# Design of a battery-charger controller for electric vehicle based on RST controller

S. LACROIX, Student Member, IEEE, M. HILAIRET, Member, IEEE, E. LABOURE
Laboratoire de Génie Electrique de Paris (LGEP)/ SPEE-Labs;
CNRS UMR8507; SUPELEC; Université Pierre et Marie Curie P6; Université Paris-Sud 11;
11, rue Joliot Curie, Plateau de Moulon F91192 Gif Sur Yvette Cedex
(samantha.lacroix, mickael.hilairet, eric.laboure)@lgep.supelec.fr

Abstract- In order to find better solution for the development of Electric Vehicles (EVs), many research are performed by companies, universities and research laboratories. The main goal of these works is to provide some solutions to reduce the cost, increase the compactness, improve the autonomy and the reliability as well as the vehicle energetic efficiency and last but not least, reduce the global carbon emissions impact of the vehicle. This paper deals with an on-board battery charger for EVs or PHEVs where power electronics are reduced thanks to the use of same power converters in traction and charger mode. RST controllers in discrete time are design to control the PWM rectifier for the single-phase case.

#### I. INTRODUCTION

In the last few years, several solutions for electric and plugin-hybrid electric vehicles have been tested [1]-[3]. A lot of charger topologies are found in papers. In [4], some off-board chargers are presented, but the proposed topologies have many drawbacks, such as the inability to recharge batteries anywhere, or the high cost induced by multiple converters. Other solutions are proposed such as a bidirectional ac-dc converter designed to power the batteries of a PHEV. For such bidirectional converter, there is also the possibility to use a specific converter but another solution is also presented using the motor driving inverter and the motor windings as filtering inductors [4]-[6]. Nevertheless, the proposed topologies are only suitable for a single-phase grid connection and cannot be used as fast charger in three-phase configuration. Retaining the idea of using the converter and the windings of the machine, in [7], a special electric machine is designed to become an isolated three-phase transformer. The idea is to connect three windings of the machine to the grid and to connect the other three to the inverter. In this configuration, because of the existence of a rotating field, the electric motor rotates during the operation of recharging batteries.

In this paper a particular topology is studied. This topology ([8]-[10]) uses the electric machine as a set of filter inductors and the same power converter for charging and traction mode. Two ways to connect to the grid are possible in single-phase or three-phase. Details on this topology can be found in [10]. The converter behaves like two three-phase interleaved PWM boost converters in parallel in charging mode and like three H-bridges inverters in traction mode. In a single-phase configuration, the two converters legs corresponding to the

third bridge are not connected and the power circuit is equivalent to two interleaved PWM rectifiers.

By controlling the PWM converters, the DC link voltage can be maintained at a defined constant value, and the AC current waveforms can be controlled in order to obtain the required power factor. In this topology, each phase of the AC grid is connected to two parallel PWM boost converters. This connection is realized through the midpoint of each winding of the electrical machine. In case of balanced currents in each half windings of a given phase, the rotating magnetic field components generated by the stator are eliminated. Therefore, there is no electromagnetic torque and the motor remains stationary. In this paper, only the single phase case is studied.

In most examples cited above, the controllers used to control the currents and the DC link voltage, are PI controllers. In this paper, a linear control, based on the pole placement method (RST), is developed. This method is based on the resolution of a Diophantine equation, allowing the rejection of low-frequency disturbances for polynomial type of any order, and ensuring, for the closed loop, a unity static gain [11]-[12]. RST controller allows a best compromise between speed and performances of a system. A RST controller designed for sine wave references is developed in [13]. This proposed design method ensures a steady state error between reference and controlled output close to zero.

This article is organized as follow: in section I, the power conversion topology is introduced and a model is proposed. This model is established for the single-phase configuration. Then, in the second section, the RST controller design is described and implemented, first in a simple study case and, then for the PWM rectifier. In section IV, the control of currents on the grid and the DC bus voltage are tested through numerical simulations. A discussion on the properties and limits of this control method is then given. In a last section, conclusion and discussions about future works are provided.

