



Frankfurt School

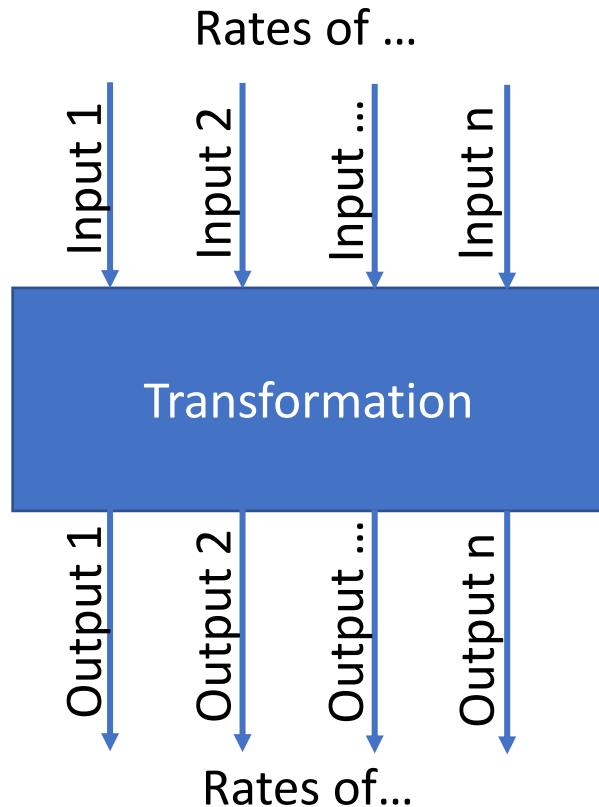
2023 Operations Management



INTRODUCTION TO OPERATIONS MANAGEMENT

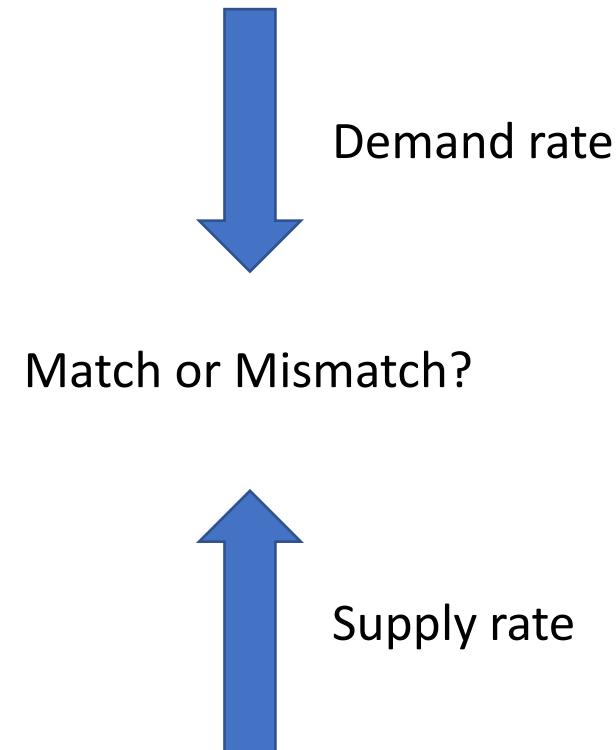
What is Operations Management (OM)?

- Managing...
- ...the transformation of an input rate into an output rate (=supply rate)...
- ...to match a demand rate and make money or achieve non-for-profit objectives.

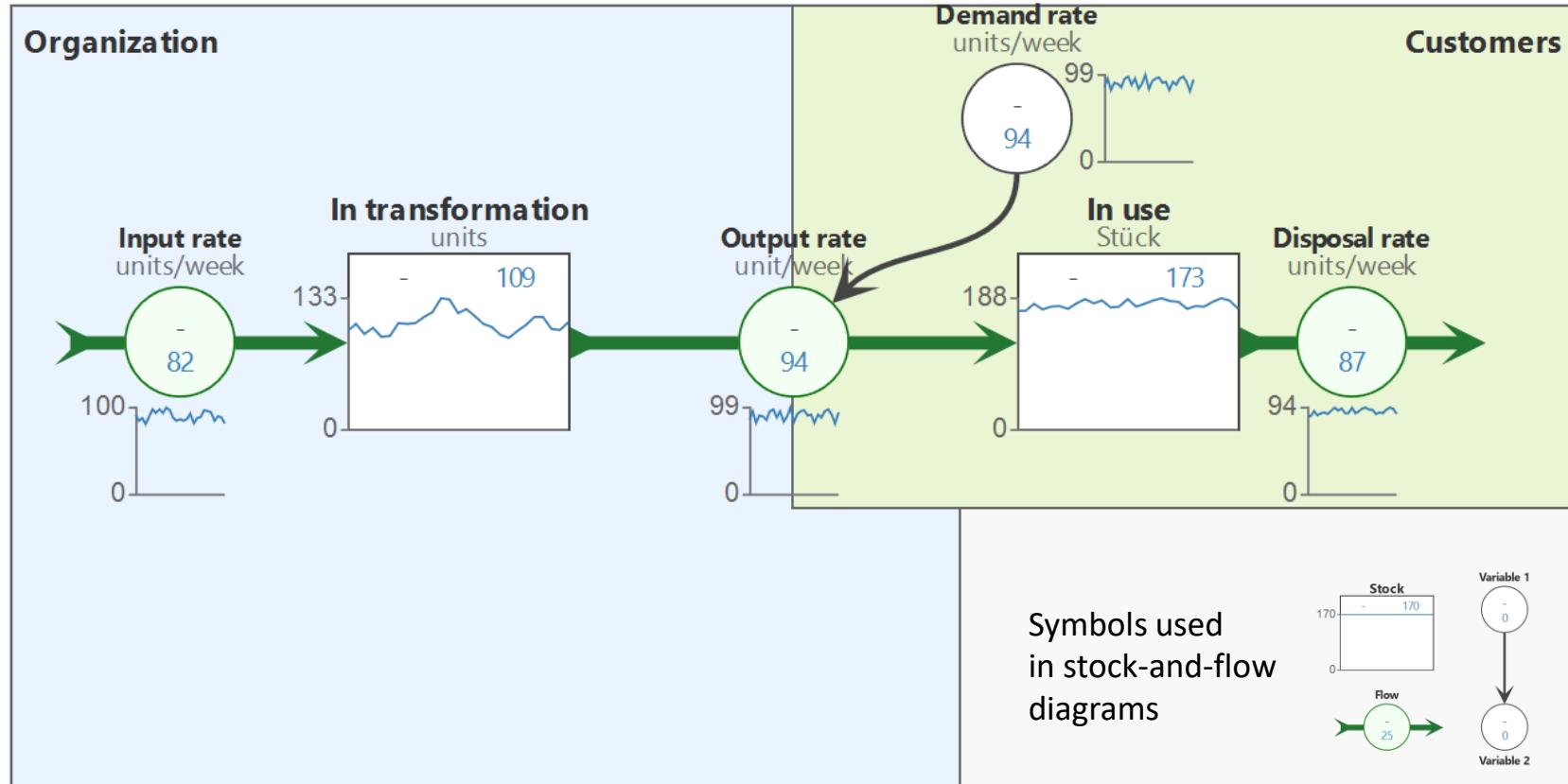


Supply and Demand

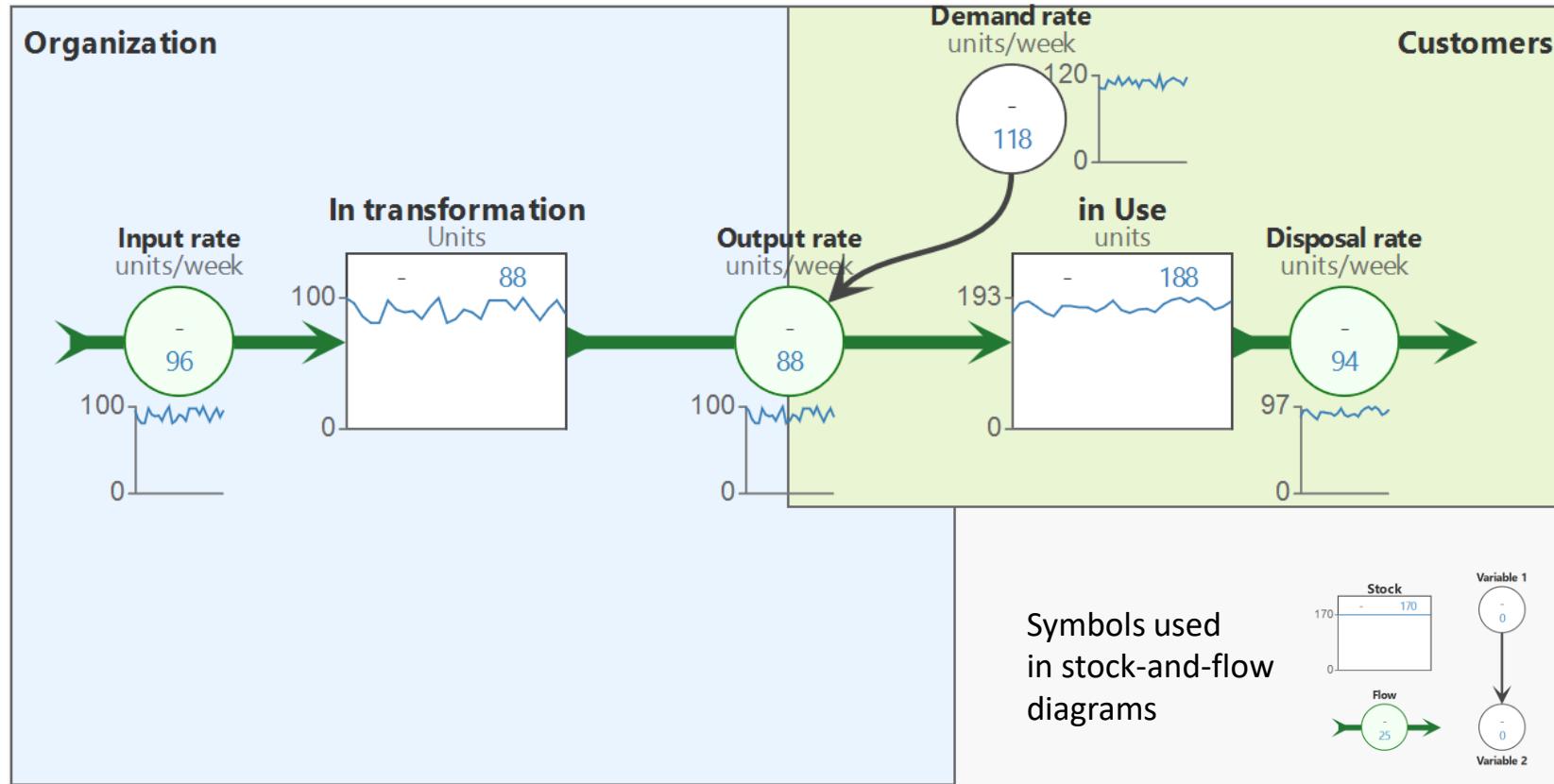
- Customer demand:
rate of products or services
requested by customers.
- Supply:
rate of products or services
provided by organizations (for-profit and non-for profit).



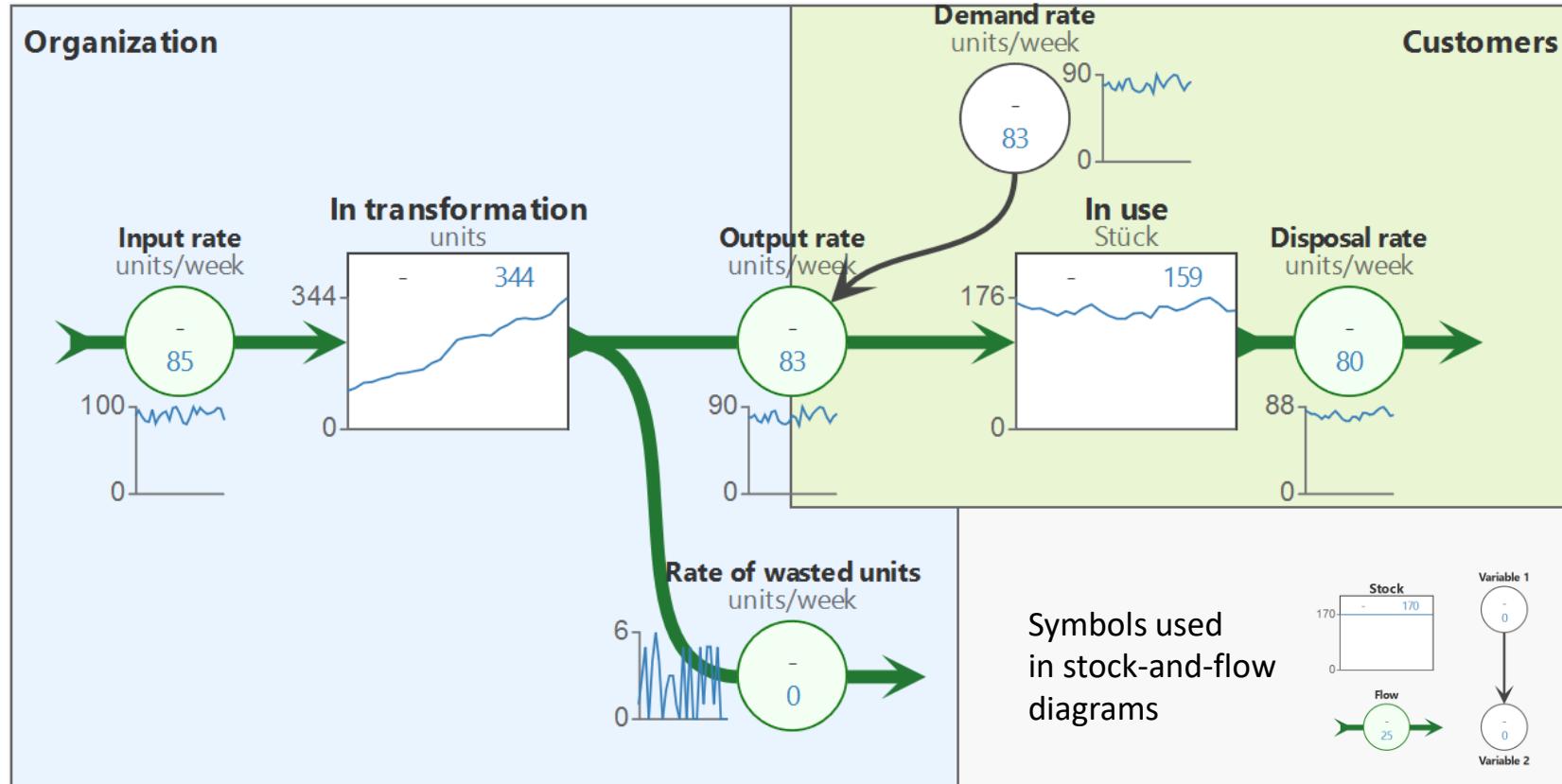
OM – Managing the „Flow“



OM – Managing the „Flow“



OM – Managing the „Flow“



HAIREXPRESS

- Professional hairdressing services at a very good price-performance ratio and quite spontaneous – while shopping;
- Enjoy your visit to the hairdresser without registration, in a friendly and fresh atmosphere;
- Our hairdressers specialize in fast cutting and service techniques and ensure that only the price, but not the quality, is saved.



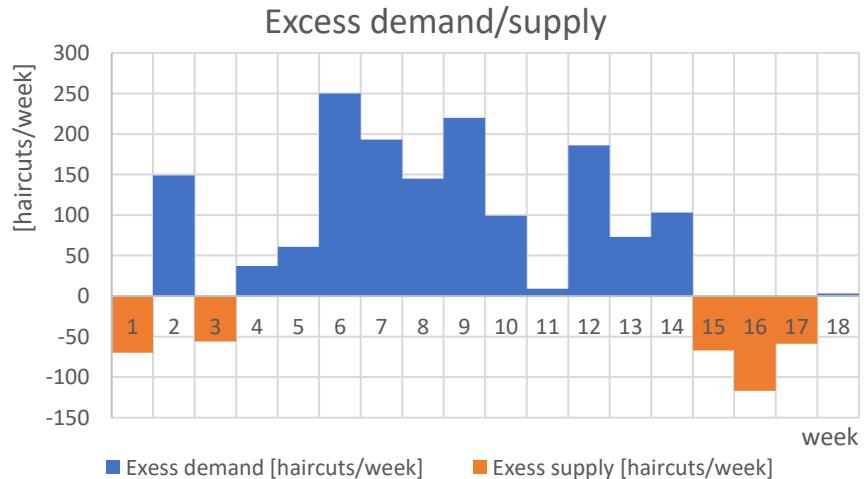
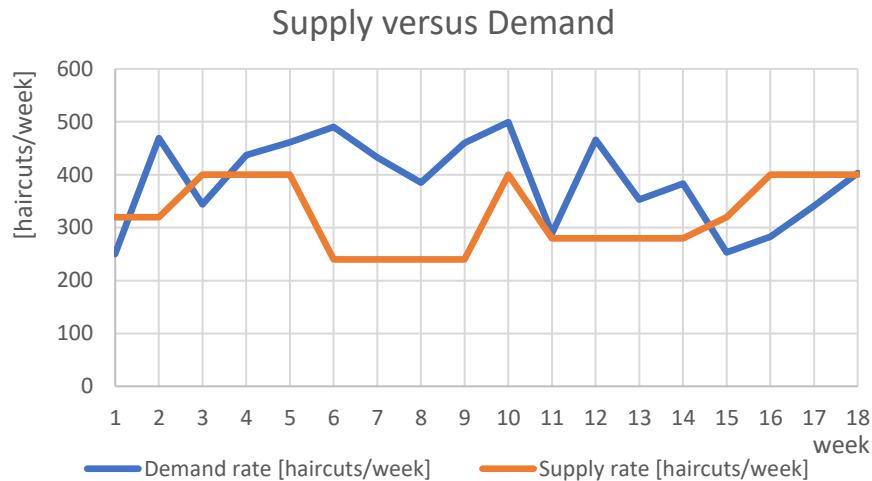
HAIREXPRESS Challenges

- What is the consequence of supply-demand mismatch?
- What does the customer want out of the operations of such a type of hairdressing saloon?



Supply-Demand Mismatch

- Excess demand
 - Theory: Price rise
 - Practice: Lost revenue
- Excess supply
 - Theory: Price fall
 - Practice: Resources are being wasted.



HAIREXPRESS Challenges

- What is the consequence of supply-demand mismatch?
- What does the customer want out of the operations of such a type of hairdressing saloon?



Consumer Utility at the Hairdresser

- Short hair, hair dye, perm;
- Hairstyling according to individual preferences;
- (Fair) price;
- A saloon nearby;
- Short waiting time.



A Second Example: Customer's Perspective

What does the customer want out of the operations of a mill (specialized on organic flour)?

