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On the design of stochastic RST controllers based on the Generalized Minimum Variance

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***Abstract*—This paper presents a general framework based on the RST structure for performing fair comparisons between deterministic and stochastic digital linear controllers. Due to the chosen RST formulation, it can be shown that any linear Single-Input Single-Output controller may be designed as a Generalized Minimum Variance Controller, *i.e.* as a stochastic controller. This in particular reduces the variance of the control signal and in turn leads to a better output characteristic. The method is applied to the controllers of an exemplary Single-Machine Infinite Bus system, namely the AVR and governor, where it is shown that the voltage and power regulation for the non-stochastic and the GMVC are similar whereas the control signal given by the latter is much smother than for the former, even using the same set of gains for both.**

I. Introduction

In spite of being well-established since the 1960s, the Stochastic Control Theory is in several cases neglected by practitioners throughout the controller design. In fact, Åström states that the usage of the Deterministic Control Theory has led to an important discussion about the gap between theory and practice in the control field during the very beginning of its study [1]. The Stochastic Control Theory was introduced to cover this gap, and the importance of designing controllers based on stochasticity has been brilliantly presented by several authors, see *e.g.* [1–4].

One of the simplest member of the Stochastic Control family is the Generalized Minimum Variance Controller (GMVC). It is based on the Minimum Variance (MV) regulator and was first introduced by Clarke and Gawthrop in 1975 [5]. Besides of being a SISO (*Single-Input Single-Output*) controller, GMVC's main difference to the most common Model-based Predictive Controllers (MPCs) is that the former is based, in its original form, on ARMAX (*Auto-Regressive Moving Average with eXternal inputs*) models and the latter only on ARX (*Auto-Regressive with eXternal inputs*) ones, *i.e.* the GMVC considers a full description of system's deterministic and stochastic parts. This important feature, inherited from the Kalman Filter synthesis [6], enables the controller to distinguish between noise and uncertainty, from deterministic cause and effect.

As observed by Bitmead et al. [7], practitioners were lured by the strong potential of MPC and its *tuning knobs* in terms of prediction horizons and many kept tuning their stochastic MPCs bounded to the aim of washing out high frequency dynamics and noise with a low-pass filtering idea instead of using the full potential of the stochastic prediction.

Despite being very practical and efficient from a robust-deterministic point of view, this concept of washing high frequency dynamics and noise out reduces the bandwidth of the control-loop and consequently the performance in terms of response velocity and low frequency disturbance recovery, similar to a *unorthodox* (non-stochastic) robust Linear Quadratic Gaussian (LQG) designed via loop-shaping [8].

Also important to remark is that, as the GMVC is a polynomial linear controller, it can be easily written in the so-called RST structure. In other words, the deterministic transfer function of a closed-loop plant controlled by the GMVC might match its counterpart controlled by any other linear controller, such as a PID, for instance. On the other hand, since the GMVC is a stochastic controller, the sensitivity function of both controlled systems might differ, leading the plant controlled by the GMVC to a more stable behavior with less influence of the measurement noise.

This interesting feature has caught our attention for a deeper investigation on a fair comparison of deterministic and stochastic controllers, resulting in the present work. Fair comparison means converting both controllers to the RST structure and hence setting the same gains into both in order to achieve a similar deterministic closed-loop behavior. Further, this paper also expands the GMVC equating from the ARMAX to a *complete* generalized polynomial form in order to gain accuracy in system's representation.

The plant selected for this study is the so-called *Single-Machine Infinite-Bus* (SMIB) system. In brief, this system is composed by a generator connected to the power grid through a transmission line, and it is very suitable for power systems stability analysis [9].

Generator's terminal voltage is controlled by an Automatic Voltage Regulator, short AVR, which is currently the most

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common used device for regulating the terminal voltage in power plants [9, 10]. It applies a voltage signal to the excitation system which is proportional to the error of generator's reference and measured terminal voltages, the latter being contaminated with measurement noise. Another important controller applied to generators is the so-called turbine governor, or just governor. It has the property of regulating either generator's speed or power.

AVRs have typically either a simple P or a PI form [10–12] whereas governors may have several different configurations. For the latter, in this paper we will use an Output Feedback controller for controlling generator's power. In summary, both controllers here applied have a RST structure so that they are suitable for the analysis to be carried out in this paper. It is important to highlight that the presented method can be extended to any SISO linear controller, as shown in the first two sections of the paper.

