

IMPLEMENTATION OF R.S.T. CONTROLLERS FOR A FLEXIBLE SERVO CONSIDERING PRACTICAL LIMITATIONS

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Abstract : In performant positioning systems, complex control structures are necessary to properly reject disturbances while correctly following reference laws. However, the higher the dynamics, the more important flexion problems are. In the paper, we focus on the control of a system with elasticity between the actuator and the load. Flexible servo tuning methods have been widely presented in the litterature. This paper only considers the case when limitations occur on the control variable. Approach is based on poles and zeros placement design. It compares the behaviour of different structures of R.S.T. controllers in a view of *practical implementation* (theoretical R, S, T structure, S/R controller and (T-S)/R feedforward term, controller with the addition of an integration antiwindup loop, controller with fully controlled state in the limitation mode).

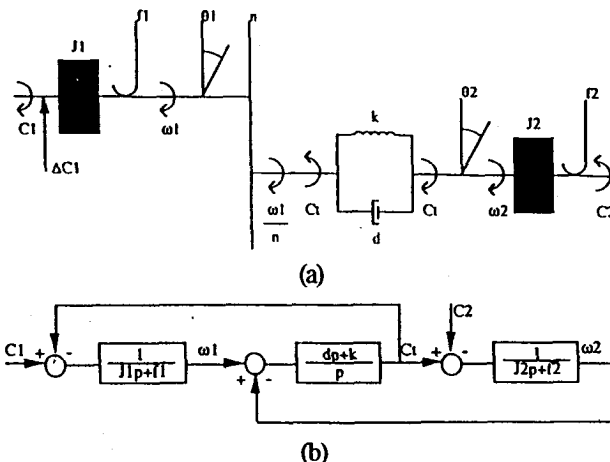
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0. Introduction

The control of flexible servo by state variable feedback controller combined with an explicit observer or, more generally, by a RST controller, has been widely discussed in the litterature, but in the linear case. The paper focuses on the problems caused by the limitations of the control variable and the constraints on the torsion. It proposes RST implementations avoiding them.

1. Model of a flexible transmission

Motor and load are separated by flexible transmission, characterised by a torsion coefficient k and a damping coefficient d . The model of a flexible transmission is shown on figure 1.



Parameters

Motor

C_1 =motor torque

ω_1 and θ_1 =speeds and positions

J_1 and f_1 =inertias and viscous friction coefficients

Load

C_2 =load torque

ω_2 and θ_2 =speeds and positions

J_2 and f_2 =inertias and viscous friction coefficients

Transmission

C_1 =torsion torque

n =reduction rate

k =torsion coefficient

d =damping coefficient

Figure 1 : Model of the flexible transmission
(a) Model (b) Block diagram

2. Identification of the parameters

The system is composed of a continuous current motor with permanent magnets[motor], associated with a magnet synchronous motor autopiloted by analogical current loops[load], and a flexible transmission. The dynamics of current loops are well known and faster than mechanicals phenomenons. Inertias and viscous friction coefficients are identified separately because parametrical identification of global system is not necessary. To identify elastic parameters, we can exploit the ω_2/ω_1 transfer function, considering no load torque (figure 2).

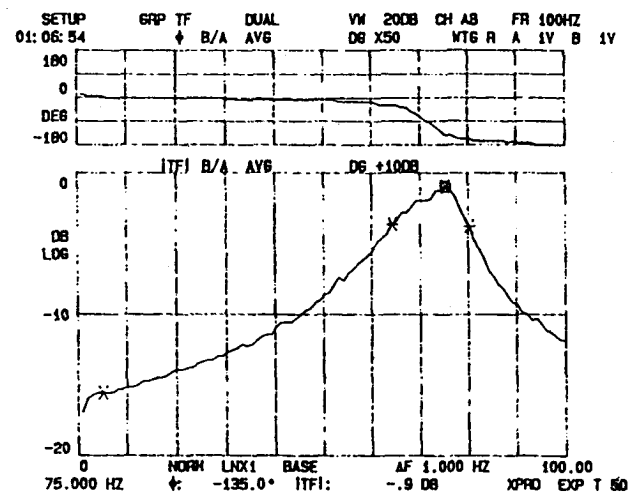


Figure 2 : ω_2/ω_1 transfer function(no load torque)

