RST-CONTROLLER DESIGN: A RATIONAL TEACHING METHOD BASED ON TWO DIOPHANTINE EQUATIONS

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Abstract: The RST-structure for discrete-time controllers provides an elegant design method, both for reference tracking and disturbance rejection, once the discrete model of the process is known and a model transfer function for the closed loop is chosen. Complete cancellation of steady state errors in response to polynomial or sinusoidal reference signals, or a combination of both, without any signal lag at the output, is achieved by the solution of an auxiliary Diophantine equation. The flexibility of this design makes it easily adaptable to various kinds of publics, to match their own requirements without going into all details.

Keywords: digital control, RST controller synthesis, pole placement, Diophantine equation, sinusoidal reference signal, pedagogic aspects of the control design.

1. INTRODUCTION

The RST controller structure, which appeared in the early 1980's, is an extension to the more classical series compensator, to which it gives an additional degree of freedom, enabling thus the designer to specify different responses of the closed loop to input references and disturbance signals. The most commonly used control design associated with such a structure consists essentially in a pole placement method relying on pole-zero cancellations and the resolution of a Diophantine equation. In its early presentations, only the stable zeros of the open loop were taken into account in the cancellation process (Åström and Wittenmark, 1984; Landau, 1993). Also, the achievement of a vanishing steady-state error, even though properly solved for the elimination of polynomial disturbances, was improperly claimed to hold also in the case of the tracking of polynomial references, this being true only for step references. A more realistic design approach, placing on an equal level the cancellation of stable open-loop poles and zeros and handling more thoroughly the response to disturbances was given later (Åström and Wittenmark, 1997). A true error-free tracking of polynomial inputs of any order was achieved by the introduction of a second, or *auxiliary*, Diophantine equation (Ostertag, 1999). This method has been successfully applied to the digital control of a PWM inverter (Godoy and Ostertag, 2003a), and has been recently extended to the error-free tracking of sinusoidal references (Ostertag and Godoy, 2005).

The teaching method presented here is the result of several years of improvement and rationalization of the material teached to our students (Godoy and Ostertag, 2003b) and includes also our latest research developments. A description of the full presentation will be the subject of Section 2, followed in Section 3 by a simple academic example, in order to illustrate the richness of this approach. Limited presentations of RST-structure based designs, adapted to three different kinds of publics according to their needs and to their mathematical background, will then be given in Section 4, and finally Section 5 will conclude our work.

2. COMPLETE DISCRETE RST-CONTROLLER DESIGN APPROACH

Let us assume that a discrete-time plant or a continuous-time plant sampled at a given period T_s, described by its discrete transfer function $G(z) = B(z^{-1})/A(z^{-1})$, is fed by a control signal $u(k)$ at its input, and by measurement and load disturbances $e(k)$ and $v(k)$ respectively. The z-transform of its measured output signal $y(k)$ is:

$$Y(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} U(z^{-1}) + \frac{B(z^{-1})}{A(z^{-1})} V(z^{-1}) + \mathrm{E}(z^{-1}) \,. \quad (1)$$

The choice to represent all the transfer functions in negative powers of z brings about several advantages, as will appear later.

As apparent in Fig. 1, an RST-controller, consisting of the polynomials $R(z^{-1})$, $S(z^{-1})$ and $T(z^{-1})$, provides the control law $u(k)$ with the following z-transform:

$$U(z^{-1}) = \frac{T(z^{-1})}{S(z^{-1})} Y_r(z^{-1}) - \frac{R(z^{-1})}{S(z^{-1})} Y(z^{-1}) \,, \quad (2)$$

where $y_r(k)$ is the reference input.

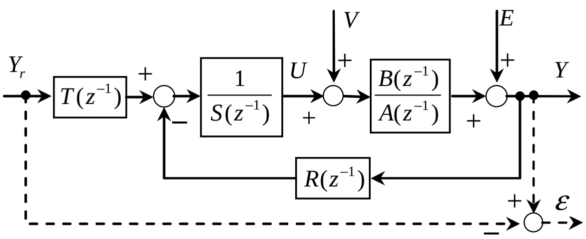

Fig. 1. RST-controlled discrete-time system

The closed-loop equations, as derived from (1) and (2) or from this figure by inspection, are:

$$U = \frac{AT}{AS+BR} Y_r - \frac{BR}{AS+BR} V - \frac{AR}{AS+BR} E \,, \quad (3)$$

$$Y = \frac{BT}{AS+BR} Y_r + \frac{BS}{AS+BR} V + \frac{AS}{AS+BR} E \,. \quad (4)$$

Note that, for the sake of simplicity, the dependence of z^{-1} has been omitted as will be done in the following whenever no ambiguity exists. The controller synthesis occurs then in two phases.

