Probability

↳ Computer scientist use ~~proper~~ probability to design algorithms for sorting data, detecting problems in computer system or predicting user behavior.

Experiment

An experiment is an act that can be repeated under given conditions. For example - coin tossing experiment, dice throwing experiment etc.

Trial and Event

If an experiment be repeated under essentially the same condition giving several possible outcomes than the experiment is called trial and the possible outcomes are known as events or cases. For example - tossing of a coin is a trial and getting Head (H) or tail (T) is an event.

Exhaustive cases :-

The total number of possible outcomes of any trial are exhaustive cases. For example - in tossing a coin there are two exhaustive cases ~~mo~~ namely occurrence of head and tail.

Equally likely cases :-

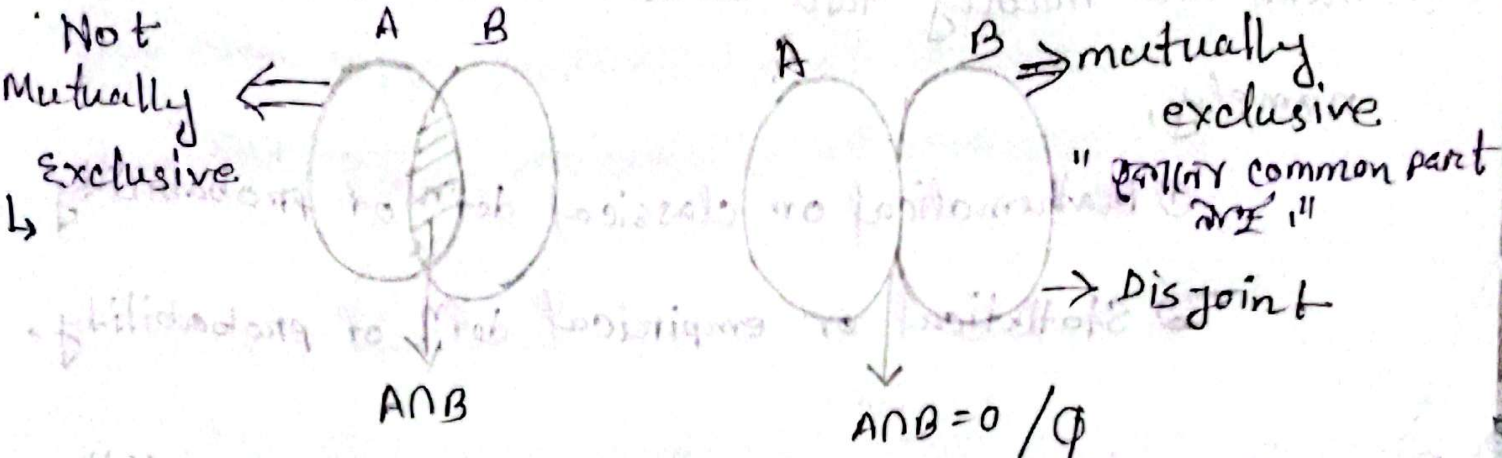
Cases are said to be equally likely when none of them is expected to occure more frequently than the other. For example - from an unbiased coin the case of appearing head or tail is equally likely. Similarly from an unbiased dice, all the six points are equally likely.

Mutually Exclusive cases :-

Cases are said to be mutually exclusive if the occurrence of one of them excludes the occurrence of all the others. For example -

CamScanner

in tossing an unbiased coin, the case of appearing head and tail and mutually exclusive



<u>Favourable cases:-</u>

The number of cases favourable to an event is called favourable cases or events. Let an unbiased dice has been thrown. A be the event which denotes even number. Then the favourable case to A is $\{2, 4, 6\}$.

<u>Sample Space:-</u>

The set of all possible outcomes is an experiment is called sample space. If we throw a dice, the sample space is $\{1, 2, 3, 4, 5, 6\}$.

Definition of probability :-

There are mainly two definitions of probability, namely,

① Mathematical or classical defn. of probability

② Statistical or empirical defn. of probability.

[1] Mathematical or classical definition of probability:-

→ If a trial result in n exhaustive, mutually exclusive and equally likely cases and m of them are favourable to an event A, then the probability p of the given ~~happing~~ happening of A is given by

$$P(A) = \frac{\text{Favourable num of cases}}{\text{Total number of cases}} = \frac{m}{n}$$

" obviously p be a positive number, not greater than unity, so that $0 \leq p \leq 1$.

$P(A) = 0 \rightarrow$ impossible event

$P(A) = 1 \rightarrow$ sure/certain event.

$p \rightarrow$ even হলে probability of success.

$p \rightarrow$ odd হলে probability of failure.

