EEE342 LAB 1:
Frequency Domain Analysis of an Electric Motor

Efe Tarhan

Department of Electrical and Electronics Engineering, Bilkent University, 06800 Ankara, Turkey

1. Introduction

Mechanical and electrical systems have transfer functions in s-domain that give information about the systems properties and their response to impulse responses.
As studied earlier some the BIBO stability analysis of a system can be done by checking if the integral of the transfer function of a filter is bounded. This stable behavior appears on systems when the system has no poles at the right side of the s-domain and therefore the impulse response converges to zero as t goes to infinity.

$$h_{ss} = \int_0^\infty h(\tau)d\tau < \infty$$

If the transfer function of a system is stable the sinusoidal steady state can be found by inserting $j\omega$ to the transfer function and analyze the system by the amplitude and phase change that it applies to different frequencies during the steady state period of its response.

$$H(j\omega) = H(s)|\ s = j\omega$$

The same result can also be achieved by using the eigenfunction properties of complex exponentials for LTI systems as the following:

$$x(t) = e^{j\omega t}$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) \cdot e^{j\omega(t-\tau)}d\tau$$

$$= e^{j\omega t} \cdot \int_{-\infty}^{\infty} h(\tau) \cdot e^{-j\omega\tau}d\tau = e^{j\omega t} \cdot H(j\omega)$$

This result shows that the complex exponentials and therefore sinusoidals are eigenfunctions of LTI systems with real impulse responses where the system introduces a phase and a magnitude effect to the eigenfunction which will be helpful for analyzing the frequency response of the filter and can be found discretely by applying sinusoidals with different frequencies.

$$H(j\omega) = |H(j\omega)| \cdot e^{j\phi(H(j\omega))}$$

2. Laboratory Content

Part 1)

In the first lab, the transfer function of the motor system was found as the following:

$$H(s) = \frac{128.25}{s + 9.5}$$

Since the system is stable with a pole at s = -9.5 the sinusoidal steady state response can be found by the following calculations:

$$H(j\omega) = \frac{128.25}{j\omega + 9.5}$$

Then the magnitude and phase plots can be found by calculating the following with MATLAB:

$$|H(j\omega)| = \frac{128.25}{\sqrt{\omega^2 + 9.5^2}}$$

That gives the magnitude plot that can be seen in Fig.1

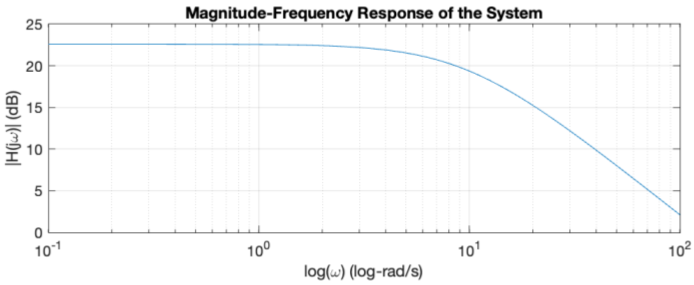

Fig. 1: Magnitude – Frequency response plot of the motor system where the logarithmic scale is used for both x and y axes.

The phase plot can be found by the following calculations:

$$\phi(j\omega) = \arctan\left(\frac{Im\{H(j\omega)\}}{Re\{H(j\omega)\}}\right)$$

That gives the phase plot of the system which can be seen in Fig.2

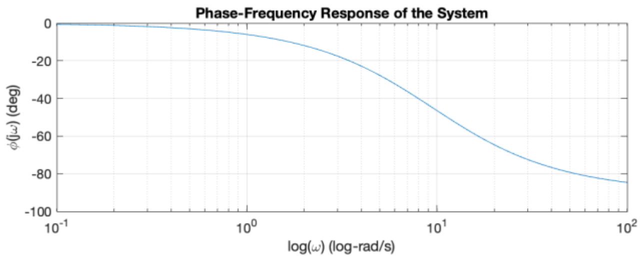

Fig. 2: Phase – Frequency response plot of the motor system where the logarithmic scale is used for x axis and degrees is the unit for y axis.