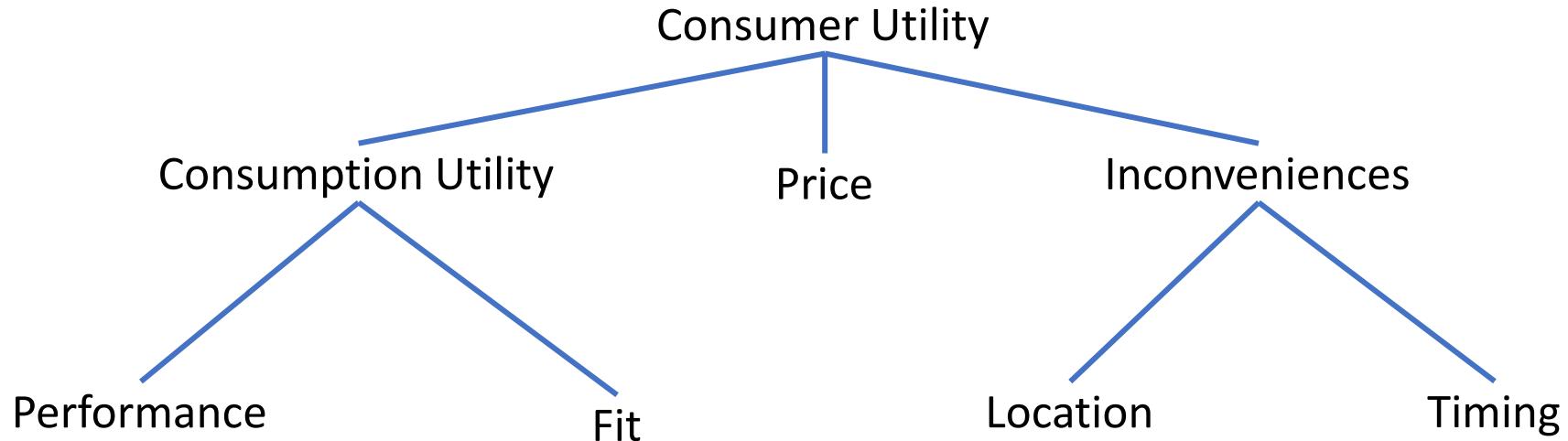


Consumer Utility at the Mill Shop

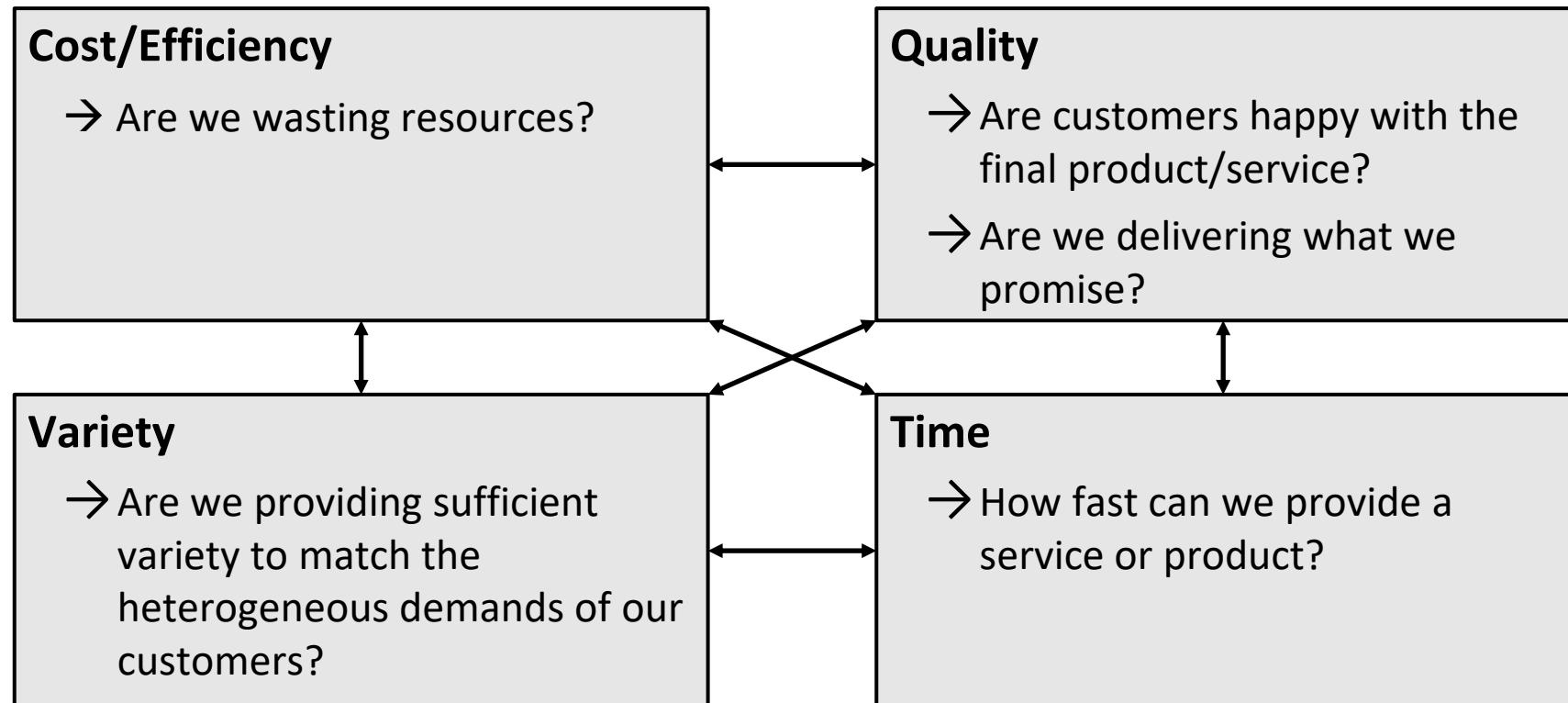
- Quantity (more is better, up to a certain limit);
- Fit of product attributes with individual preferences;
- (Fair) price;
- Short distance;
- Short lead time.



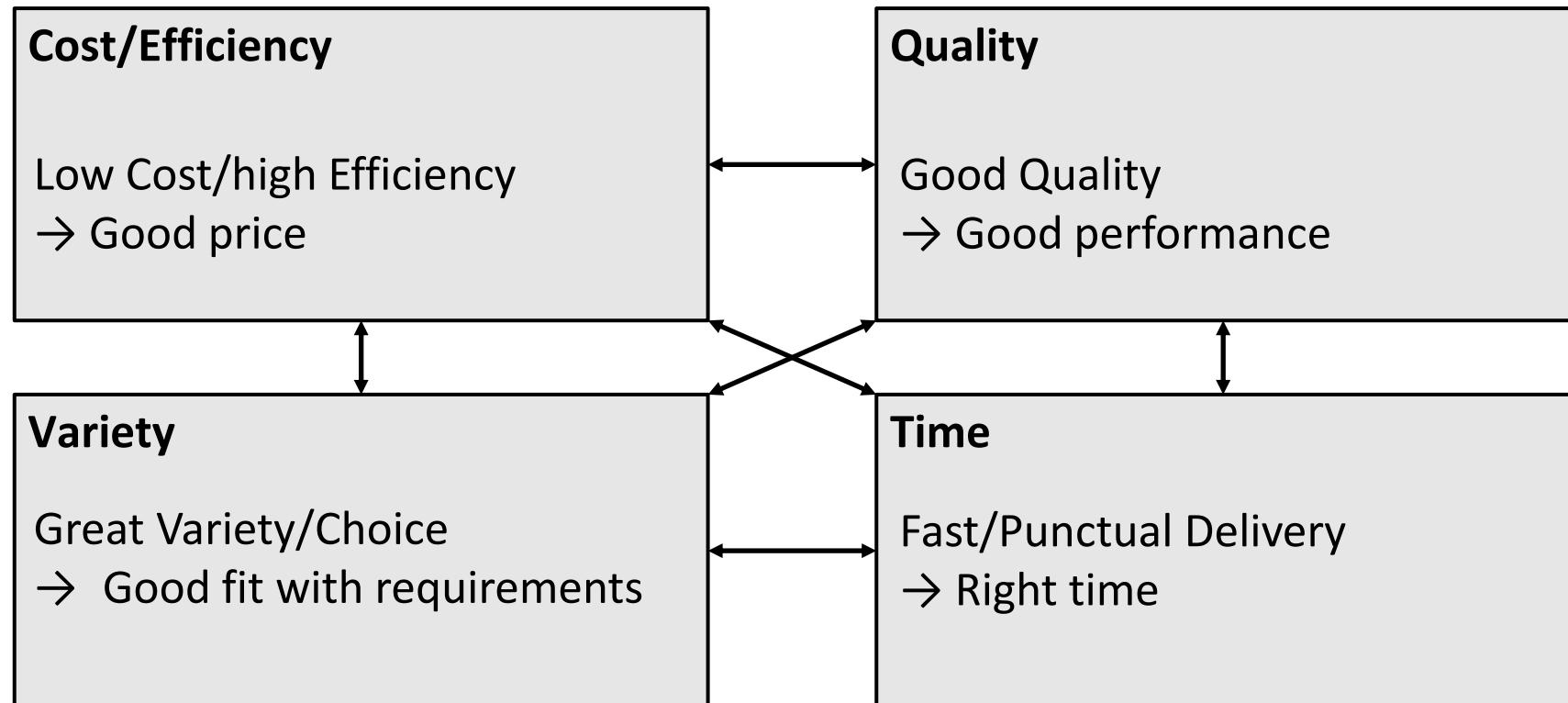
Structuring Consumer Utility



Classic Operations Performance Quadrangle



From Operations Performance to Consumer Utility

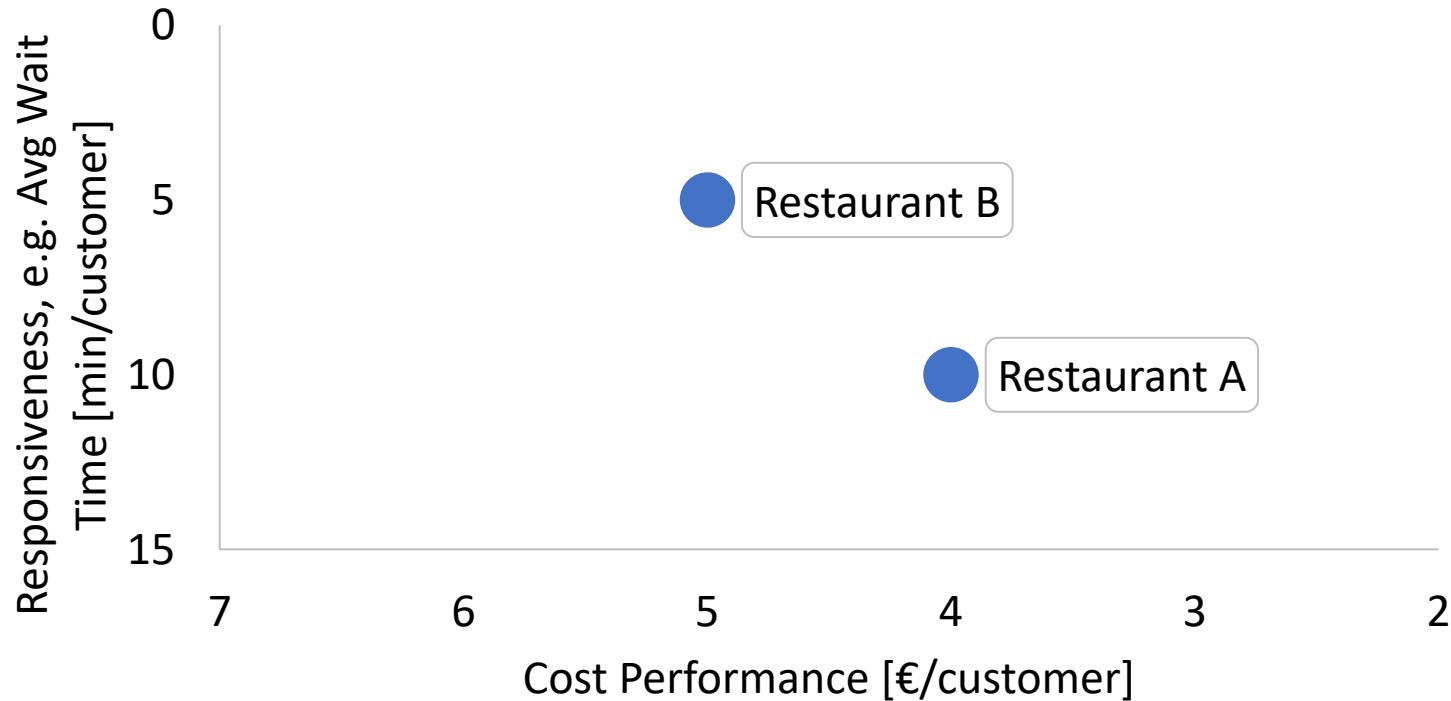


Restaurant Challenges

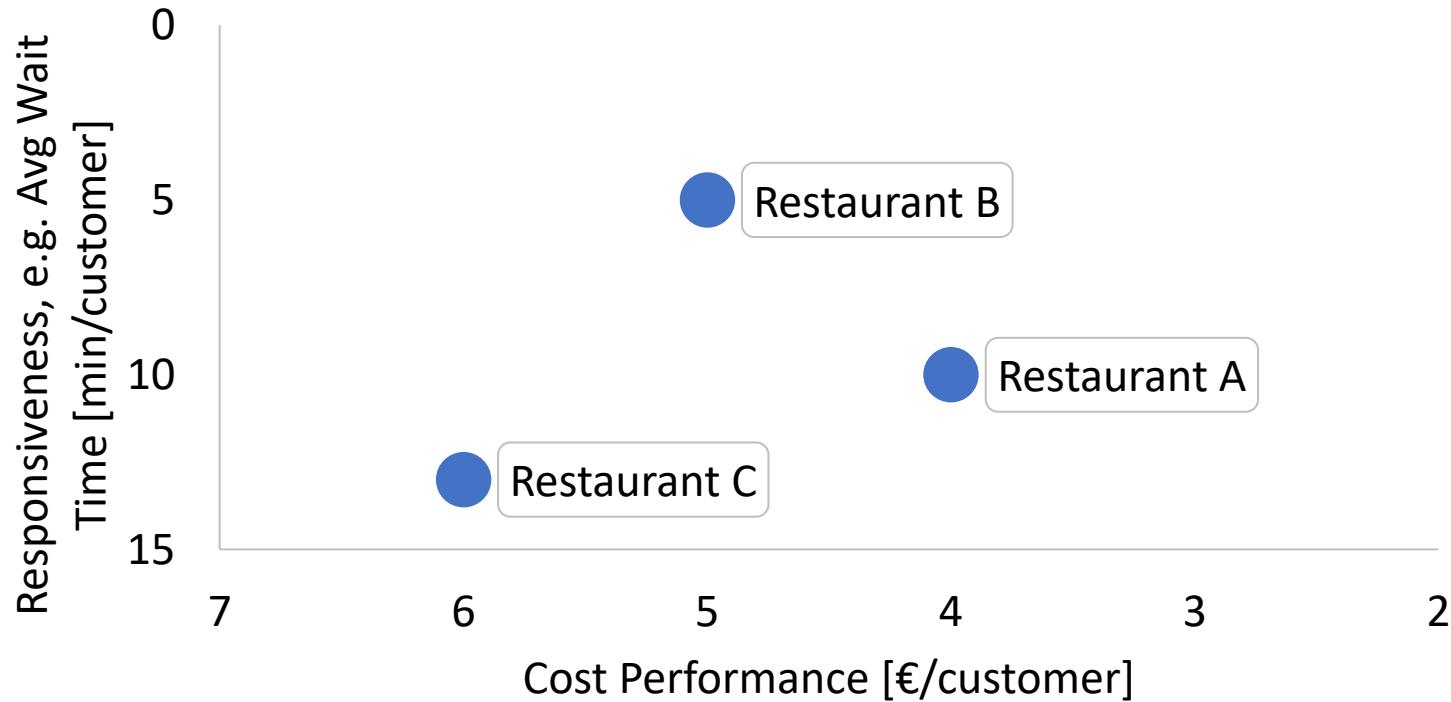
- How to identify and overcome inefficiencies?
- How to manage strategic trade-offs?
- How to improve and innovate?



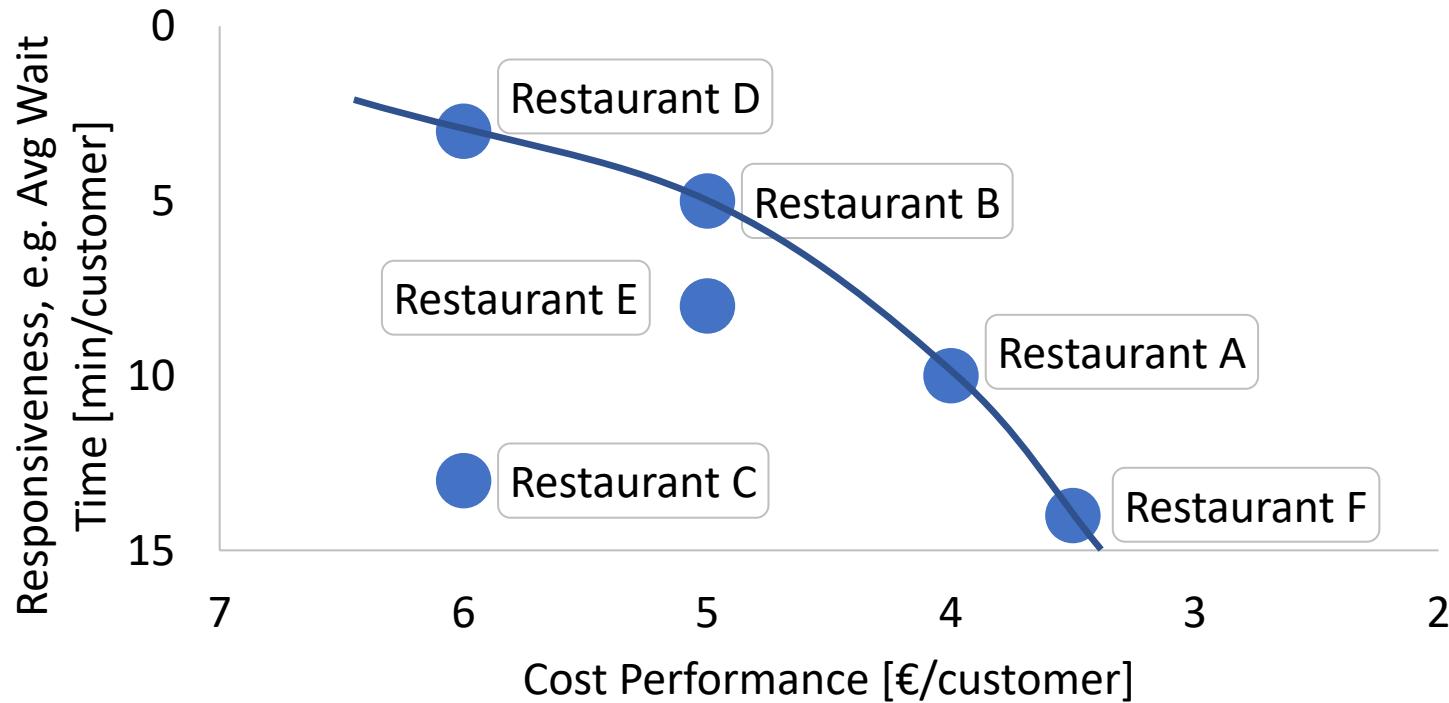
Which Restaurant is in the Better Position?