# II. MODELING OF SINGLE-PHASE PWM RECTIFIER

# A. State space model

The power circuit corresponding to the single-phase configuration in charging mode is shown in Fig. 1. The role of the current controller is to regulate the 4 windings currents in order to obtain a global unity power factor on the grid, balanced currents in the windings and to control the DC bus

voltage  $U_{dc}$  whatever the load variation. In order to achieve these objectives, the system must be first identified. The Kirchhoff laws give:

$$U_{dc} \mathbf{u} = (\mathbf{T} \mathbf{L}_{s} \mathbf{T}) \mathbf{i} s + \mathbf{R}_{s} \mathbf{i} + V_{grid} \mathbf{T}_{1} + V_{cm} \mathbf{T}_{2}$$
 (1)

$$-\mathbf{i}^t \mathbf{u} = i_{load} + CU_{dc} \mathbf{s} \tag{2}$$

where,  $L_s$  is the matrix inductance of the PMSM (Permanent Magnet Synchronous Motor), which depends on the rotor position  $\theta$ :

$$L_s(\theta) = \begin{bmatrix} L_1 & M_{12} & M_{13} & M_{14} \\ M_{12} & L_1 & M_{14} & M_{13} \\ M_{13} & M_{14} & L_2 & M_{34} \\ M_{14} & M_{13} & M_{34} & L_2 \end{bmatrix}$$
(3)

where  $\mathbf{R_s}$  is the matrix resistance of the machine, i is the current vector  $[i_a, i_b, i_c, i_d]^{\mathsf{t}}$ . The currents  $i_1, i_2, i_3$  and  $i_4$  represent the currents convention chosen to obtain only negative mutual inductances. T is the transformation matrix giving  $[i_1, i_2, i_3, i_4]$  currents from  $[i_a, i_b, i_c, i_d]$ :

$$T = diag(1, -1, 1, -1)$$
 (4)

 $\boldsymbol{u}$  is the control vector, i.e. the duty cycles, equal to  $[d_1 \ d_2 \ d_3 \ d_4]^t$ ,  $V_{cm}$  is the common mode voltage,  $\boldsymbol{T}_I$  and  $\boldsymbol{T}_2$  are respectively  $[\frac{1}{2} \ \frac{1}{2} \ -\frac{1}{2}]^t$  and  $[1\ 1\ 1\ 1]^t$ .

If the load is a resistance, the state space model of the system is given by the following expression.

$$\dot{X} = A_c X + B_c(X) u + D_c 
Y = CX, \quad (C = I_4)$$
(5)

with,

$$X = [i_a i_b i_c i_d U_{dc}]^t$$

$$\boldsymbol{A_c} = -R_s \begin{bmatrix} Y_1 & Y_{12} & Y_{13} & Y_{14} & 0 \\ Y_{12} & Y_1 & Y_{14} & Y_{13} & 0 \\ Y_{13} & Y_{14} & Y_2 & Y_{34} & 0 \\ Y_{14} & Y_{13} & Y_{34} & Y_2 & 0 \\ 0 & 0 & 0 & 0 & 1/r_{load}R_sC \end{bmatrix}$$

and

$$\boldsymbol{B}_{c} = \begin{bmatrix} Y_{1} & Y_{12} & Y_{13} & Y_{14} & 0 \\ Y_{12} & Y_{1} & Y_{14} & Y_{13} & 0 \\ Y_{13} & Y_{14} & Y_{2} & Y_{34} & 0 \\ Y_{14} & Y_{13} & Y_{34} & Y_{2} & 0 \\ 0 & 0 & 0 & 0 & 1/C \end{bmatrix} \begin{bmatrix} U_{dc} & 0 & 0 & 0 \\ 0 & U_{dc} & 0 & 0 \\ 0 & 0 & U_{dc} & 0 \\ 0 & 0 & 0 & U_{dc} \\ -i_{a} & -i_{b} & -i_{c} & -i_{d} \end{bmatrix}$$
(9)

$$D_{c} = -V_{grid} \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix} - V_{grid} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

 $D_c$  is a measurable disturbance to be compensated by the controller. By an analysis of the control matrix  $B_c$ , it is possible to see that each control signal  $d_k$  has an influence on

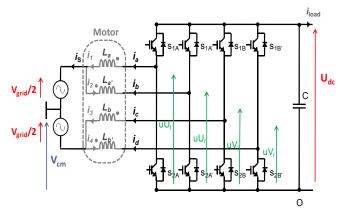


Fig. 1. Equivalent circuit in charging mode — Single-phase grid connection.

the state vector and there is a product between the state variables and the control vector. Therefore the system is non-linear and also strongly coupled due to *Y* matrix.