2.1. *Phase I: stability and transient response of the closed loop (dynamic behaviour)*

The first phase of the design consists in determining the polynomials R and S, such that the closed-loop transfer function from reference input to measured output, Y/Y_r, is equal to some *model* transfer function,

$$F_m(z^{-1}) = \frac{B_m(z^{-1})}{A_m(z^{-1})} \,, \quad (5)$$

which will be partially specified and partially determined, as will be seen below. The equation to be satisfied is thus

$$\frac{BT}{AS+BR} = \frac{B_m}{A_m} \,. \quad (6)$$

The closed-loop order being usually greater than that of the model, (6) will be satisfied by pole-zero cancellation in its left hand member. To that purpose, the plant transfer function is factored as follows:

$$G(z) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{B^+(z^{-1})B^-(z^{-1})}{A^+(z^{-1})A^-(z^{-1})} \,. \quad (7)$$

The polynomials $B^-(z^{-1})$ and $A^-(z^{-1})$ regroup, as will be seen shortly, all the terms which will not be cancelled in the process. They must thus contain all the zeros and poles of $G(z)$ which lie outside the unit circle or on it, in order not to violate the external and internal stability requirements, and can also include some other terms, which the designer chooses freely not to cancel for some reasons which will become clear later. The plant's pure delay, which appears as a factor z^{-d} at the numerator of $G(z)$ once expressed in powers of z^{-1}, must of course also be embedded in B^-: it is worthwhile to note that this results automatically from the above rule, since this term represents a zero at infinity in the complex plane, thus obviously outside the unit disk. This constitutes one of the arguments for having chosen z-transform representations in negative powers of z. It is also advantageous, though not mandatory, to include the constant coefficient which appears at the numerator of $G(z)$ into B^-. All the other factors constitute respectively $B^+(z^{-1})$ and $A^+(z^{-1})$: they are the terms that will be cancelled.

Let us now go back to the pole-zero cancellation process of the first member of (6). B^- cannot be a factor of $AS+BR$, since that would cause partial transfer functions such as U/Y_r or Y/E to possess unstable poles, as shown by (3) and (4). B^- must thus divide B_m :

$$B_m = B^- B'_m \,. \quad (8)$$

B^+ however is allowed to be a factor of $AS+BR$. Since it divides already the second term of that sum, it must also divide the first one, thus be a factor of S:

$$S = B^+ S' \,. \quad (9)$$

For identical reasons, A^+ can be a factor of $AS+BR$; since it already divides the first term of that sum, it must be a factor of the second one, thus of R :

$$R = A^+ R' \,. \quad (10)$$

As a consequence of those factorizations, and after simplification by B^-, (6) becomes:

$$\frac{T}{A^- S' + B^- R'} = \frac{A^+ B'_m}{A_m} .$$

This yields in turn the following system of equations:

$$\begin{cases} A^- S' + B^- R' = A_0 A_m \\ T = A_0 A^+ B'_m . \end{cases} \qquad (11) \begin{cases} (a) \\ (b) \end{cases}$$

(11)(a) will be referred to hereafter as the *primary* Diophantine equation. Multiplying its two members by $A^+ B^+$ yields:

$$AS + BR = A^+ B^+ A_0 A_m$$

The closed-loop characteristic polynomial contains thus the two compensated terms, A^+ and B^+, but these terms disappear from the transfer function Y/Y_r, as expected.

The desired characteristic polynomial A_m of the closed-loop is chosen according to the dynamic specification imposed to it.

If no other condition is imposed, $A_0 = 1$ is chosen by default. Otherwise, the polynomial A_0 can play an interesting role as a disturbance filter, since the modes it contains are not excited by the reference signal. Consider for instance the transfer function from *E* to *Y*. From (4) and (9),

$$\frac{Y}{E} = \frac{A^+ A^- B^+ S'}{A^+ B^+ A_0 A_m} = \frac{A^- S'}{A_0 A_m} ,$$

whereas from (11)(b),

$$\frac{Y}{Y_r} = \frac{B A_0 A^+ B'_m}{A^+ B^+ A_0 A_m} = \frac{B^- B'_m}{A_m} = \frac{B_m}{A_m} ,$$

as required. A low-pass term A_0 can thus be used to filter measurement disturbances such as high frequency noise, without having any influence on the input-output transient response.