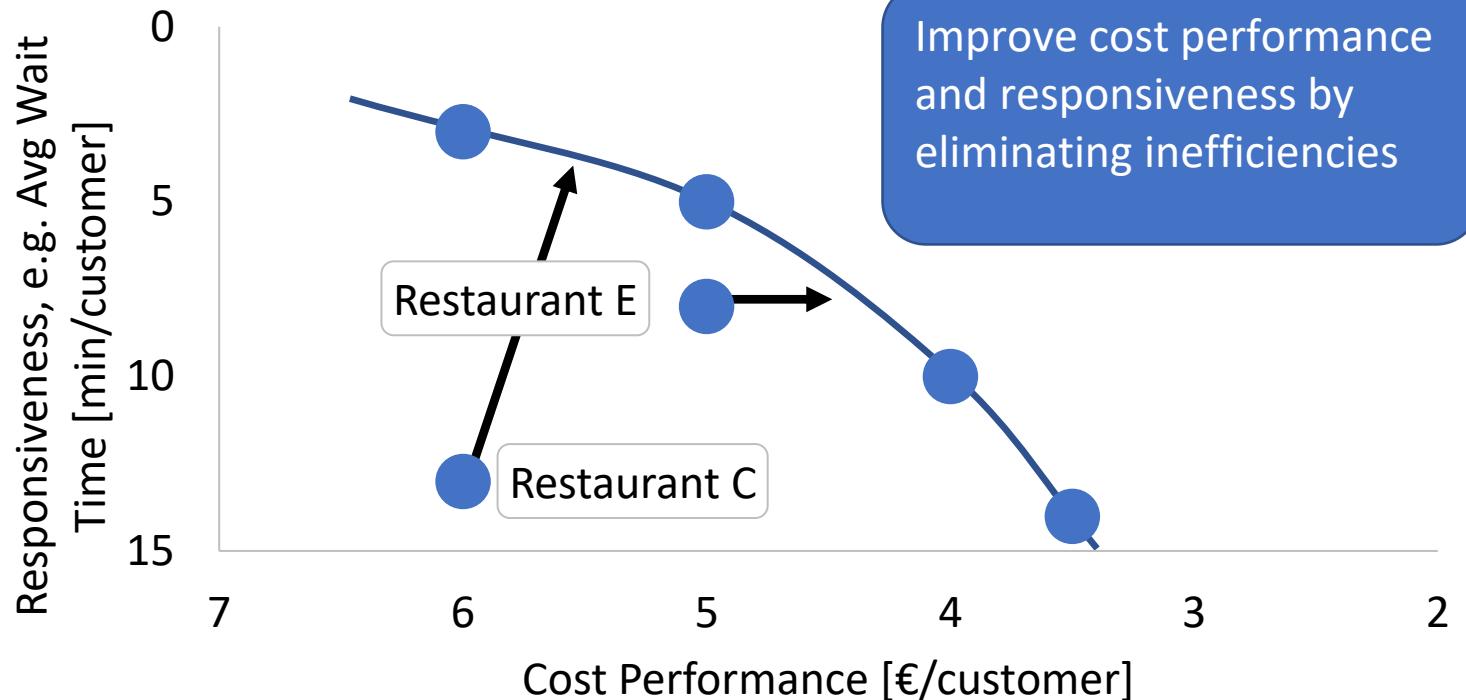
What About Restaurant C?



The Efficient Frontier



Use OM to Overcome Inefficiencies

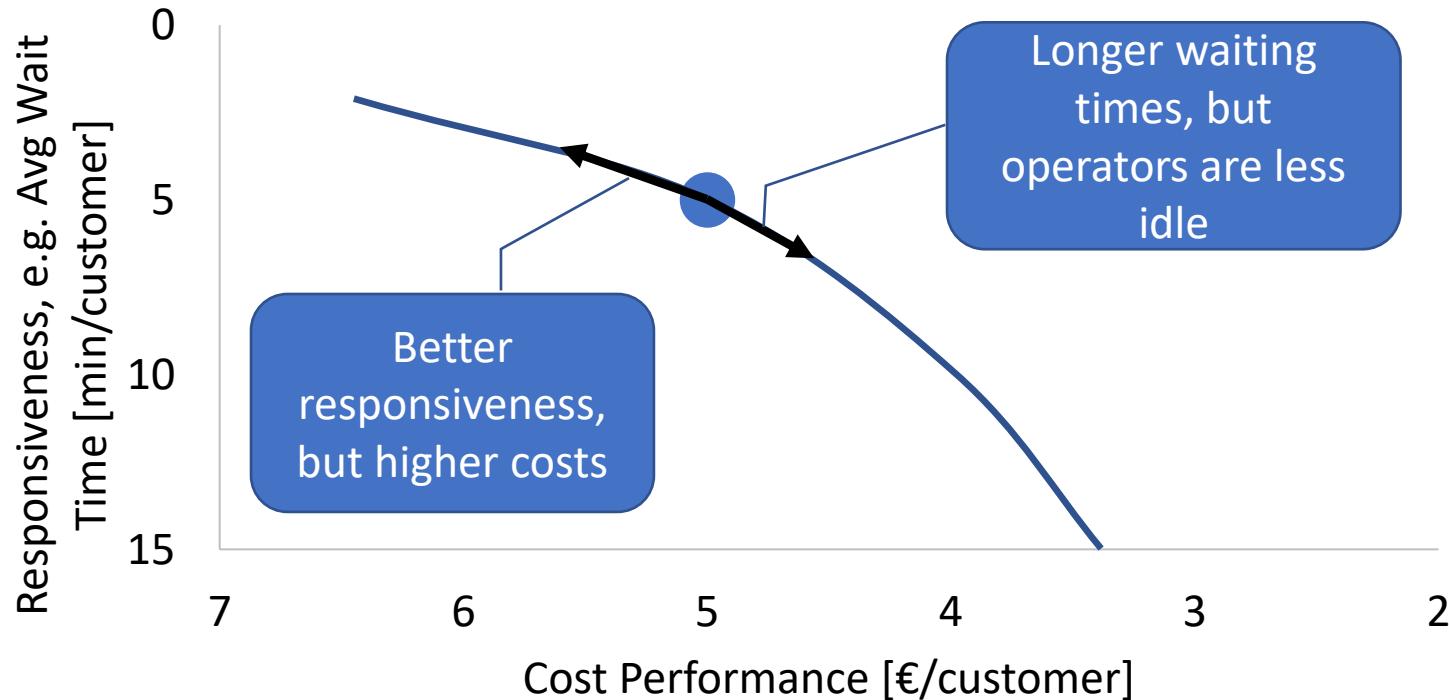


Restaurant Challenges

- How to identify and overcome inefficiencies?
- How to manage strategic trade-offs?
- How to improve and innovate?



Use OM to Support Strategic Trade-offs

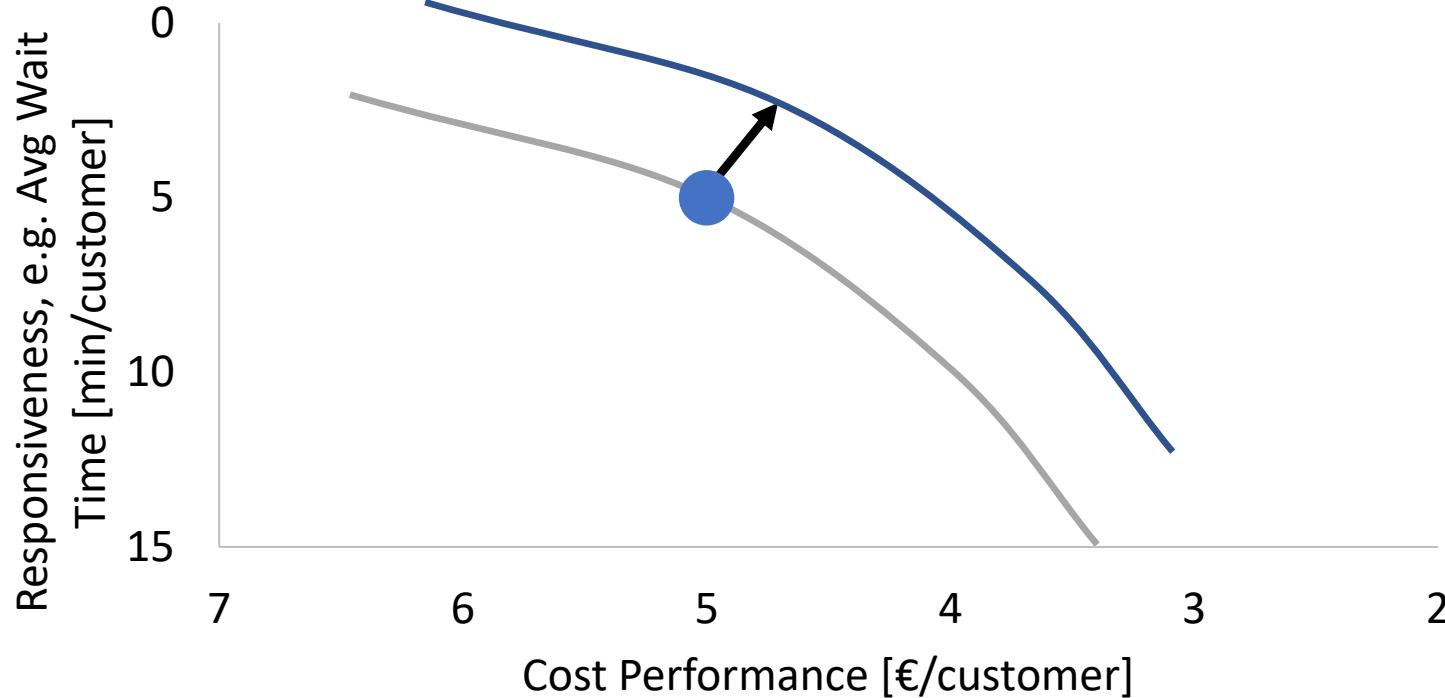


Restaurant Challenges

- How to identify and overcome inefficiencies?
- How to manage strategic trade-offs?
- How to improve and innovate?



Use OM to Innovate and Improve





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2023 Operations Management

INTRODUCTION TO PROCESSES

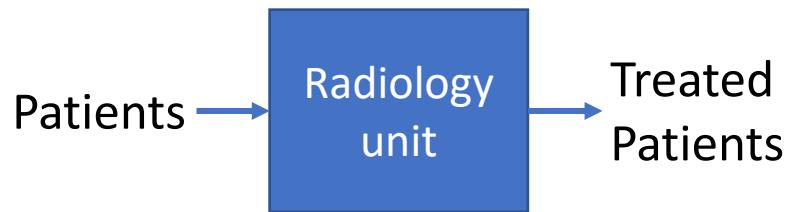
Key Questions of a Manager of a Process

- Is the process performing well?
→ this session
- How can we make the process better?
→ subsequent sessions



Characterizing a Process

- A process is an activity or a set of activities that accepts inputs and produces outputs.
- Processes require resources (employees, machines, etc.) to perform these activities.
- Processes can involve both goods and services.
- Processes can have multiple inputs and/or multiple outputs.



Diagramming a Process

→ Inputs flowing into a process

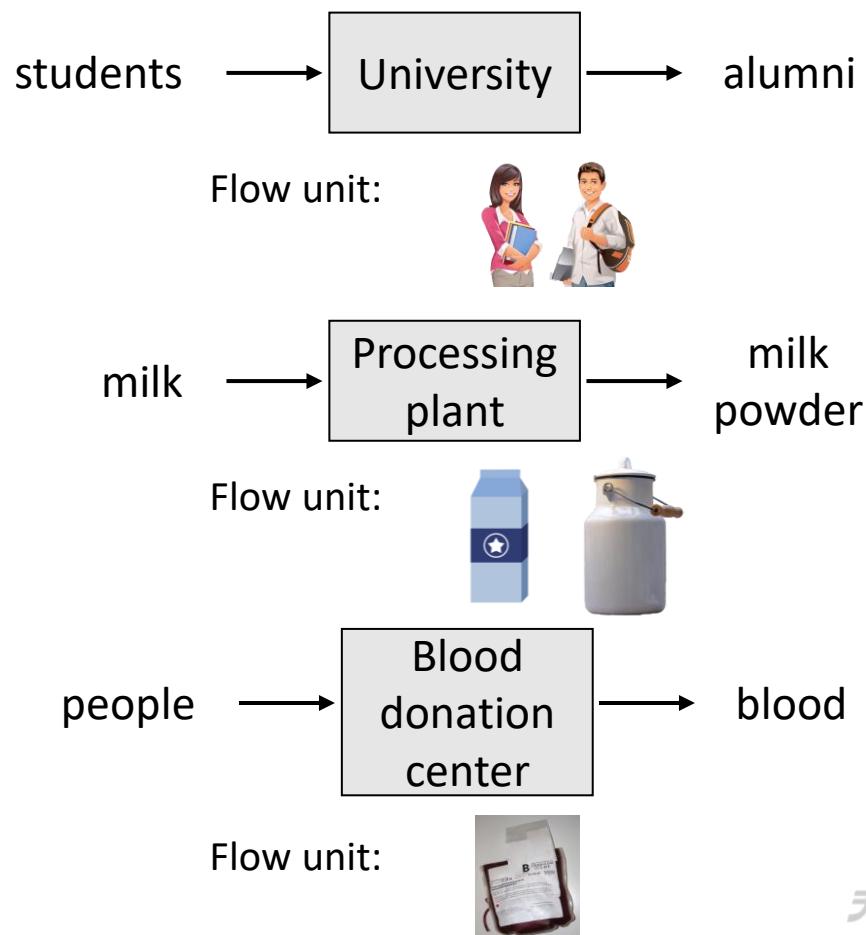
→ Outputs flowing out of a process

■ Single or multiple resources transforming inputs into outputs



Process Flow Unit

- Basic unit that “flows” through the process.
- Start as input and later leave the process as output.
- Unit that gets processed using at least one resource.
- Generally associated with the outputs of a process.



Rules to Define Good Flow Units

A suitable flow unit...

- ... corresponds to what you want to track and measure (with respect to the process).
- ... remains fixed throughout the process (e.g., do not mix kg and tons of flour).
- ... can be used to measure and describe all the activities within the process (e.g., minutes of workout could be better than km run/jumps accomplished).



Process Analysis: Three Basic Measures

- **Inventory:** the number of flow units in the process at a given moment in time;
- **Flow Rate:** rate at which flow units leave the process (or enter* it);
- **Flow Time:** total time a flow unit spends in the process from start to finish.

* in dynamic equilibrium

	Radiology	MBA Program	Wine Production
Flow unit	Patient	Student	Bottle
Inventory	[Patient]	[Student]	[Bottle]
Flow rate	$\frac{[Patient]}{Day}$	$\frac{[Student]}{Year}$	$\frac{[Bottles]}{Quarter}$
Flow time	[Days]	[Years]	[Quarter]



Radiomedicum

- Reliable and fast diagnosis
- Pleasant atmosphere and modern practice rooms
- Focus on people
- At 7 locations in the Rhine-Main area and the Wetterau region



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radiomedicum
Gemeinschaftspraxis für
Radiologie und Nuklearmedizin
Mainzer Landstraße 191 . 60327 Frankfurt
Untersuchungen



Radiomedicum Challenges

- How are flow time and flow rate determined?
- How is the stock level determined?



Kompetenz . Standorte . Termi



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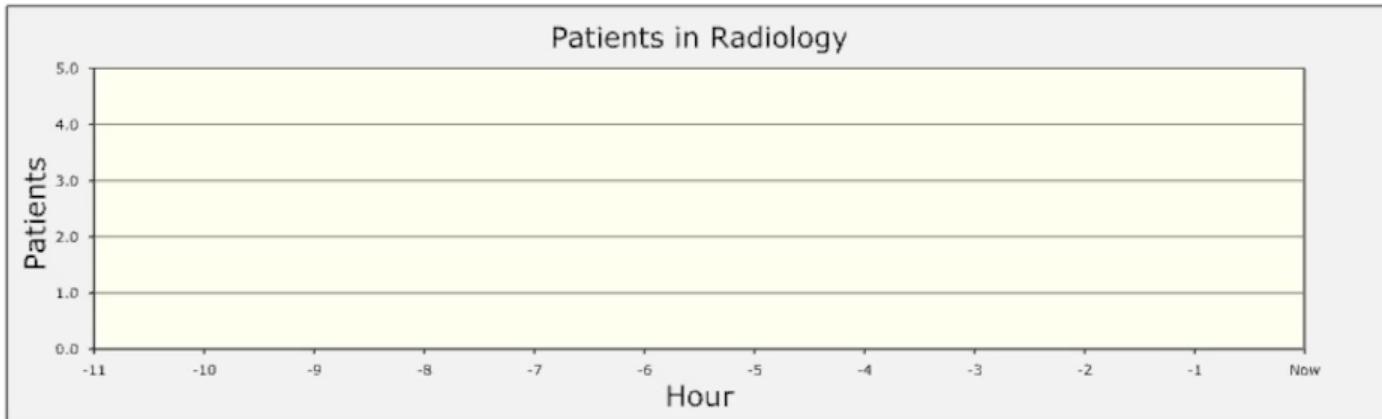
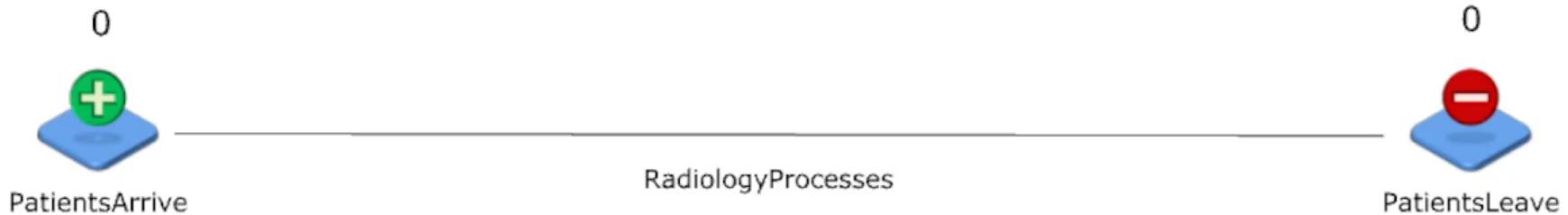
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Radiology

Patients arrive, get treated, and leave

07:30:00



How to Determine the Measures for the Radiology?

Use a watch and capture arrival and departure times:



Patient	Arrival	Departure
1	07:35:00	08:50:00
2	07:45:00	10:05:00
3	08:10:00	10:10:00
4	09:30:00	11:15:00
5	10:15:00	10:30:00
6	10:30:00	13:35:00
7	11:05:00	13:15:00
8	12:35:00	15:05:00
9	14:30:00	18:10:00
10	14:35:00	15:45:00
11	14:40:00	17:20:00

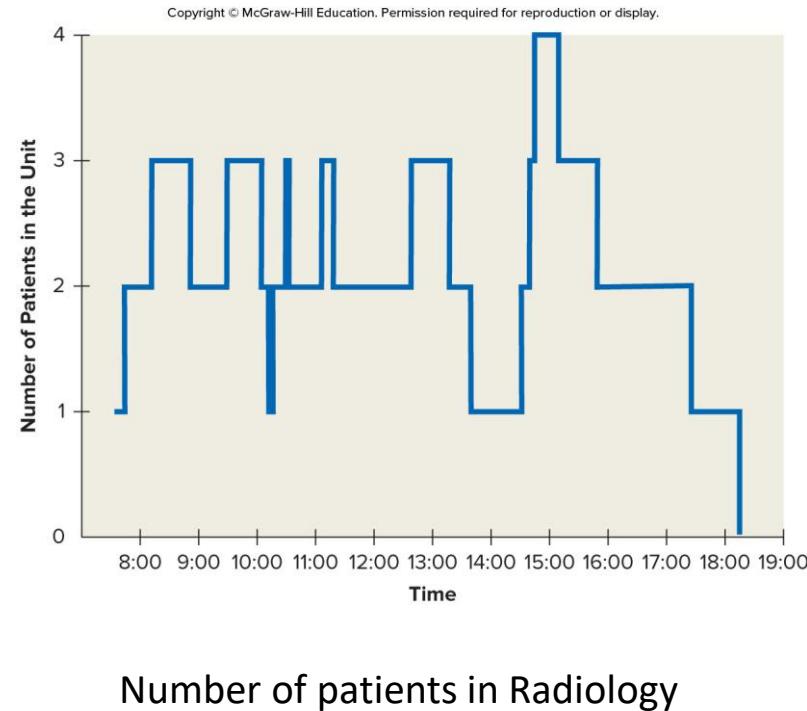
Average Flow Time and Average Flow Rate

Patient	Arrival [hh:mm:ss]	Departure [hh:mm:ss]	Flow Time [hh:mm:ss]	Arrival of first patient	07:35:00 [hh:mm:ss]
1	07:35:00	08:50:00	01:15:00	Departure of last patient	18:10:00 [hh:mm:ss]
2	07:45:00	10:05:00	02:20:00		
3	08:10:00	10:10:00	02:00:00		
4	09:30:00	11:15:00	01:45:00	Length of day	10:35:00 [hh:mm:ss]
5	10:15:00	10:30:00	00:15:00	Length of day	10,583 [hours]
6	10:30:00	13:35:00	03:05:00	Patient per day	11 [patient/day]
7	11:05:00	13:15:00	02:10:00	Ø Average flow rate	1,04 [patient/hour]
8	12:35:00	15:05:00	02:30:00	Ø Average flow time	02:04:33 [hh:mm:ss]
9	14:30:00	18:10:00	03:40:00	Ø Average flow time	2,08 [hours]
10	14:35:00	15:45:00	01:10:00		
11	14:40:00	17:20:00	02:40:00		



Average Inventory

- Straightforward to graph inventory of patients using the arrival and departure data;
- Not straightforward to calculate average inventory;
- Weighing the number of patients in radiology with the time span leads to 2.16 [patients].