$\therefore \boxed{p + q = 1}$

* combination, $^nC_r = \dfrac{n!}{r!(n-r)!}$

* $^nC_0 = 1 \rightarrow r$ '0' হলে

* $^nC_n = 1 \rightarrow n$ এবং r সমান হলে

* $^nC_1 = n \rightarrow r$ '1' হলে

* $^4C_2 = \dfrac{3 \times 4}{1 \times 2} = \dfrac{12}{2} = 6$ " ~~যেখানে r = 2 নিলে combination~~ ~~ও n~~

* $^9C_3 = \dfrac{7 \times 8 \times 9}{1 \times 2 \times 3}$

* $n! = n(n-1)!$

* A bag contains 15 identical balls of which 5 are white and the rest are black. Two balls are drawn at random from the bag. What is the probability that both balls are white?

Soln

Let A be the even that both the balls are white. The total number of cases - 2 balls from 15 balls is

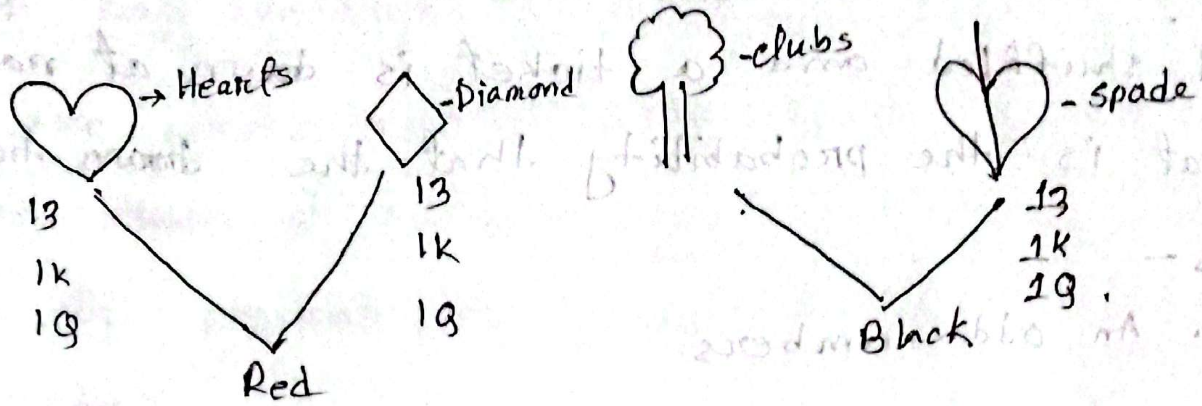
$$^{15}C_2 = \frac{14 \times 15}{2} = 105$$

The favourable number of cases of getting 2 white balls from 5 white ball is

$$^{5}C_2 = \frac{4 \times 5}{2} = 10$$

Therefore, the required probability is

$$P(A) = \frac{m}{n} = \frac{10}{105} = \frac{2}{21}$$



A card is drawn from a pack of 52 cards. Find the probability that the selected card is

(a) a red card b) a spade c) Not a spade
d) a king or a queen.

soln

(a) $P(A) = \frac{m}{n} = \frac{26C_1}{52C_1} = \frac{26}{52} = \frac{1}{2}$

(b) $P(B) = \frac{m}{n} = \frac{13C_1}{52C_1} = \frac{13}{52} = \frac{1}{4}$

(c) $P(C) = \frac{m}{n} = \frac{39C_1}{52C_1} = \frac{39}{52} = \frac{3}{4}$.

(d) $P(D) = \frac{m}{n} = \frac{8C_1}{52C_1} = \frac{8}{52} = \frac{2}{13}$.

* Tickets are numbered from 1 to 100. They are well shuffled and a ticket is drawn at random. What is the probability that the drawn ticket has –

1. An odd numbers
2. A number 4 or multiple of 4
3. A number which is greater than 70.
4. A number which is square.

Soln

1. $P(A) = \frac{m}{n} = \frac{50_{C_1}}{100_{C_1}} = \frac{50}{100} = \frac{1}{2}$

2. $P(B) = \frac{m}{n} = \frac{25_{C_1}}{100_{C_1}} = \frac{25}{100} = \frac{1}{4}$.

3. $P(C) = \frac{m}{n} = \frac{30_{C_1}}{100_{C_1}} = \frac{30}{100} = \frac{3}{10}$.

4. $P(D) = \frac{m}{n} = \frac{10_{C_1}}{100_{C_1}} = \frac{10}{100} = \frac{1}{10}$

Toss a coin and a dice simultaneously. Show the sample points with a tree diagram and construct the sample space.

Sample space $\{H, T\}$

soln

There are two possible outcomes for the coin and six possible outcomes for the dice, the total number of outcomes will be

$2 \times 6 = 12$.

The sample space of the experiment is

$S = \{H_1, H_2, H_3, H_4, H_5, H_6, T_1, T_2, T_3, T_4, T_5, T_6\}$.

