Frankfurt School of Finance & Management
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Examination in the Bachelor of Science
Course title: Data Collection & Games and Incentives
Lecturers: Prof. Dr. Frederik Schwerter
Examination date: 14th December 2021

Aids: non-programmable pocket calculator

Please enter your student ID (matriculation number)!

Student ID

Please note:

The exam consists of 5 questions of which you will have to answer **all** questions. You have **120** minutes to complete the examination. The maximum of points to be reached is **120**. Please use the enclosed answer sheet to answer your questions and add your student ID on its cover.

We wish you all the best for your examination!

Internal use only!

Question	1	2	3	4	5	**Total**
Possible points:	24	30	34	12	20	**120**
Points achieved:						

Signature of corrector

Question 1 – General Equilibrium (24 points)

(a) What are the definitions of Pareto improvement and Pareto efficiency? (4 points)

Assume there are two consumers, A and B, and two goods, X und Y. The initial endowment is such that consumer A is endowed with $(X_A,Y_A)=(2,4)$ and consumer B is endowed with $(X_B,Y_B)=(4,2)$. Both consumers have standard preferences, and their utility functions are different and equal to

$$U_A(X_A, Y_A) = X_A^2\, Y_A \quad \text{and} \quad U_B(X_B, Y_B) = X_B Y_B^2$$

(b) Determine consumer A and B's utility at their initial endowment. (2 points)

(c) Determine (mathematically) the slopes of the indifference curves for the two consumers. (4 points)

(d) What is the definition of a competitive equilibrium? (3 points)

(e) Assume for this subquestion that the following prices were given:

$$p_X = 2 \quad and \quad p_Y = 1.$$

Given these prices and the consumers preferences, consumer A would maximize utility by consuming the bundle $(X_A = \frac{9}{2}, Y_A = \frac{9}{8})$ and consumer B would maximize utility by consuming the bundle $(X_B = \frac{10}{3}, Y_B = \frac{10}{3})$. Determine the excess supply and excess demand for the two goods. (3 Points)

(f) Assume for this subquestion that the following prices were given:

$$p_X = 2 \quad and \quad p_Y = 1.$$

Given these prices and the consumers preferences, consumer A would maximize utility by consuming the bundle $(X_A = 4, Y_A = 2)$ and consumer B would maximize utility by consuming the bundle $(X_B = 2, Y_B = 4)$. Show that the given prices implement a competitive equilibrium. (3 Points)

(g) State the first welfare theorem as well as the three key assumptions behind the first welfare theorem. (5 points)

Question 2 – Oligopoly (30 points)

Consider a market with two firms, firm 1 and firm 2, which produce a homogeneous product. Suppose that the firms simultaneously set their quantities (**Cournot competition**). The inverse demand function is $p(Q) = 20 - 2Q$, where $Q = q_1 + q_2$. The cost function of firm 1 is $C_1 = 4q_1$ and the cost function of firm 2 is $C_2 = 0$.

(a) Set up the profit functions of firm 1 and firm 2. Determine the first-order conditions of firm 1 and firm 2. (6 points)

(b) Determine the quantities of both firms in the Nash equilibrium. (6 points)

(c) Determine the profits of both firms in the Nash equilibrium. Explain why they differ. (4 points)

(d) Assume for this subquestion that the production technology of firm 2 is available also to firm 1 as well as to two additional firms that entered the market. Consequently, all four firms produce at no cost, such that $C_1 = C_2 = C_3 = C_4 = 0$. The inverse demand function maintains to be $p(Q) = 20 - 2Q$, where we now have $Q = q_1 + q_2 + q_3 + q_4$. Determine the quantities of the four firms in the Nash equilibrium. (6 points)

(e) Discuss verbally the differences between standard Cournot competition and standard Bertrand competition. (8 points)

Question 3 – Tacit Collusion (34 points)

Consider a situation with two firms (i.e., firm 1 and firm 2), which produce a homogenous product at the same marginal costs c. Competition occurs in prices (**Bertrand competition**). The two firms share the demand of their product equally in case they choose the same price. The firm with the lowest price supplies the entire demand in case their prices differ.

