EEE342 LAB 3:
Phase, Gain and Delay Margin Measurements of the DC Motor

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1. Introduction

In mechanical and electrical system design stability is one of the most crucial issues to be addressed. Stability of a system can be determined by using different algorithms like the Routh-Hurwitz Criterion, Root-Locus Method or the Nyquist plots. An example Nyquist plot can be seen in Fig.1

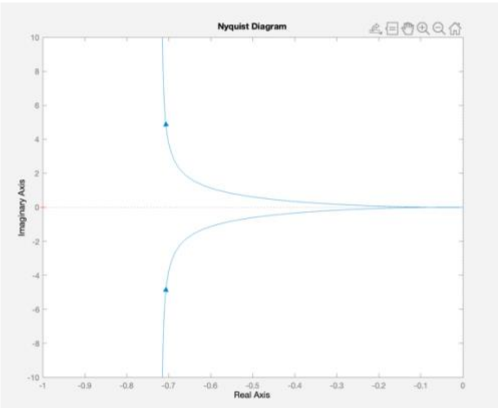

Fig. 1: Nyquist plot of the described closed-loop control system which is $G = G_c \cdot G_p$

The given figure describes the steady state frequency response of the DC motor and the controller combination which can be mathematically written as the following.

$$G_p = \frac{0.1481 \cdot s + 11.85}{s^2 + 28.5 \cdot s}$$

$$G_c = \frac{128.2}{s + 9.5}$$

$$G = G_c \cdot G_p = \frac{19 \cdot s + 1520}{s^3 + 38 \cdot s^2 + 270.8 \cdot s}$$

The relationship between the Nyquist plot and the stability can be determined by using a law called Nyquist Stability Criterion. According to the criterion the following must be satisfied to have a stable system:

$$Z = N + P$$

N denotes the number of cycles of the plot around the point -1 clockwise which is 0 in this example since the Nyquist plot extends to infinity gain around point 0.7 with frequency w = 0 rad/s. Therefore, the plot does not cycle around the point -1. The variable P represents the open loop poles that has $Re\{s\} > 0$, which is again 0 for this system. The final variable Z represents the open loop zeros that are also the close loop poles with $Re\{s\} > 0$ which is the sum of N and P which are both 0. Therefore, the system is stable for the given design parameters.

It is also important for a system to be analyzed by their tolerance to parameter changes like a constant gain K. The concepts gain margin, phase margin and delay margin are all related concepts to this property of the systems.

A) Gain Margin (GM)

Gain margin is the measure of how much can the gain of the system is available to change to still be kept in the boundaries of stability conditions. Gain margin can be found both from the Nyquist Plot and the Bode Plot of the system. In the introduction part the Nyquist Plot technique will be addressed, and the Bode Plot technique will be further described in the second chapter of the report. For the Nyquist Plot technique, the gain margin (GM) is the amount of K that stretches the plot so that a cycle around -1 occurs. This can be seen in Fig.2

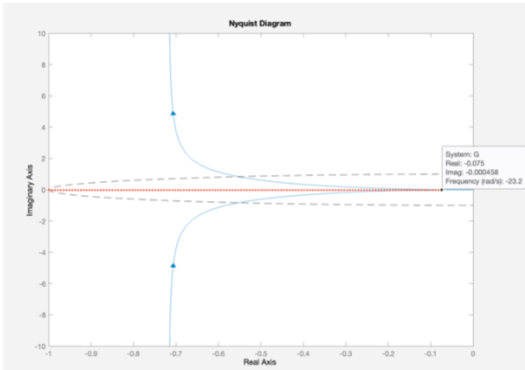

Fig. 2: GM of the system that requires the amount of K to stretch the real axis crossing point to -1 to create a full cycle.