Moreover, the single-phase topology introduces a natural relation between the 4 currents in the windings of the machine. So, the system of equation can be reduced and it is not necessary to control the 4 currents. The control of three currents is sufficient. The latter current is then completely defined by:

$$i_a + i_b + i_c + i_d = 0 (10)$$

#### B. Problem statement

Using the windings of the motor instead of independent inductances introduces a coupling between each current equation due to the natural magnetic coupling of the PMSM.

Moreover, the effects of saliency associated with the fact that the motor uses buried permanent magnets causes variations of inductances and mutual inductances with rotor position.

In equations (5) to (9), the terms used are not the inductances but the permeances and the computations show that the variation for different positions of permeances is not negligible.

From these elements, i.e. the mathematical non-linearity, the coupling of the system and the variations associated with the rotor position, the idea is to develop a robust controller, which will be able to regulate the currents and the DC-bus voltage regardless the rotor position even in case of estimations errors of inductances.

This will be achieved using a RST controller. Although the RST controller is linear, its performances and limits will be tested on this complex system.

#### III. CONTROLLER DESIGN

# A. Control structure

The RST polynomial regulator seems to be an interesting alternative to the regular PI controller, because it allows a best compromise between speed and performances. Based on the pole placement theory, it is possible to impose poles in closed loop and to carry out in separate way the objectives for tracking and for regulation.

The control block diagram of the PWM rectifier is shown in Fig. 2, where the external loop is the voltage loop and the internal one represents the currents loops. In this work, the DC link voltage is assumed nearly constant according to the currents dynamic. This allows designing this control architecture.

Block 1 and block 2 are respectively voltage and currents controllers. For the currents loop, 4 currents are controlled through 4 RST controllers.

## B. RST controller design

The block diagram of a regular system with RST controller is presented in Fig. 3, where  $B(z^{-1})/A(z^{-1})$  represents the open-loop transfer function. The closed loop transfer function is given by

$$y = \frac{BT}{AS + BR} y_c + \frac{S}{AS + BR} d \tag{11}$$

where, y is the output,  $y_c$  the reference and d the disturbance. Polynomials  $R(z^{-1})$  and  $S(z^{-1})$  are obtained by a pole placement strategy and are solutions of the Diophantine equation:

$$A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1}) = D(z^{-1})$$
 (12)

To ensure a unity gain of the transfer  $y/y_c$  at steady state, Equ. (11) shows that polynomial  $T(z^{-1})$  must verify the constraint T(1) = R(1). Moreover, the degree of  $T(z^{-1})$  is a parameter that is defined by the designer. It can be used to simplify the tracking transfer  $y/y_c = BT/D$ . Another constraint is that S(1) = 0 in steady state in order to reject the perturbation. In this case,  $S(z^{-1}) = (1 - z^{-1}) S'(z^{-1})$ .

Degrees of S and R depend on the type of controller: proper or strictly proper. The choice of a strictly proper controller is done. In fact, this latter is best suited for high frequency noise rejection. So, if the degree of polynomial A is equal to n:

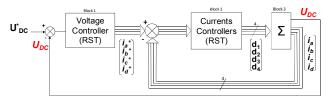


Fig. 2. Control block diagram.

