + q_L + b_L - (x_m - x_R + q_R + b_R)\} \\ &= \frac{1}{2} + \frac{x_m}{\alpha} + \frac{(q_R + b_R) - (q_L + b_L)}{2\alpha} \end{aligned} \tag{6}$$

Under our assumptions about α, this probability is interior at the optimal choice of candidate platforms.

For each primary election, the party decisive voter's decision is straightforward. Given the irrelevance of elite status from her perspective, the party R primary voter simply chooses the faction whose platform and electability shock maximizes (1) for the median voter. This is faction E if $q_R^E + b_R^E \geq q_R^N + b_R^N$, and faction N otherwise.[8] Similarly, the party L primary voter chooses the faction that maximizes (2). If factions choose identical platforms, then each will win and achieve nomination with probability 1/2.

4.2 Platforms

Optimal platforms depend on the profile of candidate selection mechanisms, c. An important property that simplifies our analysis is that for any c, the equilibrium platforms are simply those that maximize the factions' utilities in the current period, under the assumption that

[7] General election voters might have a greater strategic role if factional control somehow depended on the outcome of the general election, or if factions were differentiated ideologically.

[8] Observe that a primary voter has no incentive to choose strategically in order to induce the median voter to choose the opposing party's candidate. Even if the party R primary voter preferred a party L faction to either party R faction, the median voter would also choose the party L faction because $x_m \in (x_L, x_R)$.

primary voters nominate the faction with the high electability shock. In other words, we may treat platform selection as if it occurs in a single period, using the simplest assumption for how primary voters behave.

To illustrate the platform choice, consider what happens when there are no primaries ($c = \emptyset$). A faction's objective in a single period is simply the sum of her utility from each election outcome, weighted by its probability. Since there is no primary to lose, future elite status is assured. Thus, the elite faction's payoff depends only on whether her party wins or loses the current election. Using (6) and aggregating over each realization of the electability shock, party R's probability of victory is:

$$\pi^{\emptyset}(q_L^E, q_R^E) = \frac{1}{4}\left(\pi(q_L^E + b, q_R^E + b) + \pi(q_L^E, q_R^E + b) + \pi(q_L^E + b, q_R^E) + \pi(q_L^E, q_R^E)\right).$$

Combining this with the appropriate election result from equation (3), the party R elites's maximization problem is as follows. (The L elite's problem is symmetric.)

$$\max_{q_R^E} \pi^{\emptyset}(q_L^E, q_R^E)\left(q_R^E + v\right) + (1 - \pi^{\emptyset}(q_L^E, q_R^E))\left(q_L^E - \Delta\right) - (q_R^E)^2 \tag{7}$$

This function is concave in q_R^E, and the first order conditions of the objectives form a system of best response equations:

$$q_R^E = \frac{-2q_L^E + v + \Delta + 2x_m + \alpha}{4\alpha - 2} \tag{8}$$

$$q_L^E = \frac{-2q_R^E + v + \Delta - 2x_m + \alpha}{4\alpha - 2} \tag{9}$$

Solving this system produces the best response platforms. Observe that in this case, elite factions need to care only about the general election. When there are primaries, each faction within a party should also worry about beating the other faction. In particular, by "outbidding" the opposing faction's public goods platform by b, a faction can assure itself of a primary victory. However, this possibility never arises because elite selection — which also guarantees victory, but without distorting platform choices — would produce at least as good of a result for the elites. Thus, whenever a faction might be tempted to outbid an opponent, the elites would prefer to implement elite selection instead. As the following result shows, it follows that whenever the elites opt for primaries, primary elections are determined by the electability shock. All results are proved in Appendix A.

Remark 1. Primary Win Probabilities. *In equilibrium, each faction wins any primary election with probability* $1/2$.

Remark 1 ensures that current platforms will not affect future elite status in equilibrium. This allows us to derive in a straightforward manner the unique symmetric platforms for any combination of candidate selection mechanisms.

Remark 2. Platforms. *Equilibrium platforms for each profile* c *of candidate selection mechanisms are as follows.*

For $c = \emptyset$*:*

$$q_R^{E*}(\emptyset) = \frac{1}{4} + \frac{v+\Delta}{4\alpha} + \frac{2x_m}{4(\alpha-1)} \tag{10}$$

$$q_L^{E*}(\emptyset) = \frac{1}{4} + \frac{v+\Delta}{4\alpha} - \frac{2x_m}{4(\alpha-1)} \tag{11}$$

For $c = R$*:*

$$q_R^{E*}(R) = q_R^{N*}(R) = \frac{1}{4} + \frac{v+\Delta}{4\alpha} + \frac{4x_m+b}{8(\alpha-1)} \tag{12}$$

$$q_L^{E*}(R) = \frac{1}{4} + \frac{v+\Delta}{4\alpha} - \frac{4x_m+b}{8(\alpha-1)} \tag{13}$$

For $c = L$*:*

$$q_R^{E*}(L) = \frac{1}{4} + \frac{v+\Delta}{4\alpha} + \frac{4x_m-b}{8(\alpha-1)} \tag{14}$$

$$q_L^{E*}(L) = q_L^{N*}(L) = \frac{1}{4} + \frac{v+\Delta}{4\alpha} - \frac{4x_m-b}{8(\alpha-1)} \tag{15}$$

For $c = LR$:

$$q_R^{E*}(LR) = q_R^{N*}(LR) = \frac{1}{4} + \frac{v+\Delta}{4\alpha} + \frac{2x_m}{4(\alpha-1)} \tag{16}$$

$$q_L^{E*}(LR) = q_L^{N*}(LR) = \frac{1}{4} + \frac{v+\Delta}{4\alpha} - \frac{2x_m}{4(\alpha-1)} \tag{17}$$

Several features of these platforms are immediately noteworthy. By the symmetry of the elite and non-elite factions' objectives, platforms within a party are identical whenever there is a primary. In addition, platforms under two primaries are identical to those in the no-primary case. Finally, adopting primaries increases public goods offerings within a party, but reduces them in the opposing party. This reflects changes across parties in the returns to effort in developing platforms.[9]

4.3 Candidate Selection Mechanisms

The preceding derivations allow us to present the conditions under which different configurations of candidate selection mechanisms arise. Stationarity and symmetry imply that in each period, elite factions will choose the same mechanisms. It will therefore be useful adopt the following notation. For faction j of party i and mechanisms c, let $V_i^j(c)$ denote the equilibrium expected per period payoff. As is standard, the optimality of a candidate selection mechanism c_i is verified by evaluating whether elites in one party can do better than $V_i^j(c)$ by deviating once to an alternative mechanism.

No Primaries. In the simplest case, elites in both parties retain indefinite control. We illustrate the party R elite's incentive to maintain this control in equilibrium. Since there are no changes in factional power, the long-run average payoff for either faction j is its current period expected payoff:

$$V_R^j(\emptyset) = \mathbb{E}[U_R^j(\mathbf{q}^*(\emptyset))]. \tag{18}$$

[9]Holding the opposing party's candidate selection method constant, introducing primaries raises the probability of victory by $b/(16\alpha(\alpha-1))$ over an opponent with a favorable electability shock.

A deviation introduces primaries for one period, after which the elite faction (i.e., the primary winner) returns to elite selection. This improves expected ideological outcomes, but at the cost of possibly losing elite status. The expected payoff from deviation is:

$$(1-\delta)\mathbb{E}[U_R^E(\mathbf{q}^*(R))] + \frac{\delta}{2}\left(V_R^E(\emptyset) + V_R^N(\emptyset)\right). \tag{19}$$

By taking the difference between these expressions, we obtain the condition for no primaries to be sustainable as an equilibrium for the R elite. A symmetric calculation characterizes the party L elite's incentives. Proposition 1 shows these conditions.

Proposition 1. No Primaries. *An equilibrium with elite selection in both parties exists if and only if:*

$$\Delta < \frac{1}{4\alpha-3}\left[\frac{4v((\alpha-1)\alpha-(2\alpha-1)|x_m|)}{b(1-\delta)} - \frac{(3\alpha-2)(b-8|x_m|)}{4(\alpha-1)} - 2v(\alpha-1)+\alpha\right]. \tag{20}$$

This condition holds only if x_m is sufficiently moderate.

Proposition 1 states that an equilibrium with no primaries coincides with both low polarization and a moderate constituency. The intuition is that low polarization reduces the ideological stakes of the election, thus reducing the relative benefit of a more electable candidate. In addition, a moderate constituency ensures that both parties' elites have a reasonable chance of winning and receiving private goods.

Two Primaries. Given that low polarization cements elite control, a sensible conjecture would be that high polarization encourages mutual primary adoption. The ideological consequences of a general election defeat make elites in both parties willing to risk the loss of elite status in order to maximize the chances of victory.