It is easy to verify by similar calculations that $U/Y_r = (AB'_m)/(B^+ A_m)$ and $Y/V = (BS')/(A^+ A_0 A_m)$. Knowing that poles with a negative real part in a transfer function yield alternating responses, it may be advisable, e.g. if a smooth control signal u is required, to avoid compensating plant zeros having such real parts, even though lying inside the unit disk, and to place them in B^- instead of B^+. Similarly, it may be judicious to avoid the compensation of some slow stable plant poles and to put them into A^- instead of A^+, e.g. if the rejection speed of transient load disturbances υ matters. These remarks explain the *free* choice of the designer not to compensate some of the *compensable* poles or zeros of the plant mentioned previously.

So far, (11)(a) can be solved for the two unknown polynomials S' and R' for the smallest possible degree in z^{-1}, from which *S* and *R* are then deduced by (9) and (10). What remains to be determined is the numerator B_m of the model transfer function. Since it must contain B^- in factor, according to (8), the remaining unknown is in fact the polynomial B'_m. This is the aim of the second design phase.

2.2. *Phase II: steady-state response of the closed loop (static behaviour)*

Two steady-state errors are to be canceled:

With respect to perturbations (full rejection problem).
Polynomial perturbations of order *n* are completely eliminated in steady-state (vanishing steady-state error) by introducing a number $m = n + 1 - l$ of integrators (poles at $z = 1$) into the polynomial *S*, where l is the number of such poles already contained in the part of $G(z)$ comprised between the output of the comparator and the perturbation point of application. This is done by letting

$$S' = (1 - z^{-1})^m S'_1 ,$$

and solving

$$A^- (1 - z^{-1})^m S'_1 + B^- R' = A_0 A_m ,$$

which replaces (11)(a).

Sinusoidal perturbations can be eliminated in the same way, by inserting in *S* a second order factor, of the form given hereafter in (17) (Åström and Wittenmark, 1997).

With respect to the reference (tracking or servoing problem).
For constant references (step inputs), it is sufficient to impose $F_m(1) = 1$, i.e. $B_m(1) = A_m(1)$, so that the closed loop has unity static gain. This can be obtained by choosing for B'_m a constant: $B'_m = b'_0 = A_m(1)/B^-(1)$.

Steady state errors of higher orders, e.g. in response to a ramp reference, are however not cancelled by that solution, unless the two polynomials $R(z^{-1})$ and $T(z^{-1})$ are chosen to be the same, which is the case only for the series controller embedded in the more general RST structure.

To remedy that situation, a different way of determining the polynomial $T(z^{-1})$ has been proposed (Ostertag, 1999; Godoy and Ostertag, 2003b) , which in turn imposes $B_m(z^{-1})$. The partial *z*-transfer function from the reference Y_r to the true error signal $\mathcal{E}(z^{-1}) = Y_r(z^{-1}) - Y(z^{-1})$ between reference and measured output is given, with the use of (5), by:

$$\frac{\mathcal{E}(z^{-1})}{Y_r(z^{-1})} = 1 - \frac{Y}{Y_r} = 1 - F_m = \frac{A_m - B_m}{A_m} . \qquad (12)$$

In order to cancel, at the sampling times, steady-state errors in response to polynomial references of the form $y_r(t) = t^m$, having thus a *z*-transform

$$Y_r(z) = \frac{Y_{r1}}{(1 - z^{-1})^{m+1}} , \qquad (13)$$

where Y_{r1} is some polynomial in z^{-1}, it is necessary and sufficient, according to the final limit theorem of the *z*-transform, that $(1 - z^{-1})^{m+1}$ divides $A_m - B_m$, i.e.

$$A_m - B_m = (1 - z^{-1})^{m+1} L(z^{-1}) , \qquad (14)$$

where $L(z^{-1})$ is some unknown polynomial, to be determined. With (8) it is straightforward that both $L(z^{-1})$ and $B'_m(z^{-1})$ become now solutions to the equation

$$(1 - z^{-1})^{m+1} L + B^- B'_m = A_m , \qquad (15)$$

which has been designated as *auxiliary Diophantine equation* by Ostertag (1999). The case of the DC input-output unity gain, corresponding to $m = 0$, is embedded in this derivation, since it results from (14) that, for $m \geq 0$, $A_m(1) = B_m(1)$.

