Supervisory Control Based on Closed-Loop Adaptive Control Approach of Nonlinear Continuous Stirred Tank Reactor Process (NCSTRP)

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Abstract- **In the last few years there was a great deal of interest in the application of adaptive and supervisory control of linear systems using multi-models and multi-estimators based on switching and tuning. The aim of this paper is to apply this methodology to highly nonlinear continuous stirred tank reactor process (NCSTRP), which is very useful in chemical and biochemical engineering industries, and in the other hand to tackle some drawbacks of closed-loop adaptive control. Closed-loop adaptive control algorithm has been used as a technique for identification of models set and in the same time to calculate the corresponding set of RST controllers. The simulation results with different scenarios show the effectiveness of this switching control approach in terms of stability and performance.**

Keywords: *Closed-loop adaptive control; supervisory control; switching criterion; hysteresis factor; nonlinear continuous stirred tank reactor process (NCSTRP).*

I. INTRODUCTION

In the field of process control system, a challenge is to get good performances such as response time, accuracy and stability despite nonlinearities of the plant. Another challenge is to design controllers that can be easily implemented and tuned for a specific process. Adaptive control is appealing as the controller tunes itself to the specific plant by means of a combination of identification and control. The closed-loop identification adaptive control proposed in [1, 2] is particularly interesting as it overcomes the difficulties linked to open-loop identification and it may be applied to a wide range of processes.

The exothermic continuous stirred tank chemical reactor (NCSTR) described by several authors such as [3-6], is a system that is typical of chemical processes for which this adaptive control would be interesting. However it's highly nonlinear behaviour (for example non affinity, where the input appeared as exponential in the temperature equation) leads to the limits of the adaptive controller when large set-point variations are considered, as the dynamic characteristics of the plant change too quickly with respect to the adaptation time.

An alternative approach to cope with this sudden change is proposed which is, supervisory control or switching controllers [7-8]. In this approach at each instant a controller is selected from a bank of controllers according to a switching scheme. One criterion to select the controller is based on a bank of models of the plant and the determination of the best fitted model in order to apply the corresponding controller.

The aim of this paper is to study the combination of adaptive and switching control in order to improve the transient response of the NCSTRP. This idea to combine switching and tuning has also been considered in [7, 9, 10] for example, but herein adaptive controller is not used as a specific controller of the bank but as a mean to automatically design and tune the model and the controller adapted around a specific set-point that will be included in the bank of the switching scheme.

The rest of this paper is organized as follows: Section 2 formulises the mains ideas of the closed-loop adaptive controller as in [1-2]. Then in section 3 the supervisory control based on this closed-loop adaptive algorithm is presented with the switching scheme. The model of the NCSTR plant is given in section 4 and simulation results of this control algorithm applied to NCSTRP with their discussion are well discussed in section 5.

II. CLOSED-LOOP ADAPTIVE CONTROL ALGORITHM

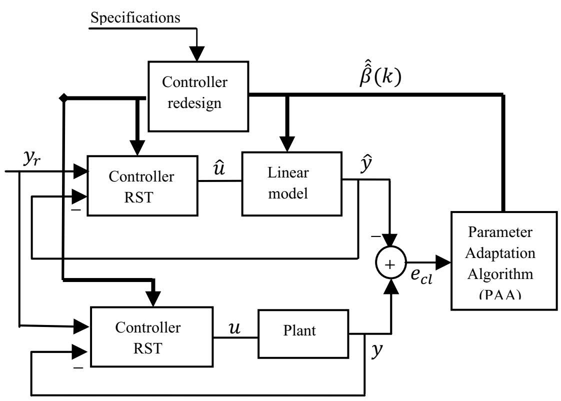

Fig. 1: Closed-loop adaptive control architecture

As shown in fig.1, closed-loop adaptive control algorithm can be seen as a combination of parameter adaptation algorithm (PAA) and controller tuning by minimum-degree pole placement technique [11-12]. This algorithm requires the specification of.