By symmetry, we again focus on party R elites. Accounting for changes in factional power, the elite's long-run average payoff is given by:

$$
\begin{aligned}
V_R^E(LR) &= (1-\delta)\mathbb{E}[U_R^E(\mathbf{q}^*(LR))] + \frac{\delta}{2}\left(\mathbb{E}[U_R^E(\mathbf{q}^*(LR))] + \mathbb{E}[U_R^N(\mathbf{q}^*(LR))]\right) \\
&= \mathbb{E}[U_R^E(\mathbf{q}^*(LR))]. \qquad (21)
\end{aligned}
$$

The continuation value reflects the fact that in every period with a primary, a given faction's expected payoff will be that of the elite or non-elite, with equal probability. Since primaries equalize expected payoffs across factions, the average payoff simplifies to the elite's per-period payoff under primaries.

A deviation introduces elite control for one period, followed by a return to primaries. Since the elite remains in control following this return, the expected payoff is that from a single period of elite control followed by $V_R^E(LR)$:

$$
(1-\delta)\mathbb{E}[U_R^E(\mathbf{q}^*(L))] + \delta V_R^E(LR). \qquad (22)
$$

Whether a deviation is profitable therefore depends only on the immediate payoff of elite control. The next result performs this comparison and derives the conditions for the long run persistence of primaries.

Proposition 2. Two Primaries. *An equilibrium with primaries in both parties exists if and only if:*

$$
\Delta > \frac{1}{4\alpha-3}\left[\frac{4v((\alpha-1)\alpha+(2\alpha-1)|x_m|)}{b} + \frac{(3\alpha-2)(b-8|x_m|)}{4(\alpha-1)} + \alpha\right] - v. \qquad (23)
$$

This condition holds only if x_m is sufficiently moderate.

In contrast with Proposition 1, two primaries require high polarization. Polarization magnifies potential ideological losses, and therefore make elites more willing to give up private goods. A moderate constituency is still required, since an extreme constituency would reduce the potential electability gain from primary competition.

One Primary. An equilibrium with one primary combines elements of both preceding cases: one party must have an incentive to maintain primaries, while the other stays with elite selection. The intuitions of Propositions 1 and 2 then suggest that such an equilibrium will require Δ to be "intermediate." We consider here the case where party R holds primaries. The equilibrium with primaries only in party L is symmetric.

For party L, expected payoffs are similar to those in expressions (18) and (19). With no variation in factional control, the elite's long-run average equilibrium payoff is simply her current period expected payoff:

$$V_L^E(R) = \mathbb{E}[U_L^E(\mathbf{q}^*(R))]. \tag{24}$$

A deviation introduces primaries for one period, after which the possibly new elite faction returns to elite selection. The expected payoff from deviation is:

$$(1-\delta)\mathbb{E}[U_L^E(\mathbf{q}^*(LR))] + \frac{\delta}{2}\left(V_L^E(R) + V_L^N(R)\right). \tag{25}$$

For party R, expected payoffs are analogous to those in expressions (21) and (22). The elite's long-run average payoff is again her expected payoff in each period under primaries, since both factions expect to do equally well in any primary election:

$$V_R^E(R) = \mathbb{E}[U_R^E(\mathbf{q}^*(R))]. \tag{26}$$

Finally, a deviation results in a single period of elite control in both parties, followed by a return to primaries in party R:

$$(1-\delta)\mathbb{E}[U_R^E(\mathbf{q}^*(\emptyset))] + \delta V_R^E(R). \tag{27}$$

Proposition 3 combines these expressions to derive the conditions for an equilibrium with only party R primaries.[10]

Proposition 3. One Primary. *An equilibrium with a primary only in party R exists if and only if:*

$$\Delta \in \left[\frac{1}{4\alpha - 3} \left(\frac{4v((\alpha-1)\alpha + (2\alpha-1)x_m)}{b} - \frac{(3\alpha-2)(b+8x_m)}{4(\alpha-1)} - 2(\alpha-1)v + \alpha \right), \right.$$
$$\left. \frac{1}{4\alpha-3} \left(\frac{4v((\alpha-1)\alpha - (2\alpha-1)x_m) - bv(2\alpha(2-\delta)+2\delta-3)}{b(1-\delta)} + \frac{(3\alpha-2)(b+8x_m)}{4(\alpha-1)} + \alpha \right) \right]. \tag{28}$$

This condition holds only if x_m is sufficiently low.

The "symmetric" cases with zero or two primaries existed when the electorate was relatively moderate. By contrast, a biased electorate encourages asymmetric primary adoption. Generally, the disadvantaged party is more inclined to hold primaries, since it has more to gain from an improvement in its electoral prospects.[11] Moderate levels of polarization are also needed, since extremely high or low levels will result in the previously examined symmetric adoption equilibria.

Taken together, the conditions in Propositions 1-3 ensure that at least one equilibrium exists for any set of parameters.

Proposition 4. Existence. *A pure strategy equilibrium exists.*

Figure 5 illustrates the adoption of primaries as a function of polarization (Δ) and partisan bias (x_m). To convey some intuition for how repetition affects the set of equilibria, the left part of the figure plots equilibrium candidate selection mechanisms when $\delta = 0$, which is equivalent to a one-shot game. Here, the one primary equilibria are unique when they exist, but for some moderate x_m and Δ the zero and two primary equilibria can both exist.

[10] The conditions for an equilibrium with only party L primaries are identical to those in Proposition 3, but replacing x_m with $-x_m$.

[11] If factions instead received private goods regardless of the general election outcome, then the electorally favored party becomes more likely to hold primaries. In this case, electorally unfavored elites would prefer simply to preserve their private goods, rather than somewhat improving their chances at an unlikely general election victory.

An infinite horizon generally makes elite control more sustainable in equilibrium. As δ increases, the set of parameters supporting both no primaries and one primary expands "upwards" toward higher values of Δ. This reflects the concern for receiving private goods over the longer term. Notably, however, the set of parameters supporting two primaries is unaffected. As shown above, the profitability of deviating from primaries boils down to a comparison of payoffs in the current period, since primaries equalize payoffs across factions. This can also be seen in Proposition 2, where the conditions for the two primary equilibrium are independent of δ. One interpretation of the effect of repeated interaction is that mutual primary adoption is only robust when polarization is high and electorates are very moderate, as these are the conditions under which parties would be unable to "renegotiate" to an equilibrium with elite selection.

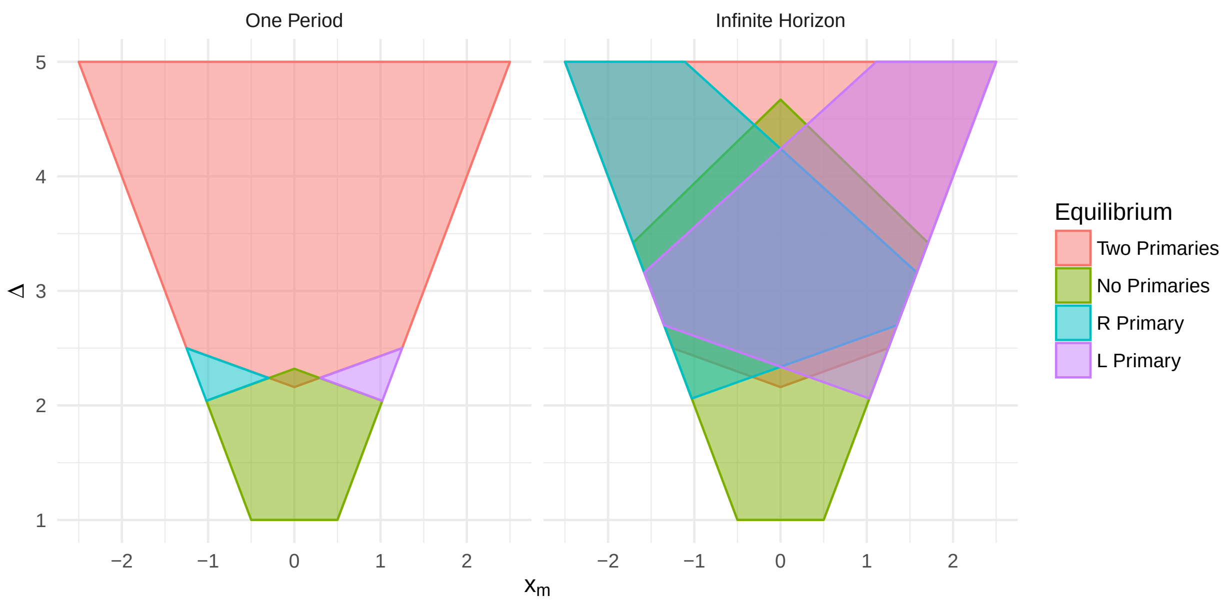

Figure 5: Equilibrium regions as a function of x_m and Δ. Here $v = 0.5$, $b = 2.5$, and $\alpha = 12$. The left panel plots equilibrium regions of the stage game, and the right panel plots the equilibrium regions of the infinite horizon game with $\delta = 0.5$.

