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RST Controller Design by Convex Optimization Using Frequency-Domain Data [1]

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Abstract: A new robust fixed-order RST controller design approach by convex optimization is proposed in this paper. Only frequency-domain data are considered in the approach for linear time-invariant single-input single-output systems. It is shown that performance specifications given as the infinity norm of the weighted sensitivity functions can be represented as convex constraints in the Nyquist diagram. One of the advantage of the proposed method is that it can deal directly with multi-model uncertainty. Moreover, a solution to a real industrial problem is given showing the effectiveness of the method.

1. INTRODUCTION

Most of the industrial control problems have specifications in terms of robustness, disturbance rejection and tracking of a given reference signal. It is very difficult to achieve all the specifications using one-degree-of-freedom controllers, therefore, 2DOF controllers are preferred for this type of systems. The degree-of-freedom of a controller defines the number of closed-loop transfer functions that can be shaped independently (Horowitz [1963]), which allows good performance to be achieved whilst preserving robustness.

Typically, a two-step strategy is chosen in order to design the feedback and feedforward terms of 2DOF controllers. In the first step, the feedback term is designed to guarantee stability, robustness and disturbance rejection specifications. Then, the feedforward term is designed to achieve the desired tracking specifications. Many different methods can be found in the literature to tune the feedback or feedforward terms separately. An example is shown in Sidi [2002] where a combined QFT/H_∞ design technique is proposed. The classical H_∞ method is used to tune the feedback controller to minimize the maximum value of the sensitivity function and the noise amplification for a desired frequency range. Then, the QFT technique is applied to design the feedforward term to assure certain tracking specifications.

This paper is interested in solving the general H_∞ control problem, where infinity norm constraints are defined for different weighted closed-loop transfer functions. Furthermore, it can be desirable to minimize one or more of these norms to achieve better performances. The classical H_∞ optimization method can deal with the mixed sensitivity controller design problem which is a particular case of the previously mentioned control problem. As it is shown in Zhou [1996], this is achieved based on the linear fractional transformation (LFT). A controller design problem for a SISO system is transformed to an augmented MIMO system and the infinity norm of the closed-loop weighted transfer function matrix is minimized. This matrix contains many cross transfer functions between the different outputs and inputs of the augmented system which affects the norm to be minimized. Therefore, it is not possible to minimize the infinity norm of one of the weighted closed-loop transfer functions of the original SISO problem under some constraints on the other weighted sensitivity functions. It should be mentionned that this method tunes both degrees of freedom simultaneously instead of tuning them in two different steps.

Recently, a fixed-order H_∞ controller design method for spectral models has been proposed in Galdos et al. [2007] and Karimi and Galdos [2010]. The method uses only the frequency response of the system and no parametric model is required. It can also deal with multi-model uncertainty and with systems containing a time-delay. Basically, a two step strategy can be considered to extend the proposed method to tune 2DOF controllers. First, a linearly parameterized feedback controller can be obtained by applying directly the method proposed in Galdos et al. [2007] and Karimi and Galdos [2010]. Then, by a second optimization, a linearly parameterized feedforward controller can be tuned subject to convex constraints assuring desired upper bounds on the magnitude of the closed-loop sensitivity functions. The main drawback of tuning a 2DOF controller in two different steps is that there is no guarantee of achieving the optimal solution for the original problem. The first optimization result influences the solution of the second optimization problem. Consequently, it is desirable to tune both degrees of freedom simultaneously.

In this paper, the idea presented in Karimi and Galdos [2010] is extended to design of robust fixed-order RST controllers satisfying some infinity norm constraints on the weighted closed-loop transfer functions. An RST controller is a 2DOF polynomial controller (see Fig. 1), where R, S

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and T are polynomials in q^{-1} (time shift operator) . It can be shown that any 2DOF controller can be represented in an RST form by choosing the correct feedback and feedforward terms. In this approach, to obtain a convex optimization problem, we consider that the R polynomial is fixed a priori, for example as an integrator. The performance specifications are presented as H_∞ constraints on the weighted closed-loop sensitivity functions. The set of all fixed-order stabilizing RST controllers satisfying these conditions is a nonconvex set. An inner approximation of this set is represented by a set of convex constraints in the Nyquist diagram. The multi-model uncertainty can be directly considered by repeating the set of convex constraints. In this paper, a discrete-time approach is considered, however, the results are also applicable to continuous-time systems.