* The parameters are :

$$J_1 = 0,82 \cdot 10^{-3} \text{ kgm}^2 \quad J_2 = 0,31 \cdot 10^{-3} \text{ kgm}^2$$

$$f_1 = 0,16 \cdot 10^{-3} \text{ Nm/rad s}^{-1} \quad f_2 = 0,15 \cdot 10^{-3} \text{ Nm/rad s}^{-1}$$

$$k = 68,8 \text{ Nm/rad} \quad d = 29 \cdot 10^{-3} \text{ Nm/rad s}^{-1}$$

* The roots of continuous transfer function are :

- Poles

$$p_1 = -0,27$$

$$p_{2,3} = -65 \pm 550 j \quad \omega_r = 550 \text{ rad s}^{-1}$$

(frequency)

$m_r = 0,11$ (damping ratio)

Zeros

$$G_{11} : z_{1,2} = -47 \pm 470j \quad \omega_{a1} = 470 \text{ rad s}^{-1} \\ m_{a1} = 0,1$$

$$G_{12} = G_{21} : z = -2400$$

$$G_{22} : z_{1,2} = -18 \pm 290j \quad \omega_{a2} = 290 \text{ rad s}^{-1} \\ m_{a2} = 0,06$$

3. Transfer functions Bode diagrams

Continuous transfer function are :

$$\omega_1 = \frac{J_2 p^2 + (f_2 + d)p + k}{A(p)} C_1 - \frac{dp + k}{A(p)} C_2 \quad \text{and}$$

$$\omega_2 = \frac{dp + k}{A(p)} C_1 - \frac{J_1 p^2 + (f_1 + d)p + k}{A(p)} C_2$$

$$\theta_1 - \theta_2 = \frac{J_2 p + f_2}{A(p)} C_1 + \frac{J_1 p + f_1}{A(p)} C_2 \quad \text{with}$$

$$A(p) = J_1 J_2 p^3 + [(J_1 f_2 + J_2 f_1) + d(J_1 + J_2)] p^2 \\ + [k(f_1 + f_2) + d(f_1 + f_2) + f_1 f_2] p + k(f_1 + f_2)$$

$$\text{so} \quad \omega_1 = G_{11} C_1 - G_{12} C_2 \\ \omega_2 = G_{21} C_1 - G_{22} C_2 \quad G_{21} = G_{12}$$

On G_{11} et G_{12} , we note anti-resonance frequency :

$$\omega_{a1} = \sqrt{\frac{k}{J_2} - \frac{1}{2} \left(\frac{f_2 + d}{J_2} \right)^2} \quad \text{et} \quad \omega_{a2} = \sqrt{\frac{k}{J_1} - \frac{1}{2} \left(\frac{f_1 + d}{J_1} \right)^2}$$

and resonance frequency ω_r when transmission is not rigid. The speed/torque transfer functions Bode diagrams are shown on figure 3.

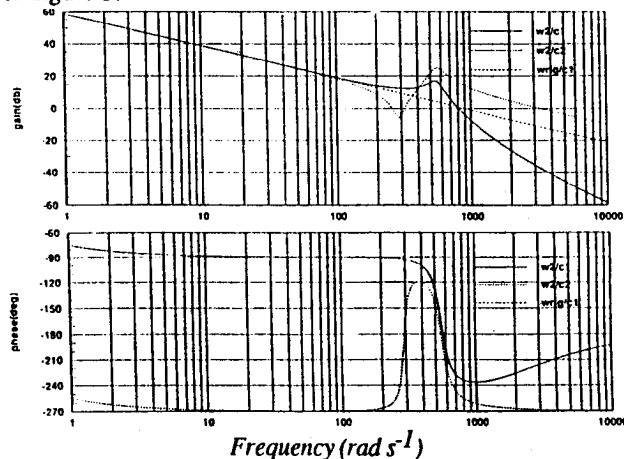


Figure 3 : Speed/torque transfer functions $\omega_2/C_{1,2}$

It appears that flexibility causes a resonance that cannot be ignored because it is often located in the frequency bandwidth of the servo.

