In this book our objective is to present an introduction of the basic analysis tools and techniques for digital processing of signals. We begin by introducing some of the necessary terminology and by describing the important operations associated with the process of converting an analog signal to digital form suitable for digital processing. As we shall see, digital processing of analog signals has some drawbacks. First, and foremost, conversion of an analog signal to digital form, accomplished by sampling the signal and quantizing the samples, results in a distortion that prevents us from reconstructing the original analog signal from the quantized samples. Control of the amount of this distortion is achieved by proper choice of the sampling rate and the precision in the quantization process. Second, there are finite precision effects that must be considered in the digital processing of the quantized samples. While these important issues are considered in some detail in this book, the emphasis is on the analysis and design of digital signal processing systems and computational techniques.

1.1 SIGNALS, SYSTEMS, AND SIGNAL PROCESSING

A *signal* is defined as any physical quantity that varies with time, space, or any other independent variable or variables. Mathematically, we describe a signal as a function of one or more independent variables. For example, the functions

$$\begin{aligned} s_1(t) &= 5t \\ s_2(t) &= 20t^2 \end{aligned} \tag{1.1.1}$$

describe two signals, one that varies linearly with the independent variable t (time) and a second that varies quadratically with t. As another example, consider the function

$$s(x, y) = 3x + 2xy + 10y^2 \tag{1.1.2}$$

This function describes a signal of two independent variables x and y that could represent the two spatial coordinates in a plane.

The signals described by (1.1.1) and (1.1.2) belong to a class of signals that are precisely defined by specifying the functional dependence on the independent variable. However, there are cases where such a functional relationship is unknown or too highly complicated to be of any practical use.

For example, a speech signal (see Fig. 1.1) cannot be described functionally by expressions such as (1.1.1). In general, a segment of speech may be represented to a high degree of accuracy as a sum of several sinusoids of different amplitudes and frequencies, that is, as

$$\sum_{i=1}^{N} A_i(t) \sin[2\pi F_i(t)t + \theta_i(t)] \tag{1.1.3}$$

where $\{A_i(t)\}$, $\{F_i(t)\}$, and $\{\theta_i(t)\}$ are the sets of (possibly time-varying) amplitudes, frequencies, and phases, respectively, of the sinusoids. In fact, one way to interpret the information content or message conveyed by any short time segment of the

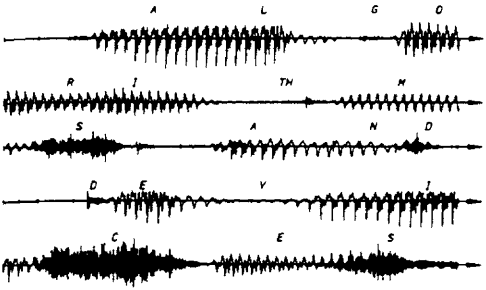

Figure 1.1 Example of a speech signal.

speech signal is to measure the amplitudes, frequencies, and phases contained in the short time segment of the signal.

Another example of a natural signal is an electrocardiogram (ECG). Such a signal provides a doctor with information about the condition of the patient's heart. Similarly, an electroencephalogram (EEG) signal provides information about the activity of the brain.

Speech, electrocardiogram, and electroencephalogram signals are examples of information-bearing signals that evolve as functions of a single independent variable, namely, time. An example of a signal that is a function of two independent variables is an image signal. The independent variables in this case are the spatial coordinates. These are but a few examples of the countless number of natural signals encountered in practice.

Associated with natural signals are the means by which such signals are generated. For example, speech signals are generated by forcing air through the vocal cords. Images are obtained by exposing a photographic film to a scene or an object. Thus signal generation is usually associated with a *system* that responds to a stimulus or force. In a speech signal, the system consists of the vocal cords and the vocal tract, also called the vocal cavity. The stimulus in combination with the system is called a *signal source*. Thus we have speech sources, images sources, and various other types of signal sources.

A *system* may also be defined as a physical device that performs an operation on a signal. For example, a filter used to reduce the noise and interference corrupting a desired information-bearing signal is called a system. In this case the filter performs some operation(s) on the signal, which has the effect of reducing (filtering) the noise and interference from the desired information-bearing signal.