While our results depend on possible future changes in elite status, the equilibria studied here feature no over-time variation in the parties' choices. This reflects in part the model's static electoral setting. A natural way to induce changes in the incentives to hold primaries along the equilibrium path would be to introduce shifts in electoral variables such polarization or constituency ideology. We examine the latter possibility in section 5.

4.4 Public Goods

Do candidate selection mechanisms affect public goods provision? For each configuration c, let $\Pi(c)$ denote the expected level of public goods delivered by winning candidates in a given period. This calculation excludes the value of any electability benefits, except insofar as they affect candidates' chances of victory. For the case of no primaries, this can be written as:

$$\begin{aligned}\Pi(\emptyset) =& \frac{1}{4}\left[\pi(q_L^{E*}(\emptyset), q_R^{E*}(\emptyset))q_R^{E*}(\emptyset) + (1-\pi(q_L^{E*}(\emptyset), q_R^{E*}(\emptyset)))q_L^{E*}(\emptyset) + \right.\\ & \pi(q_L^{E*}(\emptyset)+b, q_R^{E*}(\emptyset))q_R^{E*}(\emptyset) + (1-\pi(q_L^{E*}(\emptyset)+b, q_R^{E*}(\emptyset)))q_L^{E*}(\emptyset) + \\ & \pi(q_L^{E*}(\emptyset), q_R^{E*}(\emptyset)+b)q_R^{E*}(\emptyset) + (1-\pi(q_L^{E*}(\emptyset), q_R^{E*}(\emptyset)+b))q_L^{E*}(\emptyset) + \\ & \left.\pi(q_L^{E*}(\emptyset)+b, q_R^{E*}(\emptyset)+b)q_R^{E*}(\emptyset) + (1-\pi(q_L^{E*}(\emptyset)+b, q_R^{E*}(\emptyset)+b))q_L^{E*}(\emptyset)\right].\end{aligned}$$

$\Pi(c)$ can be calculated in a similar fashion for each other value of c. Simplifying these expressions then produces:

$$\Pi(\emptyset) = \Pi(LR) = \frac{1}{4} + \frac{\Delta + v}{4\alpha} + \frac{(2\alpha-1)x_m^2}{2(\alpha-1)^2\alpha} \quad (29)$$

$$\Pi(R) = \frac{1}{4} + \frac{\Delta + v}{4\alpha} + \frac{2\alpha-1}{2(\alpha-1)^2\alpha}\left(x_m^2 + \frac{b^2+8bx_m}{16}\right) \quad (30)$$

$$\Pi(L) = \frac{1}{4} + \frac{\Delta + v}{4\alpha} + \frac{2\alpha-1}{2(\alpha-1)^2\alpha}\left(x_m^2 + \frac{b^2-8bx_m}{16}\right) \quad (31)$$

The equivalence of the $c = \emptyset$ and $c = LR$ public good levels follows from the fact that all platforms are identical across these cases. In addition, the symmetry between the $c = L$ and $c = R$ cases implies that public goods are identical for values of x_m symmetric around 0.

Expressions (29)-(31) imply a simple comparative statics result on expected public goods provision. Under each candidate selection regime c, ideological polarization Δ and private benefits from victory v increase public goods provision. This follows from the fact that increases in these parameters raise the stakes of winning, and therefore cause all candidate platforms to increase. The result can also be derived by differentiating $\Pi(c)$ with respect to Δ or v, and is stated without proof.

Proposition 5. Public Goods. *Expected public goods production is increasing in* Δ *and* v.

Public goods provision can also be compared across equilibria, since as Figure 5 shows there exist parameters under which all configurations of candidate selection mechanisms can exist in equilibrium. It is also clear from expressions (29)-(31) that for moderate values of x_m, the one-primary equilibria maximize $\Pi(c)$. This reflects the higher platforms chosen by primary candidates, as well as their party's higher probability of victory. Thus the model predicts that partial primary adoption in moderate electorates to be most conducive to public goods.

5 Electoral Volatility and Factional Entrenchment

Our basic model shows how factional control of a party can condition party elites' decisions over whether to hold primaries. In this section we consider two important factors that may affect the adoption of primaries over time. First, we examine the role of electoral volatility, modeled as stochastic median voter ideology. Second, we consider the role of entrenchment of the candidate selection mechanism across elections, by assuming that winning parties must retain the candidate selection mechanism used in the past election, whereas losing parties may choose freely. For tractability purposes, we assume that all players are myopic. Factions therefore do not concern themselves with future elite status or candidate selection mechanisms. These modifications allow us to study the patterns of adoption and persistence of candidate selection profiles over time.

We model a simple form of electoral volatility. Let the median voter's position x_m be stochastic, taking on possible positions $\{x_m^l, x_m^m, x_m^h\}$ in each period, where $x_m^l < x_m^m < x_m^h$. In period 1, $x_m = x_m^m$, and in each subsequent period t, x_m does not change with probability $1-2\epsilon$ and assumes each of the other two values with probability $\epsilon \in [0, 1/2]$. Thus ϵ serves as a crude measure of party-level incumbency *dis*advantage. These locations are consequential for the equilibrium of the single period game: where x_m^l and x_m^h are sufficiently extreme, at $x_m = x_m^l$ (respectively, $x_m = x_m^h$) only party R (respectively, party L) holds a primary in the one-period game, while at $x_m = x_m^m$ either both parties or no parties hold primaries.

Allowing only the losing party to choose whether to hold primaries captures the idea that losing parties face better opportunities for "starting over," while winning parties have leaderships who might resist changing structures that brought recent success. Consequently, in any given period, the resulting profile of candidate selection mechanisms might not be an equilibrium of the single period game. This happens if the winning party's preceding choice is mismatched with the new ideological environment.

These assumptions allow us to analyze long-run distribution of candidate selection mechanisms as the limiting distribution of a simple finite-state Markov chain. States of the Markov chain are of the form $(x_m, p, c_{\neg p})$, where $x_m \in \{x_m^l, x_m^m, x_m^h\}$ represents the current location of the median voter, $p \in \{L, R\}$ is the losing party from the previous period (which can switch candidate selection mechanisms), and $c_{\neg p} \in \{elite, primary\}$ is the current incumbent's (i.e., previous period winner's) fixed candidate selection mechanism. This produces a state space with twelve elements. Further details on the construction of the Markov chain can be found in Appendix B.1.

The probabilities of state transitions are simplified greatly by entrenchment. Since the winner cannot change her selection mechanism, the probability of transitioning from a state $(x_m', p, c_{\neg p})$ to $(x_m'', p, \neg c_{\neg p})$ for any x_m' and x_m'' is zero. The remaining elements are determined by the probability of median voter transitions, the candidate selection best response of the current non-incumbent at x_m given $c_{\neg p}$, and the probability of victory resulting from the two parties' platform choices. It is straightforward to verify that the Markov chain is aperiodic and irreducible, and thus has a unique stationary distribution over states that is independent of the starting state.

As an example, consider the setting examined in Figure 5, where $v = 0.5$, $b = 2.5$, and

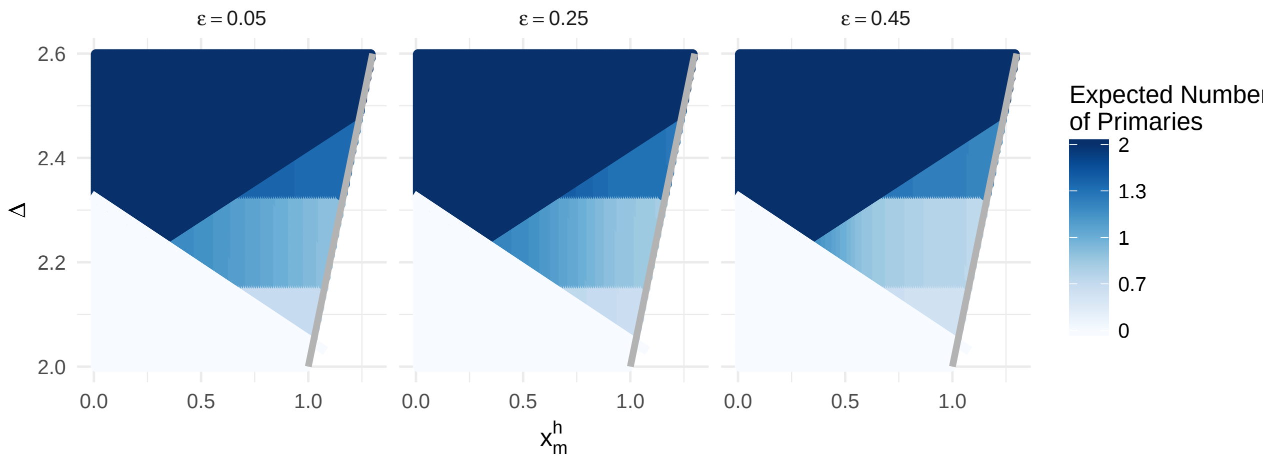

Figure 6: Expected number of primaries in terms of Δ, x_m^h, and for differing values of ϵ (panels). Here, $v = 0.5$, $b = 2.5$, $\alpha = 12$, $x_m^m = 0$, and $x_m^l = -x_m^h$. The expected number of primaries is calculated based on the probability of each equilibrium (no primary, L or R primary, and two primary) at each point.