Fortunately, there is...

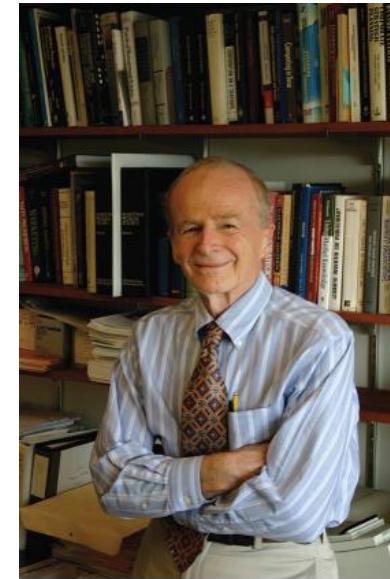
...a relationship between flow rate, flow time and inventory named...

$$\text{Average Flow Rate } R = 1.04 \left[\frac{\text{patients}}{\text{hour}} \right]$$

$$\text{Average Flow Time } T = 2.08 \text{ [hour]}$$

$$\text{Average Inventory } I = 2.16 \text{ [patients]}$$

$$I = R \cdot T$$



©John D. Little

...Little's Law.

Little, J.D., 1961. A proof for the queuing formula.
Operations Research, 9(3), pp.383-387.



Little's Law

- Descriptively simple, but remarkably powerful;
- It holds for any distributions of R and T ;
- It applies to any system, particularly, it applies to systems within systems;
- Only requirement: system must be in dynamic equilibrium (stable).

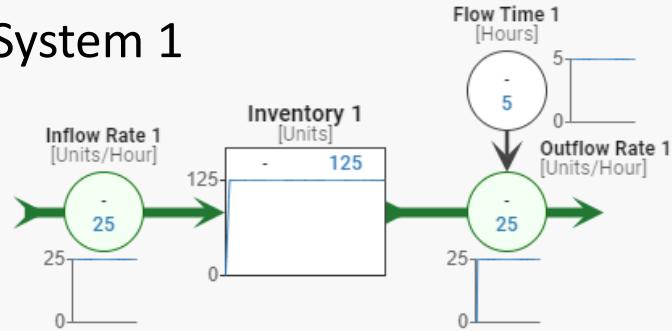
The long-term average number l of flow units in a stationary system is equal to the long-term average flow rate R multiplied by the average flow time T that a flow unit spends in the system.



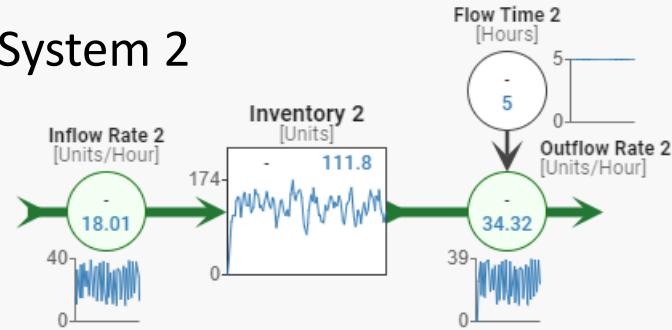
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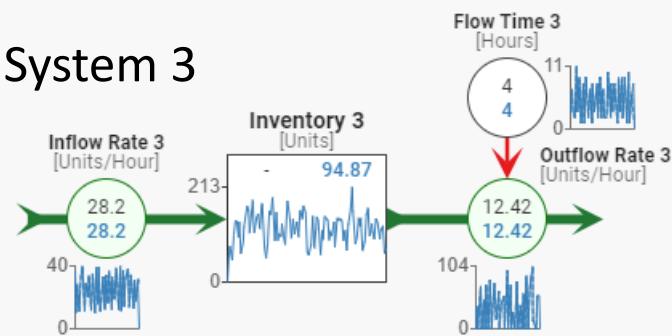
System 1



System 2



System 3



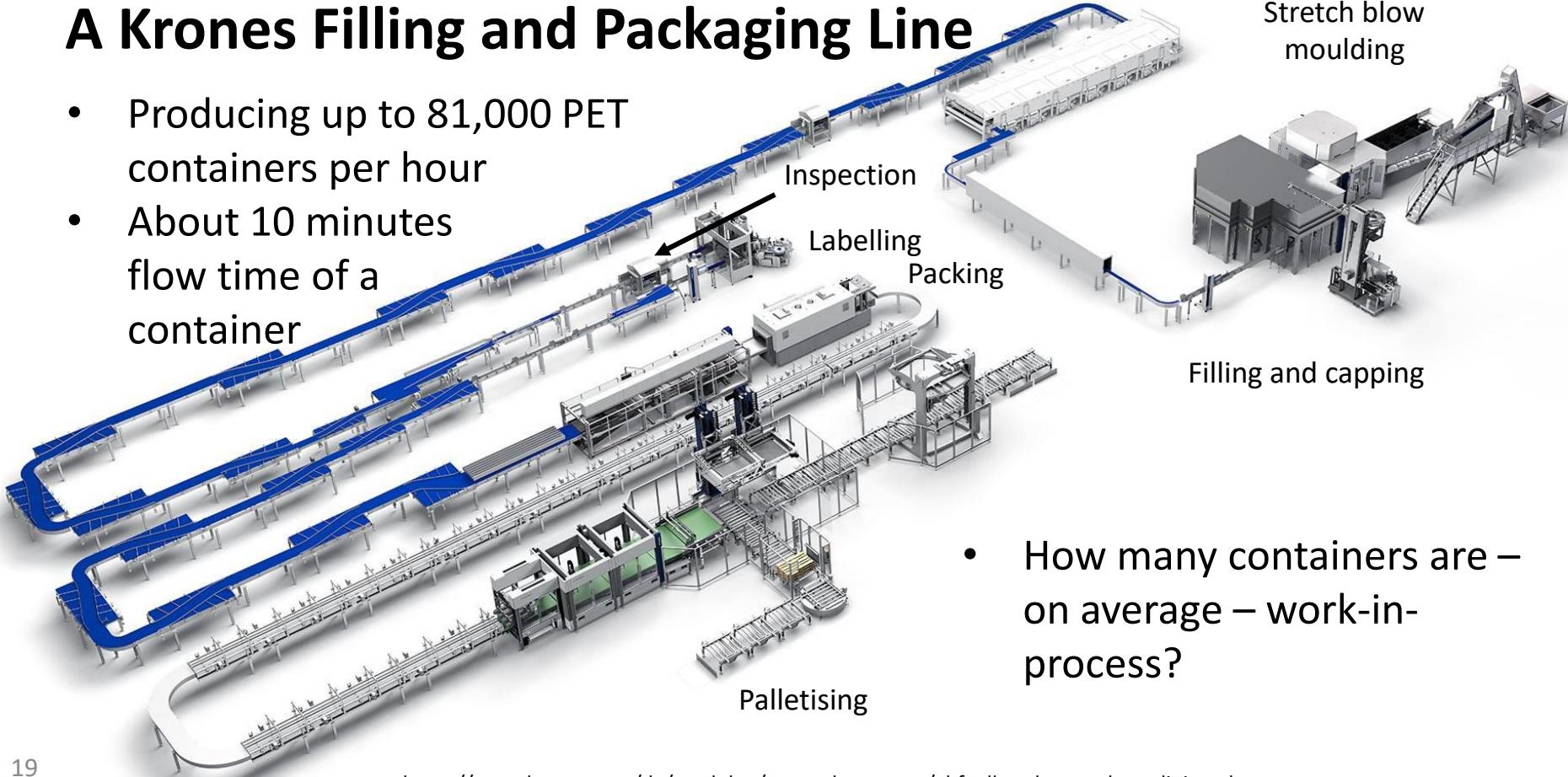
Krones

- Founded in 1951 as a manufacturer of labelling machines.
- Headquarters of Krones AG are located in Neutraubling near Regensburg.
- Today, Krones delivers leading technologies for filling and packaging



A Krones Filling and Packaging Line

- Producing up to 81,000 PET containers per hour
- About 10 minutes flow time of a container



- How many containers are – on average – work-in-process?

BACKUP



Little's Law Tested

System 1 Descriptive Data (N=190)

Average Inventory	125 [units]
Average Inflow Rate	25 [units/hour]
Average Outflow Rate	25 [units/hour]
Average Flow Time	5.00 [hour]

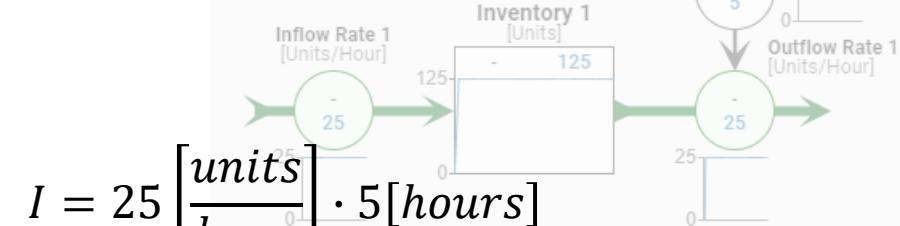
System 2 Descriptive Data (N=190)

Average Inventory	127.16 [units]
Average Inflow Rate	25.37 [units/hour]
Average Outflow Rate	25.47 [units/hour]
Average Flow Time	5.00 [hour]

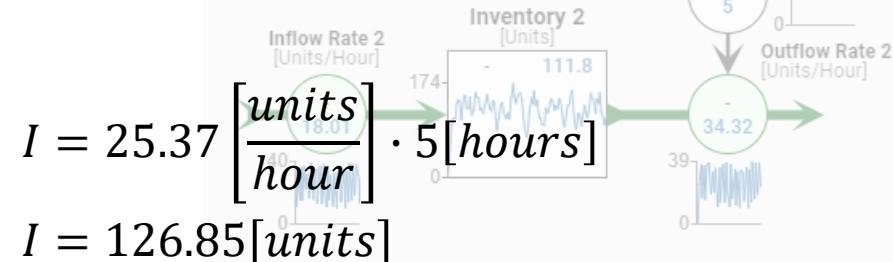
System 3 Descriptive Data (N=190)

Average Inventory	121.08 [units]
Average Inflow Rate	24.31 [units/hour]
Average Outflow Rate	24.58 [units/hour]
Average Flow Time	5.06 [hour]

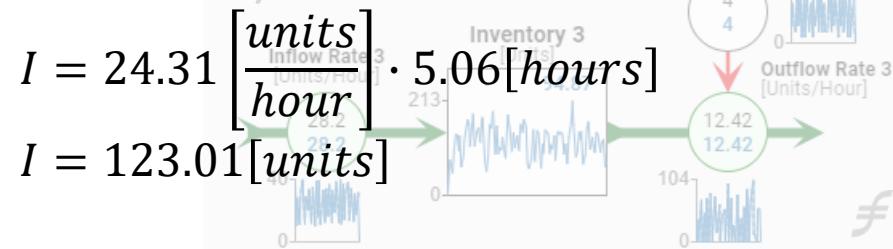
System 1



System 2



System 3





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PROCESS ANALYSIS

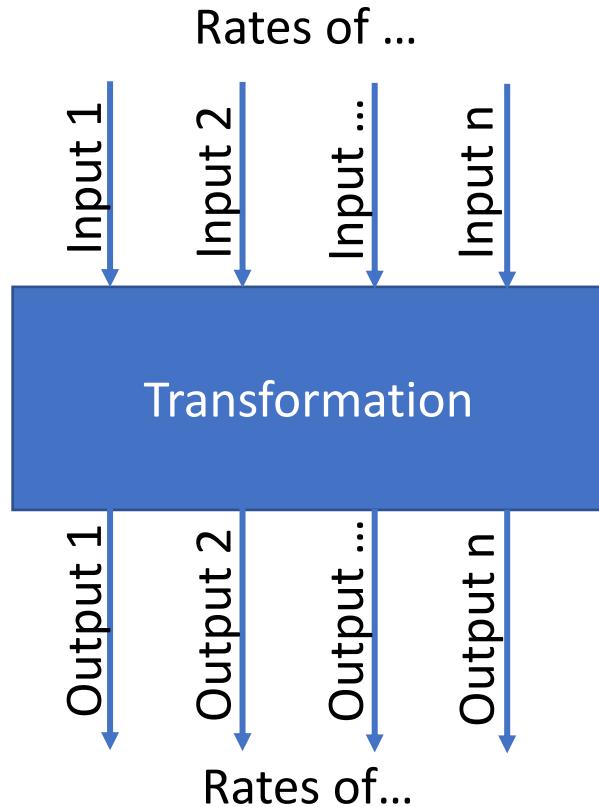


PROCESS ANALYSIS AND PROCESS FLOW DIAGRAM



Process Analysis

- Addressing the question
“How can we improve a process?”
requires to open the “black box”
of transformation.
- Process analysis provides a
rigorous framework for
understanding the detailed
operations of an organization.



Process Flow Diagram

- The best way to begin any process analysis: drawing a Process Flow Diagram.
- A process flow diagram is a graphical way to describe important (not all) details of a process.
- It focuses on flows, resources and inventory.

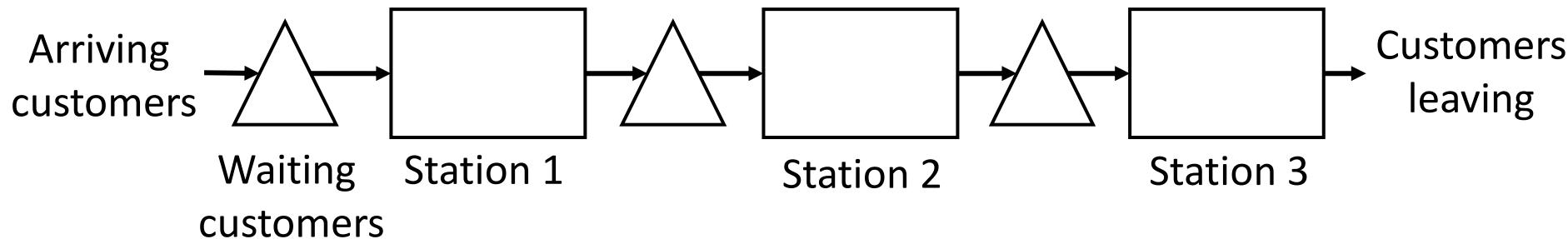
Boxes for
resources used in
activities



Arrows for flows



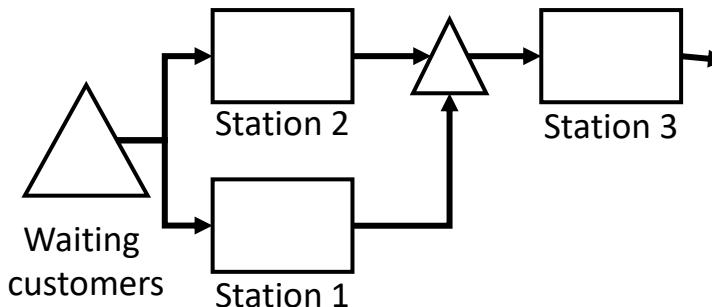
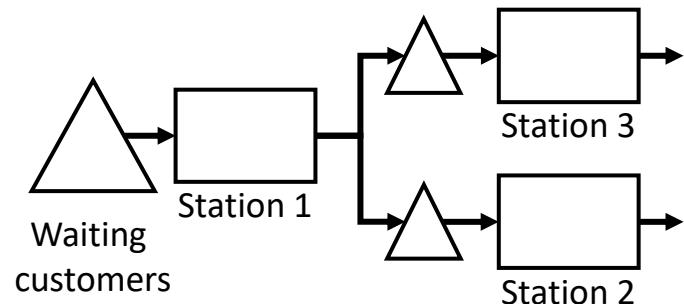
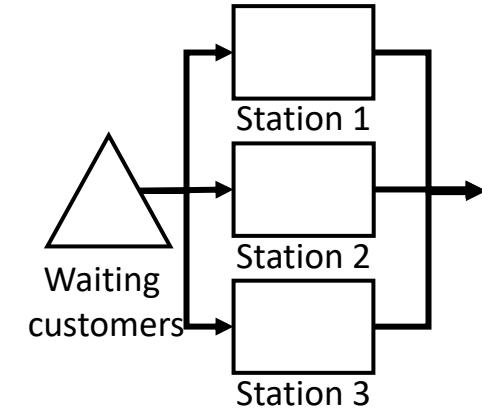
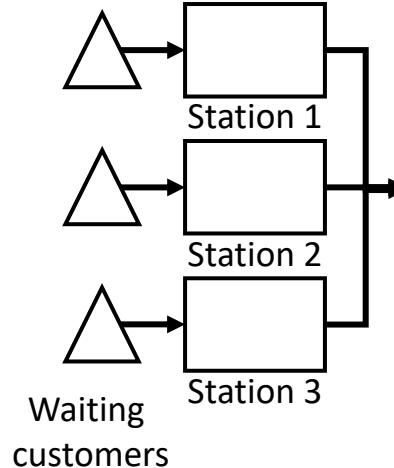
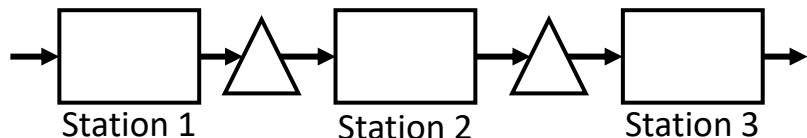
A Simple Example



Upstream – the parts of the process that are at the beginning of the process flow

Downstream – the parts of the process that are at the end of the process flow

Alternate Process Designs

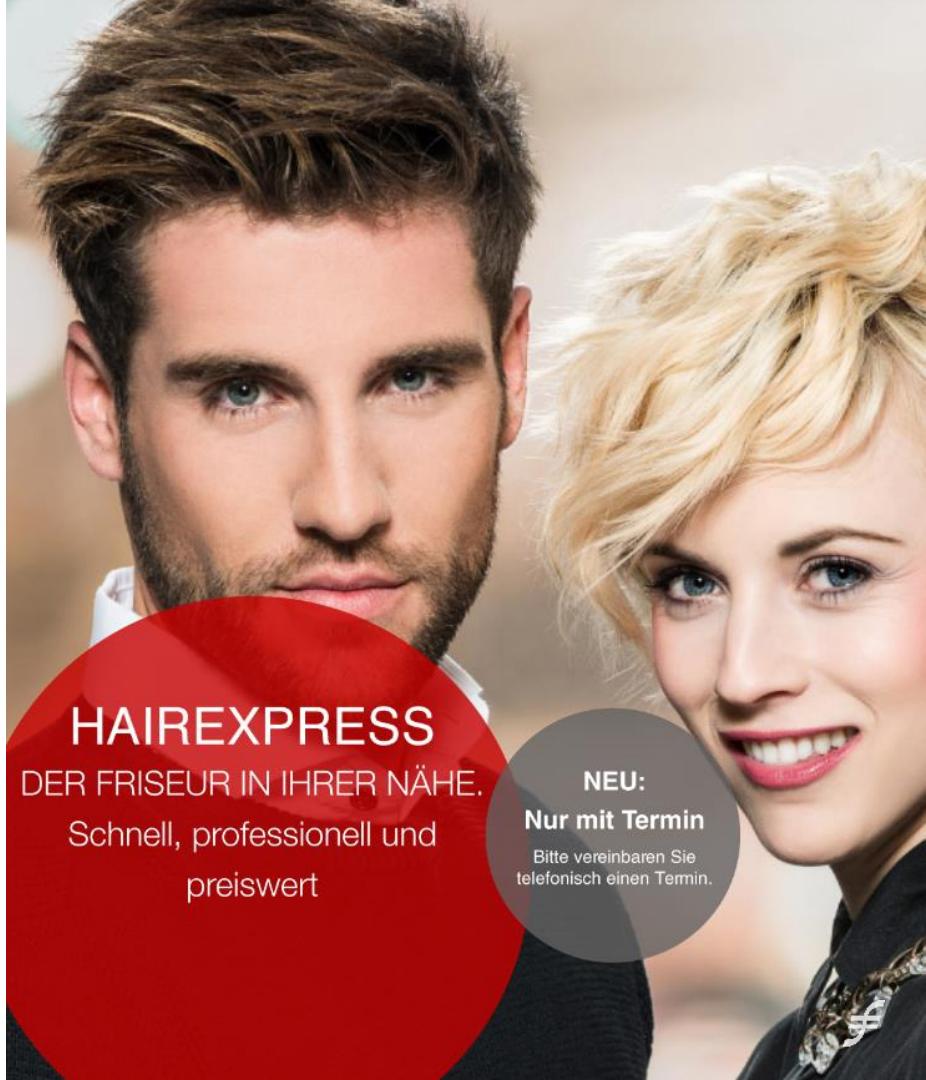


ONE-STEP PROCESSES

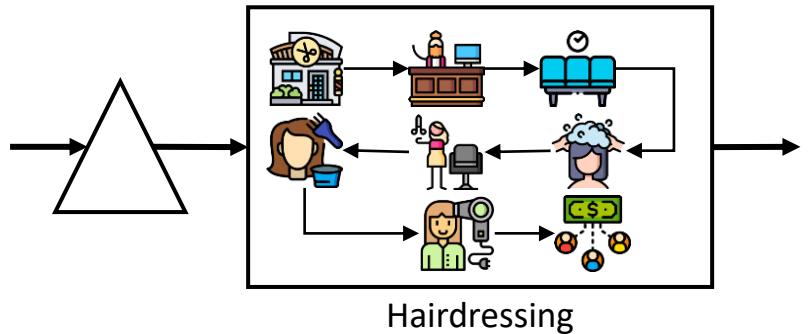


HAIREXPRESS Challenges

- What is the capacity of a one-step process?
- What is the flow rate, the utilization, and the cycle time of a process?



