

$\theta$ , parameters we are trying to optimize

- parameters:  $\mu, \Sigma, \pi$

$$\text{maximize} - \sum_{n=1}^N \ln \left\{ \sum_{k=1}^K \pi_k \mathcal{N}(x_n | z_n, \Sigma_k) \right\} \quad (1)$$

Use EM algorithm to optimize (1)

$$\ln \left\{ \prod_n p(z_n | \pi) p(x_n | z_n, \theta) \right\}$$

$$E_{\text{step}} \rightarrow p(z_k = k | x_n, \theta) = \frac{\pi_k \mathcal{N}(x_n | \mu_k, \Sigma_k)}{\sum_{j=1}^K \pi_j \mathcal{N}(x_n | \mu_j, \Sigma_j)}$$

$$\mu_{k, \text{new}} = \frac{1}{N_k} \sum_{n=1}^N p(z_k = k | x_n, \theta) x_n$$

$$\Sigma_{k, \text{new}} = \frac{1}{N_k} \sum_{n=1}^N p(z_k = k | x_n, \theta) (x_n - \mu_{k, \text{new}})(x_n - \mu_{k, \text{new}})^T$$

$$\pi_{k, \text{new}} = \frac{N_k}{N} \quad N_k = \sum_{n=1}^N p(z_k = k | x_n, \theta)$$

$$\text{New likelihood} = \sum_{n=1}^N \ln \left\{ \sum_{k=1}^K \pi_k \mathcal{N}(x_n | \mu_k, \Sigma_k) \right\}$$