$\alpha = 12$. When the possible values of x_m are $\{-0.9, 0, 0.9\}$ and $\Delta = 2.4$, Proposition 4 implies that the stage game candidate selection strategies are $c = R$ if $x_m = -0.9$, $c = LR$ if $x_m = 0$, and $c = L$ if $x_m = 0.9$. Party L's best response mechanism in each "out of equilibrium" case is:

$$\begin{cases} primary & \text{if } c_R = primary,\ x_m = 0.9 \\ primary & \text{if } c_R = elite,\ x_m = 0 \\ elite & \text{if } c_R = elite,\ x_m = -0.9 \end{cases} \tag{32}$$

Party R's best responses are symmetric.

Using these best responses, the long-run probability of having both parties use primaries is the sum of the long-run probability of the states (x_m^l, R, $primary$), (x_m^m, L, $primary$), (x_m^m, R, $primary$) and (x_m^h, L, $primary$). Note that other parameter settings require summing probabilities for different states, depending on the party p elite faction's best response at those parameters.

Unfortunately, even with this assumption, the Markov chain is complex enough to rule out analytical solutions, and we therefore turn to numerical results. We assume $x_m^m = 0$ and $x_m^h = -x_m^l$, and examine the relationship between primary adoption and volatility. Figure 6 depicts the expected number of primaries in terms of polarization and the extent of electoral

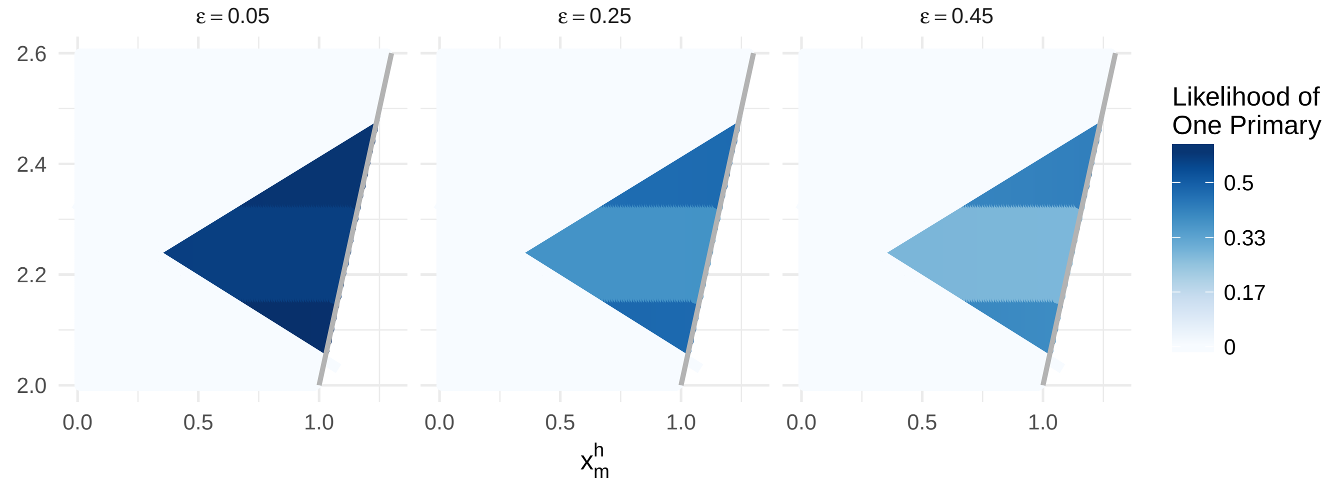

Figure 7: This figure depicts the probability of either one-primary equilibrium state as a function of Δ and x_m^h, and for differing values of ϵ (panels). Here, $v = 0.5$, $b = 2.5$, $\alpha = 12$, $x_m^m = 0$, and $x_m^l = -x_m^h$.

volatility (measured by x_m^h and ϵ). Some patterns from the stage game persist here: where the median voter volatility is low and fluctuates within the range of the zero primary or two primary equilibria, parties do not deviate from the stage game predictions.[12] Outcomes are less trivial where the upper and lower median voter positions x_m^l and x_m^h fall into the R and L primary regions, respectively. As previewed above, entrenchment and changing voter ideology can result in mutual adoption of primaries in these cases. The figure shows that the expected number of primaries is decreasing in both the extremeness of possible median voters and in ϵ.

We next examine the prevalence of one-primary outcomes. Figure 7 shows that lower values of ϵ are associated with an increase in the probability of a single primary. This implies a lower likelihood of mutual adoption or non-adoption of primaries. Interpreting low values of ϵ as a measure of incumbency advantage, we summarize the observations of our numerical results as follows. Further examples can be found in Appendix B.2.

Comment 1. Simulation Results on Entrenchment and the Incumbency Advantage. *In the game with myopic factions, entrenched incumbent candidate selection mechanisms, and*

[12]Note that when all possible median voter locations fall within the region where both no primary and two primary equilibria exist in the stage game, we make the simplifying assumption that parties employ elite selection in the first phase, which persists to all subsequent periods.

$x_m \in \{-x_m^h, 0, x_m^h\}$, we observe the following:

1. *The likelihood that only one party will employ primaries is weakly increasing in the incumbency advantage.*

2. *The expected number of primaries across both parties is weakly increasing in the incumbency advantage. For x_m^h sufficiently high, the effect of incumbency advantage on the expected number of primaries is weakly increasing in x_m^h.*

What explains these patterns? Winning in this setting is associated with stickiness of the selection mechanism. As ϵ increases (or the incumbency advantage decreases), winners are doubly disadvantaged in the next round: incumbents are less likely to face a friendly median voter and are unable to optimize their candidate selection mechanism. Comparing Figures 5 and 7 shows that the result is an increasing divergence from the equilibrium prediction from the stage game.

The effect of volatility in the position of the median voter, x_m^h is more ambiguous. While Figure 6 suggests that the expected number of primaries is decreasing in this type of volatility, this relationship can actually be either positive or negative, as we document in Appendix B.2. However, the "cross partial" of expected number of primaries with respect to incumbency advantage (low ϵ) and x_m^h is positive across the parameter space: a high incumbency advantage increases the number of primaries, and this effect is heightened as x_m^h increases (Figure 6). As possible median voter locations become more extreme, the probability of victory for the advantaged party increases regardless of candidate selection mechanism. This reduces the likelihood of having the option of switching to an optimal selection mechanisms in the next period.

6 Conclusions

In most democracies, primary elections are adopted in a decentralized manner. And yet, parties consist of self-interested actors who are mindful of their electoral prospects in a competitive environment. To explain the adoption of primaries, it is therefore important to develop a model of party competition in which all parties face an internal governance choice.

Our game focuses on the basic ideological incentives surrounding the adoption of primaries. Mutual adoption of primaries will result when ideological polarization is high, while mutual rejection of primaries will happen when polarization is low. Somewhat surprisingly, the possible loss of elite status does not necessarily attenuate the incentive for parties to hold primaries. At moderate levels of polarization, ideological imbalance in the constituency will result in primary adoption only by the electorally unfavored party. These findings are consistent with the stylized facts drawn from data on Latin American presidential elections. To the extent that polarization has been measured in the region, primaries are more likely at higher levels of polarization. Moreover, primaries in these elections have disproportionately been adopted by disadvantaged major parties engaged in competitive races.

We also examine the roles of intra-party rigidity and a changing electoral environment, where only losing parties have a "mandate" to reconsider candidate selection mechanisms. This setting illustrates how a higher incumbency advantage increases the number of primaries and favors asymmetric (one-party) primary adoption. Further, high levels of extremity and volatility reduce the use of primaries altogether. The model also offers a framework for analyzing the long-run evolution of party governance in complex electoral settings.