The paper is organized as follows: section II and III review respectively the RST structure and the GMVC; IV deals with the RST structure both for the PI AVR and the Output Feedback governor, besides introducing their conversion to the GMVC structure; section V shows controllers' evaluation through computer simulations; and section VI presents paper's conclusions and proposals for future works.

II. Review of the RST structure

In order to formulate our idea, this section reviews the RST structure. This formulation introduces a standard structure for linear digital controllers [13]. It counts on three arbitrary polynomial filters given by,

$$\begin{aligned} R(q^{-1}) &= r_0 + r_1 q^{-1} + \ldots + r_{n_a} q^{-n_r} \\ S(q^{-1}) &= s_0 + s_1 q^{-1} + \ldots + s_{n_b} q^{-n_s} \\ T(q^{-1}) &= t_0 + t_1 q^{-1} + \ldots + t_{n_c} q^{-n_t}, \end{aligned}$$

which are responsible for plant's output closed-loop behavior and are posed in the way shown in Fig. 1.

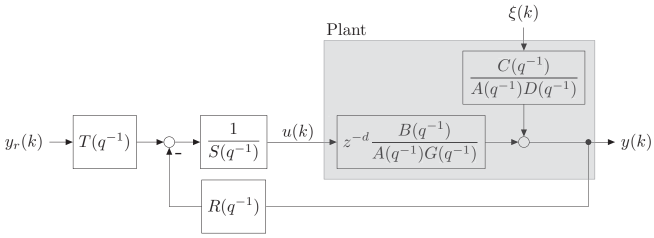

Fig. 1. RST structure for a plant represented by a generic SISO polynomial model.

In the figure, $y(k)$, $y_r(k)$ and $u(k)$ are sampled output, controller reference and control signal, respectively. The plant is represented by generic polynomial model with coefficients,

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \ldots + a_{n_a} q^{-n_a} \\ B(q^{-1}) &= b_0 + b_1 q^{-1} + \ldots + b_{n_b} q^{-n_b} \\ C(q^{-1}) &= 1 + c_1 q^{-1} + \ldots + c_{n_c} q^{-n_c} \\ D(q^{-1}) &= 1 + d_1 q^{-1} + \ldots + d_{n_d} q^{-n_d} \\ G(q^{-1}) &= 1 + g_1 q^{-1} + \ldots + g_{n_g} q^{-n_g}, \end{aligned}$$

being d the sample delay, q^{-1} is the backward shift operator and $\xi(k)$ is a Gaussian white noise sequence. Important to remark that for OE (*Output Error*) models $C(q^{-1}) = A(q^{-1})$ and $D(q^{-1}) = G(q^{-1}) = 1$, for ARX (*Auto-Regressive with eXogenous input*) models $C(q^{-1}) = D(q^{-1}) = G(q^{-1}) = 1$, for ARMAX (*Auto-Regressive Moving Average with eXogenous input*) models $D(q^{-1}) = G(q^{-1}) = 1$, and finally, for Box-Jenkins models $A(q^{-1}) = 1$.

According to the figure, the RST control law is given by,

$$u(k) = \frac{T(q^{-1})y_r(k) - R(q^{-1})y(k)}{S(q^{-1})}, \tag{1}$$

which results in the following closed-loop expression,

$$y(k) = \frac{q^{-d}BT}{AGS + q^{-d}BR} y_r(k) + \frac{CGS}{ADGS + q^{-d}BDR}\xi(k), \tag{2}$$

where the argument q^{-1} is omitted for the sake of readibility.

To be noticed is the two closed-loop transfer functions, namely the deterministic and the sensitivity ones. For the former, the sample delay d appears explicitly in the numerator, which means that the closed-loop system does not compensate it. One must keep it in mind when analyzing the GMVC, which is presented in the next section.

III. Review of the GMVC

This section reviews briefly the theory behind the GMVC. For a deeper understanding of it we recommend the reading of [5, 14, 15].

