Aerospace Math Appendix — UAV LiDAR + MPC Lab

Aerodynamics

Lift: $L = \frac{1}{2} \rho V^2 C_L S$

Lift equals dynamic pressure times lift coefficient and wing planform area (N).

Drag: $D = \frac{1}{2} \rho V^2 C_D S$

Drag scales with dynamic pressure, drag coefficient, and reference area (N).

Drag polar: $C_D = C_{D0} + k C_L^2$, $k = 1 / (\pi e AR)$

Total drag coefficient is parasitic plus induced; k depends on aspect ratio AR and Oswald efficiency e (–).

Stall speed: $V_s = \sqrt{2 W / (\rho S C_{L,max})}$

Minimum steady flight speed when lift equals weight using maximum lift coefficient (m/s).

Turn relations: $R = V^2 / (g \tan \varphi)$, $\omega = g \tan \varphi / V$

For a coordinated turn at bank angle φ, turn radius R and yaw rate ω depend on speed V and gravity g.

Propulsion

Power required: $P_{req} = D V$

Propulsive power to overcome drag equals drag times airspeed (W).

Shaft power: $P_{shaft} = T V / \eta_p$

Shaft power is thrust–power divided by propulsive efficiency η_p (W).

Fuel flow: $\dot{m}_f = P_{shaft} / (\eta_t \cdot LHV)$

Fuel mass flow equals shaft power over efficiency and fuel heating value (kg/s).

Battery energy: $E_{batt} = V_{nom} Q \eta$ [Wh or J]

Available energy = $V_{nom} \times Q \times \eta$ (Wh if Q in Ah, J if Q in Coulombs).

Electrical power: $P \approx P_0 + k_v V^3$ (multirotor $\approx T^{3/2}/\sqrt{A}$)

Parasitic (V^3) dominates at cruise; hover/low-speed induced power scales $\propto T^{3/2}$ with disk loading.

Performance

Thrust–weight & wing loading: T/W, W/S

Metrics comparing thrust-to-weight and wing loading for design and performance sizing.

Rate of climb: $RC = (P_{avail} - P_{req}) / W$
Excess specific power divided by weight gives climb rate (m/s).

Glide ratio (for range): $(L/D) = C_L / C_D$
Lift-to-drag ratio determines best range; best endurance occurs at minimum power speed.

Endurance: $E \approx (\eta_p E_{fuel}) / P_{req}$
Endurance approximates usable energy divided by required power (s).

RTB trigger: $E_{rem} \le E_{RTB}(d,V,wind) + E_{reserve}$
Return-to-base condition reserves energy for distance d under wind and speed with reserve margin.

Navigation / Stability

Kinematics: $p_{k+1} = p_k + V_k \Delta t$, $V_{k+1} = \mathrm{clip}(V_k + a_k \Delta t, V_{max})$
Discrete motion update with bounded velocity (m, m/s).

Coordinated turn: $\dot{\psi} = g \tan \varphi / V$
Heading rate in coordinated turn depends on bank angle φ and speed V (rad/s).

MPC cost: $J = \Sigma \|p_k - p_{goal}\|^2 + \lambda \|a_k\|^2$
Predictive control cost balances goal tracking and effort weight λ (–).

LiDAR coverage: $\text{coverage} = |\text{cells}_{hit}| / |\text{cells}_{total}|$
Coverage fraction of map cells observed by LiDAR rays (–).

EKF update: $\hat{x} = \hat{x}^- + K(z - h(\hat{x}^-))$, $K = P^-H^T(HP^-H^T + R)^{-1}$
Kalman filter update corrects the state estimate $\hat{x}^-$ with gain K (SI units per model).

Variable Definitions (SI Units)

ρ: air density (kg/m³), V: true airspeed (m/s), S: reference area (m²), C_L, C_D: coefficients (–).

C_D0: zero-lift drag (–), e: Oswald efficiency (–), AR: aspect ratio (–), W: weight (N), $g = 9.81$ m/s².

T: thrust (N), η_p: propulsive efficiency (–), η_t: thermal efficiency (–), LHV: heating value (J/kg).

E_batt: battery energy (J or Wh), Q: capacity (Ah or C), V_nom: voltage (V), a: acceleration (m/s²), Δt: time step (s).

φ: bank angle (rad), ψ: heading (rad), angles in radians unless stated otherwise, J: objective (–), λ: weight (–).