1) A transfer function $F_m(q^{-1})$ and an observed polynomial $A_0(q^{-1})$are given to specify the desired behaviour, which are;

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$F_m(q^{-1}) = q^{-d}\dfrac{B_m(q^{-1})}{A_m(q^{-1})}$; where

$A_m(q^{-1}) = 1 + a_{m1}q^{-1} + \cdots + a_{mn}q^{-n}$

$B_m(q^{-1}) = b_{m1}q^{-1} + b_{m2}q^{-2} + \cdots + b_{mr}q^{-r}$

$A_0(q^{-1}) = (1 - a_0 q^{-1})^{\ell}$;

2) The parametric model transfer function $\hat{F}(q^{-1})$ and the orders of its numerator and denominator are given in the form.

$\hat{F}(q^{-1}) = q^{-d}\dfrac{\hat{A}(q^{-1})}{\hat{B}(q^{-1})}$; where

$\hat{A}(q^{-1}) = 1 + \hat{a}_1 q^{-1} + \cdots + \hat{a}_n q^{-n}$

$\hat{B}(q^{-1}) = \hat{b}_1 q^{-1} + \hat{b}_2 q^{-2} + \cdots + \hat{b}_r q^{-r}$

Therefore the estimated output is

$$\hat{y}(k+1) \;=\; \hat{\beta}^T(k)\hat{\vartheta}(k) \tag{1}$$

where

$\hat{\beta}^T(k) = [\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_n, \hat{b}_1, \hat{b}_2, \ldots, \hat{b}_r]$

$\hat{\vartheta}^T(k) = [-\hat{y}(k), -\hat{y}(k-1), \ldots, -\hat{y}(k-n+1),$
$\hat{u}(k-d), \hat{u}(k-1-d), \ldots, \hat{u}(k-r-d+1)]$

where $\hat{\beta}(k)$ and $\hat{\vartheta}(k)$ are the vectors of parameters and regression respectively. At each sampling time the three following steps are performed

a) Identification of the parameters of $\hat{F}(q^{-1})$ by means of recursive parameters adaptation algorithm such as described in [1, 2, 11] where the stability and convergence of the algorithm are also given. This algorithm can be presented briefly as

$$\hat{\beta}(k+1) = \hat{\beta}(k) + P(k)\hat{\vartheta}(k)e_{cl}(k+1) \tag{2}$$

$$P(k+1) = \frac{1}{\lambda_1(k)}\left[P(k) - \frac{P(k)\hat{\vartheta}(k)\hat{\vartheta}^T(k)P(k)}{\frac{\lambda_1(k)}{\lambda_2(k)} + \hat{\vartheta}^T(k)P(k)\hat{\vartheta}(k)}\right] \tag{3}$$

$$0 < \lambda_1(k) \le 1,\; 0 \le \lambda_2(k) < 2$$

$$e_{cl}(k+1) = y(k+1) - \hat{y}(k+1) \tag{4}$$

$P(0) = \alpha I$; $\alpha > 0$; $where\ I\ is\ the\ unit\ diagonal\ matrix$

$P^{-1}(k) > \eta P^{-1}(0); 0 < \eta < +\infty$

The vector $\hat{\beta}(k)$ is identified using recursive least square parameter adaptation algorithm (RLS-PAA*)* given previously, and $P(k)$ is the decreasing covariance matrix or the adaptation gain, to ensure the convergence of the adaptation, α should be taken sufficiently large. The terms $\lambda_1(k)$ and $\lambda_2(k)$ are constants. These terms have opposite effects on the variation of $P(k)$. However, $\lambda_1(k) \le 1$ is called the forgetting factor used to increase the adaptation gain, but $\lambda_2(k) > 0$ tends to decrease it, the bound $\lambda_2(k) < 2$ results from the stability analysis.

b) Using the polynomials $\hat{A}(q^{-1})$ *and* $\hat{B}(q^{-1})$ estimated by (2)-(4) at each instant and $A_m(q^{-1})$ and $A_0(q^{-1})$, the three polynomials, $R(q^{-1})$ $S(q^{-1})$ and $T(q^{-1})$ of the controller are calculated according to the minimum-degree pole placement technique, or by solving the following polynomial Diophantine equations see [11-12].

