Theory Exercise 10

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Task 1: Maekawa's Voting Algorithm

a)

Does the algorithm work correctly with these voting sets? Explain your statement, e.g., by performing the essential test steps.

Test steps:

- ✅ Number of processes (8) is equal to the number of voting sets (8)
- ✅ Each process is member of its own voting set, i.e. $p_1 \in V_1, \dots, p_8 \in V_8$
- ✅ The pairwise intersection of the voting sets is not empty:

$V_1 \cap V_2 = \{ 1, 2, 3 \}$ $V_1 \cap V_3 = \{ 1, 2, 3 \}$ $V_1 \cap V_4 = \{ 1, 4, 7 \}$ $V_1 \cap V_5 = \{ 2, 4 \}$ $V_1 \cap V_6 = \{ 3, 4 \}$ $V_1 \cap V_7 = \{ 2, 4, 7 \}$ $V_1 \cap V_8 = \{ 1, 4, 7 \}$	$V_2 \cap V_3 = \{ 1, 2, 3 \}$ $V_2 \cap V_4 = \{ 1, 5 \}$ $V_2 \cap V_5 = \{ 2, 5, 8 \}$ $V_2 \cap V_6 = \{ 3, 5 \}$ $V_2 \cap V_7 = \{ 2, 8 \}$ $V_2 \cap V_8 = \{ 1, 5, 8 \}$	$V_3 \cap V_4 = \{ 1, 6 \}$ $V_3 \cap V_5 = \{ 2, 6 \}$ $V_3 \cap V_6 = \{ 3, 6 \}$ $V_3 \cap V_7 = \{ 2 \}$ $V_3 \cap V_8 = \{ 1 \}$
$V_4 \cap V_5 = \{ 4, 5, 6 \}$ $V_4 \cap V_6 = \{ 4, 5, 6 \}$ $V_4 \cap V_7 = \{ 4, 7 \}$ $V_4 \cap V_8 = \{ 1, 4, 5, 7 \}$	$V_5 \cap V_6 = \{ 4, 5, 6 \}$ $V_5 \cap V_7 = \{ 2, 4, 8 \}$ $V_5 \cap V_8 = \{ 4, 5, 8 \}$	$V_6 \cap V_7 = \{ 4 \}$ $V_6 \cap V_8 = \{ 4, 5 \}$ $V_7 \cap V_8 = \{ 4, 7, 8 \}$

All conditions are met, so the algorithm works correctly with the given voting sets.

b)

What are the two specific fairness conditions of Maekawa's algorithm? Do they hold in this case? Explain your answer!

The first fairness condition is called 'equal effort'. This condition is met if the number of members in all voting sets is equal, i.e., $|V_i| = K$ for some integer K. This conditions tries to distribute the voting-effort equally amongst all voting sets.

The second fairness condition is called 'equal responsibility'. This condition is met if each process is member in the same number of voting sets. This condition avoids that some processes are able to vote more often than others and therefore have a higher 'responsibility' or 'power'.

The first condition is obviously not met as the voting sets are of different size, e.g. $|V_1| = 5$ and $|V_3| = 4$

Also the second condition is not met, e.g. because $p_1 \in V_1, V_2, V_3, V_4, V_8$ and $p_3 \in V_1, V_2, V_3, V_6$

c)

Is it possible to change the voting sets (e.g., by adding or deleting processes) such that both fairness conditions hold? If not, why? If yes, state the modified voting sets.

Yes, it is. The most simple way of doing it is to add each process to each voting set. Because the two conditions of Maekawa's algorithm are already met, we know, that we can extend the sets without worrying about it.

V_3, V_6, V_7, V_8 do need one more item in order to have the same number of items. 3, 6, 7, 8 are only members of 4 sets, all others are part of five sets.

We can add 3 to V_7, 6 to V_8, 7 to V_3, 8 to V_6. If we do it, all constrains are hold and each process is part of 5 sets and each set has the same size. Hence, the fairness is given.

The resulting voting sets are as follows:

$V_1 = \{1,2,3,4,7\}$	$V_5 = \{2,4,5,6,8\}$
$V_2 = \{1,2,3,5,8\}$	$V_6 = \{3,4,5,6,8\}$
$V_3 = \{1,2,3,6,7\}$	$V_7 = \{2,3,4,7,8\}$
$V_4 = \{1,4,5,6,7\}$	$V_8 = \{1,5,6,7,8\}$