Two-degree-of-freedom Control of a PMSM Drive without Mechanical Sensor

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Abstract – **For high performance application, fast acting current loops using the PI controllers are usually employed for PMSM drive. Although this leads to a simple design, it is difficult to achieve good command tracking and load regulation characteristics simultaneously. In this paper, we investigate a new approach for both the current loops and the speed loop based on the RST method. In the proposed scheme, a feedback controller and a feedforward compensator have been used to accomplish two-degree-of-freedom of control. To estimate the rotor position, a minimal order observer has been developed in the $\alpha\beta$ co-ordinate. By the elimination of the expensive mechanical sensors, we have achieved the mechanical sensorless control of the drive. Simulation and experimental results have verified the effectiveness of both the control and estimation schemes.**

I. INTRODUCTION

The permanent magnet synchronous motor (PMSM) is used widely for servo applications due to its high efficiency, low inertia, compact size and ease of maintenance. Usually the cascade control scheme is employed to control the motor. The outer one is the speed control loop whose output provides the desired current reference for the inner current loop. Since the outer loop is constrained to response more slowly than the inner loop, the dynamic performance of the drive is highly dependent on the bandwidth of the current control loop. For a PMSM drive, the d-axis and q-axis current loops are coupled with each other. Moreover, the q-axis current loop is influenced by the back emf of the motor. To control the currents in these loops accurately, the current controllers should be insensitive to these disturbances.

Historically, current control was exercised either with the hysteresis controllers in the 3-phase stator axes or the PI controllers in the stationary or rotary co-ordinate. As it is difficult to eliminate the steady state errors in the stationary reference frame, current controllers that are implemented in the rotor reference frame give better performance. For the PI controller, it is designed to place the dominant poles of the system at desired locations. However, it is difficult to achieve good command tracking and load regulation property simultaneously as the closed-loop zeros cannot be placed arbitrarily. To overcome this shortcoming, this paper uses the two-degree-of-freedom (2DOF) structure to design the current controller in the *dq* reference frame. The preferred closed-loop characteristic is attained with a feedback controller and the accurate tracking is acquired with a feedforward compensator. The adoption of the 2DOF structure to the speed control of electrical machines has been reported [1,2], while the 2DOF control of the current loops has not been well investigated. In this paper, we demonstrate the usefulness of the RST controller [3], which can provide 2DOF of control to both the current and speed loops. The resultant system is insensitive to disturbances including the inter-loop couplings and the load torque.

For a PMSM drive, the position of the rotor is required for commutation and control. This is usually achieved with an absolute encoder or a resolver. This shaft sensor has such drawbacks as high cost, space occupation, additional wires and susceptibility to temperature. Therefore, the eradication of such sensors has become a subject of wide interest [4-8]. In [8], a position observer using the measured motor currents and voltages was developed based on the assumption that the velocity is approximately constant over the transient period of the inner current loop. In this paper, we use the same assumption but design the observer in the discrete-time domain. A truncated discrete-time model is adopted and the resultant modelling error is analysed. Both the control and estimation algorithms have been implemented with a DSP. The experimental results have verified that the proposed schemes work well for our mechanical sensorless PMSM drive.

II. DYNAMIC MODEL OF THE PMSM

The motor model in the *dq* co-ordinate attached to the rotor is presented in (1),

$$
\begin{aligned}
v_q &= ri_q + L_q \frac{di_q}{dt} + pL_d i_d \omega_m + k_e \omega_m \\
v_d &= ri_d + L_d \frac{di_d}{dt} - pL_q i_q \omega_m
\end{aligned} \tag{1}
$$

where v_q, v_d, i_q, i_d, L_q and L_d are the voltages, currents and inductances in the *dq* reference frame. r is the stator resistance, k_e is the back emf constant, ω_m is the mechanical speed and p is the number of pole-pairs of the motor. The electromagnetic torque is given as,

$$
T_e = \tfrac{3}{2}\left[k_e i_q + \left(L_d - L_q\right) i_d i_q\right]. \tag{2}
$$

In this paper, the motor under consideration is a surface mounted PMSM. Consequently, $L_d = L_q = L_s$. Thus, the electromagnetic torque is proportional to the q-axis current. As i_d does not contribute to the torque, it is to be controlled near zero to achieve maximum torque per ampere in the speed range up to the rated speed. i_q is controlled to track the desired reference demanded by the speed controller.

