**PRACTICE 4 – FOURIER SERIES**

**OBJECTIVES:**

1. Calculate Fourier coefficients of a -periodic signal x(t).
2. Calculate the N-order partial sum of Fourier series.
3. Calculate Fourier coefficients of a T-periodic signal x(t).

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1. **Fourier coefficients of a 2π-periodic signal x(t)**

Given an integrable -periodic function x(t) in the interval [], the Fourier series of x(t) is defined as:

where are the Fourier coefficients:

The following MATLAB script let you calculate the Fourier coefficients (from N to -N) of the -periodic signal x(t)=:

% Fourier coefficients from c(-5) to c(5)

syms t

Coef = [];

N = 5;

for n=-N:N

Coef = [Coef int(t^2\*exp(-i\*n\*t),-pi,pi)/(2\*pi)];

end

disp('Coefficients: ')

Coef

1. Define a MATLAB function that obtains the **Fourier coefficients**  (from N to -N) of a -periodic signal x(t) in the interval [].

function [Coef] = F\_coef(x,N)

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| **F\_coef.m** |
| function [Coef] = F\_coef(x,N)  syms t  Coef = []; %zeros(1, 2 \*N+1);  %i = 1;  for n=-N:N  Coef = [Coef int(x(t)\*exp(-1i\*n\*t),-pi,pi)/(2\*pi)];  %Coef(i) = [int(x(t)\*exp(-1i\*n\*t),-pi,pi)/(2\*pi)];  %i = i +1;  end  disp('Coefficients: ')  end |

1. Use the previous MATLAB function to calculate **the Fourier coefficients**  (from 8 to -8) of the following -periodic signal x(t) in the interval []:

x(t) = t x(t) = heaviside(t) x(t)= (abs(t))

heaviside([x](https://www.mathworks.com/help/releases/R2019b/symbolic/heaviside.html?doclanguage=es&nocookie=true&prodfilter=ML%20SL%205G%20AE%20AT%20AA%20AU%20DR%20AS%20BI%20CM%20VP%20CT%20CF%20DA%20DB%20DF%20NN%20DS%20ET%20EC%20FH%20IT%20FI%20PO%20FL%20GD%20GC%20HD%20ES%20IA%20IP%20IC%20LH%20LS%20MG%20ME%20CO%20MJ%20MR%20AM%20MP%20MB%20NV%20OT%20OP%20DM%20PD%20AR%20PW%20PM%20RL%20RB%20RF%20RK%20RO%20RC%20RR%20TF%20SX%20SG%20SB%20SE%20SS%20LD%20PS%20SH%20MS%20VR%20VV%20CI%20RT%20SD%20CV%20SO%20DV%20WT%20PL%20XP%20SR%20RQ%20SZ%20HW%20EL%20SF%20ST%20SM%20ZC%20ID%20TA%20TR%20VE%20VN%20VT%20WA%20WL&docviewer=helpbrowser&docrelease=R2019b&s_cid=pl_webdoc&loginurl=https%3A%2F%2Flocalhost%3A31515%2Ftoolbox%2Fmatlab%2Fmatlab_login_framework%2Fweb%2Findex.html%3Fsnc%3DRLNARA&container=jshelpbrowser#bt3r7ry-x)) returns the value 0 for x < 0, 1 for x > 0, and 1 for x = 0.

oldparam = sympref('HeavisideAtOrigin',1);

heaviside(sym(0))

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| **x(t) = t**  Coef = [ -1i/8, 1i/7, -1i/6, 1i/5, -1i/4, 1i/3, -1i/2, 1i, 0, -1i, 1i/2, -1i/3, 1i/4, -1i/5, 1i/6, -1i/7, 1i/8]  **x(t) = heaviside(t)**  Coef = [0, 1i/(7\*pi), 0, 1i/(5\*pi), 0, 1i/(3\*pi), 0, 1i/pi, 1/2, -1i/pi, 0, -1i/(3\*pi), 0, -1i/(5\*pi), 0, -1i/(7\*pi), 0]  **x(t)=**  Coef = [0, -2/(49\*pi), 0, -2/(25\*pi), 0, -2/(9\*pi), 0, -2/pi, pi/2, -2/pi, 0, -2/(9\*pi), 0, -2/(25\*pi), 0, -2/(49\*pi), 0] |

1. **N-order partial sum of Fourier series**

Given an integrable -periodic function x(t) in the interval [], the N-order partial sum of Fourier series of x(t) is defined as:

where are the Fourier coefficients.