This paper is organized as follows: In Section 2 the class of models, controllers, the design specifications and the control problem are defined. Section 3 introduces the RST control design methodology based on the convex constraints in the Nyquist diagram. The proposed method is applied to a double-axis Linear Permanent Magnet Synchronous Motor (LPMSM) in Section 4. Some conclusions are given in Section 5.

2. PROBLEM FORMULATION

2.1 Class of models

The class of causal discrete-time LTI-SISO systems is considered. It is assumed that the plant model belongs to a set $\mathcal{G}$ containing m spectral models:

$$\mathcal{G} = \left\{G_i(e^{-j\omega}); \quad i = 1, \ldots, m; \quad \forall\omega \in [0, \pi]\right\} \tag{1}$$

which are presented in normalized frequencies. This type of models can be obtained from a parametric model or by spectral analysis from a set of input/output data.

In the sequel, for the sake of simplicity, we consider a nominal model $G \in \mathcal{G}$ and a discrete-time controller will be designed. However, the results are also applicable to the multi-model case.

2.2 Class of controllers

The controller to be designed is a discrete-time 2DOF controller of the RST-type (Figure 1). The R polynomial is fixed a priori, which is typically chosen as an integrator:

$$R(q^{-1}) = 1 - q^{-1} \tag{2}$$

The S and T polynomials are given by :

$$S(q^{-1}, \rho) = \rho_1 + \rho_2 q^{-1} + \ldots + \rho_{n_S} q^{-n_S+1} \tag{3}$$

$$T(q^{-1}, \rho) = \rho_{n_S+1} + \rho_{n_S+2} q^{-1} + \ldots + \rho_{n_S+n_T} q^{-n_T+1} \tag{4}$$

where n_S and n_T are the number of parameters for S and T polynomials, respectively, and

$$\rho^T = [\rho_1, \rho_2, \ldots, \rho_{n_S+n_T}] \tag{5}$$

is the vector of controller parameters.

The main property of this parameterization is that every point on the Nyquist diagram of the open-loop transfer function

$$L(q^{-1}, \rho) = \frac{S(q^{-1}, \rho)}{R(q^{-1})} G(q^{-1}) \tag{6}$$

can be written as a linear function of the parameters ρ.

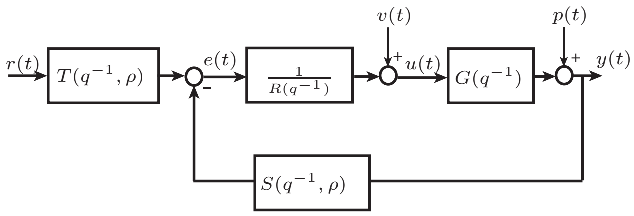

Fig. 1. Structure of two-degrees-of-freedom controller of the RST-type

2.3 Design Specifications

Following the controller structure given in Figure 1, different sensitivity functions can be defined between external inputs: $r(t)$ (the reference signal), $v(t)$ (the input disturbance) and $p(t)$ (the output disturbance) and outputs: $y(t)$ (the plant output) and $u(t)$ (the control input).

The following 5 sensitivity functions can be considered:

$$\mathcal{S}_{yr}(q^{-1}, \rho) = \mathcal{S}_1(q^{-1}, \rho) = \frac{\frac{T(q^{-1},\rho)}{R(q^{-1})} G(q^{-1})}{1 + \frac{S(q^{-1},\rho)}{R(q^{-1})} G(q^{-1})} \tag{7}$$

$$\mathcal{S}_{yv}(q^{-1}, \rho) = \mathcal{S}_2(q^{-1}, \rho) = \frac{G(q^{-1})}{1 + \frac{S(q^{-1},\rho)}{R(q^{-1})} G(q^{-1})} \tag{8}$$

$$\mathcal{S}_{yp}(q^{-1}, \rho) = \mathcal{S}_{uv}(q^{-1}, \rho) = \mathcal{S}_3(q^{-1}, \rho) = \frac{1}{1 + \frac{S(q^{-1},\rho)}{R(q^{-1})} G(q^{-1})} \tag{9}$$