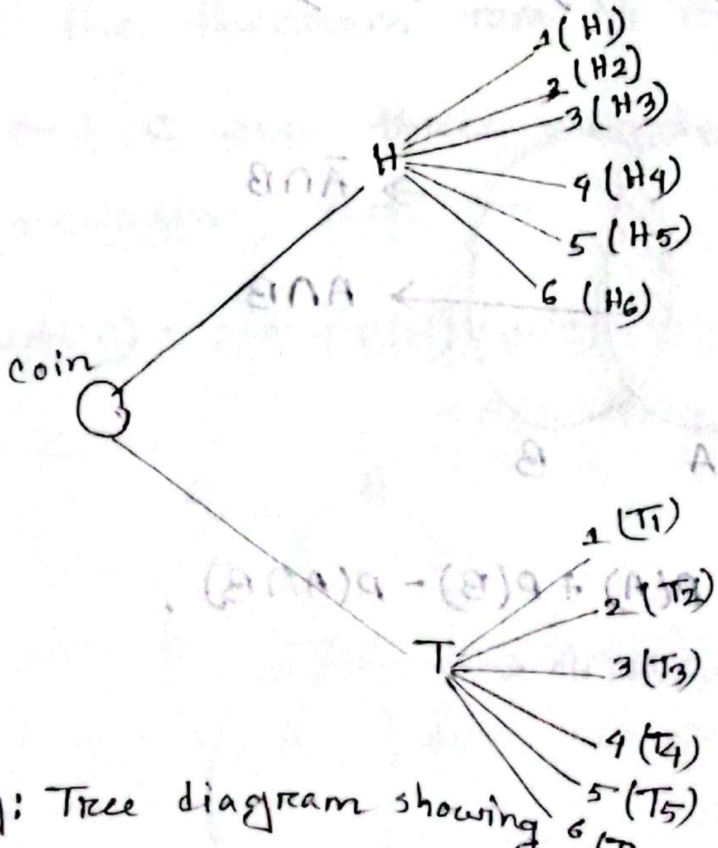


Fig: Tree diagram showing outcomes of a coin and a dice.

Laws of probability:-

→ Additive law of probability or theorem of total probability.

→ Multiplicative law of probability or theorem of compound.

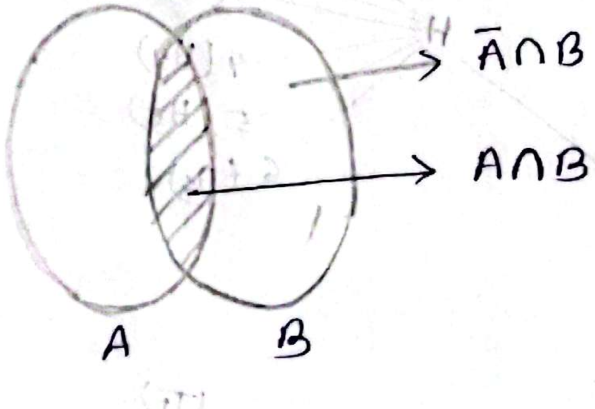
[1] Additive laws of probability for two event:-

⇒ If A and B are two events which are not mutually exclusive, then

যাদের মধ্যে common point থাকবে

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

Proof:-



$$P(A\cup B) = P(A) + P(B) - P(A\cap B).$$

proof :- Consider the following venn-diagram

$$A \cup B = A \cup (\bar{A} \cap B)$$

$$\Rightarrow P(A \cup B) = P\{A \cup (\bar{A} \cap B)\}$$

$$\Rightarrow P(A \cup B) = P(A) + P(\bar{A} \cap B) \quad [\because A \;\&\; (\bar{A} \cap B) \text{ are disjoint}]$$

$$= P(A) + P(A \cap B) + P(\bar{A} \cap B) - P(A \cap B)$$

$$\cancel{= P(A) + P(A \cap B)}$$

$$= P(A) + P\{(A \cap B) \cup (\bar{A} \cap B)\} - P(A \cap B)$$

$[\because (A \cap B) \cup (\bar{A} \cap B)$ are disjoint$]$

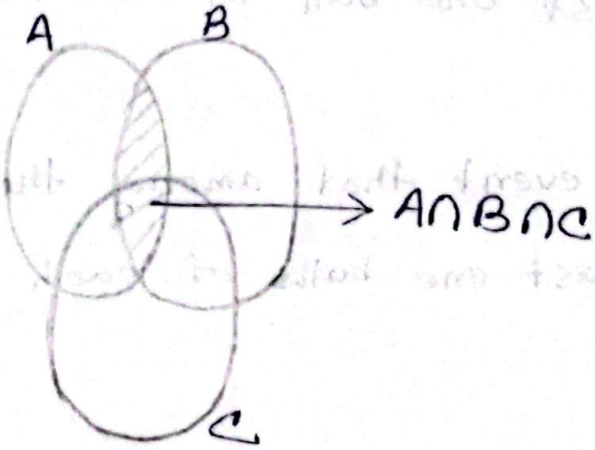
$$= P(A) + P(B) - P(A \cap B)$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Extend the theorem for 3 events :-

⇒ If A, B and C are three events which are not mutually exclusive, then,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$



proof

$$P(A\cup B\cup C) = P(D\cup C) \qquad [\because D = A\cup B]$$

$$= P(D\cup C)$$

$$= P(D) + P(C) - P(D\cap C)$$

$$= P(A\cup B) + P(C) - P[(A\cup B)\cap C]$$

$$= P(A) + P(B) - P(A\cap B) + P(C) - P\{(A\cap C)\cup(B\cap C)\}$$

$$= P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) - P(A\cap B\cap C)$$

[$\because$ $A\cap C$ & $B\cap C$ are not mutually exclusive]

A bag contains 5 red, 7 white and 6 blue balls. Four balls are drawn at random from the bag. Find the probability that among the four balls there is at least one ball of each colour.

soln

Let, A be the event that among the four balls, there is at least one balls of each colour.