B) Phase Margin (PM)

Phase margin is the measure of how much can the phase of the system is tolerant to change to still be kept in the boundaries of stability conditions. Phase margin can be found both from the Nyquist Plot and the Bode Plot of the system. In the introduction part the Nyquist Plot technique will be addressed, and the Bode Plot technique will be further described in the second chapter of the report. For the Nyquist Plot technique, the phase margin (PM) is the amount of rotation that moves the point that the Nyquist plot crosses the unit circle so that a cycle around -1 occurs. This can be seen in Fig.2

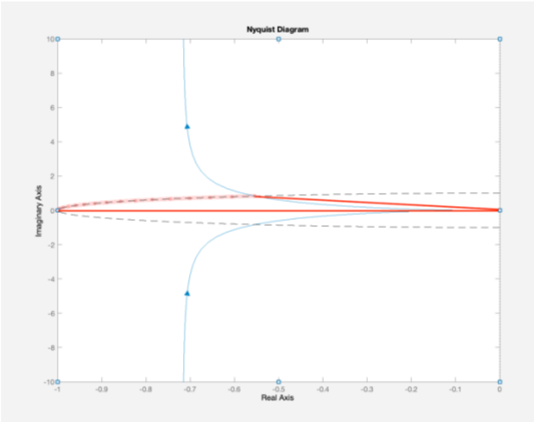

Fig. 2: PM of the system that requires the amount of rotation to move the unit circle crossing to point -1 to create a full cycle.

C) Delay Margin (PM)

Delay introduces a linear phase to the system and can be modelled with the following:

$$D(T) = e^{-sT}$$

With this delay component the new phase margin becomes the following:

$$\Phi_{PM} - \omega_x \cdot T$$

Therefore, there is a limit in the amount of delay that the system can be kept stable. This value can be found by calculating the point where the phase margin is zero.

$$\pi - \Phi_{PM} - \omega_x \cdot T = \pi$$

$$T_{DM} = \frac{\Phi_{PM}}{\omega_x}$$

In the next chapter the experiment procedure will be described.

2. Laboratory Content

Part 1)

At the first part of the laboratory a controller has been designed for the parameters of the approximated transfer function of the DC motor found in Lab 1. The approximation of the DC motor is the following:

$$G_p(s) = \frac{\frac{128.25}{9.5}}{s + \frac{1}{9.5}} = \frac{13.5}{s + 0.105}$$

From this result the values of the parameters K_g, K_c, τ_P and τ_{LPF} has been found and the controller was designed by following the lab guideline.

$$G_c(s) = \frac{0.1481s + 11.85}{s^2 + 28.5s}$$

The final system becomes the combination of these two transfer functions:

$$G = G_c \cdot G_p = \frac{19s + 1520}{s^3 + 38s^2 + 270.8s}$$

The bode plot of this transfer function has been plotted by using the ‘bode’ function of the MATLAB. The corresponding plots for gain and phase can be seen in Fig.3.

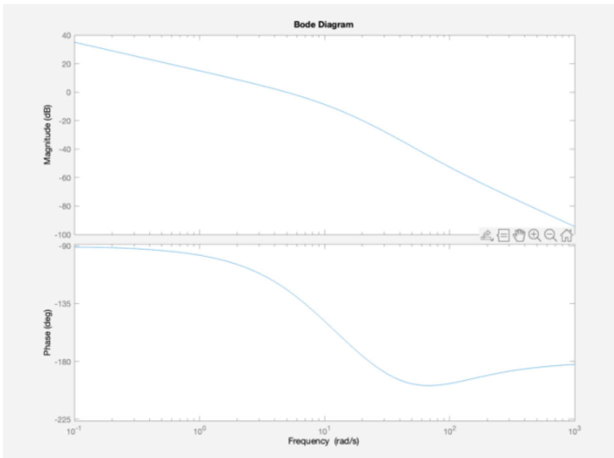

Fig. 3: Bode plot of the plant-controller system G.

Phase margin, gain margin and delay margin of this system can be found by the following procedure.

A) Gain Margin (GM) of the System G

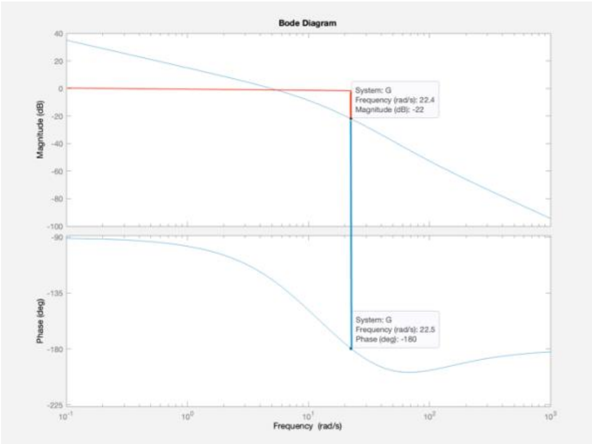

Fig. 4: Gain Margin (GM) of the system

From Fig.4 the gain margin can be determined by finding the difference between the 0 dB line and the magnitude of the frequency response at point where the phase response is $-180°$. By using this method, the gain margin has found approximately 22dB or 12.59 as a constant gain multiplier.