From the figures, the system acts like a low pass filter for the magnitude response and creates the most phase difference at higher frequencies. These are tested in the lab environment with the hardware set up and results will be shared in the next part.

Part 2)

In this part the magnitude and the phase responses of the system will be found by applying discrete frequency sinusoidal signals and observing the output. The applied frequencies and their durations can be seen in Table.1.

Angular Frequency $\left(\frac{rad}{s}\right)$	Simulation Duration (s)
0.1	70
0.3	70
1	25
3	25
10	10
30	10
100	10

Table. 1: Sinusoidal Frequency – Simulation Duration table that are being used for hardware implementation.

a. $\underline{\omega = 0.1:}$

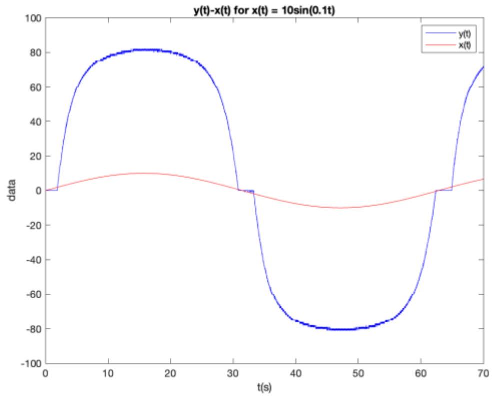

Fig. 3: Plot of y(t) and x(t) for x(t) = $10\sin(0.1t)$

The plot of the input output relationship of the signal can be seen in Fig.3 for $\omega = 0.1\ rad/s$. As it can be seen the magnitude response is proper for a low frequency signal which has been amplified accordingly. Besides there is no apparent phase shift. The magnitude and phase responses has been found by following the steps described in the introduction. The frequency components of both the input and the output signal are calculated with FFT.

$$|H(j0.1)| = 19.47\ dB$$

$$\phi(0.1j) = -2.425\,°$$

b. $\underline{\omega = 0.3:}$

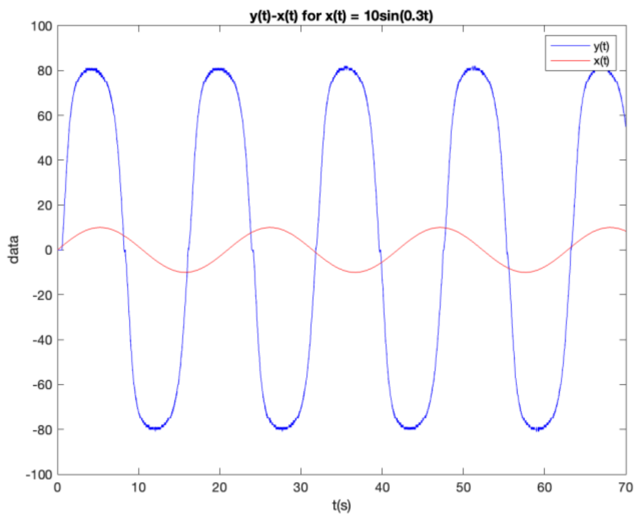

Fig. 4: Plot of y(t) and x(t) for x(t) = $10\sin(0.3t)$

The plot of the input output relationship of the signal can be seen in Fig.4 for $\omega = 0.3\ rad/s$. As it can be seen the magnitude response is proper for a low frequency signal which has been amplified accordingly.
Besides there is a slight appearance of a phase shift which is expected since the phase changes rapidly compared to magnitude. But there is also an anomaly that the phase difference becomes positive and increased rather than the

expected decrease, this might be due to random errors or uncertainties occurred during measurement.
The magnitude and phase responses has been found by following the steps described in the introduction. The frequency components of both the input and the output signal are calculated with FFT.

$$|H(j0.3)| = 18.01\ dB$$

$$\phi(j0.3) = 10.58°$$

c. $\underline{\omega = 1:}$

The plot of the input output relationship of the signal can be seen in Fig.5 for $\omega = 1\ rad/s$. As it can be seen the magnitude response is proper for a low frequency signal which has been amplified accordingly.
There is some slight phase change occurred during measurement as expected since the test frequency of the input increases.