$$\hat{A}(q^{-1})R(q^{-1}) + \hat{B}(q^{-1})S(q^{-1}) = A_m(q^{-1})A_0(q^{-1}) \tag{5}$$

$$T(q^{-1}) = \frac{A_m(1)}{\hat{B}(1)}A_0(q^{-1}) \tag{6}$$

c) Computation of the control inputs $u(k)$ and $\hat{u}(k)$ according to the new RST controller using the following equalities.

$$R(q^{-1})u(k) = T(q^{-1})y_r(k) - S(q^{-1})y(k) \tag{7}$$

$$R(q^{-1})\hat{u}(k) = T(q^{-1})y_r(k) - S(q^{-1})\hat{y}(k) \tag{8}$$

For nonlinear systems this approach can be used to control the system around a given set-point. However the global structure is modified in order to take into account the static points according to Fig.2, as it can be seen in the last section of the application, therefore the control inputs of (7)-(8) are given by these modified equations.

$$R(q^{-1})u(k) = \frac{1}{1-q^{-1}}[T(q^{-1})y_r(k) - S(q^{-1})y(k)] \tag{9}$$

$$R(q^{-1})\hat{u}(k) = \frac{1}{1-q^{-1}}[T(q^{-1})y_r(k) - S(q^{-1})\hat{y}(k)] \tag{10}$$

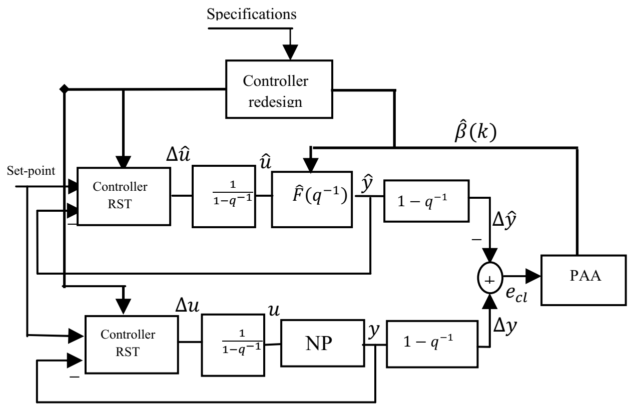


Fig. 2: General architecture of closed-loop adaptive control nonlinear plant (NP) around a given set-point

III. SUPERVISORY CONTROL USING CLOSED-LOOP ADAPTIVE CONTROL

The structure used in this paper for supervisory control is based on multi-models and multi-RST controllers and is presented on Fig 3. The plant is controlled by the input signal $u(k)$, his output signal is denoted $y(k)$ and the reference signal $y_r(k)$. N models $M = \{M_1, M_2, \ldots, M_N\}$ of the plant are run in parallel with the same input signal. At each sample time each output signal $y_i(k)$ of the model M_i is compared to the actual output of the plant to compute the estimation error of the model M_i $e_i(k) = y(k) - \hat{y}_i(k)$. The set of errors is then used to select the active mode of the switching controller. From the square of each error a performance index is computed by filtering and using a hysteresis factor in order to avoid small variations that may lead to quick switching signals. The active mode is then chosen from the model with the best performance index that the one with the smallest value. For each mode RST controller has been designed in order to fulfil the requirements and is then applied when the mode is active.

