R-S-T Conventional Controller Design for Slow Processes

Mircea Dulău, Stelian-Emilian Oltean, and Adrian-Vasile Duka

Abstract— **Discrete time control laws can be synthesized based on the shape of the system's response in a specified number of samples, as well as by digitizing continuous time controllers (e.g. PI, PD, PID). For SISO linear systems, solving the Diophantine equation from the poles placement method leads to discrete controllers with R-S-T canonical structure.**

The paper shows the design procedure of discrete time R-S-T controllers for slow processes, particularly for level control processes. From the discrete time controller the equivalent continuous controller is rebuilt and the behavior of the system to parameter variations is studied. The results and simulations are made using the Matlab environment.

***Index Terms*—slow process control, level control, poles placement method, R-S-T robust controller**

I. Introduction To R-S-T Controller

THE methods for designing discrete time controllers, the canonical structures, computing the R-S-T parameters, identification, the polynomial approach and the sensitivity functions are shown in [1],[2],[3],[4],[10],[12],[15].

A methodology of synthesis of an R-S-T controller, enabling to achieve steady-state error cancellation with respect to harmonic references, is presented in [5]. A control strategy for a double fed induction machine based on the regulator R-S-T is developed in [6] and a technique for designing a robust polynomial R-S-T controller for parametric uncertain systems are presented in [7]. A R-S-T controller, implemented on a real-time operating system as a control task, and a comparison between conventional RST polynomial control by poles placement and R-S-T flatness-based control are proposed in [8],[9].

Fig. 1 shows a canonical structure of a digital R-S-T controller, in which the control signal is [3]:

$$U\left(z^{-1}\right)=\left(T\left(z^{-1}\right)/S\left(z^{-1}\right)\right)\cdot Y_r\left(z^{-1}\right)-\left(R\left(z^{-1}\right)/S\left(z^{-1}\right)\right)\cdot Y\left(z^{-1}\right). \tag{1}$$

This paper is based upon work supported by the Energy and Electrotechnologies Management research center, Department of Electrical and Computer Engineering, “Petru Maior” University of Tîrgu-Mureş.

M. Dulău is with the Department of Electrical and Computer Engineering, “Petru Maior” University of Tîrgu-Mureş, Mureş, Romania (e-mail: mircea.dulau@ing.upm.ro).

S. E. Oltean is with the Department of Electrical and Computer Engineering, “Petru Maior” University of Tîrgu-Mureş, Mureş, Romania (e-mail: stelian.oltean@ing.upm.ro).

A. V. Duka is with the Department of Electrical and Computer Engineering, “Petru Maior University” of Tîrgu-Mureş, Mureş, Romania (e-mail: adrian.duka@ing.upm.ro).

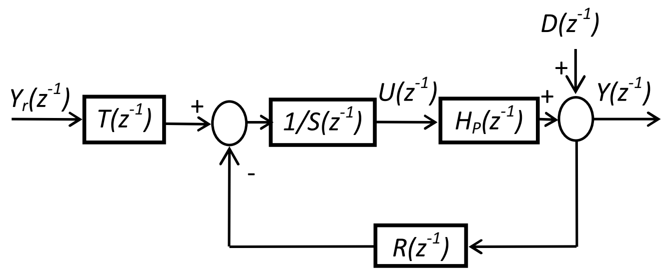

Fig. 1. The R-S-T control structure.

The discrete time controller synthesis aims at finding the coefficients of $R(z^{-1})$, $S(z^{-1})$ and $T(z^{-1})$, in order to obtain the setpoint to output and disturbance to output closed loop transfer functions which satisfy the performance requirements.