When we pass a signal through a system, as in filtering, we say that we have processed the signal. In this case the processing of the signal involves filtering the noise and interference from the desired signal. In general, the system is characterized by the type of operation that it performs on the signal. For example, if the operation is linear, the system is called linear. If the operation on the signal is nonlinear, the system is said to be nonlinear, and so forth. Such operations are usually referred to as *signal processing*.

For our purposes, it is convenient to broaden the definition of a system to include not only physical devices, but also software realizations of operations on a signal. In digital processing of signals on a digital computer, the operations performed on a signal consist of a number of mathematical operations as specified by a software program. In this case, the program represents an implementation of the system in *software*. Thus we have a system that is realized on a digital computer by means of a sequence of mathematical operations; that is, we have a digital signal processing system realized in software. For example, a digital computer can be programmed to perform digital filtering. Alternatively, the digital processing on the signal may be performed by digital *hardware* (logic circuits) configured to perform the desired specified operations. In such a realization, we have a physical device that performs the specified operations. In a broader sense, a digital system can be implemented as a combination of digital hardware and software, each of which performs its own set of specified operations.

This book deals with the processing of signals by digital means, either in software or in hardware. Since many of the signals encountered in practice are analog, we will also consider the problem of converting an analog signal into a digital signal for processing. Thus we will be dealing primarily with digital systems. The operations performed by such a system can usually be specified mathematically. The method or set of rules for implementing the system by a program that performs the corresponding mathematical operations is called an *algorithm*. Usually, there are many ways or algorithms by which a system can be implemented, either in software or in hardware, to perform the desired operations and computations. In practice, we have an interest in devising algorithms that are computationally efficient, fast, and easily implemented. Thus a major topic in our study of digital signal processing is the discussion of efficient algorithms for performing such operations as filtering, correlation, and spectral analysis.

1.1.1 Basic Elements of a Digital Signal Processing System

Most of the signals encountered in science and engineering are analog in nature. That is, the signals are functions of a continuous variable, such as time or space, and usually take on values in a continuous range. Such signals may be processed directly by appropriate analog systems (such as filters or frequency analyzers) or frequency multipliers for the purpose of changing their characteristics or extracting some desired information. In such a case we say that the signal has been processed directly in its analog form, as illustrated in Fig. 1.2. Both the input signal and the output signal are in analog form.

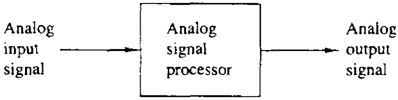

Figure 1.2 Analog signal processing.

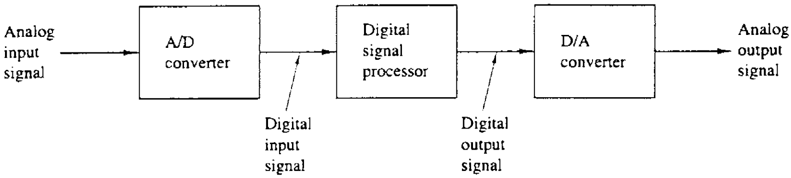

Figure 1.3 Block diagram of a digital signal processing system.

Digital signal processing provides an alternative method for processing the analog signal, as illustrated in Fig. 1.3. To perform the processing digitally, there is a need for an interface between the analog signal and the digital processor. This interface is called an *analog-to-digital (A/D) converter*. The output of the A/D converter is a digital signal that is appropriate as an input to the digital processor.

The digital signal processor may be a large programmable digital computer or a small microprocessor programmed to perform the desired operations on the input signal. It may also be a hardwired digital processor configured to perform a specified set of operations on the input signal. Programmable machines provide the flexibility to change the signal processing operations through a change in the software, whereas hardwired machines are difficult to reconfigure. Consequently, programmable signal processors are in very common use. On the other hand, when signal processing operations are well defined, a hardwired implementation of the operations can be optimized, resulting in a cheaper signal processor and, usually, one that runs faster than its programmable counterpart. In applications where the digital output from the digital signal processor is to be given to the user in analog form, such as in speech communications, we must provide another interface from the digital domain to the analog domain. Such an interface is called a *digital-to-analog (D/A) converter*. Thus the signal is provided to the user in analog form, as illustrated in the block diagram of Fig. 1.3. However, there are other practical applications involving signal analysis, where the desired information is conveyed in digital form and no D/A converter is required. For example, in the digital processing of radar signals, the information extracted from the radar signal, such as the position of the aircraft and its speed, may simply be printed on paper. There is no need for a D/A converter in this case.

