Texas Hold ’Em Poker as a Public Announcement Logic Model

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Click here for the GitHub repository.

1 Introduction

This project explores the application of public announcement logic (PAL) to model the dynamics of Texas hold’em poker. This topic is particularly interesting because poker involves strategic reasoning and incomplete information. This means that players must make decisions based on limited knowledge. New information is revealed as the game progresses and players continuously update their knowledge, making it a good scenario for studying multi-agent epistemic logic. Using PAL, one can effectively model how players update their knowledge after each revealed card or betting action.

1.1 How to play texas hold’em poker

Texas hold’em is a probability-based card game played with a 52-card deck. Each player is dealt two private cards (other players cannot see them) and five additional cards are revealed incrementally as community cards (everyone can see them). The players have to form their best five-card poker hand by combining their two private cards with three community cards. The game consists of four betting rounds (PokerNews, 2025):

1. Pre-flop: Players place their bets after received their private cards.
2. Flop: The first three community cards are revealed and another round of betting occurs.
3. Turn: A fourth community card is revealed and the third round of betting occurs.
4. River: The final community card is revealed and the last round of betting takes place.

At the end of the betting rounds, the players reveal their hands and the player with the highest ranking hand wins the pot (Shown in Figure 1). Alternatively, a player can win if all others fold, indicating that they are unwilling to match the bet.

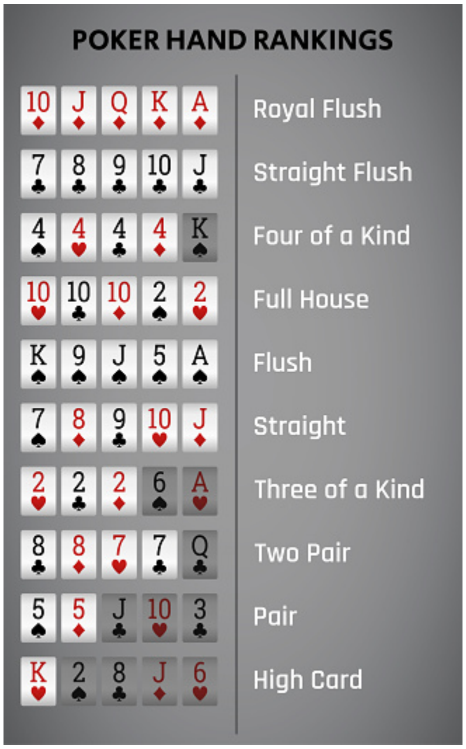

Figure 1: Ranking of hands in poker from the strongest (top) to the weakest (bottom) (iStockPhoto, n.d.)

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1.2 Simplifications

To make the logical modelling of poker feasible, we introduced several simplifications. The game was reduced to two perfectly logical players and the deck was limited to 12 cards (Ace, King and Queen of each suit). Using a full deck of cards would result in an exponential growth in the number of possible world (card combinations), making computations and Kripke models impractical. With this 12-card deck, the hands flush, straight flush, straight, and royal flush are no longer possible (shown in Figure 1). This specific deck was chosen because it results in a manageable number of states, making it suitable for exploring public announcement logic in poker. Furthermore, instead of implementing the traditional betting mechanism, we modelled betting as a process where agents publicly and truthfully announce their perceived probability of winning based on all possible worlds accessible in the Kripke model. These public announcements simulate betting behaviour and allow us to focus on the epistemic aspects of the game without adding unnecessary complexity. These simplifications allow us to effectively explore the epistemic logic behind poker and how players update their knowledge through public announcements.

2 Epistemic logic behind the project

2.1 Syntax

The revealing of common cards and betting are modelled using Public Announcement Logic in the language $L_{KC[]}(A, P)$, the syntax of which is defined in BNF as

$$\varphi, \psi ::= p|\neg\varphi|(\varphi \wedge \psi)|K_a\varphi|C_B\varphi|[\psi]\varphi$$

In the example resulting from applying our simplifications, the set of agents $A = 1, 2$. The atomic propositions for revealing of the cards will be of the form Kd='The King of Diamonds is in the common cards', the set of such atomic propositions will include propositions of this form for all the cards in the deck. The set of atomic propositions for betting will be of the form HB_1='Agent 1 placed a high bet', where the possible values for the bet are HB=high bet, LB=low bet.