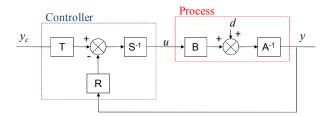


Fig. 3. System with RST controller.

$$deg (S) = deg (A) +1$$

$$deg (R) = deg (A)$$

$$deg (D) = 2n + 1$$

Assuming a first order plant

$$H(s) = \frac{K}{1+\tau s} \tag{13}$$

which results from the discretization with a sampling period  $T_c$ :

$$H(z^{-1}) = K \frac{(1-a)z^{-1}}{1-az^{-1}}$$
 (14)

with  $a = e^{-T_S/\tau}$ . Polynomials  $R(z^{-1})$  and  $S(z^{-1})$  are defined as follow:

$$R(z^{-1}) = r_1 z^{-1} + r_0$$
  
 $S(z^{-1}) = (1 - z^{-1})(s_1 z^{-1} + s_0)$ 

The poles of the closed-loop are imposed in the z-plane such as:

$$z_{1,2} = e^{(-\xi_0 \omega_0 \pm j\omega_0 \sqrt{1 - \xi_0^2})T_S}$$

$$z_3 = e^{-\omega_0 T_S}$$

that leads to the polynomial  $D(z^{-1}) = (1-z_1z^{-1})(1-z_2z^{-1})(1-z_3z^{-1})$ .

Therefore, the resolution of the Diophantine equation gives the linear system, as follows:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -(1-a) & 1 & K(1-a) & 0 \\ 0 & -(1-a) & 0 & K(1-a) \end{bmatrix} \begin{bmatrix} s_0' \\ s_1' \\ r_0 \\ 0 & a & 0 \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_2 \end{bmatrix}$$
(15)

where  $D(z^{-1})$  is defined as  $d_0 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3}$ .

In practice, the outputs of controllers are often saturated in order to protect the components. Therefore, a saturation associated with an anti-windup device is implemented, as shown on Fig. 4.

As stated before, polynomial T is a free tuning parameter with two constraints: (i) the transfer BT/D is at least proper, (ii) T(1) = R(1) in order to ensure a unity gain of the transfer  $y/y_c$  at steady state. Polynomial T is defined as follow:

$$T(z^{-1}) = \frac{R(1)}{F(1)}F(z^{-1}) = hF(z^{-1})$$
 (16)

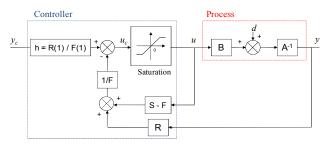


Fig. 4. System with RST anti-windup controller.

where polynomial F is a tuning parameter that allows to modify the type of closed-loop transfer function: third, second and first order closed-loop system. For this example three possibilities are allowed:

- 1<sup>st</sup> case:  $F(z^{-1})$  is scalar. In this case, polynomial T is equal to  $r_0 + r_1$ .
- $2^{\text{nd}}$  case:  $F(z^{-1})$  is a  $1^{\text{st}}$  order polynomial equal to  $(1 z_1 z^{-1})$ . Therefore, T is equal to

$$T(z^{-1}) = \frac{(r_0 + r_1)}{(1 - z_1)} (1 - z_1 z^{-1})$$

•  $3^{\text{rd}}$  case:  $F(z^{-1})$  is a  $2^{\text{nd}}$  order polynomial equal to  $(1 - z_1 z^{-1})(1 - z_2 z^{-1})$ . Therefore, T is equal to

$$T(z^{-1}) = \frac{(r_0 + r_1)}{(1 - z_1)(1 - z_2)} (1 - z_1 z^{-1})(1 - z_2 z^{-1})$$

Finally, the poles of the closed-loop are specified according to the desired response time  $t_r$ :

- For a first order system,  $\omega_0 t_r = 3$
- For a second order system,  $\omega_0 t_r = 4.8$  with  $\xi = 1$
- For a third order system,  $\omega_0 t_r = 6.3$  (this relation is true only for a cascade of three 1<sup>st</sup> order systems).

Fig. 5.a, 5.b and 5.c show the step response for the 3 different cases.

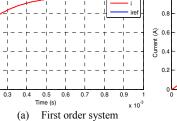
# C. Application to single phase charger

The state space model of the system is given in section II. Firstly, this model is non-linear due to product between the control input and the state vector (currents and voltage), secondly, states are coupled due to mutual inductances.

