A New Directional Weighted Median Filter for Removal of Random-Valued Impulse Noise

Yiqiu Dong* and Shufang Xu

Abstract

The known median-based denoising methods tend to work well for restoring the images corrupted by random-valued impulse noise with low noise level, but poorly for highly corrupted images. This paper proposes a new impulse detector, which is based on the differences between the current pixel and its neighbors aligned with four main directions. Then, we combine it with the weighted median filter to get a new directional weighted median (DWM) filter. Extensive simulations show that the proposed filter not only can provide better performance of suppressing impulse with high noise level, but can preserve more detail features, even thin lines. As extended to restoring corrupted color images, this filter also performs very well.

Index Terms

random-valued impulse noise, impulse detector, image denoising.

I. Introduction

Impulse noise is often introduced into images during acquisition and transmission. Based on the noise values, it can be classified as the easier-to-restore salt-and-pepper noise and the more difficult random-valued impulse noise [1]. There have been much more methods for removing the former, and some of them have performed very well [2]–[5]. So here, we only focus on removing the latter.

Among all kinds of methods for impulse noise, the median filter [6] is used widely because of its effective noise suppression capability and high computational efficiency. However, it uniformly replaces the gray-level value of every pixel by the median of its neighbors. Consequently, some desirable details

The EDICS category is IMD-ANAL.

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are also removed, especially when the window size is large. In order to improve the median filter, many filters with an impulse detector are proposed, such as signal dependent rank order mean (SD-ROM) filter [7], multi-state median (MSM) filter [1], adaptive center weighted median (ACWM) filter [8], the pixel-wise MAD (PWMAD) filter [9], and iterative median filter [10], etc.. These filters usually perform well, but as the noise level is higher than 30%, they tend to remove many features from the images, or retain too much impulse noise.

In this paper, we propose a new impulse detector, which makes use of the differences between the current pixel and its neighbors aligned with four main directions. After impulse detection, we do not simply replace noisy pixels identified by outputs of median filter, but continue to use the information of the four directions to weight the pixels in the window in order to preserve the details as removing noise. Our directional weighted median (DWM) filter performs much better than the other median-based filters in removing random-valued impulse noise, especially when the noise level is as high as 60%. Furthermore, it can preserve more detail features, even thin lines. We also extended the DWM filter to restore the color images corrupted by random-valued impulse noise, in which case, it also performs very well.

The organization of this paper is as follows. The new impulse detector is formulated in Section 2. Section 3 shows the filtering framework. Section 4 provides a number of experimental results to demonstrate the performance of the new filter. Finally, conclusions are drawn in Section 5.

II. Impulse Detector

Before introducing the new impulse detector, we review a basic assumption, that is, a noise-free image consists of locally smoothly varying areas separated by edges. Here, we only focus on the edges aligned with four main directions shown in Fig. 1.

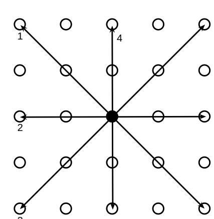

Fig. 1. The four directions for impulse detection.

Let S_k $(k = 1$ to $4)$ denotes a set of coordinates aligned with the kth direction centered at $(0, 0)$, i.e.,

$$\begin{aligned} S_1 &= \{(-2,-2),(-1,-1),(0,0),(1,1),(2,2)\}, \\ S_2 &= \{(0,-2),(0,-1),(0,0),(0,1),(0,2)\}, \\ S_3 &= \{(2,-2),(1,-1),(0,0),(-1,1),(-2,2)\}, \\ S_4 &= \{(-2,0),(-1,0),(0,0),(1,0),(2,0)\}. \end{aligned} \tag{1}$$