2.2 Semantics

Given a Kripke model $M = \langle S, \sim, V\rangle$ he semantics is the usual semantics for PAL:

$M, s \models p$ iff $s \in Vp$
$M, s \models \neg\varphi$ iff $M, s \models \varphi$
$M, s \models \varphi \wedge \psi$ iff $M, s \models \varphi$ and $M, s \models \psi$
$M, s \models Ka\varphi$ iff for all $t \in S : s \sim_a t$ implies $M, t \models \varphi$
$M, s \models CB\varphi$ iff for all $t \in S : s \sim_B t$ implies $M, t \models \varphi$
$M, s \models [\varphi]\psi$ iff $M, s \models \varphi$ implies $M|\varphi, s \models \psi$

where $M|\varphi = \langle S', \sim', V'\rangle$ is defined as:
$S' = [[\varphi]]M$
$\sim' a = \sim_a \cap([[\varphi]]M \times [[\varphi]]M)$
$V'p = Vp \cap [[\varphi]]M$

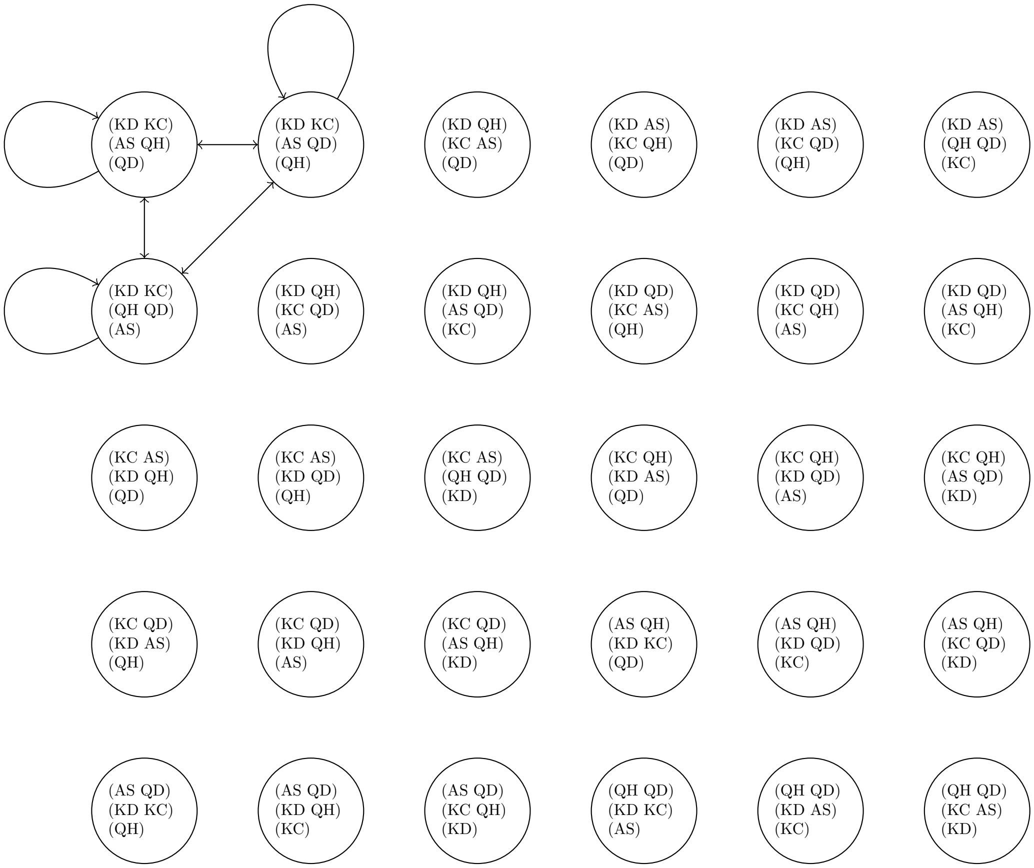
$\langle\varphi\rangle$ is defined as the dual of $[\varphi]$ as: $M, s \models \langle\varphi\rangle\psi$ iff $M, s \models \varphi$ and $M|\varphi, s \models \psi$

2.3 Card Revealing

In this section we present an example of card revealing modelled as a PAL event. We begin by reminding the reader of the details of the scenario we are trying to model. This is as follows:

- We have two players, each with two cards in their hands
- The deck is composed of 12 cards (Ace, King, Queen of each of the four suits)
- All of the common cards have been revealed, except for the River
- Before revealing the river, all of the cards remaining in the deck have also been revealed. Thus only 5 cards remain to be revealed in the deck (4 in the players' hands, 1 being the river)