The GMVC was developed from the Minimum Variance Regulator (MVR) first introduced by Åström in the 1970s. Its generalized output is,

$$\phi(k) = P(q^{-1})y(k) - T(q^{-1})y_r(k+d) + Q(q^{-1})u(k), \tag{3}$$

with $P(q^{-1})$, $T(q^{-1})$ and $Q(q^{-1})$ being arbitrary weighting filters for system's output, reference and control signal respectively. The generalized output $\phi(k)$ is posed into a stochastic optimization problem of minimizing the GMV cost function,

$$\mathbf{J} = \mathbb{E}\left[\phi^2(k+d)\right], \tag{4}$$

while $\mathbb{E}[.]$ denotes the mathematical expectation operator.

In this work we expand GMV's common equating from ARMAX to generalized stochastic plant models of the form,

$$A(q^{-1})y(k) = q^{-d}\frac{B(q^{-1})}{G(q^{-1})}u(k) + \frac{C(q^{-1})}{D(q^{-1})}\xi(k). \tag{5}$$

Shifting Eq. 5 d-steps ahead and remembering that $P(q^{-1})$ multiplies the output $y(k)$ in Eq. 3, one obtains,

$$P(q^{-1})A(q^{-1})y(k+d) = \frac{P(q^{-1})B(q^{-1})}{G(q^{-1})}u(k) + \frac{P(q^{-1})C(q^{-1})}{D(q^{-1})}\xi(k+d). \quad (6)$$

Clearly $\xi(k+d)$ is unknown because it represents the future of the noise, and therefore it can be represented by present and future parts:

$$\frac{P(q^{-1})C(q^{-1})}{A(q^{-1})D(q^{-1})}\xi(k+d) = \frac{F(q^{-1})}{A(q^{-1})D(q^{-1})}\xi(k) + E(q^{-1})\xi(k+d), \quad (7)$$

whereas its Diophantine equation, *i.e.* a polynomial equality which arises, is given by,

$$P(q^{-1})C(q^{-1}) = A(q^{-1})D(q^{-1})E(q^{-1}) + q^{-d}F(q^{-1}). \quad (8)$$

being $E(q^{-1}) = 1 + e_1q^{-1} + \cdots + e_{n_{d-1}}q^{-d+1}$ and $F(q^{-1}) = f_0 + f_1q^{-1} + \cdots + f_{n_f-1}q^{-n_f+1}$, with $n_f = n_p + n_c$.

Using only the known data, the predicted output $\hat{y}(k+d|k)$ is,

$$P(q^{-1})\hat{y}(k+d|k) = \frac{P(q^{-1})B(q^{-1})}{A(q^{-1})G(q^{-1})}u(k) + \frac{F(q^{-1})}{A(q^{-1})D(q^{-1})}\xi(k). \quad (9)$$

Thus, the current stochastic signal $\xi(k)$, obtained from the estimation error is,

$$\xi(k) = \frac{P(q^{-1})}{E(q^{-1})}\left[y(k) - \hat{y}(k|k)\right]. \quad (10)$$

Substituting Eq. 10 into 9 and after some algebraic manipulations, the d-steps ahead Minimum Variance Predictor (MVP) turns to,

$$\hat{y}(k+d|k) = \frac{B(q^{-1})D(q^{-1})E(q^{-1})}{P(q^{-1})C(q^{-1})G(q^{-1})}u(k) + \frac{F(q^{-1})G(q^{-1})}{P(q^{-1})C(q^{-1})G(q^{-1})}y(k). \quad (11)$$

The GMVC control law is thus obtained through the minimization of $\mathbf{J}$ w.r.t $u(k)$ with $\mathbb{E}\left[\phi^2(k+d)\right] = \hat{\phi}^2(k+d)$, resulting in,

$$u(k) = \frac{TCGy_r(k+d) - FGy(k)}{BDE + CGQ}, \quad (12)$$

with the argument q^{-1} omitted again for the sake of readibility.

At last, the closed-loop polynomial is obtained through the substitution of Eq. 12 into 6. Using also Eq. 8, after some algebraic manipulations, one finds:

$$y(k) = \frac{BT}{AGQ + BP}y_r(k) + \frac{CGQ + BDE}{ADGQ + BDP}\xi(k). \quad (13)$$

Comparing equations 2 and 13 one may clearly see that, for the GMVC, the sample delay d is entirely compensated if the reference is known d-steps in advance. Despite this fact, both deterministic transfer functions are equivalent if $Q(q^{-1}) = S(q^{-1})$ and $P(q^{-1}) = R(q^{-1})$. On the other hand, the sensitivity functions differ since for the GMVC the term corresponding to the solution of the polynomial $E(q^{-1})$, which is gained from the solution of the Diophantine equation (Eq. 8), appears explicitly on it and, according to Eq. 7, is directly responsible for the stochasticity effect of the controller.