Hairdressing as One-Step Process



- One-step processes can include multiple activities.
- One employee (one resource) is in charge of the complete service process.



Activities Included in the Hairdressing Process

Activity	Activity Time [min/customer]
Welcome the customer	2.0
Check-in the customer and place them in the waiting area	6.5
Seat the customer and wash the hair	7.0
Cut customer's hair	9.5
Color customer's hair	10.0
Dry and style the hair and check overall appearance	15.0
Check-out (advertising and selling of hairdressing products, paying of the bill, new appointment)	4.5
Total	54.5

All activities are performed by the same employee (resource).

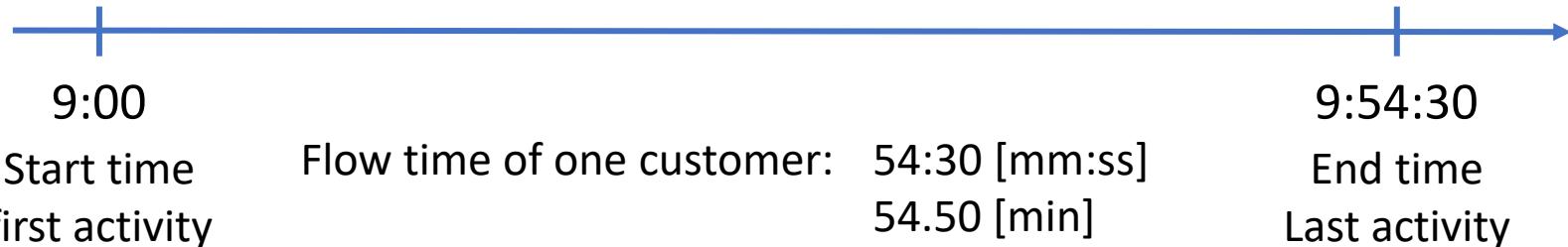
Total activity time = processing time = 54.5 min/customer.

Be very careful with units!



Processing Time versus Flow Time

- Flow time of one flow unit = End time of last activity – Start time of first activity



- Processing time of a resource:
Time required by a resource to process flow units

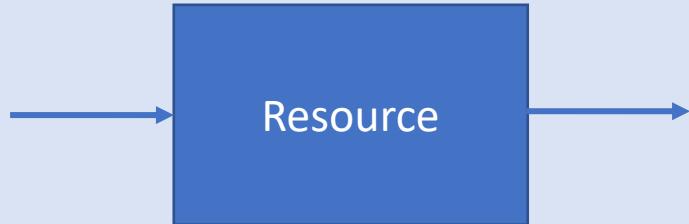
Example Hairexpress:

A hairdresser works 54.50 min to serve a customer from start to end.

$$\text{Processing time} = 54.50 \frac{\text{hairdresser} \cdot \text{min}}{\text{customer}}$$

Capacity of a Resource

- Maximum amount that a resource can produce in a given amount of time.
(= maximum flow rate).
- Measured in flow units per time period.
- Different time units can be used (seconds, minutes, hours, days, weeks, month, quarters, years, ...).
- Basis for a multitude of important performance measures.



$$\text{Capacity} = \frac{1 \text{ resource unit}}{\text{resource processing time}}$$

$$\text{Capacity} = \frac{1}{\text{Processing time}}$$

The "1" in the formula means that one unit of a resource is used, so that one flow unit can be processed/served at the same time.

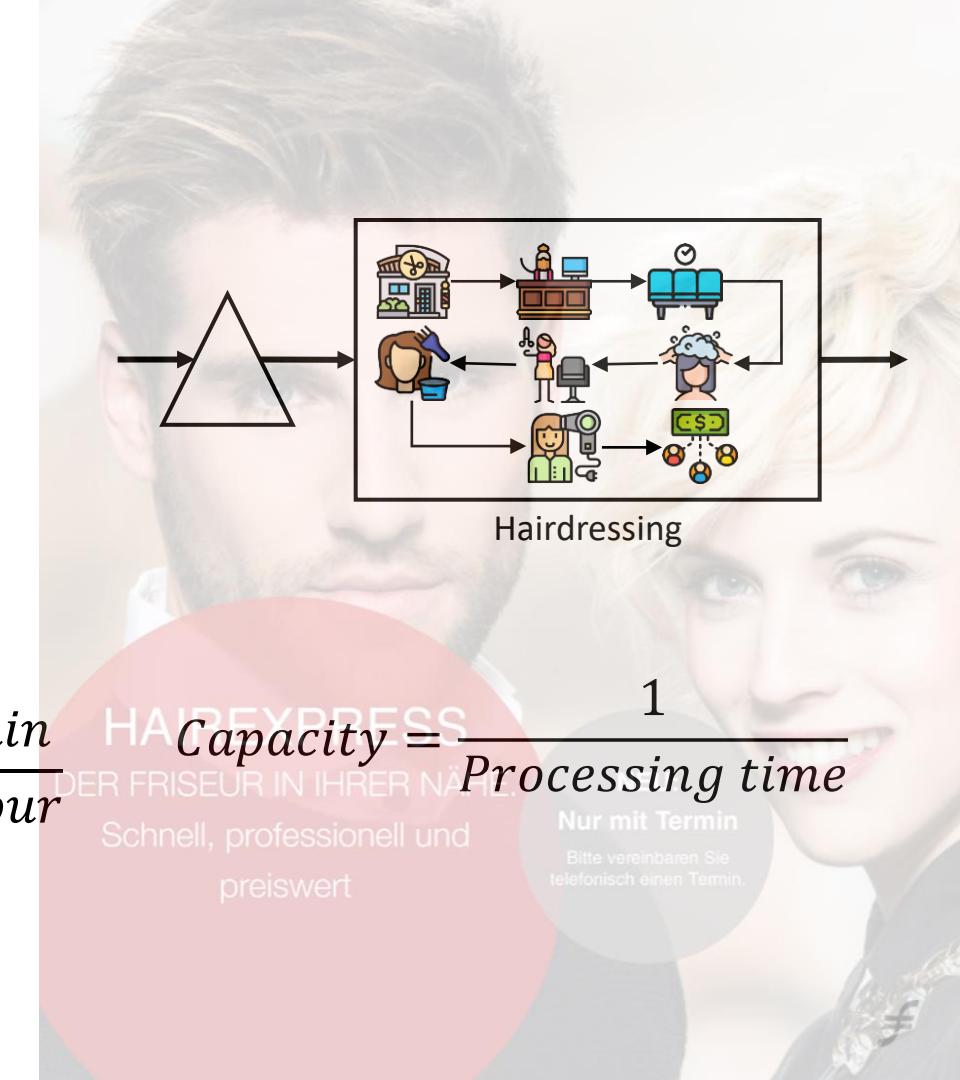
Capacity of a Hairexpress-hairdresser

$$\text{Capacity} = \frac{1 \text{ hairdresser}}{54.5 \frac{\text{hairdresser} \cdot \text{min}}{\text{customer}}}$$

$$\text{Capacity} = 0.0183 \frac{\text{customers}}{\text{min}}$$

$$= 0.0183 \frac{\text{customers}}{\text{min}} \cdot 60 \frac{\text{min}}{\text{hour}}$$

$$= 1.1009 \frac{\text{customers}}{\text{hour}}$$



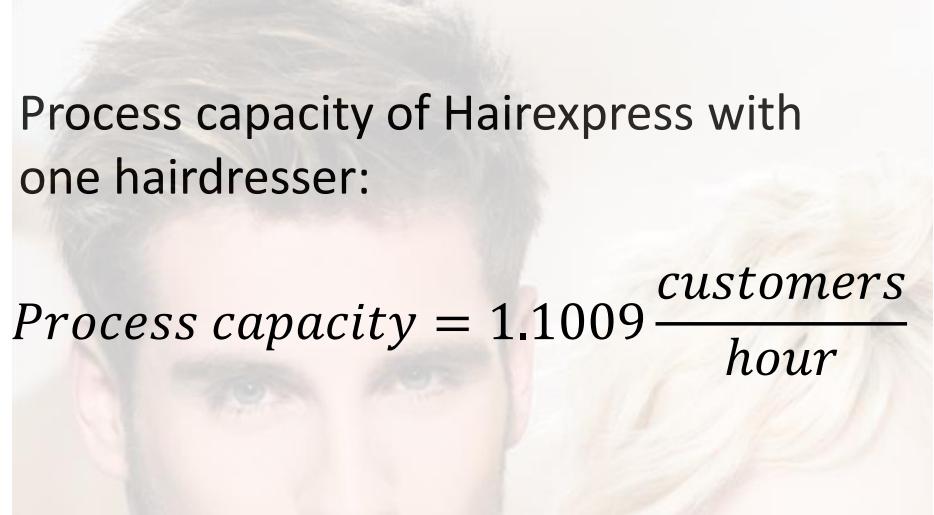
Process Capacity

- Maximum amount that a process can produce in a given amount of time.
 (= maximum flow rate).
- One-step process with one resource:

$$\text{Process capacity} = \text{Capacity of the resource}$$

- One-step process with multiple resources:

$$\text{Process capacity} = \min \left\{ \begin{array}{l} \text{Capacity}_1, \\ \text{Capacity}_2, \\ \dots \\ \text{Capacity}_n \end{array} \right\}$$

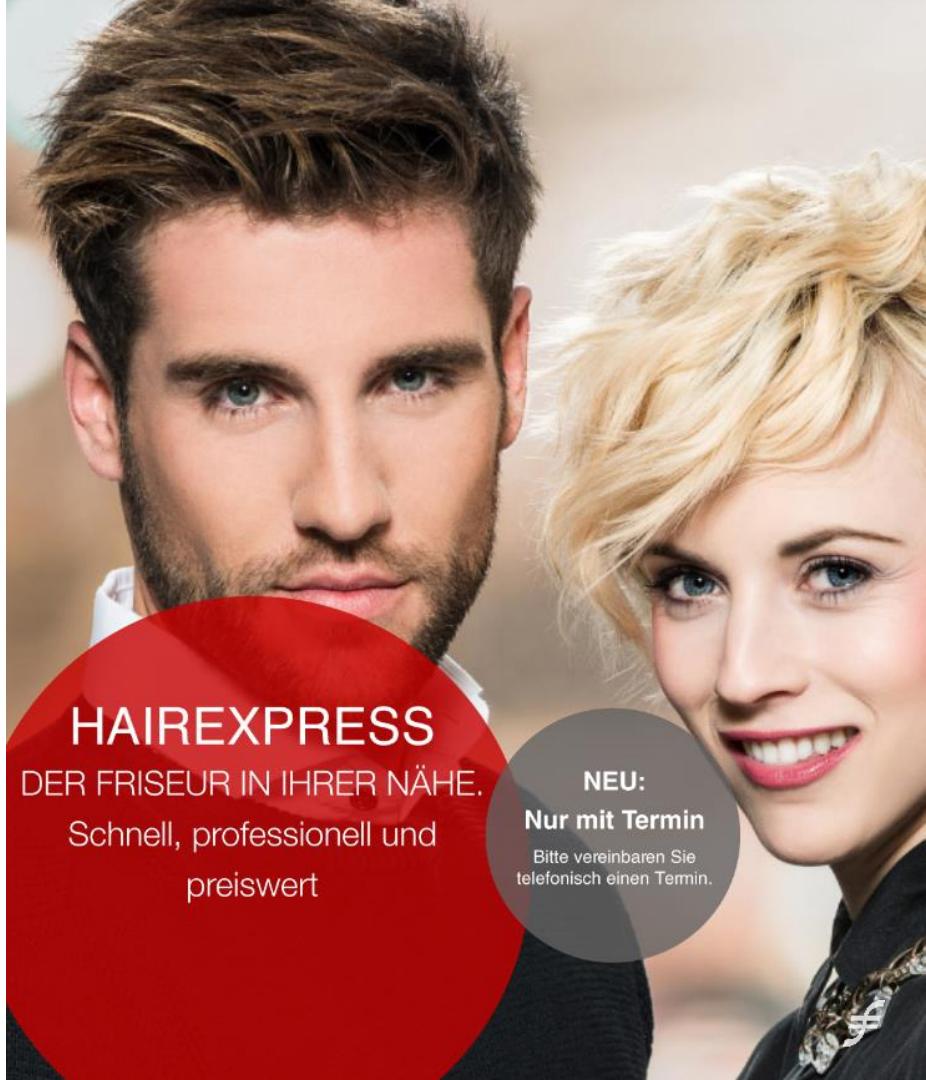


Process capacity of Hairexpress with one hairdresser:

$$\text{Process capacity} = 1.1009 \frac{\text{customers}}{\text{hour}}$$

HAIREXPRESS Challenges

- What is the capacity of a one-step process?
- **What is the flow rate, the utilization, and the cycle time of a process?**



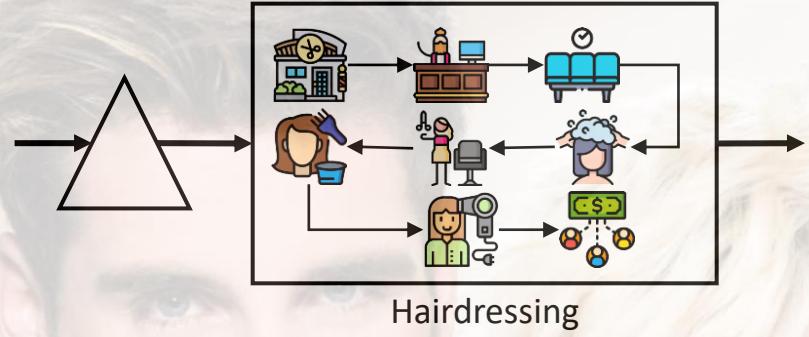
Flow Rate

$$\text{Demand rate} = 12 \frac{\text{customers}}{\text{hour}}$$

$$\text{Process capacity} = 1.1009 \frac{\text{customers}}{\text{hour}}$$

$$\text{Flow rate} = \min \left\{ 12 \frac{\text{customers}}{\text{hour}}, 1.1009 \frac{\text{customers}}{\text{hour}} \right\}$$

$$\text{Flow rate} = 1.1009 \frac{\text{customers}}{\text{hour}}$$



Flow rate =

$$\min \left\{ \begin{array}{l} \text{Demand rate,} \\ \text{Process capacity} \end{array} \right\}$$

HAIREXPRESS

DER FRISEUR IN IHRER NÄHE.

Schnell, professionell und

preiswert

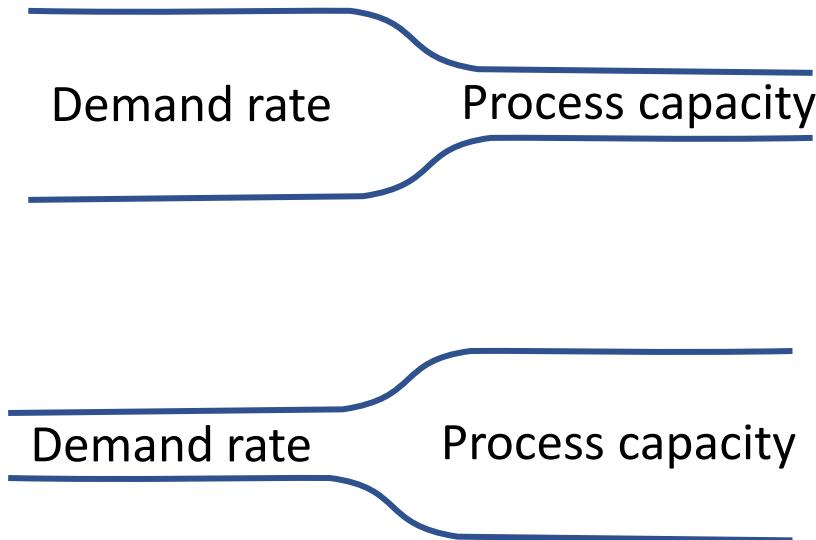
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Treated as synonyms:

Flow rate = Throughput rate

Two Types of Constraints



Capacity-constrained:
Flow rate is limited by
process capacity

Demand-constrained:
Flow rate is limited by
demand rate

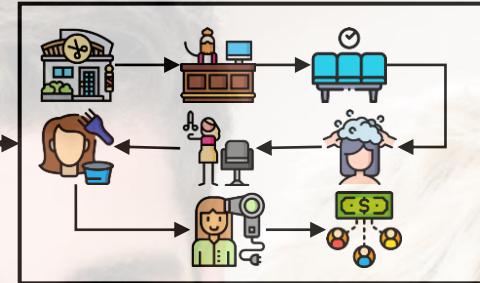
Utilization

$$\text{Flow rate} = 1.1009 \frac{\text{customers}}{\text{hour}}$$

$$\text{Process capacity} = 1.1009 \frac{\text{customers}}{\text{hour}}$$

$$\text{Utilization} = \frac{1.1009 \frac{\text{customers}}{\text{hour}}}{1.1009 \frac{\text{customers}}{\text{hour}}}$$

$$= 1.0000 = 100\%$$



$$\text{Utilization} = \frac{\text{Flow rate}}{\text{Capacity}}$$

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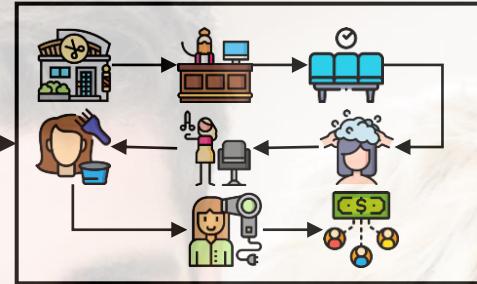
Pay attention to units!

