# **UAV Battery Efficiency Estimator**

Aerospace Math Appendix (Unicode Safe)

Author: Tareq Omrani • GPT-UAV Planner

#### 0) Symbols & Notation

ρ	Air density (kg·m <sup>-3</sup> )
ρο	Sea-level density (1.225 kg·m <sup>-3</sup> )
V	True airspeed (m·s <sup>-1</sup> )
q	Dynamic pressure = $\frac{1}{2}\rho V^2$ (Pa)
S	Wing planform area (m²)
b	Wingspan (m)
AR	Aspect ratio = $b^2/S$ (—)
W	Weight (N) = m g
Cr	Lift coefficient (—)
C□	Drag coefficient (—)
$C^{D_0}$	Parasite drag coefficient (—)
е	Oswald efficiency factor (—)
$\eta_{\text{p}}$	Propulsive efficiency (—)
Р	Shaft/electrical power (W)
BSFC	Brake-specific fuel consumption (g·kWh <sup>-1</sup> )
ΔΤ	Skin/air temperature rise (°C)

## 1) Standard Atmosphere (ISA Troposphere ≤ 11 km)

Sea-level reference:  $\rho_0$ =1.225 kg·m<sup>-3</sup>,  $P_0$ =101325 Pa,  $T_0$ =288.15 K, lapse L=0.0065 K·m<sup>-1</sup>, R=287.05 J·kg<sup>-1</sup>·K<sup>-1</sup>, g=9.80665 m·s<sup>-2</sup>.

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T(h) = max(1, T_0 - Lh)
P(h) = P_0 \cdot (1 - Lh/T_0)^{(g/(R \cdot L))}
\rho(h) = P(h)/(R \cdot T(h))
\sigma = \rho/\rho_0
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#### 2) Rotorcraft Induced-Power Scaling

Ideal induced power  $\propto 1/\sqrt{\rho}$ . Model factor:  $f_{\rho} = 1/\text{max}(0.3, \sqrt{\sigma})$ .

 $P_hover_scaled = P_hover_nominal \times f_p$ 

## 3) Fixed-Wing Aerodynamics & Power Required

Lift:  $C^L = W/(qS)$ . Aspect ratio AR =  $b^2/S$ . Parabolic drag polar:  $C^D = C^{D_0} + kC^{L^2}$  with  $k = 1/(\pi e AR)$ .

 $D = q S C^{D}$ 

 $P_req = (D \cdot V)/max(0.3, \eta_p)$ 

Safeguards:  $C^{D_0} \ge 0.05$ ; e ≤ 0.70; 0.55 ≤  $\eta_p \le 0.65$ .

#### 4) Battery Fixed-Wing Power Model (App Implementation)

 $P_{tot} = hotel + (1+f_{install}) \cdot P_{req}$ , hotel  $\approx 15$  W,  $f_{install} \approx 0.15$ ; mission penalties and gust penalties applied multiplicatively.

#### 5) Multirotor Power Model (App Implementation)

Baseline draw  $P_0$  from profile, scaled by mass and density plus parasitic  $\propto V^2$ .

 $P \approx P_0 \cdot (m/m_base) \cdot f_\rho + 0.018 \cdot (V_km/h)^2$ 

Then gust/mission penalties applied.

#### 6) Thermal Model (Convective + Radiative)

Waste heat  $Q_w \approx \text{total electrical (battery)}$  or shaft+hotel (ICE).

 $h \approx (10.45 - V + 10\sqrt{V}) \cdot (\rho/\rho_0)$ 

 $k \text{ rad} = 4\epsilon\sigma T_a^3$ 

 $K = (h + k rad) \cdot A$ 

 $\Delta T = Q w / max(1,K)$ 

 $\Delta T \leftarrow \Delta T \cdot (1 - 0.35 \cdot CC)$ 

#### 7) Gust Penalty

Fractional penalty  $\phi_g \le 0.35$ .

$$\begin{split} \phi\_g &= clamp[0,0.35]\{1.5\cdot(g\_ms/V)^2\cdot(WL\_ref/WL)^{-}0.7\,+\,0.03\cdot(W\_ms/8)\}\\ g\ ms &\approx 0.6\cdot g,\ WL\ ref=70\ N\cdot m^{-2} \end{split}$$

## 8) Climb/Descent Energy

Battery: E\_climb = (m g h)/3600 Wh; Descent recovery ~20%. ICE: convert mgh to kWh, multiply by BSFC, divide by fuel density.

## 9) Endurance & Reserve Policy

Battery:  $E_use = 0.85 \cdot E_pack$ .  $t_raw = 60 \cdot E_use/P_tot$ ;  $t_dispatch = 0.70 \cdot t_raw$ . ICE: usable fuel=0.90 \cdot tank-climb+assist savings.

## 10) Wind-Vector Range

Given V\_air, W: distance\_best =  $(V_air+W)\cdot t_h$ ; distance\_upwind =  $max(0,(V_air-W)\cdot t_h)$ . If  $W \ge V$  air, upwind=0.

## 11) Hybrid Assist

Fraction f of total power from battery for  $\tau$  minutes. Fuel saved  $\approx$  LPH(P\_tot·f)·( $\tau$ /60). Thermal:  $\Delta T \leftarrow \Delta T \cdot (1 - 0.3f)$ .

## 12) Detectability Scores

Al visual: altitude, speed, gusts, clouds, stealth. IR:  $\Delta T$  norm, altitude attenuation, cloud attenuation, ICE bias, stealth.

#### 13) Uncertainty & Safeguards

Endurance  $\pm 10\%$ . Clamps:  $\sigma$  floor;  $\eta_P$  range; e bound;  $C^{D_0}$  floor;  $\Delta T \geq 0.2^{\circ}C$ .

#### 14) Constants

App default constants:

RHO0	1.225 kg·m <sup>-3</sup>
P0	101325 Pa
TOK	288.15 K
LAPSE	0.0065 K·m <sup>−1</sup>
R_AIR	287.05 J·kg <sup>-1</sup> ·K <sup>-1</sup>
G0	9.80665 m·s <sup>-2</sup>
SIGMA	5.670374419×10 <sup>-8</sup> W·m <sup>-2</sup> ·K <sup>-4</sup>
USABLE_BATT_FRAC	0.85
USABLE_FUEL_FRAC	0.90
DISPATCH_RESERVE	0.30
HOTEL_W_DEFAULT	15 W
INSTALL_FRAC_DEF	0.15

#### 15) Code Cross-Reference

air\_density(), density\_ratio(), rotorcraft\_density\_scale(), drag\_polar\_cd(), aero\_power\_required\_W(), realistic\_fixedwing\_power(), gust\_penalty\_fraction(), convective\_radiative\_deltaT(), climb\_energy\_wh(), climb\_fuel\_liters(), heading\_range\_km(), bsfc\_fuel\_burn\_lph(), compute\_ai\_ir\_scores(), render\_detectability\_alert().

## 16) Assumptions & Limitations

• Valid for small/medium UAV regime. • No compressibility corrections (V  $\ll$  transonic). • Detectability heuristic only. • Hybrid assist neglects internal resistance and thermal lags. • Terrain/stealth penalties applied multiplicatively.