B) Phase Margin (PM) of the System G

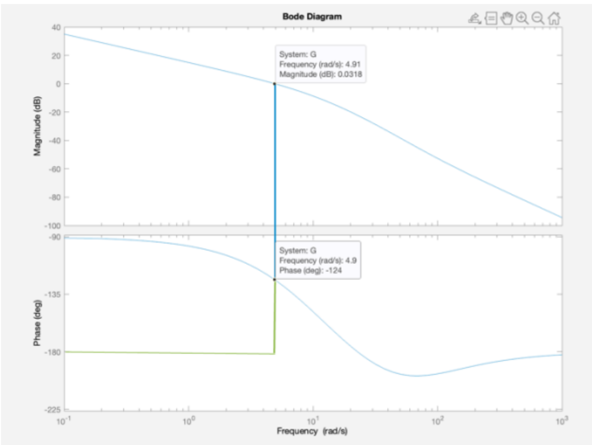

Fig. 5: Phase Margin (PM) of the system

From Fig.5 the gain margin can be determined by finding the difference between the $-180°$ line and the phase of the frequency response at point where the magnitude response is 0 dB By using this method the phase margin has found approximately $56°$ or 0.98 radians.

C) Delay Margin (DM) of the System G

From Fig.5 the delay margin can be determined by dividing the phase margin to the frequency value of the point where the phase margin is obtained. From this description delay margin can be found with the following calculations:

$$DM = \frac{0.98}{4.9} = 0.2\ sec.$$

The found numbers can be compared to the results of 'allmargin()' function of the MATLAB. The results of the function can be seen in Fig.6

```
ans =

  struct with fields:

      GainMargin: [12.8934 Inf]
     GMFrequency: [22.7097 Inf]
     PhaseMargin: 56.3368
     PMFrequency: 4.9214
     DelayMargin: 0.1998
     DMFrequency: 4.9214
          Stable: 1
```

Fig. 6: Results of the 'allmargin()' function of MATLAB to the denoted system.

From Fig.6, the values are mostly determined correctly and therefore the MATLAB function has been satisfied the found simulation results.

Part 2)

In the first stage of the second part of the lab the gain margin has been manually tested by changing the parameter K until number of oscillations begin to increase through the end of the output signal. The Figures 7-9 shows the values that has been determined to be satisfy slightly less than the gain margin, nearly the gain margin and slightly more than the gain margin can be seen respectively.

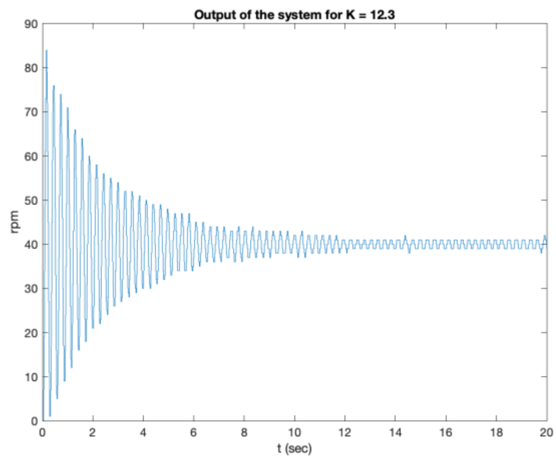

Fig. 7: Output of the system for r(t) = 40u(t) and $K_1 = 12.3$

In Figure 7, we observe that the system's oscillations have not yet initiated, which points to the presence of sustained stability. For K_1, a value of 12.3 could be an optimal choice since it's slightly below the gain margin, thus contributing to the system's overall stability.