4. Structural solution for speed control

To choose and tune the controller, the global model of the shaft is considered: therefore the model and the controller are of third order and all the modes of the system are controlled.

4.1 Principle

The problem is formulated as zeros and poles placement. RST controller is used to get the desired closed loop tracking and regulation behaviours. The conditions on RST polynomials and the way to get them will not be discussed in the paper : R and S polynomials are chosen to minimize the load torque disturbance effects. It must be noted that, in addition to the integrator introduced to eliminate steady state errors, some controller state variables (eg R polynomials roots) may be unstable. General structure of control is shown on figure 4.

Remark : Continuous approach has been considered for theoretical developments but results are easily extrapolated in discrete domain.

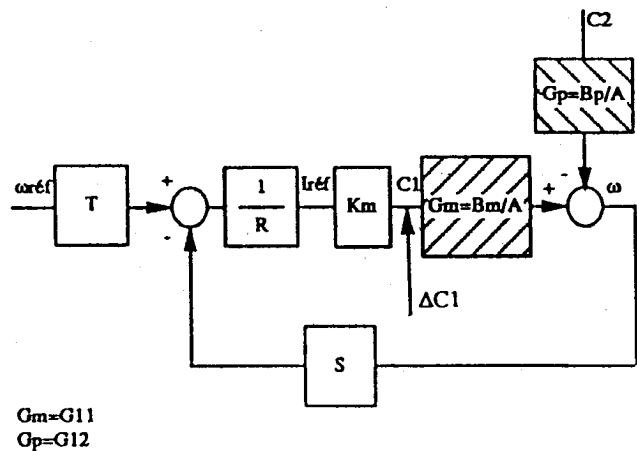
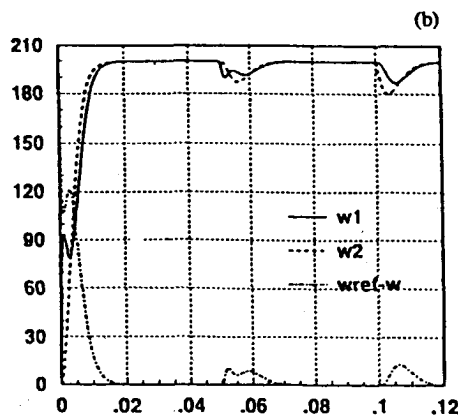
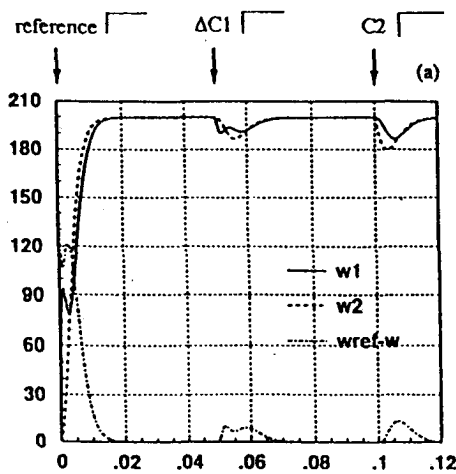


Figure 4 : General structure with RST controller

ω_{ref} : reference speed
 I_{ref} : reference current
 k_m : motor torque coefficient
 C_1 : motor torque
 ΔC_1 : motor torque disturbance
 C_2 : load torque (disturbance)

