EEE424 Homework 3
(Clearly justify all answers.)

(Due 3 April 2023)

In this homework, you will investigate some DFT properties.

Let N be an even integer, and $x[n]$ be a finite length signal over the interval $n \in [0, N-1]$; let $X[k]$ be the N-pt DFT of $x[n]$.

1- Analytically find the DFT of each sequence below in terms of $X[k]$. (The DFT size is N unless specified otherwise.):

a) $x_1[n] \;=\; x[N-1-n]$

b) $x_2[n] \;=\; (-1)^n x[n]$

c) DFT size is $2N$.

$$x_3[n] \;=\; \begin{cases} x[n] & \text{if} \quad n \in [0, N-1] \\ x[n-N] & \text{if} \quad n \in [N, 2N-1] \\ 0 & \text{else} \end{cases}$$

d) DFT size is $N/2$.

$$x_4[n] \;=\; \begin{cases} x[n] + x[n+N/2] & \text{if} \quad n \in [0, N/2-1] \\ 0 & \text{else} \end{cases}$$

e) DFT size is $2N$.

$$x_5[n] \;=\; \begin{cases} x[n] & \text{if} \quad n \in [0, N-1] \\ 0 & \text{else} \end{cases}$$

f) DFT size is $2N$.

$$x_6[n] \;=\; \begin{cases} x[n/2] & \text{if } n \text{ is even} \\ 0 & \text{else} \end{cases}$$

g) DFT size is $N/2$. $x_7[n] \;=\; x[2n]$

Choose an arbitrary 16-pt complex valued array $x[n]$. Make sure that the array is not too simple (for example, do not choose a constant $x[n]$.) Plot the real and imaginary parts of $x[n]$

Write a MATLAB code to compute the 16-point DFT of your $x[n]$; do not use the fft or related commands of the MATLAB; rather directly implement the DFT definition. Then, computationally find the DFT's of each item above (with their indicated sizes (8, 16 or 32)), again using MATLAB and direct implementation of the DFT. Finally, numerically compute your analytical solutions for each item, again using MATLAB.

For each item, print the direct DFT implementation and the numerical implementation of your analytical results as two side-by-side arrays. If you find any mismatch between those two arrays, either your analytic solutions are wrong, or your MATLAB DFT implementation has a problem. In that case, go over your solutions and the implementation untill the mismatches are resolved. Plot the real and imaginary parts of your MATLAB outputs.

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EEE424 - Digital Signal Processing
Homework 3

Let N be an even integer, and $x[n]$ be a finite length signal over the interval $n \in [0, N-1]$; let $X[k]$ be the N-pt DFT of $x[n]$.

1- Analytically find the DFT of each sequence below in terms of $X[k]$. (The DFT size is N unless specified otherwise.):

a) $x_1[n] = x[N-1-n]$

Let $\tilde{x}_1[n] = x_1[n_{\text{mod } N}] = x[N-1-n_{\text{mod } N}] = x[(-1-n)_{\text{mod } N}]$

Let $\tilde{x}[n] = x[n \text{ mod } N] \xrightarrow{DFS} \tilde{X}[k] = \frac{1}{N}.DFT_N\{x[n]\} = \frac{X[k_{\text{mod } N}]}{N}$

$\tilde{x}[-1-n] = \tilde{x}_1[n]$

$$DFS\{\tilde{x}_1[n]\} = \frac{1}{N}.\sum_{n=0}^{N-1} x[-1-n].\,e^{-j\frac{2\pi k}{N}n} = \frac{1}{N}\sum_{n=0}^{N-1} x[n-1].\,e^{j\frac{2\pi k}{N}n}$$

$$= \frac{e^{j\frac{2\pi k}{N}}}{N}\sum_{n=0}^{N-1} x[n].\,e^{j\frac{2\pi k}{N}n} = \tilde{X}[-k].\,e^{j\frac{2\pi k}{N}} = \tilde{X}_1[k]$$

$\tilde{X}_1[k] = X[(-k)_{\text{mod } N}].\,e^{j\frac{2\pi k}{N}\text{mod}N}$. Then,

$$\boxed{X_1[k] = X[(-k) \text{ mod } N].\,e^{j\frac{2\pi k}{N}\text{mod } N} \quad \text{for } k = 0, 1, \ldots, N-1}$$

b) $x_2[n] = (-1)^n x[n]$

$$x[n] \xrightarrow{DFT_N} X[k]$$

$$x_2[n] \xrightarrow{DFT_N} \sum_{n=0}^{N-1} x_2[n] \cdot e^{-j\frac{2\pi k}{N} n} = \sum_{n=0}^{N-1} x[n] \cdot \left(-e^{-j\frac{2\pi k}{N}}\right)^n$$

$$= \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi(k-N/2)}{N} n} = \boxed{X\left[\left(k+\tfrac{N}{2}\right)_{\text{Mod } N}\right] = X_2[k] \quad \text{for } k = 0, 1, 2, \ldots, N-1}$$

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c) DFT size is $2N$.

$$x_3[n] = \begin{cases} x[n] & \text{if} \quad n \in [0, N-1] \\ x[n-N] & \text{if} \quad n \in [N, 2N-1] \\ 0 & \text{else} \end{cases}$$

$$x[n] \xrightarrow{DFT_N} X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi k}{N} n}$$

$$x_3[n] \xrightarrow{DFT_{2N}} X_3[k] = \sum_{n=0}^{2N-1} x_3[n] \cdot e^{-j\frac{2\pi k}{2N} n}$$

$$= \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi k}{2N} n} + \sum_{n=N}^{2N-1} x[n-N] \cdot e^{-j\frac{2\pi k}{2N} n}$$

$$= X\left(\tfrac{k}{2}\right) + \left(\sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi k}{2N} n}\right) \cdot \underbrace{e^{-j\pi k}}_{(-1)^k} = X\left[\tfrac{k}{2}\right] \cdot \left[1 + (-1)^k\right]$$

$$\boxed{X_3[k] = \begin{cases} 2X\left(\frac{k}{2}\right) & k \text{ even} \\ 0 & k \text{ odd} \end{cases} \quad \text{for } k = 0, 1, \ldots, 2N-1}$$

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d) DFT size is $N/2$.

$$x_4[n] = \begin{cases} x[n] + x[n+N/2] & \text{if} \quad n \in [0, N/2-1] \\ 0 & \text{else} \end{cases}$$

$$x[n] \xrightarrow{DFT_N} X(k) = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi k}{N} n}$$

$$x_4[n] \xrightarrow{DFT_{N/2}} X_4(k) = \sum_{n=0}^{N/2-1} x_4[n] \cdot e^{-j\frac{2\pi k}{N/2} n}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n] \cdot e^{-j\frac{4\pi k}{N} n} + \sum_{n=0}^{\frac{N}{2}-1} x[n+\tfrac{N}{2}] \cdot e^{-j\frac{4\pi k}{N} n}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n] \cdot e^{-j\frac{4\pi k}{N} n} + \left(\sum_{n=\frac{N}{2}}^{N-1} x[n] \cdot e^{-j\frac{4\pi k n}{N}} \right) \cdot \underbrace{e^{j\frac{\overset{2}{\cancel{4}}\pi k \cancel{N}}{\cancel{2N}}}}_{1}$$

$$= \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{4\pi k}{N} n} = X(2k)$$

$$\boxed{X_4(k) = X(2k) \qquad \text{for} \quad k = 0, 1, \ldots, \tfrac{N}{2}-1}$$

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e) DFT size is $2N$.

$$x_5[n] = \begin{cases} x[n] & \text{if} \quad n \in [0, N-1] \\ 0 & \text{else} \end{cases}$$

$$x[n] \xrightarrow{DFT_N} \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi k}{N} \cdot n} = X[k]$$

$$x_5[n] \xrightarrow{DFT_{2N}} \sum_{n=0}^{2N-1} x_5[n] \cdot e^{-j\frac{2\pi k}{2N} \cdot n} = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi k}{2N} \cdot n} = \underbrace{X_5[k] \overset{?}{=} X[\tfrac{k}{2}]}$$

Not a valid answer since $X[k]$ is only defined for integer k. Then try sampling DTFT instead.

$$x_5[n] \xrightarrow{DTFT} \sum_{n=-\infty}^{+\infty} x_5[n] \cdot e^{-j\omega n} = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\omega n}$$

$$X_5(e^{j\omega}) = \sum_{n=0}^{N-1} \left(\frac{1}{N} \sum_{k=0}^{N-1} X[k] \cdot e^{j\frac{2\pi k}{N}} \right) e^{-j\omega n}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X[k] \sum_{n=0}^{N-1} e^{j(\frac{2\pi}{N}k - \omega)n} \xrightarrow{DFT_{2N}} X_5[m] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \sum_{n=0}^{N-1} e^{j(\frac{2\pi}{N}k - \omega)n} \Bigg|_{\omega = \frac{2\pi m}{2N}}$$

$$X_5[m] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] \boxed{\sum_{n=0}^{N-1} e^{j\left(\frac{2\pi}{N}k - \frac{\pi m}{N}\right)n}} = \begin{cases} \dfrac{1-(-1)^{m-2k}}{1-e^{\frac{j\pi(m-2k)}{N}}} & \text{if } (m-2k)_{\bmod N} \neq 0 \\ N & \text{if } (m-2k) = 0 \end{cases}$$