III. DESIGN OF THE RST CONTROLLERS

A. i_q controller

The block diagram of the overall control system is displayed in Fig. 1. The typical cascade control structure consisting of the outer speed loop and the inner current loop is used in the system. As shown in Fig. 1, the output of the speed controller sets the desired current commands for the current loops. The d-axis and q-axis voltages are calculated with the RST controllers. In the set-up, no mechanical sensors are employed. The rotor position is estimated with a minimal order observer using the measured currents and voltages.

From the model of the PMSM, the q-axis current can be derived as,

$$I_q(s) = \frac{(Js+D)}{L_sJs^2+(L_sD+Jr)s+(k_ek_t+Dr)}V_q(s) - \frac{(Js+D)pL_s}{L_sJs^2+(L_sD+Jr)s+(k_ek_t+Dr)}I_d(s)\Omega_m(s) - \frac{k_e}{L_sJs^2+(L_sD+Jr)s+(k_ek_t+Dr)}T_L(s), \tag{3}$$

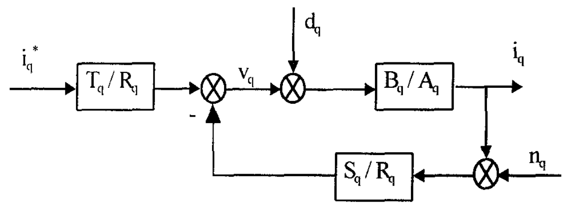

Fig. 2. Block diagram of the q-axis current loop

where J is the moment of inertia, D is the damping coefficient, T_L is the load torque and k_t is the torque constant. Equation (3) shows that the q-axis current is influenced by a non-linear product term of i_d and ω_m. Moreover, it also surfers from the load disturbance T_L. To realise accurate command tracking, the i_q controller must be insensitive to these disturbances.

The block diagram of the i_q loop with a RST controller is shown in Fig. 2, where B_q / A_q , S_q / R and T_q / R_q are pulse-transfer functions of the plant, the feedback controller and the feedforward compensator respectively. Under the control law $R_q v_q = T_q i_q^* - S_q i_q$, the closed-loop system is described by (4),

$$i_q = \frac{B_qT_q}{A_qR_q+B_qS_q}i_q^* + \frac{B_qR_q}{A_qR_q+B_qS_q}d_q - \frac{B_qS_q}{A_qR_q+B_qS_q}n_q, \tag{4}$$