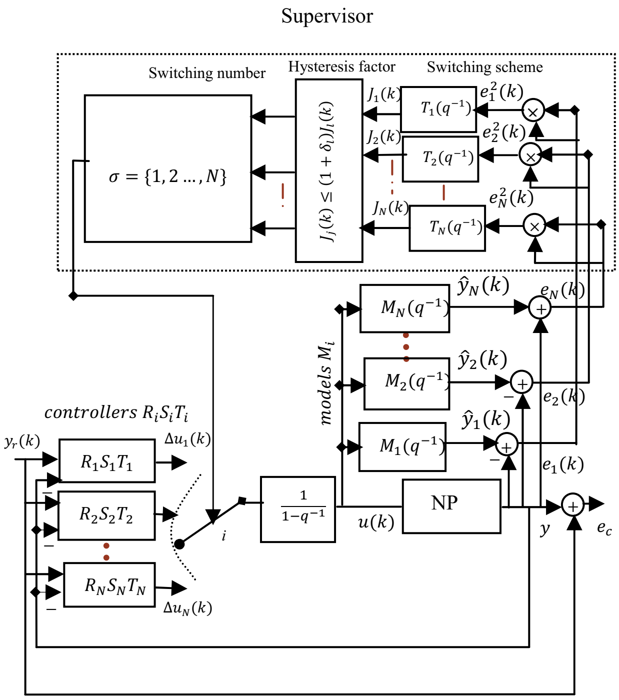


Fig. 3: Structure of supervisory control of nonlinear plant (NP) using multi-models and multi-controllers

To apply this strategy it is then necessary to compute a bank of models and the associated controllers. As the previous adaptive algorithm can be applied around a given set-point and give a model of the plant and an associated RST controller, its results for a given set of operating points are used to define the models and the controllers to use them in switching controller. In a first phase some experiments near the operating points are performed in order to automatically tune the controllers and the models. Because of the structure used to tune the controllers see Fig.2, it is not possible to directly use the resulting model because of some differences between the static values of the control for the plant and the model. For each operating point the model that is embedded is then specified by Fig.4 where u_i is a constant value that takes into account the difference between the static value of the control for the plant and the model of the adaptive algorithm $\hat{F}(q^{-1})$.

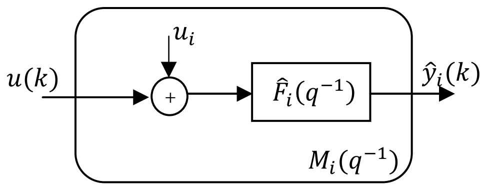


Fig. 4: Structure of the embedded models of the switching controllers

The switching scheme that is used is a first order stable and minimum phase transfer operator $T_i(q^{-1})$ between the error squared $e_i^2(k)$ and the output $J_i(k)$ for each model $i \in \{1 \ldots N\}$. This function is given by

$$T_i(q^{-1}) = K\frac{1-z_i q^{-1}}{1-p_i q^{-1}};\ (i \in \{1, 2, \ldots, N\}) \tag{11}$$

$$K > 0, 0 \leq p_i \leq 1 \ and\ 0 \leq z_i \leq p_i$$

Where K is the gain and z_i is the zero, are chosen in order to combine instantaneous and long-term accuracy measure, and p_i is the pole or the forgetting factor used to ensure bounded $J_i(k)$ for bounded errors. In otherwise at each instant, the square of the error is filtered by (11) this strategy is then used to select the model with the minimum performance index, which results to the appropriate controller. However to avoid the problem of fast repeating switches, which may cause the loss of stability by selecting an erroneous controller, a hysteresis algorithm is integrated see [13]. In the other way, when the model j is active and the model l is the one which correspond to the minimum value of this performance index then the active model will change to l if the inequality $J_j(k) \leq (1+\delta_l)J_l(k)$ is satisfied, where $\delta_l > 0$ is the hysteresis factor for mode l.

IV. NCSTR MODELING

Nonlinear continuously stirred tank reactor (NCSTR) process studied in this paper is a typical nonlinear and non-affine (is nonlinear with respect to the input) dynamical system used in chemical and biochemical industry. NCSTRP consists of an irreversible, exothermic reaction, $A \rightarrow B$, in a constant volume reactor cooled by a single coolant stream. The process is described by the following continuous time nonlinear, simultaneous, differential equations as given in [3, 5, 6, 14, 15].

$$\frac{dC_a(t)}{dt} = \frac{Q}{V}\left(C_{a0} - C_a(t)\right) - k_0 C_a(t) e^{-\frac{E}{RT(t)}} \tag{12}$$

$$\frac{dT(t)}{dt} = \frac{Q}{V}\left(T_0 - T(t)\right) + k_1 C_a(t) e^{-\frac{E}{RT(t)}} + \\ +k_2 q_c(t)\left(1 - e^{-\frac{k_3}{q_c(t)}}\right)\left(T_{c0} - T(t)\right) \tag{13}$$

