EPFL Semestr Project

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October 2024

$$p(\mathbf{x}) =: \mathbf{1}_{[-0.5,0.5]^2}(\mathbf{x}), \tag{1}$$

$$\phi(\mathbf{x}) =: \mathbf{1}_{[0,1]^2}(\mathbf{x}), \tag{2}$$

$$\hat{\phi}(\mathbf{f}) =: \text{sinc}(f_1) \cdot \text{sinc}(f_2) \cdot e^{-j\pi(f_1+f_2)} \tag{3}$$

$$|FDx|^2 = \mathcal{F}\Big(p(2^w \cdot - \mathbf{k}) \sum_{\mathbf{n}\in[0,2^\zeta-1]^2} c_\mathbf{n} \cdot \phi(2^\zeta \cdot - \mathbf{n})\Big)\{\mathbf{s}\} \quad \text{where } \mathbf{s},\mathbf{k} \in \mathbb{R}^2 \tag{4}$$

$$\hat{c}_\mathbf{n} = c_\mathbf{n} \cdot p(2^w \cdot - \mathbf{k}) \quad \textbf{for } w > s \tag{5}$$

Then,

$$\begin{aligned} |FDx|^2 &= \mathcal{F}\Big(\sum_{\mathbf{n}\in[0,2^\zeta-1]^2} \hat{c}_\mathbf{n} \cdot \phi(2^\zeta \cdot - \mathbf{n})\Big)\{\mathbf{s}\} \\ &= \sum_{\mathbf{n}\in[0,2^\zeta-1]^2} \hat{c}_\mathbf{n} \cdot \mathcal{F}(\phi(2^\zeta \cdot - \mathbf{n}))\{\mathbf{s}\} \\ &= \sum_{\mathbf{n}\in[0,2^\zeta-1]^2} \hat{c}_\mathbf{n} \cdot 2^{-2\zeta} \cdot \hat{\phi}(2^{-\zeta}\mathbf{s}) \cdot e^{-j2\pi\frac{<\mathbf{n},\mathbf{s}>}{2^\zeta}} \end{aligned} \tag{6}$$

Where,

$$\sum_{\mathbf{n}\in[0,2^\zeta-1]^2} \hat{c}_\mathbf{n} \cdot e^{-j2\pi\frac{<\mathbf{n},\mathbf{s}>}{2^\zeta}}\Bigg|_{\mathbf{s}=\mathbf{m}} = \text{DFT}(\hat{c}_\mathbf{n})\{\mathbf{s}\} \quad \text{for } \mathbf{m} \in [0, 2^\zeta - 1]^2 \tag{7}$$

Then the discretized result is,

$$\begin{aligned} &\sum_{\mathbf{n}\in[0,2^\zeta-1]^2} \hat{c}_\mathbf{n} \cdot 2^{-2\zeta} \cdot \hat{\phi}(2^{-\zeta}\mathbf{s}) \cdot e^{-j2\pi\frac{<\mathbf{n},\mathbf{s}>}{2^\zeta}}\Bigg|_{\mathbf{s}=\mathbf{m}} \quad \text{for } \mathbf{m} \in [0, 2^\zeta - 1]^2 \\ &= 2^{-2\zeta}\text{DFT}_{2^\zeta}(\hat{c}_\mathbf{n}) \odot \hat{\phi}(\mathbf{2}^{-\zeta}) \end{aligned} \tag{8}$$