1.1.2 Advantages of Digital over Analog Signal Processing

There are many reasons why digital signal processing of an analog signal may be preferable to processing the signal directly in the analog domain, as mentioned briefly earlier. First, a digital programmable system allows flexibility in reconfiguring the digital signal processing operations simply by changing the program.

Reconfiguration of an analog system usually implies a redesign of the hardware followed by testing and verification to see that it operates properly.

Accuracy considerations also play an important role in determining the form of the signal processor. Tolerances in analog circuit components make it extremely difficult for the system designer to control the accuracy of an analog signal processing system. On the other hand, a digital system provides much better control of accuracy requirements. Such requirements, in turn, result in specifying the accuracy requirements in the A/D converter and the digital signal processor, in terms of word length, floating-point versus fixed-point arithmetic, and similar factors.

Digital signals are easily stored on magnetic media (tape or disk) without deterioration or loss of signal fidelity beyond that introduced in the A/D conversion. As a consequence, the signals become transportable and can be processed off-line in a remote laboratory. The digital signal processing method also allows for the implementation of more sophisticated signal processing algorithms. It is usually very difficult to perform precise mathematical operations on signals in analog form but these same operations can be routinely implemented on a digital computer using software.

In some cases a digital implementation of the signal processing system is cheaper than its analog counterpart. The lower cost may be due to the fact that the digital hardware is cheaper, or perhaps it is a result of the flexibility for modifications provided by the digital implementation.

As a consequence of these advantages, digital signal processing has been applied in practical systems covering a broad range of disciplines. We cite, for example, the application of digital signal processing techniques in speech processing and signal transmission on telephone channels, in image processing and transmission, in seismology and geophysics, in oil exploration, in the detection of nuclear explosions, in the processing of signals received from outer space, and in a vast variety of other applications. Some of these applications are cited in subsequent chapters.

As already indicated, however, digital implementation has its limitations. One practical limitation is the speed of operation of A/D converters and digital signal processors. We shall see that signals having extremely wide bandwidths require fast-sampling-rate A/D converters and fast digital signal processors. Hence there are analog signals with large bandwidths for which a digital processing approach is beyond the state of the art of digital hardware.

1.2 CLASSIFICATION OF SIGNALS

The methods we use in processing a signal or in analyzing the response of a system to a signal depend heavily on the characteristic attributes of the specific signal. There are techniques that apply only to specific families of signals. Consequently, any investigation in signal processing should start with a classification of the signals involved in the specific application.

1.2.1 Multichannel and Multidimensional Signals

As explained in Section 1.1, a signal is described by a function of one or more independent variables. The value of the function (i.e., the dependent variable) can be a real-valued scalar quantity, a complex-valued quantity, or perhaps a vector. For example, the signal

$$s_1(t) = A \sin 3\pi t$$

is a real-valued signal. However, the signal

$$s_2(t) = Ae^{j3\pi t} = A \cos 3\pi t + jA \sin 3\pi t$$

is complex valued.

In some applications, signals are generated by multiple sources or multiple sensors. Such signals, in turn, can be represented in vector form. Figure 1.4 shows the three components of a vector signal that represents the ground acceleration due to an earthquake. This acceleration is the result of three basic types of elastic waves. The primary (P) waves and the secondary (S) waves propagate within the body of rock and are longitudinal and transversal, respectively. The third type of elastic wave is called the surface wave, because it propagates near the ground surface. If $s_k(t)$, $k = 1, 2, 3$, denotes the electrical signal from the kth sensor as a function of time, the set of $p = 3$ signals can be represented by a vector $\mathbf{S}_3(t)$, where

$$\mathbf{S}_3(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix}$$

We refer to such a vector of signals as a *multichannel signal*. In electrocardiography, for example, 3-lead and 12-lead electrocardiograms (ECG) are often used in practice, which result in 3-channel and 12-channel signals.

Let us now turn our attention to the independent variable(s). If the signal is a function of a single independent variable, the signal is called a *one-dimensional* signal. On the other hand, a signal is called *M-dimensional* if its value is a function of M independent variables.