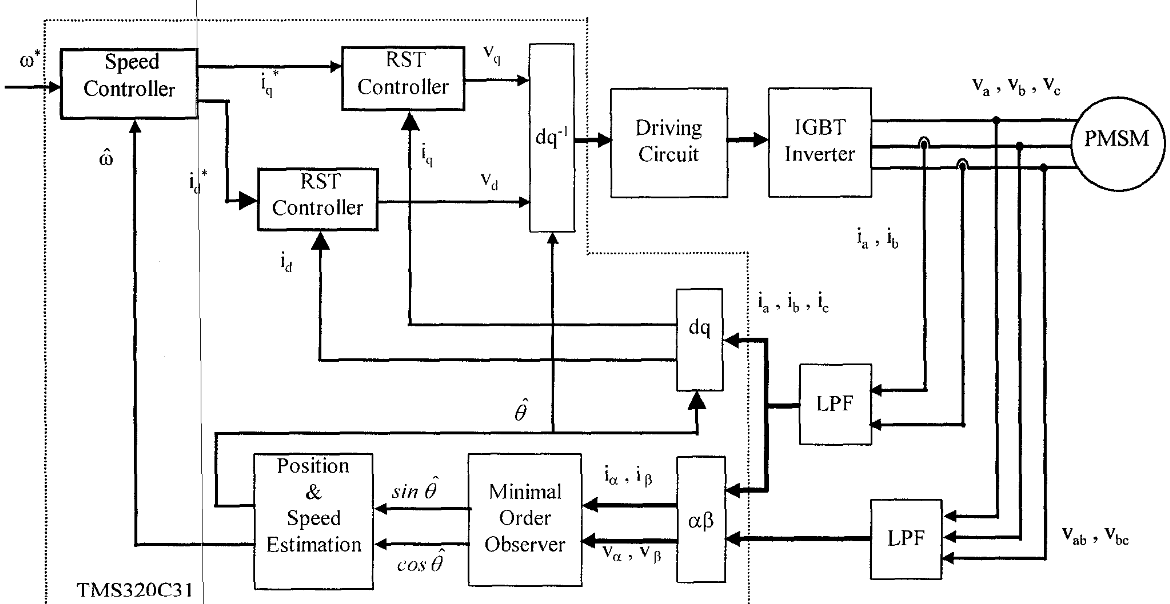

Fig. 1. Block diagram of the PMSM control system

where i_q^* is the current reference, d_q is the disturbance including the inter-loop coupling and the load torque, n_q is the measurement noise. Equation (4) shows that the responses of i_q to the disturbance and measurement noise are determined by the feedback controller S_q / R_q, while the response to the reference signal i_q^* can be tuned with T_q. Consequently, we can design S_q and R_q to acquire good regulation property, and design T_q to improve the tracking performance.

Let the desired closed-loop pulse-transfer function be B_{qm} / A_{qm}, then the design problem is to find suitable polynomials R_q, S_q and T_q to satisfy,

$$\frac{B_{qm}}{A_{qm}} = \frac{B_q T_q}{A_q R_q + B_q S_q} \tag{5}$$

To solve (5), we factorize B_q into $B_q^+ B_q^-$. Let $B_q^+ = z - z_{q0}$ where z_{q0} is a minimum-phase zero of the open-loop system which is close to 1. We select B_q^+ as a factor of R_q to cancel out this slow zero. We also introduce $(z-1)$ to R_q as an integration to reduce the influence of low-frequency perturbations. Consequently, R_q is formulated as $R_q = \overline{R}_q B_q^+ (z-1)$. From the Euclid's algorithm, there exists a causal solution to (5) if the following conditions are satisfied [3],

$$\begin{aligned} &\deg A_{qm} - \deg B_{qm} \geq \deg A_q - \deg B_q, \\ &\deg S_q < \deg A_q + l_q, \\ &\deg \overline{R}_q = \deg A_{qm} - \deg A_q - l_q, \end{aligned} \tag{6}$$

where l_q is the order of the integration in R_q.

If we choose the desired closed-loop system to be a 3rd-order system, then $\deg S_q = 2$ and $\deg \overline{R}_q = 0$. Let $S_q = S_{q2} z^2 + S_{q1} z + S_{q0}$ and substitute it into the denominator polynomial of the desired closed loop pulse transfer function $A_{qm} = A_q (z-1) + B_q^- S_q$, the coefficients of S_q can be easily determined. After the cancellation of B_q^+ in (5), $T_q = B_{qm} / B_q^-$. Thus, we get the feedback controller S_q / R_q and the feedforward compensator T_q / R_q.