Next section presents an application for the just shown theory using an AVR and a turbine governor attached to a SMIB system as example.

IV. GMVC SETUPS AIMING POWER SYSTEMS CONTROL

As stated in the Introduction, this paper regards the AVR and the governor as a PI and an Output Feedback controllers, respectively.

This section demonstrates how to setup the GMVC in order to obtain similar deterministic behaviors as for the cited ones.

It is important to remark that this is an exemplary section aimed to power systems simulations, however the procedure here utilized may be derived for any linear SISO control structure.

A. GMVC-based PI controller

The PI controller might be represented by in the Laplace complex domain as,

$$\frac{U(s)}{E(s)} = k_p\left(1 + \frac{1}{T_is}\right), \quad (14)$$

with $U(s)$, $E(s) = Y_r(s) - Y(s)$, k_p and T_i being controller's output, error signal, proportional gain and integration time respectively. $Y_r(s)$ and $Y(s)$ are the reference and measured output.

Defining,

$$s := \frac{1}{t_s}(1 - q^{-1}) \qquad \text{(implicit Euler method)},$$

for converting Eq. 14 to the digital domain and being t_s the sample time, the RST structure is obtained after some algebraic manipulations, which results in,

$$\underbrace{\frac{1}{k_p}}_{s_0}u(k) + \underbrace{\left(-\frac{1}{k_p}\right)}_{s_1}u(k-1) = \underbrace{\left(1 + \frac{t_s}{T_i}\right)}_{r_0}e(k) + \underbrace{(-1)}_{r_1}e(k-1), \quad (15)$$

being,

$$R(q^{-1}) = r_0 + r_1q^{-1}$$
$$S(q^{-1}) = s_0 + s_1q^{-1}$$
$$T(q^{-1}) = R(q^{-1}).$$

Now, from Eq. 13 of the previous GMVC section one might notice that,

$$Q(q^{-1}) = R(q^{-1})$$
$$P(q^{-1}) = S(q^{-1}),$$

which basically means that, despite the sample delay compensation, both PI- and GMV-based AVR controllers will have similar closed-loop behaviors.

B. GMVC-based Output Feedback controller

The Output Feedback controller, also called 2-DOF PID [16], when considered with its integrating effect, has the following structure in the Laplace complex domain:

$$U(s) = \frac{1}{sT_i} Y_r(s) - \left(\frac{1}{sT_i} + k_p + sT_d \right) Y(s), \tag{16}$$

where T_d represents the derivative time.

Utilizing again the implicit Euler method, its RST structure results in,

$$\begin{aligned} \underbrace{t_s T_i}_{s_0} u(k) + \underbrace{(-t_s T_i)}_{s_1} u(k-1) = \underbrace{t_s^2}_{t_0} y_r(k) \\ - \underbrace{\left[(T_d + k_p t_s) T_i + t_s^2 \right]}_{r_0} y(k) - \underbrace{(-2T_d - k_p t_s) T_i}_{r_1} y(k-1) \\ - \underbrace{T_d T_i}_{r_2} y(k-2) \end{aligned} \tag{17}$$

being,

$$R(q^{-1}) = r_0 + r_1 q^{-1} + r_2 q^{-2}$$
$$S(q^{-1}) = s_0 + s_1 q^{-1}$$
$$T(q^{-1}) = t_0.$$

Finally, the Output Feedback controller will have roughly the same behavior as the GMVC if,

$$Q(q^{-1}) = R(q^{-1})$$
$$P(q^{-1}) = S(q^{-1}).$$

V. Simulation Results

This section presents the evaluation of the just presented PI and Output Feedback controllers based on the GMV theory applied to a SMIB system using the stochastic RST framework developed in the former sections. The simulation is carried out using the software MATLAB® along with its toolbox SimPowerSystems™, whereas the simulation data is obtained from [10] and shown in the Appendix. Figure 2 shows the system with its controllers, where P_r, P_e, u_P and ξ_P are power's reference, output, control signal and noise signals, whilst v_r, v_t, u_v and ξ_v are its voltage counterparts. Besides, $\delta\omega$ is the speed deviation.