Results when control limitation is not considered are presented on figure 5a(continuous case) and 5b(discrete case) :



Primary and secondary speeds(rad s^{-1}) versus time(s)

Figure 5 : Speed response in the linear case
(a) continuous case (b) discrete case

When the output of the controller is not limited, ω_2 is correctly controlled. Equivalent results are obtained in the continuous and discrete cases. In both cases, the six closed loop poles are chosen equal and real [natural frequency $\omega_n = 628 \text{ rad s}^{-1}$, damping coefficient $m = 1$]; the T polynomial cancels three of them to reduce the order of the $\omega_1/\omega_{\text{ref}}$ transfer function and to get a good response to step like reference. R has one unstable root in addition to the integrator.

4.2 RST controller implementation

4.2.1 Scheme (figure 6)

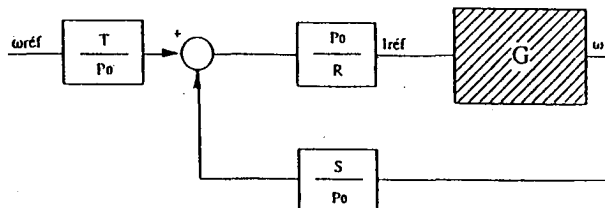


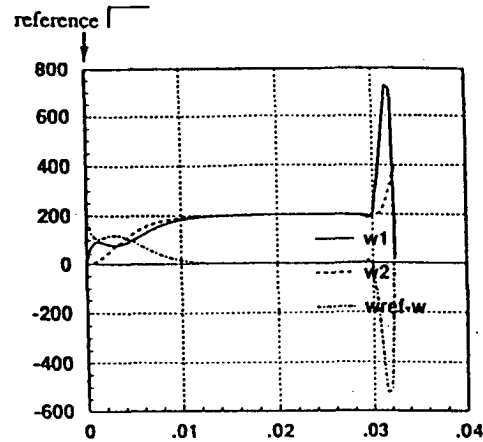
Figure 6 : T,S,R structure

With $P_o = z^3$ (discrete case)
 $P_o = (p + a)^3$ (continuous case) $a > 0$

P_o polynomial has the same order as R,S,T polynomials. The corresponding modes of P_o roots are not observable, so they have to be stable and fast.

Nevertheless, such structure presents inconvenient when T transfer error occurred: it will be reflected on controlled variable.

Another possible implementation would consist of a S/R loop controller and (T-S)/R feedforward term ; in this case, if R has unstable roots in addition to the integration, the feedforward term continuously diverges and in the long run the whole system "blows out" as shown on figure 7.



Primary and secondary speeds(rad s^{-1}) versus time(s)

Figure 7 : RST implementation with $\frac{S}{R}$ loop controller and $\frac{T-S}{R}$ feedforward term

4.2.2 Problem when control limitation is considered

Whenever limitations of the control variable (motor torque) occur, the controller is in open loop and all its unstable state variables diverge. So, T,S,R structure must be modified to have satisfactory responses, in linear and saturation cases.

4.3 Operational structures

4.3.1 Integration antiwindup Scheme is shown on figure 8 :

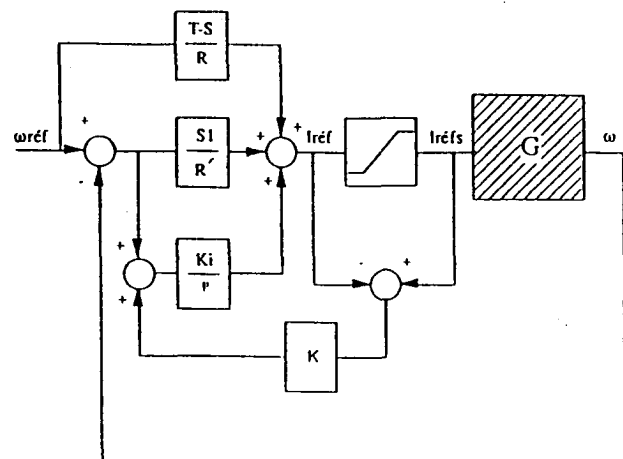


Figure 8 : Stabilization of integral term

The classical antiwindup solution only deals with problems caused by the integral term, assuming the other state variables are stable. Figure 9 shows that, in this case, ω_2 is correctly controlled : the time constant of the first order antiwindup loop is chosen equal to 2 ms.

