Aerospace Math Appendix

Autonomous UAV Networks – Digital-Green v2.3

1. International Standard Atmosphere (ISA)

The simulator applies ISA standards for air density and temperature to preserve aerodynamic accuracy. Temperature and pressure variations with altitude are governed by:

$T(h) = T_0 + L \cdot h$

$p(h) = p_0 \cdot (T/T_0)^{-g/(R \cdot L)}$

$\rho(h) = p/(R \cdot T)$

Where:

- $T_0 = 288.15$ K, $p_0 = 101325$ Pa
- $L = -0.0065$ K/m (lapse rate)
- $R = 287.058$ J/(kg·K), $g = 9.80665$ m/s^2

2. Aerodynamic Power Model

Total propulsion power combines induced, parasite, and climb components:

P_total = P_induced + P_parasite + P_climb

P_induced = $(2 \cdot W^2)/(\pi \cdot e \cdot AR \cdot \rho \cdot S \cdot V)$

P_parasite = $½ \cdot \rho \cdot S \cdot C\_D0 \cdot V^3$

P_climb = $W \cdot V \cdot \sin(\gamma)$

Parameters:

- W = m·g (weight)
- e = Oswald efficiency
- AR = Aspect ratio
- C_D0 = Zero-lift drag coefficient
- ρ = Air density at altitude

3. Electrical and Battery Model

Electrical power draw is corrected for propeller and motor efficiencies: P_elec = P_total / (η_prop· η_motor)

Battery discharge follows: ΔE_Wh = P_elec· Δt / 3600

SoC = E_remaining / E_nominal

4. Flight Kinematics

UAV motion follows 3-DOF planar-plus-altitude kinematics with stall and climb constraints:

$\psi_{t+1} = \psi_t + \Delta\psi$ (limited by turn rate)

V = clamp(V_cmd, 1.05·V_stall, V_max)

$h_{t+1} = h_t + V \cdot \sin(\gamma) \cdot \Delta t$

5. RF Propagation (Two-Ray + Rician)

Signal gain combines Free-Space Path Loss (FSPL) with ground reflection via Two-Ray modeling:
$FSPL(dB) = 32.44 + 20 \cdot \log_{10}(f\_GHz \cdot 1000) + 20 \cdot \log_{10}(d\_km)$
$E\_tot = E_1 + \Gamma \cdot E_2$
Rician fading introduces a deterministic LOS component (K, dB) with random multipath scattering, yielding gain scaling by $|g|^2$, where $g = s + n\_I + j \cdot n\_Q$.

6. LTE MCS and MAC Throughput

Effective data rate derives from Shannon or LTE MCS interpolation:
$C = B \cdot \log_2(1 + SINR) \cdot K\_MAC \cdot \eta\_PHY$
TDMA, NOMA, and RSMA adjust throughput via time, power, and rate-splitting factors respectively.

7. Graph Flow Algorithm

A directed graph G(V,E) models link capacities between UAVs. A “widest path” heuristic estimates aggregate throughput from sources to sinks, updating residual capacities iteratively (akin to Ford–Fulkerson) for network realism.