Moreover, since GMVC is a model-based controller, the system is modeled through an identified stochastic model with the form presented in Eq. 5 using $d = 1$ sample.

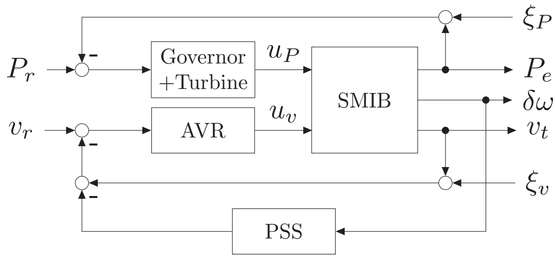

Fig. 2. Single-Machine Infinite Bus with its controllers.

A. AVR-based GMVC

1) Simulation Setup: For stability improvement, a Power System Stabilizer (PSS) is attached to the input of the AVR, according to the diagram presented in Fig. 2. In the present analysis, the exciter dynamics is neglected and a constant mechanical torque τ_m is considered, *i.e.* no governor is regarded. Further, a saturation block is applied to the output of both AVRs in order to limit its output level. The white noise signal ξ_v with a relatively small variance of $\sigma^2_{\xi_v} = 1e^{-5}$ and sample time of 0.02 s is added to the feedback loop of the AVR in order to highlight the difference of both controllers presented in this section whenever the measurement noise is considered.

Two simulation procedures are carried out in order to evaluate both AVRs presented in the former section:

(a) 5% step in reference voltage v_r at t = 0.1 s, and
(b) short circuit on grid's voltage at t = 0.1 s with duration of 120 ms.

For both cases generator's terminal voltage v_t and speed deviation $\delta\omega$, besides AVRs' control signal u_v, are evaluated.

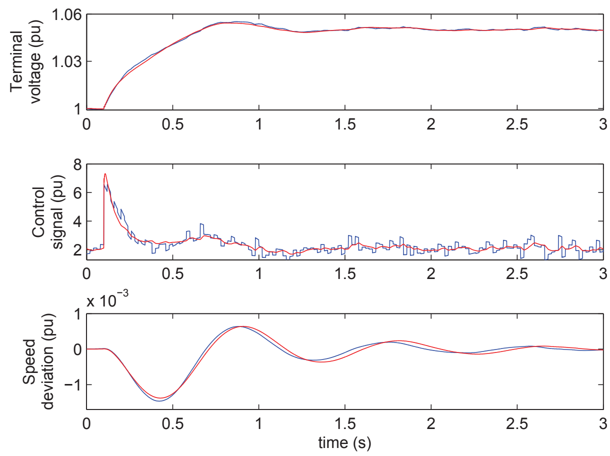

Fig. 3. Step of 5% in generator's reference voltage v_r with measurement noise for $k_p = 100$ and $T_i = 50$ s. Legend: (blue) PI-based AVR; (red) GMVC-based AVR.

2) Comparison of the Step Response: Figure 3 shows the case of a 5 % step in the reference voltage of the generator.

From the upper graph it may be seen that both controllers approach the new set point of the terminal voltage in a similar manner. Both controllers, the PI and the GMVC, lead to a stabilizing control. However, on a closer look it is apparent that in the PI case the terminal voltage has a slightly higher level of oscillation as its GMVC counterpart. Nevertheless, the settling time of both controllers is similar.

More importantly to notice are the deviations in the control signal. The PI controller results in a highly dynamic signal, with fast changing characteristics in the time scale as well in level of the control signal. In contrast the control signal of the GMVC shows a smoothed characteristic. This behavior is in favor due to its superior influence on the plant to be controlled and the little stress on the exciter. Indeed, the superior characteristics of the GMVC are a result of the underlying stochastics of the GMVC, which may also be seen in the variance of the control signal:

(a) PI Control: $\sigma^2_{u_{\text{PI}}} = 0.5914$,
(b) GMV Control: $\sigma^2_{u_{\text{GMV}}} = 0.4606$.