Where $C_a(t)$ is the concentration, $T(t)$ is the temperature and the input is the coolant flow-rate $q_c(t)$, the temperature can be changed and hence the product concentration controlled. C_{a0} is the inlet-feed concentration, Q the process flow-rate, T_0 and T_{c0} are the inlet-feed and coolant temperature respectively, all of which are assumed constants at nominal values. Likewise, k_0, E/R, V, k_1, k_2 and k_3 are thermodynamics and chemical constants relating to this particular process. Numerical values of nominal parameters of NCSTRP are given in table.1

Parameter	***Description***	***Nominal value***
Q	Process flow rate	100 ℓ/min
V	Reactor volume	100 ℓ
k_0	Reaction rate constant	7.2×10^{10} min^{-1}
E/R	Activation energy	1×10^4 K
T_0	Feed temperature	350 K
T_{c0}	Inlet coolant temperature	350 K
ΔH	Heat of reaction	-2×10^5 cal/mol

C_p, C_{pc}	Specific heats	1 cal /g/K
ρ, ρ_c	Liquid densities	1×10^3 g/ℓ
C_{a0}	Inlet feed concentration	1 mol/ℓ
h_a	Heat transfer term	7×10^5 cal min^{-1} K^{-1}

$$where\ k_1 = -\frac{\Delta H k_0}{\rho C_p}, k_2 = \frac{\rho_c C_{pc}}{\rho C_p V}, and\ k_3 = \frac{h_a}{\rho_c C_{pc}}$$

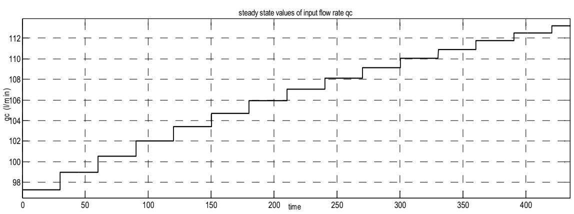
Table.1: The NCSTR nominal parameters

The sampling of this and the linearization around an operating point of this process leads to a second order model transfer function and then the same form of second order transfer functions will be used for the identification step and for the tuning phase of each controller.

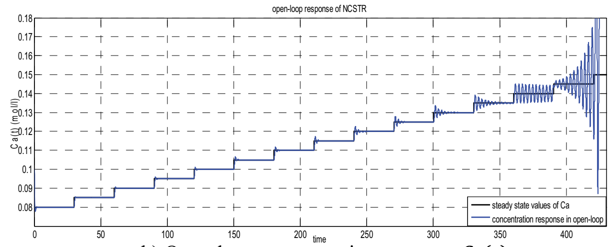
The open loop response of the NCSTRP to step stairs changes of the flow-rate $q_c(t)$ are given in fig 5 for initial conditions where $C_{a\_init} = 0.1\frac{mol}{l}$, $T_{init} = 438.54\ K$, $q_{c_{init}} = 103.41\ l/\ min$. As it can be seen the nonlinearities of the system leads to rapidly increasing of overshoot especially for high operating points, this happened either for the concentration $C_a(t)$ or for the temperature $T(t)$ responses. The response of the plant even becomes instable for high value of the concentration C_a , when exceeds 0.15*mol/l*. The steady state values of C_a and T for some given values of q_c are given in Table.2. It is then difficult to design a linear controller that controls the NCSTR process for all values of concentration in the range of 0.08 to 0.15*mol/l*. As it will be seen in the next section the purely adaptive control fails in controlling this plant and it is then interesting to study the switching control algorithm performance.

q_{c_ss} (*l/min*)	C_{a_ss} (*mol/l*)	T_{ss}(*K*)
97.264	0.08	443.309
98.964	0.085	442.015
100.547	0.09	440.794
102.025	0.095	439.640
103.406	0.100	438.544
104.699	0.105	437.501
105.908	0.110	436.506
107.042	0.115	435.553
108.104	0.120	434.641
109.099	0.125	433.763
110.031	0.130	432.920
110.904	0.135	432.106
111.722	0.140	431.320
112.487	0.145	430.560
113.202	0.150	429.824