The case of sinusoidal reference signals is somewhat different. Since the final limit theorem does not apply here, consider instead the frequency response of the error $\mathcal{E}$ versus sinusoidal inputs. Its magnitude is given from (12) by

$$\left| \frac{\mathcal{E}(z^{-1})}{Y_r(z^{-1})} \right|_{z = e^{j\omega T_s}} = \left| \frac{A_m - B_m}{A_m} \right|_{z = e^{j\omega T_s}} .$$

In order to cancel the error at a given angular frequency ω_0, it is necessary to introduce a transmission zero into this transfer function at that frequency. This can be done by letting $(1 - e^{j\omega_0 T_s} z^{-1})(1 - e^{-j\omega_0 T_s} z^{-1})$ divide $A_m - B_m$, the complex conjugate pair of zeros being chosen so as to ensure a product polynomial with pure real coefficients. Equation (14) is then replaced by

$$\begin{aligned} A_m - B_m &= (1 - e^{j\omega_0 T_s} z^{-1})(1 - e^{-j\omega_0 T_s} z^{-1}) L(z^{-1}) \\ &= (1 - 2\cos\omega_0 T_s \cdot z^{-1} + z^{-2}) L(z^{-1}), \qquad (16) \end{aligned}$$

and the auxiliary Diophantine equation which must be solved becomes here

$$(1 - 2\cos\omega_0 T_s \cdot z^{-1} + z^{-2}) L + B^- B'_m = A_m \qquad (17)$$

instead of (15). It is worthwhile to note that the cancellation can apply simultaneously to more than one sinusoidal reference. If for instance the reference is the sum of two sinewaves, of frequencies ω_1 and ω_2, the resulting input-output error will be fully cancelled in steady state if two factors are inserted in the auxiliary Diophantine equation, one for each frequency:

$$\begin{aligned} &(1 - 2\cos\omega_1 T_s \cdot z^{-1} + z^{-2})(1 - 2\cos\omega_2 T_s \cdot z^{-1} + z^{-2}) L \\ &\qquad + B^- B'_m = A_m . \qquad (18) \end{aligned}$$

Likewise, a combination of one or more sinusoidal and polynomial reference signals is being taken into account by simply introducing the corresponding factors in the auxiliary Diophantine equation, as e.g. in the case of a sinewave of frequency ω_0 added to a polynomial reference of order *m*:

$$(1 - 2\cos\omega_0 T_s \cdot z^{-1} + z^{-2})(1 - z^{-1})^{m+1} L + B^- B'_m = A_m . (19)$$

Once $B'_m(z^{-1})$ is calculated by the resolution of (15), (17), (18) or (19), $T(z^{-1})$ is given by (11)(b). This completes the design process.

3. ACADEMIC EXAMPLE

A very simple academic example will illustrate the method. Assume a plant is given by the following discrete model, which results from the sampling with zero-order hold of some continuous-time plant at a period of $T_s = 0.1$ s :

$$G(z) = \frac{2z^{-1}(1 + 2z^{-1})}{(1 - z^{-1})(1 - 0.3z^{-1})} .$$

3.1. Controller synthesis according to Section 2

According to the comments of Section 2 no zero of the plant will be cancelled here, so that $B^-(z^{-1}) = 2z^{-1}(1 + 2z^{-1})$. Assume that this plant is to be controlled so that the closed loop has a second order type response with a damping factor of 0.8 and a bandwidth of 10 rad/s, thus a discrete characteristic polynomial $A_m(z^{-1}) = 1 - 0.7417 z^{-1} + 0.2020 z^{-2}$. The primary Diophantine equation yields

$$R(z^{-1}) = 0.1031 - 0.0264 z^{-1} ; \; \mathrm{S}(z^{-1}) = 1 + 0.3521 z^{-1} .$$

