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Question 1	

## 1

### 1.1 a)

We are given three functions which describe the derivative of three functions,  $x, y$ , and  $z$ , with respect to time (i.e., differential equations). The three functions are coupled in that they depend on each other and not time. To solve the system (given  $r$  and  $s$ ) using RK2, at each time step we approximate the values of  $x, y$ , and  $z$  using the previously computed values and the values of the derivatives evaluated at these previously computed values (i.e., we use  $(x_i, y_i, z_i)$  and  $x', y', z'$  evaluated at this point to compute  $(x_{i+1}, y_{i+1}, z_{i+1})$ . The code implementing this (and which produced the figures below) is in the script `q1_rockpaperscissors_RK2.py`

We set  $r = -1$  and  $s = 1$  and plot for the initial conditions  $v_1 = (0.2, 0.3, 0.5)$ ,  $v_2 = (0.3, 0.2, 0.5)$ ,  $v_3 = (0.2, 0.5, 0.3)$ ,  $v_4 = (0.5, 0.3, 0.2)$ ,  $v_5 = (0.3, 0.5, 0.2)$ , and  $v_6 = (0.5, 0.2, 0.3)$  (figures shown in the files `vi_A_plot.png` where  $1 \leq i \leq 6$ ). Note that the initial conditions as above account for all permutations of the sizes of the numbers in each coordinate. For example  $v_1$  has  $x_0$  the smallest,  $y_0$  in-between and  $z_0$  the largest,  $v_4$  is reversed, etc.

From the figures we observe that the six initial conditions can fall into one of two types; where two initial conditions being in the same type implies they have the same overall behaviour. We see in  $v_2, v_3$ , and  $v_4$  the variable corresponding to the smallest and largest value decreases linearly, while the other variable increases linearly, until a stationary state is reached and all three become constant. In the other case though we have the reverse occurring (e.g., the variable with the largest initial condition decreases linearly until its stationary value is reached).

## 1.2 b)

Repeating the above but with  $r=1, s=1$  first and then  $r=-1, s=-1$  we obtain the plots `vi_B_pos_plot.py` and `vi_B_neg_plot.py` for  $1 \leq i \leq 6$ . We observe that when  $r+s>0$ , all three variables decrease linearly until a stationary state is reached. Moreover, the values of each of the variables in the stationary state depend on not just the initial values, but the ordering of the sizes of the initial values. For example, we see that  $v_2$  to  $v_4$  are the same up to permutation of variables and similarly for  $v_1, v_5$ , and  $v_6$ . When  $r+s<0$  we observe a similar situation as to the above except that the variables are linearly increasing instead of decreasing.