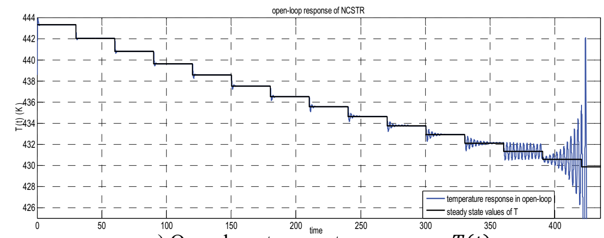
Table.2: Steady-state values of C_a and T for different of q_c



a) Input flow-rate $q_c(t)$



b) Open-loop concentration response $C_a(t)$



c) Open-loop temperature response $T(t)$

Fig. 5: Open-loop response of nonlinear CSTR process

V. ADAPTIVE AND SWITCHING CONTROL OF NCSTRP

The aim in this paper is to control the concentration $C_a(t)$ by the flow-rate $q_c(t)$ in the range of set-point from 0.08 to 0.15*mol/l*. In a first approach we will consider a purely adaptive controller and then a switching one.

5.1. Closed-loop adaptive control of NCSTR plant

The parametric linear model used for the identification is the second order operator function is given by.

$$(1 + \hat{a}_1 q^{-1} + \hat{a}_2 q^{-2})\hat{C}_a(k) = q^{-1}(\hat{b}_1 + \hat{b}_2 q^{-1})\hat{q}_c(k) \qquad (14)$$

The specifications are given by the objective transfer function $F_m(q^{-1}) = \frac{B_m(q^{-1})}{A_m(q^{-1})}$ where $A_m(q^{-1})$ and $B_m(q^{-1})$ are given below and the observable polynomial $A_0(q^{-1})$. The sampling time is T_s=0.1*min* natural frequency is ω_n = 5 *rad/min* and the damping ratio $\xi = 0.9$. The forgetting factors of the recursive PAA algorithm were fixed to $\lambda_1 = 0.956\ and\ \lambda_2 = 1$ then;

$$A_m(q^{-1}) = 1 - 1.245q^{-1} + 0.4066q^{-2}$$
$$B_m(q^{-1}) = 0.09279q^{-1} + 0.06869q^{-2}$$
$$A_0(q^{-1}) = (1 - 0.006q^{-1})^3$$

The structure of the adaptive algorithm is shown on fig.2, and takes into account the fact that the input and output data used for identification must be centred on zero.

5.2. Switching control of NCSTR process

In a first phase the closed-loop adaptive control shown in section.2, has been applied to NCSTR process to determine the bank of model/controller pairs corresponding to different set-points scaled between 0.08 *mol/l* and 0.135 *mol/l* with a step size of 0.005*mol/l* as shown in Table.3. The models identified by the adaptive control algorithm were then modified as explained in section 3 to take into account the steady value of the control introduced by the integrator. The new model and the controllers issued from the adaptive phase were then embedded in the switching controller with the switching policy described in section 3. The parameters of the filters and the hysteresis factor were empirically chosen in order to make a compromise between speed of the response, the quality of performances and the stability and convergence of the performance indices $J_i(k)$.

$model\ number\ i$	$C_{a\_set-point}$ (mol/l)
1	0.08
2	0.085
3	0.09
4	0.095
5	0.100
6	0.105
7	0.110
8	0.115
9	0.120
10	0.125
11	0.130
12	0.135

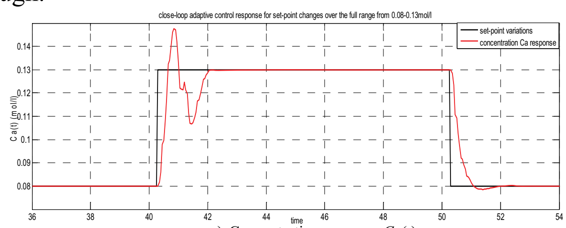
Table. 3: model number with respect to its set-point