$$\mathcal{S}_{ur}(q^{-1}, \rho) = \mathcal{S}_4(q^{-1}, \rho) = \frac{\frac{T(q^{-1},\rho)}{R(q^{-1})}}{1 + \frac{S(q^{-1},\rho)}{R(q^{-1})} G(q^{-1})} \tag{10}$$

$$\mathcal{S}_{up}(q^{-1}, \rho) = \mathcal{S}_5(q^{-1}, \rho) = -\frac{\frac{S(q^{-1},\rho)}{R(q^{-1})}}{1 + \frac{S(q^{-1},\rho)}{R(q^{-1})} G(q^{-1})} \tag{11}$$

A small tracking error for a given reference signal is usually a desired specification in many control problems. The tracking error sensitivity function can be defined as:

$$\mathcal{S}_6(q^{-1}, \rho) = \mathcal{S}_1(q^{-1}, \rho) - 1 = \frac{\frac{T(q^{-1},\rho) - S(q^{-1},\rho)}{R(q^{-1})} G(q^{-1}) - 1}{1 + \frac{S(q^{-1},\rho)}{R(q^{-1})} G(q^{-1})} \tag{12}$$

The performance and robust stability of most control problems can be defined by constraints on the infinity norm of the weighted sensitivity functions. A standard nominal performance control problem designs a controller satisfying $\|W_1 \mathcal{S}_{yp}\|_\infty < 1$ where W_1 represents the performance filter. If the system's uncertainty is represented by a multiplicative uncertainty

$$\tilde{G}(q^{-1}) = G(q^{-1})[1 + W_2(q^{-1})\Delta(q^{-1})] \text{ with } \|\Delta\|_\infty < 1 \tag{13}$$

the robust stability is given by (Doyle et al. [1992]):

$$\|W_2 \mathcal{S}_{yr}\|_\infty < 1 \tag{14}$$

In general, any upper bound condition on the previously mentioned sensitivity functions can be defined by the following constraints:

$$\|W_p \mathcal{S}_p\|_\infty < 1 \quad \text{for } p = 1, \ldots, 6 \tag{15}$$

These constraints are however non-convex on the controller parameters ρ. In this chapter, convex constraints are proposed on the Nyquist diagram to guarantee the following performance conditions:

$$|W_p(e^{-j\omega})\mathcal{S}_p(e^{-j\omega})| < 1 \quad \forall\omega \in [0,\pi] \quad \text{and for } p = 1,\ldots,6 \quad (16)$$

2.4 Control Problem

The following non-convex controller design problem is treated in this paper:

$$\begin{array}{l} \min \|W_k\mathcal{S}_k\|_\infty \\ \text{Subject to:} \\ \|W_\ell\mathcal{S}_\ell\|_\infty < 1 \quad \text{for} \quad \forall\ell \in A \end{array} \quad (17)$$

where A is a subset of the set $\{[1,\ldots,6] \setminus k\}$ where $\setminus k$ defines the exclusion of k.

Two different examples are given below:

(1) The infinity norm of the weighted tracking error sensitivity function $W_6\mathcal{S}_6$ should be minimized under an infinity norm constraint on the weighted sensitivity function $W_3\mathcal{S}_3$.
(2) The infinity norm of the weighted disturbance rejection function $W_3\mathcal{S}_3$ should be minimized under an infinity norm constraint on the weighted input sensitivity function $W_4\mathcal{S}_4$.

The main difference between these control problems is that for the first case, the norm to be minimized depends on $T(q^{-1},\rho)$ and $S(q^{-1},\rho)$, while for the second case, it depends only on $S(q^{-1},\rho)$. Consequently, if a 2 step strategy is applied, the method used to solve the problem varies for both examples.

For the first example, first $S(q^{-1},\rho)$ is designed with a feasibility problem satisfying the constraints in the weighted output sensitivity function $W_3\mathcal{S}_3$. Then, $T(q^{-1},\rho)$ is designed minimizing $\|W_6\mathcal{S}_6\|_\infty$ for a feasible $S(q^{-1},\rho)$. It is clear that, in the feasible set, it might be another polynomial $S(q^{-1},\rho)$ which gives a smaller value for $\|W_6\mathcal{S}_6\|_\infty$.

