# Mathematical Appendix — Multi-Agent UAV LiDAR + MPC

This appendix presents the mathematical formulations underlying the UAV simulation app.

## 1. UAV Dynamics

Each UAV is modeled with 2D position p and velocity v. Updates follow:
$p(t+1) = p(t) + (v(t) + w)\Delta t$
$v(t+1) = \text{clip}(v(t) + a\Delta t, v_{max})$
where a is control acceleration, $\Delta t$ is time step, and w is wind disturbance.

## 2. Model Predictive Control (MPC)

Candidate accelerations are sampled from a discrete set. For each candidate a, the cost over horizon H is:
$J(a) = \Sigma ||p(\tau) - g||^2 + \Sigma$ obstacle_penalty + $\Sigma$ separation_penalty + $\lambda||a||^2$
The best action minimizes J(a).

## 3. Obstacle Avoidance

Obstacles are modeled as squares with center o and side length s. Distance to obstacle is:
$d = || \max(|p - o| - s/2, 0) ||$
Penalty: large if $d < 0.5$, moderate if $d <$ LiDAR_range.

## 4. Multi-Agent Separation

For UAV i and j, distance is $d_{ij} = ||p_i - p_j||$.
Penalty: large if $d_{ij} < 0.8$, moderate if $d_{ij} < 2.5$.

## 5. LiDAR Ray-Casting

Each UAV emits N beams at angles $\theta_k$.
Beam endpoint: $b_k = p + \min(r_{max}, r_{hit})[\cos\theta_k, \sin\theta_k]$
where $r_{hit}$ is distance to first obstacle.
All hit points are fused into a global map.

## 6. Coverage Grid

Environment is discretized into grid cells of size c. A cell is marked covered if any LiDAR hit falls within it.
Coverage % = covered_cells / total_cells × 100.

## 7. Battery and Energy Model

Battery capacity: $E_0$ (Wh). Energy consumption per step:
$\Delta E = (P_{hover} + k||v||^3)\Delta t / 3600$
RTB triggered when $E < 0.15E_0$.

## 8. Task Allocation

Goals are assigned using a greedy nearest-neighbor strategy:
assign goal j to UAV i that minimizes $||s_i - g_j||$ subject to unassigned goals.

## 9. Latency Profiling

For each UAV decision step:
$\tau = t_{end} - t_{start}$
Metrics: average, 95th percentile, and maximum latency.

## 10. AI Suggestions Logic

The app generates AI suggestions based on metrics:
• Coverage low/high → adjust LiDAR or drone count.
• Latency high/low → tune MPC horizon or rays.
• Separation too close/wide → adjust avoidance cost or formation.
• Battery low/high → increase capacity or trade endurance.
• Speed saturation → adjust max speed/acceleration.