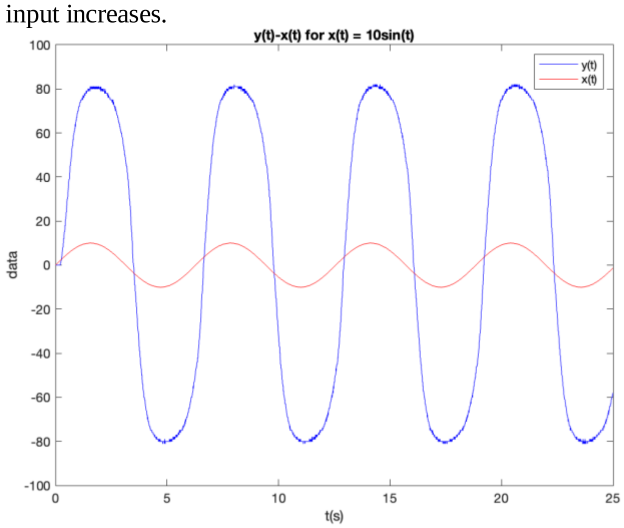

Fig. 5: Plot of y(t) and x(t) for x(t) = $10\sin(t)$

The magnitude and phase responses has been found by following the steps described in the introduction. The frequency components of both the input and the output signal are calculated with FFT.

$$|H(j1)| = 19.25\ dB$$

$$\phi(1j) = -18.96°$$

d. $\underline{\omega = 3:}$

The plot of the input output relationship of the signal can be seen in Fig.6 for $\omega = 3\ rad/s$. As it can be seen the magnitude response is proper for a low frequency signal which has been amplified accordingly.
There is again some slight phase change occurred during measurement as expected since the test frequency of the input increases.

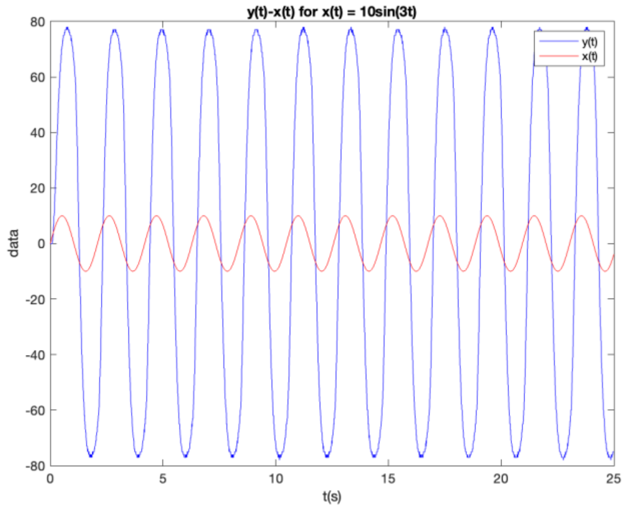

Fig. 6: Plot of y(t) and x(t) for x(t) = $10\sin(3t)$

The magnitude and phase responses has been found by following the steps described in the introduction. The frequency components of both the input and the output signal are calculated with FFT.

$$|H(j3)| = 18.65\ dB$$

$$\phi(j3) = -40.53°$$

e. $\underline{\omega = 10:}$

The plot of the input output relationship of the signal can be seen in Fig.7 for $\omega = 10\ rad/s$. As it can be seen the magnitude response has been decreased since the signal frequency shifts to intermediate frequencies.
Now the phase change becomes visually and easily detectable during measurement as expected since the test frequency of the input increases. But the difference of the filter response for this graph is that there is also a noticeable transient response since the input frequency increases.

