APPLICATION OF CONSTRAINED RECEDING HORIZON PREDICTIVE CONTROL ON A BRUSHLESS MOTOR

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Abstract

The effect of constraints in generalized predictive control (G.P.C.) is studied with application to the speed control of motor drives. The problem of minimising the G.P.C. cost function subject to equality constraints is handled by programming a polynomial RST controller, designed by using the Constrained Receding Horizon Predictive Control (C.R.H.P.C.). The main difference to G.P.C. consists in imposing constraints on the final output values, with the important result that theorems guarantee stable closed loop behaviours for particular sets of tuning parameters. This method applied on a benchmark including a brushless motor proves that the effective use of constraints enables better results for severe conditions of use.
Key Words : predictive control, constrained optimization, motor control

1. Introduction

Generalized predictive control (G.P.C.), as proposed in [1] and [3], has shown to be an effective strategy for high performance applications such as the control of motor drives in speed or position [6]. This paper presents the results obtained using C.R.H.P.C. (Constrained Receding Horizon Predictive Control) [2], [4], [5], [7] and [8] for determining a controller able to ensure good performances on a brushless motor.
In the first part of the paper, the standard formulation of a G.P.C. cost function is reminded and constraints are defined. Then the C.R.H.P.C. structure is developed. This control law minimises a quadratic cost function, with the condition that the output coincides to the reference value over a further constraint range. It will be seen that the computation of the optimal control sequence can be formulated in a polynomial form.
In the second part the proposed algorithm is tested on a brushless motor. After showing the way of proceeding and a brief description of the system, the results of speed control of motor drives are presented and discussed.

2. Cost function and constraints formulation

The heart of a model-based predictive controller is the plant model. The Controlled Autoregressive Integrated Moving Average model (CARIMA) is commonly used in G.P.C., as it is applicable to many single-input single-output plants:

$$A\left(q^{-1}\right)y(t)=B\left(q^{-1}\right)u(t-1)+\frac{\xi(t)}{\Delta\left(q^{-1}\right)} \qquad (1)$$

where $u(t)$, $y(t)$ are the plant input and output. A and B are polynomials in the backward shift operator q^{-1} :

$$A\left(q^{-1}\right)=1+a_1\,q^{-1}+\ \dots\ +a_{n_a}\,q^{-n_a}$$

$$B\left(q^{-1}\right)=b_0+b_1\,q^{-1}+\ \dots\ +b_{n_b}\,q^{-n_b}$$

$\xi(t)$ is an uncorrelated random sequence and the use of the operator $\Delta\left(q^{-1}\right)=1-q^{-1}$ ensures an integral control law.
Equivalently to the procedure of Clarke *et al.* [3], an optimal j- step ahead predictor is given by :

$$\hat{y}(t+j)=\underbrace{F_j\left(q^{-1}\right)y(t)+H_j\left(q^{-1}\right)\Delta u(t-1)}_{\text{free response}} \\ +\underbrace{G_j\left(q^{-1}\right)\Delta u(t+j-1)}_{\text{forced response}} \qquad (2)$$

where the F_j, G_j and H_j polynomials are solutions of the following diophantine equations :

$$\Delta(q^{-1})A(q^{-1})J_j(q^{-1})+q^{-j}F_j(q^{-1})=1 \qquad (3)$$

$$G_j(q^{-1})+q^{-j}H_j(q^{-1})=B(q^{-1})J_j(q^{-1}) \qquad (4)$$

The first term of equation (2) is called 'free response', as it represents the plant predicted output $\hat{y}(t+j)$,when there is no future control action. The second term is called 'forced response', as it represents the output prediction due to the hypothetical future control actions $u(t+j-1)$, $j\geq 1$.