5.3. Discussion and comparison of results

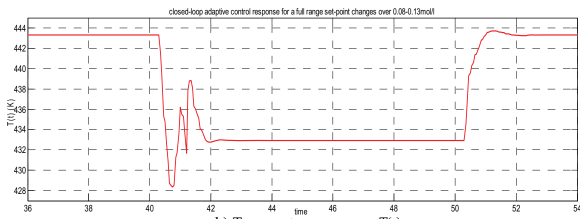
To evaluate the performance and the effectiveness of this control methodology, two scenarios that depend on different scaling of set-points are proposed each of them will be studied with the two types of controllers. These scenarios are formulated as follows; 1) using full range changes of set-point, 2) scaling the set-point from 0.08mol/l to 0.15mol/l with step-size of 0.005 mol/l.

A. Set-point changes over the full range from 0.08mol/l to 0.15 mol/l

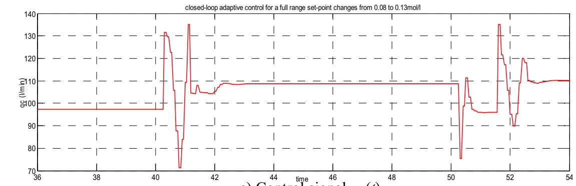
As shown in Fig.6 and Fig.7, closed-loop adaptive control alone is not satisfactory when the set-point changes over the full range, otherwise; when the set-point exceeded 0.13*mol/l* the output concentration and the temperature of the process became unstable. To overcome this kind of problem, supervisory control architecture is applied. From Fig.8, we observe that, the transient response of either the concentration $C_a(t)$ or the temperature $T(t)$ response is good enough.



a) Concentration response $Ca(t)$



b) Temperature response $T(t)$



c) Control signal $qc(t)$

Fig. 6: Closed-loop adaptive control over the range of set-point from 0.08 to 0.13mol/l

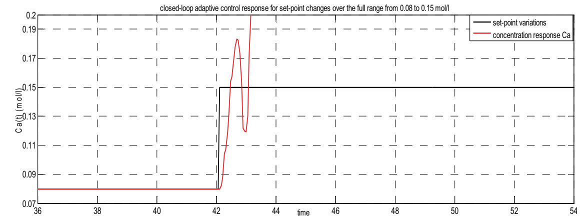
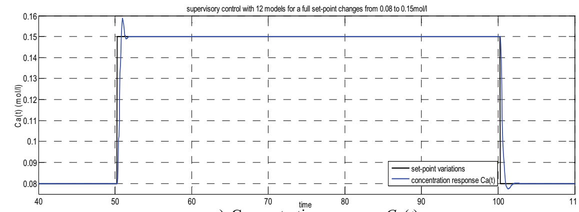
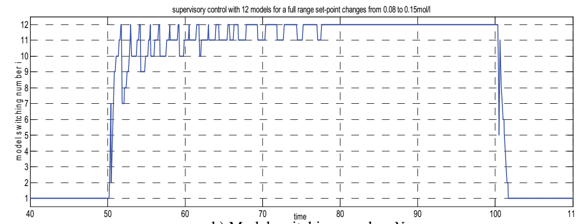


Fig. 7: Closed-loop adaptive control with the full range of set-point changes (first scenario)