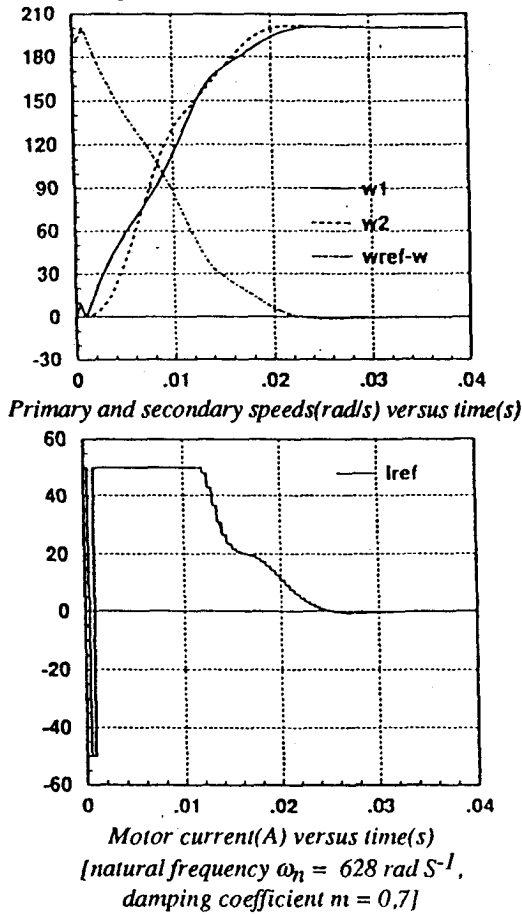


Figure 9 : Integration antiwindup (control limitations)

The performances are satisfactory because the other R state variables are stable. However, we can observe some rebounds on the current reference Iref. If R has unstable roots, we can note that precedent structure is not available (figure 10) :

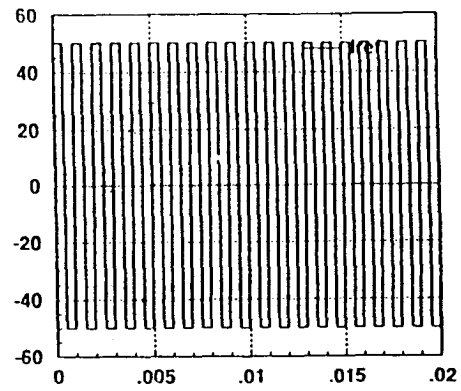
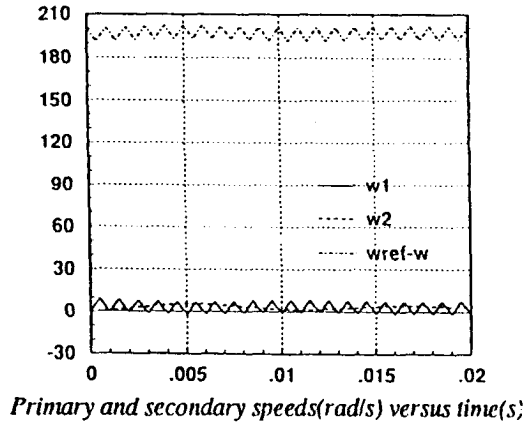


Figure 10 : Integration antiwindup (one root of R is unstable)

4.2.2 Global antiwindup

It is better maintaining all the controller states variables in a global third order stabilization loop (figure 11).

