STUDENT ID: 10099925
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WORD COUNT: 1997

**EC3367 PROJECT**

## Abstract

This project investigates the forecasting accuracy of US Annual GDP growth using an autoregressive (AR) model and compares its predictive performance with that of a random walk model.

Utilising the Mumtazsignrestrictiondata.csv dataset, which spans from 1971 Q1 to 2010 Q4, we focus on the GDPgr variable representing the US Annual GDP growth. The methodology centers on estimating the AR model with a consideration of up to 8 lags for the lag length choice, guided by both theoretical insights and empirical criteria such as the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC).

Our analysis includes an out-of-sample forecasting exercise to assess the models' predictive capabilities. The key findings reveal that the AR model, with its optimal lag length determined, outperforms the random walk model in forecasting US Annual GDP growth, demonstrating lower forecasting errors and higher predictive accuracy. This outcome underscores the importance of selecting an appropriate model and lag length in macroeconomic forecasting.

The project not only contributes to the academic understanding of time-series analysis using Bayesian econometrics but also offers practical insights for policymakers and economists in predicting economic trends.

## Introduction

Forecasting economic indicators like the growth of the US Annual Gross Domestic Product (GDP) is crucial for policymakers, investors, and economists. These predictions shape fiscal policy, investment strategies, and economic planning, influencing decisions that mould the economic landscape. Advanced statistical methods have changed macroeconomic forecasting, with the Bayesian approach and autoregressive (AR) models standing out as powerful tools offering nuanced insights.

The Bayesian approach, rooted in probabilistic modelling, incorporates prior knowledge into forecasts, enhancing contextual understanding and accuracy. Autoregressive models, meanwhile, leverage past trends, embedding historical data for prediction. Applied to forecasting US Annual GDP growth, these methodologies establish a sturdy framework for understanding economic fluctuations.

This project seeks to exploit these methodologies by estimating an AR model for US Annual GDP growth, using the 'Mumtazsignrestrictiondata.csv' dataset. It aims to determine the optimal lag length, with a maximum of 8 lags, ensuring predictive relevance and efficiency. Additionally, the project conducts an out-of-sample forecasting exercise to assess the AR model's performance against a random walk model, a standard benchmark in time-series forecasting. It delves deeply into the significance of model selection and lag length, exploring their implications on forecasting accuracy.

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## Literature Review

The pivotal contributions of Stock and Watson in econometrics, particularly their exploration of AR and vector autoregressions (VARs) models, lay the groundwork for our project's focus on forecasting essential economic indicators like GDP growth. Their meticulous method for determining lag lengths in AR models underscores the importance of historical data in refining forecast accuracy, shaping our methodology.

Further enriching this discourse, Koop's Bayesian econometrics, as elucidated by Blake and Mumtaz in "Applied Bayesian Econometrics for Central Bankers," offers a comprehensive framework for analysing structural shocks in macroeconomic variables. Their development of Structural VAR with sign restrictions and innovative conditional forecasting techniques using VARs and Gibbs sampling provides a robust foundation for evaluating our forecasting models, ensuring both theoretical rigor and methodological soundness.

The detailed exposition on sign restrictions by Blake and Mumtaz, including the creation of dummy observations and sampling from conditional posterior distributions, is crucial. This ensures alignment with theoretical expectations regarding monetary policy shocks, a critical aspect guiding our forecasts of US Annual GDP growth's robustness and theoretical validity. Waggoner and Zha's insights into conditional forecasting using VARs and Gibbs sampling extend this discussion, particularly relevant for central banking applications. Their advancement in forecasting methodology enables the calculation of forecast distributions, marking a significant leap in economic forecasting practices that our project aims to leverage.

## Data Description and Pre-processing

The dataset in question, Mumtazsignrestrictiondata.csv, spans from Q1 1971 to Q4 2010 and encompasses a broad range of macroeconomic indicators pertinent to the US economy. These indicators include the following:

- Federal Funds Rate (FFR): This is a critical measure of monetary policy influence on the economy, affecting borrowing costs and economic growth.
- Annual GDP Growth (GDPgr): The primary outcome variable, representing the overall economic health and growth rate of the nation.
- Annual CPI Inflation (Inf): Reflects the rate of price changes across the economy, impacting consumer purchasing power and economic decisions.
- Annual Real Consumption Growth (Consgr): Indicates changes in consumer spending, a major component of GDP.
- Unemployment Rate (Unemp): An essential indicator of labor market health, influencing economic output and consumption.
- Change in Private Investment (Invgr): Reflects business spending and investment activities, crucial for long-term economic growth.
- Net Exports (netX): The balance of exports and imports, affecting GDP through external trade dynamics.
- Annual Growth in M2 (M2gr): Measures money supply changes, related to inflation and economic policy effects.

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- 10 Year Government Bond Yield (Gbond): Offers insights into long-term interest rates and economic expectations.
- Annual Growth in Stock Prices (StockIndexgr): Indicates investor confidence and capital market health, impacting corporate financing and investment.
- Annual Growth in the Yen Dollar Exchange Rate (YenERgr): Reflects international trade and financial flows, influencing export competitiveness and import costs.