If the discrete time transfer function of the controlled process, which includes the A/D converter (with the zero order hold element), the plant and the D/A converter, is of the following form:

$$H_p\left(z^{-1}\right)=B\left(z^{-1}\right)/A\left(z^{-1}\right), \tag{2}$$

where: $B\left(z^{-1}\right)=1+b_1z^{-1}+b_2z^{-1}+\ldots$; $A\left(z^{-1}\right)=1+a_1z^{-1}+a_2z^{-1}+\ldots$, then, the closed loop transfer (setpoint to output) function is:

$$H_0\left(z^{-1}\right)\stackrel{\Delta}{=}Y\left(z^{-1}\right)/Y_r\left(z^{-1}\right)=\frac{B\left(z^{-1}\right)\cdot T\left(z^{-1}\right)}{P\left(z^{-1}\right)}. \tag{3}$$

The polynomial:

$$P\left(z^{-1}\right)=A\left(z^{-1}\right)S\left(z^{-1}\right)+B\left(z^{-1}\right)R\left(z^{-1}\right)=1+p_1z^{-1}+p_2z^{-1}+\ldots. \tag{4}$$

defines the system's closed loop poles.

The required performance for the closed loop can be expressed by the poles of the polynomial $P(z^{-1})$ (and by the imposed zeroes).

The disturbance $D(z^{-1})$ is linked to the output signal and to the control signal through sensitivity functions, which are used to analyze the closed loop system's stability and robustness.

Therefore:

- *the disturbance to output sensitivity function*, shown in (5), describes the system's performance to disturbances, namely, it specifies the form of the polynomial $S(z^{-1})$, with the purpose of achieving a satisfactory disturbance rejection [3];

978-1-4799-7650-8/15/$31.00 ©2015 IEEE

$$S_{yd}\left(z^{-1}\right)=\frac{A\left(z^{-1}\right)S\left(z^{-1}\right)}{A\left(z^{-1}\right)S\left(z^{-1}\right)+B\left(z^{-1}\right)R\left(z^{-1}\right)} \tag{5}$$

- *the disturbance to control signal sensitivity function*, shown in (6) is used to analyze the influence of the disturbance on the process input signal and to specify a part of the polynomial $R(z^{-1})$ [3].

$$S_{ud}\left(z^{-1}\right)=\frac{-A\left(z^{-1}\right)R\left(z^{-1}\right)}{A\left(z^{-1}\right)S\left(z^{-1}\right)+B\left(z^{-1}\right)R\left(z^{-1}\right)} \tag{6}$$

II. THE STRUCTURE OF THE DISCRETE TIME PI CONTROLLER

Based on the continuous time PI controller, given by [1],[3],[11],[14],[18],[19], and considering the delay operator $q^{-1}=z^{-1}$ for determining the discrete control signal, the equation of the controller is defined in (7) as:

$$U\left(z^{-1}\right)=\frac{k_p\left(1-z^{-1}\right)+k_pT_s/T_i}{1-z^{-1}}\left[Y_r\left(z^{-1}\right)-Y\left(z^{-1}\right)\right] \tag{7}$$

where: $Y(z^{-1})$ is the output signal, $Y_r(z^{-1})$ the setpoint, k_p proportional gain, T_i integral time, T_s sampling rate.

Denoting:

$$S\left(z^{-1}\right)=s_0+s_1z^{-1}=1-z^{-1} \tag{8}$$
$$R\left(z^{-1}\right)=T\left(z^{-1}\right)=k_p\left(1+T_s/T_i\right)-k_pz^{-1}=r_0+r_1z^{-1} \tag{9}$$

the control signal is determined as:

$$\begin{aligned}U\left(z^{-1}\right)&=z^{-1}U\left(z^{-1}\right)+r_0Y_r\left(z^{-1}\right)+r_1z^{-1}Y_r\left(z^{-1}\right)-\\&-r_0Y\left(z^{-1}\right)-r_1z^{-1}Y\left(z^{-1}\right).\end{aligned} \tag{10}$$

This control signal is obtained by summing the values of the setpoint and output, respectively previous values of the output signal weighted by the coefficients of the controller (r_0,r_1), as shown in Fig. 2.