This can happen the following mutually exclusive ways:

i) 1R, 1W, 2B $\longrightarrow$ Event A_1

ii) 1R, 2W, 1B $\longrightarrow$ Event A_2

iii) 2R, 1W, 1B $\longrightarrow$ Event A_3.

$$\therefore P(A) = \cancel{P(A_1)} + P(A_1 \cup A_2 \cup A_3)$$

$$= P(A_1) + P(A_2) + P(A_3)$$

$$\therefore P(A_1) = \frac{5c_1 + 7c_1 + 6c_2}{18c_4} = \frac{35}{204}$$

$$P(A_2) = \frac{5c_1 + 7c_2 + 6c_1}{18c_4} = \frac{7}{34}$$

$$P(A_3) = \frac{5c_2 + 7c_1 + 6c_1}{18c_4} = \frac{7}{51}$$

$$\therefore P(A) = P(A_1) + P(A_2) + P(A_3)$$

$$= \frac{35}{204} + \frac{7}{34} + \frac{7}{51}$$

$$= \frac{35}{68}.$$

CamScanner

A bag contains 3 red, 3 black, and 6 white identical balls. These balls are drawn at random. What is the probability that all the balls are of same colour?

Soln

$3R \rightarrow A_1$ event

$3 Black \rightarrow A_2$ event

$6 \text{ White} \rightarrow A_3$

$\therefore P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3)$

$\therefore P(A_1) = \frac{{}^3C_3}{{}^{12}C_3} = \frac{3 \times 2 \times 1}{10 \times 11 \times 12} = \frac{1}{220}$

$P(A_2) = \frac{{}^3C_3}{{}^{12}C_3} = \frac{1}{220}$

$P(A_3) = \frac{{}^6C_3}{{}^{12}C_3} = \frac{4 \times 5 \times 6}{10 \times 11 \times 12} = \frac{1}{11}$

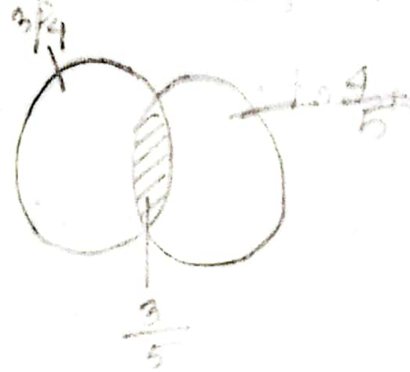
$\therefore P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3)$

$= \frac{1}{220} + \frac{1}{220} + \frac{1}{11}$

$= \frac{1}{10}$ Ans

Mr. Y feels that the probability that he will get A in calculus is 3/4, A in statistics is $\frac{4}{5}$ and A in both the courses is $\frac{3}{5}$. What is the probability that Mr. Y will get -

a) at least one A, b) No A's.



Soln

Given, $P(C) = \frac{3}{4}$, $P(S) = \frac{4}{5}$ and $P(C \cap S) = \frac{3}{5}$

$\therefore$ P(at least one A) $= P(C \cup S)$

$= P(C) + P(S) - P(C \cap S)$

$= \frac{3}{4} + \frac{4}{5} - \frac{3}{5}$

$= \frac{19}{20}$

b. No A's $= 1 - P(C \cup S)$

$= 1 - \frac{19}{20}$

$= \frac{1}{20}$ Ans

* The probability that a contractor will get a plumbing contract is $\frac{2}{3}$ and the probability that he will get an electric contract is $4/9$. The probability of getting both the contract is $14/45$, what is the probability that he will get at least one contract.

Soln

Given,

$$P(A) = \frac{2}{3},$$

$$P(B) = \frac{4}{9}$$

$$P(A \cap B) = \frac{14}{45}$$

$\therefore$ P (at least one contract) $= P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{3} + \frac{4}{9} - \frac{14}{45}$$

$$= \frac{36}{45}$$

$$= \frac{4}{5}$$

Independent and dependent Events (Multiplicative laws)

↳ Events are said to be independent if the happening or non happening of an event is not affected by the number of remaining event. otherwise, the events are said to be dependent.

$$P(AB) = P(A) \cdot P(B) \rightarrow \text{independent}.$$

$$P(A/B) = P(A) \text{ ; independent}$$

$$P(B/A) = P(B) \text{ ; independent}.$$

Multiplicative law of probability (for dependent event)

→ The probability of the simultaneous occurrence of two dependent events A and B is equal to the probability of A multiplied by the conditional probability of B given that has already occured.

$$P(AB) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B).$$

$$P(B/A) = \frac{P(AB)}{P(A)} \text{ ; } P(A) \quad P(A) > 0$$

$$P(A/B) = \frac{P(AB)}{P(B)} \text{ ; } P(B) \quad P(B) > 0$$

$$P(ABC) = P(A) \cdot P(B) \cdot P(C) \longrightarrow \text{independent}$$

$$P(ABC) = P(A) \cdot P(B/A) \cdot P(C/AB).$$

* A bag contains 4 white and 5 red balls which are ~~B~~ throughout mixed up. Two balls are drawn successively at random from the bag. What is the probability that both the balls are white when the drawing and make i) with replacement ii) without replacement.