Finally, the third graph of Figure 3 shows the resulting speed deviations of the applied step in the reference signal. It is interesting to note, that the GMVC demonstrates a lower oscillation frequency compared to the PI controller. Even though this comes at the cost of a slightly higher amplitude in the second interval, this behavior accounts for the superiority of the GMVC in the investigated setup.

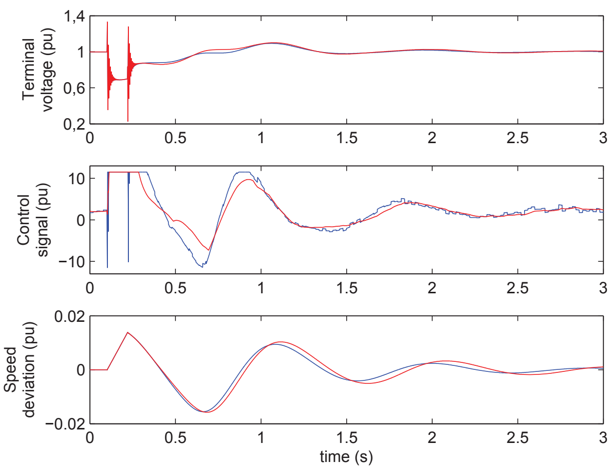

Fig. 4. Short-circuit at grid's voltage with measurement noise for $k_p = 100$ and $T_i = 50$ s. Legend: (blue) PI-based AVR; (red) GMVC-based AVR.

3) Comparison of a Short Circuit: In the second investigated case, the short circuit scenario, shows a similar behavior than before. Nevertheless, this case is interesting due to its importance in grid stability. The upper graph of Figure 4 shows the regulated output of the investigated system. Again the terminal voltage seems to be quite similar for both the PI as well as the GMV controlled system. However, particular differences occur in the control signal as well as the speed deviation. The latter one again shows a higher oscillation when controlling the system using a PI controller.

The control signal, however, shows interesting behavior. As before, the signal of the GMVC shows a smooth characteristics. The signal of the PI controller answers with spikes on the beginning and end of the short circuit. Contrary, the GMVC copes with the occurrence of the short circuit by keeping the control signal constant and hence cause only little stress on the controlled system as well as the exciter. The ongoing control signals then show similar behavior as in the case of a step response. The resulting variance again back-ups the superiority of the GMV control:

(a) PI Control: $\sigma^2_{u_{\text{PI}}} = 23.6953$,
(b) GMV Control: $\sigma^2_{u_{\text{GMV}}} = 16.0857$.

B. Governor-based GMVC

1) Simulation Setup: For this simulation, the PSS is disconnected from the loop in order to check also the damping introduced by the derivative term of the Output Feedback controller. Further, the AVR is regarded as an ordinary PI controller (non-stochastic) with gains $k_{p_{\text{AVR}}} = 20$ and $T_{i_{\text{AVR}}} = 0.4$ s. Changes in the voltage reference v_r correspond to a state disturbance on the power output P_e, so that the governor-based GMVC will be also experimented for the case of disturbance rejection. The white noise signal ξ_P with a small variance of $\sigma^2_{\xi_v} = 1e^{-8}$ and sample time of 0.02 s is added to the feedback loop of the governor.

A 5% step in the power reference P_e is applied at $t = 0.1$ s and a voltage reference step (state disturbance) is inserted at $t = 10$ s. Generator's output power P_e and its correspondent control signal u_P are evaluated, as well as the terminal voltage.

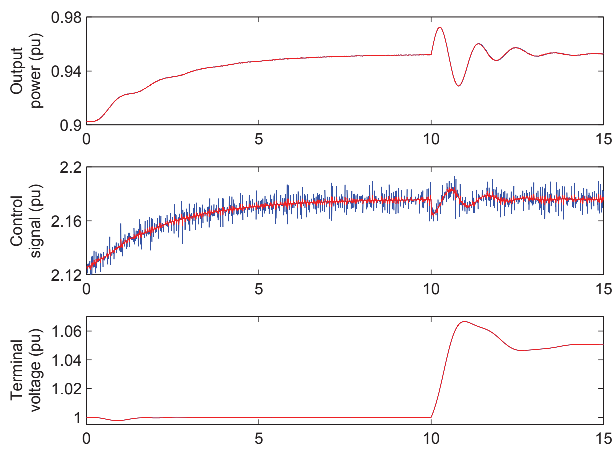

Fig. 5. Step of 5% in generator's reference power P_r with measurement noise for $k_p = 0.1$, $T_i = 2$ s and $T_d = 0.08$ s. Legend: (blue) Output Feedback-based governor; (red) GMVC-based governor.