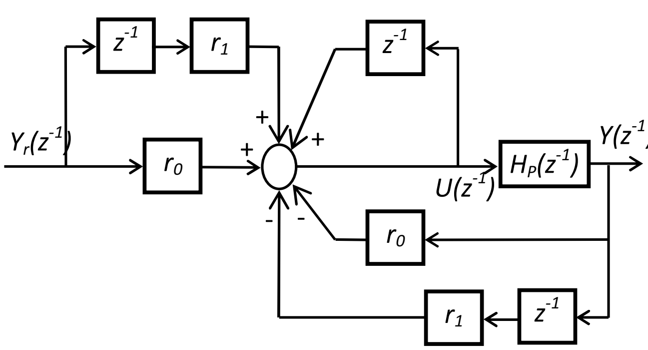

Fig. 2. PI structure for the control signal $U(z^{-1})$.

For the closed loop system expressed by (3), in which the digitized process (2) has the transfer function:

$$H_p\left(z^{-1}\right)=b_1z^{-1}/\left(1+a_1z^{-1}\right), \tag{11}$$

and the PI digital controller generates the control signal (10).

The polynomial $P(z^{-1})$ expressed in (4), corresponding to the closed loop pole placement, defines the overall performances and can be chosen as a second order type polynomial which has been digitized from a continuous time second order system, having the values ω_0 and ξ specified.

$$H_{II}\left(z^{-1}\right)=H_{II}(s)\Big|_{s=\frac{T_s}{1-z^{-1}}}=\frac{\omega_0}{s^2+2\xi\omega_0 s+\omega_0}\Bigg|_{s=\frac{T_s}{1-z^{-1}}} \tag{12}$$

As a rule of thumb, the sampling frequency f_s is chosen as:

$$f_s=(6...25)f_0^{BT}, \tag{13}$$

according to the bandwidth of the closed loop system [1],[2],[3].

Therefore, for the system in (12), for $0.7<\xi<1$, the natural pulsation and the sampling rate must satisfy the condition $0.25\le\omega_0T_s\le1.5$.

From the equality (4), by considering also (8) and (11) we obtained:

$$1+\left(a_1-1+r_0b_1\right)z^{-1}+\left(b_1r_1-a_1\right)z^{-2}=1+p_1z^{-1}+p_2z^{-2} \tag{14}$$

and the values for the coefficients of polynomial $R(z^{-1})$ are given in (15):

$$\begin{cases}r_0=\left(p_1-a_1+1\right)/b_1\\ r_1=\left(p_2+a_1\right)/b_1\end{cases}. \tag{15}$$

By using (9), the continuous time PI controller parameters contained in the control signal (7) can be computed.

$$\begin{cases}k_p=-r_1\\ T_i=-r_1T_s/\left(r_0+r_1\right)\end{cases} \tag{16}$$

III. DESIGN OF THE R-S-T STRUCTURE CONTROLLER FOR A LEVEL CONTROL PROCESS

Level control processes are usually controlled based on the error signal. They have an upstream or downstream control valve and are drained by freefall or using a pump. When the inlet flow F_a is used for controlling the level inside the tank, the output flow F_e, represents the main disturbance in the control system and vice-versa [13],[14].

When a change in level, inside the process tank R2, is sensed by the level transducer LT1, the error signal is modified, and the controller LC1 takes corrective actions by generating a control signal for valve LV1 (Fig. 3) [13].

In steady state the amount of fluid introduced in the tank is equal to the drained amount and the level inside the tank remains constant.

In dynamic regime, the difference in the inlet and outlet quantities is accumulated in the system. By performing normalization to the steady state values: the controlled signal $y(t)=\Delta L(t)/L_0$, the control signal $u(t)=\Delta F_a(t)/F_{a0}$, the final general input-output mathematical model is deducted.