$m - 2k = N$

$m - 2k = -N$

$$\boxed{X_5[m] = \begin{cases} X[k] & \text{for } (m-2k)_{\bmod 2} = 0 \\ \frac{1}{N}\sum_{k=0}^{N-1} X[k] \cdot \dfrac{1-(-1)^{m-2k}}{1-e^{\frac{j\pi(m-2k)}{N}}} & \text{for } (m-2k)_{\bmod 2} \neq 0 \end{cases}}$$

---

f) DFT size is $2N$.

$$x_6[n] = \begin{cases} x[n/2] & \text{if } n \text{ is even} \\ 0 & \text{else} \end{cases}$$

$$x[n] \xrightarrow{DFT_N} X[k] \quad \text{for } k = 0, \ldots, N-1$$

$$x_6[n] \xrightarrow{DFT_{2N}} X_6[k] = \sum_{n=0}^{2N-1} x_6[n] \cdot e^{\frac{-j2\pi kn}{2N}} \quad \text{for} \quad k = 0, 1, \ldots, 2N-1$$

$$= x[0] \cdot e^{0} + x[1] \cdot e^{\frac{-j4\pi k}{2N}} + x[2] \cdot e^{\frac{-j8\pi k}{2N}} + \cdots + x[N-1] \cdot e^{\frac{j4\pi k(N-1)}{2N}}$$

$$= \sum_{n=0}^{N-1} x[n] \cdot e^{\frac{-j4\pi kn}{2N}} = \sum_{n=0}^{N-1} x[n] \cdot e^{\frac{-j2\pi kn}{N}} \quad \text{for } k = 0, 1, \ldots, N-1$$

$$\boxed{X_6[k] = X[k \bmod N] \quad \text{for } k = 0, 1, \ldots, 2N-1}$$

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g) DFT size is $N/2$. $x_7[n] = x[2n]$

$$x_7[n] \xrightarrow{\text{DFT}_{N/2}} X_7(k)$$

$$x_0[n] = x[n]$$

$$x_1[n] = x[n] \cdot e^{\frac{-j2\pi n}{2}} = x[n] \cdot (-1)^n$$

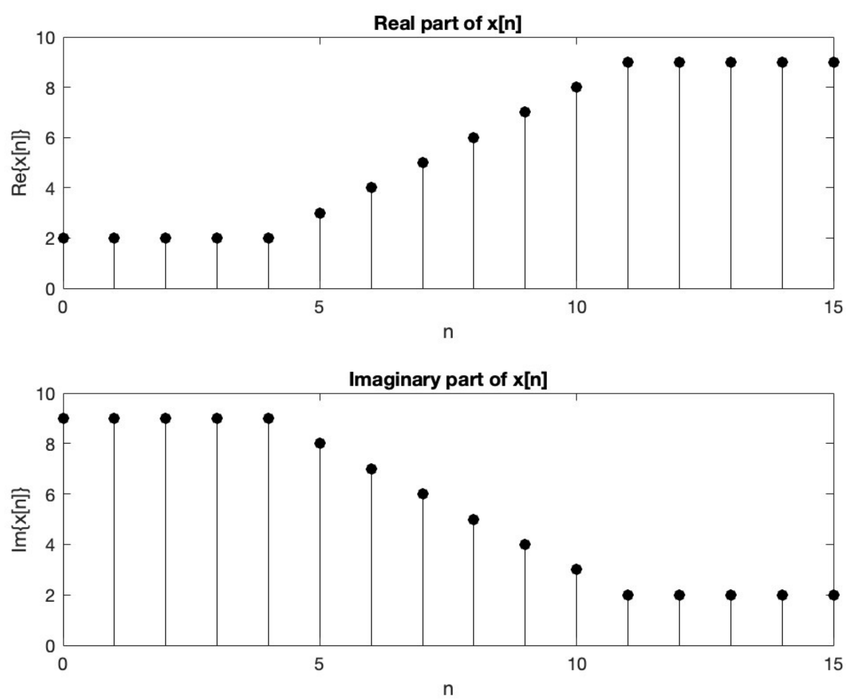
$$\frac{1}{2}\sum_{k=0}^{1} x_k[n] = \frac{x[n]\cdot(1+(-1)^n)}{2} = \begin{cases} x[2n] & \text{for even } n \\ 0 & \text{for odd } n \end{cases} = x_7'[n]$$

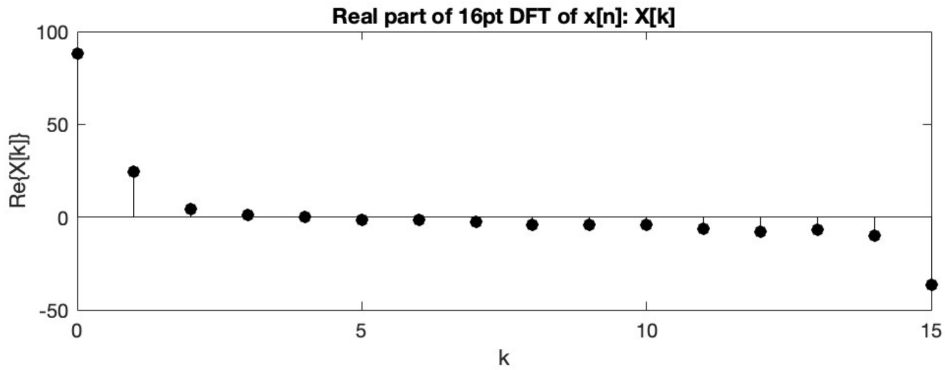
$$X_7'[k] = \frac{1}{2}\cdot\left[X[k] + X[k+\tfrac{N}{2}]\right] \quad \text{for } k = 0, 1, \ldots, N-1$$

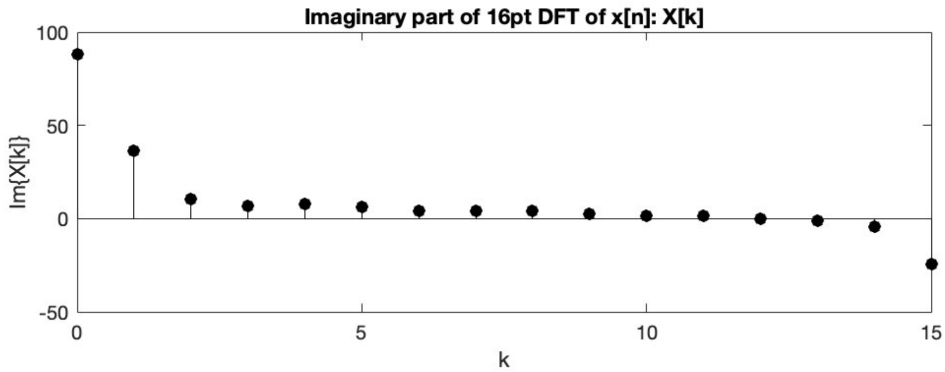
$$X_7[k] = X_7'[k] \quad \text{for } k = 0, 1, 2, \ldots, \tfrac{N}{2}-1$$

$$\boxed{X_7[k] = \frac{1}{2}\cdot\left[X[k] + X[k+\tfrac{N}{2}]\right] \quad \text{for } k = 0, 1, \ldots, \tfrac{N}{2}-1}$$

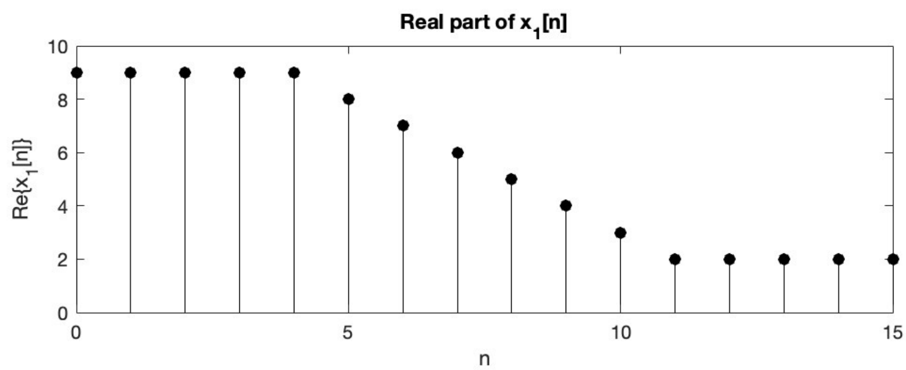
Choose an arbitrary 16-pt complex valued array $x[n]$. Make sure that the array is not too simple (for example, do not choose a constant $x[n]$.) Plot the real and imaginary parts of $x[n]$