Soln:-

Let A be the event that first ball is white and B ~~the~~ be the event that the second ball is also white.

i). With replacement, the two events become independent

$$\therefore P(AB) = \frac{{}^4C_1}{{}^9C_1} \quad P(A) \cdot P(B)$$

$$= \frac{{}^4C_1}{{}^9C_1} \cdot \frac{{}^4C_1}{{}^9C_1}$$

$$= \frac{4}{9} \cdot \frac{4}{9} = \frac{16}{81}$$

ii) Without replacement, the two events become dependent.

$$\therefore P(AB) = P(A) \cdot P(B/A) = \frac{{}^4C_1}{{}^9C_1} \cdot \frac{{}^3C_1}{{}^8C_1} = \frac{4}{9} \cdot \frac{3}{8} = \frac{1}{6}$$

A bag contains 5 gold and 6 silver coins. Two successive drawing of three coins have been made from this bag. Find the probability that the first drawn gives 3 gold and 2nd drawn gives three silver coins, if—

dependent

i) The coins are not replace before 2nd drawn.

ii) " " " " " 2nd " .

~~without~~ replace independent

Soln

i) Not replaced —— (dependent)

$$\therefore P(AB) = P(A) \cdot P(B/A)$$

$$= \frac{{}^{5}C_3}{{}^{11}C_3} \cdot \frac{{}^{6}C_3}{{}^{8}C_3}$$

$$= \frac{5}{231}$$

ii) coins are replaced, so the events become independent

$$\therefore P(AB) = P(A) \cdot P(B)$$

$$= \frac{{}^{5}C_3}{{}^{11}C_3} \cdot \frac{{}^{6}C_3}{{}^{11}C_3}$$

$$= \frac{8}{1089}.$$

Example-2

An urn has 6 black and 4 white marbles. Two of them are drawn without replacement. Find the probability of getting a black and a white marble.

Soln

$$P(A) = \frac{6_{C_1}}{10_{C_1}} = \frac{6}{10} = \frac{3}{5}$$

and $$P(B) = \frac{4_{C_1}}{10_{C_1}} = \frac{4}{10} = \frac{2}{5}$$

probability of getting white marble after drawing black, $$P(B/A) = \frac{4_{C_1}}{9_{C_1}} = \frac{4}{9}$$

$$\therefore\ P(AB) = P(A)\cdot P(B/A)$$

$$= \frac{3}{5}\cdot\frac{4}{9}$$

$$= \frac{12}{45}$$

Baye's theorem

→ Industry, Medicine, and decission making etc.

If $E_1, E_2, E_3, \ldots, E_n$ are n mutually exclusive events and if is an event such that

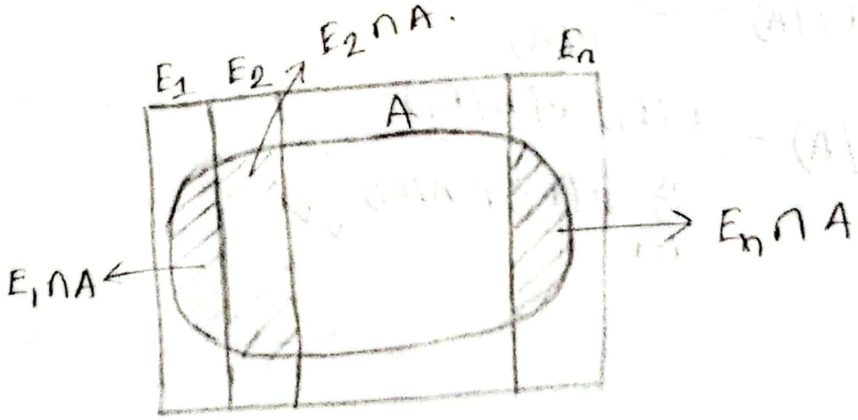
$A \subset \bigcup_{i=1}^{n} E_i$, then,

$P(E_i A)$ or $P(E_i \cap A)$

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i) P(A/E_i)}$$

Proof

consider the following venn- diagram



We have,

$E_1, E_2, \ldots, E_n$ are n mutually exclusive ~~that~~ event.