To select a good control sequence, we would wish a minimal tracking error $e(t+j)=\hat{y}(t+j)-w(t+j)$ over a certain output horizon. Furthermore, to prevent any 'explosion' of the control action, a second term is generally added, so that the final performance index has a form similar to the following :

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$$J=\sum_{j=N_1}^{N_2}(\hat{y}(t+j)-w(t+j))^2+\lambda\sum_{j=1}^{N_u}\Delta u^2(t+j-1) \quad (5)$$

subject to : $\Delta u(t+j)=0$ for $j \geq N_u$

where N_1 and N_2 are the minimum and the maximum costing horizon. N_u is the control horizon and λ is a weighting factor for the control increment sequence to be calculated. Furthermore, the constraints that the control increments are forced to zero for $j \geq N_u$ provides a better convergence of the output to the setpoint.

If we define the two following vectors formed with the polynomial solutions of equations (3) and (4):

$$\mathbf{IF}=\left[F_{N_1}\left(q^{-1}\right),\ldots,F_{N_2}\left(q^{-1}\right)\right]^{\mathrm{T}}$$

$$\mathbf{IH}=\left[H_{N_1}\left(q^{-1}\right),\ldots,H_{N_2}\left(q^{-1}\right)\right]^{\mathrm{T}}$$

$\mathbf{F}=\left[f\left(t+N_1\right),\ldots,f\left(t+N_2\right)\right]^{\mathrm{T}}$, the vector of the free response, can be written as :

$$\mathbf{F}=\mathbf{IF}\; y(t)+\mathbf{IH}\;\Delta u(t-1) \quad (6)$$

Additionally, if we denote :

$$\tilde{\mathbf{U}}=\left[\Delta u(t),\ldots,\Delta u(t+N_u-1)\right]^{\mathrm{T}}$$

$$\hat{\mathbf{Y}}=\left[\hat{y}\left(t+N_1\right),\ldots,\hat{y}\left(t+N_2\right)\right]^{\mathrm{T}}$$

$$\mathbf{W}=\left[w\left(t+N_1\right),\ldots,w\left(t+N_2\right)\right]^{\mathrm{T}}$$

and the matrix formed with the coefficients g_i^j of the G_j polynomials, which in fact correspond to the step response values $g_i=g_i^j$:

$$\mathbf{G}=\begin{bmatrix} g_{N_1} & g_{N_1-1} & \cdots & 0 \\ g_{N_1+1} & g_{N_1} & \cdots & 0 \\ \vdots & \cdots & \cdots & 0 \\ g_{N_u} & \cdots & \cdots & g_1 \\ \vdots & \cdots & \cdots & \vdots \\ g_{N_2} & g_{N_2-1} & \cdots & g_{N_2-N_u+1} \end{bmatrix} \quad (7)$$

the output prediction has the following form :

$$\hat{\mathbf{Y}}=\mathbf{G}\,\tilde{\mathbf{U}}+\mathbf{F} \quad (8)$$

Now, (5) can be rewritten in a matrix form :

$$J=(\mathbf{G}\,\tilde{\mathbf{U}}+\mathbf{F}-\mathbf{W})^{\mathrm{T}}(\mathbf{G}\,\tilde{\mathbf{U}}+\mathbf{F}-\mathbf{W})+\lambda\;\tilde{\mathbf{U}}^{\mathrm{T}}\tilde{\mathbf{U}} \quad (9)$$

and the optimal control law comes from $\dfrac{\partial J}{\partial\,\tilde{\mathbf{U}}}=0$.

C.R.H.P.C. is a method issued from G.P.C. (Generalised Predictive Control) [1], [3], but able to control applications where conventional predictive structures can fail. This is due to the fact that C.R.H.P.C. [2], [4], [5], [7] and [8] imposes the condition that the output matches the reference value over a further constraint range.
As in all receding horizon predictive control strategies, the control law providing the control increment $\Delta u(t)$ is deduced minimising a classical G.P.C. criterion (5). But, in addition to G.P.C., this function must be minimised subject to a certain set of m future equality constraints :

$$\hat{y}(t+N_2+j)=w\left(t+N_2\right) \qquad \text{for} \quad j=1,\ldots,m \quad (10)$$

where $\hat{y}$ is the predicted output and m the number of points where the predicted output has to match the setpoint w after the prediction horizon.