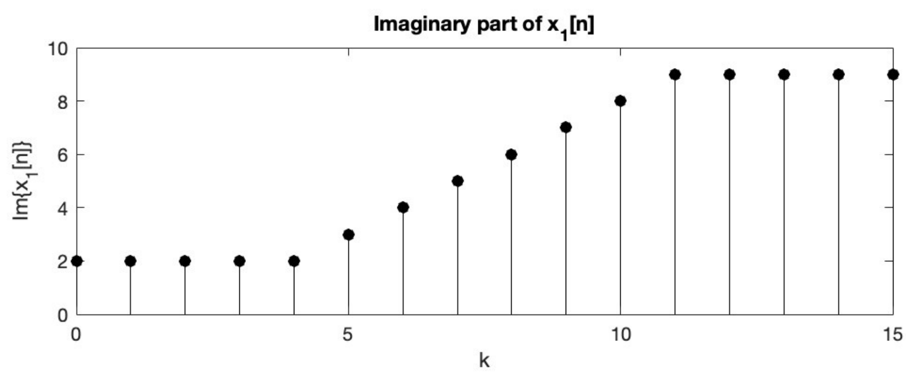


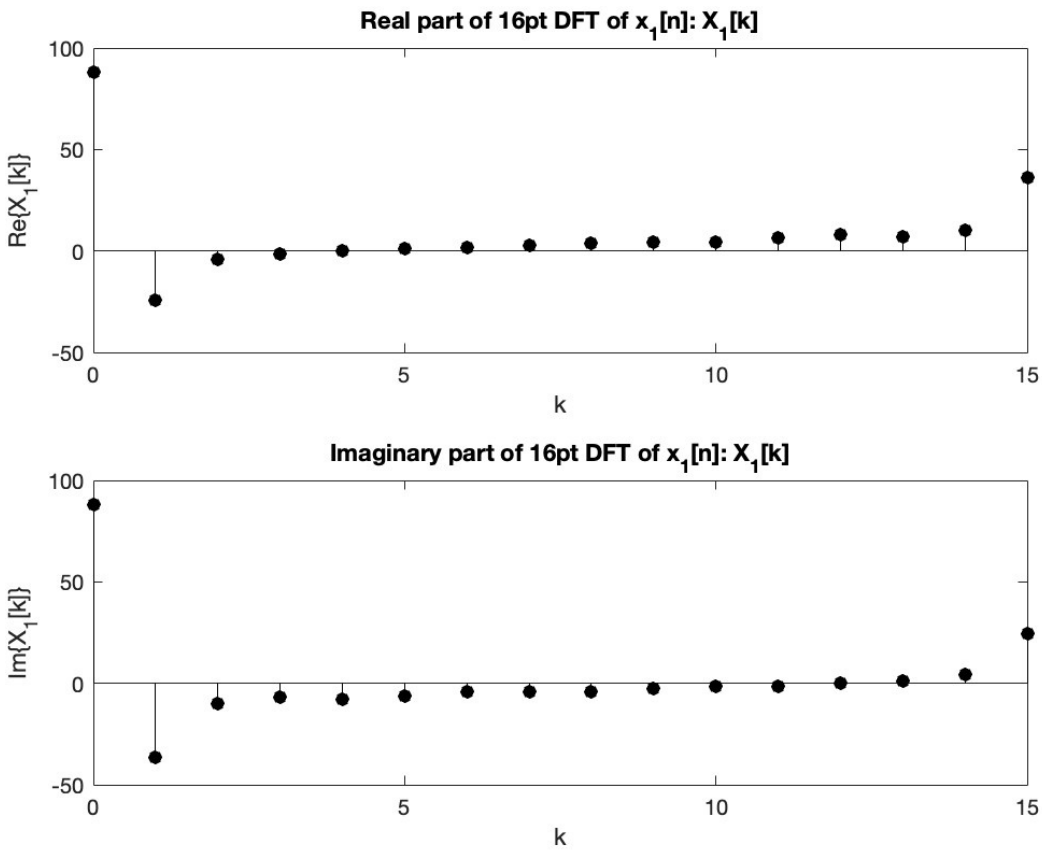

Real part of 16pt DFT of x[n]: X[k]
Re{X[k]}
100
50
0
-50
0
5
10
15
k


Imaginary part of 16pt DFT of x[n]: X[k]
Im{X[k]}
100
50
0
-50
0
5
10
15
k

Part A:


Real part of x1[n]
Re{x1[n]}
10
8
6
4
2
0
0
5
10
15
n

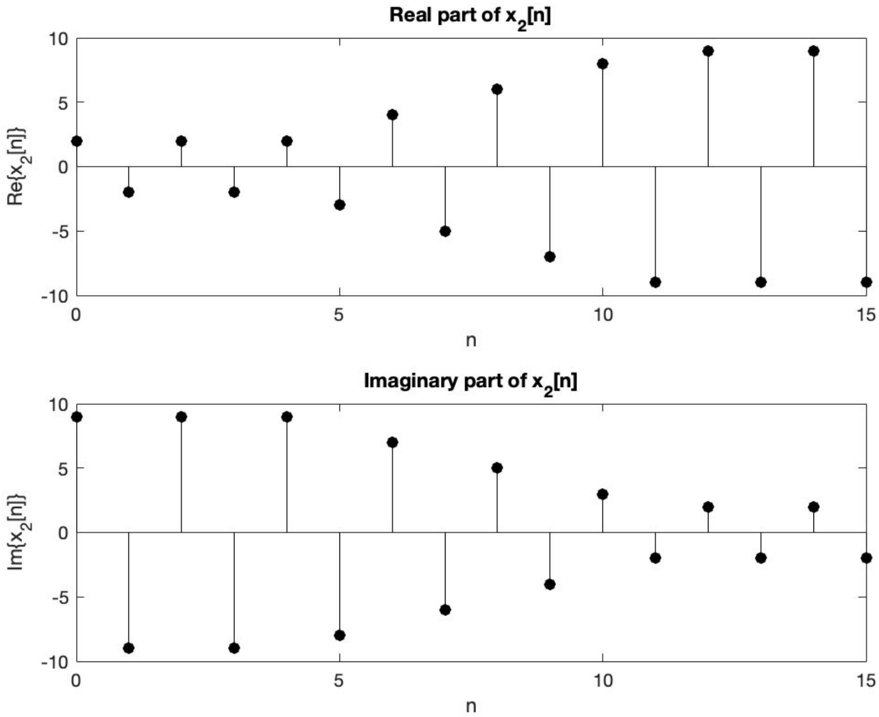

Imaginary part of x1[n]
Im{x1[n]}
10
8
6
4
2
0
0
5
10
15
k

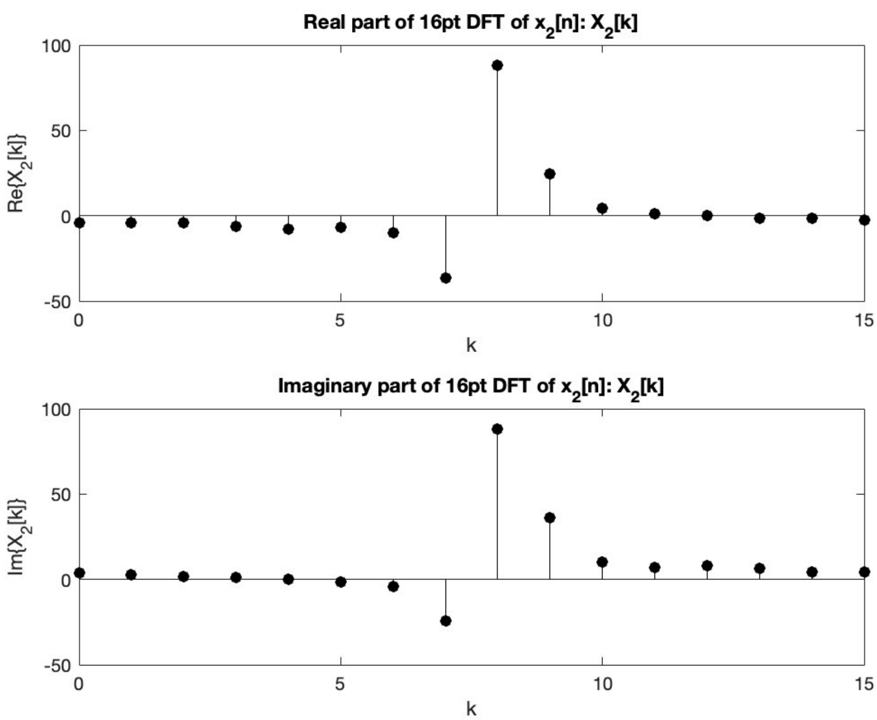


DFTx1_0 is the numerically found DFT while the DFTx1_1 is the computationally found DFT. They are equal as it can be seen.

```
DFTx1_0 =

  Columns 1 through 5

  88.0000 +88.0000i -24.2191 -36.2464i  -4.2426 -10.2426i  -1.3665 - 6.8699i  -0.0000 - 8.0000i

  Columns 6 through 10

   1.2722 - 6.3959i   1.7574 - 4.2426i   2.7241 - 4.0770i   4.0000 - 4.0000i   4.0770 - 2.7241i

  Columns 11 through 15

   4.2426 - 1.7574i   6.3959 - 1.2722i   8.0000 - 0.0000i   6.8699 + 1.3665i  10.2426 + 4.2426i

  Column 16

  36.2464 +24.2191i

>> DFTx1_1

DFTx1_1 =

  Columns 1 through 5

  88.0000 +88.0000i -24.2191 -36.2464i  -4.2426 -10.2426i  -1.3665 - 6.8699i   0.0000 - 8.0000i

  Columns 6 through 10

   1.2722 - 6.3959i   1.7574 - 4.2426i   2.7241 - 4.0770i   4.0000 - 4.0000i   4.0770 - 2.7241i

  Columns 11 through 15

   4.2426 - 1.7574i   6.3959 - 1.2722i   8.0000 + 0.0000i   6.8699 + 1.3665i  10.2426 + 4.2426i

  Column 16

  36.2464 +24.2191i
```