Then, let $S_k^0 = S_k \backslash (0,0)$ for all k from 1 to 4. In a 5×5 window centered at (i, j), for each direction, define $d_{i,j}^{(k)}$ as the sum of all absolute differences of gray-level values between $y_{i+s,j+t}$ and $y_{i,j}$ with $(s,t) \in S_k^0$. Considering that for two pixels whose spatial distance is small their gray-level values should be close, we will weight the absolute differences between the two closest pixels with a larger value w_m, before we calculate the sum. But if w_m is very large, it will cause that $d_{i,j}^{(k)}$ is mainly decided by the differences corresponding to w_m. So let $w_m = 2$, the reciprocal of distance ratio. Thus, we have

$$d_{i,j}^{(k)} = \sum_{(s,t) \in S_k^0} w_{s,t} |y_{i+s,j+t} - y_{i,j}|, \quad 1 \le k \le 4, \tag{2}$$

where

$$w_{s,t} = \begin{cases} 2, & (s,t) \in \Omega^3 \\ 1, & \text{otherwise} \end{cases}, \tag{3}$$

$$\Omega^3 = \{(s,t) : -1 \le s,t \le 1\}. \tag{4}$$

We name $d_{i,j}^{(k)}$ as a direction index. Each direction index is sensitive to the edge aligned with a given direction. Then, the minimum of these four direction indexes is used for impulse detection, which can be denoted as

$$r_{i,j} = \min\{d_{i,j}^{(k)} : 1 \le k \le 4\}. \tag{5}$$

Now, we will discuss the value of $r_{i,j}$ in three cases.

1) When the current pixel is a noise-free flat-region pixel, $r_{i,j}$ is small, because of the four small direction indexes.
2) When the current pixel is an edge pixel, $r_{i,j}$ is also small, because at least one of direction indexes is small.
3) When the current pixel is an impulse, $r_{i,j}$ is large, because of the four large direction indexes.

In definition of $r_{i,j}$, we make full use of the information aligned with four directions. So from the above analysis, we can find that by employing a threshold T we can identify the impulse from the noise-free pixels, no matter which are in a flat region, edge, or thin line. Then, we define the impulse detector as

$$y_{i,j} \text{ is a } \begin{cases} \text{noisy pixel}, & \text{if } r_{i,j} > T, \\ \text{noise-free pixel}, & \text{if } r_{i,j} \leq T. \end{cases} \tag{6}$$

III. DWM Filter

After impulse detection, most median-based filters replace the noisy pixels by median values in the window. Here, we will improve the outputs of median filter based on the information of the four directions.

At first, we calculate the standard deviation $\sigma_{i,j}^{(k)}$ of gray-level values for all $y_{i+s,j+t}$ with $(s,t) \in S_k^0$ (k =1 to 4), respectively. Let

$$l_{i,j} = \operatorname*{argmin}_k \{\sigma_{i,j}^{(k)} : k = 1 \text{ to } 4\}, \tag{7}$$

where the operator argmin is to find the minimizer of a function. Since the standard deviation describes how tightly all the values are clustered around the mean in the set of pixels, $l_{i,j}$ shows that the four pixels aligned with this direction are the closest to each other. Therefore, the center value should also be close to them in order to keep the edges (even thin lines) intact. Thus, we assign a weight $\tilde{w}_m$ to these pixels, and restore the noisy pixels as

$$m_{i,j} = \text{median}\{\tilde{w}_{s,t} \diamond y_{i+s,j+t} : (s,t) \in \Omega^3\}, \tag{8}$$

where $\tilde{w}_{s,t} = \begin{cases} \tilde{w}_m, & (s,t) \in S_{l_{i,j}}^0 \\ 1, & \text{otherwise} \end{cases}$, and operator $\diamond$ denotes repetition operation [11]. Taking account of the influence of the impulse noise, we use $\tilde{w}_m = 2$ simply.

Now, with all above denotations we can give the output of the DWM filter as

$$u_{i,j} = \alpha_{i,j} y_{i,j} + (1 - \alpha_{i,j}) m_{i,j}, \tag{9}$$

where

$$\alpha_{i,j} = \begin{cases} 0, & r_{i,j} > T \\ 1, & r_{i,j} \leq T \end{cases}. \tag{10}$$

For ensuring high accuracy of detection, we apply our method recursively and iteratively with decreasing threshold. At the early iterations, with a large threshold impulse detector only identifies pixels that are most likely to be noisy. In the subsequent iterations, we decrease the threshold to include more noise. From the simulations conducted on a broad variety of images, it has been observed that for 8-bit gray-level images the following selection of the threshold always yields satisfactory results, that is,

$$T_0 = 510, \qquad \text{and} \qquad T_{n+1} = T_n \cdot 0.8 \quad (n \geq 0), \tag{11}$$

where T_0 is the initial threshold, and T_n is the threshold in the nst step. In generally, $N_{\max}$, the maximum number of iterations, is in $[5, 10]$.