By solving the Diophantine equations, the constrained optimisation problem (9), (10) can be written in the following matrix form :

$$J=(\mathbf{G}\,\tilde{\mathbf{U}}+\mathbf{F}-\mathbf{W})^{\mathrm{T}}(\mathbf{G}\,\tilde{\mathbf{U}}+\mathbf{F}-\mathbf{W})+\lambda\;\tilde{\mathbf{U}}^{\mathrm{T}}\tilde{\mathbf{U}} \quad (11)$$

and : $\mathbf{G_c}\,\tilde{\mathbf{U}}=\mathbf{b}=\mathbf{W_c}-\mathbf{F_c}$ (12)

with :
- $\tilde{\mathbf{U}}$ future control increment sequence
- $\mathbf{G}$ matrix of step response values
- $\mathbf{F}$ vector of free response coefficients
- $\mathbf{W}$ setpoint vector
- $\mathbf{G_c},\mathbf{b}$ matrix and vector describing the endpoint equality constraints

with : $$\mathbf{G_c}=\begin{bmatrix} g_{N_2+1} & g_{N_2} & \cdots & g_{N_2-N_u+2} \\ g_{N_2+2} & g_{N_2+1} & \cdots & g_{N_2-N_u+3} \\ \vdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \vdots \\ g_{N_2+m} & g_{N_2+m-1} & \cdots & g_{N_2-N_u+m+1} \end{bmatrix}$$

$$\mathbf{W_c}=\left[w\left(t+N_2\right),\ldots,w\left(t+N_2\right)\right]^{\mathrm{T}}$$

$$\mathbf{F_c}=\left[f_c\left(t+N_1\right),\ldots,f_c\left(t+N_2\right)\right]^{\mathrm{T}}$$

$\mathbf{F_c}$ corresponds to the free response under constraints and is defined, in a similar way as (6), by :

$$\mathbf{F_c}=\mathbf{IF_c}\; y(t)+\mathbf{IH_c}\;\Delta u(t-1) \quad (13)$$

with : $$\mathbf{IF_c}=\left[F_{N_2+1}\left(q^{-1}\right),\ldots,F_{N_2+m}\left(q^{-1}\right)\right]^{\mathrm{T}}$$

$$\mathbf{IH_c}=\left[H_{N_2+1}\left(q^{-1}\right),\ldots,H_{N_2+m}\left(q^{-1}\right)\right]^{\mathrm{T}}$$

By using Lagrange multipliers the optimal solution of this problem (9), (10) can be expressed by :

$$\tilde{\mathbf{U}}^* = \left[\mathbf{H}^{-1}\mathbf{G}_c^T\left(\mathbf{G}_c\mathbf{H}^{-1}\mathbf{G}_c^T\right)^{-1}\mathbf{G}_c\mathbf{H}^{-1} - \mathbf{H}^{-1}\right]\mathbf{c} + \mathbf{H}^{-1}\mathbf{G}_c^T\left(\mathbf{G}_c\mathbf{H}^{-1}\mathbf{G}_c^T\right)^{-1}\mathbf{b} \qquad (14)$$

with : $\mathbf{H} = 2\left(\mathbf{G}^T\mathbf{G} + \lambda\ \mathbf{I}\right) \qquad \mathbf{c} = 2\ \mathbf{G}^T(\mathbf{F} - \mathbf{W})$

Similarly to GPC, only the first value of the optimal sequence has to be applied :

$$\Delta u^*(t) = \mathbf{m}_1^T\ \mathbf{b} + \mathbf{n}_1^T(\mathbf{W} - \mathbf{F}) \qquad (15)$$

with : $\mathbf{m}_1^T$ first row of $\left(\mathbf{H}^{-1}\mathbf{G}_c^T\left(\mathbf{G}_c\ \mathbf{H}^{-1}\mathbf{G}_c^T\right)^{-1}\right)$