Part B:

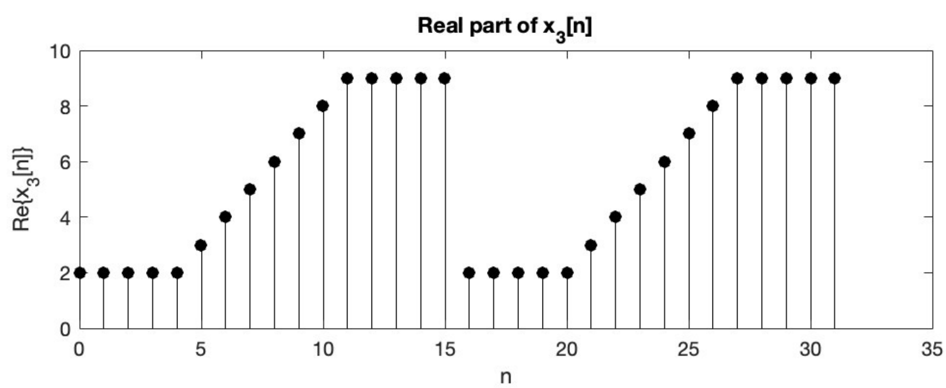

Real part of $x_2[n]$
$Re\{x_2[n]\}$
n
Imaginary part of $x_2[n]$
$Im\{x_2[n]\}$
n

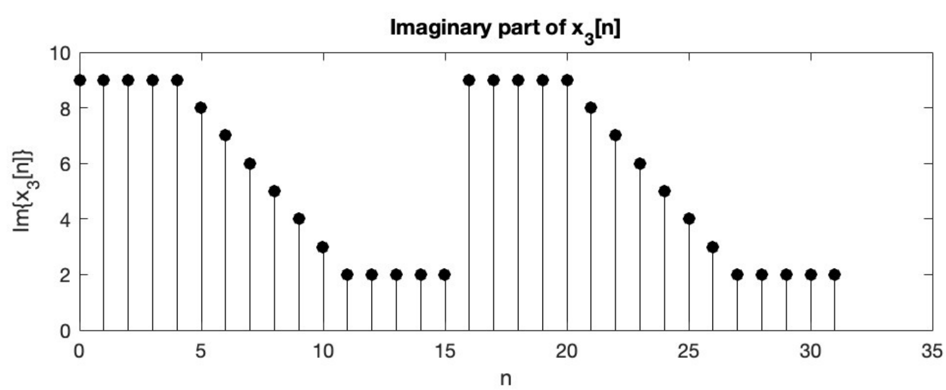

Real part of 16pt DFT of $x_2[n]$: $X_2[k]$
$Re\{X_2[k]\}$
k
Imaginary part of 16pt DFT of $x_2[n]$: $X_2[k]$
$Im\{X_2[k]\}$
k

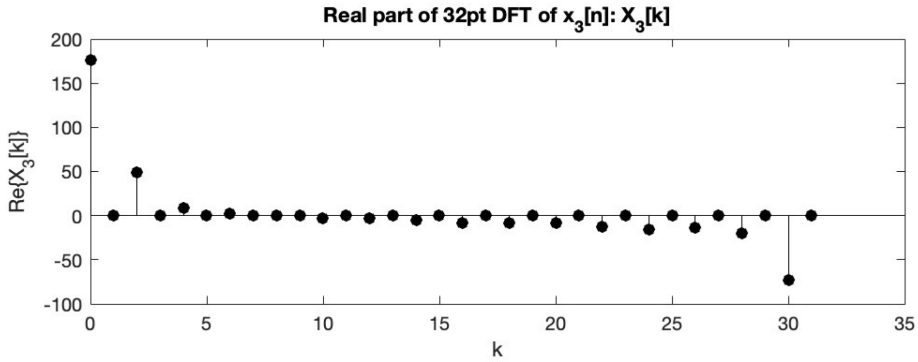
DFTx2_0 is the numerically found DFT while the DFTx2_1 is the computationally found DFT. They are equal as it can be seen.

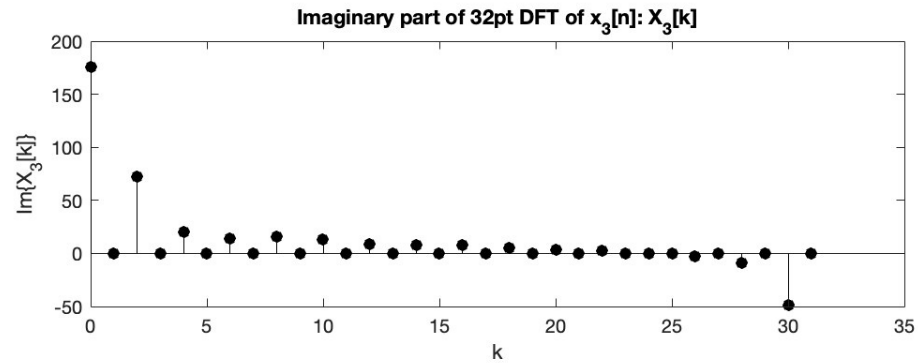
```
>> DFTx2_0

DFTx2_0 =

  Columns 1 through 5

  -4.0000 + 4.0000i  -4.0770 + 2.7241i  -4.2426 + 1.7574i  -6.3959 + 1.2722i  -8.0000 + 0.0000i

  Columns 6 through 10

  -6.8699 - 1.3665i -10.2426 - 4.2426i -36.2464 -24.2191i  88.0000 +88.0000i  24.2191 +36.2464i

  Columns 11 through 15

   4.2426 +10.2426i   1.3665 + 6.8699i   0.0000 + 8.0000i  -1.2722 + 6.3959i  -1.7574 + 4.2426i

  Column 16

  -2.7241 + 4.0770i

>> DFTx2_1

DFTx2_1 =

  Columns 1 through 5

  -4.0000 + 4.0000i  -4.0770 + 2.7241i  -4.2426 + 1.7574i  -6.3959 + 1.2722i  -8.0000 - 0.0000i

  Columns 6 through 10

  -6.8699 - 1.3665i -10.2426 - 4.2426i -36.2464 -24.2191i  88.0000 +88.0000i  24.2191 +36.2464i

  Columns 11 through 15

   4.2426 +10.2426i   1.3665 + 6.8699i  -0.0000 + 8.0000i  -1.2722 + 6.3959i  -1.7574 + 4.2426i

  Column 16

  -2.7241 + 4.0770i
```

Part C:





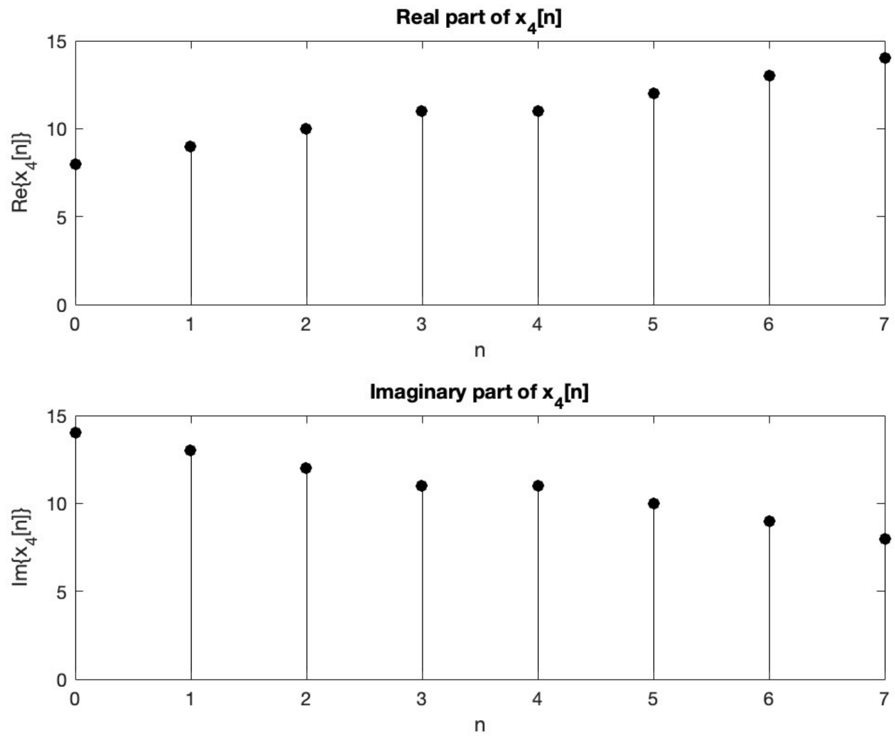




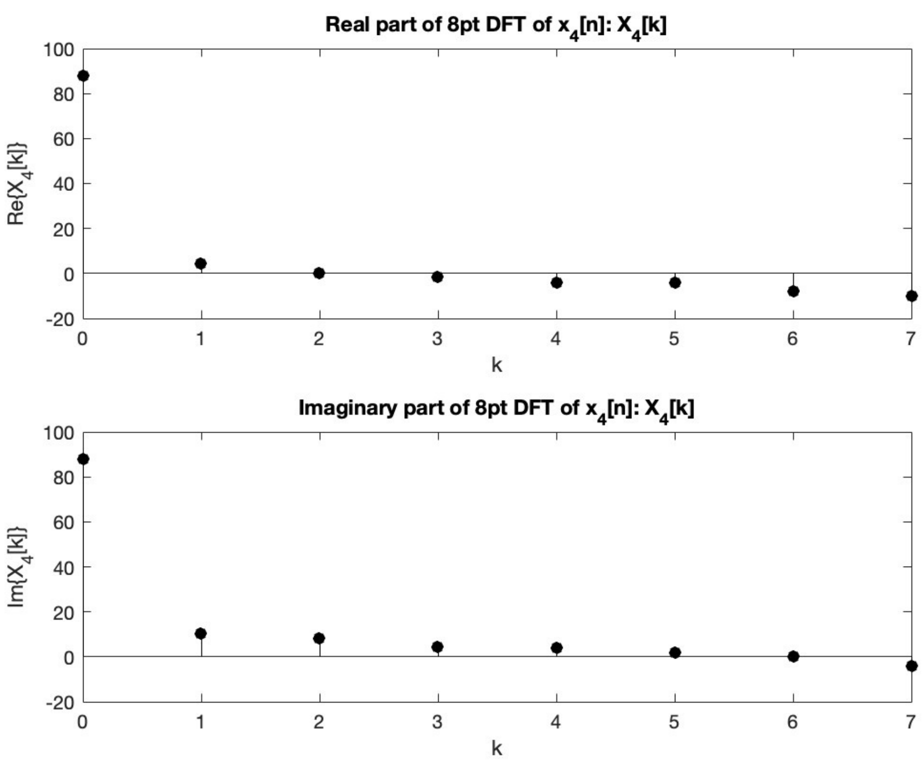
DFTx3_0 is the numerically found DFT while the DFTx3_1 is the computationally found DFT. They are equal as it can be seen.

```
DFTx3_0 =

   1.0e+02 *

  Columns 1 through 6

   1.7600 + 1.7600i  -0.0000 - 0.0000i   0.4844 + 0.7249i  -0.0000 - 0.0000i   0.0849 + 0.2049i  -0.0000 + 0.0000i

  Columns 7 through 12

   0.0273 + 0.1374i   0.0000 + 0.0000i  -0.0000 + 0.1600i   0.0000 - 0.0000i  -0.0254 + 0.1279i   0.0000 - 0.0000i

  Columns 13 through 18

  -0.0351 + 0.0849i   0.0000 + 0.0000i  -0.0545 + 0.0815i  -0.0000 - 0.0000i  -0.0800 + 0.0800i   0.0000 + 0.0000i

  Columns 19 through 24

  -0.0815 + 0.0545i  -0.0000 + 0.0000i  -0.0849 + 0.0351i   0.0000 - 0.0000i  -0.1279 + 0.0254i  -0.0000 - 0.0000i

  Columns 25 through 30

  -0.1600 - 0.0000i   0.0000 - 0.0000i  -0.1374 - 0.0273i  -0.0000 - 0.0000i  -0.2049 - 0.0849i  -0.0000 - 0.0000i

  Columns 31 through 32

  -0.7249 - 0.4844i   0.0000 + 0.0000i

DFTx3_1 =

   1.0e+02 *

  Columns 1 through 6

   1.7600 + 1.7600i   0.0000 + 0.0000i   0.4844 + 0.7249i   0.0000 + 0.0000i   0.0849 + 0.2049i   0.0000 + 0.0000i

  Columns 7 through 12

   0.0273 + 0.1374i   0.0000 + 0.0000i  -0.0000 + 0.1600i   0.0000 + 0.0000i  -0.0254 + 0.1279i   0.0000 + 0.0000i

  Columns 13 through 18

  -0.0351 + 0.0849i   0.0000 + 0.0000i  -0.0545 + 0.0815i   0.0000 + 0.0000i  -0.0800 + 0.0800i   0.0000 + 0.0000i

  Columns 19 through 24

  -0.0815 + 0.0545i   0.0000 + 0.0000i  -0.0849 + 0.0351i   0.0000 + 0.0000i  -0.1279 + 0.0254i   0.0000 + 0.0000i

  Columns 25 through 30

  -0.1600 - 0.0000i   0.0000 + 0.0000i  -0.1374 - 0.0273i   0.0000 + 0.0000i  -0.2049 - 0.0849i   0.0000 + 0.0000i

  Columns 31 through 32

  -0.7249 - 0.4844i   0.0000 + 0.0000i
```

Part D:



Real part of x4[n]
Re{x4[n]}
n
Imaginary part of x4[n]
Im{x4[n]}
n

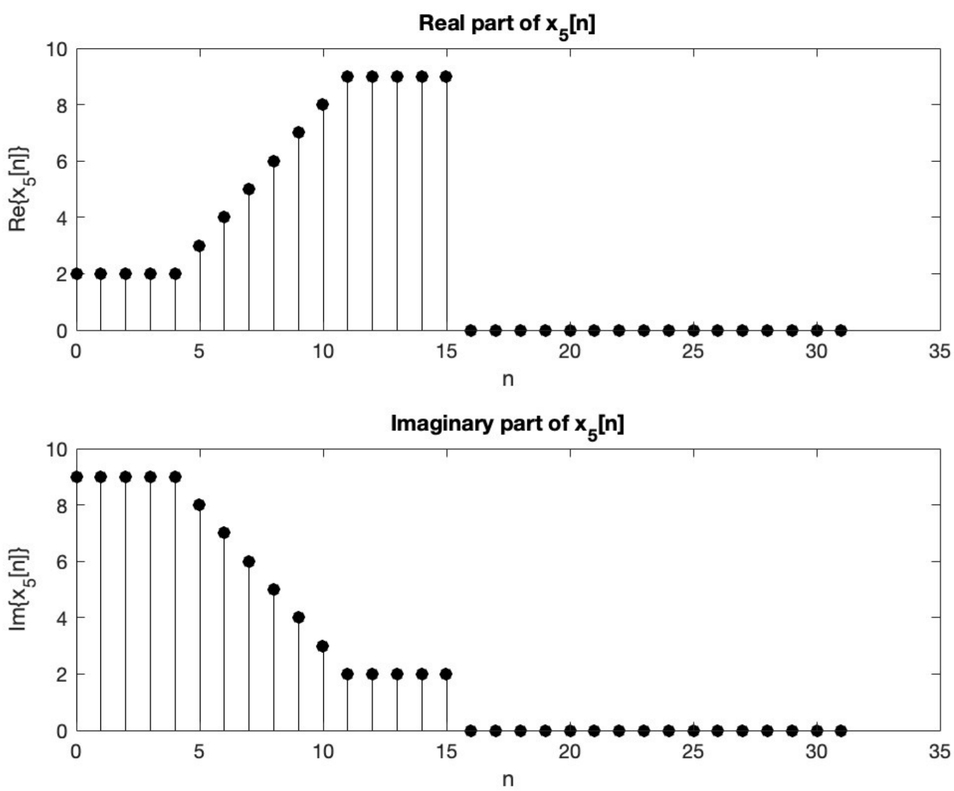


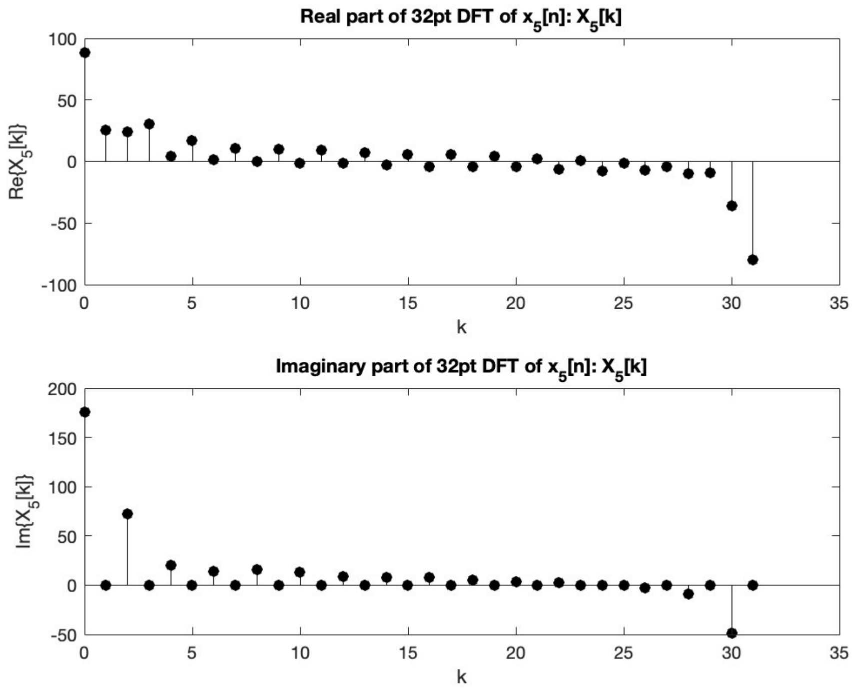
Real part of 8pt DFT of x4[n]: X4[k]
Re{X4[k]}
k
Imaginary part of 8pt DFT of x4[n]: X4[k]
Im{X4[k]}
k