B. i_d controller

The equation for the d-axis current can be written as

$$I_d(s) = \frac{1}{L_s s + r} V_d(s) + \frac{p L_s}{L_s s + r} I_q(s) \Omega(s). \tag{7}$$

Thus, i_d surfers from a disturbance caused by i_q and motor speed. Follow the similar design procedure as in the q-axis current controller, we include $(z-1)$ in R_d and let the overall system B_{dm} / A_{dm} be a 2nd-order system. Then the design is to find the polynomials R_d, S_d and T_d that satisfy the following equation,

$$\frac{B_{dm}}{A_{dm}} = \frac{B_d T_d}{A_d R_d + B_d S_d}. \tag{8}$$

C. Speed controller

For a well designed current controller, $i_q \approx i_q^*$, $i_d \approx i_d^*$. As the dynamics of the current loop is much faster than that of the speed loop, the speed loop can be described by a first-order system as follows,

$$\Omega(s) = \frac{k_t}{Js + D} I_q(s) - \frac{1}{Js + D} T_L(s). \tag{9}$$

In practice, the speed is calculated from two consecutive position data using backward difference equation. This estimated speed is usually noisy. In our experimental system, a 2nd-order Bessel filter is used to filter the noise,

$$\frac{N_f(z)}{D_f(z)} = \frac{b_{f1} z + b_{f0}}{z^2 + a_{f1} z + a_{f0}} \tag{10}$$

To include the influence of this filter, we incorporate it into the model of the speed loop. The resultant closed-loop system is of 3rd-order. The polynomials R_ω, S_ω and T_ω are obtained using the similar approach as in the previous sections.

IV DEVELOPMENT OF THE MINIMAL ORDER POSITION OBSERVER

For a PMSM drive, the position of the rotor is required for electronic commutation and vector control. To eliminate the need of an expensive position sensor, an observer has been developed to estimate the rotor position based on the measurement of the line voltages and currents of the motor. By replacing the mechanical sensors with the proposed observer, we have achieved the sensorless control of the PMSM successfully.

A. Motor model for the observer

To reconstruct the rotor position with a minimal order observer, we model the motor in the two-phase stationary reference frame ($\alpha\beta$). Unlike the dq transformation, the $\alpha\beta$ transformation is linear and independent of the rotor position to be estimated.

Define the state variables, input variables and the output variables as $\mathbf{X} = [i_\alpha, i_\beta, \sin\theta_e, \cos\theta_e]'$, $\mathbf{U} = [u_\alpha, u_\beta]'$ and $\mathbf{Y} = [i_\alpha, i_\beta]'$, the motor model in the $\alpha\beta$ frame is,

$$\begin{aligned} \dot{\mathbf{X}} &= \mathbf{A}(\omega_m) \cdot \mathbf{X} + \mathbf{B} \cdot \mathbf{U} \\ \mathbf{Y} &= \mathbf{C} \cdot \mathbf{X}. \end{aligned} \tag{11}$$

The discrete-time model can be expressed as,

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{A}(k) \cdot \mathbf{X}(k) + \mathbf{B}(k) \cdot \mathbf{U}(k), \\ \mathbf{Y}(k) &= \mathbf{C} \cdot \mathbf{X}(k). \end{aligned} \tag{12}$$

Equation (11) shows that the PMSM model is non-linear and time varying as the system matrix $\mathbf{A}(\omega_m)$ is a function of the motor speed ω_m. Assuming that ω_m varies slowly with respect to the sampling time T_s, we can regard the system as a pseudo linear time-invariant system within one sampling period.

As $A(k)$ in (12) is an infinite power series, certain form of truncation is necessary to calculate it in practice. For simplicity, we use the following truncated equations to approximate the model,

$$A(k) = I + A(\omega_m)T_s, \\ B(k) = BT_s. \tag{13}$$

The accuracy of the truncation can be evaluated using the following equation,

$$\zeta = \| e^{A(\omega_m)\cdot T_s} - (I + A(\omega_m)\cdot T_s) \|_F / \| e^{A(\omega_m)\cdot T_s} \|_F , \tag{14}$$

where the Frobenius norm is given as $\|X\|_F \hat{=} (\sum_{i,j} x_{ij}^2)^{\frac{1}{2}}$.