$\mathbf{n}_1^T$ first row of

$$\left(2\left[\mathbf{H}^{-1} - \mathbf{H}^{-1}\mathbf{G}_c^T\left(\mathbf{G}_c\mathbf{H}^{-1}\mathbf{G}_c^T\right)^{-1}\mathbf{G}_c\mathbf{H}^{-1}\right]\mathbf{G}^T\right)$$

Equation (15) represents a linear controller and its polynomial form is found in the same way as if there were no constraints. Comparing relation (15) to the polynomial RST structure we want to deduce (Fig. 1) :

$$S\left(q^{-1}\right)\Delta u(t) = -R\left(q^{-1}\right)y(t) + T(q)w(t) \qquad (16)$$

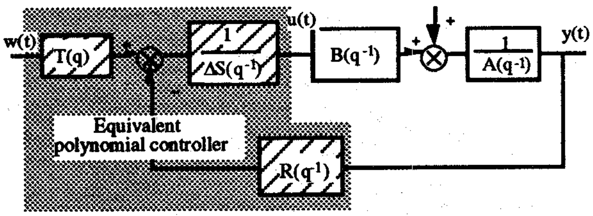

Fig. 1 - C.R.H.P.C. equivalent polynomial controller

The polynomials R, S, T can be identified in the following form :

$$R\left(q^{-1}\right) = \mathbf{n}_1^T\ \mathbf{IF} + \mathbf{m}_1^T\ \mathbf{IF}_c \qquad (17)$$

$$S\left(q^{-1}\right) = 1 + q^{-1}\left(\mathbf{n}_1^T\ \mathbf{IH} + \mathbf{m}_1^T\ \mathbf{IH}_c\right) \qquad (18)$$

$$T(q) = \left(q^{N_1}, \ldots, q^{N_2}\right)\mathbf{n}_1 + \left(q^{N_2}, \ldots, q^{N_2}\right)\mathbf{m}_1 \quad (19)$$

$\mathbf{IF}$, $\mathbf{IH}$, $\mathbf{IF}_c$, $\mathbf{IH}_c$ being polynomial expressions of the free response of the system subject to equality constraints as defined in (6) and (13).

A fundamental advantage of the C.R.H.P.C. methods is to ensure stability of the controlled system for a particular choice of the tuning parameters :

N_1	$\approx$ dead time of the system		
N_2	$\geq d°\left(A\left(q^{-1}\right)\right) + 2$	N_u	$= N_2 + 1$
m	$= d°\left(A\left(q^{-1}\right)\right) + 1$	λ	> 0

3. Proceeding way and simulation

For simulating C.R.H.P.C., a Matlab program has been developed. Given the numerator and the denominator of the transfer function, the sampling period for a choice of parameters, it calculates the polynomials R, S, T and offers utilities for examining the resulting controlled system such as Bode and Black diagrams, position of the poles of the closed loop, sensitivity functions, step response and error signals, influence of disturbances. Moreover, the main problem remains the choice of the tuning parameters which fix the three R, S, T polynomials. The following remarks can be made :

- N_1 is normally chosen equal to the system dead time.
- N_2 and N_u are to be discussed together as $N_u = N_2 + 1$ partly provides stability. Small values of (N_2, N_u) give quick step response, but produce instability and bad sensitivity functions.
  In contrast, large values of (N_2, N_u) lead to too slow step responses.
- m has to be chosen in connection to the system model with a degree of polynomial $A\left(q^{-1}\right)$.
- Small values of λ produces oscillations and lead to a maximum of the sensitivity function over 6 dB. Bigger values of λ increase the time response.

In order to find the parameters providing the best behaviour, a two dimensional parameters fixing method (N_2, λ) is used, which examines the influence of each parameter independently on the performance criteria.