Implied Utilization

$$\text{Demand rate} = 12 \frac{\text{customers}}{\text{hour}}$$

$$\text{Process capacity} = 1.1009 \frac{\text{customers}}{\text{hour}}$$

$$\text{Implied utilization} = \frac{12 \frac{\text{customers}}{\text{hour}}}{1.1009 \frac{\text{customers}}{\text{hour}}} = 10.9$$



Hairdressing

$$\text{Implied utilization} = \frac{\text{Demand rate}}{\text{Capacity}}$$

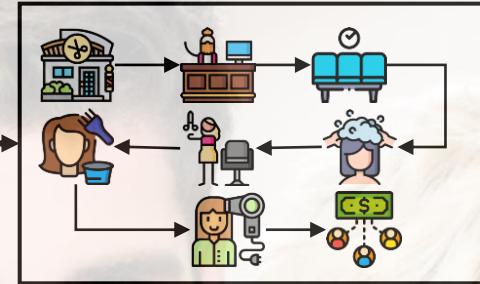
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Cycle Time

$$\text{Flow rate} = 1.1009 \frac{\text{customers}}{\text{hour}}$$

$$\begin{aligned} \text{Cycle time} &= \frac{1}{1.1009 \frac{\text{customers}}{\text{hour}}} \\ &= 0.9083 \frac{\text{hours}}{\text{customer}} \\ &= 54.5000 \frac{\text{min}}{\text{customer}} \end{aligned}$$



Hairdressing

$$\text{Cycle time} = \frac{1}{\text{Flow Rate}}$$

$$\text{Cycle time} = \frac{T}{I}$$

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Cycle time = Processing time

Only when the system is
capacity-constrained

Definition of terms

- Processing time: Time use of resources on flow units
- Flow time: Ø Time span between process start and end
- Cycle time: Ø Time span (per flow unit) between the output of two flow units
- Process capacity: Maximum flow rate through a process


$$\text{Processing time} = 54.50 \frac{\text{hairdresser} \cdot \text{min}}{\text{customer}}$$

$$\text{Flow time} = 54.50 \text{ min}$$

$$\text{Cycle time} = 54.50 \frac{\text{min}}{\text{customer}}$$

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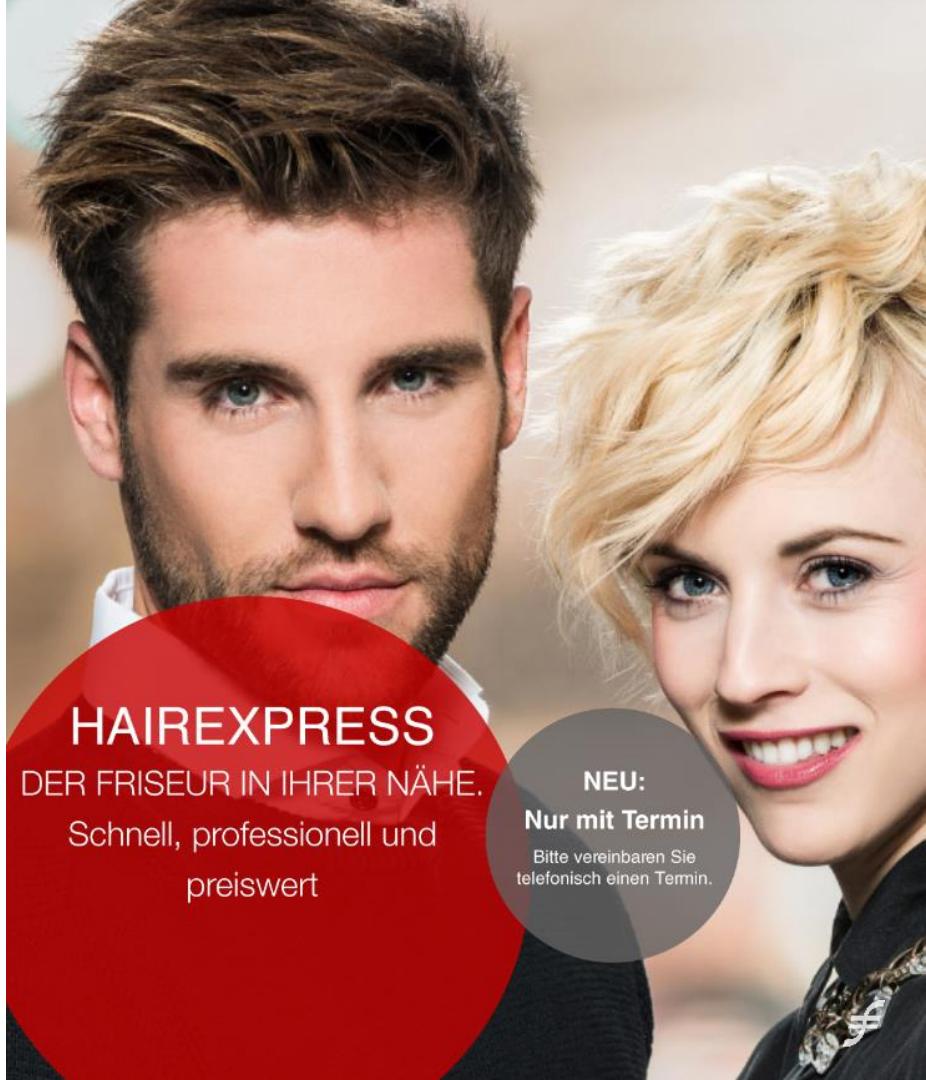
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$$\text{Process capacity} = 1.1009 \frac{\text{customers}}{\text{hour}}$$

HAIREXPRESS Challenges

- What is the capacity of parallel one-step processes?
- How many resources (workstations) are needed to match demand ?



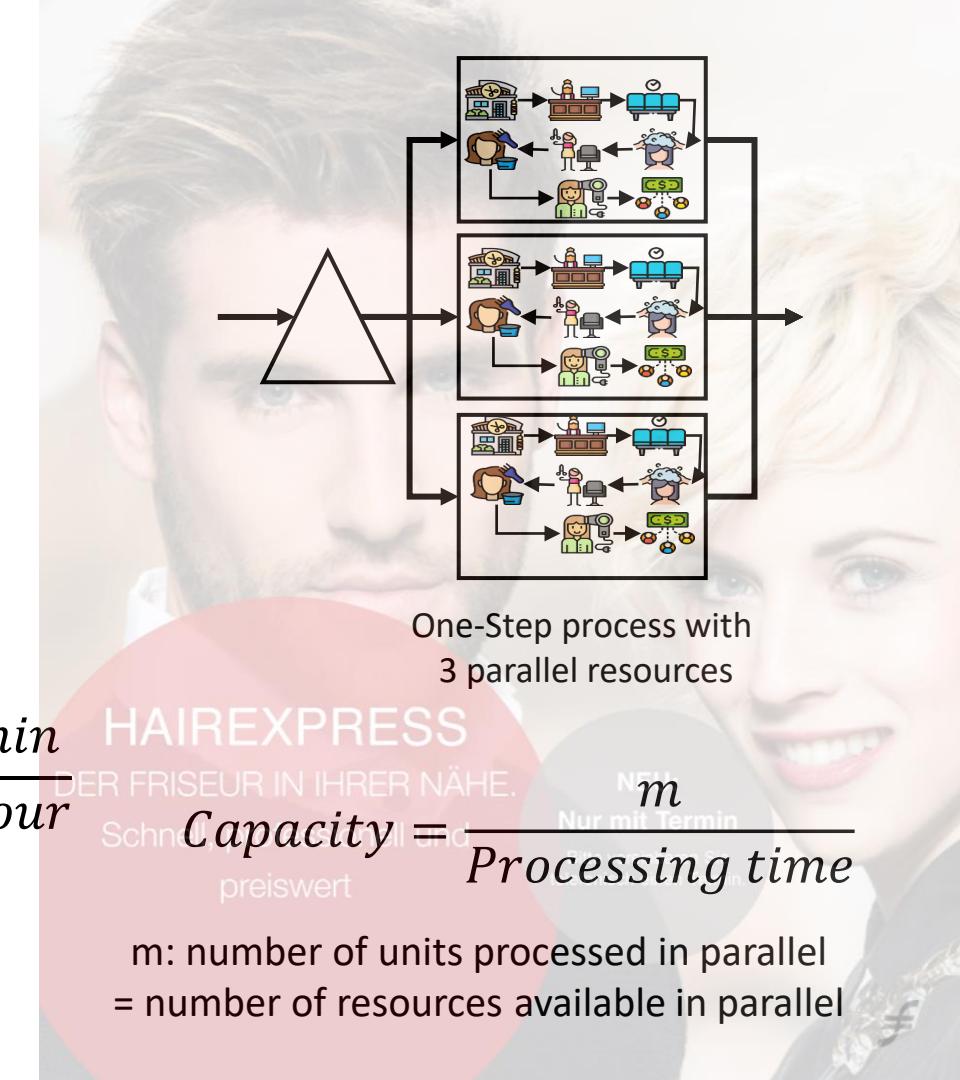
Parallel Service Multiple Resources

$$\text{Capacity} = \frac{3}{54.5 \frac{\text{min}}{\text{customer}}}$$

$$\text{Capacity} = 0.0550 \frac{\text{customers}}{\text{min}}$$

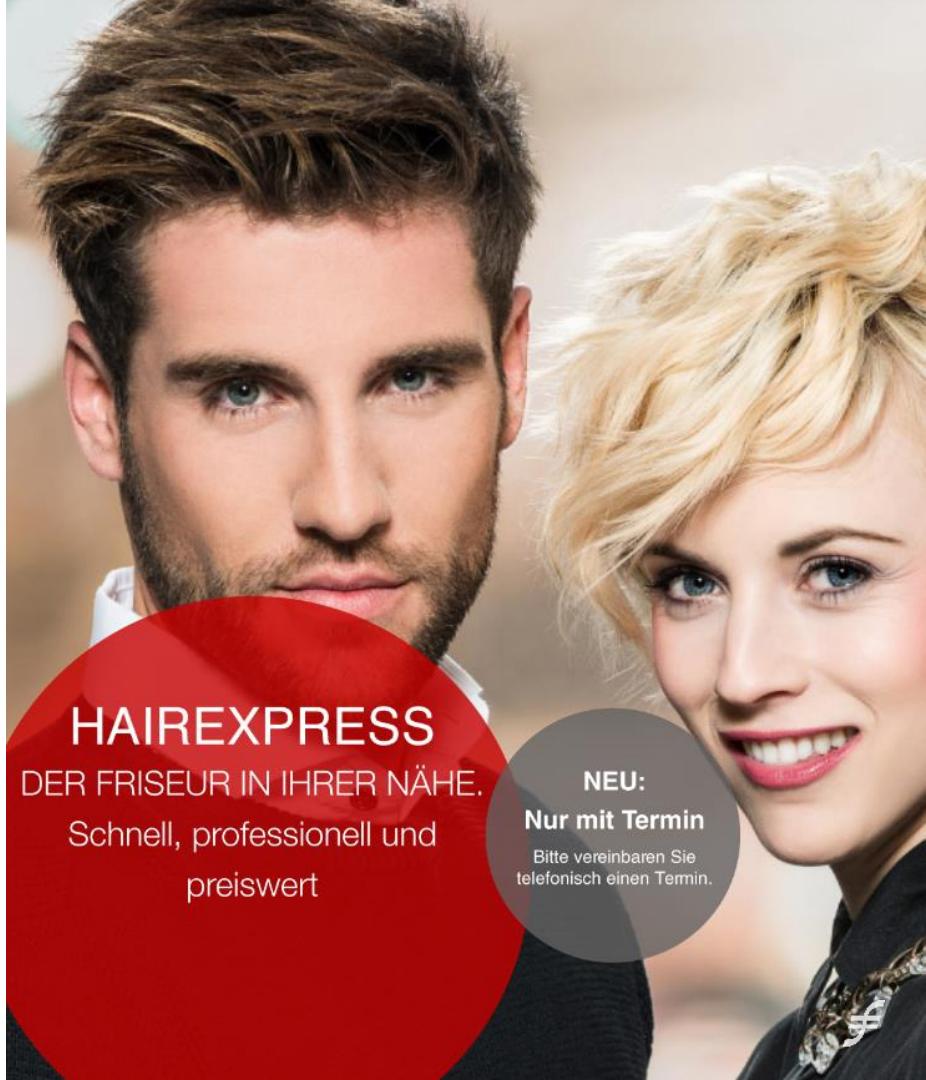
$$= 0.0550 \frac{\text{customers}}{\text{min}} \cdot 60 \frac{\text{min}}{\text{hour}}$$

$$= 3.3028 \frac{\text{customers}}{\text{hour}}$$



HAIREXPRESS Challenges

- What is the capacity of parallel one-step processes?
- **How many resources (workstations) are needed to match demand ?**



How Many Workstations Needed to Match Demand?

$$\frac{12 \frac{\text{customers}}{\text{hour}}}{60 \frac{\text{min}}{\text{hour}}} = \frac{m}{54.5 \frac{\text{hairdresser} \cdot \text{min}}{\text{customer}}}$$

$$m = \frac{12 \frac{\text{customers}}{\text{hour}}}{60 \frac{\text{min}}{\text{hour}}} \cdot 54.5 \frac{\text{hairdresser} \cdot \text{min}}{\text{customer}}$$

$$m = 10.9 \text{ hairdresser}$$

$$\text{Capacity} = \frac{m}{\text{Processing time}}$$

- Demand rate = 12 customers/hour
- Capacity = Demand rate
- Pay attention to time units!

HAIREXPRESS

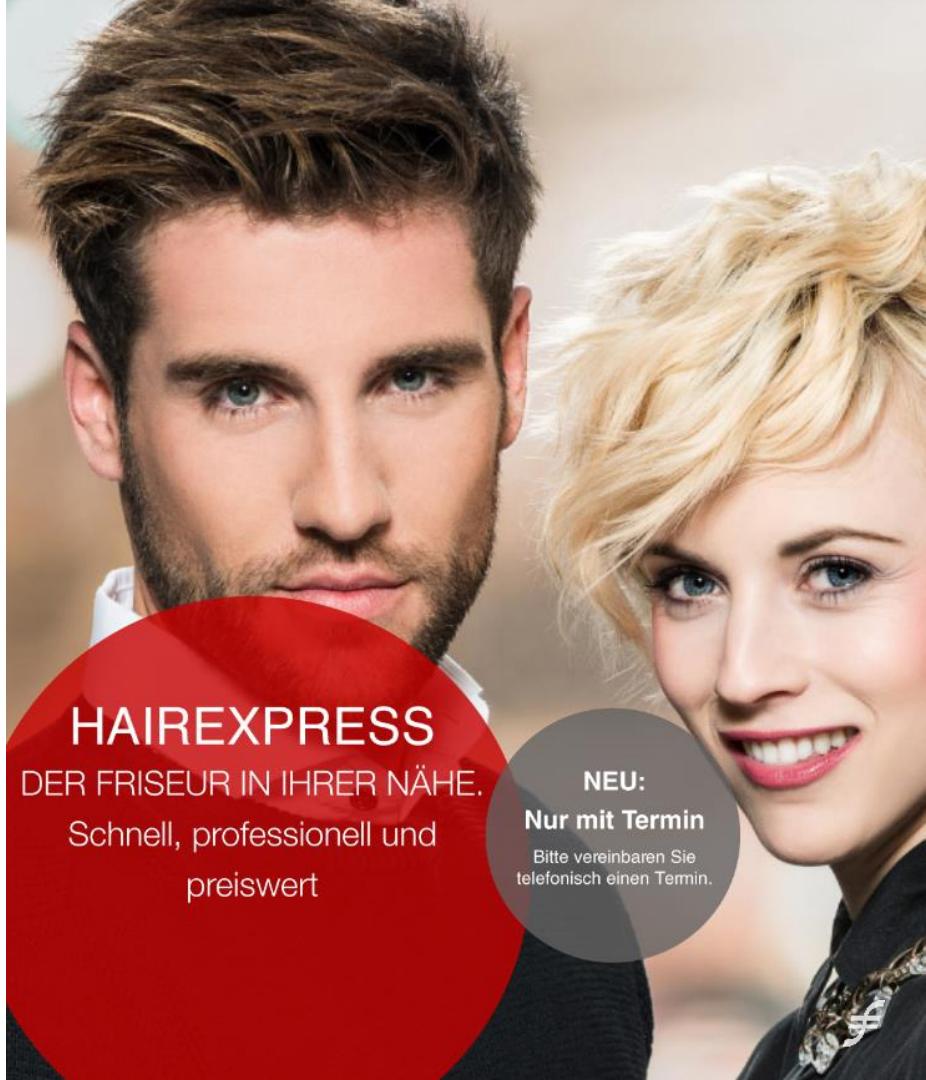
- If part-time work is not possible, 11 stations with one hairdresser each are required to match demand.

MULTISTEP PROCESS

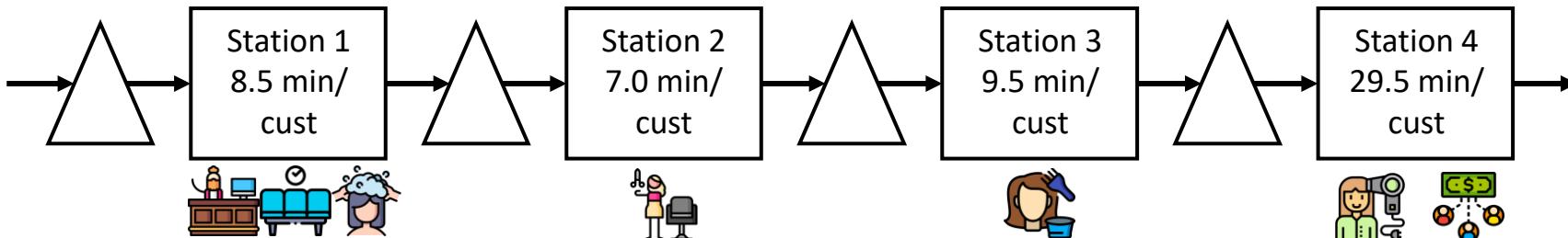


HAIREXPRESS Challenges

- How to determine the capacity of a multistep process?
- Where is the bottleneck in a multistep process?
- How to determine utilization and flow rate in a multistep process?
- How long does it take to serve a certain number of customers with a multistep process?



Analysis of a Multistep Process (multiple resources)



Capacity

$$\frac{1}{8.5 \frac{\text{min}}{\text{customer}}}$$

$$\frac{1}{7.0 \frac{\text{min}}{\text{customer}}}$$

$$\frac{1}{9.5 \frac{\text{min}}{\text{customer}}}$$

$$\frac{1}{29.5 \frac{\text{min}}{\text{customer}}}$$

$$= 0.1176 \frac{\text{customers}}{\text{min}}$$

$$= 0.1429 \frac{\text{customers}}{\text{min}}$$

$$= 0.1053 \frac{\text{customers}}{\text{min}}$$

$$= 0.0339 \frac{\text{customers}}{\text{min}}$$

$$= 7.06 \frac{\text{customers}}{\text{hour}}$$

$$= 8.57 \frac{\text{customers}}{\text{hour}}$$

$$= 6.32 \frac{\text{customers}}{\text{hour}}$$

$$= 2.03 \frac{\text{customers}}{\text{hour}}$$

Process Capacity & Bottleneck

Process capacity

$$= \min \left\{ 7.06 \frac{\text{customers}}{\text{hour}}, 8.57 \frac{\text{customers}}{\text{hour}}, \right. \\ \left. 6.32 \frac{\text{customers}}{\text{hour}}, 2.03 \frac{\text{customers}}{\text{hour}} \right\}$$

$$= 2.03 \frac{\text{customers}}{\text{hour}}$$

Bottleneck is station 4.