The picture shown in Fig. 1.5 is an example of a two-dimensional signal, since the intensity or brightness $I(x, y)$ at each point is a function of two independent variables. On the other hand, a black-and-white television picture may be represented as $I(x, y, t)$ since the brightness is a function of time. Hence the TV picture may be treated as a three-dimensional signal. In contrast, a color TV picture may be described by three intensity functions of the form $I_r(x, y, t)$, $I_g(x, y, t)$, and $I_b(x, y, t)$, corresponding to the brightness of the three principal colors (red, green, blue) as functions of time. Hence the color TV picture is a three-channel, three-dimensional signal, which can be represented by the vector

$$\mathbf{I}(x, y, t) = \begin{bmatrix} I_r(x, y, t) \\ I_g(x, y, t) \\ I_b(x, y, t) \end{bmatrix}$$

In this book we deal mainly with single-channel, one-dimensional real- or complex-valued signals and we refer to them simply as signals. In mathematical

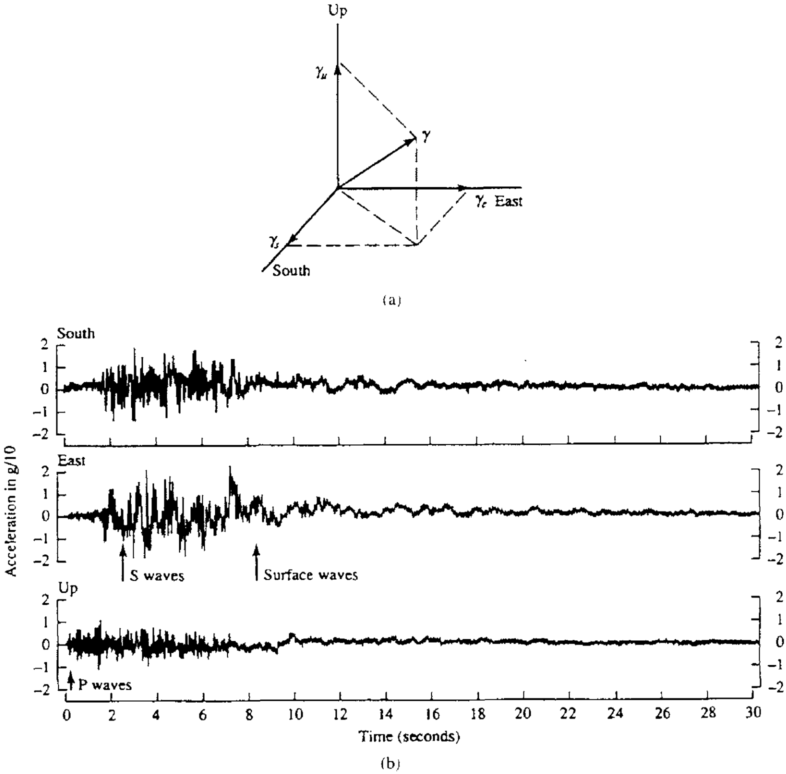

Figure 1.4 Three components of ground acceleration measured a few kilometers from the epicenter of an earthquake. (From *Earthquakes*, by B. A. Bold, ©1988 by W. H. Freeman and Company. Reprinted with permission of the publisher.)

terms these signals are described by a function of a single independent variable. Although the independent variable need not be time, it is common practice to use t as the independent variable. In many cases the signal processing operations and algorithms developed in this text for one-dimensional, single-channel signals can be extended to multichannel and multidimensional signals.

1.2.2 Continuous-Time Versus Discrete-Time Signals

Signals can be further classified into four different categories depending on the characteristics of the time (independent) variable and the values they take. *Continuous-time signals* or *analog signals* are defined for every value of time and

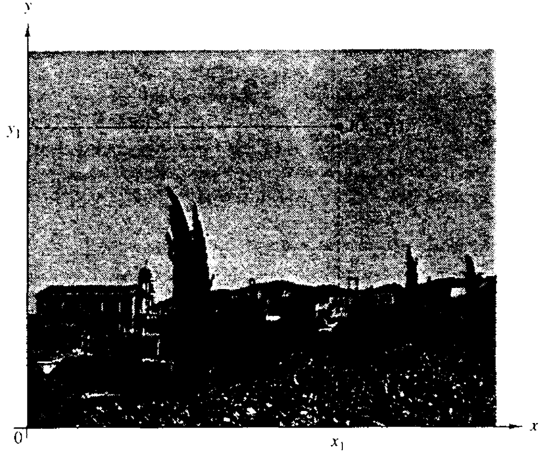

Figure 1.5 Example of a two-dimensional signal.