4. Experimental results

To show the performances of this approach, tests have been performed on a brushless motor MASAP Télémécanique.

4.1. Structure of the benchmark

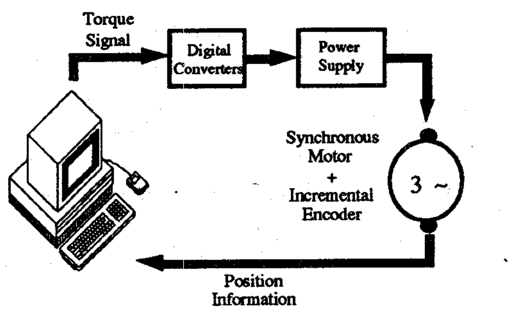

Fig. 2 - Experimental benchmark

The structure of the benchmark is the following :

⊳ Brushless motor : Nominal velocity : 3000 rpm
Nominal torque : 9,3 Nm
Power : 3 kW

- ⊳ Incremental encoder : 32 bits encoder with 20000 points/revolution
- ⊳ Digital converters : 12 bits converters coupled to a fast data acquisition module performing a conversion in less than 80 μs
- ⊳ Computer : classical 486 PC
- ⊳ Power supply : Three current loops with P.I. controllers and P.W.M.

All the following results are obtained with the same benchmark configuration and with no load for the motor. This choice enables to reach very important speeds without saturation of the current.

4.2. Identification

Before the design of the control law, an identification of the system is necessary to define the CARIMA model. The identification algorithm is simple and fast, and can be executed at the beginning of the test, or when an important change of the system is detected.
The basic idea is first to generate two pseudo random binary sequences (PRBS), the first being used to initialize the system, the second to calculate the correlation functions. In this case a step to initialize the system instead of the first PRBS is impossible due to the integrative behaviour of the motor drive. After that, to obtain the transfer function between the torque and the velocity of the brushless motor, a recursive least squares method provides the unknown parameters of the model with no matrix inversion.
For a sampling period of 4 ms, the identified transfer function between the control signal (torque signal) and the velocity is :

$$\frac{\Omega(t)}{\Gamma(t)} = \frac{4.1393\, q^{-1} + 5.2901\, q^{-2}}{1 - 1.0620\, q^{-1} + 0.2046\, q^{-2} - 0.1426\, q^{-3}}$$

4.2.1. First experiment

A polynomial predictive structure has been applied on this benchmark to efficientlty control the velocity of the motor. According to the guaranteed stability recommandations given above, $m = 4$ has been selected.
The *autocalibration* procedure is first used to determine the set of tuning parameters of the loop. The results of this two dimensional research are given Fig. 3 to 6.
It provides the following "best" set of parameters :

$$[N_1, N_2, N_u, m, \lambda] = [1,\ 10,\ 11,\ 4,\ 12890]$$

and the most fitting RST polynomial controller:

$$R(q^{-1}) = 0.05420 - 0.04926q^{-1} + 0.01058q^{-2} - 0.00654q^{-3}$$

$$S(q^{-1}) = 1 + 0.24705q^{-1}$$

$$T(q) = 0.000196q + 0.000556q^2 + 0.000696q^3 + 0.000649q^4 + 0.000522q^5 + 000359q^6 + 0.000189q^7 + 0.000055q^8 + 0.005749q^{10}$$