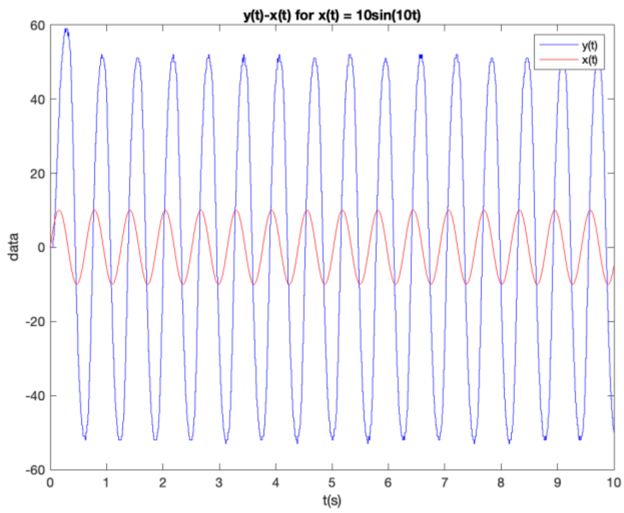

Fig. 7: Plot of y(t) and x(t) for x(t) = $10\sin(10t)$

The magnitude and phase responses has been found by following the steps described in the introduction. The frequency components of both the input and the output signal are calculated with FFT.

$$|H(j10)| = 14.67\ dB$$

$$\phi(j10) = -70.63°$$

f. $\underline{\omega = 30:}$

The plot of the input output relationship of the signal can be seen in Fig.8 for $\omega = 30\ rad/s$. As it can be seen the magnitude response has been significantly decreased since the input signal frequency shifts to high frequency components.
The phase change is visually and easily detectable during measurement as expected since the test frequency of the input is at the high frequency spectrum of possible inputs. Also, the first and ongoing sinusoidals have a modulation or oscillating amplitude behavior since the peaks reach different amplitude levels.

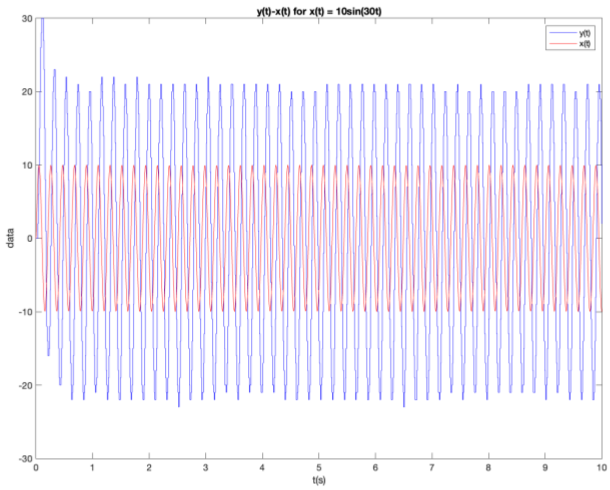

Fig. 8: Plot of y(t) and x(t) for x(t) = $10\sin(30t)$

The magnitude and phase responses has been found by following the steps described in the introduction. The frequency components of both the input and the output signal are calculated with FFT.

$$|H(j30)| = 6.75\ dB$$

$$\phi(j30) = -98.16°$$

g. $\underline{\omega = 100:}$

The plot of the input output relationship of the signal can be seen in Fig.9 for $\omega = 100\ rad/s$. As it can be seen the magnitude response has been significantly decreased below of the amplitude level of input. This is due to the lowpass behavior of the system.
The phase change nearly becomes -180 degree as it can be seen from the graph since the red output data fit between the blue data points. Also, both the detected input signal and the measured output signal distorted due to physical limitations of the system since what we have used is a first order approximation.

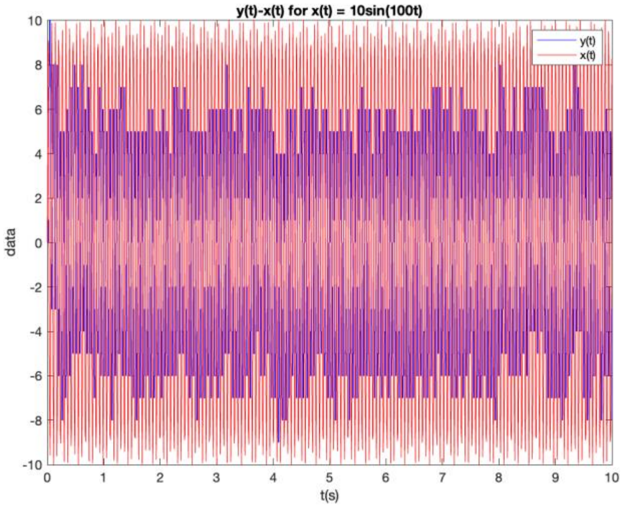

Fig. 9: Plot of y(t) and x(t) for x(t) = $10\sin(100t)$

The magnitude and phase responses has been found by following the steps described in the introduction. The frequency components of both the input and the output signal are calculated with FFT.