they take on values in the continuous interval (a, b), where a can be $-\infty$ and b can be ∞. Mathematically, these signals can be described by functions of a continuous variable. The speech waveform in Fig. 1.1 and the signals $x_1(t) = \cos \pi t$, $x_2(t) = e^{-|t|}$, $-\infty < t < \infty$ are examples of analog signals. *Discrete-time signals* are defined only at certain specific values of time. These time instants need not be equidistant, but in practice they are usually taken at equally spaced intervals for computational convenience and mathematical tractability. The signal $x(t_n) = e^{-|t_n|}$, $n = 0, \pm 1, \pm 2, \ldots$ provides an example of a discrete-time signal. If we use the index n of the discrete-time instants as the independent variable, the signal value becomes a function of an integer variable (i.e., a sequence of numbers). Thus a discrete-time signal can be represented mathematically by a sequence of real or complex numbers. To emphasize the discrete-time nature of a signal, we shall denote such a signal as $x(n)$ instead of $x(t)$. If the time instants t_n are equally spaced (i.e., $t_n = nT$), the notation $x(nT)$ is also used. For example, the sequence

$$x(n) = \begin{cases} 0.8^n, & \text{if } n \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{1.2.1}$$

is a discrete-time signal, which is represented graphically as in Fig. 1.6.

In applications, discrete-time signals may arise in two ways:

1. By selecting values of an analog signal at discrete-time instants. This process is called *sampling* and is discussed in more detail in Section 1.4. All measuring instruments that take measurements at a regular interval of time provide discrete-time signals. For example, the signal $x(n)$ in Fig. 1.6 can be obtained

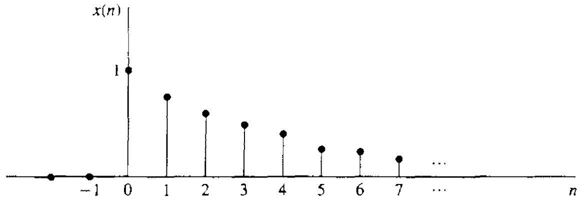

Figure 1.6 Graphical representation of the discrete time signal $x(n) = 0.8^n$ for $n > 0$ and $x(n) = 0$ for $n < 0$.

by sampling the analog signal $x(t) = 0.8^t$, $t \geq 0$ and $x(t) = 0$, $t < 0$ once every second.

2. By accumulating a variable over a period of time. For example, counting the number of cars using a given street every hour, or recording the value of gold every day, results in discrete-time signals. Figure 1.7 shows a graph of the Wölfer sunspot numbers. Each sample of this discrete-time signal provides the number of sunspots observed during an interval of 1 year.

1.2.3 Continuous-Valued Versus Discrete-Valued Signals

The values of a continuous-time or discrete-time signal can be continuous or discrete. If a signal takes on all possible values on a finite or an infinite range, it

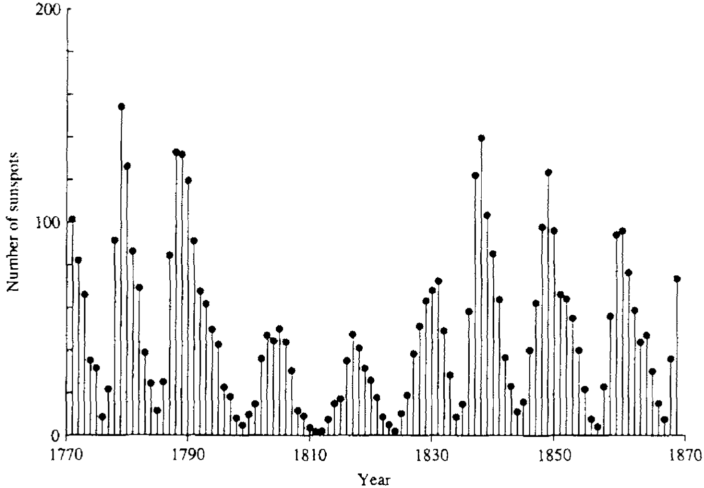

Figure 1.7 Wölfer annual sunspot numbers (1770–1869).

is said to be continuous-valued signal. Alternatively, if the signal takes on values from a finite set of possible values, it is said to be a discrete-valued signal. Usually, these values are equidistant and hence can be expressed as an integer multiple of the distance between two successive values. A discrete-time signal having a set of discrete values is called a *digital signal*. Figure 1.8 shows a digital signal that takes on one of four possible values.