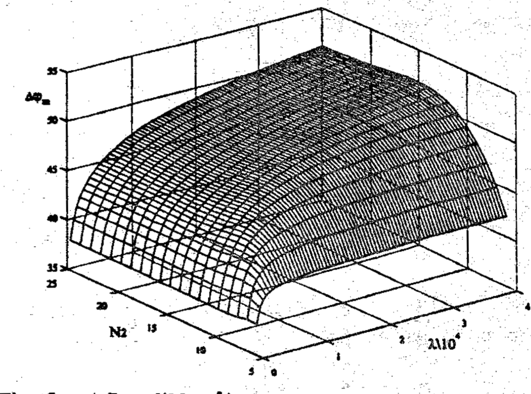

Fig. 3 - $\Delta\Phi = f(N_2, \lambda)$

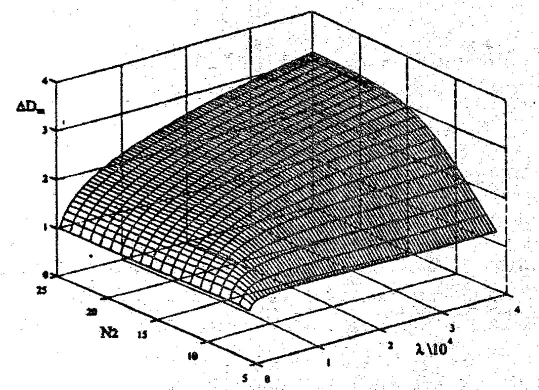

Fig. 4 - $\Delta\tau = f(N_2, \lambda)$

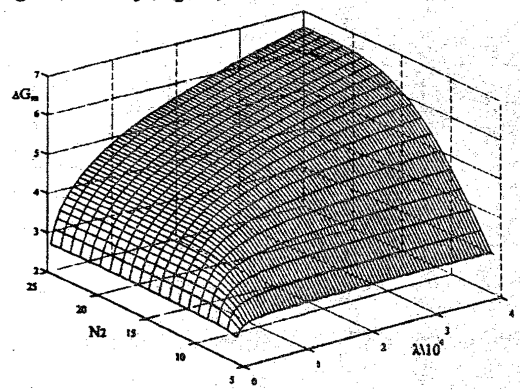

Fig. 5 - $\Delta G = f(N_2, \lambda)$

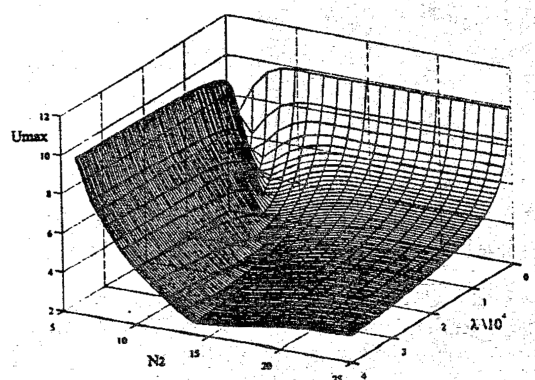

Fig. 6 - $u_{max} = f(N_2, \lambda)$

The diagrams of fig. 7 and 8 show the frequency responses of the velocity in the Bode and Black planes.

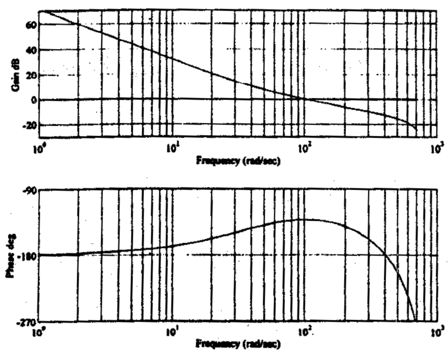

Fig. 7 - Bode diagram of the velocity loop

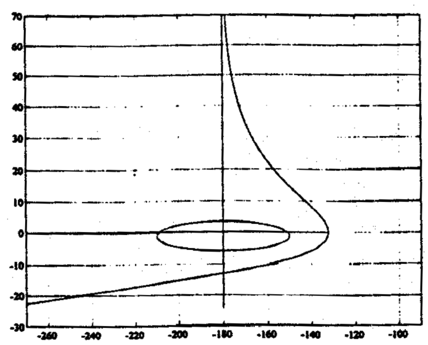

Fig. 8 - Black diagram of the velocity loop

It can be noticed here that the phase and gain margins provide good stability, furthermore ensuring to the step response small overshoot. The regulation bandwith is about 100 rd/s. The sensitivity function remains less than 6 dB, as shown Fig. 8.