$$|H(j100)| = 0.6159\ dB$$

$$\phi(j100) = -148.31°$$

All these data can be plotted on top of the theoretical and approximated frequency response graphs in Fig.1 and Fig.2 to inspect the relevance of the experimental and theoretical data. The 7 frequency points obtained experimentally can be seen in Fig.10 on top of the previously defined theoretical frequency responses:

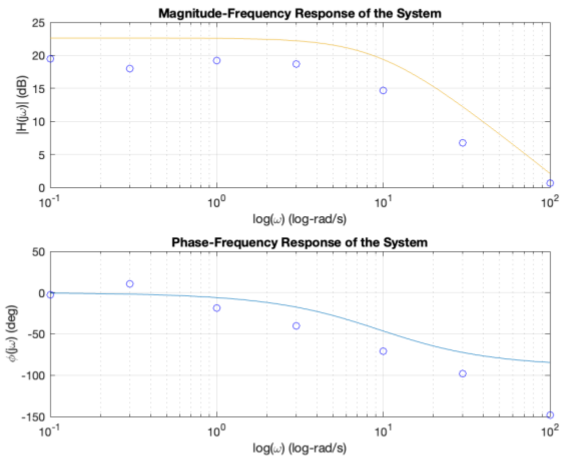

Fig. 10: Magnitude – Frequency and Phase – Frequency response plot of the motor system where the logarithmic scale is used for both x and y axes.

Magnitude of the frequency response is correlated with the theoretical results since it follows the plot from a little below amplification levels. This might be due to the physical properties or disturbances that affect the motor system.
On the other hand, the phase response exhibits a strange behavior at the latest data point where the frequency is equal to 100 rad/s. Rather than following the plot the phase response at that frequency has decreased significantly that the difference between the theoretical and experimental results become large.

This problem is because the system has 10ms delay when processing the input and therefore it appears mostly at the high frequencies as a large phase shift. This problem will be investigated at the next part of the lab report.

Part 3)

In the lab manual it has been told that the system has a delay of 10 ms that affect the general phase response. This causes the difference between theoretical and experimental results at high frequencies. To overcome this issue the theoretical G(s) was multiplied with a Pade delay approximation and became the following:

$$G_{delayed}(s) = G(s) \cdot \frac{1 - 0.005s}{1 + 0.005s}$$

This approximation doesn't change the magnitude of the frequency response of the filter since the magnitude of the approximator is one but changes the phase response of the filter. After the approximation both three plots on top of each other can be examined in Fig.11.

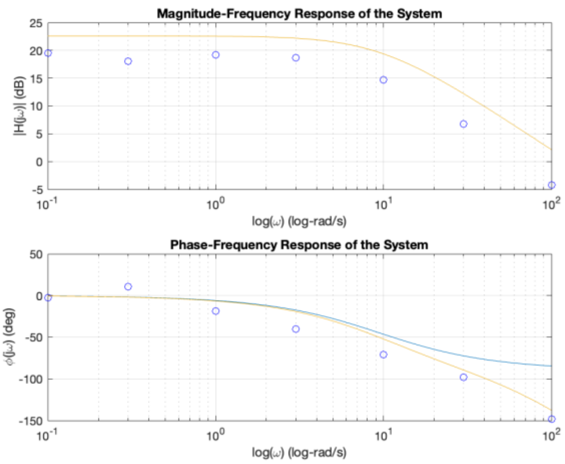

Fig. 11: Magnitude – Frequency and Phase – Frequency response plot of the motor system where the logarithmic scale is used for both x and y axes.

After the change applied to the theoretical data, it fits to the experimental results and therefore the reason of the error between the theoretical and experimental data are proved to be due to lack of knowledge about the system itself.