On the other hand, in the second example, first the $S(q^{-1},\rho)$ polynomial is designed by minimizing $\|W_3\mathcal{S}_3\|_\infty$. Then, the feedforward term $T(q^{-1},\rho)$ is designed with a feasibility problem assuring the weighted sensitivity function constraints in $W_4\mathcal{S}_4$. In this example, the feasibility problem of the second step might not have a solution. However, in the case that is feasible, the solution would be optimal.

Obviously, if $S(q^{-1},\rho)$ and $T(q^{-1},\rho)$ polynomials are designed in the same optimization problem better results can be obtained. This is shown in Section 4 with an experimental example.

Note that the classical H_∞ optimization solution using LFT cannot directly solve this problem. In this method, the infinity norm of the weighted transfer function matrix of the augmented system is minimized. Consequently, it is impossible to minimize the infinity norm of a specific weighted sensitivity transfer function together with constraints on other sensitivity functions which is typically the case on most of the practical problems.

3. CONVEX APPROXIMATION OF THE BOUNDS ON SENSITIVITY FUNCTIONS

The constraints presented in (16) and in the optimization problem (17) are non-convex on controller parameters ρ. The idea is to approximate these constraints by a set of convex constraints in the Nyquist diagram that guarantees the robustness and/or performance conditions .

Let the inequality in (16) be multiplied by

$$|1 + \frac{S(e^{-j\omega},\rho)}{R(e^{-j\omega})}G(e^{-j\omega})|.$$

Then, define:

$$\begin{aligned} &\tilde{W}_p(e^{-j\omega})X_p(e^{-j\omega},\rho) \equiv \\ &\qquad W_p(e^{-j\omega})\mathcal{S}_p(e^{-j\omega},\rho)\left[1 + \frac{S(e^{-j\omega},\rho)}{R(e^{-j\omega})}G(e^{-j\omega})\right] \\ &\qquad\qquad \omega \in [0,\pi] \quad \text{and for } p = 1,\ldots,6 \quad (18) \end{aligned}$$

where $\tilde{W}_p(e^{-j\omega})$ and $X_p(e^{-j\omega},\rho)$ are, respectively, the fixed term and the term depending linearly on ρ. Then, the constraint in (16) can be re-written as:

$$\begin{aligned} &|\tilde{W}_p(e^{-j\omega})X_p(e^{-j\omega},\rho)| < |1 + \frac{S(e^{-j\omega},\rho)}{R(e^{-j\omega})}G(e^{-j\omega})| \\ &\qquad \forall\omega \in [0,\pi] \quad \text{and for } p = 1,\ldots,6 \quad (19) \end{aligned}$$

Note that $|1 + \frac{S(e^{-j\omega},\rho)}{R(e^{-j\omega})}G(e^{-j\omega})|$ is the distance in the Nyquist diagram between the critical point $(-1+0j)$ and $L(e^{-j\omega},\rho)$. Hence, the constraints in (19) are satisfied if and only if, in the Nyquist diagram, the circle centered at the critical point $(-1+0j)$ with a radius of $|\tilde{W}_p(e^{-j\omega})X_p(e^{-j\omega},\rho)|$ does not contain the point $L(e^{-j\omega},\rho)$ for all ω.

The condition that the point $L(e^{-j\omega},\rho)$ is outside the circle centered at the critical point with a radius of $|\tilde{W}_p(e^{-j\omega})X_p(e^{-j\omega},\rho)|$ is a non-convex constraint on controller parameters ρ. This non-convex constraint can be approximated with a convex one if we replace the circle of radius $|\tilde{W}_p(e^{-j\omega})X_p(e^{-j\omega},\rho)|$ by a fixed line $d(\omega)$ tangent to the circle (Fig. 2). The line $d(\omega)$ divides the Nyquist complex plane into two parts at each frequency ω. Now, the condition that the point $L(e^{-j\omega},\rho)$ is at the side of the line $d(\omega)$, that excludes the critical point, is a convex constraint on the controller parameters ρ. The conservatism of this approximation can be reduced if the slope of $d(\omega)$ changes with frequency. A progressive variation of the slope is proposed using the frequency response of an available desired open-loop transfer function $L_d(e^{-j\omega})$. The line $d(\omega)$ at each frequency ω can be defined orthogonal to the line connecting the critical point $(-1+0j)$ to $L_d(e^{-j\omega})$ and tangent to the circle centered at the critical point with a radius of $|\tilde{W}_p(e^{-j\omega})X_p(e^{-j\omega},\rho)|$. The choice of $L_d(e^{-j\omega})$ should be coherent with respect to the plant, controller structure and design specifications. A discussion on the choice of $L_d(e^{-j\omega})$ is given in Karimi and Galdos [2010].