- Multistep process with multiple resources:

Process capacity =

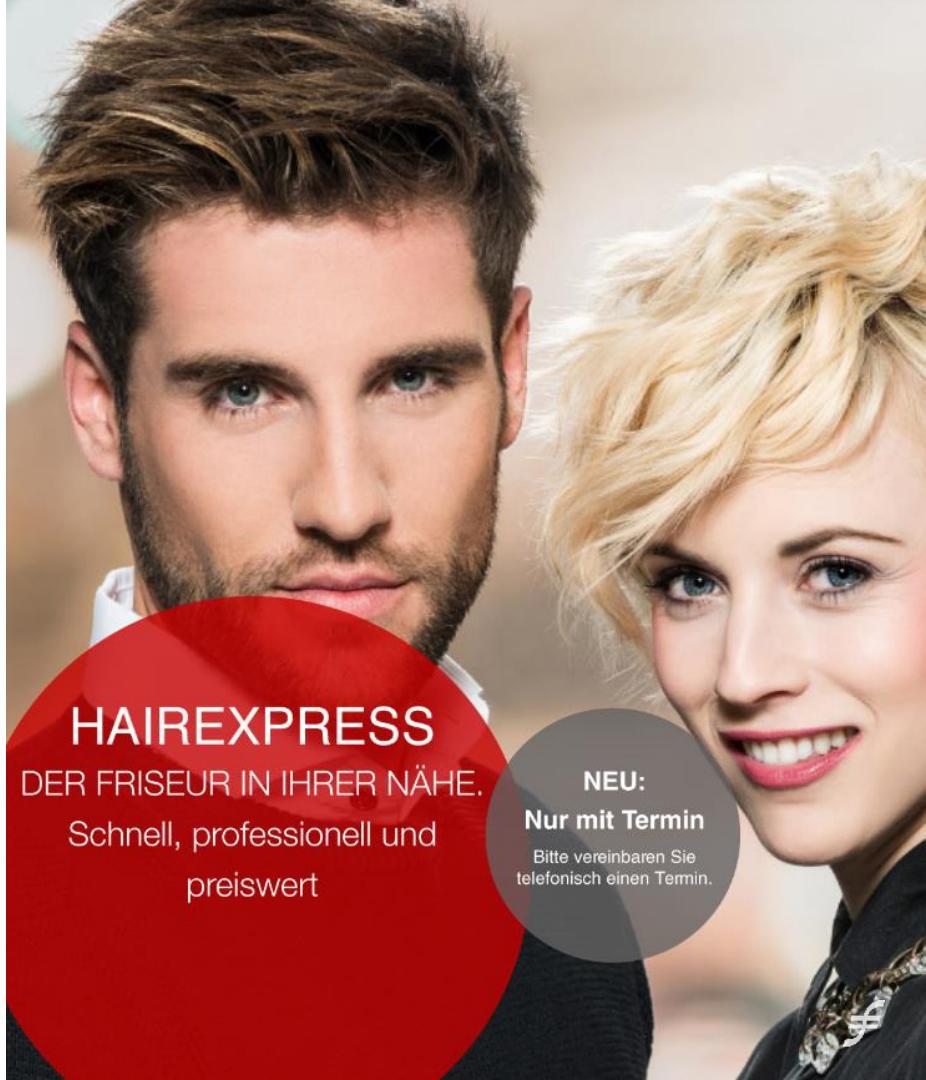
$$\min \left\{ \begin{array}{l} \text{Capacity}_1, \\ \text{Capacity}_2, \\ \dots, \\ \text{Capacity}_n \end{array} \right\}$$

- Bottleneck:
resource with the lowest capacity.

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HAIREXPRESS Challenges

- How to determine the capacity of a multistep process?
- Where is the bottleneck in a multistep process?
- **How to determine utilization and flow rate in a multistep process?**
- How long does it take to serve a certain number of customers with a multistep process?



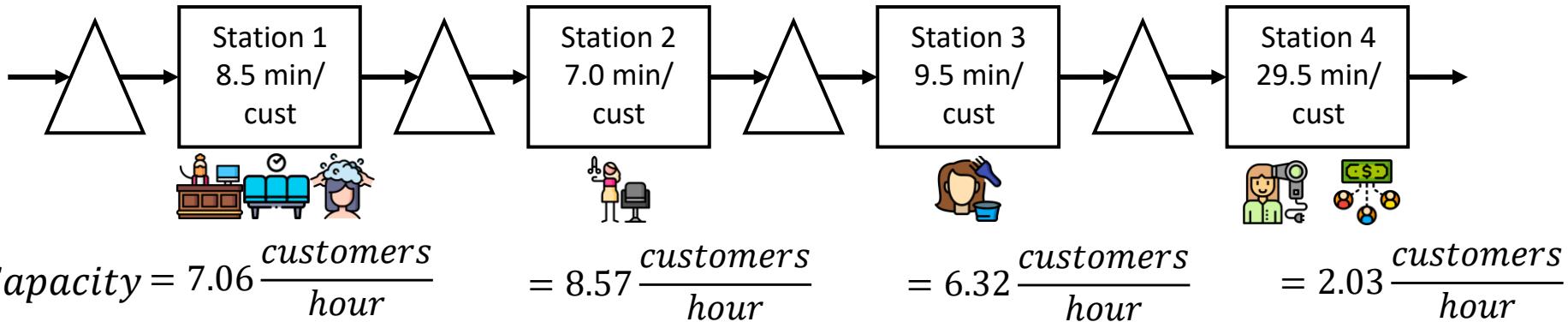
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Flow Rate and Utilization



$$\text{Demand} = 12 \frac{\text{customers}}{\text{hour}}$$

$$\text{Flow rate} = \min\left(12 \frac{\text{customers}}{\text{hour}}, 2.03 \frac{\text{customers}}{\text{hour}}\right)$$

$$\text{Utilization} = 0.2876$$

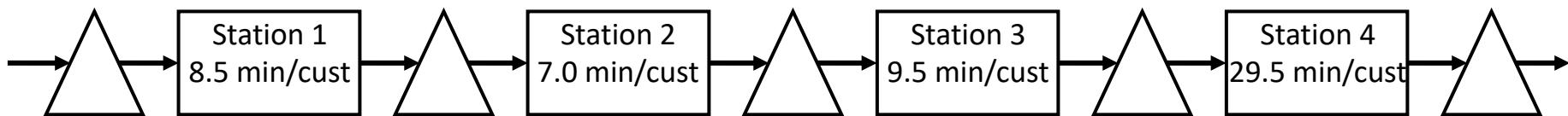
$$= 0.2368$$

$$= 0.3214$$

$$= 1.0000$$

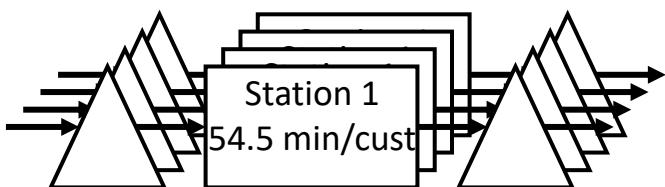
$$\text{Demand} = 12 \frac{\text{customers}}{\text{hour}}$$

Downside of Specialization



$$\text{Average utilization} = \frac{0.2876 + 0.2368 + 0.3214 + 1}{4} = 0.4610$$

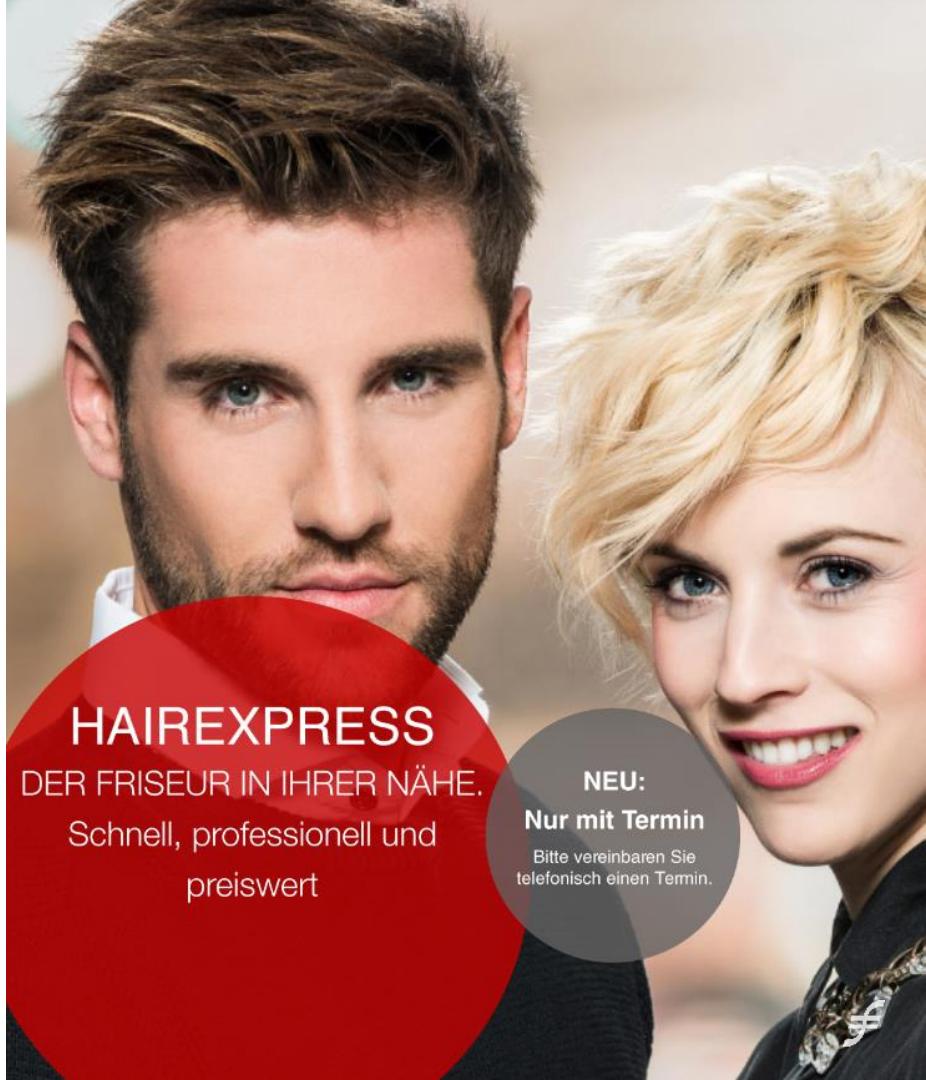
Parallel process:



$$\text{Average utilization} =$$

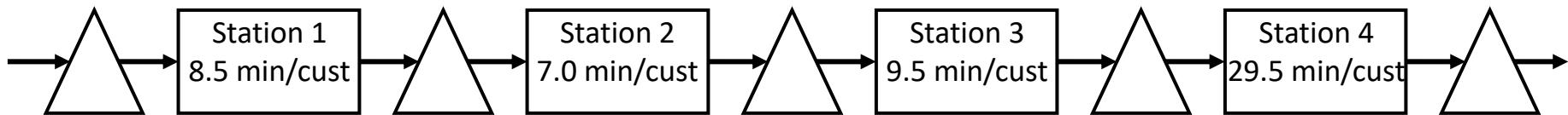
HAIREXPRESS Challenges

- How to determine the capacity of a multistep process?
- Where is the bottleneck in a multistep process?
- How to determine utilization and flow rate in a multistep process?
- **How long does it take to serve a certain number of customers with a multistep process?**



Serial vs Parallel Process (Working)

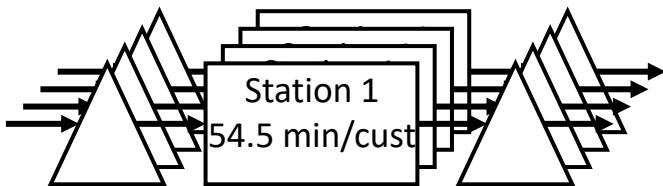
Serial process with specialists



Cycle time = processing time at the bottleneck = 29.5 min/customer

$$\text{Time to serve 10 customers} = 29.5 \frac{\text{min}}{\text{customer}} \cdot 10 \text{ customers} = 295 \text{ min}$$

Parallel process with generalists (manufacturing cell)



$$\begin{aligned} \text{Cycle time} \\ = 1 / \frac{4 \text{ stations}}{54.5 \frac{\text{station} \cdot \text{min}}{\text{customer}}} = 13.625 \frac{\text{min}}{\text{customer}} \end{aligned}$$

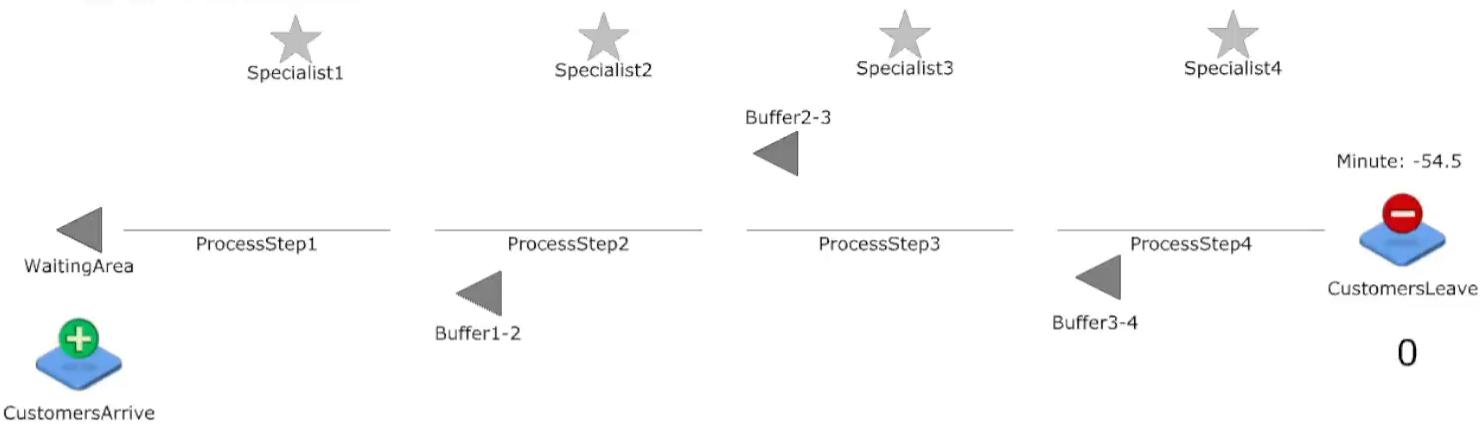
$$\text{Time to serve 10 customers} = 13.625 \frac{\text{min}}{\text{customer}} \cdot 10 \text{ customers} = 136.25 \text{ min}$$



Serial versus Parallel Process - Serial Manufacturing versus Manufacturing Cell

Time to serve Q customers when operating

00:00:00



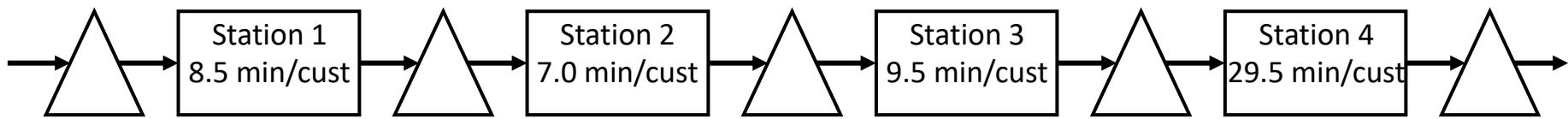
$Q = 10$



Serial Process without Takt with Buffers (Starts Empty)

Serial production (without takt):

- serial system with buffers between the stations
- no automated transport of flow units



$$\text{Time to serve first customer} = 8.5 \text{ m} + 7.0 \text{ m} + 9.5 \text{ m} + 29.5 \text{ m} = 54.5 \text{ min}$$

$$\text{Time to serve 9 more customers} = 29.5 \frac{\text{min}}{\text{customer}} \cdot 9 \text{ customers} = 265.5 \text{ min}$$

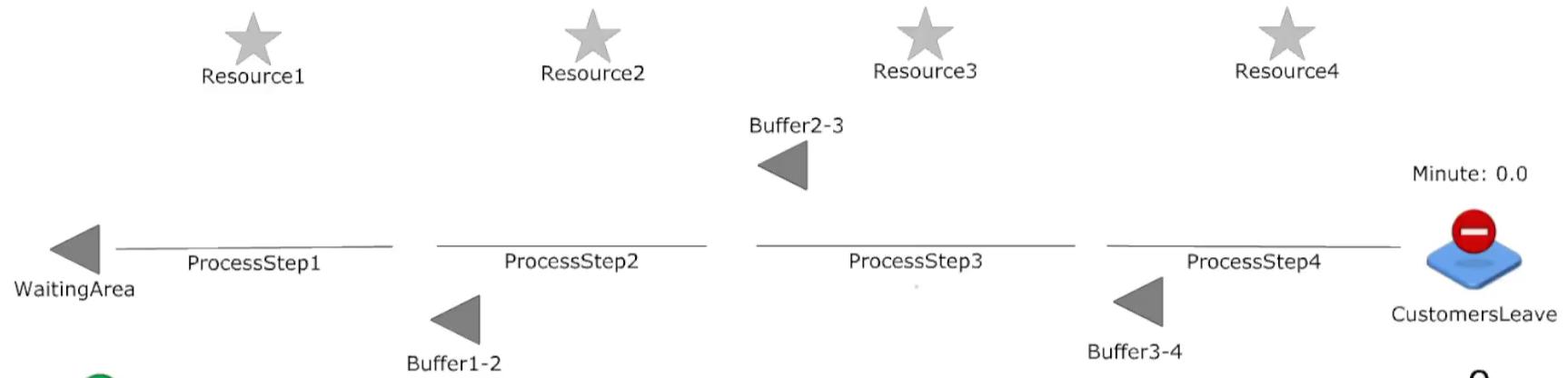
$$\text{Time to serve 10 customers} = 265.5 \text{ min} + 54.5 \text{ min} = 320 \text{ min}$$

Serial Worker Paced Process with Buffers (Empty)



Time to serve Q customers

00:00:00



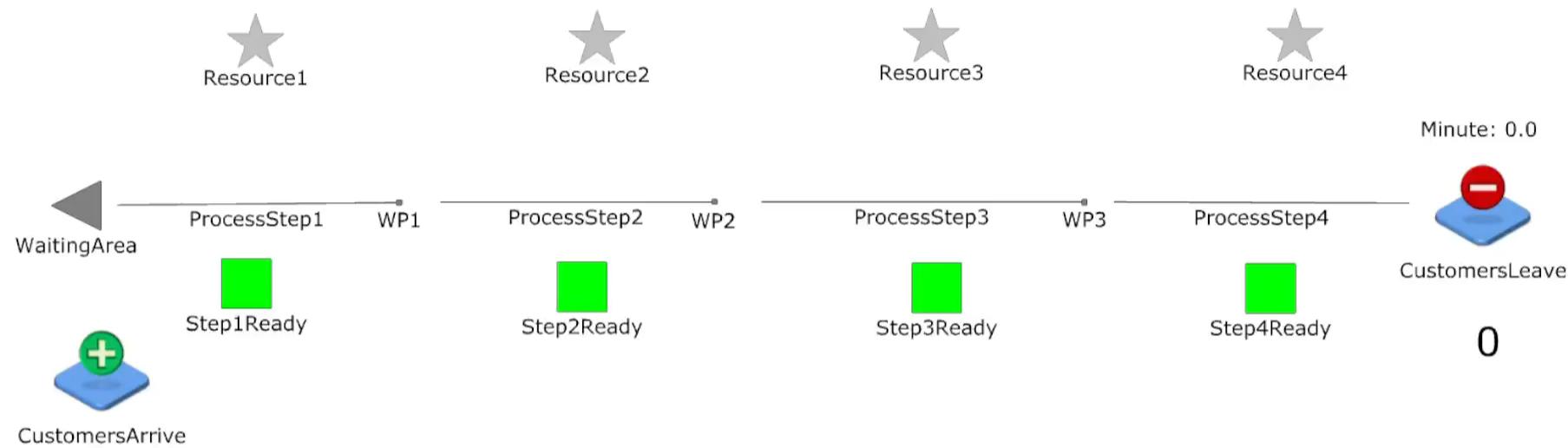
$$Q = 10$$



Serial Worker Paced Process without Buffers (Empty)

Time to serve Q customers

00:00:00



$$Q = 10$$

Serial Process with Takt without Buffers (Starts Empty)

Serial production (with takt):

- automated transport between the stations (conveyor belt)
- no buffers, no possibility to build up inventories



Time to serve first customer = $29.5 \text{ m} + 29.5 \text{ m} + 29.5 \text{ m} + 29.5 \text{ m} = 118 \text{ min}$

Time to serve 9 more customers = $29.5 \frac{\text{min}}{\text{customer}} \cdot 9 \text{ customers} = 265.5 \text{ min}$