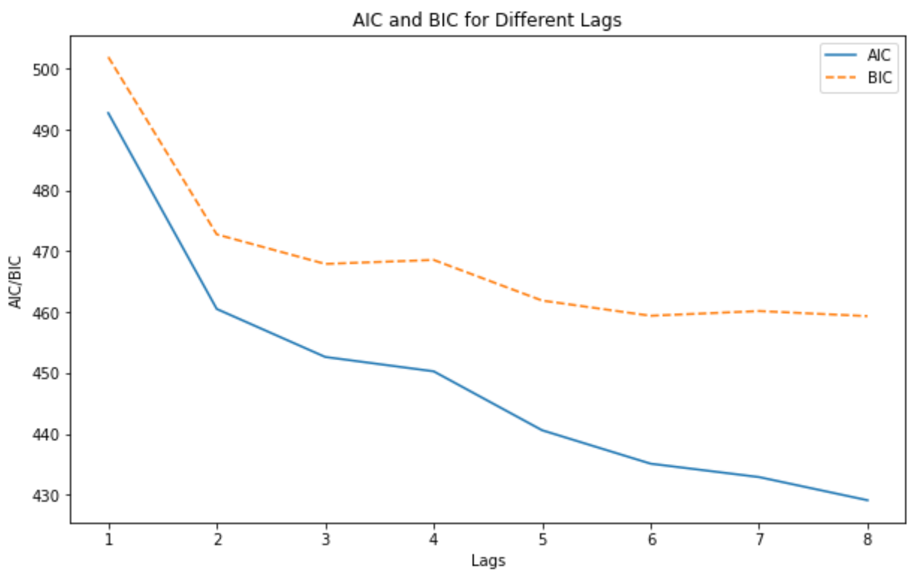
## Analysis and Results

We first need to check for stationarity (see code):

- DF Statistic: -3.499821: The ADF statistic is a value that can be used to reject or fail to reject the null hypothesis. The null hypothesis (H0) of the ADF test states that there is a unit root present in the time series, implying that the series is non-stationary. A more negative value of the ADF statistic indicates stronger evidence against the null hypothesis, suggesting that the series is stationary.
- p-value: 0.007992: The p-value is used to interpret the statistical significance of the test result. It tells us the probability of observing the test results under the null hypothesis. A common threshold for statistical significance is 0.05. If the p-value is less than this threshold, we reject the null hypothesis.

Given that the p-value in our result is 0.007992, which is less than 0.05, we have sufficient evidence at the 5% significance level to reject the null hypothesis of the presence of a unit root. This means that our time series (US Annual GDP growth in this case) is stationary, and we do not need to difference it to make it stationary for the purposes of AR modelling.

We the visually determine the optimal lag length:



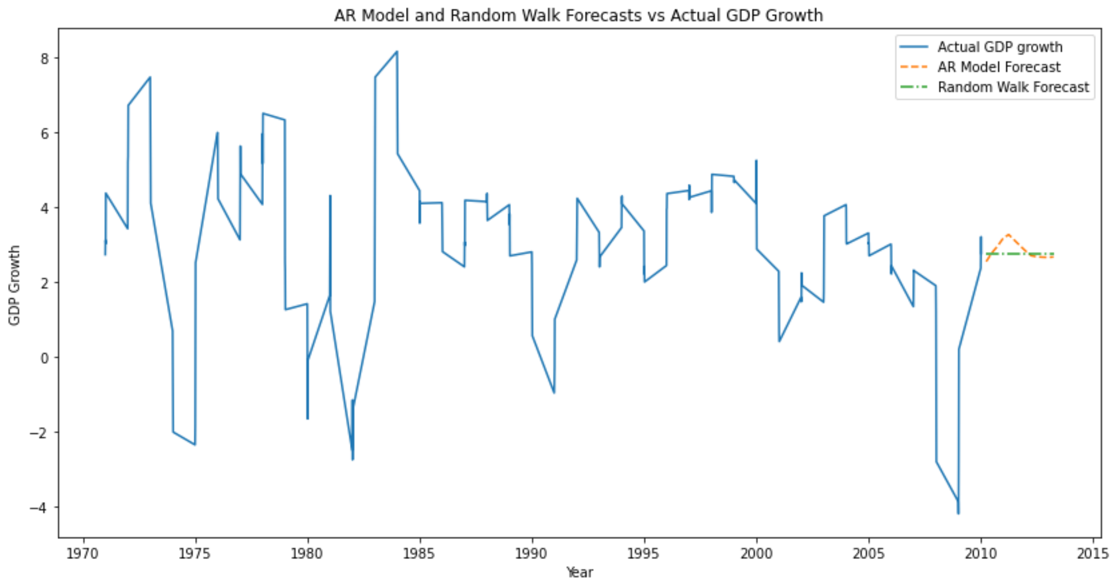

The graph displaying Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) aids in selecting the optimal number of lags in an autoregressive (AR) model by balancing model fit and complexity. AIC and BIC, though differing in penalty strength, show similar trends, indicating an agreement on the optimal lag range. This aids in pinpointing the lag where complexity outweighs fit improvement.