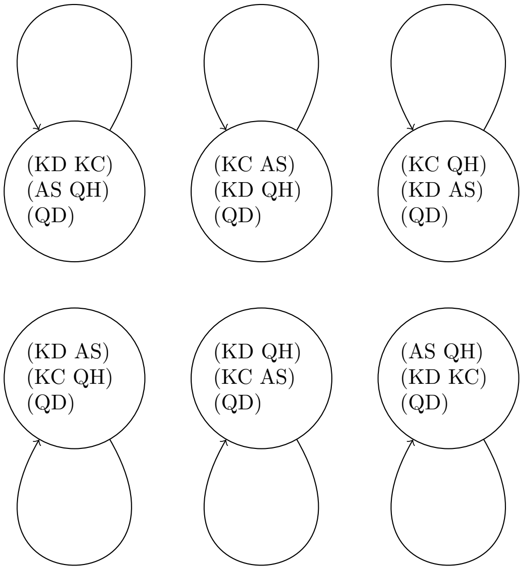
This gives us a total of $\binom{5}{2} \cdot \binom{3}{2} \cdot 1 = 10 \cdot 3 \cdot 1 = 30$ possible states before revealing the river. Suppose the 5 cards remaining are the King of Diamonds, the King of Clubs, the Ace of Spades, the Queen of Hearts, and the Queen of Diamonds. This gives us the following Kripke model Given a Kripke model $M = \langle S, \sim, V \rangle$:



Where every state corresponds to a possible distribution of the cards. Here we have drawn only the accessibility relations for player 1 from 3 of the states. The accessibility relations for the players are defined as follows:

$$(s, t) \in R_n \text{ iff } H_n s = H_n t$$

where $H_n s$ indicates the cards in the hand of player n in state s. The intuitive idea is that in every state each player has access only to the states in which he has the same cards in his hands.
If we define H_n as the content of the hand of player n, we can see that $M, s \models \langle\varphi\rangle CH_n$ for every state and every $n \in A$, where $\langle\varphi\rangle$ is a public announcement corresponding to the revealing of the River.
Suppose now that the River is revealed as the Queen of Diamonds. Following the semantic definitions, the initial model will be restricted to the model $M|Qd = \langle S, \sim, V\rangle$ (remember that according to our definition the atomic proposition Qd stands for "the Queen of Diamonds is in the common cards"):



We can see that in this scenario, after the card is revealed all the remaining states only have the reflexive accessibility relation for both players, meaning that the distribution of the cards becomes common knowledge after the public announcement that the Queen of Diamonds is in the common cards. This is intuitively right, given that we modelled the situation in such a way that only one card was left to be revealed.

2.4 Betting

With respect to betting, we are also going to model it as a PAL event. In particular, we are going to consider only truthful betting as truthfully representing the player's subjective chances of winning, as follows:

- $B_n x$="Player n placed a bet of value x"
- $B_n x$ iff Player n's chances of winning are x

For simplicity, we assume that bets are in the range $0 \leq x \leq 1$ and thus that there is a one-to-one correspondence between bets and chances of winning. This is all common knowledge between the player.
Suppose we are in a situation in which all the common cards have been revealed, and they are as follows:

