# Statistical Arbitrage Project

# Pairs-Trading Based on Co-integration in US Equity Market

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### **Executive Summary**

We implement and optimize the pairs-trading strategy based on co-integration relationship in the US equity market. Co-integration indicates that although each stock price series may not be stationary, their linear combination exists long-term stationarity with short-term deviation and mean-reversion trend following, which is favorable to pairs-trading strategies. We use all NYSE stocks daily data from Jan 2001 to August 2016 to conduct the in-sample (Jan 2001 to Dec 2010) and out-of-sample (Jan 2011 to August 2016) tests.

There are two core problems related to this strategy: stock pairs selection and trading rules determination. We cut the data into consecutive formation periods and trading periods with different lengths, and use the formation period data to select stock pairs and use the trading period data to generate trading signals based on per-determined trading rules.

We perform a three-stage selection procedure based on the logarithm of stock closing prices: first, through "Correlation matrix", "Clustering", or "Graphical Lasso" pre-select stock groups with similar price movement patterns; second, through "Co-integration test" choose stock pairs or baskets that pass the statistical test with 99% significant level; third, through in-sample simulated trading choose the top 20 pairs with highest Sharpe Ratio as the profitability measure. We compare the performances of each selection method with and without the Sharpe Ratio test.

In the trading period, we utilize the estimated coefficients from the formation period to linearly combine the logarithm of selected stock prices. We call the linear combination "spread" in the following context. We observe the standardized spread each trading day using the closing bid and ask prices, and compare it with six pre-set thresholds (short bail-out, short open, short close, long close, long open and long bail-out from top to bottom) to determine when to enter into the market and when to exit the market. To calculate the mean and standard deviation to normalize the spread, we utilize two methods: moving look-back windows and formation periods.

Combining the selection and trading rules, we totally have 8 strategies: correlation matrix with and without Sharpe Ratio test, Graphical Lasso with and without Sharpe Ratio test, and for each of the

above four selection methods, we consider using moving windows or formation periods to calculate the statistics to standardize the spread.

There are two important sets of parameters we need to tune: one is the lengths of formation and trading periods, the other is the level of the six thresholds. We consider 9 combinations for the first set of parameters, 5 combinations for the second set and in total 45 possible sets for each trading strategy.

To approximate the real trading environments, we use the bid and ask prices instead of the closing prices to buy and sell the stocks and take into consideration transaction costs including slippage, commission fees and rental costs for short stocks. In addition, to avoid look-ahead bias, we treat unavailable price data differently for formation periods and trading periods, and only use the historical data to design strategies and generate the trading signals.

Comparing in-sample and out-of-sample results for different trading strategies with optimal parameters, we conclude that, Sharpe Ratio test can help to significantly improve the Sharpe Ratio in the trading period under all cases. Although Graphical Lasso can pre-select stocks with higher profit potential, the performance is less stable and the drawdown is larger. Comparatively, performances of correlation matrix are more stable and more consistent in sample and out of sample. Using moving average windows to standardize spreads can realize a higher winning percentages than using formation periods. The higher cumulative profits by using formation periods are largely due to a few extremely large daily profits. Therefore, we prefer the strategy with correlation matrix as pre-selection measure and moving look-back windows as standardizing methods.

We further conduct statistical arbitrage tests based on t and t + AR(1) residuals and find that our strategy results do not reject the null hypothesis, which means that it is not satisfies statistical arbitrage strategy even though it could have positive profits.

There are still some issues worth further investigation: first, we should try different data

frequencies to find the optimal choice for co-integration pairs-trading strategy to improve the Sharpe Ratio; second, we need to find the reasons of extreme values in the profit series to come up with possible solutions to reduce the volatility; third, we should further consider trading strategies with hedging options to determine whether hedging would improve the strategy performance; fourth, we can use other information to determine the trade size instead of the price series, like volume data and minimum profit level we would like to realize.

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### 1. Motivation

Pairs trading is a classical statistical arbitrage strategy that is still popular along the street, where we take a long position and a short position simultaneously. There are many measures to select the assets to buy and sell. In our strategy, we focus mainly on the co-integration method and its utilization in US equity market. We try to find the long-term stationary relationship between stocks so that once the linear combination deviates too much from the threshold, we will long or short the spread accordingly and wait for it to return to the stable level. For this strategy, there are two main parts: pairs or baskets selection and trading rules determination.

We use a three-stage stock selection procedure. Considering such a large pool of candidate stock pairs, we try to pre-select stock groups with similar price movement pattern before conducting the co-integration tests. We consider three preselection methods: correlation, clustering and Graphical Lasso. Then we perform co-integration tests to choose the stock pairs or baskets which pass the co-integration test, indicating there exists a long-term stable relationship among the pairs. Based on that, we can make investment decision. Finally, through in-sample simulated trading we choose the top 20 pairs with highest Sharpe Ratio. We introduce the third selection criterion since the final purpose of our strategy is to realize a high profitability measure (Sharpe Ratio is one of the most important measures), but highly co-integrated pairs do not necessarily achieve high Sharpe Ratios, and adding this filter enables us to focus on the ones that tend be more profitable in the real trading.

As for the trading rules, we preset the open, close, and bail-out thresholds and look at the path of the standardized spread to determine when to enter into the market and when to exit. To standardize the spreads, we need to have mean and standard deviation. Here, we employ two different methods: calculating the statistics using moving look-back window and using formation period data.

There are some parameters we need to tune: the length of formation period and that of trading period and the six levels of thresholds. Then we use the cumulative profit-and-loss, volatility of returns, Sharpe Ratio, maximum drawdown and winning percentage to compare different strategies with different sets of parameters in-sample and out-of-sample.

### 2. Pairs Selection Methods

We choose co-integration test to select stock pairs or baskets. There are two problems we need to solve before that. First, what is the target time series. Second, which pairs or baskets of stocks should we test on.

For the first problem, since the distribution of stock price has heavier tails than normal distribution and cannot be negative, we take the logarithm of the stock price as our target time series, which is approximately normal distributed.

For the second problem, one possible answer is to test on all possible pairs or baskets. However, the number of tests would explode as the number of candidate stocks increases. Therefore, we utilize some methods to reduce the pool size first and then perform co-integration tests. We tried three methods: correlation, clustering and Graphical Lasso.

## 2.1. Correlation and Co-Integration Test

Before performing co-integrating test, we pre-select prospective pairs based on their correlation. The correlation threshold is pre-set to be 0.97. However, this parameter could be adjusted or tuned. Then, we conduct co-integration test and if the ADF test value is below -2.57 under alpha = 0.01, we denote this pair passes this test and the residual display somewhat co-integration property. The coefficients in the regression are used for long short amount calculation.

## 2.2. Clustering and Co-Integration Test

Before performing co-integrating test, we pre-select prospective pairs based on their clustering group. In order to better approximate the real equity market, we choose the clustering group number to be 11, which is the total sector number in S&P 500. Again, in each clustering group, we conduct co-integration test on each pair and if the ADF test value is below -2.57 under alpha = 0.01, we denote this pair passes this test and the residual display somewhat co-integration property. The coefficients in the regression are used for long short amount calculation.

### 2.3. Graphical Lasso and Co-Integration Test

Graphical Lasso is another method we use to preselect stock pairs. It treats the whole collection of candidate stocks as a network and finds a small collection of stocks that appear to be co-integrated. We set  $\rho = 0.88$  in the Graphical Lasso function and only choose the baskets with 2-5 stocks to further conduct co-integration test. Then we use Johansen test to choose the stocks we are going to trade in the following trading period. We set the significant level as 1% and select the baskets satisfying two criteria: 1) at least one coefficient is negative and 2) there is only one relation.

## 2.4. In-sample Sharpe Ratio

In order to better select the top performance stock pair, we add another in sample Sharpe ratio criteria and generate trade list based on top 20 pairs.

We introduce the extra selection criterion since the final purpose of our strategy is to realize a high profitability measure (Sharpe Ratio is one of the most important measures), but highly co-integrated pairs do not necessarily achieve high SRs. Adding this filter enables us to focus on the ones that tend to be more profitable in real trading. We are going to compare the results with and without this criterion to see whether it is helpful.

## 3. Trading Rules

## 3.1. Basic Trading Idea

Co-integration means linear combination of two or more series is a stationary process, which has a desirable property of mean-reversion for pairs trading strategy. Therefore, we can look at the dynamics of the linear combination of logarithm price processes (we call it "spread" in the following context). If the spread is too low, we expect it would move up toward the mean level and take a long position of the spread (long in stocks with positive coefficients and short in stocks with negative coefficients); when the spread returns to the mean or near the mean, we close the position and realize the profit. Similarly, if the spread is too high, we expect it would move down toward the mean level and take a short position of the spread (short in stocks with positive coefficients and long in stocks with negative coefficients); when the spread returns to the mean or near the mean, we close the position and realize the profit.

To apply the same thresholds as consistent measures of whether the spread is too high or too low, we first standardize the spread with its mean and standard deviation since the spreads are in different scales for different stock pairs or baskets.

### 3.2. Possible Risks

One primary risk related to this kind of strategy is that it is possible that the spread goes further and further away from its mean level and never goes back. To reduce the possible loss size, one way is to set some stop-loss or bail-out levels. When the spread crosses and goes beyond the levels, we close our position by force.

### 3.3. Transaction Costs

There are three types of transaction costs: bid-ask spread, commission fee, and market impact.

To take into consideration the bid-ask spread in the market, we use ask prices when we buy stocks, and bid prices when we sell stocks.

As for the other two kinds of transaction cost, we charged 0.05% to open or close any

trade. This is comprised of 2\*0.01% brokerage fee, slippage 2\*0.005% and 0.02% rental cost for short positions.

Another issue we should bear in mind is that in real trading, we cannot trade with the close prices that triggers the trading signals. However, we do not have access to open bid and ask prices, the length of waiting periods is too long if we wait to the end of next day. Therefore, we assume that the open bid and ask prices are very close to closing bid and ask prices, and when we receive the trading signals, we use current day's closing prices to enter into the market with 0.02% slippage.

### 3.4. Trading Rules Formulation

Therefore, the trading rules can be formulated as follows. Let  $S_1$  and  $S_2$  be the two stocks we select. Define  $LR = log(ask_1) - \beta log(bid_2)$  and  $SR = log(bid_1) - \beta log(ask_2)$  as the log price spreads of long pair and short pair.  $sLR = \frac{LR - Lm}{Ls}$  and  $sSR_s = \frac{SR - Sm}{Ss}$  are the standardized log price spreads, where Lm, Ls, Sm and Ss are the mean, standard deviation of long pair and mean and standard deviation of short pair. There are six thresholds in the trading rules and from top to bottom are short bail-out level (Sb), short enter level (So), short exist level (Sc), long exist level (Lc), long enter level (Lo) and long bail-out level (Lb). The trading rules are:

- If we do not hold any position currently, when  $sLR_t < Lo$  and  $sLR_t > Lb$ , we take a long position of the spread;
- If we hold a long spread position currently, when  $sSR_t \ge Lc$  or  $sSR_t \le Lb$ , we close the position;
- If we do not hold any position currently, when  $sSR_t > So$  and  $sSR_t < Sb$ , we take a short position of the spread;
- If we hold a short spread position currently, when  $sLR_t \leq Sc$  or  $sLR_t \geq Sb$ , we close the position.

Here, we have two remaining questions to be answered, 1) how to determine the mean

<sup>&</sup>lt;sup>1</sup> This can be easily extended to the case of more than two stocks.

and standard deviation, and 2) how many shares to trade.

#### 3.4.1. How to determine the mean

For the first question, we come up with two solutions: one is to use moving look-back windows and the other is to use data in the formation periods.

### Moving Look-back Window

On each day t, we use the past k days' spread (we set k = 20 in our case) to calculate the mean and standard deviation. Then we use them to normalize today' spread, and compare it with the predetermined thresholds of trading signals. Once we enter into the market, we freeze the mean and standard deviation as the ones when we open the position. After we close the position, we unfreeze the mean and standard deviation and recalculate it on a rolling basis.

### Formation Period

We cut our data into formation period and trading period. We use the formation period to select stock pairs and determine the co-integration coefficients. One natural way to use formation data to calculate the mean and standard deviation and then use these numbers to standardize the spread in the trading period.

### **3.4.2.** How many shares to trade

After we conduct the co-integration test, we can get the coefficients to combine the logarithm of stock price processes. Although these coefficients are related to logarithm of stock prices, which are not tradable, we can actually use them to determine the number of shares we are going to buy and sell. The reason is as follows.

Let  $S_1$  and  $S_2$  be the two stocks we select and their co-integration relationship is  $log(P_1(t_0)) - \beta log(P_2(t_0))$ . Assuming at  $t_0$  the spread is too small, we decide to open a long position and expect it would move up toward the mean level. Later on, at time  $t_1$ , the spread returns and we close the position. If we could trade the log spread, we

would get

$$\log (P_1(t_1)) - \beta \log (P_2(t_1)) - (\log (P_1(t_0)) - \beta \log (P_2(t_0))) > 0.$$
 Using  $\log (1+x) \sim x$ , we have 
$$\log (P_1(t_1)) - \log (P_1(t_0) - \beta (\log (P_2(t_1)) - \log (P_2(t_0)))$$

$$\log (P_1(t_1)) - \log (P_1(t_0) - \beta(\log (P_2(t_1)) - \log (P_2(t_0)))$$

$$= (P_1(t_1) - P_1(t_0))/P_1(t_0) - \beta (P_2(t_1) - P_2(t_0))/P_2(t_0) > 0,$$

which means that if we buy  $1/P_1(t_0)$  shares of stock 1 and sell  $\beta/P_2(t_0)$ , we could earn a profit. Therefore, assuming we can trade in fractional shares, the number of shares we are going to buy or sell is equal to the coefficients from co-integration test over the initial price when we decide to enter into the market.

### 3.5. Close types

There are 8 types to close a position, 4 for long position and 4 for short positions. We summarize as follows.

Position	Symbol	Meaning					
Long	du	Down and up, return to mean.					
pairs	dd	Down and down, bail out.					
	dexc	Down and exceed the maximum holding length.					
	dna	Down and encounter unavailable data.					
Short	ud	Up and down, return to mean.					
pairs	uu	Up and up, bail out.					
	uexc	Up and exceed the maximum holding length.					
	una	Up and encounter unavailable data.					

