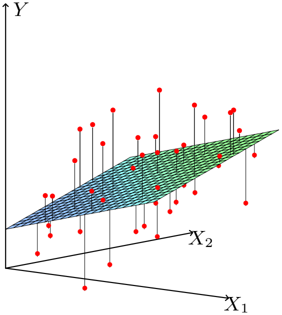
Geometric Interpretation of LS



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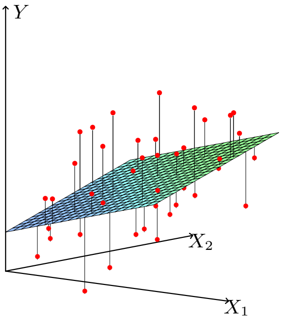

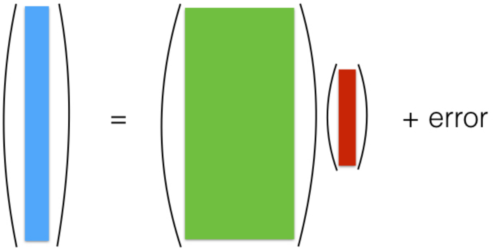

Vectors

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \in \mathbb{R}^2, \quad \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \in \mathbb{R}^3, \quad \mathbf{v}_{n\times 1} = \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_n \end{pmatrix} \in \mathbb{R}^n$$

Vector = Point

A point $\in \mathbb{R}^n$ corresponds to a vector starting from the origin and pointing to that point.

addition and scalar multiplication

$$\begin{aligned} 2\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + 3\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} &= \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 9 \\ 3 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 11 \\ 7 \\ 3 \end{pmatrix} \end{aligned}$$

Linear Subspace

Let $\mathcal{M}$ be a collection of vectors from $\mathbb{R}^n$. $\mathcal{M}$ is a linear subspace if $\mathcal{M}$ is closed under linear combinations.

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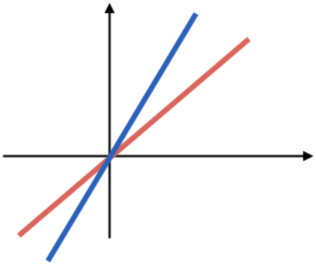
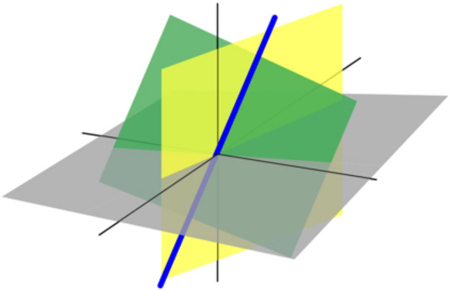
- You can image a linear subspace as a bag of vectors. For any two vectors in of that bag $(\mathbf{u}, \mathbf{v})$, their linear combinations (e.g., $\mathbf{u} - 2\mathbf{v}$), are also in the bag.
- The two vectors could be the same (i.e., you are allowed to create copies of vectors in that bag). So $\mathbf{0} = \mathbf{u} - \mathbf{u}$ is in any linear subspace (i.e., any linear subspace should pass the origin).

Examples of Linear Subspaces





Column Space $C(\mathbf{X})$

Columns of $\mathbf{X}$ form a linear subspace in $\mathbb{R}^n$, denoted by $C(\mathbf{X})$, which consists of vectors that can be written as linear combinations of columns of $\mathbf{X}$, i.e.,

$$C(\mathbf{X}) = \{\mathbf{X}\boldsymbol{\beta},\ \boldsymbol{\beta} \in \mathbb{R}^{p+1}\}.$$