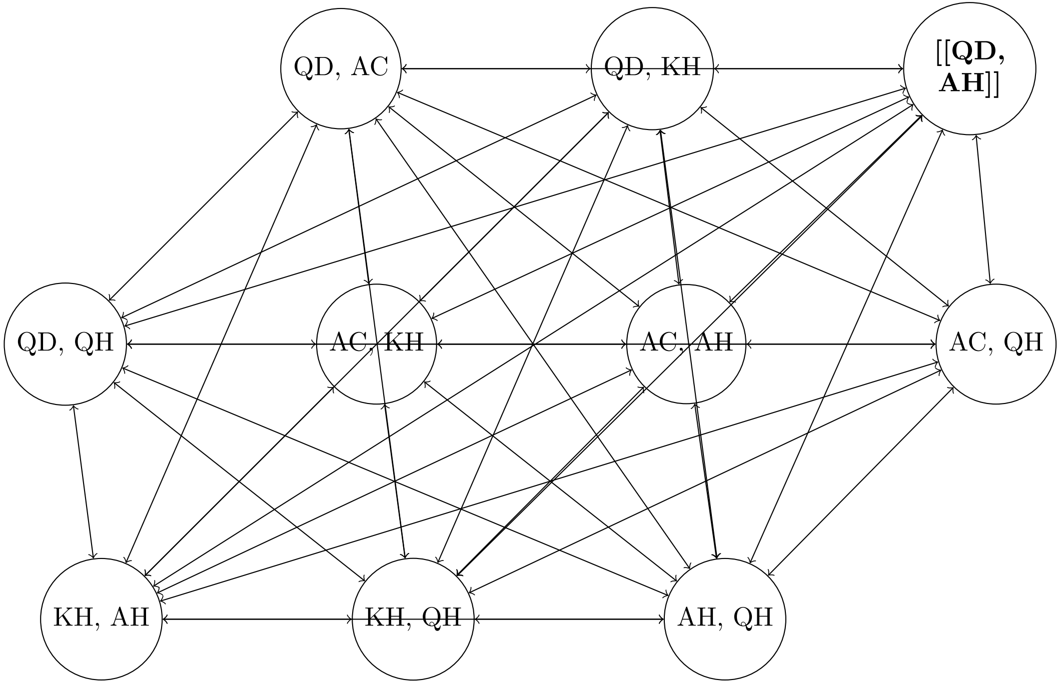
- As
- Ks
- Qs

- Kd
- Ad

And suppose the hands of the players are:

- $H_1 = Kc, Qc$
- $H_2 = Qd, Ah$

We look at this from the perspective of Player 1 so as to keep the number of states in the model to an intelligible amount.
From the perspective of player 1, player 2 has $\binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{5\times4}{2\times1} = 10$ possible hands, represented in the following Kripke model:



Where (QD, AH) is the actual world. In this world, Player 2 has a perceived chance of winning of 0.5 (they have a better hand than Player 1 in exactly half of the accessible worlds). Thus, he bets 0.5, letting Player 1 know that Player 2's perceived chances of winning are 0.5.
After this public announcement, Player 1 will eliminate all the possible worlds in which Player 2's chances of winning are other than 0.5, leaving only worlds in which Player 2 is in fact winning and Player 1 is losing.
Thus, we have $M, (QD, AH) \models [HB_2]K_1\varphi$ where φ is either "Player 1 is winning" or "Player 2 is losing".
If this process is repeated for both players, the epistemic state of both will converge on a single world (the actual world) giving in this case that $M, (QD, AH) \models [B_m y]\langle B_n x\rangle C\varphi$ where φ indicates the actual state of the world.

3 Code implementation

The code implements the simple version of poker described earlier. We initialize two agents with two main properties: a set of cards, and knowledge about the opponent's potential cards.

These agents each get a hand of two cards and then print the number of possible hands the opponent might have. After this, the community cards get dealt (flop, turn, and river). After each part of the community cards get dealt the agents also print the possible hands their opponent might have. The possible hands the opponent might have is a list of all the combinations of 2 cards that can be made with the cards left in the deck. After the final community card gets dealt the agents will announce their perceived probability of winning. As these announcements are truthful, the agents will use this to recalculate their probability of winning based on the opponent's announcement. These recalculations are done based on rounding the reported probability to the nearest 0.1, in practice this does not change the reported probabilities for the simplified model, but this does model the limitation of reporting probabilities as a set of discrete atoms. The recalculation of the opponent's possible hands is done by looping through each set of cards an agent thinks their opponent might have. Then for each set of cards, the agent calculates the opponent's belief on which cards the agent might have. With this information the agent can calculate the win chance the opponent would report for each possible hand they have and compare this to the actual reported win percentage. Each hand that does not match the reported percentage gets filtered out. The output the simulation gives can be seen in Figure 2.

Two alternative recalculation functions can be found in the *altProbabilityCalculations.py*. The first of the alternative functions matches reported with expected probabilities exactly to determine which hands to filter out and thus models perfect logical agents. The second function uses a threshold to determine a high/low bet and uses this to distinguish hands that need to be filtered out.

By Monte Carlo simulation we found that a full house and two pairs would not lead to more interesting results in our simplification of the game, as these would be guaranteed to occur. We keep the simulation limited to calculate the best hand based on multiples of the same cards, thus limiting the simulation to calculating only the occurrences of pairs, three of a kind, and four of a kind. In our simplified version of the game, only three and four of a kinds will occur. However, an "extended" mode is added to the code for testing purposes, in which two pairs are possible. This "extended" mode however is was not analysed using Kripke models and is there just for testing purposes.