"Du" and "ud" are the close types we are looking for since it means after deviation, the spread returns back to the normal range and we close the position with a positive profit. "Dd" and "uu" are due to the fact that we set the stop-loss levels and when the spread deviates from its mean level too much, we close the position we hold with a controllable loss.

We cut the data into non-overlapping rolling trading periods, and for each trading period we use the formation period just before it to select stock pairs to trade in this period. Sine we can open a position at any point in the trading period, it is possible that we enter into the market at the end of this trading period. Considering this possibility, we extend the holding period to the end of the trading period plus 50 days by default. That means we are permitted to hold an open position after the current trading period with maximum 50 days following the end of the trading period, while we are not permitted to open a position after the current trading periods with the selected pairs from the previous formation period. There are 2 kinds of close types with this setting: "dexc"and "uexc", which represent that we get to the last day of the extended holding period and close the position at the last day.

"Una" and "dna" mean that after we open the position, we encounter NA prices, and under this condition we close the position with last day's price data.

## 4. Data Description

We downloaded data from CRSP. We used quarterly updates, NYSE listed stocks from 2001/01/02 to 2016/08/31. We identify stocks by their unique Ticker information. We collected time series information of each stock including Price, Ask or High, Bid or Low, Closing Bid, Closing Ask and Cumulative Factor. We use Price/Cumulative Factor as the adjusted closing price. There are in total 2104 stocks and 2944 trading days.

In order to utilized data into our model, we reshape each time series information into a separate matrix format. Each row represents that information on a specific date. Each matrix has 2944 rows according to length of trading dates and 2104 columns according to total stocks.

We split whole data set into training and testing parts. Training data is from 2001/01/02 to 2010/12/31, approximately 60% of our whole dataset. The rest is used for testing.

### 4.1. Data cleaning

For closing prices, there are some negative values, and according to the documentation of CRSP it is due to missing data and comes from negating the average of closing bid and ask price. Therefore, we take the absolute value of the closing price. There is no negative ask and bid prices.

### 4.2. Unavailable data

When dealing with real data, we would find there are some unavailable data and our solutions are different based on whether we are in formation period and trading period. In the formation period, since we have all the historical data, and we only focus on the stocks with full records during this period and select stock pairs or baskets from them. In the trading period, to avoid look-ahead bias, we do not delete any price series and deal with the unavailable data as follows.

- If we do not hold any position and today's price data is unavailable, we move to the next day.
- If we have already opened a position and today's price data is unavailable, in theory there are three possible conditions: 1) the stock is delisted that day, and we should clear our position; 2) the stock does not have trading activity that day, and we could hold the position until there are available trading data; 3) missing records. However, we could not distinguish these conditions and therefore, once we encounter NA data after we open a position, we close the position using last day's price information.

## 5. Empirical Results

## **5.1.** Training period and testing period

We cut our data into two parts: training data and testing data. The training period is from the beginning of Jan 2001 to the end of Dec 2010, and the testing period is from the beginning of Jan 2011 to the end of June 2016. For each period, we further cut the data into formation period and trading period. We use the data in formation periods to select

stock pairs and use the data in trading periods to conduct simulation trading. Trading periods are not overlapped while formation periods are likely to overlap with each other. For example, if we take 6 months of data to form stock pairs and 3 months of data to simulate trading, then the first trading period is from 2002.1.1 to 2002.3.31 and the second trading period is from 2002.4.1 to 2002.6.30, where the first formation period is from 2001.7.1 to 2001.12.31 and the second formation period is from 2001.10.1 to 2002.3.31.

### 5.2. Parameters to tune

The first set of tuning parameters is the length pf of formation and that of trading periods. Denote J as the length of formation period in unit of days and K as the length of trading period in unit of days. J and K both can take value of 60, 90 and 120, leading to a and there are totally of 9 possible combinations.

Another set of important parameters to tune is the six trading rule thresholds: short bail-out level (Sb), short enter level (So), short exist level (Sc), long exist level (Lc), long enter level (Lo) and long bail-out level (Lb). We consider the following possible 5 combinations. Combined with J and K, there are in total 45 cases for each selection and trading strategy.

Lb	Lo	Lc	Sc	So	Sb
-6	-2	0	0	2	6
-10	-2	0	0	2	10
-3.5	-2	0	0	2	3.5
-10	-2	-0.5	0.5	2	10
-6	-1	0	0	1	6

## **5.3.** Stock selection and trading rules combination

We consider 4 stock selection methods: correlation + co-integration test with and without in-sample Sharpe Ratio test, and graphical lasso + co-integration test with and without

in-sample Sharpe Ratio test; 2 trading rules: spread standardized using moving windows and spread standardized using formation statistics. In total, we have 8 possible combinations of stock selection and trading strategies.

### **5.4.** Performance measurement

We use the following statistics to measure the performance of our trading strategies.

Name	Calculation Method
Average PnL	Mean of monthly PnL series
Standard Deviation of PnL	Standard deviation of monthly PnL series
Cumulative PnL	Total PnL during the whole trading periods
Winning Percentage	Number of days with positive profit over total number of
	days with non-negative profit
Maximum Drawdown	The maximum of the difference series between historical
	maximum cumulative PnL and today's cumulative PnL
Total Long Numbers	Total number of trades opened for taking a long position
	of the pairs
Total Short Numbers	Total number of trades opened for taking a short position
	of the pairs
Average Long Days	Average number of holding period for taking a long
	position of the pairs
Average Short Days	Average number of holding period for taking a short
	position of the pairs
Close Types	ud, uu, uexc, una for short position in pairs
	du, dd, dexc, dna for long position in pairs

## **5.5.** In-sample results

Using data from 2001.1.2 to 2010.12.31, the best set of parameters using Sharpe Ratio as the criteria for each combination of stock selection and trading strategy is as follows. We also list the average Profit-and-Loss (PnL), standard deviation of PnL, cumulative PnL (with and without Sharpe transaction cost), winning percentage, maximum drawdown,

total number of long pairs and short pairs, average holding period pf long pairs and short pairs, and average pairs of stocks selected for trading period.

Pairs	Corre	lation + C	Co-integrati	on Test	Graphica	l Lasso + (	Co-integration Test		
Selection Methods	with S	with SR test		without SR test		with SR test without SR tes		SR test	
Trading Rule	mv	rolling	mv	rolling	mv	rolling	mv	rolling	
J	120	120	120	90	120	90	120	90	
K	120	90	60	90	120	60	120	120	
Sb	10	6	10	6	6	6	6	6	
So	2	2	2	2	1	2	1	2	
Sc	0	0	0	0	0	0	0	0	
Lc	0	0	0	0	0	0	0	0	
Lo	-2	-2	-2	-2	-1	-2	-1	-2	
Lb	-10	-6	-10	-6	-6	-6	-6	-6	
Sharpe Ratio	0.09046	0.0521	-0.0637	-0.1017	-0.0054	0.2535	-0.006	0.1844	
Average PnL	0.08	0.03	-0.81	-12.67	-0.08	1.23	-0.09	2.29	
PnL Volatility	0.87	0.52	12.84	124.55	14.77	4.84	14.77	12.40	
Cumulative PnL	9.79	3.44	-102.31	-1583.12	-10.00	153.61	-11.12	286.09	
Sharpe Ratio With TC	0.0903	0.0520	-0.064	-0.1017	-0.0056	0.2535	-0.006	0.1844	
Average PnL With TC	0.078	0.0274	-0.821	-12.671	-0.082	1.228	-0.091	2.286	
PnL Volatility With TC	0.866	0.527	12.842	124.612	14.76	4.845	14.77	12.39	
Cumulative PnL With TC	9.770	3.428	-102.675	-1583.92	-10.21	153.53	-11.33	285.79	

Winning	0.65	0.30	0.63	0.26	0.51	0.41	0.51	0.39
Percentage	0.02	0.30	0.03	0.20	0.51	0.41	0.51	0.39
Maximum	7.67	3.22	183.63	1584.27	183.71	6.25	185.13	45.76
Drawdown	7.07	3.22	165.05	1301.27	103.71	0.23	105.15	45.70
Total Long	576	36	22976	791	881	164	923	670
Number		30	22910	791	001	104	923	070
Total Short	643	56	22392	1301	927	147	973	773
Number	043	30	22372	1301	721	117	713	775
Average Long	23.08	28.03	27.28	25.68	16.65	9.30	16.39	11.65
Days	25.00	20.03	27.20	25.00	10.03	7.50	10.57	11.03
Average Short	29.00	48.63	27.90	31.07	16.99	8.32	17.00	8.98
Days	29.00	+0.05	21.90	31.07	10.99	0.32	17.00	0.90
Average Pairs	16.70	16.44	548.775	650 184	14.25	13 30	140	14 00
Number	10.70	70   16.44	548.775	659.184	14.25	13.39	14.9	14.09

### From the above table, we can see that:

- With the additional Sharpe Ratio test, all kinds of strategies' performance have improved.
- While longer formation period has better selection results compared to shorter formation period, the best trading period can take any value of 60, 90 and 120.
- We prefer higher bail-out levels, which implies that although sometimes the spread can become very large, there still exists a high possibility that it will return back to the normal level.
- Comparing across different strategies, the Sharpe Ratio is highest for Graphical
   Lasso + co-integration test with SR test using rolling windows to standardize price
   spreads and trigger trading signals.
- The largest average and cumulative PnL is achieved when using Graphical Lasso + co-integration test without SR test using rolling windows. However, its PnL's standard deviation is very high and therefore its Sharpe Ratio is less than the corresponding one with SR test.

- The winning percentages of strategy using moving windows are all higher than those of strategies using rolling statistics. The former ones' winning percentages are higher than 50% and the latter ones' are lower than 50%.
- Using correlation methods to pre-select stock pairs, the resulting volatility of PnL is much smaller than those using Graphical Lasso preselection method, and the holding periods are longer than the latter ones.
- On average, each period we select 15 to 20 stock pairs to trade except for correlation
   + co-integration test, since we set the minimum correlation as the selection criteria
   instead of limits of maximum pair numbers.
- The performance is worst if we use correlation to preselect stock pairs and without additional SR test.

In summary, we prefer Graphical Lasso as the pre-selection method to Correlation method, which implies that higher correlation does not imply higher performance for pairs-trading strategies. Moving average method can guarantee higher winning percentages but due to large negative returns it may lead to, the average PnL is not as good as rolling method.

## **5.6.** Out-of-sample results

For each trading strategy, we select the best 3 sets of parameters to conduct out-of-sample tests and compare the results with the in-sample results.

The following chart shows the in-sample and out-of-sample performance comparison of the best in-sample set of parameters for each trading strategy.

Pairs Selection Methods	Corre	lation + (	Co-integra	ntion Test		Graphical Lasso + Co-integration Test			
	with	SR test	withou	t SR test	with S	SR test	withou	t SR test	
Trading Rule	mv	rollin	mv	rolling	mv	rollin	mv	rolling	
	IIIV	g	IIIV	Tonnig	IIIV	g	IIIV	Tolling	

Sharpe	In-sample	0.09	0.05	-0.06	-0.10	-0.01	0.25	-0.01	0.18
Ratio	Out-of-sampl	0.12	0.12	0.25	-0.12	-0.03	-0.08	-0.03	-0.21
With TC	e								
Average	In-sample	0.08	0.03	-0.82	-12.67	-0.08	1.23	-0.09	2.29
PnL	Out-of-sampl	0.20	0.26	1.22	-0.09	-0.32	-0.31	-0.32	-2.25
With TC	e	0.20	0.20	1.22	-0.09	-0.52	-0.51	-0.52	-2.23
PnL	In-sample	0.87	0.53	12.84	124.61	14.76	4.85	14.77	12.39
Volatility	Out-of-sampl	1.72	2.07	4.80	0.81	9.24	3.91	9.24	10.94
With TC	e	1.72	2.07	4.00	0.01	9.24	3.91	9.24	10.84
Cumulativ	In-sample	9.77	3.43	-102.6	-1583.9	-10.2	153.53	-11.3	285.79
e PnL	in-sample	9.11	J.43	8	2	1	155,55	3	203.19
With TC	Out-of-sampl	14.1	18.27	86.42	-6.61	-22.7	-22.18	-22.7	-159.72
WITH IC	e	0	10.27	00.42	-0.01	9	-22.18	9	-139.72

From the above table, we can see that:

- Graphical Lasso has highest in-sample Sharpe Ratio with TC and average PnL with TC but the out-of-sample Sharpe Ratio and average PnL with TC are negative, indicating this is not a trustable strategy.
- Correlation Co-integration with SR Test has relative stable positive Sharpe Ratio and Average PnL with TC in both in-sample and out-of-sample, implying this is a reliable strategy. In addition, it has quite low standard deviation and fairly large out-of-sample PnL.
- Maximum out-of-sample Sharpe Ratio lies in the Correlation Co-integration Test, however, the in-sample results are quite disappointing. Hence, we couldn't select this strategy from the in-sample statistics.
- Generally speaking, compared with mv method, rolling method has quite discrepant results in-sample and out-of-sample. It's very volatile in terms of Sharpe Ratio, average PnL and Cumulative PnL with TC.