The model finally suggests a few areas for further theoretical progress. For example, we have posited perhaps the simplest possible internal structure for parties. Party factions may be heterogenous along several dimensions, and party leaders have more alternatives than elite control versus closed primaries. Short-lived candidates for office may also differ from long-lived factions. Such features will be worth exploring with the increasing volume of data on the adoption of primary elections worldwide.

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Appendices

A Proofs

Proof of Remark 1. Within party i, faction j defeats faction j' in a primary election with probability one if $q_i^j > q_i^{j'} + b$, and with probability 1/2 otherwise. We therefore rule out the former possibility.

Suppose to the contrary that in equilibrium, faction E chooses primary and $q_i^E \geq q_i^N + b$. This ensures that faction E wins the primary, receives v, and stays faction E in the subsequent period. Then faction E would do at least as well by choosing elite selection, as this ensures both a payoff at least as high in the current period and staying elite in the subsequent period for any platform.

Now suppose faction N chooses $q_i^N \geq q_i^E + b$. By the symmetry of factions in primary elections, this implies that the preceding case is also an equilibrium outcome, which implies that faction E would prefer elite selection. $\blacksquare$

Proof of Remark 2. Notationally, recall that $c \in \{\emptyset, L, R, LR\}$ denotes the parties holding primaries. Let $V_i^j(c)$ denote the equilibrium continuation value for faction j of party i under profile c. Finally, let $\pi^c(\mathbf{q})$ denote the probability of a party R victory under platforms $\mathbf{q}$ and profile c.

We first show that myopic platform strategies are optimal for any equilibrium c. Under elite control, faction E's expected utility is:

$$E\left[U_i^E(\mathbf{q})\right] + \delta V_i^E(c),$$

where the expectation over $U(\cdot)$ is taken over values of b_i^j and ϕ. Similarly, using Remark 1, faction j's expected utility under primaries is:

$$E\left[U_i^j(\mathbf{q})\right] + \frac{\delta}{2}\left(V_i^E(c) + V_i^N(c)\right),$$

In both cases, it is clear that factions choose q_i^j to maximize utility in the current period.

We now derive myopic platforms for each value of c.

(i) $c = \emptyset$. The objective function for R elite candidates is given by expression (7). For parties R and L, these expand respectively to the following:

$$\frac{1}{2\alpha}\left[(q_L^E)^2 + (1-2\alpha)(q_R^E)^2 - q_L^E(2q_R^E + v + \Delta + 2x_m - \alpha) + q_R^E(v + \Delta + 2x_m + \alpha) + \right.$$
$$\left. \alpha(v-\Delta) + 2x_m(v+\Delta)\right]$$
$$\frac{1}{2\alpha}\left[(q_R^E)^2 + (1-2\alpha)(q_L^E)^2 - q_L^E(2q_R^E - v - \Delta + 2x_m - \alpha) - q_R^E(v + \Delta - 2x_m - \alpha) + \right.$$
$$\left. \alpha(v-\Delta) - 2x_m(v+\Delta)\right]$$

The best responses are given by expressions (8) and (9), and the solutions to this system are given by expressions (10) and (11).

What remains is to verify concavity of the objective. For R, the second order condition with respect to q_R is $1/\alpha - 2$, which is clearly negative. The second order condition for L is identical.

(ii) $c = R$. We consider the maximization problem for the R elite, R non-elite, and L (elite) candidates in turn. To write the objective, we first determine the probability of nomination and victory for each faction. Using (6) and Remark 1, this probability for faction j of party R is:

$$\pi^R(q_L^E, q_R^j) = \frac{1}{2}\left(\frac{1}{2}\pi(q_L^E + b, q_R^j + b) + \frac{1}{2}\pi(q_L^E, q_R^j + b)\right).$$

The maximization problem for party R elites is then as follows.

$$\max_{q_R^E} \pi^R(q_L^E, q_R^E)\left(q_R^E + v\right) + \pi^R(q_L^E, q_R^N)q_R^N + (1 - \pi^R(q_L^E, q_R^E) - \pi^R(q_L^E, q_R^N))\left(q_L^E - \Delta\right) - (q_R^E)^2 \quad (33)$$

This expands to:

$$\frac{1}{8\alpha}\left[b(q_R^E + q_R^N - 2q_L^E + v + 2\Delta) + 2\left(2(q_L^E)^2 - q_L^E(2q_R^E + 2q_R^N + v + 2\Delta + 4x_m - 2\alpha) + \right.\right.$$
$$\left.\left.(1-2\alpha)\left((q_R^E)^2 + (q_R^N)^2\right) + q_R^E v + (q_R^E + q_R^N)(\Delta + 2x_m + \alpha) + \alpha(v - 2\Delta) + 2x_m(v + 2\Delta x_m)\right)\right].$$

The non-elite R faction's objective is symmetric to the above, exchanging q_R^E and q_R^N.

Similarly, the maximization problem for party L elites is:

$$\max_{q_L^E} \pi^R(q_L^E, q_R^E)\left(q_R^E - \Delta\right) + \pi^R(q_L^E, q_R^N)\left(q_R^N - \Delta\right) +$$
$$(1 - \pi^R(q_L^E, q_R^E) - \pi^R(q_L^E, q_R^N))\left(q_L^E + v\right) - (q_L^E)^2 \tag{34}$$

This expands to:

$$\frac{1}{8\alpha}\left[b(q_R^E + q_R^N - 2q_L^E - 2v - 2\Delta) - 2\left((4\alpha - 2)(q_L^E)^2 + 2q_L^E(q_R^E + q_R^N - v - \Delta + 2x_m - \alpha) - \right.\right.$$
$$\left.\left.(q_R^E)^2 - (q_R^N)^2 + (q_R^E + q_R^N)(v + \Delta - 2x_m - \alpha) - 2\alpha(v - \Delta) + 4x_m(v + \Delta)\right)\right].$$

The second derivative for the elite and non-elite factions of R with respect to q_R^E and q_R^N is $1/(2\alpha) - 1$, which is clearly negative. The second derivative for L (elite faction) with respect to q_L is $1/\alpha - 2$, which is clearly negative.

The first order conditions of these expressions form the following system of equations:

$$q_R^E = \frac{b - 2(2q_L^E - v - \Delta - 2x_m - \alpha)}{8\alpha - 4}$$
$$q_R^N = \frac{b - 2(2q_L^E - v - \Delta - 2x_m - \alpha)}{8\alpha - 4}$$
$$q_L^E = \frac{-b - 2(q_R^E + q_R^N - v - \Delta + 2x_m - \alpha)}{8\alpha - 4}$$

Solving this system produces expressions (12) and (13).

(iii) $c = R$. This case is symmetric to the right-primary case, and produces expressions

(15) and (14).

(iv) $c = LR$. In the case where both parties hold primaries, we use (6) and Remark 1 to write the probability of nomination and victory for faction j of parties R and L, respectively:

$$\pi^{LR}(q_L^E, q_L^N, q_R^j) = \frac{1}{2}\left(\frac{1}{2}\pi(q_L^E + b, q_R^j + b) + \frac{1}{2}\pi(q_L^N + b, q_R^j + b)\right).$$

Noting that by Remark 1, each party L faction wins with probability 1/2, the maximization problem for party R elites is then as follows.

$$\begin{aligned}\max_{q_R^E} \pi^{LR}(q_L^E, q_L^N, q_R^E)\left(q_R^E + v\right) + \pi^R(q_L^E, q_L^N, q_R^N)q_R^N + \\ (1 - \pi^R(q_L^E, q_R^E) - \pi^R(q_L^E, q_R^N))\left(\frac{q_L^E + q_L^N}{2} - \Delta\right) - (q_R^E)^2. \qquad (35)\end{aligned}$$

This expression expands to:

$$\begin{aligned}&\frac{1}{8\alpha}\left[2(q_L^E)^2 + 2(q_L^N)^2 - (q_L^E + q_L^N)(2q_R^E + 2q_R^N + v + 2\Delta + 4x_m - 2\alpha) + \right. \\ &\left. \quad 2\left((1-2\alpha)((q_R^E)^2 + (q_R^N)^2) + (q_R^E + q_R^N)(\Delta + 2x_m + \alpha) + q_R^E v + \alpha(v - 2\Delta) + 2x_m(v + 2\Delta)\right)\right].\end{aligned}$$

The non-elite R faction's objective is symmetric to the above, exchanging q_R^E and q_R^N. The calculation for the party L factions is symmetric to those of the party R factions.