4 Discussion

4.1 Limitations

This project provides insights into poker dynamics using PAL, but there are some limitations that may prevent our model from capturing the full complexity of real-life poker. First, the model simplifies the game by using a 12-card deck, which limits the variety of possible card combinations (worlds in the Kripke model) and reduces the complexity of the game. The model is also restricted to a scenario with only two agents, which oversimplifies the interaction dynamics that can be seen in multi-agent poker games. In reality, poker is often played with more agents which makes the interactions and epistemic states considerably more complex. We restricted the game so that only two agents play, which simplifies the interaction dynamics that can be seen in multi-agent poker games. In reality, poker is often played with more agents, making the interactions and epistemic states considerably more complex.

Another limitation lies in the betting mechanism, which was simplified by assuming truthful betting based on the agents' perceived probabilities of winning. In real-life poker games, agents do not always bet more if they think they think they have higher chances of winning, but can also use bluffing. Bluffing is a key strategic element of poker that can be used to deceive opponents. Without incorporating the possibility of bluffing, the model misses a significant aspect of the game's complexity. Furthermore, the model does not simulate diverse strategies, as all agents use the same method of publicly announcing their probability of winning and update their knowledge based on the new information (cards revealed and opponent's announced probability of winning) until common knowledge is reached and the game ends.

4.2 Future research

As previously mentioned, our current implementation is limited. Some parts can be expanded more easily than others. Increasing the deck size or the number of players is an easily manageable task, but analysing the results would be more difficult, so the 'return on investment' is not great.

A more interesting direction to explore, is the introduction of betting strategies. Currently, our model only includes the pure strategy of truthfulness. It would be interesting to see how the pure strategy of lying would modify the results (i.e. slower convergence towards the real world in the Kripke model). Of course this only works in our model if the agents' strategies are common knowledge. But even with that assumption, we would need to change the model to accommodate mixed strategies (e.g. agent 1 'tells the truth' with probability 0.7, and 'lies' otherwise), as this introduces the need for a strategy for belief for each Agent. Modelling false public announcement logic and exploring how it affects the Kripke models could be an interesting direction for future research. This is well outside of the scope of the course, but worthy of research, for two reasons: from a game-theory perspective, is there a dominant strategy?; and from the logic perspective what axioms do we need to model this, and what is the logic of those axioms?. The last question can be expanded towards frame defineability theory (as the frame for the game of poker has some defining characteristics), possibly establishing a connection between different types of games.

5 Conclusion

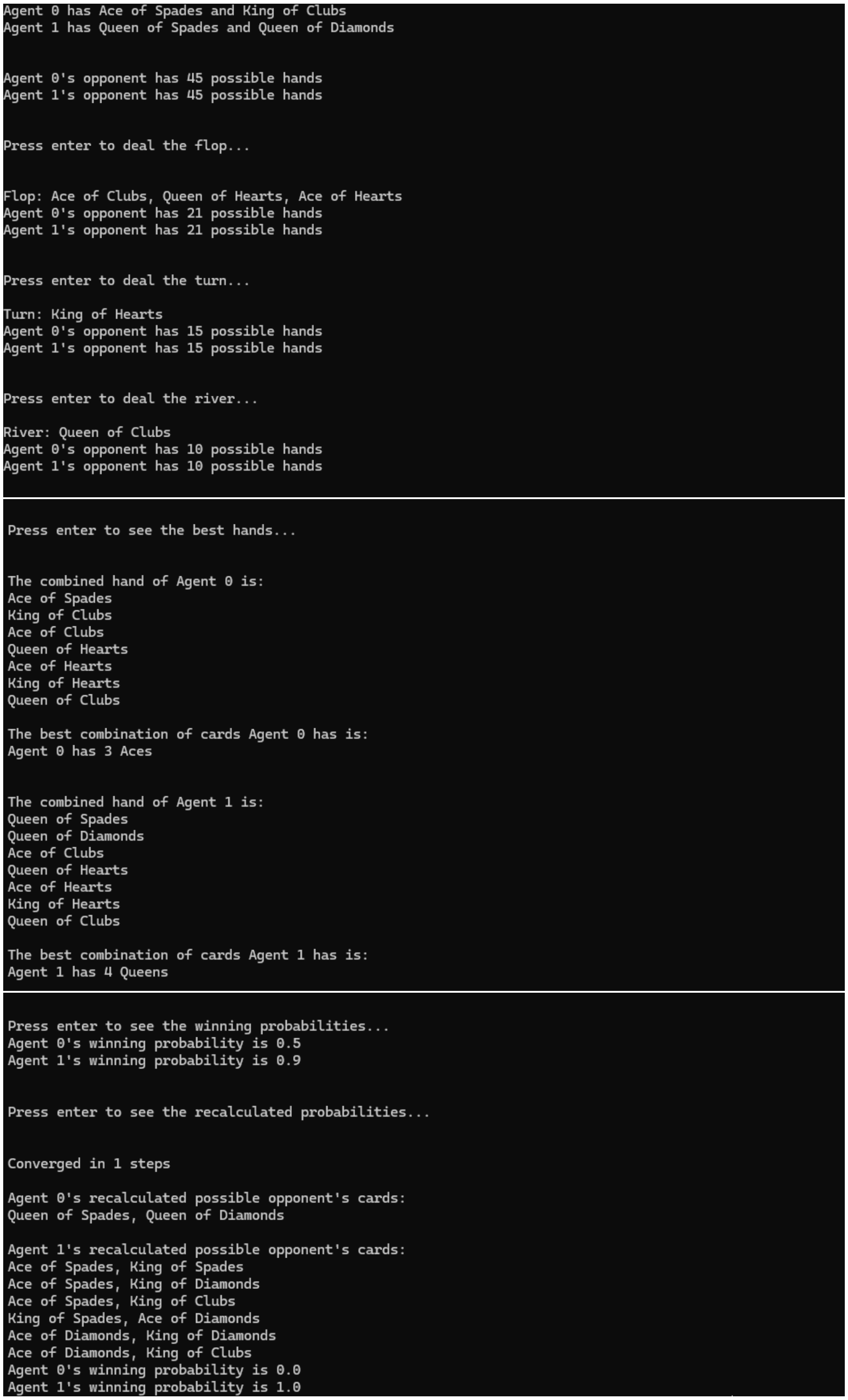
This project demonstrates how PAL can be applied to model Texas hold'em poker. Although the version of the game modelled in this project was simplified, the core logical framework behind the game was effectively represented. The approach used in this study can be used as a basis for extending the model to explore more complex aspects of poker and epistemic logic.