If we name x and y respectively, the real and imaginary parts of a point on the complex plane, the equation of $d(\omega)$ is given by:

$$|\tilde{W}_p(e^{-j\omega})X_p(e^{-j\omega},\rho)[1+L_d(e^{-j\omega})]| \\ -I_m\{L_d(e^{-j\omega})\}y-[1+R_e\{L_d(e^{-j\omega})\}](1+x)=0 \\ \forall\omega\in[0,\pi] \quad \text{and} \quad \text{for } p=1,\ldots,6 \quad (20)$$

where $R_e\{\cdot\}$ and $I_m\{\cdot\}$ represent respectively the real and imaginary parts of a complex value. The condition that the point $L(e^{-j\omega},\rho)$ is on the side of the line $d(\omega)$ excluding the critical point $(-1+0j)$ is given by the following convex constraints:

$$|\tilde{W}_p(e^{-j\omega})X_p(e^{-j\omega},\rho)[1+L_d(e^{-j\omega})]| \\ -I_m\{L_d(e^{-j\omega})\}I_m\{L(e^{-j\omega},\rho)\} \\ -[1+R_e\{L_d(e^{-j\omega})\}](1+R_e\{L(e^{-j\omega},\rho))<0 \\ \forall\omega\in[0,\pi] \quad \text{and} \quad \text{for } p=1,\ldots,6 \quad (21)$$

These convex constraints can be simplified to:

$$|\tilde{W}_p(e^{-j\omega})X_p(e^{-j\omega},\rho)[1+L_d(e^{-j\omega})]|- \\ R_e\{[1+L_d^*(e^{-j\omega})][1+\frac{S(e^{-j\omega},\rho)}{R(e^{-j\omega})}G(e^{-j\omega})]\}<0 \\ \omega\in[0,\pi] \quad \text{and for } p=1,\ldots,6 \quad (22)$$

where $L_d^*(e^{-j\omega})$ is the complex conjugate of $L_d(e^{-j\omega})$.

The objective function of the control problem in (17) is also a non-convex function on ρ. Let modify the optimization problem in (17) into:

$$\min \gamma$$

Subject to:

$$|\frac{\tilde{W}_k(e^{-j\omega})}{\gamma}X_k(e^{-j\omega},\rho)[1+L_d(e^{-j\omega})]|- \\ R_e\{[1+L_d^*(e^{-j\omega})][1+\frac{R(e^{-j\omega},\rho)}{S(e^{-j\omega})}G(e^{-j\omega})]\}<0 \\ \text{for } \omega\in[0,\pi] \quad (23)$$

$$|\tilde{W}_\ell(e^{-j\omega})X_\ell(e^{-j\omega},\rho)[1+L_d(e^{-j\omega})]|- \\ R_e\{[1+L_d^*(e^{-j\omega})][1+\frac{R(e^{-j\omega},\rho)}{S(e^{-j\omega})}G(e^{-j\omega})]\}<0 \\ \text{for } \omega\in[0,\pi] \quad \text{and} \quad \forall\ell\in A$$

This optimization problem can be solved with an iterative bisection algorithm. At i-th iteration, a feasibility optimization problem is solved with the constraints given in (23) for a fixed value of γ_i. If the problem is feasible, γ_{i+1} will be chosen smaller than γ_i, and if the problem is infeasible, γ_{i+1} will be increased. This is solved using standard solvers. Note that for fixe values of γ, the constraints in the optimization problem (23) are convex on controller parameters ρ.