The following chart shows the best parameter among the best three in-sample parameters

## and their results.

Pairs Selection	Correl	ation + Co	-integra	tion Test	Graphic	cal Lasso +	Co-integ	ro-integration Test           without SR test           mv         rolling           90         90           120         120           10         6           2         1           0         0.5           0         -0.5           -2         -1           -10         -6           0.10         -0.21           0.21         -2.25           2.15         10.83           14.59         -159.56           0.10         -0.21           0.20         -2.25           2.15         10.84           14.55         -159.72	
Methods	with SR test without S			t SR test	with	SR test	withou	it SR test	
Trading Rule	mv	rolling	mv	rolling	mv	rolling	mv	rolling	
J	120	120	90	90	90	90	90	90	
K	120	90	120	90	120	60	120	120	
Sb	10	6	10	6	10	6	10	6	
So	2	2	2	2	2	2	2	1	
Sc	0	0	0	0	0	0	0	0.5	
Lc	0	0	0	0	0	0	0	-0.5	
Lo	-2	-2	-2	-2	-2	-2	-2	-1	
Lb	-10	-6	-10	-6	-10	-6	-10	-6	
Sharpe Ratio	0.12	0.12	0.25	-0.12	0.10	-0.08	0.10	-0.21	
Average PnL	0.20	0.26	0.46	-0.09	0.21	-0.31	0.21	-2.25	
PnL Volatility	1.73	2.07	1.79	0.81	2.15	3.91	2.15	10.83	
Cumulative	14 11	18.29	32.41	-6.60	14.59	-22.14	14 50	150 56	
PnL	14.11	10.29	32.41	-0.00	14.33	-22.14	14.39	-139.30	
Sharpe Ratio	0.12	0.12	0.25	-0.12	0.10	-0.08	0.10	-0.21	
With TC	0.12	0.12	0.23	0.12	0.10	0.00	0.10	0.21	
Average PnL	0.20	0.26	0.46	-0.09	0.20	-0.31	0.20	-2.25	
With TC									
PnL Volatility	1.72	2.07	1.79	0.81	2.15	3.91	2.15	10.84	
With TC									
Cumulative									
PnL	14.10	18.27	32.38	-6.61	14.55	-22.18	14.55	-159.72	
With TC									
Winning	0.78	0.47	0.81	0.31	0.71	0.43	0.71	0.37	
Percentage									
Maximum	4.01	7.87	5.53	12.67	9.22	40.49	9.22	178.45	

Drawdown								
Total Long	213	64	1498	56	177	61	177	274
Number	213	04	1490	30	177	01	177	274
Total Short	221	53	1341	90	181	38	181	348
Number	221	55	1341 90	101	50	101	540	
Average Long	28.35	35.44	24.28	30.23	28.57	11.05	28.57	9.95
Days	20.33	33.44	24.20	30.23 28.37 11.0	11.05	20.57	9.93	
Average Short	32.40	38.21	23.12	31.44	31.61	11.16	31.61	10.57
Days	32.40	30.21	23.12	31.44	31.01	11.10	31.01	10.57
Average Pairs	11.91	11.87	62.00	40.20	8.50	7.70	8.58	8.58
Num	11.71	11.0/	02.00	40.20	06.0	7.70	00.00	0.70

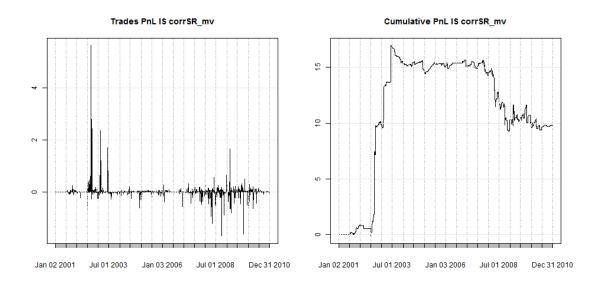
It's not difficult to reach the following findings:

- Compared to in-sample best parameters, out-of-sample parameters don't change for Correlation Co-integration with SR Test (both mv and rolling trading rules), Correlation Co-integration without SR test Rolling rule and Graphical Lasso with SR test Rolling rule.
- Compared to in-sample best parameters, Correlation Co-integration methods' Sharpe
  Ratio, average PnL and Winning Percentage improved in the out-of-sample result.
  Other methods' out-of-sample results are somewhat not in line with the in-sample
  test results.
- Compared with mv trading rule, the rolling rule has smaller winning percentage, smaller Sharpe Ratio as well as average PnL in general. Thus mv trading rule is better empirically.
- Conclusion: Correlation Co-integration method with SR test based on mv trading rule should be the optimal choice considering in-sample out-of-sample consistency, overall profitability and reliability.

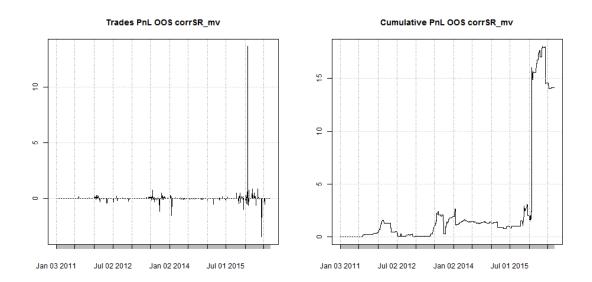
### 5.7. Result visualization

For comparison, here we pasted the best and worst pairs of profit and cumulative profit from in-sample test and out-of-sample test.

Best in-sample instance



Corresponding out-of-sample instance



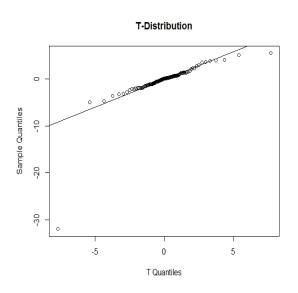
We can see that significant portion of cumulative profit is contributed by jumps of the profit sequence, for the out-of-sample corresponding plots, we see the profit is quite random.

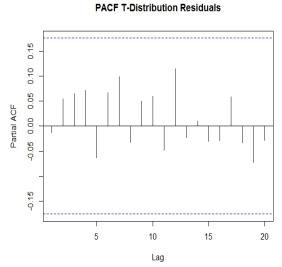
# 6. Statistical Arbitrage Test

The In-Sample CorrSR\_mv T-Statistical give the following p-value

	mu	lambda	df
CorrSR_mv	0.9746943	0.8619055	0.4813791

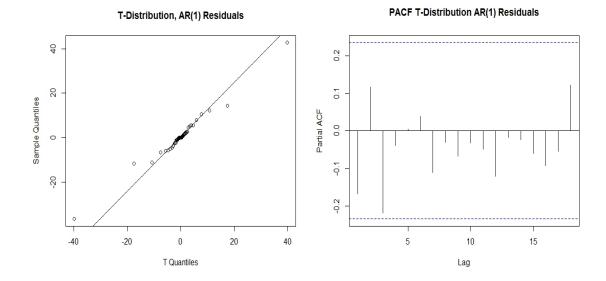
The residual QQplot and PACF plot are





Out-Of-Sample CorrSR\_mv T-Statistical AR(1) give the following P-value

	mu	lambda	df
Corr_MV	0.5653275	0.9616601	0.999771



The in-sample and out-of-sample tests cannot make us confident about the fact that this is a profitable arbitrage strategy. The sudden spikes still make us believe high frequency would be a better choice. And we will dig more into why this happens if time allowed.

## 7. Conclusion and Future Research

We executed our model as proposed and simulated our trading strategy based on the best tuning parameters. That is, using correlation as our first stage stocks screening, using Sharpe Ratio as additional criterion and using moving window to estimate mean and standard deviation for both in-sample and out-of-sample.

The in-sample Sharpe Ratio is 0.09, maximum drawdown is 7.67 and winning percentage is 65%. The out-of-sample Sharpe Ratio is 0.25, maximum drawdown is 5.53 and winning percentage is 81%.

To further update our model, we intended to increase the trading frequency. Besides, we also intend to adjust our trading strategy to require a minimum profit of K. The theoretic minimum profit strategy is following:

Suppose  $S_1$  and  $S_2$  is the selected pair following the co-integration process

$$\log S_1(t) + \beta \log S_2(t) = \epsilon(t)$$

Let a and b be our open and close position threshold, and K is the minimum profit we want to obtain in the end. Suppose we long the pair, that means we short stock  $S_1$  and long  $S_2$ .

Suppose at one trading day,  $\epsilon(t_0) > a$ , we long  $n_2$  shares of  $S_2$ . At this point, we have

to short  $n_1 = \frac{n_2 P_2(t_0)}{|\beta| P_1(t_0)}$  shares of  $S_1$ . Then at the closing date, the profit is

$$n_{2}(P_{2}(t_{c}) - P_{2}(t_{0})) + n_{1}(P_{1}(t_{0}) - P_{1}(t_{c}))$$

$$= \frac{n_{2}P_{2}(t_{0})}{\beta} * \left(\beta * \left(\frac{P_{2}(t_{c})}{P_{2}(t_{0})} - 1\right) + \left(\frac{P_{1}(t_{c})}{P_{1}(t_{0})} - 1\right)\right)$$

$$= \frac{n_{2}P_{2}(t_{0})}{\beta} * (\beta * logP_{2}(t_{c}) + logP_{1}(t_{0}) - (\beta * logP_{2}(t_{0}) + logP_{1}(t_{0})))$$

$$= \frac{n_{2}P_{2}(t_{0})}{\beta} * (b - a) > K$$

To satisfy the above condition, at the start period, we need to long

$$K * \frac{\beta}{(b-a)P_2(t_0)}$$
 shares of  $S_2$  and short  $\frac{K}{b-a} * P_1(t_0)$  shares of  $S_1$ .

## 8. Reference

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# Appendix:

Table 1 Tuning Parameters Combination

No.	Lb	Lo	Lc	Sc	So	Sb	formation	holding
:	-6	-2	0	0	2	6	90	60
	-10	-2	0	0	2	10	90	60
;	-3.5	-2	0	0	2	3.5	90	60
	-6	-1	0	0	1	6	90	60
	-10	-2	-0.5	0.5	2	10	90	60
	-6	-2	0	0	2	6	90	90
	-10	-2	0	0	2	10	90	90
:	-3.5	-2	0	0	2	3.5	90	90
	-6	-1	0	0	1	6	90	90
1	-10	-2	-0.5	0.5	2	10	90	90
1:	-6	-1	-0.5	0.5	1	6	90	120
1:	-6	-2	0	0	2	6	90	120
1:	-10	-2	0	0	2	10	90	120
14	4 -3.5	-2	0	0	2	3.5	90	120
1	-6	-1	0	0	1	6	90	120
10		-2	-0.5	0.5	2	10	60	60
1	7 -6	-1	-0.5	0.5	1	6	60	60
1		-2	0	0	2	6	60	60
19		-2	0	0	2	10	60	60
20		-2	0		2	3.5	60	60
2:	-6	-1	0	0	1	6	60	90
2:		-2	-0.5	0.5	2	10	60	90
2:		-1	-0.5	0.5	1	6	60	90
24		-2	0	0	2	6	60	90
2!		-2	0	0	2	10	60	90
21		-2	0	0	2	3.5	60	120
2		-1	0	0	1	6	60	120
28		-2	-0.5	0.5	2	10	60	120
29		-1	-0.5	0.5	1	6	60	120
30		-2	0	0	2	6	60	120
3:		-2 -2	0		2	3.5	120 120	60
3;		-2	0	0	1	6	120	60
34		-2	-0.5	0.5	2	10	120	60
3!			-0.5	0.5	1	6	120	60
3(					2		120	90
3					2		120	90
38		-2			2	3.5	120	90
39					1	6	120	90
4(				0.5	2	10	120	90
4:			-0.5	0.5	1	6	120	120
42			0.5		2	6	120	120
4;			0		2	10	120	120
44		-2			2		120	120
4!					1		120	120

Table 2 Correlation + Co-integration + Moving Look-back Window (mv)