The model used in this project highlights how all possible card combinations can be represented as worlds in the Kripke model. The agents update their knowledge as new information is revealed, such as the new community cards or updates to the opponent's win probabilities. As the game progresses, worlds that are no longer accessible based on the current world are removed from the Kripke model, effectively updating the agent's knowledge. The game finishes when when common knowledge is reached and all agents are aware of who has the best hand.

References

iStockPhoto. (n.d.). *Royalty-free stock images, photos, and videos* [Accessed: 2025-01-26].

PokerNews. (2025). *Texas hold'em poker rules* [Accessed: 2025-01-26]. https://www.pokernews.com/poker-rules/texas-holdem.htm



```
Agent 0 has Ace of Spades and King of Clubs
Agent 1 has Queen of Spades and Queen of Diamonds


Agent 0's opponent has 45 possible hands
Agent 1's opponent has 45 possible hands


Press enter to deal the flop...


Flop: Ace of Clubs, Queen of Hearts, Ace of Hearts
Agent 0's opponent has 21 possible hands
Agent 1's opponent has 21 possible hands


Press enter to deal the turn...

Turn: King of Hearts
Agent 0's opponent has 15 possible hands
Agent 1's opponent has 15 possible hands


Press enter to deal the river...

River: Queen of Clubs
Agent 0's opponent has 10 possible hands
Agent 1's opponent has 10 possible hands
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```
Press enter to see the best hands...


The combined hand of Agent 0 is:
Ace of Spades
King of Clubs
Ace of Clubs
Queen of Hearts
Ace of Hearts
King of Hearts
Queen of Clubs

The best combination of cards Agent 0 has is:
Agent 0 has 3 Aces


The combined hand of Agent 1 is:
Queen of Spades
Queen of Diamonds
Ace of Clubs
Queen of Hearts
Ace of Hearts
King of Hearts
Queen of Clubs

The best combination of cards Agent 1 has is:
Agent 1 has 4 Queens
```

```
Press enter to see the winning probabilities...
Agent 0's winning probability is 0.5
Agent 1's winning probability is 0.9


Press enter to see the recalculated probabilities...


Converged in 1 steps

Agent 0's recalculated possible opponent's cards:
Queen of Spades, Queen of Diamonds

Agent 1's recalculated possible opponent's cards:
Ace of Spades, King of Spades
Ace of Spades, King of Diamonds
Ace of Spades, King of Clubs
King of Spades, Ace of Diamonds
Ace of Diamonds, King of Diamonds
Ace of Diamonds, King of Clubs
Agent 0's winning probability is 0.0
Agent 1's winning probability is 1.0
```

Figure 2: Figure showing command line output of the program