DFTx4_0 is the numerically found DFT while the DFTx4_1 is the computationally found DFT. They are equal as it can be seen.

```
DFTx4_0 =

  Columns 1 through 6

  88.0000 +88.0000i   4.2426 +10.2426i  -0.0000 + 8.0000i  -1.7574 + 4.2426i  -4.0000 + 4.0000i  -4.2426 + 1.7574i

  Columns 7 through 8

  -8.0000 - 0.0000i -10.2426 - 4.2426i

>> DFTx4_1

DFTx4_1 =

  Columns 1 through 6

  88.0000 +88.0000i   4.2426 +10.2426i  -0.0000 + 8.0000i  -1.7574 + 4.2426i  -4.0000 + 4.0000i  -4.2426 + 1.7574i

  Columns 7 through 8

  -8.0000 - 0.0000i -10.2426 - 4.2426i
```

Part E:



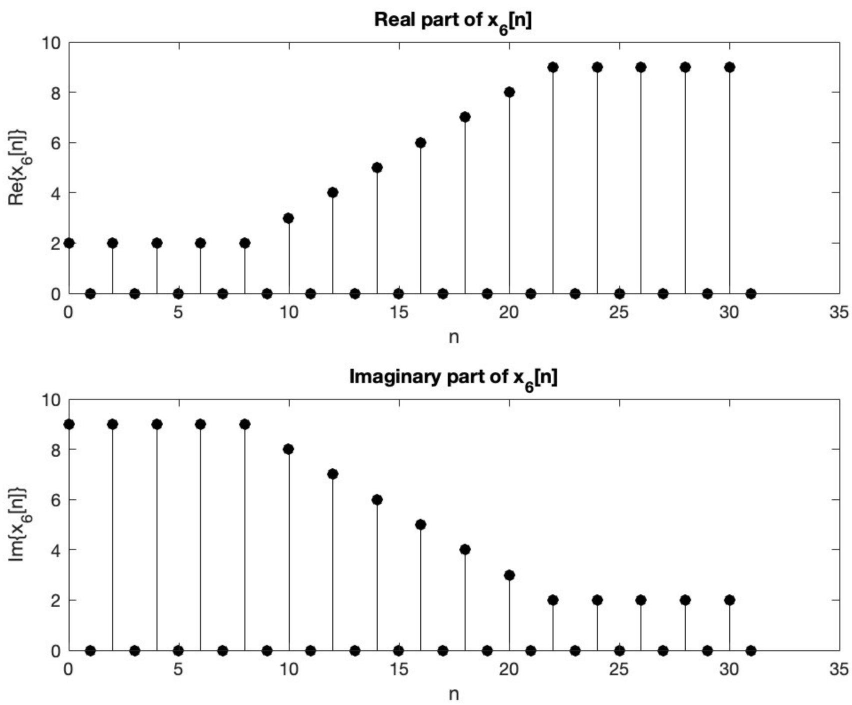


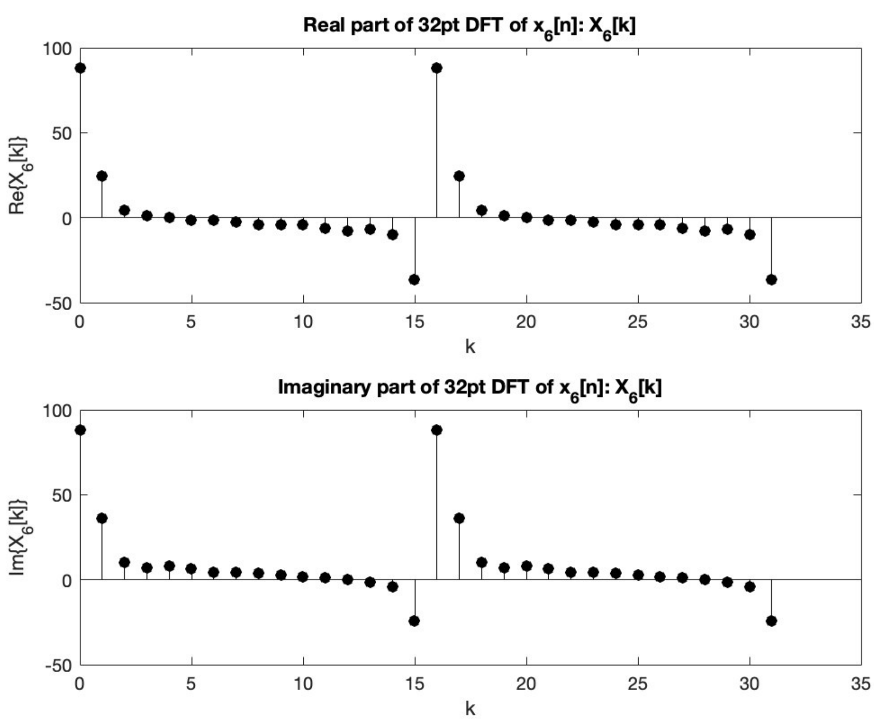
DFTx5_0 is the numerically found DFT while the DFTx5_1 is the computationally found DFT. They are equal as it can be seen.

```
DFTx5_0 =

  Columns 1 through 6

  88.0000 +88.0000i  25.2493 -20.7215i  24.2191 +36.2464i  30.1578 -16.1197i   4.2426 +10.2426i  16.6737 - 5.0579i

  Columns 7 through 12

   1.3665 + 6.8699i  10.4617 - 1.0304i  -0.0000 + 8.0000i  10.1292 + 0.9976i  -1.2722 + 6.3959i   9.2723 + 2.8127i

  Columns 13 through 18

  -1.7574 + 4.2426i   6.8474 + 3.6600i  -2.7241 + 4.0770i   5.6151 + 4.6082i  -4.0000 + 4.0000i   5.3084 + 6.4683i

  Columns 19 through 24

  -4.0770 + 2.7241i   4.0032 + 7.4894i  -4.2426 + 1.7574i   2.3076 + 7.6073i  -6.3959 + 1.2722i   0.9749 + 9.8983i

  Columns 25 through 30

  -8.0000 - 0.0000i  -1.3732 +13.9419i  -6.8699 - 1.3665i  -4.5216 +14.9058i -10.2426 - 4.2426i  -9.1425 +17.1043i

  Columns 31 through 32

 -36.2464 -24.2191i -79.9633 +97.4356i
```

```
DFTx5_1 =

  Columns 1 through 6

  88.0000 +88.0000i  25.2493 -20.7215i  24.2191 +36.2464i  30.1578 -16.1197i   4.2426 +10.2426i  16.6737 - 5.0579i

  Columns 7 through 12

   1.3665 + 6.8699i  10.4617 - 1.0304i  -0.0000 + 8.0000i  10.1292 + 0.9976i  -1.2722 + 6.3959i   9.2723 + 2.8127i

  Columns 13 through 18

  -1.7574 + 4.2426i   6.8474 + 3.6600i  -2.7241 + 4.0770i   5.6151 + 4.6082i  -4.0000 + 4.0000i   5.3084 + 6.4683i

  Columns 19 through 24

  -4.0770 + 2.7241i   4.0032 + 7.4894i  -4.2426 + 1.7574i   2.3076 + 7.6073i  -6.3959 + 1.2722i   0.9749 + 9.8983i

  Columns 25 through 30

  -8.0000 - 0.0000i  -1.3732 +13.9419i  -6.8699 - 1.3665i  -4.5216 +14.9058i -10.2426 - 4.2426i  -9.1425 +17.1043i

  Columns 31 through 32

 -36.2464 -24.2191i -79.9633 +97.4356i
```

Part F:


Real part of x6[n]
Re{x6[n]}
n
Imaginary part of x6[n]
Im{x6[n]}
n

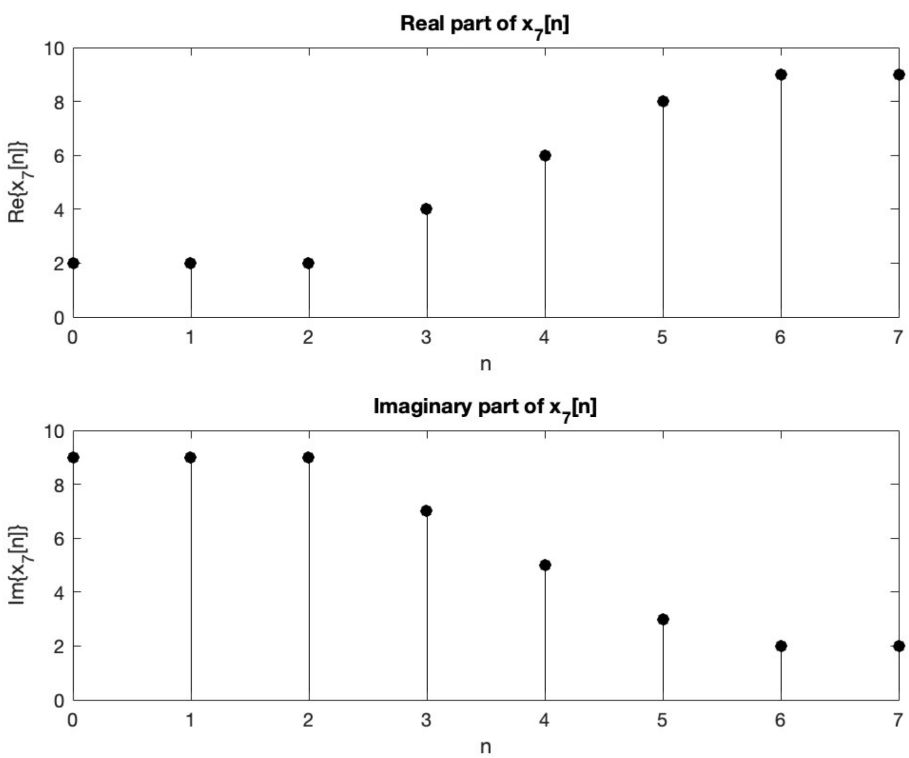

Real part of 32pt DFT of x6[n]: X6[k]
Re{X6[k]}
k
Imaginary part of 32pt DFT of x6[n]: X6[k]
Im{X6[k]}
k