A is another event. so that

$A \subset (E_1 \cup E_2 \cup \cdots \cup E_n)$

i.e. $A \subset \bigcup_{i=1}^{n} E_i$

$\therefore A = (E_1 \cap A) \cup (E_2 \cap A) \cup \cdots \cup (E_n \cap A)$

$\Rightarrow P(A) = P[(E_1 \cap A) \cup (E_2 \cap A) \cup \cdots \cup (E_n \cap A)]$

$= P(E_1 \cap A) + P(E_2 \cap A) + \cdots + P(E_n \cap A)$ [$\because E_1 \cap A, E_2 \cap A, E_n \cap A$ are mutually exclusive]

$= P(E_1) P(A/E_1) + P(E_2) P(A/E_2) + \cdots$
$P(E_n) P(A/E_n)$ [$\because$ multiplicative law]

$$\Rightarrow P(A) = \sum_{i=1}^{n} P(E_i) P(A/E_i)$$

Now, $$P(E_i/A) = \frac{P(E_i \cdot A)}{P(A)}$$

$$\therefore P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i) P(A/E_i)}$$

There are two identical boxes containing respectively 4 white and 3 red balls, 3 white and 7 red balls. A box is chosen at random and a ball is drawn at random from it. If the ball is white, what is the probability that it is from the first box?

Soln:- Let, E_1 be the event that the first box is chosen and E_2 be the event that the second box is chosen and A be the event of getting a white ball, As the boxes are identical and are chosen at random, we get,

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$\therefore P(A/E_1) = \frac{{}^4C_1}{{}^7C_1} = \frac{4}{7}$$

$$P(A/E_2) = \frac{{}^3C_1}{{}^{10}C_1} = \frac{3}{10}$$

for 2nd box

$$P(E_2/A) = 1 - \frac{40}{61} = \frac{21}{61}$$

$$\therefore P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)\cdot P(A/E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{4}{7}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{3}{10}} = \frac{40}{61}$$

In a bolt factory, machine a produces 35%, B produces 40% and C producs 25% of the total. Among them 2, 4 and 3 parcents are defective respectively. A bolt is drawn from the product and found defective. What is the probability that it has produced by machine A

Soln

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$P(E_1) = \frac{35}{100}$ $\quad \therefore P(A/E_1) = \frac{2}{100}$

$P(E_2) = \frac{40}{100}$ $\quad P(A/E_2) = \frac{4}{100}$

$P(E_3) = \frac{25}{100}$ $\quad P(A/E_3) = \frac{3}{100}$

$$\therefore P(E_1/A) = \frac{35/100 \times \frac{2}{100}}{\frac{35}{100} \times \frac{2}{100} + \frac{40}{100} \times \frac{4}{100} + 25/100 \times \frac{3}{100}}$$

$$= \frac{14}{61}$$

[1] probability Distribution.

↳ Discrete Distribution → countable / finite

↳ i) Binomial distribution

ii) poisson "

→ Continuous " → Infinite

↳ normal distribution.

[1] Binomial Distribution :-

→ A random variable x is said to follow binomial distribution if its probability mass function is given by

$\binom{n}{x} = {}^nC_x$

$$p(X=x) = \binom{n}{x} p^x q^{n-x} \quad ; \quad x = 0, 1, 2, \cdots n$$

where p be the probability of success and q be the probability of failure, so that $p+q=1$.

[1] Bernoulli trial :-

The trial which has two outcomes is called bernoulli trial.

p → probability of success

q → probability of failure.

$$\left.\begin{array}{l}\sum P(x)=1 \\ \sum_{x=0}^{n}\binom{n}{x}p^{x}q^{n-x}=1\end{array}\right\}\text{condition of probability}$$

[ii] Conditions for binomial

1. There should be a fixed number of trials (n).
2. There are two outcomes for each trial.
3. The trial are independent of each other.
4. The probability of success and the probability of failure remain same or constant for each trial.

* n এর মান ছোট এবং p এর মান যদি বড় হয় তাহলে Binomial distribution হবে।

* n এর মান বড় এবং p যদি ছোট হয় তাহলে poisson distribution হবে।

Eight unbiased coins are tossed. Find the probability of getting i) 4 heads ii) at least 2 heads iii) at best 3 heads.

at least থাকলে minimum and best থাকলে max

soln

Given,

$$P(H) = P(T) = \frac{1}{2}.$$

$$\therefore P(x) = \binom{8}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{8-x} \quad ; (x = 0, 1, 2, \cdots, 8)$$

$$= \binom{8}{x}\left(\frac{1}{2}\right)^{8}.$$

i) $$P(X=4) = \binom{8}{4}\left(\frac{1}{2}\right)^{8} = \frac{35}{128}.$$

ii) $$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - [P(X=0) + P(X=1)]$$

$$= 1 - \left[\binom{8}{0}\left(\frac{1}{2}\right)^{8} + \binom{8}{1}\left(\frac{1}{2}\right)^{8}\right]$$

$$= \frac{247}{256}.$$

iii) $$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= \left(\frac{1}{2}\right)^{8}\left[\binom{8}{0} + \binom{8}{1} + \binom{8}{2} + \binom{8}{3}\right]$$

$$= \frac{93}{256}.$$ Ans

During war 1 ship out of 10 was sunk, on an average in making a certain voyage. Find the probability that out of 8 ships i) a convoy of 4 ships will arrive safely, ii) at least 4 ships will arrive safely.