1 1	4	42	41	40	39	38	37	36	35	22	33	32	31	3	29	22	26	25	24	23	22	21	20	19	18	17	16	15	14	3 6	13 11	10	9	00	7	6	ر ت	4	ω	2		N <sub>o</sub> .
2 47						-3.38	-1.35	-1.89	-5.52	-1.31	-4.79	-3.87				-0.01					-2.40	-3.78									-1.38		-5,46		-2.26					-1.75	-2.52	Average PnL.Vol
27			66			38	35	89	52										55		40										2 8											ge PnLV
0.79	10.33	7.10	8.56	17.02	18.08	13.87	18.79	13.64	17.68	11.77	18.10	16.81	12.84	13.80	19.81	14 28	15.23	7.98	8.81	11.12	7.85	11.55	17.67	15.31	15.75	21.37	14.38	10.53	7.87	10.56	9.57	17.33	20.59	16.25	17.96	14.20	10.37	12.89	10.83	10.97	62	
0.35	-0.15	-0.25	-0.31	-0.09	-0.19	-0.24	-0.07	-0.14	-0.31	-0.11	-0.26	-0.23	-0.06	-0.30	-0.37	-0.00	-0.34	-0.28	-0.29	-0.37	-0.31	-0.33	-0.33	-0.21	-0.27	-0.36	-0.25	-0.22	-0.35	-0.09	-0.32	-0.16	-0.27	-0.28	-0.13	-0.21	-0.19	-0.32	-0.39	-0.16	-0.26	Sharpe. Ratio
200 65	-188.10	-220.25	-332.91	-195.31	-421.41	-422.05	-169.21	-236.58	-689.78	-163.73	-598.66	-483.22	-102.31	-518.74	-910.71	-448 56	-654.05	-276.60	-318.57	-515.79	-300.61	-472.55	-729.01	-394.10	-526.19	-962.89	-441.86	-290.75	-339.26	-114.38	-378.13	-340.47	-682.33	-565.65	-283.11	-377.40	-240.45	-515.32	-524.62	-219.13	-314.87	Cumulat ive.PnL
2 47	TC.T-	-1.76	-2.66	-1.57	-3.37	-3.38	-1.36	-1.90	-5.52	-1.31	-4.79	-3.87	-0.82	-4.15	-7.29	25.0	-5.24	-2.21	-2.55	-4.13	-2.41	-3.78	-5.84	-3.16	-4.21	-7.71	-3.54	-2.33	-2.72	-0.92	-1.38	-2.73	-5.46	-4.53	-2.27	-3.02	-1.93	-4.13	-4.20	-1.76	-2.52	Average PnLVol Sharpe. Cumulat .PnL.Plu atility.Pl Ratio.Pl ive.PnL s.Cost us.Cost us.Cost
0.70	L					13.87	18.80	13.64	17.69	11.77	18.11	16.82				14 28				11.13	7.86	11.55	17.68	15.31	15.76	21.38	14.38			_	9.57		20.59									Average PnLVol .PnL.Plu atility.Pl s.Cost us.Cost
-0.38						-0.24	-0.07	-0.14	-0.31	-0.11	-0.26	-0.23				-0.00					-0.31	-0.33	-0.33	-0.21							-0.32		-0.27								-0.26	PnLVol Sharpe. atility.Pl Ratio.Pl us.Cost us.Cost
200 00			Τ.	١,		-422.38	-169.66	-236.93	-690.20	-164.06	-599.10	-483.56				-103.44				-516.10	-300.85	-472.87		-394.53							-172 47										-315.17	Cumulati ve.PnL.Pl us.Cost
0.30	T		ĺ		ĺ	8 0.34	6 0.61	3 0.48	0 0.42	6 0.61	0 0.48	6 0.36				7 056		ĺ		0 0.42	5 0.58	7 0.44									0.41		6 0.44								7 0.46	Cumulati Winnin ve.PnL.Pl g.Perce us.Cost ntage
200.00						4 499.58	1 197.96	8 304.77	2 690.39	1 212.57	8 600.29	6 503.95				6 451 37			6 341.36	2 529.81	8 318.54	4 489.88	3 736.93								1 396.99		4 684.29								6 352.15	
22757						38825	20431	25852	49400	25221	39118	38427				20145			Т	47563	24690	37492	53229								20320										29723	Average Maximu Total.Lo Total.Sh .Long.D m.DD ng.Num ort.Num ays
22027						37766	19640	24804	49140	24758	38718	37776				20106				47638	24437	37590		31709							20733		41372								29496	Total.Sh ort.Num
10.24	29.29	20.32	13.44	25,44	20.14	10.15	31.29	21.56	12,65	22.46	18.06	9.64	27.28	20.02	13.70	24.05	9.57	28.33	19,47	13.39	23.27	18.74	9.01	25,48	18.02	12.36	21.12	20.02	9.96	31.09	13.93	23.84	18.83	9.65	28.89	19.78	22.74	18.10	9.75	27.50	19.39	Average .Long.D ays
10.00	30.51	20.81	13.70	25.19	20.09	10.23	31.03	21.30	12.72	22.72	18.26	9.92	27.89	20.38	14.20	25.34	9.51	28.94	19.63	13.57	24.01	18.88	9.07	26.31	18.53	12.78	21.96	20.42	9.86	31.41	21 09	23.86	19.05	9.69	29.16	19.86	22.67	18.26	9.72	27.65	19.53	Average Average Long.D .Short.D .Pairs.N ays ays um
303.30	393.30	393.30	393.30	549.52	549.52	549.52	549.52	549.52	548.78	548.78	548.78	548.78	548.78	788.86	788.86	788 86	788.86	563.54	563.54	563.54	563.54	563.54	703.66	703.66	703.66	703.66	703.66	463.67	463.67	463.67	463.67	659.15	659.15	659.15	659.15	659.15	564.00	564.00	564.00	564.00	564.00	
E13	191	389	327	1377	1196	218	1714	861	995	1840	1580	306	2351	720	728	1288	1065	1656	785	810	1372	1132	359	2911	1449	1463	2328	909	162	1179	65 65 65 65 65 65 65 65 65 65 65 65 65 6	1638	1466	305	2044	1045	2157	1921	409	2668	1365	uexc
1/620	0010	9048	21745	15656	22224	13328	12778	13551	36917	17884	25869	14006	14818	16801	39290	10270	16874	13334	14090	32860	16307	22708	17480	18884	19042	45797	22608	16582	10662	9723	25050	18290	25692	15135	14936	15757	18460	26466	14497	15207	15596	nd
5555	2903	6554	6800	3887	9341	23447	3830	9288	9721	3741	9590	22613	3802	14693	15247	6500	35846	5074	11403	12051	5069	11652	35032	6939	15204	15990	6777	7879	20001	3257	8315	4879	11570	28313	4847	11611	4331	10944	25387	4379	10600	e e
1200	CEOT	923	1102	1222	1416	773	1318	1104	1507	1293	1679	851	1421	2745	3373	2020	2048	1859	1547	1917	1689	2098	1874	2975	2611	3230	2741	1899	1066	1631	1740	2062	2644	1467	2215	1957	2017	2639	1395	2196	1935	una
CTT	107	3/2	360	1467	1312	212	1853	921	1081	1950	1653	325	2423	802	792	1345	264	1677	852	820	1338	1230	456	2944	1485	1493	2333	964	269	1348	707	1862	1565	393	2265	1172	2253	2005	474	2721	1404	dexc du
1/220	8432	9040	21450	16007	22453	13393	13035	13796	36334	17681	25485	13893	14821	16619	39164	19098	16662	13524	14317	32975	16608	22597	17394	18672	18914	45499	22363	16613	10574	9720	24901	18198	25720	15195	14957	15785	18589	26691	14463	15445	15687	
E0707	2002	6583	6880	4280	10079	24407	4248	10016	10285	4103	10093	23194	4096	13479	14040	5008	32816	4992	11264	11919	4988	11580	33475	6807	14799	15543	6684	7791	19518	3224	7900	4792	11356	27704	4740	11363	4451	11263	25568	4485	10890	dd
1252	T143	989	1109	1164	1399	813	1295	1119	1700	1487	1887	1015	1636	2564	3088	2697	1905	1945	1596	1849	1756	2085	1904	2991	2613	3128	2753	1561	912	1383	1468	1984	2511	1425	2164	1946	1832	2318	1226	2052	1742	dna

Table 3 Correlation + Co-integration + Formation Period (rolling)

25.	3 4	2 4	4.	40	39	88	37	36	ӄ	32	జ	32	22	8	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	15	1 5	3 0	0 00	, _	7 0	0	4 n	. ω	2	ь	S <sub>o</sub>
0 0	-0.27	-0.27	-0.27	-7.90	-7.90	-7.90	-7.90	-7.90	-1.17	-1.17	-1.17	-1.17	-1.17	-8.24	-8.24	-8.24	-8.24	-8.24	-0.28	-0.28	-0.28	-0.28	-0.28	-1.54	-1.54	-1.54	-1.54	-1.54	-3.36	-3.36	-3.36	-3.26	-3.36	10.00	-12.6b	- 12.00	12.00	10.66	3.25	-3.25	-3.25	-3.25	PnL tility atio ve.PnL Cost Cost us.Cost ge m.DD
2 F	1.05	1.05	1.05	76.10	76.10	76.10	76.10	76.10	5.80	5.80	5.80	5.80	5.80	78.10	78.10	78.10	78.10	78.10	1.64	1.64	1.64	1.64	1.64	6.32	6.32	6.32	6.32	6.32	26.65	26.65	26.65	28.65	26.65	104.00	124.55	124.55	124.55	104 55	26.36	26.36	26.36	26.36	tility
0 0	-0.25	-0.25	-0.25	-0.10	-0.10	-0.10	-0.10	-0.10	-0.20	-0.20	-0.20	-0.20	-0.20	-0.11	-0.11	-0.11	-0.11	-0.11	-0.17	-0.17	-0.17	-0.17	-0.17	-0.24	-0.24	-0.24	-0.24	-0.24	-0.13	-0.13	-0.13	-0 13	-0.13	0.10	-0.10	-0.10	0.10	010	-0.12	-0.12	-0.12	-0.12	atio
00000	-33.35	-33.35	-33.35	-987.66	-987.66	-987.66	-987.66	-987.66	-145.68	-145.68	-145.68	-145.68	-145.68	-1030.36	-1030.36	-1030.36	-1030.36	-1030.36	-34.48	-34.48	-34.48	-34.48	-34.48	-192.70	-192.70	-192.70	-192.70	-192.70	-420.30	-420.30	-420.30	-420 30	-420.30	1500.10	-1583.12	- LOOS. LZ	1593.12	1500 10	-405.86	-405.86	-405.86		ve.PnL
0 1	-0.27	-0.27	-0.27	-7.91	-7.91	-7.91	-7.91	-7.91	-1.17	-1.17	-1.17	-1.17	-1.17	-8.25	-8.25	-8.25	-8.25	-8.25	-0.28	-0.28	-0.28	-0.28	-0.28	-1.54	-1.54	-1.54	-1.54	-1.54	-3.36	-3.36	-3.36	- 2 26	-3.36	10.67	-12.67	10.5T-	12.57	10.67	3.25	-3.25	-3.25	-3.25	Cost
2 F	1.05	1.05	1.05	76.14	76.14	76.14	76.14	76.14	5.80	5.80	5.80	5.80	5.80	78.13	78.13	78.13	78.13	78.13	1.64	1.64	1.64	1.64	1.64	6.33	6.33	6.33	6.33	6.33	26.66	26.66	26.66	26.66	26.66	104.01	124.61	124.01	124.61	10.07	26.37	26.37	26.37	26.37	.Cost (
0 0	-0.25	-0.25	-0.25	-0.10	-0.10	-0.10	-0.10	-0.10	-0.20	-0.20	-0.20	-0.20		•	_	-			-0.17	-0.17	-0.17	-0.17	-0.17	-0.24	-0.24	-0.24	-0.24	-0.24	-0.13	-0.13	-0.13	-0 13	-0.13						0.13	-0.12	-0.12	-0.12	Cost
2 2 2	-33.37	-33.37	-33.37	-988.17	-988.17	-988.17	-988.17	-988.17	-145.78	-145.78	-145.78	-145.78	-145.78	-1030.89	-1030.89	-1030.89	-1030.89	-1030.89	-34.52	-34.52	-34.52	-34.52	-34.52	-192.81	-192.81	-192.81	-192.81	-192.81	-420.52	-420.52	-420.52	-420 52	-420.52	1500.00	-1583.92	-T202.92	1592.92	100.07	406.07	-406.07	-406.07	-406.07	us.Cost (
0 0	0.21	0.21	0.21	0.23	0.23	0.23	0.23	0.23	0.26	0.26	0.26	0.26		25	25	25	0.25	0.25	0.22	0.22	0.22	0.22	0.22	0.23	0.23	0.23	0.23	0.23	0.21	0.21	0.21	0 21	0.20	_	_			$\overline{}$	0.22	0.22	0.22	0.22	ge
27.00	37.65	37.65	37.65	990.36	990.36	990.36	990.36	990.36	152.83	152.83	152.83	152.83	152.83	1030.36	1030.36	1030.36	1030.36	1030.36	36.31	36.31	36.31	36.31	36.31	193.24	193.24	193.24	193.24	193.24	420.39	420.39	420.39	420 39	420.39	1504.27	1584.27	17.4007	1594.27	400.90	405.93	405.93	405.93	405.93	m.DD r
000	268	268	268	807	807	807	807	807	807	807	807	807	807	1067	1067	1067	1067	1067	397	397	397	397	397	923	923	923	923	923	653	653	653	653	653	701	701	16,	701	704	674	674	674	674	ng.Num ort.Num
475	475	4/5	4/5	986	986	986	986	986	1209	1209	1209	1209	1209	1155	1155	1155	1155	1155	475	475	475	475	475	1112	1112	1112	1112	1112	682	682	682	683	682	1301	1301	TOCT	1301	1001	0 8	881	881	881	ort.Num ys
VC &C	28.24	28.24	28.24	37.63	37.63	37.63	37.63	37.63	27.74	27.74	27.74	27.74	27.74	18.54	18.54	18.54	18.54	18.54	10.29	10.29	10.29	10.29	10.29	16.56	16.56	16.56	16.56	16.56	31.69	31.69	31.69	31 69	31.69	20.00	25.68	25.00	25.00	25.04	27.54	27.54	27.54	27.54	'S 'S'
29.74	29.74	29.74	29.74	44.94	44.94	44.94	44.94	44.94	34.03	34.03	34.03	34.03	34.03	27.79	27.79	27.79	27.79	27.79	16.46	16.46	16.46	16.46	16.46	22.64	22.64	22.64	22.64	22.64	39.61	39.61	39.61	30 61	39.61	31.07	31.07	01.07	31.07	31 00	33.50	33.50	33.50	33.50	ort.Num ys ys m
303 30	393.30	393.30	393.30	549.52	549.52	549.52	549.52	549.52	548.78	548.78	548.78	548.78	548.78	788.86	788.86	788.86	788.86	788.86	563.54	563.54	563.54	563.54	563.54	703.66	703.66	703.66	703.66	703.66	463.67	463.67	463.67	463.67	463.67	000 1 E	650 15	019.LO	650 15	650 15	564.00	564.00	564.00	564.00	
45	45	45	45	211	211	211	211	211	250	250	250	250	250	95	95	95	95	95	29	29	29	29	29	138	138	138	138	138	99	99	99	9	99	100	103	L PC	103 103	102	201	201	201	201	uexc ud
71	71	71	71	212	212	212	212	212	212	212	212	212	212	265	265	265	265	265	92	92	92	92	92	206	206	206	206	206	133	133	133	23	133	220	328	022	322	200	153	153	153	153	
304	304	304	304	456	456	456	456	456	441	441	441	441	441	541	541	541	54.	541	322	322	322	322	322	546	546	546	546	546	358	358	သ္ဟ မ	22.00	358	n 00	565 505	2000	л Б Б Б	404	404	404	404	404	una
л 5	55 8	5	3 8	107	107	107	107	107	306	306	306	306	306	254	254	254	254	254	32	32	32	32	32	222	222	222	222	222	92	92	92	8	9 2	O C	2 L	010	2 2	31 5	123	123	123	123	
1 6	16	16	16	114	114	114	114	114	111	111	111	111	111	58	58	55	58	58	4	4	4	4	4	69	69	69	69	69	63	63	63	20	63	1 4	7,4	1 1	7,4	7.4 T/00	108	108	108	108	xc du
1 6	42	42	42	140	140	140	140	140	115	115	115	115	115	185	185	185	185	185	69	69	69	69	69	159	159	159	159	159	83	88	œ (	20	83 ‡	1 47	14/	14/	147	1 04	0 04	84	84	84	dd
100	100	100	100	314	314	314	314	314	341	341	341	341	341	435	435	435	435	435	196	196	196	196	196	366	366	366	366	366	249	249	249	249	249	0 00	329	675	320	200	253	253	253	253	dna
ى د	ಬ ಜ	3 6	2 6	71	71	71	71	71	79	79	79	79	79	40	40	40	40	40	12	12	12	12	12	32	32	32	32	32	61	61	61	51	5 2	70	70	3 2	700	700	3 6	ω ω	ယ္ထ	33	

Table 4 Correlation + Co-integration + In-sample SR + Moving Look-back Window (mv)