Remarks:

(1) It should be mentioned that $X_\ell(e^{-j\omega},\rho) = 1$ for $\ell = 2,3$ which gives linear constraints on controller parameters ρ.
(2) The multi-model uncertainty can be directly considered by repeating the constraints for each model. Moreover, if the same weighting filters are considered for all models, the stability and performance will be satisfied for all models in the convex combinations of the models in $\mathcal{G}$.
(3) The convex optimization problem in (23) has an infinite number of constraints for $\omega \in [0,\pi]$. By an appropriate frequency gridding the number of constraints becomes finite and the problem can be solved by a standard SDP solver. A full discussion on the frequency gridding for this approach can be found in Galdos et al. [2010].

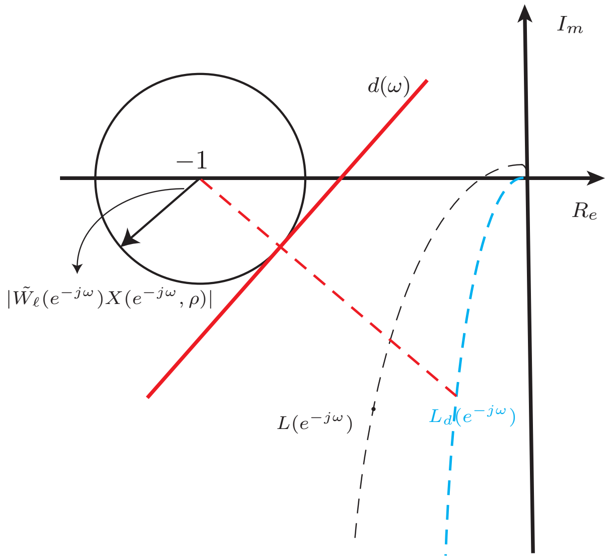

Fig. 2. Convex constraint for performance condition in Nyquist diagram

4. EXPERIMENTAL RESULTS

In this section, the proposed approach is applied to a double-axis LPMSM. The goal is to control the position of the mentioned system shown in Fig. 3, using an RST discrete-time controller with a sampling frequency of 6 kHz. This kind of systems have high dynamics (high accelerations and decelerations), high mechanical stiffness, reduced friction and high accuracy as there is no backlash and no mechanical transmission. For simplicity, only the system identification and controller design for the higher axis is analyzed when the motor is in three different positions at -0.16m, 0m and 0.16m of the x axis.

4.1 Non-parametric identification of the dynamics of the higher axis

A spectral model is obtained by exciting the higher axis with a sum of sinusoidal signals from 4.4 to 3000 Hz for each position of the x axis. Three different frequency response functions (FRF) $G_i(e^{-j\omega})$ are obtained for $i = 1,2,3$ which are shown in Fig. 4. Though the model uncertainty is not significant, the controller is designed using the multi-model uncertainty to show the effectiveness of the method with this type of systems.

4.2 RST controller design of the higher axis

The objective is to design an RST controller by tuning the polynomials S and T given in (3) and (4) using the proposed method. A second-order feedback controller and a second-order pre-compensator are designed by choosing n_S and n_T equal to 3. The controller should stabilize the three models and reduce the infinity norm of the tracking error sensitivity function $\mathcal{S}_{6_i}$.