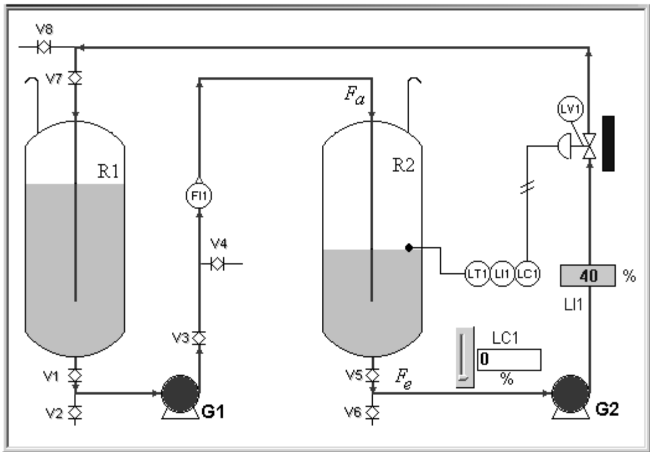

Fig. 3. Level control unit, R1 – fluid supply tank and R2 – process tank.

This model depends on the tank's section A, the section of the outlet pipe a, the gravitational acceleration g, the steady state inlet flow F_{a0} and the steady state level L_0.

$$AL_0\frac{d}{dt}y(t)+\frac{a\sqrt{2gL_0}}{2\sqrt{L_0}}y(t)=F_{a0}u(t) \tag{17}$$

Denoting:

$T_p=2A\sqrt{L_0}/\left(a\sqrt{2g}\right)$; $k_p=\left(2\sqrt{L_0}/\left(a\sqrt{2gL_0}\right)\right)\cdot F_{a0}$

the transfer function of the process results from (17) as:

$$H_p(s)=k_p/\left(T_ps+1\right). \tag{18}$$

The inherent errors resulting from modeling the process define the system's uncertainties. For the level control process, if there are parameter variations, the transfer function in (18) can be written as:

$$H_p(s)=H_{pn}(s)+\Delta H_p(s), \tag{19}$$

where: $H_{pn}(s)$ represents the nominal plant and $\Delta H_p(s)$ represents the uncertainties block [21].

In the case of an *uncertain system*, the effect of the feedback connection is investigated, that is, the errors with respect to the reference signal and the disturbance, as well as the effect of the open loop parameter change upon the closed loop.

IV. Experimental Results

In order to determine the way in which the variation of the parameters from the nominal process affect the closed loop behavior of the system, Matlab is used to generate samples of uncertain parameters inside some specified confidence range [16],[17],[21].

For the level control process described by (18), the uncertain model considers a variation of the process parameters within a range of $k_p=1\pm 25\%$ and $T_p=15\pm 25\%$. The digitized nominal process with form (11) is $H_p\left(z^{-1}\right)=0.2835z^{-1}/\left(1-0.7165z^{-1}\right)$. If a standard behavior of a second order system is desired, for ω_0=0.05, T_s=5 and ξ=0.707, the discrete time model (12) has the form $H_{II}\left(z^{-1}\right)=\left(0.02773z^{-1}+0.02464z^{-2}\right)/\left(1-1.65z^{-1}+0.7022z^{-2}\right)$ and the PI controller, reconstructed according to (16), is $H_R(s)=(0.06892s+0.05047)/(1.3657s)$.

The time responses to a step change in the setpoint of the specified second order discrete-time model and of the process controlled by the discrete PI controller are shown in Fig. 4. The behavior of the system in relation to the disturbance, which is based on the sensitivity function in (5), is shown in Fig. 5. The behavior of the closed loop continuous system to a step change is shown in Fig. 6.

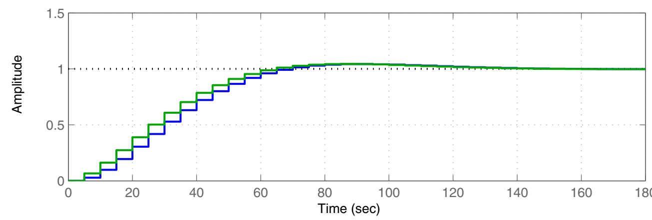

Fig. 4. The step responses of the process and second order model.

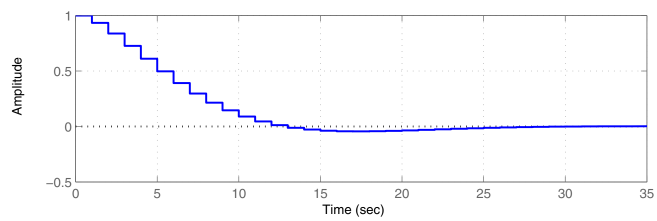

Fig. 5. The sensitivity function, $S_{yd}(z^{-1})$.