Assume furthermore that the closed loop should exhibit zero steady-state error in response to a reference signal, obtained by the sum of two sinewaves, with frequencies $\omega_1 = 7$ rad/s and $\omega_2 = 5$ rad/s, and a ramp. The polynomial T is determined by an auxiliary Diophantine equation, resulting from a combination of (18) and (19), i.e.

$$\begin{aligned} &(1 - z^{-1})^2 (1 - 1.530 z^{-1} + z^{-2})(1 - 1.755 z^{-1} + z^{-2}) L + \\ &\quad + 2z^{-1}(1 + 2z^{-1}) B'_m = 1 - 0.7417 z^{-1} + 0.2020 z^{-2}, \end{aligned}$$

which yields

$$\begin{aligned} T(z^{-1}) = B'_m(z^{-1}) &= 1.419 - 4.358 z^{-1} + 6.237 z^{-2} \\ &\quad - 5.011 z^{-3} + 2.216 z^{-4} - 0.4263 z^{-5} . \end{aligned}$$

A simulation of the time response of the closed loop is shown in Fig. 2, for a reference composed of two sinusoids with frequencies ω_1 and ω_2 and amplitudes of 1 and 2 arbitrary units, and a ramp with a slope of 2 arbitrary units per second. The error vanishes completely at the sampling times in steady state.

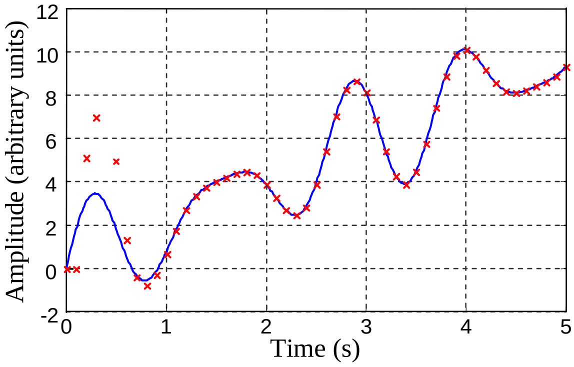

Fig. 2. Response of the closed loop to a reference made of two sinewaves and a ramp.

3.2. *Comparison with an internal-model principle based design*

Designs based on the well known internal model principle allow to reject all kinds of *disturbances*, provided a model of the disturbance is contained in the controller. This applies of course also to RST designs. However, the steady-state error-free tracking of *references*, e.g. of polynomial or sinusoidal type, is obtained by such a design only in the case of a one-degree-of-freedom structure, also called *error feedback* control, i.e. when $T = R$. This choice leads to a single Diophantine equation to be solved, $AS + BR = A_0 A_m$, which yields directly the polynomials R and S of the series compensator $C(z) = R(z^{-1})/S(z^{-1})$.

Stability robustness

Fig. 3 represents the Bode amplitude diagrams of the sensitivity function

$$\mathcal{S} = \frac{1}{1+\mathcal{L}} = \frac{A(z^{-1})S(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

for $z = \exp(j\omega T_s)$, calculated from the loop transfer function $\mathcal{L} = BR/AS$ for the two designs: the RST-design (I) and the error-feedback design (II), with the same parameters as in Subsection 3.1. For the second design, the two second degree factors and the term $(1 - z^{-1})$, corresponding to the two sinusoids and the added integrator, are included in the loop as factors of the polynomial *S*.

As apparent on this figure, the RST design yields a much better modulus margin (-3.7 dB) than the error-feedback design (-30 dB). This is not a surprise, since the inclusion of the two second order terms in the loop by means of the polynomial S unavoidably deteriorates the stability margins (Table 1).

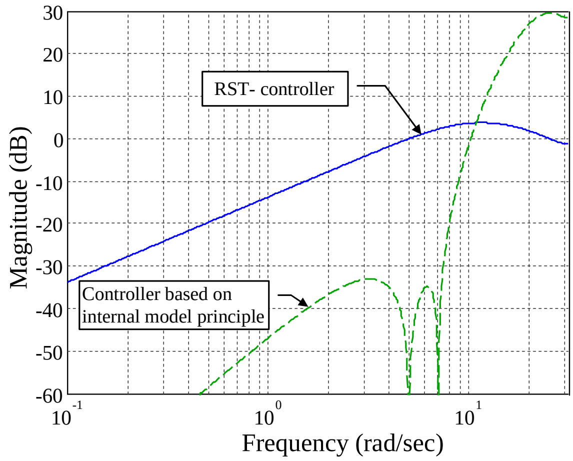

Fig. 3. Sensitivity Bode diagram for the two designs.