TABLE I

COMPARISON OF RESTORATION RESULTS IN PSNR (DB)

Method	"Lena" image				
	20%	30%	40%	50%	60%
Med Filter [6]	32.37	30.00	27.64	24.28	21.58
SD-ROM [7]	35.72	32.77	29.85	26.80	23.41
PSM Filter [13]	35.09	30.85	28.92	26.12	22.06
MSM Filter [1]	35.44	31.67	29.26	26.11	22.14
ACWM Filter [8]	36.07	32.59	28.79	25.19	21.19
PWMAD Filter [9]	36.50	33.44	31.41	28.50	24.30
Iterative Median Filter [10]	36.90	31.76	30.25	24.76	22.96
DWM Filter	**37.15**	**34.87**	**32.62**	**30.26**	**26.74**

IV. SIMULATION

In this section, we compare the DWM filter with a number of existing median-based filters for removal of random-valued impulse noise.

A. *Comparison of Image Restoration*

Restoration results are quantitatively measured by peak signal-to-noise ratio (PSNR), which is defined as in [12]. Table I shows the PSNR values of the best results obtained by different filters, where the 512-by-512 image "Lena" corrupted with different noise ratios are used. In the DWM filter, we use $N_{\max} = 8, 9, 9, 10, 10$, respectively.

It can be seen that our proposed filter provides the best results in PSNR. In particular, when the noise ratio is larger than 30%, the DWM filter produces PSNR values full one decibels higher than the closest competing filter. In order to compare the results subjectively, we give some restored images in Fig. 2. It is obvious that the DWM filter performs better. There are still a lot of noise patches in the images restored by the others, but by the DWM filter, noticeable noise is much less.

We also compare the performance to restore the images with full features, "bridge" and "boat" as example. The restoration results in PSNR are listed in Table II. For the DWM filter, $N_{\max} = 7, 8, 10, 8, 9, 10$, respectively. From Table II, the DWM filter still provides better results than the others. Fig. 3 gives enlarged areas of image "boat" to show how our filter performs on image details. It is easy

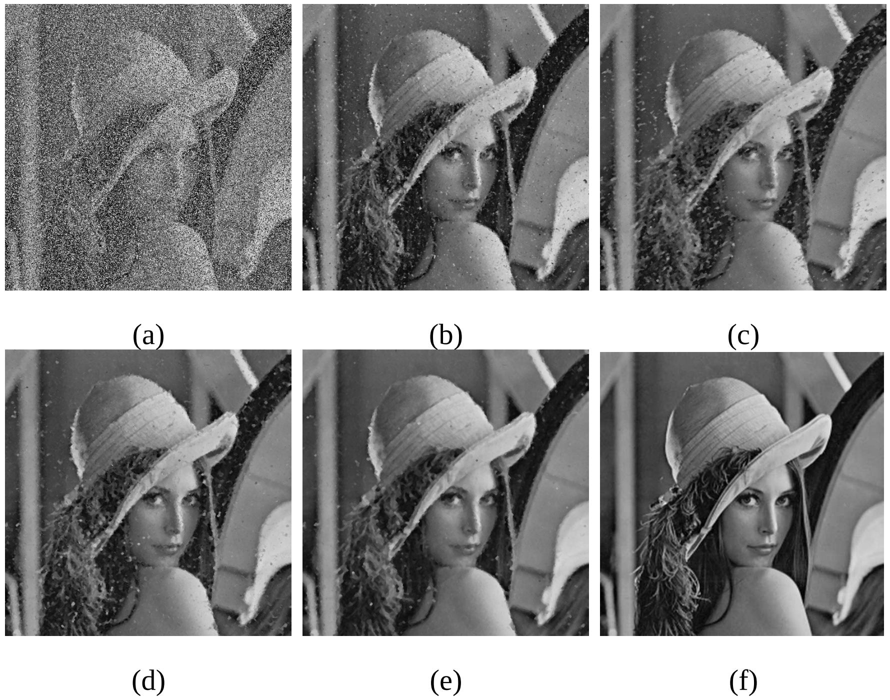

Fig. 2. Results of different filters in restoring 60% corrupted image “Lena”: (a) the noisy image, (b) the SD-ROM filter, (c) the MSM filter, (d) the PWMAD filter, (e) the DWM filter, (f) original image.

to see that excellent restoration results are obtained by the DWM filter. It can remove most noise while preserving details very well, even thin lines.