In this work, we do not consider coupling effects to design the RST controller. Therefore, in these conditions, the system equations can be rewritten as follow:

$$(L_1s + R_s)i_a = U_{dc}d_1 - \frac{V_{grid}}{2} - V_{cm}$$
 $(L_1s + R_s)i_b = U_{dc}d_2 - \frac{V_{grid}}{2} - V_{cm}$ 
 $(L_2s + R_s)i_c = U_{dc}d_3 + \frac{V_{grid}}{2} - V_{cm}$ 
 $(L_2s + R_s)i_d = U_{dc}d_4 + \frac{V_{grid}}{2} - V_{cm}$ 
 $(1 + r_{load}Cs)U_{dc} = i_{dc}r_{load}$ 

with, 
$$i_{dc} = -\mathbf{i}^t \mathbf{u} = i_a d_1 + i_b d_2 + i_c d_3 + i_d d_4$$
 (17)



(b) Second order system
Fig. 5. Step response of the simple example

where  $i_{dc}$  is the DC link voltage. Terms  $V_{grid}$  and  $V_{cm}$  are considered like measurable perturbations and the DC link voltage is assumed nearly constant according to the currents dynamic. By neglecting mutual inductances and compensating perturbations the system is represented by 5 first order transfer functions. The 5 controllers are computed according section III.B. Fig.6 represents the DC link voltage control, where grid current reference  $i_s$  is obtained by multiplying a sinus waveform by the control voltage output. The reference windings currents are equal to half of this reference current  $i_s$ .

## IV. SIMULATION RESULTS

Simulations have been done based on the following parameters:

TABLE I SIMULATION PARAMETERS

$V_{grid}$	RMS grid voltage (V)	240
$U_{dc}ref$	Init DC link reference voltage (V)	400
$V_{cm}$	Common mode voltage (V)	U <sub>dc</sub> ref / 2
$r_{load}$	Load resistance $(\Omega)$	213
$R_s$	Stators resistances ( $\Omega$ )	0.33
P	Output power (kW)	3.3
$T_{ri}$	Current loops response time (ms)	0.8
$T_{ro}$	Voltage loop response time (ms)	16
$L_{mean}$	Mean value of the inductance (mH)	3.3
Θ	Mechanical rotor position (°)	22

Fig. 7 represents the voltage regulation for a variation of the reference. The DC link voltage is well regulated despite the ripple voltage (10% of the DC link voltage) at 100 Hz due to the natural single-phase alimentation. The AC source current is in phase with the source voltage with a time delay equal to 0.5 ms. However, this delay is acceptable in practice.

Some robustness tests have also been conducted. In table II, all cases of inductances and resistances windings variations are presented and the maximum relative error between current reference  $i_{ref}$  and current measured  $i_a$  is done. The control is ensured even for a variation of  $\pm$  50% of the inductances and resistances. Fig. 8 shows the results obtained for the worst case.

(c) Third order system

Current

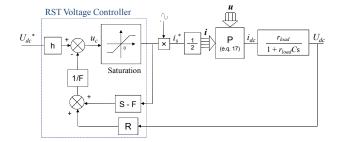


Fig. 6. Voltage control loop.

TABLE II
MAXIMUM CURRENT RELATIVE ERROR (%)

	$L_{1/2}$	$L_{1/2} + 50\%$	$L_{1/2} - 50\%$
r	16.7	21.1	12.4
r + 50%	17.1	21.3	12.9
r – 50%	16.4	20.9	11.9

#### V. CONCLUSION

This paper has presented a RST controller for single-phase battery charger for EV application. The converter has been described and its particularity has been pointed out. The controller based on a RST controller is designed without considering the mutual inductances of the machine. The simulation results show, despite this particular assumption and parameters variations in robustness test, that the PWM rectifier works properly. This validates the proposed architecture.

A time delay has been observed for the currents control, this delay is acceptable in practice.

### ACKNOWLEDGMENT

This work was supported by the French automotive cluster MOVEO in a FUI program, through the project SOFRACI.