The relative error under different sampling times and speeds are depicted in Fig. 3. The curves indicate that ζ is smaller than *2%* at a speed of *1200* rpm with a sampling period of *0.2* ms.

B. Design of the minimal order observer

To design a minimal order observer, we partition the motor model:

$$\begin{bmatrix} \mathbf{X}_A(k+1) \\ \mathbf{X}_B(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11}(k) & \mathbf{A}_{12}(k) \\ \mathbf{A}_{21}(k) & \mathbf{A}_{22}(k) \end{bmatrix} \begin{bmatrix} \mathbf{X}_A(k) \\ \mathbf{X}_B(k) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} \mathrm{u}(k) , \tag{15}$$

where $X_A = [i_\alpha(k),\ i_\beta(k)]^T$ and $X_B = [sin\theta_e(k),\ cos\theta_e(k)]^T$. Defining $Z(k)$ as an auxiliary state vector, we obtain the minimal order observer as,

$$\hat{Z}(k+1) = D(k)\cdot \hat{Z}(k) + E(k)\cdot Y(k) + G(k)\cdot U(k), \\ \hat{X}_B(k) = \hat{Z}(k) + V_2(k)Y(k), \tag{16}$$

where $D(k)$, $E(k)$ and $G(k)$ are design matrices [9]. With a properly chosen matrix $D(k)$, the estimation error will be reduced to zero exponentially. From the estimated states $\hat{sin}(\theta_e(k))$ and $\hat{cos}(\theta_e(k))$, the electrical position of the motor is obtained as,

$$\hat{\theta}_e(k) = tan^{-1}\left(\frac{\hat{sin}(\theta_e(k))}{\hat{cos}(\theta_e(k))}\right) . \tag{17}$$

V. SIMULATION STUDY

In the experiments, we formulate the closed-loop systems according to the ITAE criterion [10]. The simulation results for a step change of the speed command from *300* rpm to *1200* rpm are presented in Fig. 4. In this case, the motor is loaded at *2* Nm. Fig. 4(b) shows that a current of *4.2 A* is produced to accelerate the motor. When the speed reference drops to 300 rpm at 1.2 s, i_q decreases and the speed settles down to *300* rpm. It is shown in Figs. 4(c) that i_d is very small due to the regulation of the i_d controller.

VI. EXPERIMENTAL RESULTS

An *8* poles PMSM rated at *1.78* kW, *334* V and *3.4* A is used in the experimental system. A torque brake is connected to the PMSM as a load. The motor is driven by a 3-phase IGBT inverter. The motor position is estimated by the proposed observer. All the control and estimation algorithms have been implemented with a DSP.

The dynamic response of the PMSM under the step change of the speed reference is depicted in Fig. 5 under a load of *1.9* Nm. The steady-state value of i_q is *2* A under a speed reference of *300* rpm. At t = *0.2* s, the speed reference is increased from *300* rpm to *1200* rpm. From Fig. 5(b), we observe that i_q rises to accelerate the motor. It takes about *60* ms for the motor to reach the new speed