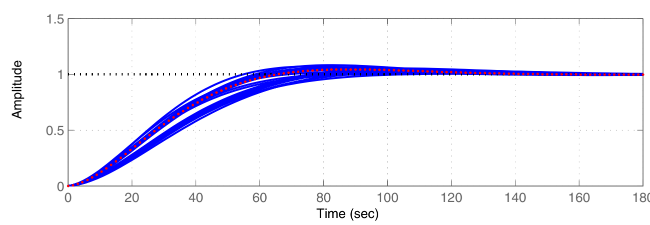

Fig. 6. The step responses of the nominal an uncertain processes.

The nominal plant, as well as, the plants affected by uncertainties are considered. The frequency analysis of the sensitivity function provides a measure of the control performances and is shown in Fig. 7.

The experimental setup, shown Fig. 3, consists of the following equipments and devices: 20 litres feed tank (R1), 600mm water column (R2), pneumatic control valve (LV1), electropneumatic converter (4-20)mA / (0-1.25)bar, two centrifugal feed pumps (G1,G2) with a flow rate of 4 m^3/h, electronic level transmitter (LT1) with (4-20)mA output, ABB Digitric 500 Level Controller (LC1). Fig. 8 shows the experimental results, considering [13],[20], for controlling the level inside the R2 tank, to a setpoint change from 65% to 25%, which empties the tank.

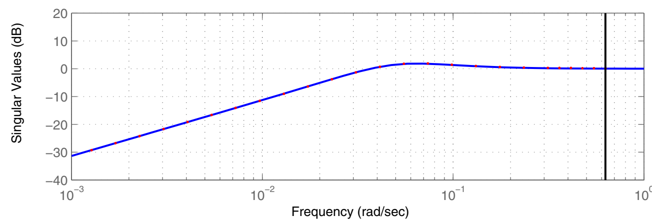

Fig. 7. The frequency responses of the sensitivity function.

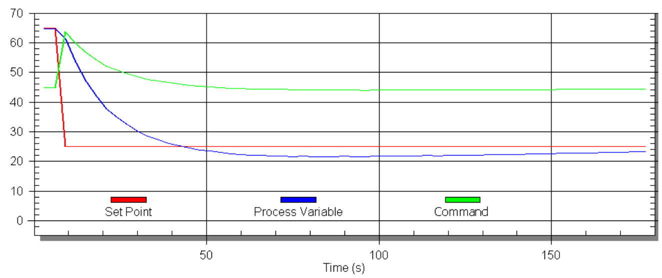

Fig. 8. The step response of the experimental level process.

V. CONCLUSIONS

The stable behavior of the slow process depends on the controller's capacity to maintain and re-establish, almost optimally, the system's steady state.

The Matlab simulation results show that the discrete-time controller is able to assure control characteristics which are (almost) identical to the ones of the second order model, namely, 4% overshoot, around 120 seconds settling time, zero steady-state error, including in the presence of disturbances. When comparing, the experimental results show that the level inside the tank becomes stable in the limits of a 2% steady state error in about 130 seconds.

If we consider the computational needs, the PI controller can be easily tuned and implemented. At the same time, the continuous time controller can be reconstructed from the discrete-time controller.

In relation to (9), (10), all classical digital control algorithms have virtually the same structure. All that is changed, from one type to another, is their "memory", namely, their coefficients and the sole operations they can perform are addition, multiplication, memory and delay.

The control of an uncertain system is done by tuning a stabilizing controller for the nominal plant and for a class of systems which fit a specified range around the nominal plant.

In the next stage, a control structure can be determined using robust tuning techniques and compared to R-S-T controllers. Also, an eventual delay time should be considered for a new mathematical model of the level process.

