VTOL Precision Landing Simulator — Compact Math Appendix

1. Camera Geometry

- Focal length (pixels): $f_{px} = W / [2 \cdot \tan(\theta_{HFOV} / 2)]$
- Marker pixels vs altitude: $p(z) = (f_{px} \cdot s) / \max(z, \varepsilon)$
- In-FOV condition: $r \le z \cdot \tan(\theta_{HFOV} / 2)$

These define pixel size and visibility given altitude and camera geometry.

2. Vision Detection & Lock

- Base detection: $base(p) = 1 / [1 + e^{-(p - T) \cdot k}]$
- Adjusted: $P_{det} = base \cdot (1 - 0.6b) \cdot (0.6 + 0.4\ell) \cdot \beta$
- Lock: held after ≥ dwell frames of detection
- Beacon correction: $p \leftarrow p - g \cdot p$ (gain $g \in [0, 1)$)

3. Kalman Filter (CV Model)

State: $x = [x, y, vx, vy]^T$

$A = [[1,0,\Delta t,0],[0,1,0,\Delta t],[0,0,1,0],[0,0,0,1]]$

$H = [[1,0,0,0],[0,1,0,0]]$

$Q = q \cdot [[\Delta t^4/4,0,\Delta t^3/2,0],[0,\Delta t^4/4,0,\Delta t^3/2],[\Delta t^3/2,0,\Delta t^2,0],[0,\Delta t^3/2,0,\Delta t^2]]$

Predict: $x^- = A \cdot x$, $P^- = A \cdot P \cdot A^T + Q$

Update: $K = P^- \cdot H^T (H \cdot P^- \cdot H^T + R)^{-1}$, $x = x^- + K(z - Hx^-)$

4. Measurement Noise Logic

GNSS available: $\sigma_{meas} \approx kf_{r\_base}$ (≈0.02–1.0 m)

GPS-denied: $\sigma_{meas} \approx 3.0$ m (unlocked) , or $\max(0.05, \min(0.25, 0.8 / \max(p,1)))$ when locked

Simulated INS drift = random walk + slow bias.

5. Altitude Models

Barometer: random walk (σ≈0.25 m per 40 samples)

Lidar: low-variance (σ≈0.02–0.05 m)

Used for descent shape and cone geometry.

6. Landing Cone & Metrics

Cone rule: $r_{allow}(z) = \alpha \cdot (z / z_{max})$, $\alpha = 1.0$ for $z_{max} = 10$ m

Touchdown error: $||[x, y]||_2$

Vertical speed: $v_{TD} = \max(0, (z_{n-k} - z_n) / (k\Delta t))$

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7. Landing Success Score

Score = 100·[0.4·e^(−XY/0.20) + 0.2·e^(−max(0,v_TD−0.5)/0.5)
+ 0.2·e^(−5ρ) + 0.2λ]
XY = touchdown error (m), v_TD = vertical speed (m/s), ρ = cone violation rate, λ = final lock stability.
Soft thresholds: 20 cm (XY), 0.5 m/s (V-speed).

8. Auto-Tuner Search Ranges

beacon_gain ∈ [0.15, 0.60] | lock_thresh_px ∈ [18, 48] | lock_dwell_frames ∈ [4, 14]
kf_q ∈ [3×10⁻⁵, 10⁻²] | kf_r_base ∈ [0.02, 0.60] m
Goal: maximize mean landing score over seeds.

9. Key Symbols

θ_HFOV – horizontal field of view (deg) | f_px – focal length (px)
s – marker size (m) | p – marker pixels | T – pixel threshold | k – sigmoid slope
ℓ – illumination | b – blur | g – beacon gain | q – process noise | σ_meas – measurement std dev
r_allow – allowable cone radius | ρ – cone violation rate | λ – lock stability | Δt – time step