3. Conclusion

Aim of this lab is understanding and modelling the frequency response of a motor by inputting sinusoidals with various frequencies. After obtaining experimental results for several discrete input signals, theoretical and experimental results are compared.

Because of uncertainties and physical loss embedded inside the system, the magnitude responses are lower than expected and the phase shifts are larger than expected since the additional features of the system cause more error. Also the delay becomes the unknown property of the system that has been introduced in part 3 and the theoretical results are updated accordingly.

This lab was a good chance for understanding how a frequency response plot can be obtained experimentally and what are possible outcomes like the severe distortions on the plot with the input signal that has 100 rad/s frequency. It shows that real world implementations don't always satisfy theoretical results even though they are realistic.

4. MATLAB Code

```
%% Q.1
figure(1)
w = logspace(-1,2,100);
for k = 1:100
s = 1i * w(k);
G(k) = 128.25 / (s+9.5);
end
subplot(2,1,1)
semilogx(w,20*log10(abs(G)));
title('Magnitude-Frequency Response of the
System')
xlabel('log(\omega) (log-rad/s)')
ylabel('|H({j\omega})| (dB)')
grid on
subplot(2,1,2)
semilogx(w,angle(G)*180/pi)
title('Phase-Frequency Response of the System')
xlabel('log(\omega) (log-rad/s)')
ylabel('\phi({j\omega}) (deg)')
grid on
```

```
%% Q.2
y1 = load('0.1rad.mat');
y2 = load('0.3rad.mat');
y3 = load('1rad.mat');
y4 = load('3rad.mat');
y5 = load('10rad.mat');
y6 = load('30rad.mat');
y7 = load('100rad.mat');

%%
figure(2)
plot(y1.a,'b');
duration = 70;
time = 0:0.01:duration;
x1 = 10*sin(0.1.*time);
hold on
plot(time,x1,'r');
title('y(t)-x(t) for x(t) = 10sin(0.1t)')
xlabel('t(s)')
legend('y(t)','x(t)')
magnitudes(1:7) = 0;
phases(1:7) = 0;

[y1fft,indy1] = max(abs(fft(y1.a.data)));
[x1fft,indx1] = max(abs(fft(x1)));
K1 = y1fft/x1fft;

y1angle = angle(fft(y1.a.data));
x1angle = angle(fft(x1));

phi1 = y1angle(indy1)-x1angle(indx1);

magnitudes(1) = K1;
phases(1) = phi1;
%%
figure(3)
plot(y2.a,'b');
duration = 70;
time = 0:0.01:duration;
x2 = 10*sin(0.3.*time);
hold on
plot(time,x2,'r');
title('y(t)-x(t) for x(t) = 10sin(0.3t)')
xlabel('t(s)')
legend('y(t)','x(t)')
[y2fft,indy2] = max(abs(fft(y2.a.data)));
[x2fft,indx2] = max(abs(fft(x2)));
K2 = y2fft/x2fft;

y2angle = angle(fft(y2.a.data));
x2angle = angle(fft(x2));

phi2 = y2angle(indy2)-x2angle(indx2);

magnitudes(2) = K2;
phases(2) = phi2;

%%
figure(4)
plot(y3.a,'b');
duration = 25;
time = 0:0.01:duration;
x3 = 10*sin(time);
hold on
plot(time,x3,'r');
title('y(t)-x(t) for x(t) = 10sin(t)')
xlabel('t(s)')
legend('y(t)','x(t)')
[y3fft,indy3] = max(abs(fft(y3.a.data)));
[x3fft,indx3] = max(abs(fft(x3)));
K3 = y3fft/x3fft;
```

```
y3angle = angle(fft(y3.a.data));
