**PRACTICE 3 – Z-TRANSFORM AND CAUSAL LTI DISCRETE -TIME SYSTEMS DESCRIBED AS LDECC**

**OBJECTIVES:**

1. Calculate the z-transform of special sequences x[n].
2. Calculate the inverse z-transform of rational functions.
3. Obtain the system function H(z) of causal LTI discrete-time systems described as LDEcc.

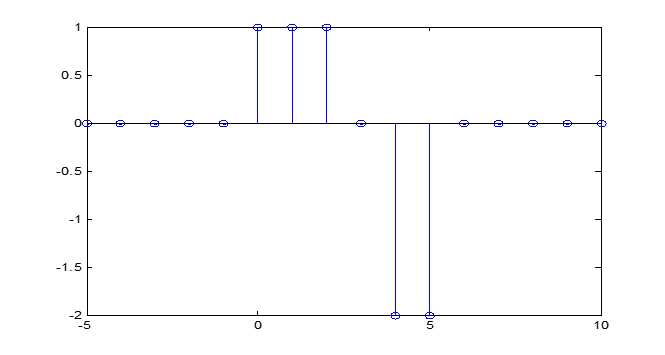
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1. **Z-transform of special sequences.**

The Z-transform of a sequence x[n] is defined as:

The set of values of z for which the infinite sum converges is called the *region of convergence* (ROC).

1. Calculate the Z-transform and ROC of the following finite sequence x[n]:



>> n = -5:10;

>> x = [0, 0, 0, 0, 0, 1, 1, 1, 0, -2, -2, 0, 0, 0, 0, 0];

>> stem(n,x)

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| 1 + 1/z + 1/z^2 - 2/z^4 - 2/z^5  ROC = {z ≠ 0} |

1. Define a MATLAB function with input parameters, the finite sequence x[n], and output parameter, the Z-transform symbolic function X(z).

Remember that the MATLAB instruction “syms” creates symbolic variables and functions. For example:

syms X(z) % Create the symbolic function X(z)

X(z) = z/(z-1) % Specify the formula for X

>> X(z) = z/(z - 1)

To evaluate the function for some z=z0, write X(z0).

function [X] = z\_trans\_finite(x,n)

% x is the input sequence and n is the range of the discrete independent variable

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| **z\_trans\_finite.m** |
| function [X] = z\_trans\_finite(x,n) % x is the input sequence and n is the range of the discrete independent variable  syms z  X(z) = sum(x.\* z.^(-n))  end |

1. Try to find the z-transforms of the sequences below by using the MATLAB function “ztrans”, defining first the symbolic variable n (syms n). Specify the ROC too.

impulse = n==0;

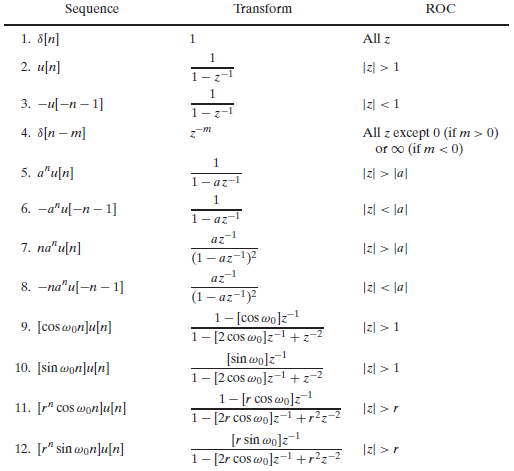
unitstep = n>=0;

ramp = n.\*unitstep;

quad = n.^2.\*unitstep;