Table 1 Stability margins for the previous example and the two designs

Design	Gain margin	Phase margin
I	10.2 dB	63 °
II	0.3 dB	2.5 °

Sensitivity to model uncertainty

The highest required accuracy of the model is given by the relation (Åström and Wittenmark, 1997):

$$\alpha = \frac{|\Delta G(z)|}{|G(z)|} \le \frac{1}{|\mathcal{T}|},$$

where $\mathcal{T} = 1 - \mathcal{S} = BR/(AS + BR)$ is the complementary sensitivity function. The values obtained for the previous example are $\alpha = 100\,\%$ (design I) and $\alpha = 3.4\,\%$ (design II). Here again the true RST design shows a clear advantage.

4. PEDAGOGIC ASPECTS, PRESENTATION TO DIFFERENT PUBLICS

Let us first emphasize that the term “RST” represents a controller *structure*, not a controller *design method* as sometimes abusively stipulated. Since the design method used with that structure is the closed-loop pole-placement, many other design methods which can be interpreted as pole-placement designs can be embedded in an RST structure, such as e.g. the Smith predictor (Åström and Wittenmark, 1997) and the internal model controller (Åström and Wittenmark, 1997; Landau and Zito, 2005).

From our teaching experience, in front of various kinds of learning publics, the synthesis of an RST controller can be declined according to several parameters, such as the public’s theoretical and practical levels, the nature of the course (initial or continuing education) and its duration. It is thus possible to distinguish between three different presentations, according to the audiences concerned.

4.1. Publics with a high theoretical background

This type of public is typically encountered in initial education. The complete design approach, as described in details in Section 2 of this paper, including the concept of internal stability underlying the factorization of the plant transfer function, is then well adapted. It can be eventually increased by additional issues, such as the robust choice of the poles assignments (De Larminat, 1993).

4.2. Publics with a low theoretical background but a high practical experience

This profile is more characteristic of learners participating in a continuing training program. It may be preferable in this case not to enter into the details of the problems associated with the pole/zero cancellations. A simplified presentation, which corresponds to the implicit assumption of letting $A^- = A,\ A^+ = 1,\ B^- = B$ and $B^+ = 1$ in (7), can then be adopted. Thus the factorization of the plant transfer function is skipped, but all the other aspects of RST control are presented, including the decoupling of the perturbation rejection and the reference tracking in steady state. This presentation leads to the two Diophantine equations

$$\begin{cases} (1-z^{-1})AS' + BR = A_0 A_m \\ \qquad\qquad\qquad T = A_0 B'_m \end{cases}$$

$$(1-z^{-1})^{m+1} L + BB'_m = A_m$$

in the case where constant disturbances should be rejected and a polynomial reference of order m should be tracked, with vanishing steady-state error in both cases. Notice the simplified notations and handling of the various terms in this presentation, thus limiting possible confusions. It is however very easy to extend it in a later lecture, in order to take into account the details of the pole/zero cancellations which have been neglected here.

4.3. Publics with limited backgrounds, both theoretical and experimental

In this case it seems preferable to use an extremely simplified presentation, limited eventually to the one-degree-of-freedom version of the RST controller ($T = R$) mentioned in Section 3. An alternative could be a true RST version, with a polynomial T determined so as to impose a closed-loop static gain of unity. In the case where A or S contain at least one integrator, this leads to the formula, $T(z^{-1}) = A_0(z^{-1})R(1)/A_0(1)$, which becomes simply $T(z^{-1}) = R(1)$ if $A_0(z^{-1}) = 1$ has been chosen.

5. CONCLUSION

Within the framework of teaching, this paper proposes a rational approach for the presentation of an RST-controller design method. As compared to other existing approaches, this design method, by introducing an auxiliary Diophantine equation, allows to take into account different types of references such as sinewave signals for example. The effectiveness of the suggested approach is illustrated by the study of an academic example where references consist in a combination of a ramp and sinusoidal signals. As far as stability and model uncertainty are concerned, the solution proposed here shows greater robustness than a design based on the internal model principle.

Finally, the possibility to teach the RST controller synthesis at different levels of difficulty, according to the theoretical and practical backgrounds of the audience, has been discussed.

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