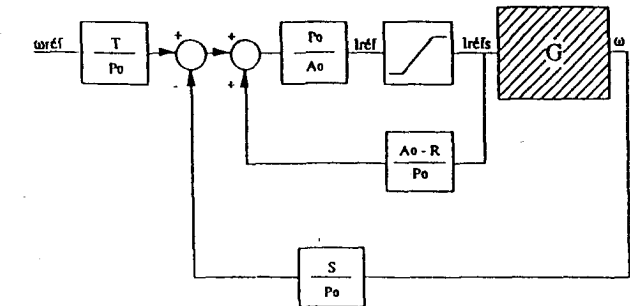
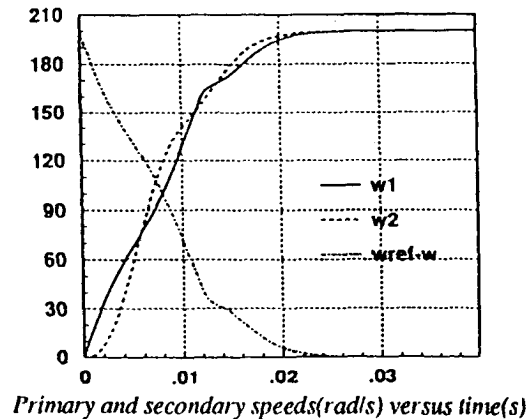


Figure 11 : Theoretical structure of an RST including a global stabilization loop

Its characteristics equation is A_0 . An equivalence can still be found with the state feedback controller combined with an explicit observer for some specific choice of A_0 . Figure 12 confirms the validity of such a solution : the conditions are those of figure 5 (one unstable root in addition to the integration); we can note that both ω_1 and ω_2 variables have very smooth dynamic evolutions. The control maximum value is chosen in order to maintain the torsion angle inside acceptable limits. With a classical RST controller or with an integration antiwindup, the loop would be totally unstable in this case.



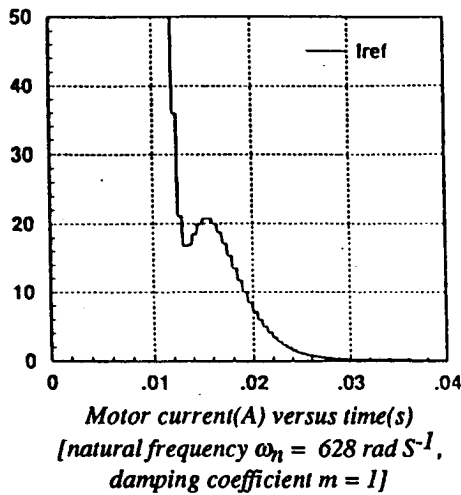


Figure 12 : Global antiwindup (control limitations)

We can note on figure 13 that torsion shaft is satisfactory :

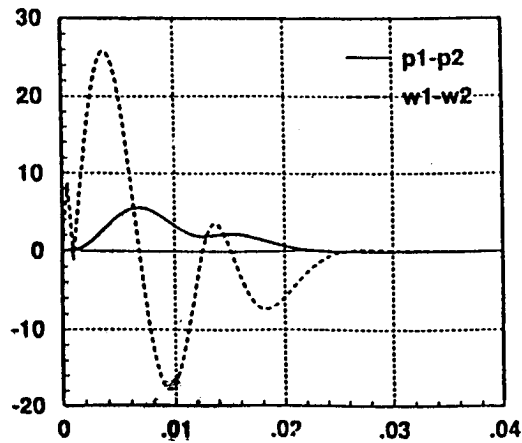


Figure 13 : Torsion shaft(p1-p2)

5 Conclusions

Elasticity on transmission can't be ignored to have satisfactory responses on ω_2 secondary speed. Therefore, to get the desired closed loop tracking and regulations behaviours, considering control limitation, it is necessary to implement a global anti-windup structure, where all modes are controlled.

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