Assume that the monopoly profit in the market is 1, $\pi^M = 2$. If both firms produce at marginal costs, $p_1 = p_2 = c$, the profit of each firm is 0, $\pi_1 = 0,\ \pi_2 = 0$. If one firm chooses to charge the monopoly price, while the other firm undercuts the monopoly price by as little as possible, the profit of firm with monopoly price is zero and the profit of the firm who undercuts the monopoly price is equal to $x \leq 2$. If firm 2 charges monopoly price and firm 1 undercuts the monopoly price, then $\pi_1 = x \; and \; \pi_2 = 0$. If firm 1 charges monopoly price and firm 2 undercuts the monopoly price, then $\pi_1 = 0 \; and \; \pi_2 = x$.

Assume that both firms have the same discount factor δ between 0 and 1, $0 < \delta < 1$.

(a) Suppose both firms compete in one period only. Determine the prices of both firms in Nash equilibrium. (4 points)

(b) Please discuss the meaning of the discount factor δ. Do smaller values of δ imply smaller or greater discounting of future periods? (2 points)

(c) Suppose the two firms compete over **a finite** number of periods that is greater than 100. In principle, could firms sustain collusion if their discount factors are large enough? If yes, (i) determine the strategy and (ii) state how it is called. If no, explain why not by (i) describing how firms behave instead and (ii) how they do so. (6 points)

(d) Suppose the two firms compete over **an infinite** number of periods. Determine a reasonable strategy that allows the firms to sustain collusion, that is all firms set their prices equal to the monopoly price and share the monopoly profit, if their discount factors are large enough. How is such a strategy called? (6 points)

(e) Suppose again the two firms compete over an infinite number of periods. For each of the following discount factors and values of x, state and determine whether collusion (via monopoly price setting) can be an equilibrium outcome or not (8 points):

i. $\delta = 0.00001, x = \frac{1}{5}$
ii. $\delta < \frac{1}{3}, x = \frac{3}{2}$
iii. $\delta = \frac{2}{5}, x = \frac{7}{4}$
iv. $\delta = \frac{3}{5}, x = 2$

(f) Now assume that 3 new firms enter the market. These new firms also produce at marginal costs c and have the same discount factor δ as the other firms. Analogous to before: (i) all five firms share demand equally if they all choose the same price; (ii) if four firms charge the monopoly price and one firm undercuts the monopoly price, the firms with monopoly prices make a profit of zero and the undercutting firm makes a profit of $x \leq 2$.
Return to the cases i., ii., iii., and iv. of subquestion 3e. Would the arrival of new firms in the market change your responses to whether collusion can be an equilibrium outcome? (8 points)

Question 4 – Asymmetric Information (12 points)

Jane is responsible for the profits of all restaurants in region Z of the restaurant chain M. How high the profits of the restaurants in region Z are depends on how hard Jane works but also on many other factors she cannot influence and which cannot be observed by her or Kim, the owner of the restaurant chain. Kim wants to induce Jane to work hard but cannot (at reasonable cost) monitor Jane's effort level.

(a) Assume both Jane and Kim are **risk neutral**. Also assume that Kim wants to offer Jane a contract that induces Jane to provide high effort. Describe and explain verbally what the optimal contract would be. (4 points)

(b) Assume Jane is **risk averse**, while Kim maintains to be **risk neutral**. Again assume that Kim wants to offer Jane a contract that induces Jane to provide high effort. (i) Describe and explain verbally what trade-off Kim faces in designing an optimal contract. (ii) Describe and explain verbally how the optimal contract will look like. (iii) Name the two restrictions/constraints that Kim would have to take care of when proposing a contract. (8 points)

Question 5 – Data Collection (20 points)

a) State the definition of a third factor. (6 points)

Assume you collected data on 50k workers of a company. All of these workers are working in factories that are all located in region A. For each worker, you collected data on (i) whether or not the worker participated in on-the-job training a year ago and (ii) how productive the workers are in their job. The training program was intended to improve worker's productivity. In analyzing the effect of the training program on worker's productivity, the status of the training program is the independent variable, while worker's productivity is dependent variable.

b) Based on the above situation, explain the difference in questions pertaining to prediction and causality. (7 points)

c) Assume that you do not know for certain why or why not workers participated in the training program in region A. You observe, however, that workers in region A who participated in the training program are significantly more productive than workers who did not participate in the training program. Assume your colleague Bob wants to convince your CEO to implement on-the-job training programs in all other regions in which your company has factories. You are not sure whether you agree with Bob. You argue that the training program would be costly. Bob argues that your collected data proves that the training program would make workers more productive and hence likely increases revenues as well. Would you agree with Bob's statement? Explain your response. (7 points)