Normal Distribution Sampling

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```
library(tidyr) #the pipe (%>%) tool is extremely useful
library(MASS) # used for mvrnorm
```

Q1

Suppose Suppose X_1, X_2, Y_1, Y_2 are mutually independent.

- X_1 and X_2 are iid from $N(\mu = 0, \sigma^2 = 2^2)$
- Y_1 and Y_2 are iid from $N(\mu = 0, \sigma^2 = 1^2)$

a) Calculate $P(|X_1 - X_2| > 4)$

- Let $X = |X_1 - X_2|$ then $P(|X| > 4) \sim N(0, 8)$
- Transform X -> Z_X then $P(|Z_X| > 4/\sqrt{8}) \sim N(0, 1)$
- Calculation in R below:

```
2 * (1 - pnorm(4/sqrt(8)))
```

```
## [1] 0.1572992
```

b) Calculate $P(|Y_1 - Y_2| > 4)$

- Let $Y = Y_1 - Y_2$ then $P(|Y| > 4) \sim N(0, 2)$
- Transform Y -> Z_y then $P(|Z_Y| > 4/\sqrt{2}) \sim N(0, 1)$
- Calculation of in R below:

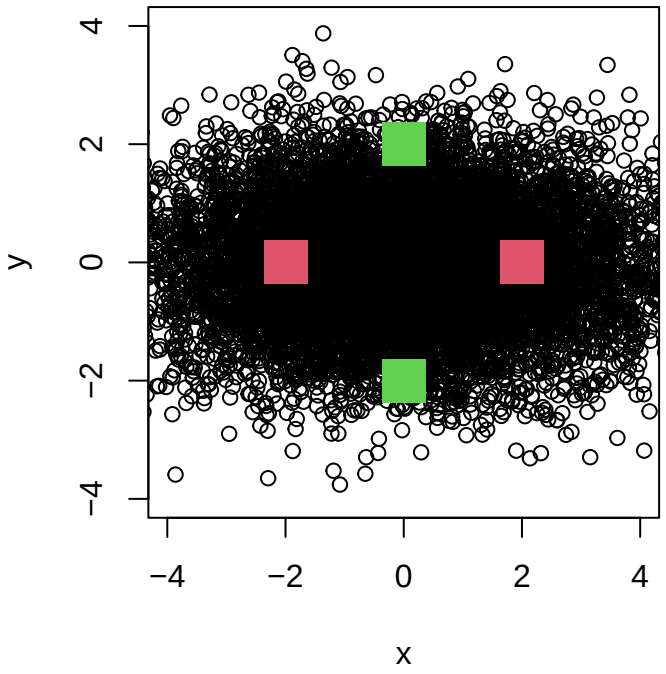
```
2 * (1 - pnorm(4/sqrt(2)))
```

```
## [1] 0.004677735
```

Q1 continued

c) Estimate the two probabilities using simulations. The code in the previous page generates 1000 random samples. Change the sample size from n=1000 to n=10000 and then estimate the two probabilities. To do that, you need to examine all pairs of data points and then calculate the proportion of pairs satisfying a certain condition.

```
set.seed(20230404)
n <- 10000 # number of samples
# bivariate normal random sample parameters
bivariate_mu <- c(0,0)
cov_matrix <- matrix(c(4,0,0,1),2,2)
# bivariate normal sample
sample <- mvrnorm(n=n, mu=bivariate_mu, Sigma=cov_matrix)
# Extract the X values, X ~ N(0, 4)
x <- sample[, 1]
# Extract the Y values, X ~ N(0, 1)
y <- sample[, 2]
# plot
par(pty="s") #to make sure the shape of figure is a square
sample %>%
  plot(xlab="x", ylab="y", xlim=c(-4,4), ylim=c(-4,4))
points(x=c(-2, 0, 0, 2), y=c(0, -2, 2, 0), pch=15, col=c(2,3,3,2),cex=3)
```



Calculate $P(|X| > 2)$ and calculate $P(|Y| > 2)$

```
# Calculate the proportion of pairs satisfying |X| > 2
mean(abs(x) > 2)
```

```
## [1] 0.3196
```

```
# Calculate the proportion of pairs satisfying |Y| > 2
mean(abs(y) > 2)
```

```
## [1] 0.05
```

Q2

Find a matrix A such that AY gives the di