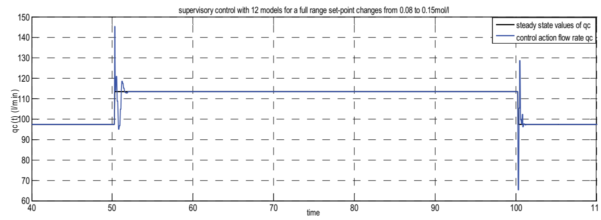


a) Concentration response $Ca(t)$



b) Model switching number N

c) Temperature response $T(t)$

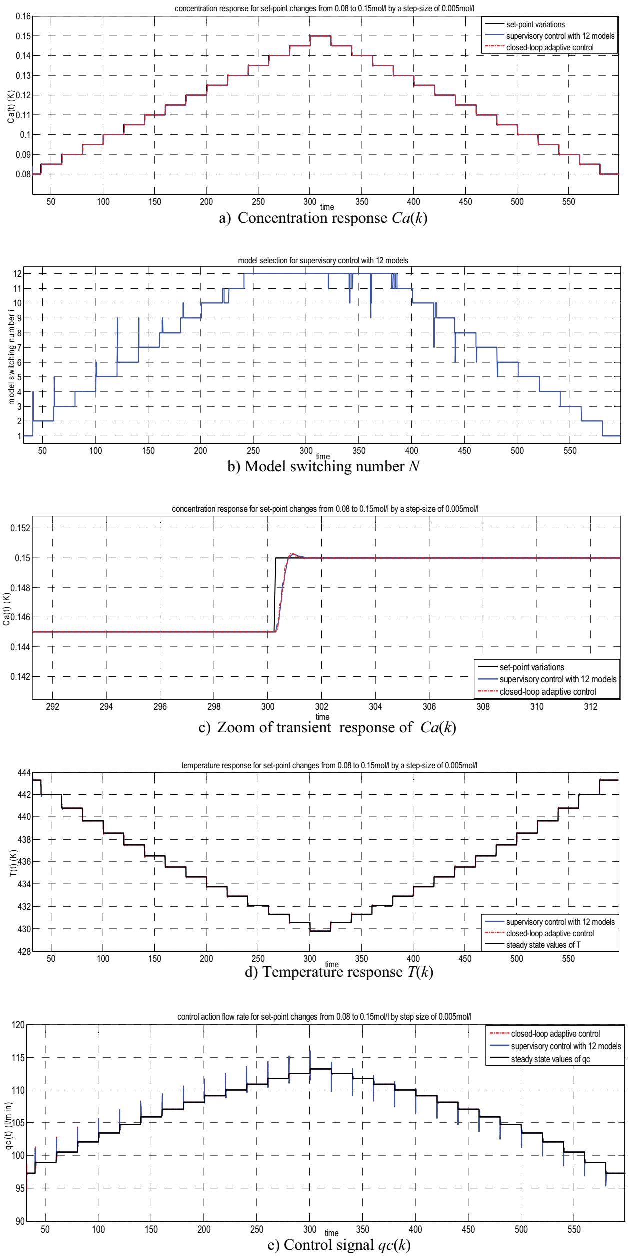


d) Control signal $qc(k)$

Fig. 8: Supervisory control for the first scenario or for the full range of set-point changes from 0.08 to 0.15mol/l

B. Set-point changes over the range from 0.08mol/l to 0.15mol/l by a step size of 0.005mol/l

Fig.9 shows the set-point tracking of the output concentration response and the temperature response for the closed-loop adaptive control and supervisory control. For each controller the closed-loop performances are satisfactory. Indeed, when the step-point changes by small step size of 0.005*mol/l*, there is only a small difference, which can be highlighted in Fig.9c between the transient response (overshoot at each step size) of closed-loop adaptive control and the one of supervisory control with 12 model/controller pairs. However, supervisory control is a little better as performances are concerned and far better for complexity of computation at each sampling time.



a) Concentration response $Ca(k)$

b) Model switching number N

c) Zoom of transient response of $Ca(k)$

d) Temperature response $T(k)$

e) Control signal $qc(k)$

Fig. 9: Closed-loop adaptive control and supervisory control of the second scenario or when the set-point changes over the range from 0.08 to 0.15mol/l by a step size of 0.005mol/l

VI. CONCLUSION

In this paper we considered the combination of adaptive algorithm and switching controller to control a nonlinear process. The adaptive algorithm is used in a first phase to tune the controllers and the models that are embedded in the switching controller in the second phase. This approach was applied in simulation to the model of a nonlinear plant and lead to satisfactory results.

However one aim is to be able to design a generic controller that can easily be adapted to specific plants In order to reach this goal one open point is the study of the adaptive or learning phase as the supervisory controller has a given number of modes that corresponds to specific operating points. The number of operating points that have to be considered and their choice is still a manual operation that has to be embedded in the controller tuning phase in order to get an autonomous learning controller.

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