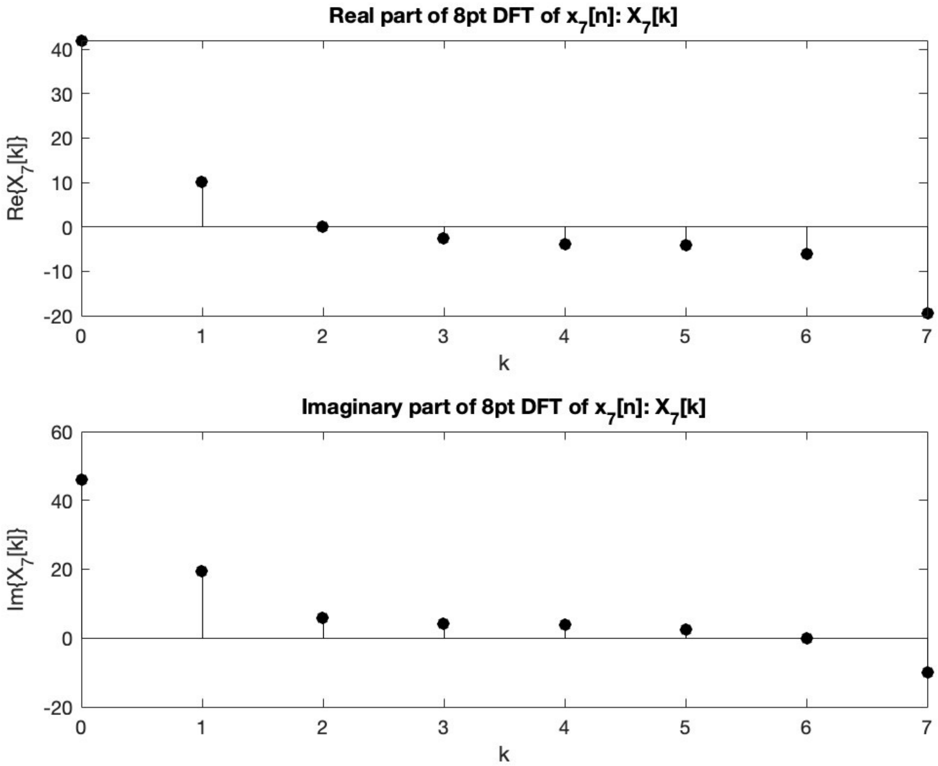
DFTx6_0 is the numerically found DFT while the DFTx6_1 is the computationally found DFT. They are equal as it can be seen.

```
DFTx6_0 =

  Columns 1 through 7

  88.0000 +88.0000i  24.2191 +36.2464i   4.2426 +10.2426i   1.3665 + 6.8699i  -0.0000 + 8.0000i  -1.2722 + 6.3959i  -1.7574 + 4.2426i

  Columns 8 through 14

  -2.7241 + 4.0770i  -4.0000 + 4.0000i  -4.0770 + 2.7241i  -4.2426 + 1.7574i  -6.3959 + 1.2722i  -8.0000 - 0.0000i  -6.8699 - 1.3665i

  Columns 15 through 21

 -10.2426 - 4.2426i -36.2464 -24.2191i  88.0000 +88.0000i  24.2191 +36.2464i   4.2426 +10.2426i   1.3665 + 6.8699i   0.0000 + 8.0000i

  Columns 22 through 28

  -1.2722 + 6.3959i  -1.7574 + 4.2426i  -2.7241 + 4.0770i  -4.0000 + 4.0000i  -4.0770 + 2.7241i  -4.2426 + 1.7574i  -6.3959 + 1.2722i

  Columns 29 through 32

  -8.0000 - 0.0000i  -6.8699 - 1.3665i -10.2426 - 4.2426i -36.2464 -24.2191i
```

```
DFTx6_1 =

  Columns 1 through 7

  88.0000 +88.0000i  24.2191 +36.2464i   4.2426 +10.2426i   1.3665 + 6.8699i  -0.0000 + 8.0000i  -1.2722 + 6.3959i  -1.7574 + 4.2426i

  Columns 8 through 14

  -2.7241 + 4.0770i  -4.0000 + 4.0000i  -4.0770 + 2.7241i  -4.2426 + 1.7574i  -6.3959 + 1.2722i  -8.0000 - 0.0000i  -6.8699 - 1.3665i

  Columns 15 through 21

 -10.2426 - 4.2426i -36.2464 -24.2191i  88.0000 +88.0000i  24.2191 +36.2464i   4.2426 +10.2426i   1.3665 + 6.8699i  -0.0000 + 8.0000i

  Columns 22 through 28

  -1.2722 + 6.3959i  -1.7574 + 4.2426i  -2.7241 + 4.0770i  -4.0000 + 4.0000i  -4.0770 + 2.7241i  -4.2426 + 1.7574i  -6.3959 + 1.2722i

  Columns 29 through 32

  -8.0000 - 0.0000i  -6.8699 - 1.3665i -10.2426 - 4.2426i -36.2464 -24.2191i
```

Part G:





DFTx7_0 is the numerically found DFT while the DFTx7_1 is the computationally found DFT. They are equal as it can be seen.

```
DFTx7_0 =

  Columns 1 through 7

  42.0000 +46.0000i  10.0711 +19.4853i  -0.0000 + 6.0000i  -2.5147 + 4.0711i  -4.0000 + 4.0000i  -4.0711 + 2.5147i  -6.0000 - 0.0000i

  Column 8

 -19.4853 -10.0711i

>> DFTx7_1

DFTx7_1 =

  Columns 1 through 7

  42.0000 +46.0000i  10.0711 +19.4853i   0.0000 + 6.0000i  -2.5147 + 4.0711i  -4.0000 + 4.0000i  -4.0711 + 2.5147i  -6.0000 + 0.0000i

  Column 8

 -19.4853 -10.0711i
```