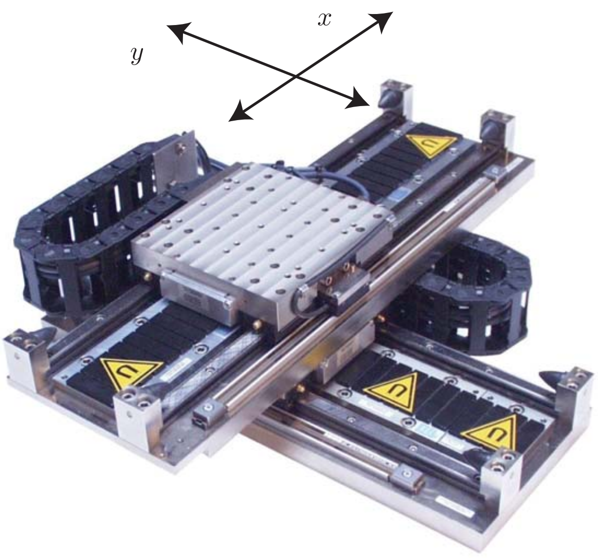

Fig. 3. Double-axis linear permanent magnet synchronous motor system

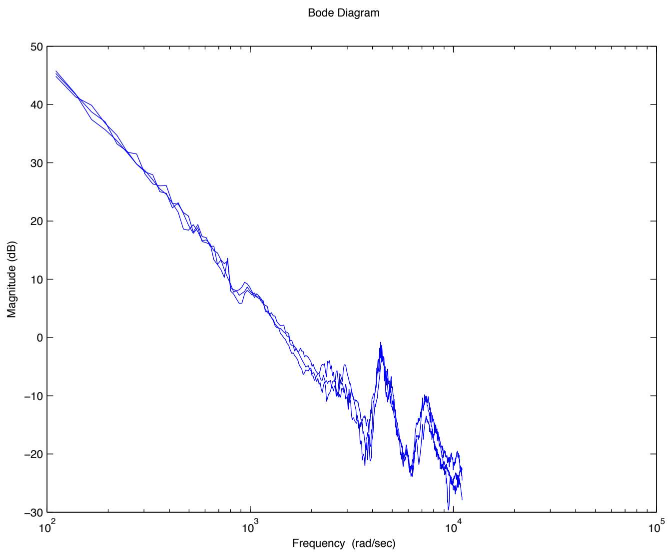

Fig. 4. FRF of the double-axis system at -0.16m, 0m and 0.16m of the x axis

Since the design of the controller is carried out in the frequency domain, the weighting filters can be defined in continuous-time. A modulus margin of 0.5 (i.e. maximum value of less than 6 dB of the magnitude of the output sensitivity functions $\mathcal{S}_{3_i}$) is desirable to assure the robustness of the controller. This is obtained by choosing $W_{3_i}(j\omega) = 0.5$ for all ω and for all i. On the other hand, due to the friction appearing as a constant input disturbance, an integrator is needed in the controller to reject it. Therefore, the open-loop transfer functions should contain three integrators, one because of the controller and two others because of the plant (the system can be considered as 2 pure integrators). It is shown in Zhou [1996] that internal cancelation between integrators and the zeros of the controller can be avoided by introducing the desired integrators in the weighting filters $W_{3_i}(j\omega)$. The desired bandwidth for the closed-loop disturbance rejection ω_c is chosen 300 rad/s. As a result, the $W_{3_i}(j\omega)$ weighting filters are designed as a triple integrator with a bandwidth of ω_c. The combination of the robustness and performance conditions defines the following weighting filters:

$$W_{3_i}(j\omega) = \begin{cases} \frac{\omega_c^3}{(j\omega)^3} & \text{for } |\frac{\omega_c^3}{(j\omega)^3}| > 0.5 \\ 0.5 & \text{for } |\frac{\omega_c^3}{(j\omega)^3}| < 0.5 \end{cases} \tag{24}$$

for $i = 1, 2, 3$.

In order to reduce the tracking error for any 2-norm bounded reference signal, constraints on the infinity norm of the weighted tracking error functions $\|W_{6_i}\mathcal{S}_{6_i}\|_\infty < \gamma$ are considered where γ should be minimized. To assure good tracking at low frequencies, a low-pass filter of a triple integrator with the previous bandwidth of ω_c is used as weighted function W_{6_i}. For frequencies higher than ω_c, a unit gain is considered:

$$W_{6_i}(j\omega) = \begin{cases} \frac{\omega_c^3}{(j\omega)^3} & \text{for } \omega < \omega_c \\ 1 & \text{for } \omega > \omega_c \end{cases} \tag{25}$$

Figure 5 and **??** show the inverse of the filters that have been chosen. A stabilizing controller tuned by the manufacturer is already available:

$$\begin{aligned} R_0(q^{-1}) &= 1 - q^{-1} \\ S_0(q^{-1}) &= 1.4661 - 2.7567q^{-1} + 1.3021q^{-2} \\ T_0(q^{-1}) &= 9.2439 - 26.0901q^{-1} \\ &\quad +24.6354q^{-2} - 7.7778q^{-3} \end{aligned} \tag{26}$$