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| **Sequence** | **Using ztrans** | **Theoretical z-transform using tables or properties** | **Domain** |
| x[n] = 0.3n·u[n] | syms n  X=ztrans((0.3)^n);  >> X = z/(z - 3/10) |  | |z|>0.3 |
| x[n] = (-1)n-1·u[n-1] = δ[n] - (-1)n·u[n] | >> syms n  >> X=ztrans(kroneckerDelta(n) -(-1.0)^n)  X = 1 - z/(z + 1) |  | |z|> |-1| |
| x[n]=u[n]-2n·u[-n-1] | WE CANNOT USE ZTRANS  (u[-n-1] cannot be computed by ztrans, since that function only computes right-sided functions while that one is left-sided) |  | |z|>|1|∩  |z|< |2| = 1<|z|<|2| |
| x[n]=u[n]-u[n-3] = δ[n]+ δ[n-1]+ δ[n-2] | >> syms n  >> X=ztrans(kroneckerDelta(n,0) + kroneckerDelta(n,1) + kroneckerDelta(n,2))    X =    1/z + 1/z^2 + 1 |  | z ≠ 0 |
| x=-2n·u[n] = -2n·1n·u[n] | >> syms n  >> X=ztrans(-2.\*n)    X =    (2\*z)/(z - 1)^2 |  | |z|>1 |
| x=cos(nπ/2)·u[n] | >> syms n  >> X=ztrans(cos(n.\*pi/2))    X =    z^2/(z^2 + 1) | = | |z|>1 |

**SOME COMMON Z-TRANSFORM PAIRS**



**PROPERTIES OF THE Z-TRANSFORM**

Una captura de pantalla de un celular con letras

Descripción generada automáticamente con confianza media

1. **The inverse Z-transform of rational functions.**
2. For the following rational z-transforms X(z):
3. Calculate X(z) as a ratio of polynomials in z-1.
4. Obtain the pole-zero plots.

'zplane' requires one of the following:

DSP System Toolbox

Signal Processing Toolbox

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| **X(z)** | **z-1 power expression** | **Pole-zero plot** |
|  |  | zplane([0,0,1],[1,-3,2]) |
|  |  | >> zplane([1,1,-3, 2],[1,-2,0,0])  Gráfico, Diagrama  Descripción generada automáticamente con confianza media |
|  |  | >> zplane([0,1],[1,-2])  Diagrama  Descripción generada automáticamente |
|  |  | >> zplane([0,0,1,-1],[8,-6,1])  Diagrama  Descripción generada automáticamente |
|  |  | >> zplane([0,0,1],[1,-2,1])  Diagrama  Descripción generada automáticamente |
|  |  | >> zplane([0,0,1],[1,0,4])  Gráfico, Diagrama  Descripción generada automáticamente |

1. Knowing that the z-transform of causal sequences x[n] are the following X(z), find the inverse z-transforms x[n]:
2. Using the MATLAB function “iztrans”.
3. Using the partial fraction expansion method. Review the MATLAB function “residuez” in the MATLAB documentation. Write: >> help residuez

**NOTE:** symbolic in MATLAB is: with “syms n”.

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| **X(z)** | **Using iztrans** | **Partial fractions expansion (z-1 powers)** | **Inverse z-transform**  **(See common z-transform pairs)** |
|  | syms n  syms z  X(z)= 1/(z^2-3\*z+2)  x(n)= iztrans(X)  >> x (n) = 2^n/2 + kroneckerDelta(n, 0)/2 – 1 | [R,p,k]= residuez([0 0 1] , [1 -3 2])  R = [0.5 -1]  p = [2 1]  k = 0.5 | x[n] = 0.5\* kroneckerDelta(n, 0) + 0.5\*2^n\*u[n] – u[n] |
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1. **Obtain the System Function H(z) of causal LTI discrete-time systems described as LDEcc**

The Linear Difference Equation with constant coefficients (LDEcc):

or

implements a causal LTI system with input sequence x[n] if the auxiliary conditions are the initial-rest conditions.

**Initial-rest conditions**: if the input x[n] is zero for n less than some time , then the output y[n] is constrained to be zero for n less than .

1. How can you obtain the impulse response h[n] of this kind of systems without using the z-transform?

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1. Using the z-transform, obtain the impulse response h[n] of the following causal LTI systems defined by LDEcc with initial-rest conditions:

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| **Causal LTI system** | **System Function H(z)** | **Pole-zero plot, ROC and stability** | **h[n]** |
|  |  | zplane([1],[1,-1])    |z|>1, unstable. | h[n]=u[n] |
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