Soln

Given,

$n = 8,$

$q = \frac{1}{10} = 0.1$

$\because p + q = 1$

$\Rightarrow p = 1 - 0.1 = 0.9.$

$\therefore P(X=x) = \binom{8}{x} \cdot (0.9)^x \cdot (0.1)^{8-x}$

i) $P(X=4) = \binom{8}{4} \cdot (0.9)^4 \cdot (0.1)^{8-4}$

$= 0.0046.$

ii) $P(X \geq 4) = 1 - P(X<4)$

$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$

$= 1 - \binom{8}{0} \cdot (0.9)^0 \cdot (0.1)^8 + \binom{8}{1} (0.9)^1 \cdot (0.1)^{8-1} + \binom{8}{2} (0.9)^2 (0.1)^{8-2}$

$+ P \binom{8}{3} (0.9)^3 (0.1)^{8-3}]$

$= 1 - 0.0043$

$= 0.99957$ Ans

H·W – 10.9, 10.12, 10.19

$4 c_2$

properties of binomial distribution

Mean (μ) / ($E(x)$) → mathematical expectation.

*** $E(x) = \sum_{x=0}^{n} x \; p(x)$

$\binom{n}{x} = \frac{n}{x}\binom{n-1}{x-1}$ বা,

$\frac{n(n-1)}{x(x-1)}\binom{n-2}{x-2}$

$= \sum_{x=0}^{n} x \binom{n}{x} p^x q^{n-x}$.

$= \sum_{x=1}^{n} x \cdot \frac{n}{\cancel{x}}\binom{n-1}{x-1} p^x q^{n-x}$

$= np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} q^{n-x}$

[p বার multiply
$p^1 \cdot p^{x-1}$
$p^{1+x-1} = p^x$]

formula

$$\boxed{(a+b)^n = {}^nc_0 a^n + {}^nc_1 a^{n-1} \cdot b + {}^nc_2 a^{n-2} b^2 + \cdots + {}^nc_n b^n}$$

$\cancel{(a+b)}^n = {}^nc_0 \; a^n + {}^nc_1$

$= np\,(q+p)^{n-1}$

$= np \times 1$

$= np$

Variance (σ^2)

$$V(x) = E(x^2) - [E(x)]^2$$

$$\Rightarrow E(x^2) = E[x(x-1) + x]$$

$$= E[x(x-1)] + E(x)$$

$$E[x(x-1)] = \sum_{x=0}^{n} x(x-1)\, p(x)$$

$$= \sum_{x=0}^{n} x(x-1) \binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=2}^{n} x(x-1) \frac{n(n-1)}{x(x-1)} \binom{n-2}{x-2} p^x q^{n-x}$$

$$= n(n-1) p^2 \sum_{x=2}^{n} \binom{n-2}{x-2} p^{x-2} q^{n-x}$$

$$= n(n-1) p^2 (q+p)^{n-2}$$

$$= n(n-1) p^2$$

$$\therefore E(x^2) = E[x(x-1) + x]$$

$$= E[x(x-1)] + E(x)$$

$$= n(n-1) p^2 + np$$

$$\therefore V(x) = n(n-1)p^2 + np - (np)^2$$
$$= n^2p^2 - np^2 + np - n^2p^2$$
$$= np(1-p)$$
$$= npq \quad , \text{ since } q = 1-p$$

~~pis~~

Poisson Distribution

A random variable x is said to follow binomial distribution if its probability of mass function is given by

$$P(X=x) = \frac{e^{-m} m^x}{x!} , x = 0,1,2,\ldots,\alpha.$$

$m \to$ mean of poisson distribution

$\therefore$ binomial dis of mean = poisson dis^n of mean.

$\therefore m = np$

Mean (μ) ***

$$E(x) = \sum_{x=0}^{\alpha} x\, p(x)$$

$$= \sum_{x=0}^{\alpha} x \cdot \frac{e^{-m} m^x}{x!}$$

$x! = x(x-1)! = x(x-1)(x-2)!$

$$= \sum_{x=1}^{\alpha} \cancel{x} \frac{e^{-m} \cdot m^x}{\cancel{x}(x-1)!}$$

$$= me^{-m} \sum_{x=1}^{\infty} \frac{m^{x-1}}{(x-1)!} \quad [\text{m ধরে multiply}]$$

$$= me^{-m} \times e^m$$

$$= m$$

$$\sum p(x) = 1$$

$$\sum_{x=0}^{\infty} \frac{e^{-m} \cdot m^x}{x!} = 1$$

$$e^{-m} \sum_{x=0}^{\infty} \frac{m^x}{x!} = 1$$

$$\sum_{x=0}^{\infty} \frac{m^x}{x!} = \frac{1}{e^{-m}} = e^m$$

The electric blub produced in an industry are found defective at the rate of 2 parcent. In an hour of a day the industry produces 500 blubs. Find the probability that, in that hour i) 4 defective blubs are produced,

ii) at least 2 defective blubs are produced.

iii) at best 2 defective blubs are produced.