Efe Tarhan
22002840

MATLAB Code of HW3

```
%% GENERATING x[n]:

arr1 = [9 9 9 9 9 8 7 6 5 4 3 2 2 2 2 2];
arr1 = 1i.*arr1;
arr2 = [2 2 2 2 2 3 4 5 6 7 8 9 9 9 9 9];
x = arr1 + arr2;
%%-------------------------------------////

%% PLOTTING THE REAL AND IMAGINARY PARTS OF x[n]:

figure(1);
tiledlayout(2,1);

ax1 = nexttile;
stem(ax1,(0:1:15),real(x),'filled','k');
title("Real part of x[n]");
ylabel("Re\{x[n]\}");
xlabel("n");

ax2 = nexttile;
stem(ax2,(0:1:15),imag(x),'filled','k');
title("Imaginary part of x[n]");
ylabel("Im\{x[n]\}");
xlabel("n");
%%------------------------------------////

%% FINDING 16 pt DFT:

DFTx = 0;
for q = (0:15)
   DFTx(1:16) = DFTx + x(q+1)*exp(-2*pi*1i*(0:15)*q/16);
```

```
end

figure(2);
tiledlayout(2,1);

ax1 = nexttile;
stem(ax1,(0:1:15),real(DFTx),'filled','k');
title("Real part of 16pt DFT of x[n]: X[k]");
ylabel("Re\{X[k]\}");
xlabel("k");

ax2 = nexttile;
stem(ax2,(0:1:15),imag(DFTx),'filled','k');
title("Imaginary part of 16pt DFT of x[n]: X[k]");
ylabel("Im\{X[k]\}");
xlabel("k");
%%----------------------------------////

%% Part a:
x1((0:15)+1) = x(15-(0:15)+1);

figure(3);
tiledlayout(2,1);

ax1 = nexttile;
stem(ax1,(0:1:15),real(x1),'filled','k');
title("Real part of x_{1}[n]");
ylabel("Re\{x_{1}[n]\}");
xlabel("n");

ax2 = nexttile;
stem(ax2,(0:1:15),imag(x1),'filled','k');
title("Imaginary part of x_{1}[n]");
ylabel("Im\{x_{1}[n]\}");
xlabel("n");
```

```
DFTx1_0 = 0;
for q = (0:15)
    DFTx1_0(1:16) = DFTx1_0 + x1(q+1)*exp(-2*pi*1i*(0:15)*q/16);
end


k = 0:15;
DFTx1_1(k+1) = DFTx(mod(-(k),16)+1).*exp(2*pi*1i*mod(k,16)/16);

figure(4);
tiledlayout(2,1);
ax1 = nexttile;
stem(ax1,(0:1:15),real(DFTx1_1),'filled','k');
title("Real part of 16pt DFT of x_{1}[n]: X_{1}[k]");
ylabel("Re\{X_{1}[k]\}");
xlabel("k");

ax2 = nexttile;
stem(ax2,(0:1:15),imag(DFTx1_1),'filled','k');
title("Imaginary part of 16pt DFT of x_{1}[n]: X_{1}[k]");
ylabel("Im\{X_{1}[k]\}");
xlabel("k");

%%-----------------------------------////

%% Part b:
x2((0:15)+1) = x((0:15)+1).*(-1).^(0:15);

figure(5);
tiledlayout(2,1);

ax1 = nexttile;
stem(ax1,(0:1:15),real(x2),'filled','k');
title("Real part of x_{2}[n]");
ylabel("Re\{x_{2}[n]\}");
xlabel("n");
```

```
ax2 = nexttile;
stem(ax2,(0:1:15),imag(x2),'filled','k');
title("Imaginary part of x_{2}[n]");
ylabel("Im\{x_{2}[n]\}");
xlabel("n");

DFTx2_0 = 0;
for q = (0:15)
    DFTx2_0(1:16) = DFTx2_0 + x2(q+1)*exp(-2*pi*1i*(0:15)*q/16);
end



k = 0:15;
DFTx2_1(k+1) = DFTx(mod((k+8),16)+1);

figure(6);
tiledlayout(2,1);
ax1 = nexttile;
stem(ax1,(0:1:15),real(DFTx2_1),'filled','k');
title("Real part of 16pt DFT of x_{2}[n]: X_{2}[k]");
ylabel("Re\{X_{2}[k]\}");
xlabel("k");

ax2 = nexttile;
stem(ax2,(0:1:15),imag(DFTx2_1),'filled','k');
title("Imaginary part of 16pt DFT of x_{2}[n]: X_{2}[k]");
ylabel("Im\{X_{2}[k]\}");
xlabel("k");

%%----------------------------------////

%% Part c:
x3((0:15)+1) = x((0:15)+1);
x3((16:31)+1) = x((0:15)+1);
```

```
figure(7);
tiledlayout(2,1);

ax1 = nexttile;
stem(ax1,(0:1:31),real(x3),'filled','k');
title("Real part of x_{3}[n]");
ylabel("Re\{x_{3}[n]\}");
xlabel("n");

ax2 = nexttile;
stem(ax2,(0:1:31),imag(x3),'filled','k');
title("Imaginary part of x_{3}[n]");
ylabel("Im\{x_{3}[n]\}");
xlabel("n");

DFTx3_0 = 0;
for q = (0:31)
   DFTx3_0(1:32) = DFTx3_0 + x3(q+1)*exp(-2*pi*1i*(0:31)*q/32);
end



DFTx3_1(2*(0:15)+1) = 2*DFTx((0:15)+1);
DFTx3_1(2*(0:15)+2) = 0;



figure(8);
tiledlayout(2,1);
ax1 = nexttile;
stem(ax1,(0:1:31),real(DFTx3_1),'filled','k');
title("Real part of 32pt DFT of x_{3}[n]: X_{3}[k]");
ylabel("Re\{X_{3}[k]\}");
xlabel("k");

ax2 = nexttile;
stem(ax2,(0:1:31),imag(DFTx3_1),'filled','k');
title("Imaginary part of 32pt DFT of x_{3}[n]: X_{3}[k]");
```

```
ylabel("Im\{X_{3}[k]\}");
xlabel("k");

%%-----------------------------------////

%% Part d:
x4((0:7)+1) = x((0:7)+1) + x((0:7)+9);

figure(9);
tiledlayout(2,1);

ax1 = nexttile;
stem(ax1,(0:1:7),real(x4),'filled','k');
title("Real part of x_{4}[n]");
ylabel("Re\{x_{4}[n]\}");
xlabel("n");

ax2 = nexttile;
stem(ax2,(0:1:7),imag(x4),'filled','k');
title("Imaginary part of x_{4}[n]");
ylabel("Im\{x_{4}[n]\}");
xlabel("n");

DFTx4_0 = 0;
for q = (0:7)
   DFTx4_0(1:8) = DFTx4_0 + x4(q+1)*exp(-2*pi*1i*(0:7)*q/8);
end



k = 0:7;
DFTx4_1(k+1) = DFTx(2*k+1);

figure(10);
tiledlayout(2,1);
ax1 = nexttile;
stem(ax1,(0:1:7),real(DFTx4_1),'filled','k');
```

```
title("Real part of 8pt DFT of x_{4}[n]: X_{4}[k]");
ylabel("Re\{X_{4}[k]\}");
xlabel("k");

ax2 = nexttile;
stem(ax2,(0:1:7),imag(DFTx4_1),'filled','k');
title("Imaginary part of 8pt DFT of x_{4}[n]: X_{4}[k]");
ylabel("Im\{X_{4}[k]\}");
xlabel("k");

%%-----------------------------------////

%% Part e:
x5((0:15)+1) = x((0:15)+1);
x5((16:31)+1) = 0+0i;

figure(11);
tiledlayout(2,1);

ax1 = nexttile;
stem(ax1,(0:1:31),real(x5),'filled','k');
title("Real part of x_{5}[n]");
ylabel("Re\{x_{5}[n]\}");
xlabel("n");

ax2 = nexttile;
stem(ax2,(0:1:31),imag(x5),'filled','k');
title("Imaginary part of x_{5}[n]");
ylabel("Im\{x_{5}[n]\}");
xlabel("n");

DFTx5_0 = 0;
for q = (0:31)
    DFTx5_0(1:32) = DFTx5_0 + x5(q+1)*exp(-2*pi*1i*(0:31)*q/32);
end
```

```
DFTx5_1 = (0:31)*0;
for m = (0:31)
    for k = (0:15)
        if 2*k-m == 0
            DFTx5_1(m+1) = DFTx(k+1);
        else
            DFTx5_1(m+1) = DFTx5_1(m+1) + DFTx(k+1)*(1-(-1)^(2*k-m))/(1-exp(1i*pi*(2*k-m)/16))/16;
        end
    end
end


figure(12);
tiledlayout(2,1);
ax1 = nexttile;
stem(ax1,(0:1:31),real(DFTx5_1),'filled','k');
title("Real part of 32pt DFT of x_{5}[n]: X_{5}[k]");
ylabel("Re\{X_{5}[k]\}");
xlabel("k");

ax2 = nexttile;
stem(ax2,(0:1:31),imag(DFTx3_1),'filled','k');
title("Imaginary part of 32pt DFT of x_{5}[n]: X_{5}[k]");
ylabel("Im\{X_{5}[k]\}");
xlabel("k");

%%----------------------------------////

%% Part f:
x6((0:2:31)+1) = x((0:15)+1);
x6((0:2:31)+2) = 0;

figure(13);
tiledlayout(2,1);

ax1 = nexttile;
```

```
stem(ax1,(0:1:31),real(x6),'filled','k');
title("Real part of x_{6}[n]");
ylabel("Re\{x_{6}[n]\}");
xlabel("n");

ax2 = nexttile;
stem(ax2,(0:1:31),imag(x6),'filled','k');
title("Imaginary part of x_{6}[n]");
ylabel("Im\{x_{6}[n]\}");
xlabel("n");

DFTx6_0 = 0;
for q = (0:31)
   DFTx6_0(1:32) = DFTx6_0 + x6(q+1)*exp(-2*pi*1i*(0:31)*q/32);
end

DFTx6_1 = DFTx(mod(0:31,16)+1);



figure(14);
tiledlayout(2,1);
ax1 = nexttile;
stem(ax1,(0:1:31),real(DFTx6_1),'filled','k');
title("Real part of 32pt DFT of x_{6}[n]: X_{6}[k]");
ylabel("Re\{X_{6}[k]\}");
xlabel("k");

ax2 = nexttile;
stem(ax2,(0:1:31),imag(DFTx3_1),'filled','k');
title("Imaginary part of 32pt DFT of x_{6}[n]: X_{6}[k]");
ylabel("Im\{X_{6}[k]\}");
xlabel("k");
%%-----------------------------------////

%% Part g:
x7((0:7)+1) = x((0:2:15)+1);
```

```
figure(15);
tiledlayout(2,1);

ax1 = nexttile;
stem(ax1,(0:1:7),real(x7),'filled','k');
title("Real part of x_{7}[n]");
ylabel("Re\{x_{7}[n]\}");
xlabel("n");

ax2 = nexttile;
stem(ax2,(0:1:7),imag(x7),'filled','k');
title("Imaginary part of x_{7}[n]");
ylabel("Im\{x_{7}[n]\}");
xlabel("n");

DFTx7_0 = 0;
for q = (0:7)
    DFTx7_0(1:8) = DFTx7_0 + x7(q+1)*exp(-2*pi*1i*(0:7)*q/8);
end



k = 0:7;
DFTx7_1(k+1) = 0.5*(DFTx(k+1)+DFTx(k+9));

figure(16);
tiledlayout(2,1);
ax1 = nexttile;
stem(ax1,(0:1:7),real(DFTx7_1),'filled','k');
title("Real part of 8pt DFT of x_{7}[n]: X_{7}[k]");
ylabel("Re\{X_{7}[k]\}");
xlabel("k");

ax2 = nexttile;
stem(ax2,(0:1:7),imag(DFTx7_1),'filled','k');
title("Imaginary part of 8pt DFT of x_{7}[n]: X_{7}[k]");
```

```
ylabel("Im\{X_{7}[k]\}");
xlabel("k");

%%-----------------------------------////
```