Homework 4

1. For the given matrices: A(2 x 3), B(2 x 2), C(3 x 3), D(3 x 4) of (n rows x m columns) find which expression are legit:

   (a) AB (b) BA (c) AC (d) ACD (e) D^TA^T (f) $C\ D^T$ (g) $(A^TB)^T\ D$

2. Find the matrix that transforms any vector **x** from 3-dimensional vector space into vector **y** = 2**x**.

3. Find the matrix that transforms any vector **x** = $[x_1, x_2, x_3]$ to vector

   y = $[x_1, -2x_2, -x_3]$

4. Find the matrix that transforms any vector **x** = $[x_1, x_2, x_3]$ to vector

   y = $[x_2, -x_1, x_3]$

5. Find the rank of the given matrices. Are these matrices invertible? Why?

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 0 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 1 & 3 \\ 6 & 3 & 9 \\ 4 & 2 & 6 \end{bmatrix}$$

6. Optional.

   The triangle corners are represented by the following vectors:

   [(1, 1, 1), (2, 3, -1), (-1, 0, 4)]

   How the area of the triangle changes if we linearly transform its vectors by the matrix $A = \begin{bmatrix} 1 & 4 & 0 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$?

7. Optional.

   Solve the system of equations below:

$$\begin{cases} x + y = 1 - z \\ x - 5 = 2z \\ 2x + 1 = y + 2z \\ 2y + z = 7 \end{cases}$$