<u>у</u> :	4	43	42	41	40	39	8	37	36	င္ဟ	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13 k	3	1 E	ی	0 00	7	0	ഗ	4	ω	2	1	N <sub>o</sub> .
-0.00	-0.07	0.08	0.01	-0.08	-0.08	-0.21	-0.16	-0.05	-0.11	-0.23	-0.04	-0.18	-0.21	-0.02	-0.13	-0.33	-0.08	-0.21	-0.28	0.03	0.00	-0.11	0.03	-0.05	-0.27	-0.09	-0.16	-0.39	-0.15	-0.17	-0.21	-0.06	900	-0.28	-0.4L	-0.37	-0.24	-0.24	-0.14	-0.24	-0.24	-0.10	0.11	Average. PnL
0 0 0	0.54	0.87	0.63	0.70	0.74	0.73	0.61	0.70	0.57	1.03	0.99	1.19	1.13	1.01	0.93	1.25	0.81	1.20	0.91	1.14	1.10	1.06	1.12	1.06	1.06	0.72	0.90	1.24	0.77	0.50	0.47	0.53	0 4 0	0.51	L.85	1.43	1.54	1.49	0.91	1.09	0.80	0.93	0.99	PnL.Vola stility
-0.05	-0.12	0.09	0.01	-0.11	-0.11	-0.29	-0.27	-0.07	-0.20	-0.22	-0.04	-0.15	-0.19	-0.02	-0.14	-0.27	-0.10	-0.17	-0.31	0.03	0.00	-0.10	0.03	-0.05	-0.25	-0.12	-0.18	-0.31	-0.20	-0.35	-0.43	-0.11	010	-0.18	-0.22	-0.26	-0.16	-0.16	-0.16	-0.22	-0.30	-0.11	0.11	Sharpe.R (
л 20 20 30 30 30 30 30 30 30 30 30 30 30 30 30	-8 14	9.79	1.11	-9.50	-10.17	-26.29	-20.38	-6.32	-14.27	-28.71	-4.86	-22.28	-26.04	-3.12	-16.56	-41.77	-10.32	-26.13	-35.27	3.74	0.06	-13.71	3.55	-6.80	-33.64	-10.80	-20.57	-48.34	-19.04	-21.86	-25.70	-7.15	10 17	-34.97	-51.20	-46.06	-29.87	-30.61	-17.84	-30.52	-29.81	-12.85	74	Cumulati PnL.Plus tility.Plus atio.Plus ve.PnL.Pl Percenta ve.PnL Cost .Cost Cost us.Cost ge
-0 0.	-0.07	0.08	0.01	-0.08	-0.08	-0.21	-0.16	-0.05	-0.11	-0.23	-0.04	-0.18	-0.21	-0.03	-0.13	-0.33	-0.08	-0.21	-0.28	0.03	0.00	-0.11	0.03	-0.05	-0.27	-0.09	-0.16	-0.39	-0.15	-0.18	-0.21	-0.06	900	-0.28	-0.41	-0.37	-0.24	-0.25	-0.14	-0.24	-0.24	-0.10	11	PnL Plus 1 Cost
0 0	0.54	0.87	0.63	0.70	0.74	0.73	0.61	0.70	0.57	1.03	0.99	1.19	1.13	1.01	0.93	1.25	0.81	1.20	0.91	1.14	1.10	1.06	1.12	1.06	1.06	0.72	0.90	1.24	0.77	0.50	0.47	0.53	0 40	0.51	1.00 0.00	1.43	1.54	1.49	0.91	1.09	0.80	0.93	.99	tility.Plus a .Cost (
0 0 0	-0.12	0.09	0.01	-0.11	-0.11	-0.29	-0.27	-0.07	-0.20	-0.22	-0.04	-0.15	-0.19	-0.02	-0.14	-0.27	-0.10	-0.17	-0.31	0.03	0.00	-0.10	0.03	-0.05	-0.25	-0.12	-0.18	-0.31	-0.20	-0.35	-0.43	-0.11	010	-0.18	-0.22	-0.26	-0.16	-0.16	-0.16	-0.22	-0.30	-0.11	11	atio.Plus.v Cost u
П ( 141)	-8 1 5 -	9.77	1.09	-9.53	-10.20	-26.32	-20.40	-6.35	-14.29	-28.74	-4.89	-22.32	-26.07	-3.15	-16.59	-41.81	-10.35	-26.17	-35.30	3.71	0.04	-13.74	3.52	-6.83	-33.67	-10.83	-20.60	-48.38	-19.07	-21.89	-25.72	-7.17	10.56	-35.00	-51.25	-46.10	-29.91	-30.65	-17.86	-30.56	-29.84	-12.88		Average, PnL. Vola Sharpe, R Cumulati PnL. Plus, tility, Plus atio ve. PnL Cost Cost us. Cost ge m. DD
0 0 0	0.42	0.65	0.56	0.54	0.67	0.54	0.39	0.65	0.53	0.56	0.69	0.56	0.40	0.67	0.57	0.52	0.67	0.54	0.38	0.66	0.53	0.54	0.66	0.53	0.39	0.65	0.54	0.53	0.66	0.53	0.35	0.66	0.00	0.50	0.52	0.37	0.63	0.52	0.69	0.55	0.37	0.67	0.56	Percenta I ge
16.00	11 66	7.68	7.99	15.07	12.53	27.68	21.89	9.17	16.09	32.98	18.32	31.62	28.10	16.74	18.03	42.40	14.78	26.34	35.92	12.70	12.86	24.46	12.11	19.17	35.85	15.75	26.52	49.60	23.52	23.08	26.22	7.72	10 05	27.00	51.95	46.06	32.70	32.74	20.67	33.52	31.90	18.34	57	
1070	986	576	704	1405	730	1169	1089	666	795	1704	859	1361	1214	799	920	1692	841	1336	1332	775	922	1751	864	1341	1401	919	1060	1987	1007	1175	1149	654	010	1507	1234	1174	709	847	909	1453	1274	842		Total.Lo 1 ng.Num (
1171	1145	643	776	1513	755	1148	1133	669	811	1764	916	1415	1320	842	880	1652	780	1313	1258	748	917	1757	835	1364	1366	853	1010	1951	921	1115	1091	590	75.4	1438	1239	1248	712	851	878	1424	1279	796	957	Total Sh Long Da Short Da Pairs Nu ort Num ys ys m
16 40	8 78	23.08	17.06	10.40	22.99	18.34	10.36	27.82	20.09	10.75	17.93	15.38	8.69	22.30	18.67	11.96	21.03	16.78	9.31	24.42	18.00	11.26	19.95	16.74	8.68	22.18	16.27	10.43	18.01	17.54	9.29	28.95	10 55	12 28	16.48	8.93	24.88	17.59	19.30	15.59	8.73	23.15	16.55	ong.Da Sh 's ys
1760	10.70	28.99	21.52	12.59	20.16	16.25	9.80	24.35	18.11	12.11	19.92	16.79	9.90	24.51	18.79	11.89	21.77	16.21	9.70	27.54	19.50	12.06	22.30	17.10	9.05	23.83	17.20	10.69	19.78	15.53	9.22	24.83	10.00	10 93	17.60	9.91	27.02	19.57	19.60	15.59	9.35	24.07	17.52	hort.Da Pa s m
16 70	16 70	16.70	16.70	16.70	16.44	16.44	16.44	16.44	16.44	16.98	16.98	16.98	16.98	16.98	18.76	18.76	18.76	18.76	18.76	18.21	18.21	18.21	18.21	18.21	17.95	17.95	17.95	17.95	17.95	16.81	16.81	16.81	16 01	16.81	17.19	17.19	17.19	17.19	16.95	16.95	16.95	16.95	16.95	
2 1	1 F	57	28	29	49	36	رى د	56	26	45	67	66	16	89	30	32	4	39	11	75	46	46	56	59	17	86	52	49	72	41	14	41	2 6	φ 2 %	4/	24	70	31	67	62	29	84	50	uexc ud
0 0	4.38	413	443	1170	527	763	422	427	447	1315	637	943	494	529	510	1245	568	899	485	461	489	1290	579	889	507	544	549	1454	635	774	421	399	4 A C	1102	1 98	424	429	447	624	955	450	517	540	E
3 0	650	109	247	243	119	270	656	117	280	320	137	315	756	140	283	290	108	283	726	135	314	326	134	317	799	145	340	352	144	222	616	94	0000	237	307	759	142	316	136	328	764	137	320	una
0 2	46	70	58	71	60	79	50	69	58	28	75	91	2	84	57	85	60	92	36	77	68	95	66	99	43	78	69	96	70	78	40	56 4	2 2	67	8 8	41	71	57	51	79	36	58	47	
5 0	t	26	14	22	64	50	11	70	33	35	44	59	11	66	35	32	49	44	15	71	45	46	57	67	18	91	44	48	75	45	14	67	o F	ω 4 1	4/	7	56	27	82	66	23	91	44	dexc du
0 .	457	442	467	1147	518	782	437	445	451	1327	633	965	524	550	522	1232	594	873	508	520	535	1338	624	893	553	605	599	1478	714	794	419	418	110	1141	841	458	459	487	631	972	481	540	551	dd
100	504	2	181	176	98	262	604	96	258	273	121	263	642	119	302	331	121	319	762	113	275	275	112	284	776	145	344	359	144	257	668	94	0.00	260	260	659	123	271	126	322	722	129	310	dna
72	22	44	42	60	50	75	37	55	53	69	61	74	37	64	61	97	77	100	47	71	67	92	71	97	54	78	73	102	74	79	48	75	60 0	75	86	50	71	62	70	93	48	82		ฉั

Table 5 Correlation + Co-integration + In-sample SR + Formation Period (rolling)

2	4	43	42	41	40	ی	3 6	20 0	37	36	35	ω	, L	3 6	3 5	بر	မ	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	œ	7	6	ري ري	4	ω	2	L	Z 0.
Ī			-0.01			T	T				-0.05		T	T	T				-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.17	-0.17	-0.17	-0.17									-0.38	-0.38	-0.38	-0.26	-0.26	-0.26	-0.26	-0.26	Averag PnL
Ì						T	T						T	T																												6	6	6	66	6	e. PnL.\
	0.35	0.35	0.35	0.35	0.53	0.53	0 0	מת	0.53	0.53	0.58	0.58	0.58	0.00	30.00	82.0	0.76	0.76	0.76	0.76	0.76	0.41	0.41	0.41	0.41	0.41	0.54	0.54	0.54	0.54	0.54	0.53	0.53	0.53	0.53	0.53	2.31	2.31	2.31	2.31	2.31	1.27	.27	.27	.27	.27	/ola Shar atio
	-0.04	-0.04	-0.04	-0.04	0.05	0.05	9 9	0 0 0	0.05	0.05	-0.09	-0.09	-0.09	0 0	0 00	-0 09	-0.03	-0.03	-0.03	-0.03	-0.03	-0.06	-0.06	-0.06	-0.06	-0.06	-0.32	-0.32	-0.32	-0.32	-0.32	-0.27	-0.27	-0.27	-0.27	-0.27	-0.16	-0.16	-0.16	-0.16	-0.16	-0.21	-0.21	-0.21	-0.21	-0.21	arpe.k C
	-1.68	-1.68	-1.68	-1.68	3.44	3.44	) c	2 9	3.44	3.44	-6.27	-6.27	-0.27	-0.27	22.3	-6 27	-2.75	-2.75	-2.75	-2.75	-2.75	-3.03	-3.03	-3.03	-3.03	-3.03	-21.59	-21.59	-21.59	-21.59	-21.59	-17.90	-17.90	-17.90	-17.90	-17.90	-47.43	-47.43	-47.43	-47.43	-47.43	-32.50	-32.50	-32.50	-32.50	-32.50	ve.PnL
	-0.01	-0.01	-0.01	-0.01	0.03	0.03	0.00	0.02	0.03	0.03	-0.05	-0.05	-0.05	0.00	0.00	-0.05	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.17	-0.17	-0.17	-0.17	-0.17	-0.14	-0.14	-0.14	-0.14	-0.14	-0.38	-0.38	-0.38	-0.38	-0.38	-0.26	-0.26	-0.26	-0.26	-0.26	Average, Pht. voia sharper cumulati Pht. Plus atio. Plus ve. Pht. Plus atio. Plus ve. Pht. Percenta waximu i otali. O i otalish Long. Da Short Da Palis Nu.  Pht. tility atio ve. Pht. Cost Cost us. Cost ge m.DD ng. Num ort. Num ys ys m
	0.35	0.35	0.35	0.35	0.53	0.53	0 00	0 70	0.53	0.53	0.58	0.58	0.58	0.00	0 70	0 58	0.76	0.76	0.76	0.76	0.76	0.41	0.41	0.41	0.41	0.41	0.54	0.54	0.54	0.54	0.54	0.53	0.53	0.53	0.53	0.53	2.31	2.31	2.31	2.31	2.31	1.27	1.27	1.27	1.27	1.27	Cost
	-0.04	-0.04	-0.04	-0.04	0.05	0.05	0.00	0.02	0.05	0.05	-0.09	-0.09	-0.09	0.00	0.00	-0.09	-0.03	-0.03	-0.03	-0.03	-0.03	-0.06	-0.06	-0.06	-0.06	-0.06	-0.32	-0.32	-0.32	-0.32	-0.32	-0.27	-0.27	-0.27	-0.27	-0.27	-0.16	-0.16	-0.16	-0.16	-0.16	-0.21	-0.21	-0.21	-0.21	-0.21	Cost Cost
	-1.69	-1.69	-1.69	-1.69	3.43	3.43	0.4.0	2 / 2	3.43	3.43	-6.28	-6.28	-0.28	02.0-	200	-6.28	-2.76	-2.76	-2.76	-2.76	-2.76	-3.04	-3.04	-3.04	-3.04	-3.04	-21.61	-21.61	-21.61	-21.61	-21.61	-17.91	-17.91	-17.91	-17.91	-17.91	-47.46	-47.46	-47.46	-47.46	-47.46	-32.52	-32.52	-32.52	-32.52	-32.52	us.Cost
I			0.25								0.27		Γ	T						0.30	0.30		0.27	0.27	0.27	0.27	0.29	0.29	0.29	0.29	0.29	0.17	0.17	0.17	0.17				0.32		0.32		0.26	0.2	0.26	0.26	ge
Ī		5 4.44	5 4.44	5 4.44		Ī	T				7 9.38	7 9.38	Ī	T						3 8.03	0 8.03		7 6.84	7 6.84	7 6.84	7 6.84	9 21.72																			6 33.04	m.DD
					2 36																3 45						2 55									9 41				1 36				4 43		4 43	ng.Num
	1 46		1 46				000														5 79																6 88		6 88	6 88					3 118	3 118	ort.Num
			6 23.29			T					7 21.43		T		T					9 23.64	9 23.64																									8 21.05	n ys
T			9 51.20		Т	Т	Т				3 59.75	Г	Т	Т	Т				40.25	4 40.25	4 40.25	5 23.02	5 23.02	5 23.02	5 23.02																					5 29.80	ys ys
			16.70		Т	Г	Т				16.98	Г	T	Т	Т											18.21	17.95		17.95	17.95									3 17.19	17.19					16.95	16.95	m m
																									11							. 7	. 7											28			uexc
																																															nd
1	7	7	7	7	1 ~	~	1 ~	7	7	7	10	10	5 5	5 5	5 5	5	17	17	17	17	17	10	10	10	10	10	11	11	11	11	11	12	12	12	12	12	14	14	14	14	14	15	15	15	15	15	E
3	22	22	22	22	22	2	3 2	3	22	22	15	15	5	1 5	<u> </u>	<u>,</u>	36	36	36	36	36	66	66	66	66	66	47	47	47	47	47	90	90	90	90	90	30	30	30	30	30	62	62	62	62	62	una
7	7	7	7	7	133	<u>۲</u>	3 5	3 6	13	13	12	12	K	3 5	3 1	13	15	15	15	15	15	9	9	9	9	9	14	14	14	14	14	7	7	7	7	7	14	14	14	14	14	13	13	13	13	13	dexc
2	ь	ь	ь	ь	4	. 4	1		4	4	5	G	U	1 0	л	л	ω	ω	ω	ω	ω	ь	ь	ь	ь	ь	4	4	4	4	4	Ľ	1	Ľ	1	1	4	4	4	4	4	4	4	4	4	4	xc du
٥	ω	ω	ω	ω	4	4	1		4	4	ω	ω	c.	o 0	ی ر	ىد	11	11	11	11	11	10	10	10	10	10	9	9	9	9	9	5	5	ۍ	5	5	œ	8	8	00	œ	6	6	6	6	6	
0	9	9	9	9	20	02	3 6	20 0	20	20	12	12		3 5	3 5	13	20	20	20	20	20	ر ت	5	ر ت	5	ر ت	19	19	19	19	19	15	15	15	15	15	00	8	8	ω	00	14	14	14	14	14	dd
								Ī							T	Ī																															dna