The second derivative of each faction's objective with respect to the relevant platform is $1/(2\alpha) - 1$, which is clearly negative. The first order conditions of these four objective functions form the following system of equations:

$$
\begin{aligned}
q_R^E &= \frac{-q_L^E - q_L^N + v + \Delta + 2x_m + \alpha}{4\alpha - 2} \\
q_R^N &= \frac{-q_L^E - q_L^N + v + \Delta + 2x_m + \alpha}{4\alpha - 2} \\
q_L^E &= \frac{-q_R^E - q_R^N + v + \Delta - 2x_m + \alpha}{4\alpha - 2} \\
q_L^N &= \frac{-q_R^E - q_R^N + v + \Delta - 2x_m + \alpha}{4\alpha - 2}
\end{aligned}
$$

Solving this system produces expressions (16) and (17). ∎

Proof of Proposition 1. The elite's long-run average payoff equals its current period expected payoff (18):

$$
\begin{aligned}
V_R^E(\emptyset) = \frac{1}{16(\alpha-1)^2\alpha^2} &\left[2(\alpha-1)v\left(4\alpha^3 + \Delta + \alpha^2(8x_m - 3) - \alpha(\Delta + 6x_m + 1)\right) + \right. \\
&\left. 4\alpha(3\alpha-2)x_m^2 + 4(\alpha-1)\alpha x_m(\alpha(4\Delta-1) - 3\Delta) - (\alpha-1)^2\left(\alpha^2(8\Delta-3) - 2\alpha\Delta + \Delta^2 + v^2\right)\right]
\end{aligned}
$$

A deviation introduces primaries for one period, after which the elite faction (i.e., the primary winner) returns to elite selection. The expected payoff from deviation is $(1-\delta)\mathbb{E}[U_R^E(\mathbf{q}^*(R))] + \delta\left(V_R^E(\emptyset) + V_R^N(\emptyset)\right)/2$, which evaluates to:

$$
\begin{aligned}
\frac{1}{64(1-\alpha)^2\alpha^2} &\left[8(\alpha-1)^2 v\left(2\alpha^2 + \alpha(b(1-\delta) + 4x_m + 1) - \Delta\right) + (3\alpha-2)\alpha\left(b(1-\delta)(b + 8x_m) + 16x_m^2\right)\right. \\
&\left. 4\left(\alpha(\alpha-1)(\alpha(4\Delta-1) - 3\Delta)(b(1-\delta) + 4x_m) - (\alpha-1)^2\left(\alpha^2(8\Delta-3) - 2\alpha\Delta + \Delta^2 + v^2\right)\right)\right]
\end{aligned}
$$

Elite control is a best response if $V_R^E(\emptyset) > (1-\delta)\mathbb{E}[U_R^E(\mathbf{q}^*(R))] + \frac{\delta}{2}\left(V_R^E(\emptyset) + V_R^N(\emptyset)\right)$. Subtracting the latter expression from the former, and solving for Δ, we have the following condition for elite control to be optimal:

$$
\Delta < M_R^{\emptyset}(x_m) \equiv \frac{1}{4\alpha-3}\left[\frac{4v((\alpha-1)\alpha + (2\alpha-1)x_m)}{b(1-\delta)} - \frac{(3\alpha-2)(b+8x_m)}{4(\alpha-1)} - 2v(\alpha-1) + \alpha\right] \quad (36)
$$

Performing the same calculation for party L elites produces the symmetric condition:

$$\Delta < M_L^{\emptyset}(x_m) \equiv \frac{1}{4\alpha-3}\left[\frac{4v((\alpha-1)\alpha-(2\alpha-1)x_m)}{b(1-\delta)}-\frac{(3\alpha-2)(b-8x_m)}{4(\alpha-1)}-2v(\alpha-1)+\alpha\right] \quad (37)$$

Combining expressions produces the condition on Δ shown in the text.

To show that these conditions hold only if x_m is sufficiently moderate, observe that the bounds $M_R^{\emptyset}(x_m)$ and $M_L^{\emptyset}(x_m)$ are symmetric around $x_m = 0$. Thus, if $M_R^{\emptyset}(x_m)$ is increasing in x_m, then it (respectively, $M_L^{\emptyset}(x_m)$) is binding for $x_m \leq 0$ (respectively, ≥ 0). Furthermore only values of x_m sufficiently close to 0 can satisfy expressions (36) and (37). We therefore show that $M_R^{\emptyset}(x_m)$ is increasing with respect to x_m. Differentiating with respect to x_m produces:

$$\frac{dM_R^{\emptyset}}{dx_m} = \frac{2}{4\alpha-3}\left[\frac{2(2\alpha-1)v}{b(1-\delta)}-\frac{3\alpha-2}{\alpha-1}\right].$$

Assumption (4) ensures that this is always positive. ∎

Proof of Proposition 2. The elite's long-run average payoff equals its current period expected payoff (21):

$$\begin{aligned} V_R^E(LR) = \frac{1}{16(\alpha-1)^2\alpha^2}&\left[-(\alpha-1)^2\left(\alpha^2(8\Delta-3)-2\alpha\Delta+\Delta^2+v^2\right)+\right. \\ &\left. 2(\alpha-1)^2 v\left(2\alpha^2+\alpha-\Delta+4\alpha x_m\right)+4\alpha(3\alpha-2)x_m^2+4(\alpha-1)\alpha x_m(\alpha(4\Delta-1)-3\Delta)\right] \end{aligned}$$

A deviation introduces primaries for one period, after which the elite faction (i.e., the primary winner) returns to elite selection. By (22) the expected payoff from deviation is $(1-\delta)\mathbb{E}[U_R^E(\mathbf{q}^*(L))]+\delta V_R^E(LR)$, which evaluates to:

$$\begin{aligned} -\frac{1}{16(\alpha-1)^2\alpha^2}&\left[(\alpha-1)v\left(2\alpha^2((2b+2)(1-\delta)-2(2-\delta)(\alpha+2x_m)+1)+\alpha(4(3-\delta)x_m-\right.\right. \\ &\left. 3b(1-\delta)+2\Delta+2)-2\Delta\right)+(\alpha-1)^2\left(\alpha^2(8\Delta-3)-2\alpha\Delta+\Delta^2+v^2\right)+ \\ &\left.\alpha(\alpha-1)(\alpha(4\Delta-1)-3\Delta)(b(1-\delta)-4x_m)+\alpha(3\alpha-2)\left(b(1-\delta)\left(2x_m-\frac{b}{4}\right)-4x_m^2\right)\right]. \end{aligned}$$

Subtracting (22) from (21), a primary is a best response if $\mathbb{E}[U_R^E(\mathbf{q}^*(LR))] > \mathbb{E}[U_R^E(\mathbf{q}^*(L))]$. Solving for Δ produces the following condition for primaries to be optimal:

$$\Delta > M_R^{LR}(x_m) \equiv \frac{1}{4\alpha - 3}\left[\frac{(3\alpha-2)(b-8x_m)}{4(\alpha-1)} + \frac{4v((\alpha-1)\alpha + (2\alpha-1)x_m)}{b} + \alpha\right] - v \quad (38)$$

Performing the same calculation for party L elites produces the symmetric condition:

$$\Delta > M_L^{LR}(x_m) \equiv \frac{1}{4\alpha - 3}\left[\frac{(3\alpha-2)(b+8x_m)}{4(\alpha-1)} + \frac{4v((\alpha-1)\alpha - (2\alpha-1)x_m)}{b} + \alpha\right] - v \quad (39)$$

Combining expressions produces the condition on Δ shown in the text.

