Pràctica 1 de MN2

October 2, 2020

Consider a n-by-n linear system $Ax = b$, where $n = 10^6$ and

$$A = \begin{pmatrix} D & B & 0 & \cdots & 0 & 0 & U \\ B & D & B & \cdots & 0 & 0 & 0 \\ 0 & B & D & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ddots & D & B & 0 \\ 0 & 0 & 0 & \cdots & B & D & B \\ L & 0 & 0 & \cdots & 0 & B & D \end{pmatrix}, \quad b = \begin{pmatrix} \frac{1}{n} \\ \frac{2}{n} \\ \frac{3}{n} \\ \vdots \\ \frac{n-2}{n} \\ \frac{n-1}{n} \\ 1 \end{pmatrix}$$

where

$$D = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad U = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

We want to find the solution with an error (in $\|\cdot\|_\infty$ norm) below 10^{-12}

1. Bound the norms of the iteration matrices BJ (Jacobi) and BGS (Gauss-Seidel).
2. Solve the linear system using the method of Jacobi.
3. Solve the linear system using the method of Gauss-Seidel.
4. Solve the linear system using SOR. Find a suitable value of ω using the results seen in theory classes. Compare with Gauss-Seidel.