Table 6 Graphical Lasso + Co-integration + Moving Look-back Window (mv)

4 6	43	42	41	40	39	38	37	36	ဌဌ	22	జ	32	32	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	E .	10	9	œ	7	6	σı	4	ω	2	₽.	No.
-1.31	-0.77	-0.68	-0.40	-2.19	-3.88	-2.58	-2.09	-2.44	-3.24	-3.16	-2.72	-3.21	-3.05	-2.47	-3.78	-2.47	-3.39	-2.84	-2.00	-2.15	-3.08	-1.92	-3.01	-2.39	-1.21	-1.53	-2.37	-1.40	-1.59	-1.12	-0.42	-0.45	-1.69	-0.80	-1.25	-1.28	-0.86	-0.82	-1.56	-2.91	-2.33	-1.64	1.55	Average PnL Vola Sharpe R Cumulati PnL Plus tility. Plus atio Plus ve. PnL Plus tility. Plus atio PnL Cost Cost Us. Cost ge m.DD
5.13	3.35	3.14	14.93	7.03	11.86	8.99	7.10	7.94	20.37	15.35	20.20	7.74	15.34	13.00	11.41	11.96	11.90	12.41	12.92	12.54	14.43	12.72	14.71	9.30	8.57	8.08	9.08	8.37	11.19	6.89	9.08	8.44	10.98	3.94	5.03	4.74	3.69	3.94	9.38	10.48	6.26	8.65	7.52	PnL.Vola tility
-0.26	-0.23	-0.22	-0.03	-0.31	-0.33	-0.29	-0.29	-0.31	-0.16	-0.21	-0.13	-0.41	-0.20	-0.19	-0.33	-0.21	-0.28	-0.23	-0.15	-0.17	-0.21	-0.15	-0.20	-0.26	-0.14	-0.19	-0.26	-0.17	-0.14	-0.16	-0.05			-0.20	-0.25	-0.27	-0.23	-0.21	-0.17	-0.28	-0.37	-0.19	0.21	Sharpe.R atio
-163.77	-95.83	-85.27	-50.60	-273.85	-485.18	-322.96	-261.23	-305.31	-404.67	-394.94	-340.58	-400.80	-381.27	-308.93	-472.87	-309.26	-424.09	-354.38	-249.86	-269.24	-385.57	-239.54	-375.80	-298.83	-150.83	-191.04	-296.80	-175.20	-199.08	-140.58	-52.10	-55.75	-210.83	-99.64	-156.81	-159.99	-107.51	-102.41	-195.52	-363.92	-290.73	-205.16	23	cumulati ve.PnL
-1.31	-0.77	-0.68	-0.41	-2.19	-3.88	-2.59	-2.09	-2.44	-3.24	-3.16	-2.73	-3.21	-3.05	-2.47	-3.79	-2.48	-3.40	-2.84	-2.00	-2.16	-3.09	-1.92	-3.01	-2.39	-1.21	-1.53	-2.38	-1.40	-1.59	-1.13	-0.42	-0.45	-1.69	-0.80	-1.26	-1.28	-0.86	-0.82	-1.57	-2.91	-2.33	-1.64	.55	Cost .
5.14	3.35	3.14	14.92	7.03	11.86	8.99	7.11	7.95	20.37	15.35	20.20	7.75	15.35	13.01	11.41	11.96	11.91	12.41	12.92	12.55	14.44	12.72	14.72	9.30	8.58	8.08	9.08	8.38	11.19	6.89	9.08	8.43	10.98	3.94	5.03	4.74	3.69	3.94	9.39	10.48	6.26	8.65	52	.Cost (
-0.26	-0.23	-0.22	-0.03	-0.31	-0.33	-0.29	-0.29	-0.31	-0.16	-0.21	-0.14	-0.41	-0.20	-0.19	-0.33	-0.21	-0.29	-0.23	-0.15	-0.17	-0.21	-0.15	-0.20	-0.26	-0.14	-0.19	-0.26	-0.17	-0.14	-0.16	-0.05	-0.05	-0.15	-0.20	-0.25	-0.27	-0.23	-0.21	-0.17	-0.28	-0.37	-0.19	).21	Cost (
-163.91	-95.95	-85.39	-50.82	-274.03	-485.47	-323.16	-261.42	-305.50	-405.14	-395.26	-341.03	-401.04	-381.59	-309.22	-473.19	-309.56	-424.43	-354.68	-250.14	-269.51	-385.89	-239.80	-376.13	-299.09	-151.10	-191.29	-297.10	-175.46	-199.35	-140.76	-52.29	-55.93	-211.11	-99.78	-156.99	-160.15	-107.64	-102.54	-195.76	-364.22	-290.94	-205.38	42	us.Cost (
0.35	0.62	0.49	0.47	0.60	0.48	0.33	0.58	0.46	0.44	0.58	0.47	0.33	0.59	0.46	0.48	0.59	0.49	0.34	0.57	0.48	0.48	0.57	0.49	0.33	0.56	0.47	0.45	0.57	0.50	0.37	0.61	0.51	0.46	0.60	0.49	0.33	0.59	0.48	0.56	0.45	0.32	0.57	0.47	ge
169.91	130.23	113.62	221.68	277.91	491.25	324.18	266.39	309.33	506.53	400.90	452.63	402.07	386.79	348.25	473.25	350.83	426.05	391.13	280.98	305.60	406.77	269.99	396.81	325.96	207.94	253.14	312.55	222.15	201.77	142.90	83.92	104.24	213.45	117.59	194.58	177.87	114.49	129.10	197.53	369.41	292.01	207.68	7	m.DD
927	531	666	1196	648	1061	985	582	728	1475	740	1216	1051	687	663	1099	607	882	936	565	677	1141	617	930	986	643	764	1287	692	955	955	567	685	1212	618	970	969	569	699	781	1241	1119	726	855	ng.Num
996	569	708	1231	704	1090	1064	625	758	1520	780	1265	1122	719	560	971	489	785	788	482	605	1055	519	867	917	587	700	1233	626	975	922	566	695	1232	632	983	971	580	718	770	1247	1072	719	854	ort.Num
9.53	24.60	17.36	11.33	21.00	15.82	8.97	24.92	17.49	10.42	18.76	14.48	8.96	22.64	17.74	12.25	20.08	16.92	9.63	23.62	18.18	12.31	20.39	16.81	9.35	22.27	15.96	11.05	18.95	16.15	9.29	24.35	17.43	11.49	21.03	16.34	9.67	25.34	17.82	18.59	14.81	9.31	22.47	16.31	ys y
9.24	25.44	17.75	12.06	18.56	15.88	9.40	24.08	16.94	10.88	18.21	14.70	8.86	22.27	18.62	11.97	21.57	16.32	9.72	25.96	18.12	11.73	21.98	15.58	8.96	23.24	16.71	10.88	20.09	15.56	9.63	22.83	17.07	11.19	20.05	16.25	9.32	23.39	17.24	17.53	13.87	8.40	20.37	15.34	ort.Num ys ys m
14.90	14.90	14.90	14.90	14.81	14.81	14.81	14.81	14.81	14.65	14.65	14.65	14.65	14.65	12.52	12.52	12.52	12.52	12.52	11.93	11.93	11.93	11.93	11.93	11.98	11.98	11.98	11.98	11.98	14.10	14.10	14.10	14.10	14.10	13.70	13.70	13.70	13.70	13.70	14.27	14.27	14.27	14.27	14.27	
9	41	25	జ	36	88	14	4	24	8	42	47	14	60	14	11	17	14	6	35	18	18	27	23	10	58	37	26	45	25	6	36	14	13	8	28	9	జ	24	39	34	11	46	25	uexc ud
336	332	353	845	456	658	364	366	375	1039	493	755	382	408	280	665	309	470	277	264	281	696	311	500	310	320	331	833	383	613	365	365	384	885	399	570	337	331	345	513	776	398	445	448	EL .
589	109	248	244	121	273	623	121	274	322	154	323	663	153	191	202	8	197	452	101	232	239	100	238	540	110	242	262	107	223	489	79	219	231	109	255	555	108	253	127	292	598	129	291	una
62	87	82	109	91	121	63	94	85	129	91	140	සි	98	75	93	83	104	53	82	74	102	82	106	57	99	90	112	91	114	62	86	78	103	94	130	70	108	96	91	145	65	99	90	la dexc
12	41	20	19	32	39	10	36	25	21	44	38	7	51	17	9	24	22	υ	29	16	24	28	29	14	49	27	29	42	13	4	24	14	11	ယ္ထ	30	11	37	24	37	43	15	54	36	xc du
357	324	359	875	414	635	342	337	359	1065	482	791	401	416	293	699	371	476	288	304	305	739	364	511	316	350	347	831	417	593	347	349	366	846	376	557	292	315	322	490	725	373	408	420	dd
500	74	201	205	108	260	571	107	248	277	113	265	583	111	251	262	110	251	568	124	256	262	123	264	592	141	292	308	134	232	549	95	228	242	107	254	597	103	255	122	308	656	118	288	dna
59	92	86	97	94	127	62	102	96	112	101	122	60	109	102	129	102	133	75	108	100	116	102	126	64	103	98	119	99	117	55	99	77	113	102	129	69	114	98	132	165	75	146	111	ជា

Table 7 Graphical Lasso + Co-integration + Formation Period (rolling)