In order for a signal to be processed digitally, it must be discrete in time and its values must be discrete (i.e., it must be a digital signal). If the signal to be processed is in analog form, it is converted to a digital signal by sampling the analog signal at discrete instants in time, obtaining a discrete-time signal, and then by *quantizing* its values to a set of discrete values, as described later in the chapter. The process of converting a continuous-valued signal into a discrete-valued signal, called *quantization*, is basically an approximation process. It may be accomplished simply by rounding or truncation. For example, if the allowable signal values in the digital signal are integers, say 0 through 15, the continuous-value signal is quantized into these integer values. Thus the signal value 8.58 will be approximated by the value 8 if the quantization process is performed by truncation or by 9 if the quantization process is performed by rounding to the nearest integer. An explanation of the analog-to-digital conversion process is given later in the chapter.

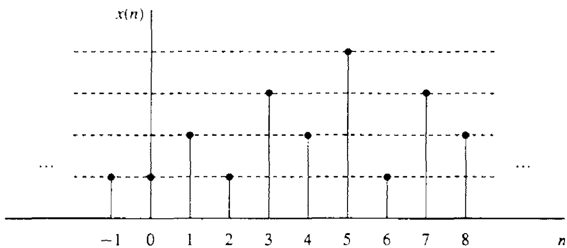

Figure 1.8 Digital signal with four different amplitude values.

1.2.4 Deterministic Versus Random Signals

The mathematical analysis and processing of signals requires the availability of a mathematical description for the signal itself. This mathematical description, often referred to as the *signal model*, leads to another important classification of signals. Any signal that can be uniquely described by an explicit mathematical expression, a table of data, or a well-defined rule is called *deterministic*. This term is used to emphasize the fact that all past, present, and future values of the signal are known precisely, without any uncertainty.

In many practical applications, however, there are signals that either cannot be described to any reasonable degree of accuracy by explicit mathematical formulas, or such a description is too complicated to be of any practical use. The lack of such a relationship implies that such signals evolve in time in an unpredictable manner. We refer to these signals as *random*. The output of a noise generator, the seismic signal of Fig. 1.4, and the speech signal in Fig. 1.1 are examples of random signals.

Figure 1.9 shows two signals obtained from the same noise generator and their associated histograms. Although the two signals do not resemble each other visually, their histograms reveal some similarities. This provides motivation for

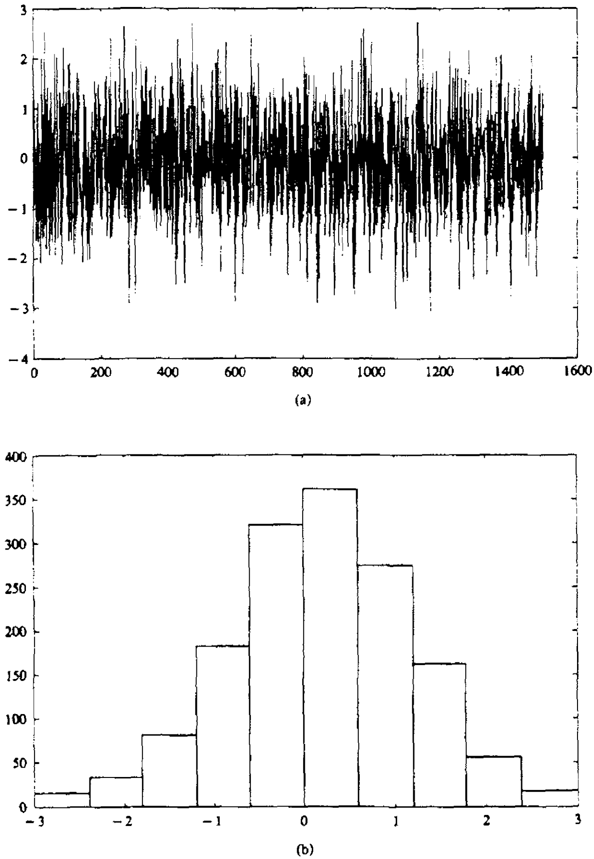

Figure 1.9 Two random signals from the same signal generator and their histograms.