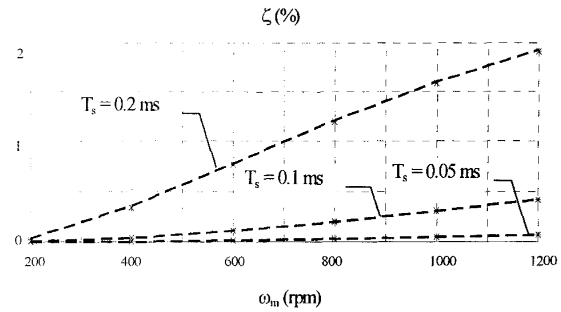

Fig. 3. Frobenius value curves of relative evaluation error

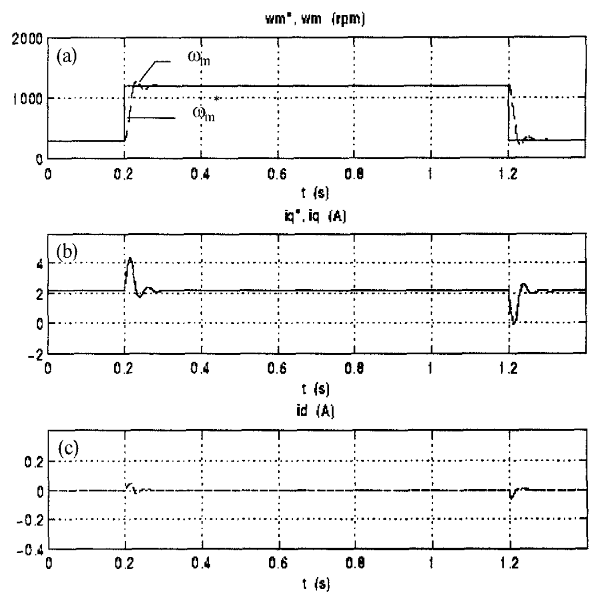

Fig. 4. Simulation results with the step change of speed reference

reference. To test the dynamic behaviour of the drive under load disturbance, we assert a dynamic load of *1.5* Nm to the motor at *0.5*s while it runs at *800* rpm. As shown in Fig. 6, the speed drops slightly while i_q increases to reject the disturbance. When we remove the load at 2.6s, i_q returns back to the previous level. In both the tests, i_d is maintained at closed to zero as shown in Figs. 5(c) and 6(c).

VII. CONCLUSIONS

The development of the RST controllers for the d-axis, q-axis current loops and the speed loop of a PMSM drive have been discussed in this paper. With the RST approach, 2DOF control has been achieved by designing the feedback and feedforward controllers separately. With the proposed approach, both the closed-loop poles and zeros can be placed arbitrarily to satisfy the preferred tracking performance and regulation property. A discrete time minimal order observer has been used in the experimental system to achieve the mechanical sensorless control of the PMSM. The effectiveness of the proposed control and estimation schemes has been verified by simulation and experimental results.

VIII. REFERENCES

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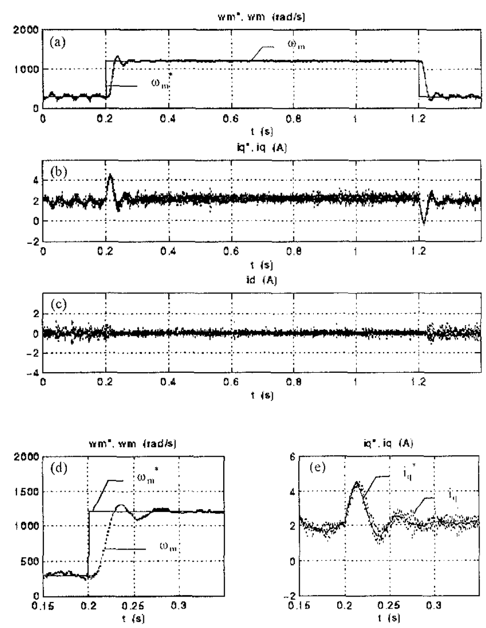

Fig. 5. Experimental results under the step change of speed reference

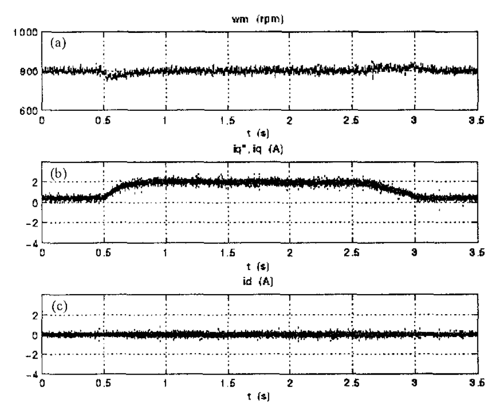

Fig. 6. Experimental results under load change