The following MATLAB script let you calculate the N-order partial sum of Fourier series of the -periodic signal x(t):

% 4-order partial sum of Fourier series Fourier

syms t

SF = Coef(N+1);

N = 5;

for n=0:N

SF = SF + Coef(N+1-n)\*exp(i\*(-n)\*t) + Coef(N+1+n)\*exp(i\*n\*t);

end

disp('Partial sum: ')

SF

1. Define a MATLAB function that obtain the **N-order partial sum** of a -periodic signal x(t) in the interval [].

function [SF] = F\_partsum(x,N)

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| **F\_partsum.m** |
| function [SF] = F\_partsum(x,N)  syms t  Coef = F\_coef(x,N);  SF = Coef(N+1);  for n=1:N  SF = SF + Coef(N+1-n)\*exp(i\*(-n)\*t) + Coef(N+1+n)\*exp(i\*n\*t);  end  disp('Partial sum: ')  end |

1. Use the previous MATLAB function to calculate the **8-order partial sum** of the following -periodic signal x(t) in the interval []:

x(t) = heaviside(t) x(t)= (abs(t))

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| **x(t) = heaviside(t)**  SF(t) = (exp(-t\*1i)\*1i)/pi - (exp(t\*1i)\*1i)/pi + (exp(-t\*3i)\*1i)/(3\*pi) - (exp(t\*3i)\*1i)/(3\*pi) + (exp(-t\*5i)\*1i)/(5\*pi) - (exp(t\*5i)\*1i)/(5\*pi) + (exp(-t\*7i)\*1i)/(7\*pi) - (exp(t\*7i)\*1i)/(7\*pi) + 1/2  **x(t) = (abs(t))**  SF(t) = pi/2 - (2\*exp(-t\*1i))/pi - (2\*exp(t\*1i))/pi - (2\*exp(-t\*3i))/(9\*pi) - (2\*exp(t\*3i))/(9\*pi) - (2\*exp(-t\*5i))/(25\*pi) - (2\*exp(t\*5i))/(25\*pi) - (2\*exp(-t\*7i))/(49\*pi) - (2\*exp(t\*7i))/(49\*pi) |

1. Plot together the function x(t) = heaviside(t) in the interval [-4,4] and the **8-order partial sum** of the following -periodic signal x(t)= heaviside(t). Use the MATLAB function “fplot”.

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| >> sympref('HeavisideAtOrigin',1);  >> x(t) = heaviside(t);  >> x(0)    ans =    1    >> N = 8  N =  8  >> [Coef1] = F\_coef(x,N)  Coefficients:    Coef1 =    [0, 1i/(7\*pi), 0, 1i/(5\*pi), 0, 1i/(3\*pi), 0, 1i/pi, 1/2, -1i/pi, 0, -1i/(3\*pi), 0, -1i/(5\*pi), 0, -1i/(7\*pi), 0]    >> [partsum1] = F\_partsum(x,N)  Coefficients:  Partial sum:    partsum1 =    (exp(-t\*1i)\*1i)/pi - (exp(t\*1i)\*1i)/pi + (exp(-t\*3i)\*1i)/(3\*pi) - (exp(t\*3i)\*1i)/(3\*pi) + (exp(-t\*5i)\*1i)/(5\*pi) - (exp(t\*5i)\*1i)/(5\*pi) + (exp(-t\*7i)\*1i)/(7\*pi) - (exp(t\*7i)\*1i)/(7\*pi) + 1/2    >> subplot(2,1,2); fplot(x,[-4 4],'b');  >> subplot(2,1,2); fplot(partsum1, [-4, 4], 'r')  Gráfico, Gráfico de líneas  Descripción generada automáticamente |

1. **Fourier coefficients of a T-periodic signal x(t)**

Given an integrable T-periodic function x(t) in the interval , the Fourier series of x(t) is defined as:

where are the Fourier coefficients:

The following script let you calculate the Fourier coefficients of the 5-periodic signal x(t)=:

% Fourier coefficients from c(-8) to c(8)

syms t

Coef = [];

T = 5;

N = 8;

for n=-N:N

Coef = [Coef int(t^2\*exp(-i\*n\*2\*pi\*t/T),-T/2,T/2)/T];

end

disp('Coefficients: ')

Coef

1. Define a MATLAB function that obtain the **Fourier coefficients**  (from N to -N) of a T-periodic signal x(t) in the interval .

function [Coef] = F\_coefT(x,N,T)

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| **F\_coefT.m** |
| function [Coef] = F\_coefT(x,N,T)  syms t  Coef = [];  for n=-N:N  Coef = [Coef int(x(t)\*exp(-i\*n\*2\*pi\*t/T),-T/2,T/2)/T];  end  disp('Coefficients: ')  Coef  end |

1. Use the previous MATLAB function to calculate the Fourier coefficients (from 8 to -8) of the following 6-periodic signal x(t) in the interval []:

x(t) = heaviside(t) x(t)= (abs(t))

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| x(t) = heaviside(t):  Coef = [0, 1i/(7\*pi), 0, 1i/(5\*pi), 0, 1i/(3\*pi), 0, 1i/pi, 1/2, -1i/pi, 0, -1i/(3\*pi), 0, -1i/(5\*pi), 0, -1i/(7\*pi), 0]  x(t)=:  Coef = [0, -6/(49\*pi^2), 0, -6/(25\*pi^2), 0, -2/(3\*pi^2), 0, -6/pi^2, 3/2, -6/pi^2, 0, -2/(3\*pi^2), 0, -6/(25\*pi^2), 0, -6/(49\*pi^2), 0] |