4.2.2 Second experiment

To prove the interest of CRHPC, a dead-beat control has been tested in simulation then implemented on the brushless motor. It is important to note that with a classical G.P.C., a dead-beat control leads to an unstable closed loop. With CRHPC, the robustness is fair, but the control is stable. The conditions to design a dead-beat controller are :

$$N_u = m = d^\circ\left(A\left(q^{-1}\right)\right) + 1 \text{ and } \lambda > 0$$

With the same model and according to the previous assumptions, the RST polynomial controller is given below :

$$\left[N_1, N_2, N_u, m, \lambda\right] = [1,\ 3,\ 4,\ 4,\ 0.1]$$

$$\begin{aligned} R\left(q^{-1}\right) &= 0.27497 - 0.19718q^{-1} + 0.05317q^{-2} \\ &\quad -0.02490q^{-3} \\ S\left(q^{-1}\right) &= 1 + 0.92388q^{-1} \\ T(q) &= 0.10605q^{3} \end{aligned}$$

The diagrams of Fig. 9 and 10 show the frequency responses of the velocity in the Bode and Black planes.

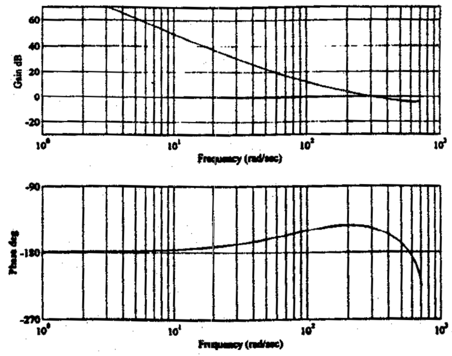

Fig. 9 - Bode diagram of the velocity loop

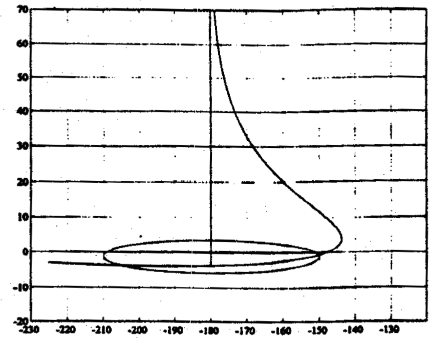

Fig. 10 - Black diagram of the velocity loop

It can be noticed here that the phase and gain margins provide stability, with a regulation bandwith greater than previously (about 300 rd/s instead of 100 rd/s). However, the maximum of the sensitivity function is greater than 6 dB.

With these two last sets of parameters, the polynomial predictive control law has been implemented on the brushless motor providing the experiments of figures 11 to 16.