Soln

$n = 500$

$p = \frac{2}{100} = 0.02$

Given, $n = 500$

$p = \frac{2}{100} = 0.02$

$\therefore m = np$

$= 500 \times 0.02$

$= 10$

$$\therefore P(X=x) = \frac{e^{-10}(10)^x}{x!}$$

i) $P(X=4) = \dfrac{e^{-10}(10)^4}{4!}$

$= 0.01892.$

ii) $P(X \geq 2) = 1 - P(X<2)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - e^{-10}\left[\dfrac{(10)^0}{0!} + \dfrac{(10)^1}{1!}\right]$

$= 1 - e^{-10} \times 11$

$= 1 - 0.000999$

$= 0.9995$

iii) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= e^{-10}\left[\dfrac{10^0}{0!} + \dfrac{(10)^1}{1!} + \dfrac{(10)^2}{2!}\right]$

$= e^{-10} \times (1 + 10 + 50)$

$= 0.00277.$

In a telephone exchange 5 wrong calls are received out of 100 calls. In an hour 250 calls received. Find the probability that, in that hour i) There are 2 wrong calls, ii) There are at least 3 wrong calls, iii) There are at best 3 wrong calls.

Soln

Given,

$$n = 250$$

$$p = \frac{5}{100} = 0.05$$

So, $m = np$

$= 250 \times 0.05$

$= 12.5$

$$\therefore P(X=x) = \frac{e^{-12.5} \cdot (12.5)^x}{x!}$$

i) $P(X=2) = \dfrac{e^{-12.5} \cdot (12.5)^2}{2!}$

$= 0.00029$.

ii) $P(X \geq 3) = 1 - P(X < 3)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - e^{-12.5}\left[\frac{(12.5)^0}{0!} + \frac{(12.5)^1}{1!} + \frac{(12.5)^2}{2!}\right]$

$= 0.99966.$

iii) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= e^{-12.5}\left[\frac{(12.5)^0}{0!} + \frac{(12.5)^1}{1!} + \frac{(12.5)^2}{2!} + \frac{(12.5)^3}{3!}\right]$

$= 0.00156.$

H.W 10.17, 10.23

Assignment

1. Baye's theorem
2. Binomial Distribution
3. Poisson "

Normal Distribution

A random variable x is said to follow normal distribution if its probability density function is given by

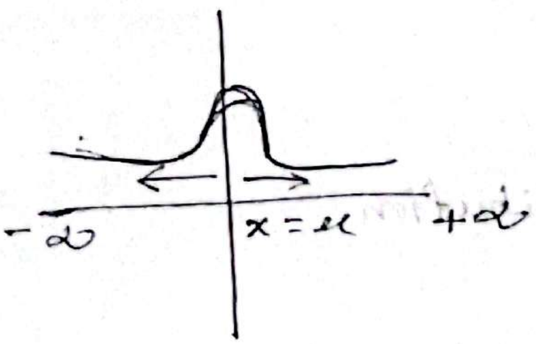
$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; \quad -\infty < x < \infty$$

$$= 0 \quad ; \text{ otherwise}$$

$\sigma^2 \Rightarrow$ Variance

$\sigma \rightarrow$ standard deviation

$\mu \rightarrow$ mean.



$x = \mu$ তে probability maximum

Probability curve of normal distribution

Standard Normal Variate

$$Z = \frac{x-\mu}{\sigma}$$

$$= \frac{x-0}{1}$$

$$Z = x$$

mean 0 এবং variance 1 হলে standard Normal variate.

State the chief characteristics of normal distribution and normal probability curve. (৮ বা ১০ point লিখ)

The per day working hours of a call center follow $N(15, 50)$. Find the probability that the working hours of a randomly selected day are i) more than 16 hr, ii) less than 20 hours, iii) 12 to 22 hours.

Soln

$X \sim N(\underset{\mu}{15}, \underset{\sigma^2}{50})$

Given, $\mu = 15$

$\sigma^2 = 50$

$\therefore \sigma = \sqrt{50}$

$= 7.07$

i) $P(X>16)$, $z = \frac{x-\mu}{\sigma} = \frac{16-15}{7.07} = 0.14$

$P(X>16) = P(Z>0.14) = 1 - P(Z<0.14)$

$= 1 - 0.5557$

$= 0.4443$

table तालिका
0.1 – coloum
0.04 – raw

ii) $P(X<20) = z = \frac{x-\mu}{\sigma} = \frac{20-15}{7.07} = 0.71$

$\therefore P(X<20) = P(Z<0.71) = 0.7611$.

iii) $P(12<x<22) = z_1 = \frac{x_1-\mu}{\sigma} = \frac{12-15}{7.07} = -0.42$

$z_2 = \frac{x_2-\mu}{\sigma} = \frac{22-15}{7.07} = 0.99$

$\therefore P(12<x<22) = P(-0.42<Z<0.99)$

$= P(Z<0.99) - P(Z<-0.42)$

$= 0.8389 - 0.3372$

$= 0.5017$

* 10.39