## REFERENCES

- [1] C.C Chan, "The state of art of electric, hybrid and fuel cell vehicles". Proc. of the IEEE, Vol. 95, No. 4, pp. 704-718, April 2007.
- [2] S. Haghbin, K. Khan, S. Lundmark, M. Alakula, O. Carlson, M. Leksell, O. Wallmark, "Integrated chargers for EVs and PHEVs: Example and new solutions". International Conference on Electrical Machines (ICEM) proceedings, Italy, September 2010.
   [3] Y. J. Lee, A. Khaligh, A. Emadi "Advanced integrated bidirectional
- [3] Y. J. Lee, A. Khaligh, A. Emadi "Advanced integrated bidirectional AC/DCand DC/DC converter for plug-in hybrid electric vehicles". IEEE Transactions on Vehicular Technology, vol. 58, no. 8, pp. 3970-3980, October 2009.
- [4] L. Solero, "Nonconventional on-board charger for electric vehicle propulsion batteries". IEEE Transactions on Vehicular Technology, vol. 50, no. 1, pp. 144-149, January 2001.
   [5] L. Shi, A. Meintz, M. Ferdowsi, "Single-phase bidirectional AC-DC
- [5] L. Shi, A. Meintz, M. Ferdowsi, "Single-phase bidirectional AC-DC converters for plug-in hybrid electric vehicle applications". IEEE Vehicle Power and Propulsion Conference, Harbin, China, September 2008.
- [6] S. K. Sul, S. J. Lee, "An integral battery charger for four-wheel drive electric vehicle". IEEE Transactions on Industry Applications, vol. 31, no. 5, pp. 1096-1099, September 1995.

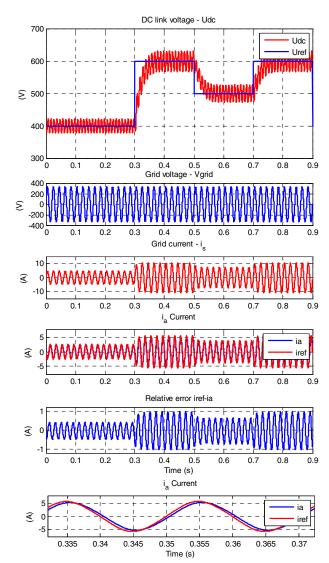


Fig. 7. Voltage and currents regulation of single phase PWM rectifier.

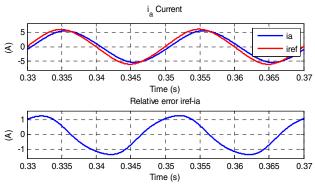


Fig. 8. Robustness result for an increase of 50% of motor's inductances and resistances.

- [7] S. Haghbin, M. Alakula, K. Khan, S. Lundmark, M. Leksell, O. Wallmark, O. Carlson, "An integrated charger for plug-in hybrid electric vehicles based on a special interior permanent magnet motor". IEEE Vehicle Power Propulsion Conference (VPPC), France, September 2010.
- [8] L. De-Souza, B. Silvestre, B. Bouchez, "A combined multiphase electric drive and fast battery charger for electric vehicles". IEEE

- Vehicle Power Propulsion Conference (VPPC), France, September
- A. Bruyère, L. De-Souza, B. Bouchez, P.Sandulescu, X. Kestelyn, E. Semail, "A multiphase traction fast –battery-charger drive for electric or plug-in hybrid vehicles". IEEE Vehicle Power Propulsion Conference (VPPC), France, September 2010.
- [10] S. Lacroix, E. Labouré, M. Hilairet, "An integrated fast battery charger for electric vehicle". IEEE Vehicle Power Propulsion Conference
- (VPPC), France, September 2010.

  [11] P. De Larminat, "Automatique Commande des systèmes linéaires".

  2nd edition, Hermès, Paris, pp. 177-187, 1998.
- [12] K. J. Astrom, B. Wittenmark, "Computer-Controlled Systems: Theory and Design". 3rd edition, Prentice Hall, 1996.
  [13] E. Ostertag, E. Godoy, "RST-Controller design for sinewave references by means of an auxiliary Diophantine equation". Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference, Spain, December 2005.