REFERENCES

[1] I. D. Landau, “The R-S-T digital controller design and applications” in Control Engineering Practice 6, 1998, pp.155-165.

[2] I. D. Landau, “Robust R-S-T Digital Control and Open Loop System Identification” presented at the IEEE Advanced Process Control Workshop, Vancouver, USA, april 29-may 1, 2002.

[3] I. D. Landau, *Identifiction et commande des systemes*, Romanian edition, Technical Publisching House, Bucharest, Romania, 1997.

[4] I. D. Landau and G. Zito, *Digital Control Systems: Design, Identification and Implementation*, Springer, 2006.

[5] E. Ostertag and E. Godoy, “RST-Controller Design for Sinewave References by Means of an Auxiliary Diophantine Equation” in Proc. of the 44th IEEE Conference on Decision and Control, and the European Control Conference 2005, Seville, Spain, 2005, pp. 6905-6910.

[6] N. Akkari, A. Chaghi and R. Abbdessemed, “Study and Simulation of RST Regulator Applied to a Double Fed Induction Machine (DFIM)”, Journal of Electrical Engineering & Technology, vol. 3, no. 3, pp. 308-313, 2008.

[7] S. Khadraoui, M. Rakotondrabe and P. Lutz, “Design of a fixed-order RST controller for interval systems: application to the control of piezoelectric actuators”, Asian Journal of Control, Chinese Automatic Control Society, vol. 15, pp.142-153, 2013.

[8] H. Gharsallaoui, M. Ayadi, M. Benrejeb and P. Borne, “Flatness-based Control and Conventional RST Polynomial Control of a Thermal Process”, Int. J. of Computers, Communications & Control, vol. IV, no. 1, pp. 41-56, 2009.

[9] D. Robert, O. Sename and D. Simon, “Sampling period dependent RST controller used in control/scheduling co-design” presented at 16th IFAC 2005 World Conference, Prague, july, 2005.

[10] F. Cabaleiro and F. Boattini, “RST controllers: General Introduction and Application on the POPS power converter” presented at RST control EPC technical seminaries, CERN, april 25, 2013.

[11] K. J. Aström and B. Wittenmark, *Computer Controlled Systems: Theory and Design*, Prentice Hall, 1996.

[12] R. N. Andriamalala, H. Razik and F. M. Sargos, “Indirect-Rotor-Field-Oriented-Control of a Double-Star Induction Machine Using the RST Controller” in Proc. 34th Annual Conference of the IEEE Industrial Electronics Society, vol. 1-5, 2008, pp. 3004-3009.

[13] *Eletronica Veneta, Flowrate Control, Pressure Control, Level Control Unit, mod. FC/EV, PC/EV, LC/EV*, Teacher/Student handbook, Italy, 2007.

[14] M. Vinatoru, *Control of industrial processes (Conducerea automată a proceselor industriale*, Romanian version), Universitaria, Craiova, Romania, 2001.

[15] M. Khanchoul, M. Hilairet, "Design and comparison of different RST controllers for PMSM control," IECON 2011 - 37th Annual Conference on IEEE Industrial Electronics Society, 2011, pp.1795-1800.

[16] Mathworks, *Matlab Control System Toolbox*, [Online]. Available: http://www.mathworks.com/products/control/

[17] C. Moler, *Numerical computing with Matlab*, [Online]. Available: http://www.mathworks.com/moler/chapters.html.

[18] N. Norman, *Control systems engineering*, International student version, 5th ed., John Wiley and Sons Inc., 2008.

[19] I. Dumitrache, *Automation (Automatica,* Romanian version), vol.1, Ed. Romania Academy, Bucharest, Romania, 2009.

[20] *ABB, Digitric 500, Controllers for process engineering. Controllers for industry, Configuration and parametrization, Manual, 42/62-50012 EN, Rev.06,* ABB Automation Products GmbH, 2007.

[21] D.W. Gu, P.Hr. Petkov, M.M. Konstantinov, *Robust Control Design with MATLAB. Advanced textbooks in control and signal processing.* Springer-Verlag London Limited, 2005.