n :	4	43	42	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	3 5	3 F	1 10	9		7	6	رى د	4	ω	2	₽	Z o
4 1	111	111	1.11	1.11	-1.08	-1.08	-1.08	-1.08	-1.08	-0.54	-0.54	-0.54	-0.54	-0.54	-1.47	-1.47	-1.47	-1.47	-1.47	0.36	0.36	0.36	0.36	0.36	2.23	2.23	2.23	2.23	2.23	2.29	2.29	2.29	2 2 2	0.79	0.79	0.79	0.79	0.79	2.83	2.83	2.83	2.83	2.83	Average PnL
700	9.87	9.87	9.87	9.87	15.22	15.22	15.22	15.22	15.22	21.83	21.83	21.83	21.83	21.83	25.89																	12.40		Т	Т		Г	Г	20.39	20.39				PnL.Vola tility
				0.11	-0.07	-0.07	-0.07	-0.07	-0.07		-0.02	-0.02																				0.18	T				0.05	0.05	0.14	0.14		0.14	0.14	Sharpe.F
100.7E	138 75	138.75	138.75	138.75	-134.39	-134.39	-134.39	-134.39	-134.39	-67.15		-67.15						.	L			45.05	45.05								П	286.09	Т	Т			98.64		353.28	353.28				Cumulat ve.PnL
1 1	1 1		5 1.11	5 1.11	-1.08	-1.08	-1.08	-1.08	-1.08		5 -0.54	5 -0.54	5 -0.54																			2 29					4 0.79		2.82	3 2.82	2			PnL.Plus tility.Plus atio.Plus ve.PnL.Pl Percenta Cost Cost us.Cost ge
0.07	9.87			9.87	15.23	15.23		15.23				1 21.83																				12.40	Τ	Т	Т		16.84	Г	20.39	20.39				cost
				0.11	-0.07	-0.07	-0.07	-0.07	-0.07		-0.02	-0.02																				0.18	Τ				0.05	0.05	0.14	0.14		0.14	0.14	s atio.Plus Cost
T				138.53	-134.70	-134.70	-134.70	-134.70	-134.70			-67.63						.														285.79	Т	Т	Т				352.78	1 352.78				Average, PnL. Vola Sharpe, R Cumulati PnL. Plus, tility, Plus atio ve. PnL Cost Cost Us. Cost ge m.DD
T					0 0.44	0 0.44	0 0.44	0 0.44	0 0.44		0.43	3 0.43	3 0.43																			9 0.39							8 0.41	8 0.41		8 0.41		9 Percent ge
T				8 72.29	4 164.04	4 164.04	4 164.04	4 164.04	4 164.04		3 217.04	3 217.04																	_			9 45.76	Т	L	Т			П		1 96.40				m.DD
Ī					4 592	4 592	4 592					4 735																				6 670	T				4 712	4 712	0 837	0 837				ng.Num
Ī												5 830																				0 773	T	2 757					7 948	7 948			7 948	o Total Sh n ort Num
T	Т		7 10.37	7 10.37	1 14.28	1 14.28	1 14.28	1 14.28	1 14.28		0 13.12	0 13.12																				3 11.65			Т		7 11.09	7 11.09	8 11.18	8 11.18			8 11.18	ort.Num ys ys m
0 0			7 9.26	7 9.26	8 10.16	8 10.16	8 10.16	8 10.16	8 10.16		2 10.46	2 10.46								Ė												5 8.97	T	Ī.,	Т	Г	9 10.07	Г	8 9.34	8 9.34				a Short.D ys
1400				6 14.90	6 14.81	6 14.81	6 14.81	6 14.81	6 14.81		6 14.65	6 14.65																				7 14.10	T		Т	Г	Г	Г	4 14.27	4 14.27		4 14.27		m m
									1	Ó	Ö	G	Óī						2	ω	ω	ū	ω	ω	8	œ	00	œ											7	7	7	7		uexc
T	(			ω	11 3	11 3	11 3	11 3	3	7 4	7 4	7 4	7 4					ω	ω ω	3	3	3	ω	3	5 3	5	<u>Б</u>					7 0	T				6 3	6 3	7 4	7 4	7 4	7 4		ud
27 -	377	377	377	377	379	379	379	379	379	412	412	412	412	412	39	339	39	339	339	348	348	348	348	348	368	368	368	368	368	405	405	405	100	388	388	388	388	388	486	486	486	486	486	E
2 6	304	324	324	324	358	358	358	358	358	343	343	343	343	343	338	338	338	338	338	367	367	367	367	367	393	393	393	393	393	328	328	328	2 000	317	317	317	317	317	393	393	393	393		una
3 8	23 8	ట్ల	జ	83	53	53	53	53	53	68	68	68	68	68	43	43	43	43	43	49	49	49	49	49	55	55	55	55	55	မ္ဘ	ဌ	SH 5	2 6	26	46	46	46	46	62	62	62	62	62	a dexc
٦ (	ט ת	ບາ	51	ഗ	13	13	13	13	13	12	12	12	12	12	ω	ω	ω	ω	ω	ω	ω	ω	ω	ω	5	ഗ	ഗ	ഗ	σı	13	13	13 5	2 5	1 14	14	14	14	14	12	12	12	12	12	xc du
100	186	186	186	186	189	189	189	189	189	250	250	250	250	250	167	167	167	167	167	181	181	181	181	181	255	255	255	255	255	173	173	173	170	184	184	184	184	184	237	237	237	237	237	dd
200	202	202	202	202	206	206	206	206	206	214	214	214	214	214	286	286	286	286	286	269	269	269	269	269	285	285	285	285	285	222	222	222	222	269	269	269	269	269	241	241	241	241	241	
2 0	<u>ب</u> ا	ယ္က	ည	ည့	4.	4.	4.	4.	4.	50	50	39	62	62	32	ည္	ည	ည္က	ည	48	48	48	4:	48	50	50	5.5	50	55	36	ω,	ی این	2 0	26	4	4	4	4	7:	7:	7:	7.5		dna

Table 8 Graphical Lasso + Co-integration + In-sample SR + Moving Look-back Window (mv)

h :	5 4	i c	42	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13 15	; E	1 10	9	8	7	6	5	4	ω	2	₽.	Z 0.
00 5	-1 3	-0.77	-0.68	-0.39	-1.12	-1.96	-1.34	-1.05	-1.29	-1.48	-2.31	-1.23	-2.44	-2.25	-2.47	-3.78	-2.47	-3.39	-2.84	-2.00	-2.15	-3.08	-1.92	-3.01	-2.14	-0.96	-1.26	-2.09	-1.16	-1.49	-0.98	-0.36	- L. 32	-0.48	-0.81	-0.89	-0.55	-0.52	-1.28	-2.50	-2.06	-1.36	1.33	Average. PnL
1 4 77	51.3	ر د د د د د د	3.13	14.93	3.90	5.13	3.33	4.18	3.34	17.60	14.35	17.68	5.63	14.40	13.00	11.41	11.96	11.90	12.41	12.92	12.54	14.43	12.72	14.71	9.19	8.28	7.77	9.21	8.09	11.16	6.84	9.03	T0.90	3.24	4.05	3.89	3.02	3.44	9.20	10.21	5.97	8.46	7.37	PnL.Vola tility
2 2	-0.25	-0.22	-0.22	-0.03	-0.29	-0.38	-0.40	-0.25	-0.39	-0.08	-0.16	-0.07	-0.43	-0.16	-0.19	-0.33	-0.21	-0.28	-0.23	-0.15	-0.17	-0.21	-0.15	-0.20	-0.23	-0.12	-0.16	-0.23	-0.14	-0.13	-0.14	-0.0-	-0.14	-0.15	-0.20	-0.23	-0.18	-0.15	-0.14	-0.24	-0.35	-0.16	0.18	Sharpe.R atio
1000	-163 23	- 05.30	-84.97	-49.29	-140.21	-245.56	-167.01	-131.52	-161.57	-184.99	-289.26	-153.76	-304.98	-281.55	-308.93	-472.87	-309.26	-424.09	-354.38	-249.86	-269.24	-385.57	-239.54	-375.80	-267.68	-120.36	-157.03	-261.24	-144.90	-186.48	-122 95	-44.51	-T90.00	-60.49	-101.45	-111.68	-69.03	-64.83	-159.68	-312.58	-257.32	-169.73	6	Cumulati ve.PnL
0 5	-1 2	-0.77	-0.68	-0.40	-1.12	-1.97	-1.34	-1.05	-1.29	-1.48	-2.32	-1.23	-2.44	-2.25	-2.47	-3.79	-2.48	-3.40	-2.84	-2.00	-2.16	-3.09	-1.92	-3.01	-2.14	-0.96	-1.26	-2.09	-1.16	-1.49	-0 98	-0.36	- L.O.	-0.48	-0.81	-0.89	-0.55	-0.52	-1.28	-2.50	-2.06	-1.36	 	PnL.Plus tility.Plus atio.Plus ve.PnL.Pl Percenta Cost Cost us.Cost ge
1 7 7 0	J (	ı ı ı	ω 11 ω	14.92	3.90	5.13	3.33	4.18	3.34	17.60	14.35	17.68	5.63	14.40	13.01	11.41	11.96	11.91	12.41	12.92	12.55	14.44	12.72	14.72	9.19	8.28	7.77	9.21	8.09	11.16	6.84	9.33	TO.90	3.24	4.05	3.89	3.02	3.44	9.20	10.22	5.97	8.46	37	tility.Plus a
0 0	-0.25	-0.22	-0.22	-0.03	-0.29	-0.38	-0.40	-0.25	-0.39	-0.08	-0.16	-0.07	-0.43	-0.16	-0.19	-0.33	-0.21	-0.29	-0.23	-0.15	-0.17	-0.21	-0.15	-0.20	-0.23	-0.12	-0.16	-0.23	-0.14	-0.13	-0.14	-0.0-	-0-14	-0.15	-0.20	-0.23	-0.18	-0.15	-0.14	-0.24	-0.35	-0.16	0.18	atio.Plus.v Cost v
10.01	-163.37	-05.00	-85.09	-49.51	-140.32	-245.73	-167.13	-131.65	-161.69	-185.36	-289.52	-154.11	-305.18	-281.81	-309.22	-473.19	-309.56	-424.43	-354.68	-250.14	-269.51	-385.89	-239.80	-376.13	-267.93	-120.61	-157.26	-261.54	-145.14	-186.75	-123 12	-44.69	- L90.00	-60.61	-101.60	-111.82	-69.14	-64.95	-159.90	-312.86	-257.52	-169.93		Average PnL.Vola Sharpe.R Cumulati PnL.Plus; tility.Plus atio.Plus ve.PnL.Pl Percenta Maximu PnL tility atio ve.PnL Cost .Cost Cost us.Cost ge m.DD
0 0	0.35	0.60	0.49	0.47	0.61	0.49	0.34	0.59	0.47	0.45	0.59	0.48	0.34	0.59	0.46	0.48	0.59	0.49	0.34	0.57	0.48	0.48	0.57	0.49	0.33	0.56	0.47	0.45	0.57	0.51	0.37	0.51	0.4.	0.62	0.51	0.33	0.61	0.49	0.57	0.47	0.33	0.58	0.49	Percenta   ge
100.71	169.42	120.84	113.02	220.07	144.17	251.59	168.37	136.59	165.55	286.56	295.03	265.51	306.38	286.89	348.25	473.25	350.83	426.05	391.13	280.98	305.60	406.77	269.99	396.81	294.81	177.47	219.12	276.99	191.84	189.17	125.68	74.62	T90.19	78.44	139.22	129.56	76.00	91.52	161.69	318.07	258.61	172.24	55	Maximu o
000	800	д г 1	641	1143	579	952	878	516	650	1356	677	1118	970	627	663	1099	607	882	936	565	677	1141	617	930	969	633	751	1269	682	889	906	537	)CIT	573	901	904	526	648	727	1147	1059	675		Total.Lo
200	0.50	544	680	1167	644	977	974	568	692	1371	711	1137	1036	657	560	971	489	785	788	482	605	1055	519	867	899	578	690	1212	616	900	867	5 0 33 0	1140	593	911	917	541	673	721	1153	1004	674	803	Total Sh Long Da Short Da Pairs Nu ort Num ys ys m
1000	9.7.E.	25.15	17.59	11.63	21.53	16.01	9.22	25.70	18.03	10.69	19.26	14.85	9.10	23.25	17.74	12.25	20.08	16.92	9.63	23.62	18.18	12.31	20.39	16.81	9.33	22.30	15.94	11.02	18.94	16.28	92.6	24.00	11.43	21.37	16.50	9.88	25.95	18.04	18.91	15.18	9.44	22.88	16.60	Long.Da Sh ys ys
2 1	9.27	25.46	17.64	12.09	18.91	16.46	9.57	24.92	17.27	11.13	18.73	15.08	8.96	22.69	18.62	11.97	21.57	16.32	9.72	25.96	18.12	11.73	21.98	15.58	9.02	23.24	16.76	10.88	20.07	16.08	9.54	23.09	11.00	20.37	16.82	9.27	23.88	17.67	17.60	14.16	8.36	20.31	15.40	hort.Da Pa s m
1 0	14 25	14.05	14.25	14.25	13.59	13.59	13.59	13.59	13.59	13.58	13.58	13.58	13.58	13.58	12.52	12.52	12.52	12.52	12.52	11.93	11.93	11.93	11.93	11.93	11.78	11.78	11.78	11.78	11.78	13.24	13.24	13.24	10.24	12.93	12.93	12.93	12.93	12.93	13.39	13.39	13.39	13.39	13.39	
3 6	ا م	40	25	ట్ర	22	37	13	42	23	28	39	43	13	55	14	11	17	14	6	35	18	18	27	23	10	58	37	26	45	25	D (	£ 25	, F	29	28	7	32	24	37	33	10	43	24	uexc ud
1 0	321	210	341	798	427	589	337	338	347	925	448	667	348	372	280	665	309	470	277	264	281	696	311	500	305	316	328	821	378	569	349	350	220	378	529	321	313	327	479	711	371	417	419	E
1 000	79.7 104	104	238	232	112	256	572	112	256	301	144	303	616	143	191	202	80	197	452	101	232	239	100	238	528	108	236	255	105	202	459	73	717	103	238	526	102	238	120	272	560	122	274	una
3 6	7 S	20 2	76	104	71	95	52	76	66	117	80	124	59	87	75	93	88	104	53	82	74	102	81	106	56	96	89	110	8	104	Z 2	75	8 8	8 8	116	63	94	84	85	137	63	92	86	
) H	<u>-</u>	41	20	19	29	34	10	33	24	21	43	38	7	50	17	9	24	22	υ	29	16	24	28	29	14	49	27	29	42	13	4	23 4	: :	32	29	11	36	22	35	42	15	51	ω	dexc du
1 0	3 2	305	339	827	370	574	313	298	319	974	437	722	366	375	293	699	371	476	288	304	305	739	364	511	308	345	340	820	411	558	ည သ သ	33 048	001	355	529	274	296	303	458	670	349	380	391	dd
200	493	73	199	203	97	235	506	96	224	258	104	247	543	102	251	262	110	251	568	124	256	262	123	264	585	138	289	303	132	216	524	90 213	022	326 99	236	563	96	239	117	290	630	113	273	dna
2 2	200	200	80	94	83	109	49	89	83	103	93	111	54	100	102	129	102	133	75	108	100	116	102	126	62	101	95	117	97	102	45	80 00	2 9	87	107	56	98	84	117	145	65	131		<u>0</u>

 $Table\ 9\ Graphical\ Lasso+Co-integration+In-sample\ SR+Formation\ Period\ (rolling)$ 

