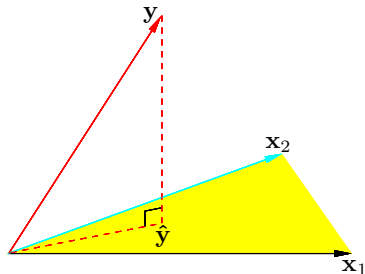


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$$\min_{\beta} \|\mathbf{y} - \mathbf{X}\beta\|^2,$$

which is equivalent to finding a vector  $\mathbf{v}$  from the subspace  $C(\mathbf{X})$  that minimizes  $\|\mathbf{y} - \mathbf{v}\|^2$ .



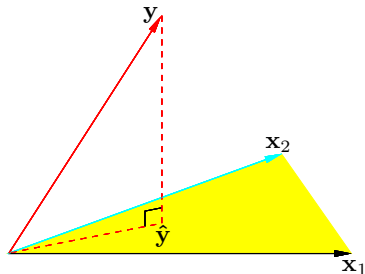
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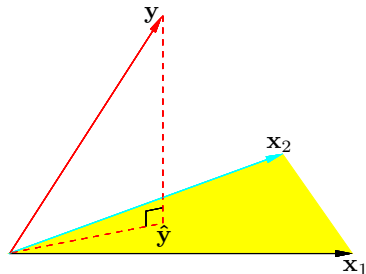
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**The essence of LS:** decompose the data vector  $\mathbf{y}$  into two orthogonal components,

$$\mathbf{y}_{n \times 1} = \hat{\mathbf{y}}_{n \times 1} + \mathbf{r}_{n \times 1}.$$