B. Comparison of Noise Detection

For good performance, the capability of noise detection is very important. Here, we lists the number of missed noisy pixels (“miss” term) and the number of noise-free pixels which are identified as noise (“false” term) in Table III.

Since some of the random-valued impulse noise values are not so different from their neighbors as in salt-and-pepper noise [12], there may be much more noise-free pixels detected as noise when detecting random-valued impulse noise. In Table III, although some methods, such as the SD-ROM and ACWM filter, produce less “false” values than the DWM filter, there are too many noisy pixels unidentified, and just these pixels lead to the presence of noticeable noise patches. Comparing with the others, the DWM filter can distinguish more noise pixels with relative fewer mistakes, even when the noise level is as high as 60%. Furthermore, if we compute the sum of the “miss” term and the “false” term, the DWM filter has the smallest value among all methods.

TABLE II

COMPARISON OF RESTORATION RESULTS IN PSNR (DB)

Method	"bridge" image			"boat" image		
	40%	50%	60%	40%	50%	60%
SD-ROM Filter	23.80	22.42	20.66	26.45	24.83	22.59
MSM Filter	23.55	22.03	20.07	25.56	24.27	22.21
ACWM Filter	23.23	21.32	19.27	26.17	23.92	21.37
PWMAD Filter	23.83	22.20	20.83	26.56	24.85	22.32
DWM Filter	**24.28**	**23.04**	**21.56**	**27.03**	**25.75**	**24.01**

(a) (b) (c) (d)

Fig. 3. Enlarged areas of different filters in restoring 40% corrupted image "boat": (a) the SD-ROM filter (with training), (b) the MSM filter, (c) the DWM filter, (d) original image.

C. Application to the Color Images

Like the BDND filter [5], the DWM filter also can be extended to remove random-valued impulse noise for color images. In impulse detector, we will treat each color component as an independent entity. After calculating the direction indexes corresponding to the three color channels, we add up them, and find the minimum value as $r_{i,j}$, like in (5). Then, impulse detector can be defined in the same way as in (6). In the noise filtering, the vector median filter [14] with the 3×3 window will be used to restore the noisy pixels identified.

Fig. 4 shows the results in restoring 50% and 60% corrupted image "Lena" by the vector median filter and DWM filter. It is easy to see that the image restored by the vector median filter is very blurry, and still includes many noticeable noise patches, especially for the result in restoring the image with 60% noise level. In contrast, the DWM filter performs much better, which can suppress the noise successfully

TABLE III

COMPARISON OF DETECTION RESULTS FOR THE IMAGE "LENA" CORRUPTED BY RANDOM-VALUED IMPULSE NOISE

Method	40%		50%		60%	
	miss	false	miss	false	miss	false
SD-ROM Filter	22842	411	32566	998	45365	2651
MSM Filter	16582	7258	20857	10288	26169	15778
ACWM Filter	16052	1759	23683	2895	32712	7644
PWMAD Filter	11817	9928	14490	15003	17760	19577
DWM Filter	9512	7761	9514	11373	12676	12351

while preserving more details.

V. CONCLUSION

In this paper, we propose a new median-based filter, the DWM filter, for removing random-valued impulse noise. It makes full use of the characteristics of impulse and edges to detect and restore noise. Simulation results show that the DWM filter performs much better than many existing median-based filters in both subjective and objective (PSNR) evaluations. Especially, it can preserve edges very well, even thin line, as removing noise. In addition, it can be extended to restore the color images corrupted by random-valued impulse noise, and also performs well. Here, we only consider the four main directions. Then, it is easy to improve it while having more information about image details.

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(a) (b)

(c) (d)

Fig. 4. Results in restoring 50% (first row) and 60% (second row) corrupted color image “Lena”: (a) and (c) the vector median filter, (b) and (d) the DWM filter (with $T_0 = 1275, N_{\max} = 10$).

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