x3angle = angle(fft(x3));

phi3 = y3angle(indy3)-x3angle(indx3);

magnitudes(3) = K3;
phases(3) = phi3;
%%

figure(5)
plot(y4.a,'b');
duration = 25;
time = 0:0.01:duration;
x4 = 10*sin(3.*time);
hold on
plot(time,x4,'r');
title('y(t)-x(t) for x(t) = 10sin(3t)')
xlabel('t(s)')
legend('y(t)','x(t)')
[y4fft,indy4] = max(abs(fft(y4.a.data)));
[x4fft,indx4] = max(abs(fft(x4)));
K4 = y4fft/x4fft;

y4angle = angle(fft(y4.a.data));
x4angle = angle(fft(x4));

phi4 = y4angle(indy4)-x4angle(indx4);

magnitudes(4) = K4;
phases(4) = phi4;
%%

figure(6)
plot(y5.a,'b');
duration = 10;
time = 0:0.01:duration;
x5 = 10*sin(10.*time);
hold on
plot(time,x5,'r');
title('y(t)-x(t) for x(t) = 10sin(10t)')
xlabel('t(s)')
legend('y(t)','x(t)')
[y5fft,indy5] = max(abs(fft(y5.a.data)));
[x5fft,indx5] = max(abs(fft(x5)));
K5 = y5fft/x5fft;

y5angle = angle(fft(y5.a.data));
x5angle = angle(fft(x5));

phi5 = y5angle(indy5)-x5angle(indx5);

magnitudes(5) = K5;
phases(5) = phi5;

%%

figure(7)
plot(y6.a,'b');
duration = 10;
time = 0:0.01:duration;
x6 = 10*sin(30.*time);
hold on
plot(time,x6,'r');
title('y(t)-x(t) for x(t) = 10sin(30t)')
xlabel('t(s)')
legend('y(t)','x(t)')
[y6fft,indy6] = max(abs(fft(y6.a.data)));
[x6fft,indx6] = max(abs(fft(x6)));
K6 = y6fft/x6fft;

y6angle = angle(fft(y6.a.data));
x6angle = angle(fft(x6));
```

```
phi6 = y6angle(indy6)-x6angle(indx6);

magnitudes(6) = K6;
phases(6) = phi6;

%%

figure(8)
plot(y7.a,'b');
duration = 10;
time = 0:0.01:duration;
x7 = 10*sin(100.*time);
hold on
plot(time,x7,'r');
title('y(t)-x(t) for x(t) = 10sin(100t)')
xlabel('t(s)')
legend('y(t)','x(t)')
[y7fft,indy7] = max(abs(fft(y7.a.data)));
[x7fft,indx7] = max(abs(fft(x7)));
K7 = y7fft/x7fft;

y7angle = angle(fft(y7.a.data));
x7angle = angle(fft(x7));

phi7 = y7angle(indy7)-x7angle(indx7);

magnitudes(7) = K7;
phases(7) = phi7;

%%
magnitudes = 20*log10(magnitudes);
phases_ = unwrap(phases);
%%
figure(9)
w = logspace(-1,2,100);
for k = 1:100
s = 1i * w(k);
G(k) = 128.25 / (s+9.5);
end
subplot(2,1,1)
semilogx(w,20*log10(abs(G)));
hold on
semilogx(0.1,magnitudes(1),'bo');
grid on
subplot(2,1,2)
semilogx(w,angle(G)*180/pi)
hold on
semilogx(0.1,phases(1),'bo');
grid on

%%
vals = [0.1 0.3 1 3 10 30 100];
figure(10)
w = logspace(-1,2,100);
for k = 1:100
s = 1i * w(k);
G(k) = 128.25 / (s+9.5);
G_n(k) = (128.25 / (s+9.5))*((1-
0.005*s)/(1+0.005*s));
end
subplot(2,1,1)
semilogx(w,20*log10(abs(G)));
title('Magnitude-Frequency Response of the
System')
xlabel('log(\omega) (log-rad/s)')
ylabel('|H({j\omega})| (dB)')
hold on
semilogx(vals,magnitudes,'bo');
hold on
semilogx(w,20*log10(abs(G_n)));
grid on
```

```
subplot(2,1,2)
semilogx(w,angle(G)*180/pi)
title('Phase-Frequency Response of the System')
xlabel('log(\omega) (log-rad/s)')
ylabel('\phi({j\omega}) (deg)')
hold on
semilogx(vals,phases_*180/pi,'bo');
%hold on
%semilogx(w,angle(G_n)*180/pi)
grid on
```