To show that these conditions hold only if x_m is sufficiently moderate, observe that the bounds $M_R^{LR}(x_m)$ and $M_L^{LR}(x_m)$ are symmetric around $x_m = 0$. Thus, if $M_R^{LR}(x_m)$ is increasing in x_m, then it (respectively, $M_L^{LR}(x_m)$) is binding for $x_m \geq 0$ (respectively, ≤ 0). Furthermore only values of x_m sufficiently close to 0 can satisfy expressions (38) and (39). We therefore show that $M_R^{LR}(x_m)$ is increasing with respect to x_m. Differentiating with respect to x_m produces:

$$\frac{dM_R^{LR}}{dx_m} = \frac{2}{4\alpha-3}\left[\frac{2(2\alpha-1)v}{b} - \frac{3\alpha-2}{\alpha-1}\right].$$

Assumption (4) ensures that this is always positive. ∎

Proof of Proposition 3. We first consider the party L elite's problem. The elite's long-run average payoff equals its current period expected payoff (24):

$$\begin{aligned} V_L^E(R) = \frac{1}{64(\alpha-1)^2\alpha^2} \big[&4(\alpha-1)v\left(8\alpha^3 - 2\alpha^2(2b+8x_m+3) + \alpha(3b - 2\Delta + 12x_m - 2) + 2\Delta\right) + \\ &\alpha(3\alpha-2)(b+4x_m)^2 - 4\alpha(\alpha-1)(4\alpha\Delta - \alpha - 3\Delta)(b+4x_m) - \\ &4(\alpha-1)^2\left(\alpha^2(8\Delta-3) - 2\alpha\Delta + \Delta^2 + v^2\right)\big]. \end{aligned}$$

A deviation introduces primaries for one period, after which the elite faction (i.e., the primary winner) returns to elite selection. The expected payoff from deviation is $(1-\delta)\mathbb{E}[U_L^E(\mathbf{q}^*(LR))] + \delta\left(V_L^E(R) + V_L^N(R)\right)/2$, which evaluates to:

$$\frac{1}{64(\alpha-1)^2\alpha^2}\left[4(\alpha-1)^2\left(3\alpha^2 - 2(4\alpha-1)\alpha\Delta + 2\alpha v(2\alpha - b\delta - 4x_m + 1) - (\Delta+v)^2\right) + \right.$$
$$\left.\delta\alpha b(4\alpha(\alpha-1) + (3\alpha-2)b) - 4\alpha(4\alpha-3)(\alpha-1)\Delta(b\delta+4x_m) + 8\alpha x_m(2\alpha(\alpha-1) + (3\alpha-2)(b\delta+2x_m))\right].$$

Elite control is a best response if $V_L^E(R) > (1-\delta)\mathbb{E}[U_L^E(\mathbf{q}^*(LR))] + \delta\left(V_L^E(R) + V_L^N(R)\right)/2$. Subtracting the latter expression from the former, and solving for Δ, we have the following condition for elite control to be optimal:

$$\Delta < M_L^R(x_m) \equiv \frac{1}{4\alpha-3}\left[\frac{4v((\alpha-1)\alpha - (2\alpha-1)x_m) - bv(2\alpha(2-\delta) + 2\delta - 3)}{b(1-\delta)} + \right.$$
$$\left.\frac{(3\alpha-2)(b+8x_m)}{4(\alpha-1)} + \alpha\right] \tag{40}$$

Now consider the party R elite's problem. The elite's long-run average payoff equals its current period expected payoff (26):

$$V_R^E(R) = \frac{1}{64(\alpha-1)^2\alpha^2}\left[4\alpha(\alpha-1)(\alpha(4\Delta-1) - 3\Delta)(b+4x_m) + \alpha(3\alpha-2)(b+4x_m)^2 - \right.$$
$$\left.4(\alpha-1)^2\left(4\alpha^2(2\Delta-1) + (\alpha-\Delta)^2\right) + 4(\alpha-1)^2 v\left(4\alpha^2 + 2\alpha(b+4x_m+1) - 2\Delta - v\right)\right].$$

A deviation introduces primaries for one period, after which the elite faction (i.e., the primary winner) returns to elite selection. By (27) the expected payoff from deviation is $(1-\delta)\mathbb{E}[U_R^E(\mathbf{q}^*(\emptyset))] + \delta V_R^E(R)$, which evaluates to:

$$\frac{1}{64(\alpha-1)^2\alpha^2}\left[4(\alpha-1)^2\left((3\alpha-\Delta)(\alpha+\Delta) - v^2 - 8\alpha^2\Delta\right) + (3\alpha-2)\alpha\left(b^2\delta + 8x_m(b\delta+2x_m)\right) - \right.$$
$$8(\alpha-1)v\left(\alpha^2(-(b+2)\delta + (2-\delta)(-(2\alpha+4x_m)) + 3) + \alpha(b\delta + \Delta + 2(3-\delta)x_m + 1) - \Delta\right) +$$
$$\left.4\alpha(\alpha-1)((4\alpha-3)\Delta - \alpha)(b\delta+4x_m)\right].$$

Subtracting (27) from (26), a primary is a best response if $\mathbb{E}[U_R^E(\mathbf{q}^*(R))] > \mathbb{E}[U_R^E(\mathbf{q}^*(\emptyset))]$. Solving for Δ produces the following condition for primaries to be optimal:

$$\Delta > M_R^R(x_m) \equiv \frac{1}{4\alpha-3}\left[\frac{4v((\alpha-1)\alpha+(2\alpha-1)x_m)}{b} - \frac{(3\alpha-2)(b+8x_m)}{4(\alpha-1)} - 2(\alpha-1)v+\alpha\right] \quad (41)$$

Combining expressions produces the condition on Δ shown in the text.

To show that these conditions hold only for x_m sufficiently small, note that both $M_R^R(x_m)$ and $M_L^R(x_m)$ are linear in x_m. The result holds if the former is increasing in x_m and the latter is decreasing. Taking derivatives produces:

$$\frac{dM_R^R}{dx_m} = \frac{2}{4\alpha-3}\left[\frac{2(2\alpha-1)v}{b} - \frac{3\alpha-2}{\alpha-1}\right]$$
$$\frac{dM_L^R}{dx_m} = \frac{2}{4\alpha-3}\left[-\frac{2(2\alpha-1)v}{(1-\delta)b} + \frac{3\alpha-2}{\alpha-1}\right].$$

It is easily verified that assumption (4) ensures that $\frac{dM_R^R}{dx_m} > 0$ and $\frac{dM_L^R}{dx_m} < 0$. ∎

Proof of Proposition 4. We show that any value of Δ not satisfying the conditions of Propositions 1 or 2 must satisfy the conditions of Proposition 3. We focus on the case where $x_m \leq 0$; the case where $x_m > 0$ follows by symmetry.

From the proof of Proposition 1, an equilibrium with zero primaries exists if $\Delta < M_R^\emptyset(x_m)$. From the proof of Proposition 2, an equilibrium with two primaries exists if $\Delta > M_L^{LR}(x_m)$. It is therefore sufficient to show that the conditions for an R party equilibrium satisfy $M_R^\emptyset(x_m) > M_R^R(x_m)$ and $M_L^{LR}(x_m) < M_L^R(x_m)$, where $M_R^R(x_m)$ and $M_L^R(x_m)$ and given by expressions (41) and (35), respectively.

For the party R condition, taking differences produces:

$$M_R^\emptyset(x_m) - M_R^R(x_m) = \frac{4\delta v\,(\alpha(\alpha-1)+(2\alpha-1)x_m)}{(1-\delta)(4\alpha-3)b}.$$

This expression is positive if $x_m > -\alpha(\alpha-1)/(2\alpha-1)$. This is satisfied by assumption (5), which ensures interior probabilities of victory.

For the party L condition, taking differences produces:

$$M_L^R(x_m) - M_L^{LR}(x_m) = \frac{\delta v\left(4\alpha(\alpha-1) - (2\alpha-1)(b+4x_m)\right)}{(1-\delta)(4\alpha-3)b}.$$

This expression is positive if $x_m < \alpha(\alpha-1)/(2\alpha-1) - b/4$. This is also satisfied by assumption (5). ∎

B Electoral Volatility and Factional Entrenchment: Supporting Materials

B.1 Markov Chain

Section 5 examined long-run candidate selection in an electorally volatile environment with factional entrenchment. For the numerical analysis in the paper, we modeled the equilibrium choice of candidate selection mechanisms by constructing a twelve-by-twelve state transition matrix and examining its stationary distribution.

Denote the transition matrix of the Markov chain $\mathbf{Q}$, where each element $Q_{s,s'}$ indicates the probability of moving from any state s in period t to state s' in period $t+1$. Aside from identifying incumbents and their selection mechanism, probabilities are independent of the history of the game. As described in the main text, states are triples of the form $(x_m, p, c_{\neg p})$, where $x_m \in \{x_m^l, x_m^m, x_m^h\}$ represents the current location of the median voter, $p \in \{L, R\}$ is the losing party from the previous period (which can switch candidate selection mechanisms), and $c_{\neg p} \in \{elite, primary\}$ is the current incumbent's (i.e., previous period winner's) fixed candidate selection mechanism. While the full transition matrix is cumbersome to write, we characterize the first row of the transition matrix (from (x_m^l, L, e) to all other states) to illustrate the construction of the Markov chain.