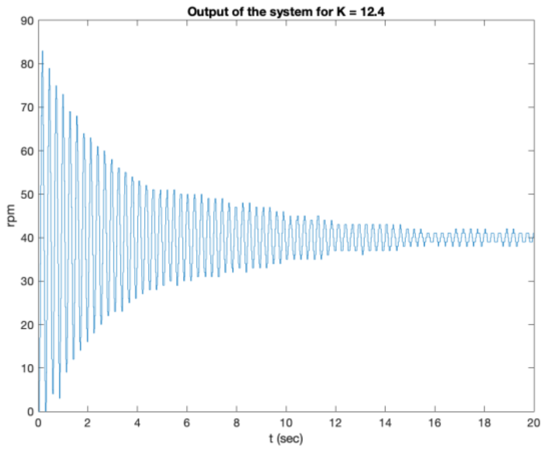

Fig. 8: Output of the system for r(t) = 40u(t) and $K_2 = 12.4$

As illustrated in Figure 8, the output signal starts to display oscillations over time, signaling a state of marginal stability. A gain value of $K_f = 12.4$ can be selected, which coincides with the gain margin. Despite this, a minor discrepancy exists with the theoretical GM value of 12.89. Consequently, a slight error is observed between the theoretical predictions and our experimental outcomes.

$$e_{GM} = \frac{0.49}{12.89} = \%3.8$$

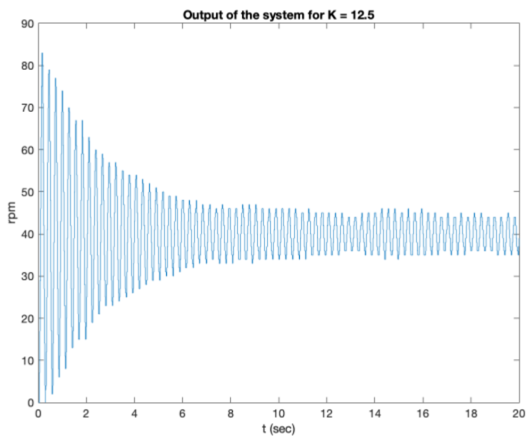

Fig. 9: Output of the system for r(t) = 40u(t) and K = 12.5

Figure 9 depicts an increase in oscillations, suggesting a considerable degree of instability and implying that the system has entered the instability region. We could opt for a gain value of $K_2 = 12.5$, which is marginally higher than the gain margin.

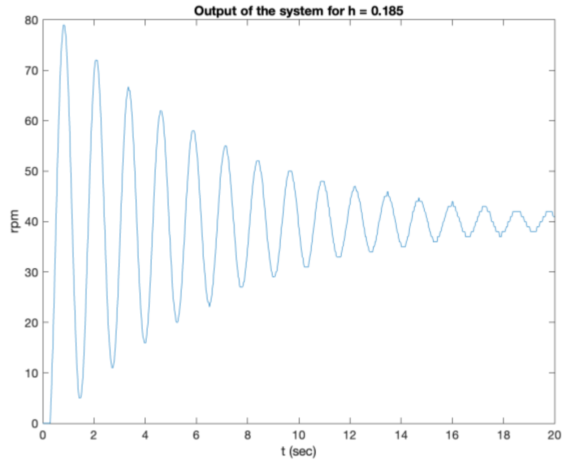

Fig. 10: Output of the system for r(t) = 40u(t) and $h = 0.185$

In Figure 10, we observe that the oscillations have yet to begin, indicating sustained stability in the system. We could consider a delay parameter, h, of 0.185, which is marginally less than the delay margin.

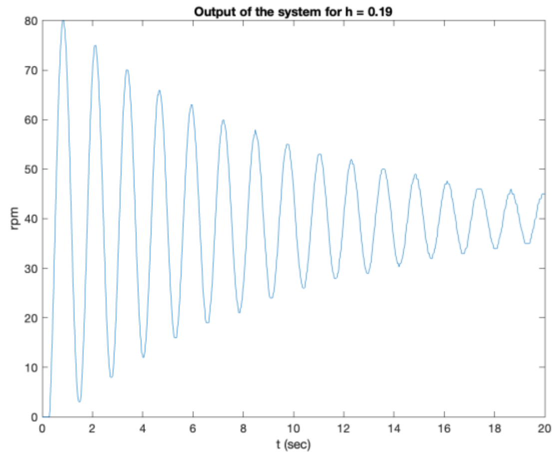

Fig. 11: Output of the system for r(t) = 40u(t) and $h = 0.19$

As depicted in Figure 11, we see the emergence of oscillations over time in the output signal, suggesting a condition of marginal stability. We might consider choosing a time delay value, h, of 0.19, which corresponds to the delay margin. Despite this, a slight discrepancy is noted with the theoretically computed GM value of 0.1998, pointing to a minor variance between theoretical expectations and experimental outcomes.