4 6	t :	1 c	43	41	40	39	38	37	36	딿	22	జ	32	ω	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	œ	7	6	رى د	4	ω	2	L	Z 0.
0.00	T		000				-0.87	-0.87	-0.87	-0.04	-0.04	-0.04	-0.04		-0.18		-0.18	-0.18	-0.18	-0.07	-0.07		-0.07		-0.32	-0.32			-0.32	1.17	1.17	1.17				-0.11	-0.11	-0.11	-0.11		1.23	1.23	1.23	1.23	Average. PnL
1.36	T						10.89	10.89	10.89	1.75	1.75	1.75	1.75		1.49	1.49	1.49	1.49	1.49	0.62	0.62	0.62	0.62	0.62	1.24	1.24		1.24	1.24	5.15	5.15	5.15				0.75	0.75	0.75	0.75	4.85		4.85	4.85	4.85	PnL.Vola
0.00	Τ						-0.08	-0.08	-0.08	-0.02	-0.02	-0.02	-0.02		-0.12		-0.12	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12	-0.26	-0.26	-0.26	-0.26	-0.26	0.23	0.23	0.23				-0.15	-0.15	-0.15	-0.15			0.25	0.25		Sharpe.F atio
0.30	T						-108.29	-108.29	-108.29	-4.45	-4.45	-4.45	-4.45		-22.99	-22.99	-22.99	-22.99	-22.99	-8.94	-8.94	-8.94	-8.94	-8.94	-39.94	-39.94			-39.94	146.22	146.22	146.22				-13.83	-13.83	-13.83	-13.83			153.61	153.61	153.61	Average, PnL. Vola Sharpe.R Cumulati PnL.Plus, tility. Plus atio.Plus ve.PnL.Pl Percenta Maximu PnL tility atio ve.PnL Cost Cost Us.Cost ge m.DD
0.00	Τ						-0.87	-0.87	-0.87	-0.04	-0.04	-0.04	-0.04		-0.18	-0.18	-0.18	-0.18	-0.18	1 -0.07	1 -0.07	-0.07	1 -0.07	1 -0.07	4 -0.32	4 -0.32	4 -0.32	4 -0.32	4 -0.32	2 1.17	2 1.17	2 1.17				-0.11	-0.11	-0.11	-0.11	_		_	L	L	PnL.Plus.tility.Plus atio.Plus.ve.PnL.Pl Percenta
1.3b	T						10.90	10.90	10.90	1.75	1.75	1.75	1.75		1.49	1.49	1.49	1.49	1.49	0.62	0.62	0.62	0.62	0.62	1.24	1.24		1.24	1.24	5.15	5.15	5.15				0.75	0.75	0.75	0.75	4.84	4	4.84	4.84	4.84	. tility.Plu
0.00	Τ						-0.08	-0.08	-0.08	-0.02	-0.02	-0.02	-0.02		-0.12	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12		-0.12	-0.12	-0.26	-0.26	-0.26	-0.26	-0.26	0.23	0.23	0.23				-0.15	-0.15	-0.15	-0.15	0.25		0.25	0.25		atio.Plus Cost
12.0	T						-108.39	-108.39	-108.39	-4.49	-4.49	-4.49	-4.49				-23.03	-23.03		-8.96		-8.96		-8.96	-39.96	-39.96	Ι.	-39.96	-39.96	146.13	146.13	146.13				-13.85	-13.85	-13.85	-13.85			153.52	153.52	153.52	us.Cost
0.34	Τ	Τ					0.44	0.44	0.44	0.37	0.37	0.37	0.37		3 0.31		0.31	0.31		0.38	0.38	0.38			0.35	0.35					0.29	0.29				0.29	0.29	0.29	0.29			0.41	0.41	0.41	ge ge
0.34 16.51	T	T					139.82	139.82	139.82	31.72	31.72	31.72	31.72			27.20	27.20	27.20	27.20		9.52	9.52	9.52	9.52	40.57	40.57		40.57	40.57		8.88	8.88				15.72		15.72	15.72	6.25		6.25	6.25	6.25	m.DD
1	1 -	77	77				64		64	70	70	70	70				118	118		143	143	143		143		87		87	87		178	178				94	94	94	94	164		164	164		ng.Num
BOT	100 TO0	108	108	108	83	83	83	83	83	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	176	176	176	176	176	104	104	104	104	104	147	147	147	147	147	Total.Sh ort.Num
8.92	0.92	26.0	8 93	8.92	10.33	10.33	10.33	10.33	10.33	11.16	11.16	11.16	11.16	11.16	6.96	6.96	6.96	6.96	6.96	9.08	9.08	9.08	9.08	9.08	11.05	11.05	11.05	11.05	11.05	8.85	8.85	8.85	8.85	8.85	14.86	14.86	14.86	14.86	14.86	9.30	9.30	9.30	9.30	9.30	ort.Num ys ys m
14.28	14.20	14.20	14 28	14.28	14.96	14.96	14.96	14.96	14.96	15.27	15.27	15.27	15.27	15.27	9.49	9.49	9.49	9.49	9.49	9.71	9.71	9.71	9.71	9.71	8.66	8.66	8.66	8.66	8.66	6.10	6.10	6.10	6.10	6.10	9.32	9.32	9.32	9.32	9.32	8.32	8.32	8.32	8.32	8.32	Short.Da ys
14.25	14.23	14.25	14 25	14.25	13.59	13.59	13.59	13.59	13.59	13.58	13.58	13.58	13.58	13.58	12.52	12.52	12.52	12.52	12.52	11.93	11.93	11.93	11.93	11.93	11.78	11.78	11.78	11.78	11.78	13.24	13.24	13.24	13.24	13.24	12.93	12.93	12.93	12.93	12.93	13.39	13.39	13.39	13.39	13.39	
Cu	0	ω c	ادر	ω	ω	ω	ω	ω	ω	رت ا	σı	ഗ	رت ا	ഗ	ω	ω	ω	ω	ω	0	0	0	0	0	2	2	2	2	2	0	0	0	0	0	ω	ω	ω	ω	ω	2	2	2	2	2	uexc ud
87	020	3 6	28	28	ည္သ	33	33	33	္ဆ	33	33	33	33	33	37	37	37	37	37	74	74	74	74	74	41	41	41	41	41	115	115	115	115	115	27	27	27	27	27	84	84	84	84	84	E
σ <u>4</u>	2 2	2 2	2	2	88	38	88	88	88	70	70	70	70	70	76	76	76	76	76	4	4	4	4	4	73	73	73	73	73	53	55	55	53	53	69	69	69	69	69	50	50	50	50	50	
L	5 5	3 5	اد	13	9	9	9	9	9	14	14	14	14	14	6	6	6	6	6	4	4	4	4	4	6	6	6	6	6	00	00	00	8	00	ഗ	ر ت	رى ت	σ <sub>1</sub>	57	11	11	11	11	12	a dexc
2		) N	0 1	2	2	2	2	2	2	4	4	4	4	4	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2	σı	Ωī	ഗ	57	57	6	6	6	6	6	σı	ű	57	ഗ	ഗ	xc du
22	22	22 6	23	22	21	21	21	21	21	25	25	25	25	25	30	30	30	30	30	36	36	36	36	36	32	32	32	32	32	29	29	29	29	29	20	20	20	20	20	32	32	32	32	32	dd
22	2 2	3 2	3	22	15	15	15	15	15	23	23	23	23	23	48	48	48	48	48	53	53	53	53	53	29	29	29	29	29	32	32	32	32	32	41	41	41	41	41	4	4	4	4	4	dna
	,	2 1	2	2	7	7	7	7	7	0	0	0	0	0	2	2	2	2	2	11	11	11	11	11	6	6	6	6	6	5	57	ഗ	5	5	11	11	11	11	11	6	6	6	6	6	D

Table 10 Out-of-Sample Testing Results

													Average	Pnl Vola	Sharne R	Cumulati	Winning	
							formatio		Average.	PnL.Vola	Sharpe.R	Cumulati						Maximu
Lb		Lo	Lc	Sc	So	Sb		holding	PnL	tility	atio	ve.PnL	Cost	.Cost	Cost			m.DD
	-10	-2	0		) 2	10		60	1.22	4.80	0.25	86.49	1.22	4.80	0.25			8.50
	-10	-2	0		) 2	10	120	90	1.69	7.55	0.22	119.78	1.69	7.55		119.69	0.80	8.07
	-10	-2	0		) 2	10	90	120	0.46	1.79	0.25	32.41	0.46	1.79	0.25	32.38	0.81	5.53
	-6	-2	0		) 2	6	90	90	-0.09	0.81	-0.12	-6.60	-0.09	0.81	-0.12	-6.61	0.31	12.67
	-10	-2	0		) 2	10	90	90	-0.09	0.81	-0.12	-6.60	-0.09	0.81	-0.12	-6.61	0.31	12.67
-	-3.5	-2	0		2	3.5	90	90	-0.09	0.81	-0.12	-6.60	-0.09	0.81	-0.12	-6.61	0.31	12.67
	-10	-2	0	- (	2	10	120	120	0.20	1.73	0.12	14.11	0.20	1.72	0.12	14.10	0.78	4.01
	-10	-2			) 2	10	60	90	0.03	0.81	0.03	1.80	0.03	0.81	0.03	1.78	0.76	5.71
	-10	-2	-0.5	0.	5 2	10	60	90	-0.01	0.64	-0.02	-0.86	-0.01	0.64	-0.02	-0.87	0.79	5.41
	-6	-2	0		) 2	6	120	90	0.26	2.07	0.12	18.29	0.26	2.07	0.12	18.27	0.47	7.87
	-10	-2	0		) 2	10	120	90	0.26	2.07	0.12	18.29	0.26	2.07	0.12	18.27	0.47	7.87
	-3.5	-2	0		2	3.5	120	90	0.26	2.07	0.12	18.29	0.26	2.07	0.12	18.27	0.47	7.87
	-6	-1	0		) 1	6	120	120	-0.32	9.24	-0.03	-22.66	-0.32	9.24	-0.03	-22.79	0.61	61.90
	-6	-1		0.	5 1	6	120	120	-0.74	5.29	-0.14	-52.89	-0.75	5.29	-0.14	-52.97	0.58	78.31
	-10	-2	0		2	10	90	120	0.21	2.15	0.10	14.59	0.20	2.15	0.10	14.55	0.71	9.22
	-6	-1		0.		6	90	120	-2.25	10.83	-0.21	-159.56	-2.25	10.84	-0.21	-159.72	0.37	178.45
	-6	-2	0		2	6	90	120	-2.25	10.83	-0.21	-159.56	-2.25	10.84	-0.21	-159.72	0.37	178.45
	-10	-2			2		90	120	-2.25	10.83	-0.21	-159.56	-2.25	10.84	-0.21	-159.72	0.37	178.45
	-6	-1			1	6	120	120	-0.32	9.24	-0.03	-22.66	-0.32	9.24	-0.03	-22.79	0.61	61.90
	-6	-1	-0.5				120	120			-0.14	-52.89		5.29		-52.97	0.58	78.31
	-10	-2			2		90	120	0.21	2.15	0.10	14.59	0.20	2.15		14.55	0.71	9.22
	-6	-2			) 2		90	60			-0.08	-22.14		3.91		-22.18	0.43	40.49
	-10	-2			) 2		90	60			-0.08	-22.14		3.91		-22.18		40.49
	-3.5	-2	0		) 2	3.5	90	60	-0.31	3.91	-0.08	-22.14	-0.31	3.91	-0.08	-22.18	0.43	40.49

	total_sh		average_										
total_nu	ort_num	long_da	short_da	pairs_nu									
mb_long	S	ys	ys	m		ud	uu	una	dexc	du	dd	dna	Trade_type
2329.00	2140.00	23.37	22.29	86.05	233.00	1776.00	104.00	27.00	257.00	1896.00	131.00	45.00	corr_mv
2604.00	2379.00	26.71	24.63	106.07	236.00	1951.00	109.00	83.00	267.00	2086.00	153.00	98.00	corr_mv
1498.00	1341.00	24.28	23.12	62.00	90.00	1129.00	78.00	44.00	103.00	1231.00	96.00	68.00	corr_mv
56.00	90.00	30.23	31.44	40.20	17.00	18.00	50.00	5.00	8.00	8.00	24.00	4.00	corr_rolling
56.00	90.00	30.23	31.44	40.20	17.00	18.00	50.00	5.00	8.00	8.00	24.00	4.00	corr_rolling
56.00	90.00	30.23	31.44	40.20	17.00	18.00	50.00	5.00	8.00	8.00	24.00	4.00	corr_rolling
213.00	221.00	28.35	32.40	11.91	22.00	163.00	26.00	10.00	30.00	151.00	20.00	12.00	corrSR_mv
231.00	203.00	28.75	31.43	9.75	25.00	144.00	31.00	3.00	31.00	155.00	35.00	10.00	corrSR_mv
262.00	229.00	22.58	24.59	9.75	20.00	174.00	33.00	2.00	25.00	191.00	37.00	9.00	corrSR_mv
64.00	53.00	35.44	38.21	11.87	15.00	12.00	19.00	7.00	9.00	14.00	17.00	10.00	corrSR_rollir
64.00	53.00	35.44	38.21	11.87	15.00	12.00	19.00	7.00	9.00	14.00	17.00	10.00	corrSR_rollir
64.00	53.00	35.44	38.21	11.87	15.00	12.00	19.00	7.00	9.00	14.00	17.00	10.00	corrSR_rollir
265.00	267.00	17.76	20.40	7.82	9.00	173.00	75.00	10.00	11.00	178.00	68.00	8.00	graph_mv
364.00	359.00	11.84	13.72	7.82	8.00	262.00	80.00	9.00	9.00	274.00	73.00	8.00	graph_mv
177.00	181.00	28.57	31.61	8.58	15.00	118.00	39.00	9.00	12.00	118.00	31.00	16.00	graph_mv
274.00	348.00	9.95	10.57	8.58	3.00	125.00	216.00	4.00	3.00	76.00	112.00	9.00	graph_rollin
274.00	348.00	9.95	10.57	8.58	3.00	125.00	216.00	4.00	3.00	76.00	112.00	9.00	graph_rollin
274.00	348.00	9.95	10.57	8.58	3.00	125.00	216.00	4.00	3.00	76.00	112.00	9.00	graph_rollin
265.00	267.00	17.76	20.40	7.82	9.00	173.00	75.00	10.00	11.00	178.00	68.00	8.00	graphSR_m\
364.00	359.00	11.84	13.72	7.82	8.00	262.00	80.00	9.00	9.00	274.00	73.00	8.00	graphSR_m\
177.00	181.00	28.57	31.61	8.50	15.00	118.00	39.00	9.00	12.00	118.00	31.00	16.00	graphSR_m\
61.00	38.00	11.05	11.16	7.70	1.00	19.00	17.00	1.00	1.00	14.00	28.00		graphSR_rol
61.00	38.00	11.05	11.16	7.70	1.00	19.00	17.00	1.00	1.00	14.00	28.00	2.00	graphSR_rol
61.00	38.00	11.05	11.16	7.70	1.00	19.00	17.00	1.00	1.00	14.00	28.00	2.00	graphSR_rol