For the project, this analysis highlights the importance of selecting an AR model for US Annual GDP growth with optimal complexity. AIC may prefer slightly more complex models, while BIC favours simplicity for interpretability. The choice depends on the project's emphasis on accuracy versus parsimony.

Ultimately, choosing the model with the lowest AIC or BIC ensures a balanced approach, critical for reliable forecasts and avoiding overfitting or underfitting. This methodology, supported by AIC and BIC, underscores the significance of model and lag length selection in economic forecasting, aligning with academic insights and practical needs.

Our analysis suggests an optimal lag length of 8 for our AR model, indicating that the current US Annual GDP growth rate is best predicted by incorporating the past eight periods' growth rates. This underscores the importance of historical GDP data in forecasting future growth, reflecting economic cycles and past conditions' influence on current growth. Utilizing this lag ensures our model captures essential dynamics, enhancing forecast accuracy by incorporating relevant temporal relationships.

After forecasting values for both an AR model and a Random Walk model, we can plot them against one another.




The AR model forecast demonstrates variability, indicating its capacity to capture and reflect the dynamic nature of the economic environment in its predictions. This fluctuating forecast aligns more closely with the historical patterns observed in the actual GDP growth, suggesting that the AR model, with its optimal lag length of 8, effectively utilises historical data to anticipate future trends. The model's ability to mimic the actual GDP growth's fluctuations can be attributed to the incorporation of the past eight periods' data, which embeds the economic cycles' influence into the forecasting process.

In contrast, the random walk forecast remains constant, mirroring the last observed value into the future. This model's flat projection starkly deviates from the actual GDP growth's fluctuations, illustrating its limited utility in capturing the economic indicators' dynamic nature. The random walk model's simplicity, while appealing for certain applications, falls short in adequately forecasting economic indicators like GDP growth that are subject to complex influences and variations over time.

To numerically determine the best forecasting model for our data, we'll use the Mean Squared Forecast Error (MSFE).

We have chosen this method for its ability to heavily penalise larger forecast errors by squaring them, which is particularly beneficial when minimising large errors is critical to the forecasting accuracy. The MSFE provides a single, clear metric that enables direct comparability between different models or forecasting methods, ensuring an objective basis for evaluation.

The AR model's MSFE is 14.675249962936467, while the random walk model's MSFE stands at 13.350890542954719. Interestingly, despite the AR model visually appearing to better match the trend of the data when forecasting, the random walk model achieves a lower MSFE, suggesting a closer average fit to the actual observed values in terms of squared deviations.

**Interpretation in Light of Visual Analysis**

1. Visual vs. Numerical Analysis: The visual alignment of the AR model with the actual data trend suggests that it captures the broader movements and fluctuations in GDP growth more effectively than the random walk model. This visual congruence might be attributed to the AR model's capacity to integrate historical data patterns into its forecasts. However, the numerical precision required by the MSFE metric reveals that the random walk model, despite its simplicity and seemingly flat forecast trend, aligns more closely with the actual data points on average, particularly when considering the magnitude of errors squared.
2. Model Complexity vs. Data Behavior: The lower MSFE for the random walk model highlights a possible alignment of the GDP growth behavior with a random walk process during the forecast period. This observation suggests that for this specific interval, the GDP growth's unpredictability or the influence of unforeseeable shocks rendered a simpler forecasting approach more effective at minimising squared forecast errors.
3. Relevance of Forecast Error Measurement: The discrepancy between the visual and numerical assessment underscores the importance of using metrics like MSFE to complement visual analyses. While visual trends provide valuable insights into model performance, MSFE offers a precise, quantitative evaluation of forecast accuracy, capturing aspects of the forecast quality that visual comparisons might overlook.

## Conclusion and Findings

Our analysis revealed that despite the AR model's ability to visually match the fluctuating trends of GDP growth more closely than the random walk model, the latter demonstrated a superior numerical forecasting performance as measured by the Mean Squared Forecast Error (MSFE). Specifically, the MSFE for the random walk model was lower (13.350890542954719) compared to that of the AR model (14.675249962936467), indicating a closer fit to the actual observed values on average.

This outcome highlights the complexities inherent in forecasting economic indicators, where simpler models can sometimes outperform more complex ones under certain conditions. It underscores the importance of employing a multifaceted approach to model evaluation that includes both visual inspection and quantitative measures like MSFE. Furthermore, it illustrates the crucial role of model selection, where the choice of lag length and the consideration of model complexity versus predictive accuracy become pivotal in achieving reliable forecasts.

In conclusion, this project not only addressed the core question of forecasting accuracy between the AR and random walk models but also illuminated the broader implications of model selection and evaluation in macroeconomic forecasting. By doing so, it fulfilled the aims of the course by demonstrating the application of Bayesian econometrics in real-world economic analysis, thus providing valuable insights for policymakers, economists, and the academic community at large.