$$e_{DM} = \frac{0.0098}{0.1998} = \%4.9$$

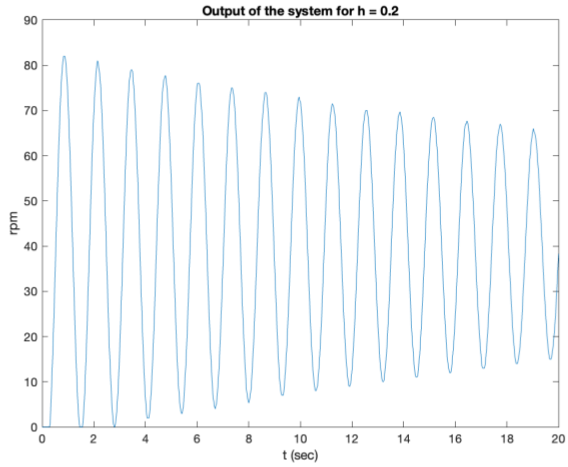

Fig. 12: Output of the system for r(t) = 40u(t) and $h = 0.2$

Figure 12 reveals an amplification in oscillations, indicative of pronounced instability and suggesting that the system has transitioned into the instability region. A time delay value of h=0.2 could be an appropriate choice, given it exceeds the delay margin slightly.

3. Conclusion

The primary objective of this laboratory exercise was to gain an understanding of key control system concepts, including gain margin, phase margin, and delay margin. We first approached this goal through theoretical analysis using Bode plots, akin to the methods typically utilized in examinations or quizzes, followed by experimental validation via parameter search.

This lab exercise underscored the importance of phase margin, gain margin, and delay margin on the stability dynamics of a control system. Notably, alterations in these parameters led to less stable step responses, further emphasizing their critical role in system stability.

However, several potential sources of error were identified during this laboratory exercise. Firstly, MATLAB's sampling precision for Bode plots introduced inaccuracies due to the discrete nature of data points. As a result, the exact locations where the gain equals zero or the phase equals -180 degrees could not be pinpointed, consequently introducing some margin of error in our calculations.

Moreover, computational limitations, such as the finite bit capacity, lead to number quantization. This implies that we cannot access all numerical values continuously, introducing another potential source of error.

Overall, this laboratory exercise proved to be highly beneficial in visualizing and evaluating the impacts of gain margin, phase margin, and delay margin on control system behavior. These hands-on experiences have undoubtedly deepened our understanding of these critical concepts.

4. MATLAB Code

```
%% PART 1:
K = 128.25/9.5;
tau_p = 1/9.5;
K_c = 2/K;
tau_lpf = 3/tau_p;
A = tf([1],[1 tau_lpf]);
B = tf([K_c K_c*80],[1 0]);
G_c= A*B;
G_p = tf([128.25],[1 9.5]);
G = G_c*G_p;
bode(G);
%PM = 180-124 = 56,w_PM = 4.95
%GM = 22, w_GM = 22.7
%DM = PM/w_PM = pi*56/(4.95*180) = 0.198
allmargin(G)
%%
nyquist(G)
hold on;

% Plot the unit circle
theta = linspace(0, 2*pi, 100);
unit_circle = exp(1i*theta);
plot(real(unit_circle), imag(unit_circle), 'k--');
hold off;
%% PART 2:
% Gain margins
figure(1)
K1 = load('K1.mat','out');
plot(K1.out.simout);
title('Output of the system for K = 12.3');
xlabel('t (sec)');
ylabel('rpm');

figure(2)
K2 = load('K2.mat','out');
plot(K2.out.simout);
title('Output of the system for K = 12.5');
xlabel('t (sec)');
ylabel('rpm');


figure(3)
Kf = load('Kf.mat','out');
```

```
plot(Kf.out.simout);
title('Output of the system for K = 12.4');
xlabel('t (sec)');
ylabel('rpm');

%% PART 3:
% Phase margins
figure(4)
h1 = load('h1.mat','out');
plot(h1.out.simout);
title('Output of the system for h = 0.185');
xlabel('t (sec)');
ylabel('rpm');

figure(5)
h2 = load('h2.mat','out');
plot(h2.out.simout);
title('Output of the system for h = 0.2');
xlabel('t (sec)');
ylabel('rpm');


figure(6)
hf = load('hf.mat','out');
plot(hf.out.simout);
title('Output of the system for h = 0.19');
xlabel('t (sec)');
ylabel('rpm');
```