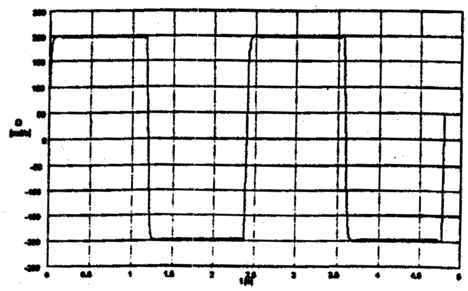

Fig. 11 - Velocity response - **First experiment**

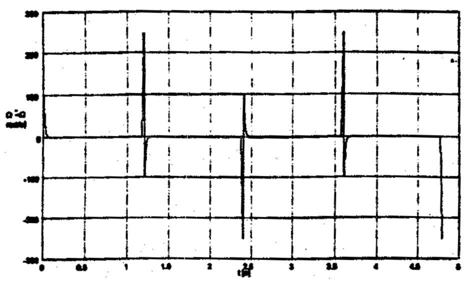

Fig. 12 - Velocity error - **First experiment**

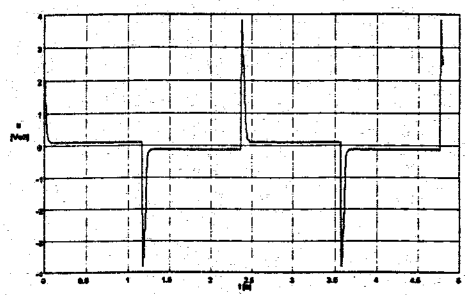

Fig. 13 - Control signal - **First experiment**

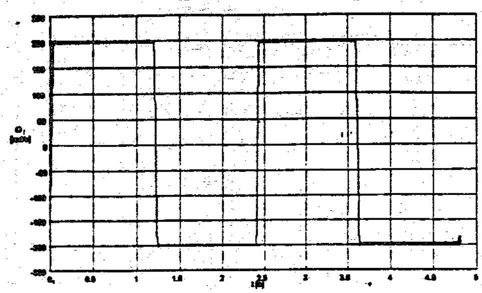

Fig. 14 - Velocity response - **Second experiment**

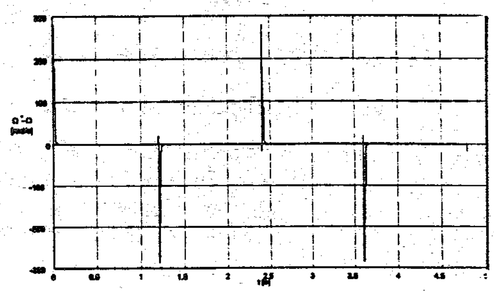

Fig. 15 - Velocity error - **Second experiment**

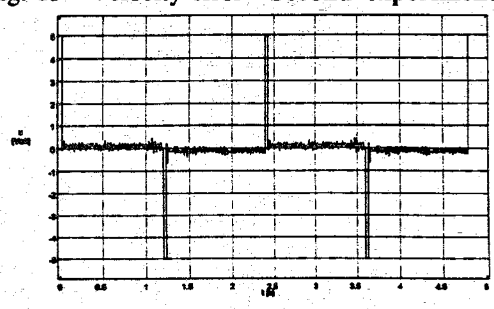

Fig. 16 - Control signal - **Second experiment**

These results obtained on industrial motors, with an identification of the system at the beginning of the program stress the importance of this method (Polynomial resolution and automatic design) and the performances of the resulting control law in term of disturbance and errors cancellations. The behaviour obtained with the *robust* controller (first case) is very interesting and proves that such an approach is well adapted to industrial problems, where performances are requested, but models not exactly known.
Even if the dead-beat controller (second case) is not realistic for an industrial application, the fact that it experimentally runs is remarkable, and shows that the consideration of terminal cons-traints increases the robustness of the predictive control law.

5. Conclusions

The control of a brushless motor drive has been examined using C.R.H.P.C.. For classical conditions of use, we mark out that all the frequency performance criteria (phase margin, gain margin, delay margin and modulus margin) are satisfied by the C.R.H.P.C. strategy, the temporal criteria are satisfied as well. Another advantage of this version, compared to basic G.P.C., is the possibility to take into account constraints which ensure a priori stability for a particular tuning. This technics provides a complete closed-loop control law, robust towards modelling errors. The resulting control algorithm is very simple with globally the same polynomial structure as classical discrete controllers, but polynomials obtained by minimization of a quadratic cost function and the creation of an anticipative effect in relation to the setpoint.
Moreover C.R.H.P.C. revealed good robustness qualities and the fact to pose endpoint constraints on the output seems to be a good strategy.
Predictive motion control improves in a significant way performances, in terms of rapidity, accuracy, cancellation of errors, mainly with severe specifications. The interest of industrials should be enlarged by an automatic design of the tuning parameters, transparent to the user, provided by the polynomial version and the stability study in frequency domains.

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