2) Comparison of the Step Response: The investigated governor control example shows interesting results and strongly supports the validity of the proposed framework to compare deterministic and stochastic controllers. The controlled variable output power shows the same behavior for both investigated controllers. Even the applied 5% step in the power reference causes the same behavior. Given the graphs, misleading conclusions could be drawn. Control practitioners could consider to devote the quality of both control strategies as equal. However, a further view on the control signal shows considerable differences. As it may be seen in Fig. 5, the GMVC-based governor control causes less stress on the control valve. This may again be seen in the control variance, which is denoted as:

(a) Output Feedback Control: $\sigma^2_{u_{\text{OF}}} = 1.8354e^{-4}$,
(b) GMV Control: $\sigma^2_{u_{\text{GMV}}} = 1.503e^{-4}$.

This shows the superior behavior of the GMVC based controller. Given that the proposed framework for comparison is used, i. e. RST-structure controllers using the same deterministic gains, the fair comparison of different controllers is possible.

VI. Conclusion

In the present paper, we have shown a framework to compare different SISO linear controllers on a unified base. By the use of the so-called RST-Structure it is possible to match the deterministic parts of the closed loop plants of different controller types. This is achieved by applying the same gain to both systems. Hence, the fair comparison of different controller types becomes possible in an easy to use setting.

The comparison framework may also be used to highlight improvement potential of controller types using stochastic in their design process. It is thereby shown that a Generalized Minimum Variance Controller has superior effects on the control when compared to standard PI and Output Feedback controllers with equivalent gains. The framework is applied in the task of an automatic voltage and power regulation setting. It could be shown that the RST formulation allows to design any linear Single-Input Single-Output controller as a Generalized Minimum Variance Controller, *i.e.* as a stochastic controller.

The GMVC based controller design improved the control of the system compared to the deterministic PI and Output Feedback ones. The application included two use cases: for the voltage regulation, a step in the voltage reference and the occurrence of a short circuit; and for the power regulations, a step in power reference and a state disturbance caused by a step in voltage reference. Especially, the control output $u(k)$ of the GMVC shows, for both cases, better characteristics/properties, due to the significantly decreased variance, *i.e.* the GMVC uses less energy causing less stress mainly on mechanical actuators such as the control valve of the governor.

The proposed method, even though being simple, is helpful regarding the comparison of different controller types. Hence, it has the potential to improve current practices in control theory and might be used as a guide by students as well as experienced practitioners.

Future works will deal with the design of a stochastic PSS and the expansion to stochastic state feedback regulators.

Appendix
Simulation Data

Generator (round rotor, 60 Hz):

$X_d = 1.81$, $X_q = 1.76$, $X_l = 0.16$, $X'_d = 0.3$,
$X'_q = 0.55$, $X''_d = 0.25$, $X''_q = 0.25$, $R_a = 0.003$,
$T'_{d0} = 8$ s, $T'_{q0} = 0.4$ s, $T''_{d0} = 0.03$ s, $T''_{q0} = 0.05$ s,
$H = 3.5$ s, $V_{\text{nom}} = 20$ kV, $P_{\text{nom}} = 0.9$, $Q_{\text{nom}} = 0.3$.

Transformers:

HV/LV $= 230/20$ kV, $X_T = 0.15$.

Transmission line (overhead):

$V_{\text{nom}} = 230$ kV, $I_{\text{nom}} = 1$ kA, $X_l = 0.5$ Ω/km.

Turbine (steam type):

$T_{\text{ch}} = 0.3$ s, $T_{\text{rh}} = 7$ s, $T_{\text{co}} = 0.3$ s, $T_{\text{valve}} = 0.2$ s,
$F_{\text{ch}} = 0.3$, $F_{\text{rh}} = 0.3$, $F_{\text{co}} = 0.4$.

PSS:

$K_{\text{pss}} = 9.5$, $T_w = 1.4$ s, $T_1 = 0.154$ s, $T_2 = 0.033$ s.

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