In order to write general expressions for state transition probabilities for all parameter inputs, we must identify the losing party's (party L in the first row) best response to the other party's fixed strategy (in this case, elite selection) at x_m^l. Define $T_i^P(x_m)$ as the difference in the losing party's expected utility of holding a primary versus not holding a primary. We define $P_i(x_m)$ as an indicator of whether the losing party holds a primary.

$$P_i(x_m) = \mathbb{I}[T_i^P(x_m) > 0]$$

The first row of $\mathbf{Q}$ is as follows:

$$\begin{aligned}
Q_{(x_m^l,L,e),(x_m^l,L,e)} &= (1-2\epsilon)[(1-P_L(x_m^l))\pi^\emptyset(q_L^{E*}(\emptyset), q_R^{E*}(\emptyset)) + P_L(x_m^l)\pi^L(q_L^{j*}(L), q_R^{E*}(L))] \\
Q_{(x_m^l,L,e),(x_m^l,L,p)} &= 0 \\
Q_{(x_m^l,L,e),(x_m^l,R,e)} &= (1-2\epsilon)(1-P_L(x_m^l))(1-\pi^\emptyset(q_L^{E*}(\emptyset)) \\
Q_{(x_m^l,L,e),(x_m^l,R,p)} &= (1-2\epsilon)(P_L(x_m^l))(1-\pi^L(q_L^{j*}(L), q_R^{E*}(L))) \\
Q_{(x_m^l,L,e),(x_m^m,L,e)} &= \epsilon[(1-P_L(x_m^l))\pi^\emptyset(q_L^{E*}(\emptyset), q_R^{E*}(\emptyset)) + P_L(x_m^l)\pi^L(q_L^{j*}(L), q_R^{E*}(L))] \\
Q_{(x_m^l,L,e),(x_m^m,L,p)} &= 0 \\
Q_{(x_m^l,L,e),(x_m^m,R,e)} &= \epsilon(1-P_L(x_m^l))(1-\pi^\emptyset(q_L^{E*}(\emptyset)) \\
Q_{(x_m^l,L,e),(x_m^m,R,p)} &= \epsilon(P_L(x_m^l))(1-\pi^L(q_L^{j*}(L), q_R^{E*}(L))) \\
Q_{(x_m^l,L,e),(x_m^h,L,e)} &= \epsilon[(1-P_L(x_m^l))\pi^\emptyset(q_L^{E*}(\emptyset), q_R^{E*}(\emptyset)) + P_L(x_m^l)\pi^L(q_L^{j*}(L), q_R^{E*}(L))] \\
Q_{(x_m^l,L,e),(x_m^h,L,p)} &= 0 \\
Q_{(x_m^l,L,e),(x_m^h,R,e)} &= \epsilon(1-P_L(x_m^l))(1-\pi^\emptyset(q_L^{E*}(\emptyset)) \\
Q_{(x_m^l,L,e),(x_m^h,R,p)} &= \epsilon(P_L(x_m^l))(1-\pi^L(q_L^{j*}(L), q_R^{E*}(L)))
\end{aligned}$$

The remaining eleven rows are constructed analogously. It is clear that all transition probabilities are strictly positive with the exception of those of the form $Q_{(\cdot,p,c_{\neg p}),(\cdot,p,\neg c_{\neg p})}$. Since state $(\cdot, p, \neg c_{\neg p})$ is accessible simply through party p winning, this implies that all states communicate. The finiteness of the Markov chain then implies the existence of a unique stationary distribution.

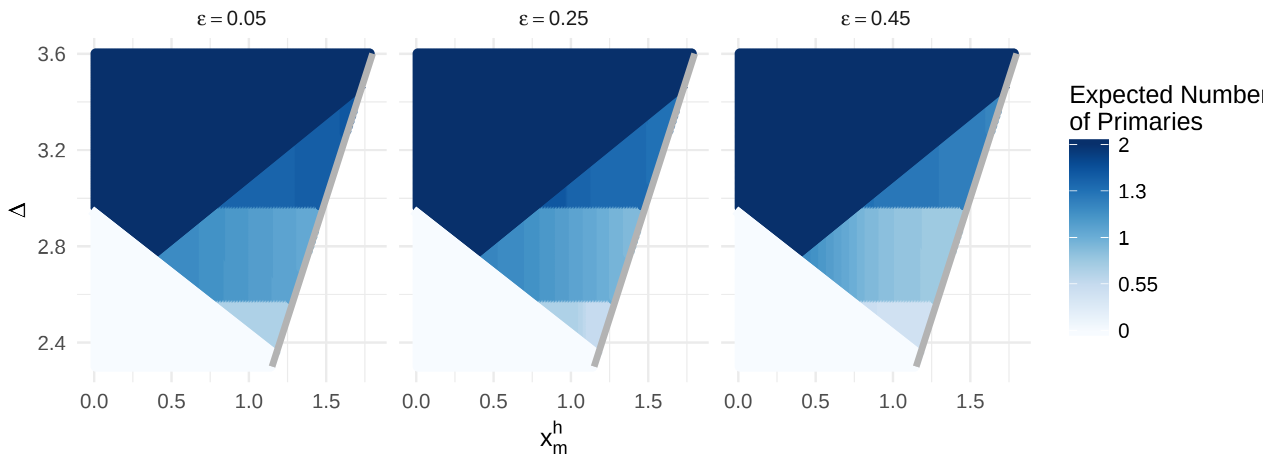

Figure 8: Expected number of primaries in terms of Δ, x_m^h, and for differing values of ϵ (panels) when $v = 1$, $b = 3$, $\alpha = 10$, and $x_m^l = -x_m^h$. The expected number of primaries is calculated based on the probability of each equilibrium (no primary, L or R primary, and two primary) at each point.

B.2 Robustness of Numerical Results

Our results in Section 5 are numerical. Here we examine the robustness of our findings to other parameter combinations. In Figure 8 we again plot the expected number of primaries, this time across a different parameter set from that graphed in Figure 6. Panels differ in the likelihood that the median voter will shift in the next period (ϵ), which we interpret as the inverse of the incumbency advantage. As before, outcomes are identical to the stage game predictions when median voter volatility is low; there are, however, deviations from the stage game where the upper and lower median voter positions x_m^l and x_m^h fall into the R- and L-only primary regions, respectively.

Recall from Comment 1 that we find the expected number of primaries between both parties is weakly increasing in incumbency advantage. This remains true for the parameter set shown here: all else equal, the number of primaries is highest in the leftmost panel where incumbency advantage is highest. Compared to Figure 6, Figure 8 reveals that the relationship between x_m^h (the possible extremity of the median voter) and the expected number of primaries can be either positive or negative.

We also examine the robustness of our findings on the likelihood of one primary (left

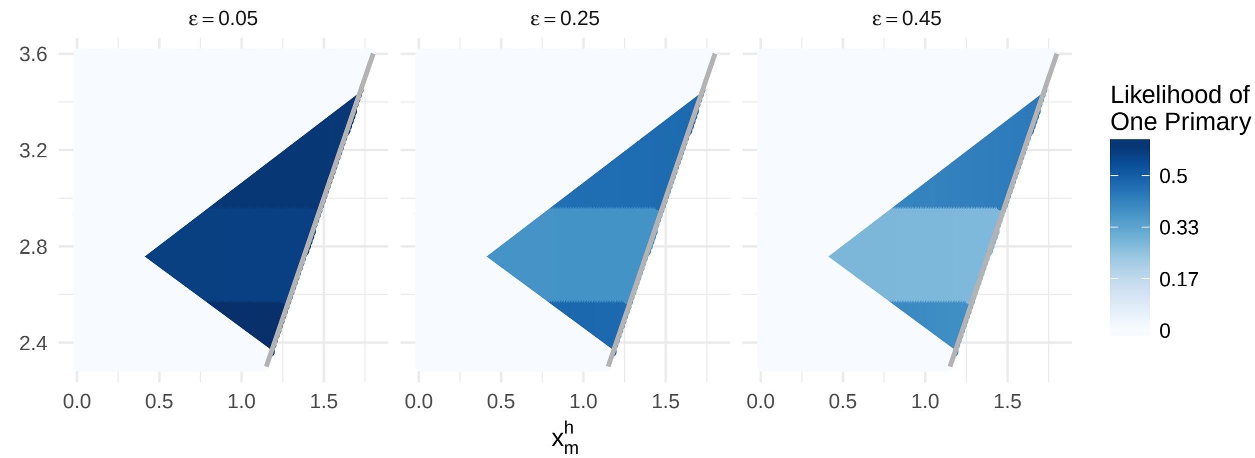

Figure 9: Probability of either one-primary equilibrium state as a function of x_m^h, Δ, and ϵ when $v = 1$, $b = 3$, $\alpha = 10$, and $x_m^l = -x_m^h$.

or right) equilibria to other parameter combinations. In Figure 9 we plot the expected number of primaries for a different parameter set from that graphed in Figure 7. As stated in Comment 1, the likelihood that only one party will employ primaries is decreasing in ϵ, or increasing in the incumbency advantage.