Therefore, $L_{d_i}(e^{-j\omega})$ is chosen as $K_0(e^{-j\omega})G_i(e^{-j\omega})$ for $i = 1, 2, 3$ where $K_0(q^{-1}) = S_0(q^{-1})/R_0(q^{-1})$. This controller leads to

$$\|W_6\mathcal{S}_{6_0}\|_\infty = \gamma_0 = 3.670445 \tag{27}$$

The goal is to minimize the maximum value of $\|W_{6_i}\mathcal{S}_{6_i}\|_\infty$ for $i = 1, 2, 3$. In order to obtain the optimal controller, the optimization problem in (23) is solved where the constraints for each system are repeated and using the weighting filters given in (24) and (25). The optimization problem leads to

$$\|W_6\mathcal{S}_6\|_\infty = \gamma_1 = 1.830908 \tag{28}$$

The resulting controller is:

$$\begin{aligned} R_1(q^{-1}) &= 1 - q^{-1} \\ S_1(q^{-1}) &= 2.392522 - 4.538816q^{-1} + 2.154203q^{-2} \\ T_1(q^{-1}) &= 2.038411 - 3.854601q^{-1} + 1.824099q^{-2} \end{aligned} \tag{29}$$

The same problem is solved based on the method proposed in Karimi et al. [2008] using the two-step strategy as explained in Section 2.4. This leads to:

$$\|W_6\mathcal{S}_{6_2}\|_\infty = \gamma_2 = 1.906136 \tag{30}$$

The resulting controller is:

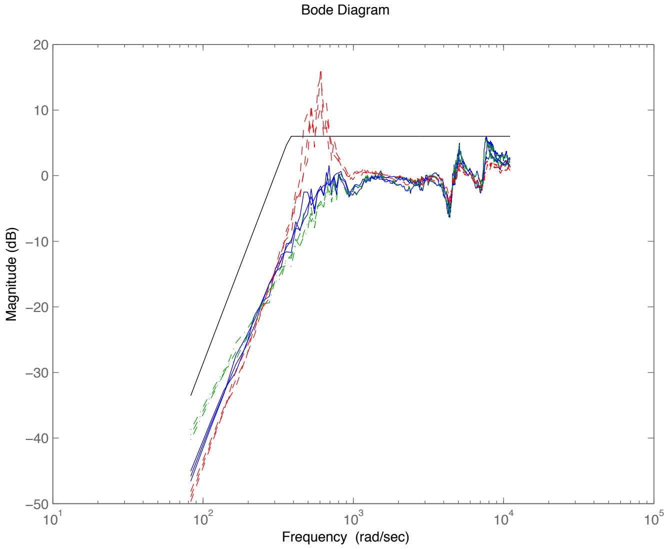

Fig. 5. The performance conditions on $\mathcal{S}_{3_i}$: Proposed controller (solid, blue), 2 Step controller (dashed-dotted, green), Manufacturer's controller (dashed, red) and $1/W_{3_i}$ (solid, black) for $i = 1, 2, 3$.

$$\begin{aligned} R_2(q^{-1}) &= 1 - q^{-1} \\ S_2(q^{-1}) &= 2.283279 - 4.298835q^{-1} + 2.017491q^{-2} \\ T_2(q^{-1}) &= 2.228780 - 4.198459q^{-1} + 1.971614q^{-2} \end{aligned} \tag{31}$$

Figure 5 shows the upper bound $1/W_{3_i}$ and the sensitivity functions $\mathcal{S}_{3_i}$ for each controller. Note that, the proposed controller and that obtained by using the 2-step approach verify the condition while the controller designed by the manufacturer does not. It should be mentioned that the controller designed by the manufacturer has not been designed for the same purpose. As expected, the proposed controller obtains the smallest maximum value for the weighted sensitivity functions.

5. CONCLUSIONS

The robust fixed-order RST controller design problem with H_∞ constraints on the closed-loop sensitivity functions is formulated as a convex optimization problem. Solving only one convex optimization problem, T and S polynomials of the RST controller with a fixed R polynomial are tuned for a SISO system. The approach is based on frequency loop shaping in the Nyquist diagram where the sensitivity function shaping conditions are approximated with convex constraints. This method requires only the frequency response of the system. In contrast to the standard H_∞ controller design method, systems with time-delay can be considered directly by this method and the multi-model uncertainty can be taken into account without any approximation. The desired infinity-norm of one weighted sensitivity functions can be minimized using a bi-section algorithm. The application of the proposed method on an industrial double-axis LPMSM system shows the effectiveness of the method.

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