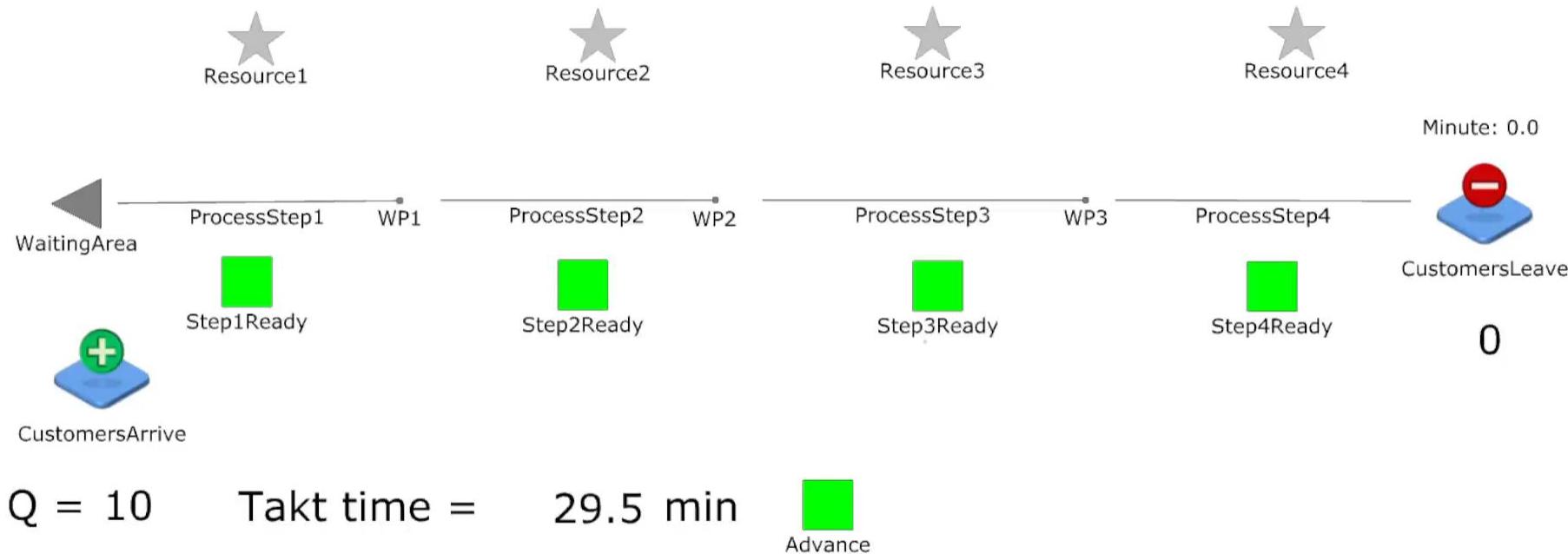
Time to serve 10 customers = $265.5 \text{ min} + 118 \text{ min} = 383.5 \text{ min}$



Serial Machine Paced Process without Buffers (Empty)

Time to serve Q customers

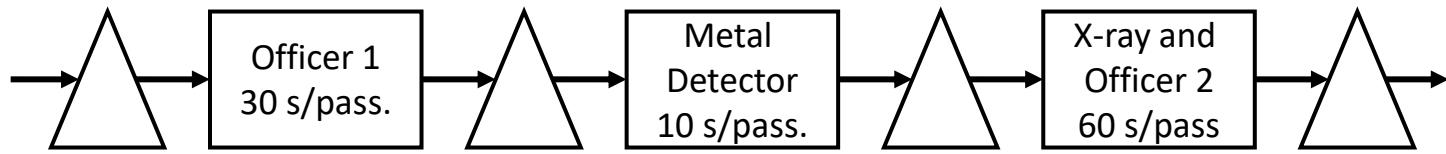
00:00:00



BACKUP



How long does it take to serve a group of 40?



The process is empty at 6:00 when a group of 40 passengers arrive.





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2023 Operations Management

PROCESS IMPROVEMENT

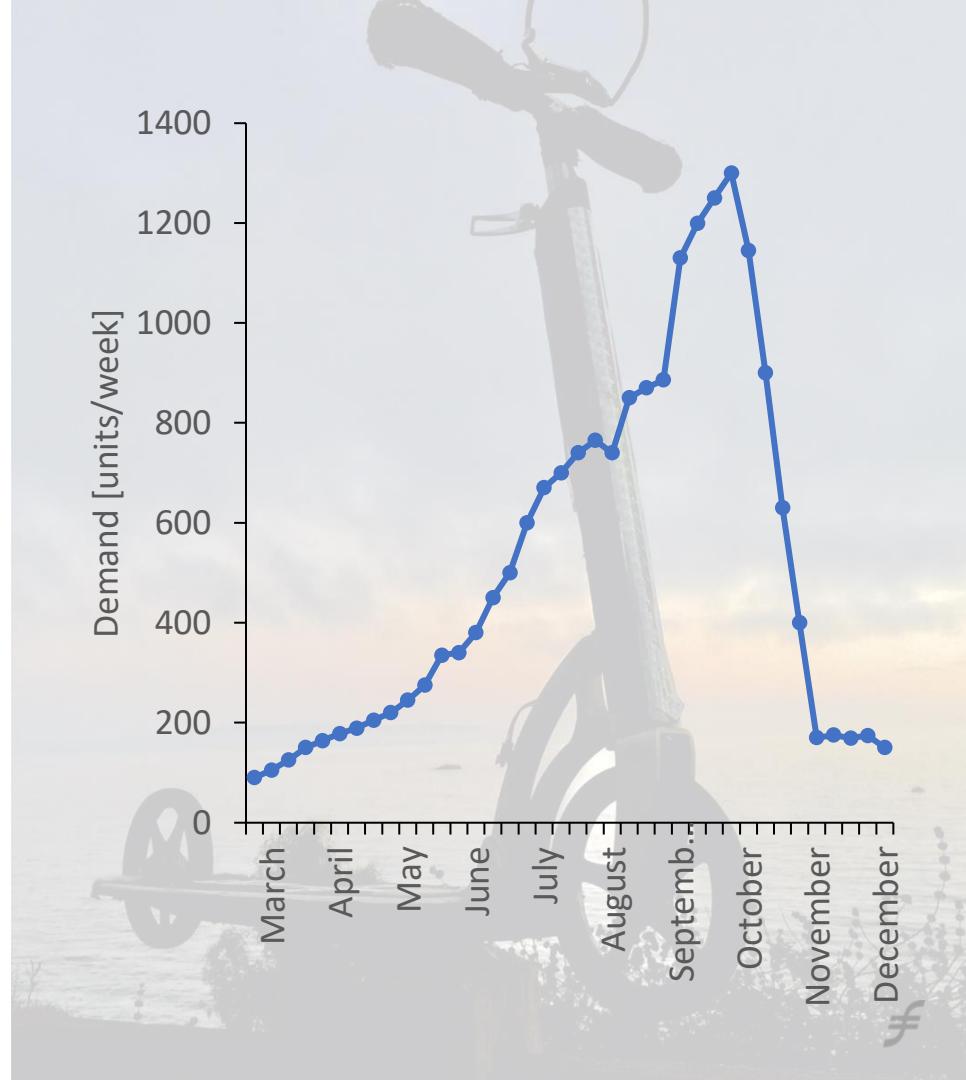
Xootr LLC

- Roots of Xootr go back to 1999, when Nova Cruz Products was founded.
- 2003 the Nova Cruz was sold, but founders were able to buy the original kick-scooter business back.
- New company has grown profitably ever since its formation.

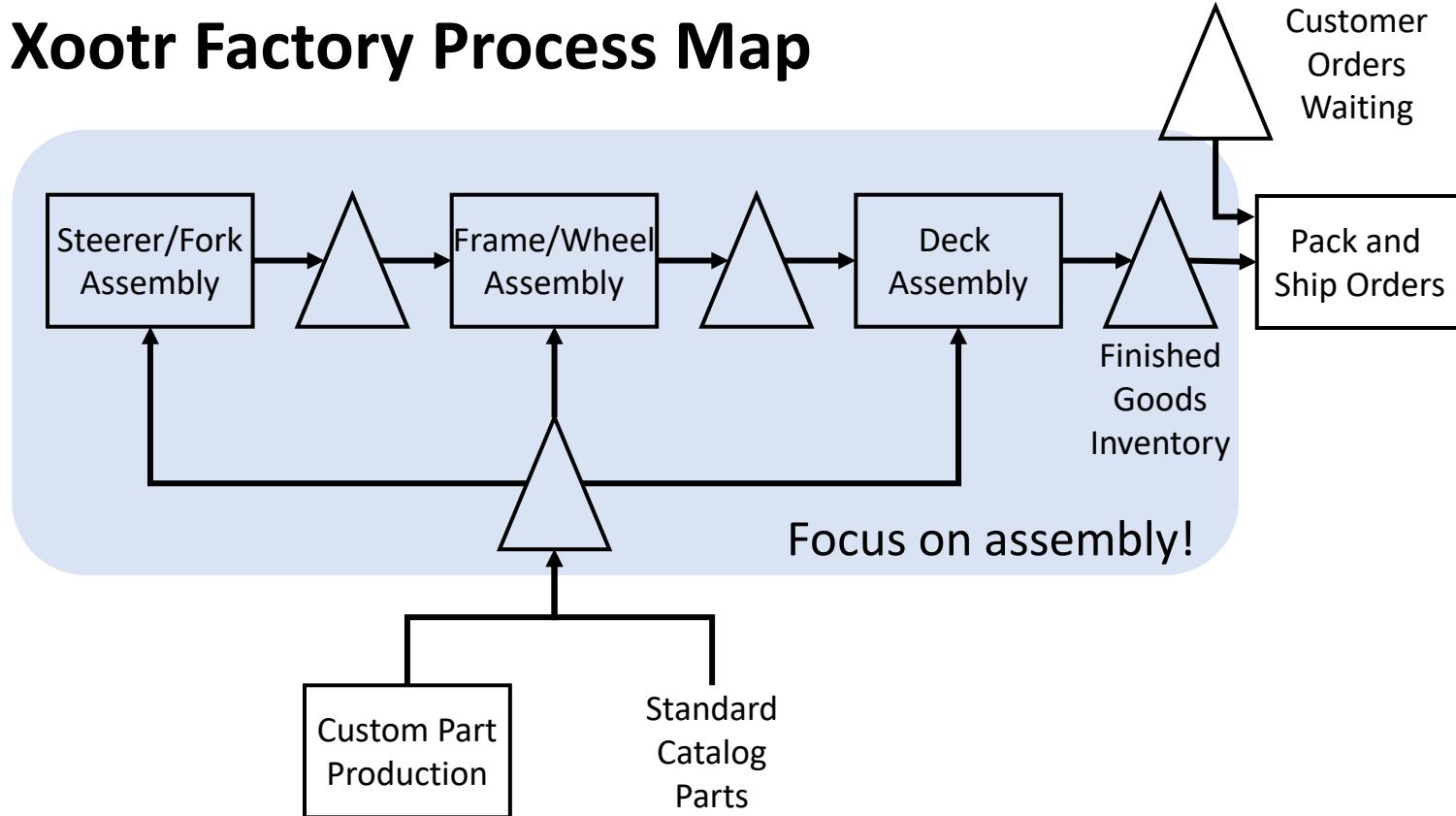


Xootr Challenges

- How to measure process efficiency?
- How to improve process efficiency?
- How to scale up production?



Xootr Factory Process Map

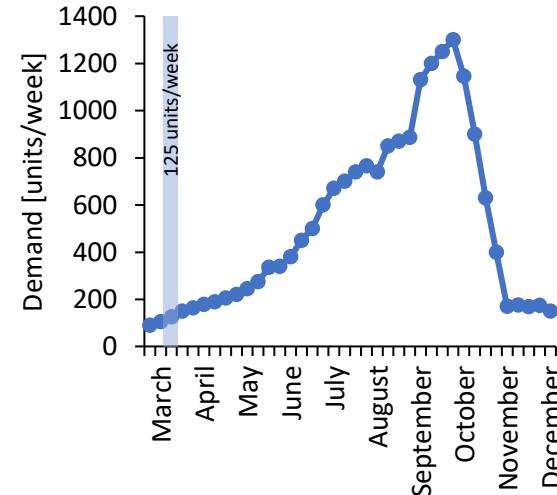
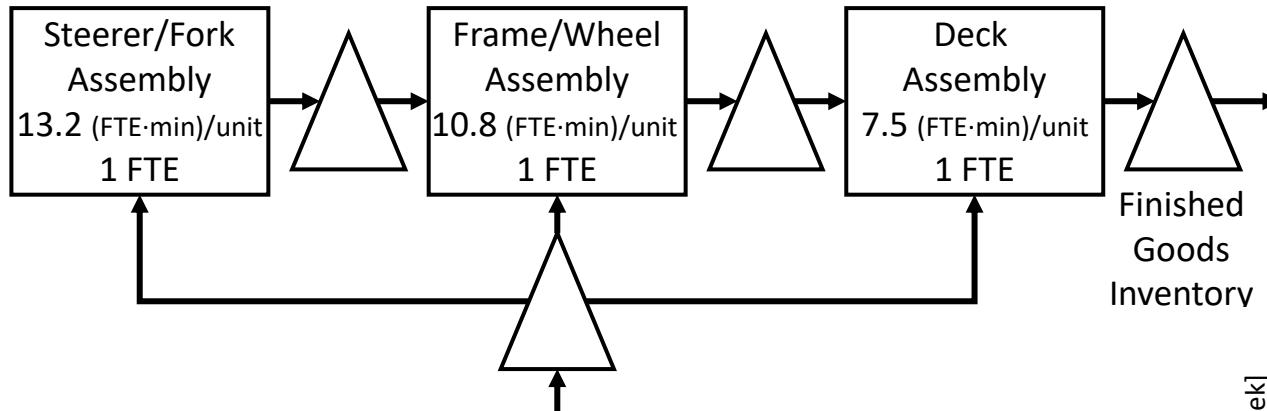


Process Efficiency

- “A process is efficient if it is able to achieve a high flow rate with few resources.”
- Focus on resources necessary, for example deck assembly processing requires
 - 1 worker (FTE = full time equivalent) for 8 minutes per unit.
 - 1 drilling machine for 3 minutes per unit.
 - ...
- Typical measures of labor efficiency
 - Cost of direct labor
 - Average labor utilization
 - Idle time



Production Situation in March



Cost of Direct Labor

$$\text{Flow rate} = 125 \frac{\text{units}}{\text{week}}$$

Total wages

$$\begin{aligned} &= 3 \text{ FTE} \cdot 12 \frac{\text{€}}{\text{hour} \cdot \text{FTE}} \cdot 35 \frac{\text{hour}}{\text{week}} \\ &= 1,260 \frac{\text{€}}{\text{week}} \end{aligned}$$

$$\text{Cost of direct labor} = \frac{1,260 \frac{\text{€}}{\text{week}}}{125 \frac{\text{units}}{\text{week}}} = 10.08 \frac{\text{€}}{\text{Unit}}$$

$$\text{Cost of direct labor} = \frac{\text{Total wages}}{\text{Flow rate}}$$

Wages and flow rate must be measured in same units of time!



Average Labor Utilization

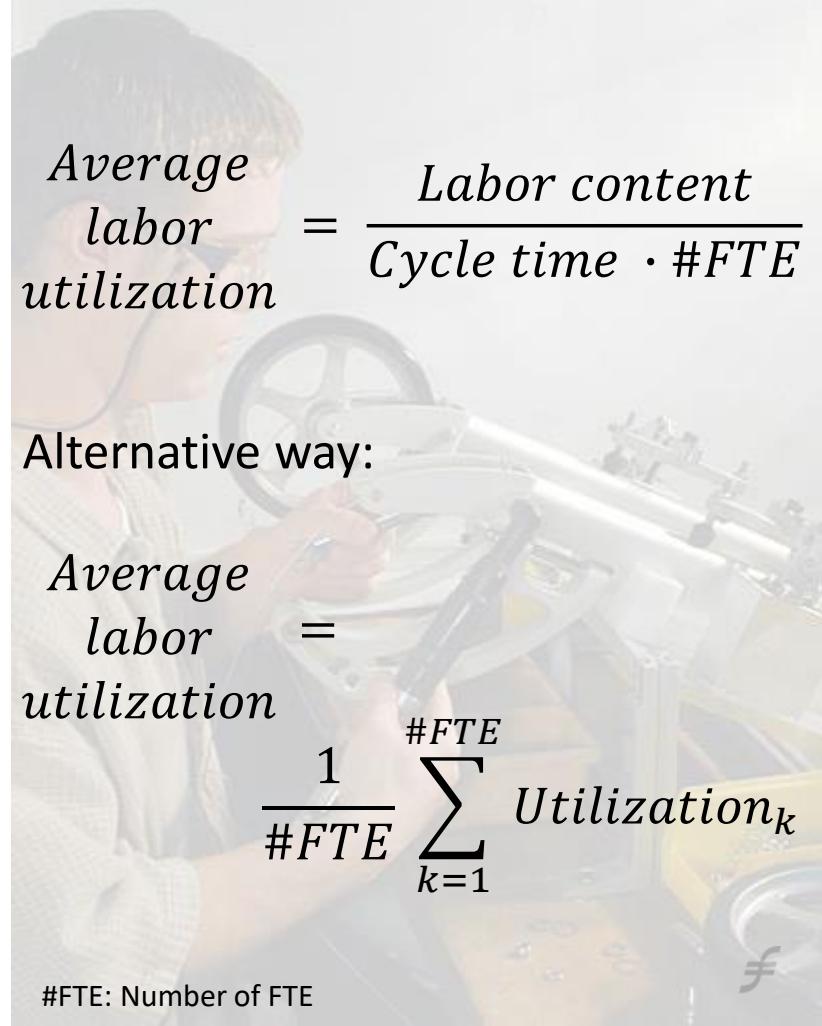
Labor content =

$$13.2 + 10.8 + 7.5 = 31.5 \left[\frac{FTE \cdot min}{unit} \right]$$

$$Cycle time = \frac{1}{125 \frac{units}{week}} \cdot 35 \frac{hour}{week} \cdot 60 \frac{min}{hour}$$

$$Cycle time = 16.8 \frac{min}{unit}$$

$$\begin{aligned} Average labor utilization &= \frac{31.5 \frac{FTE \cdot min}{unit}}{16.8 \frac{min}{unit} \cdot 3FTE} = 0.6250 \end{aligned}$$



Average labor utilization = $\frac{Labor content}{Cycle time \cdot \#FTE}$

Alternative way:

$$\begin{aligned} Average labor utilization &= \\ &\frac{1}{\#FTE} \sum_{k=1}^{\#FTE} Utilization_k \end{aligned}$$

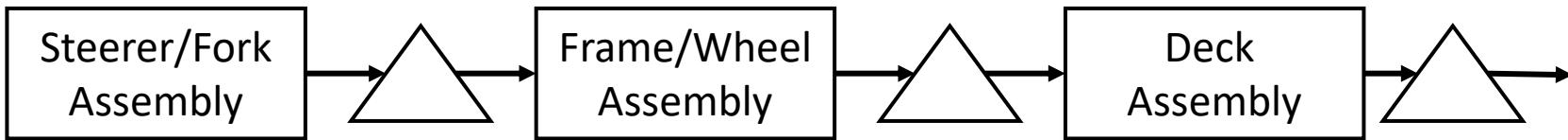
#FTE: Number of FTE

Idle Time

$$\begin{aligned} \text{Total idle time} &= 16.8 \frac{\text{min}}{\text{unit}} \cdot 3\text{FTE} \\ &- 31.5 \frac{\text{FTE} \cdot \text{min}}{\text{unit}} = 18.9 \frac{\text{FTE} \cdot \text{min}}{\text{unit}} \end{aligned}$$

The time a resource (here labor) is not working (but still be paid)

$$\begin{aligned} \text{Total idle time} &= \text{Cycle time} \cdot \text{FTE} \\ &- \text{Labor content} \end{aligned}$$



Idle time at each station =

$$3.6 \frac{\text{FTE} \cdot \text{min}}{\text{unit}}$$

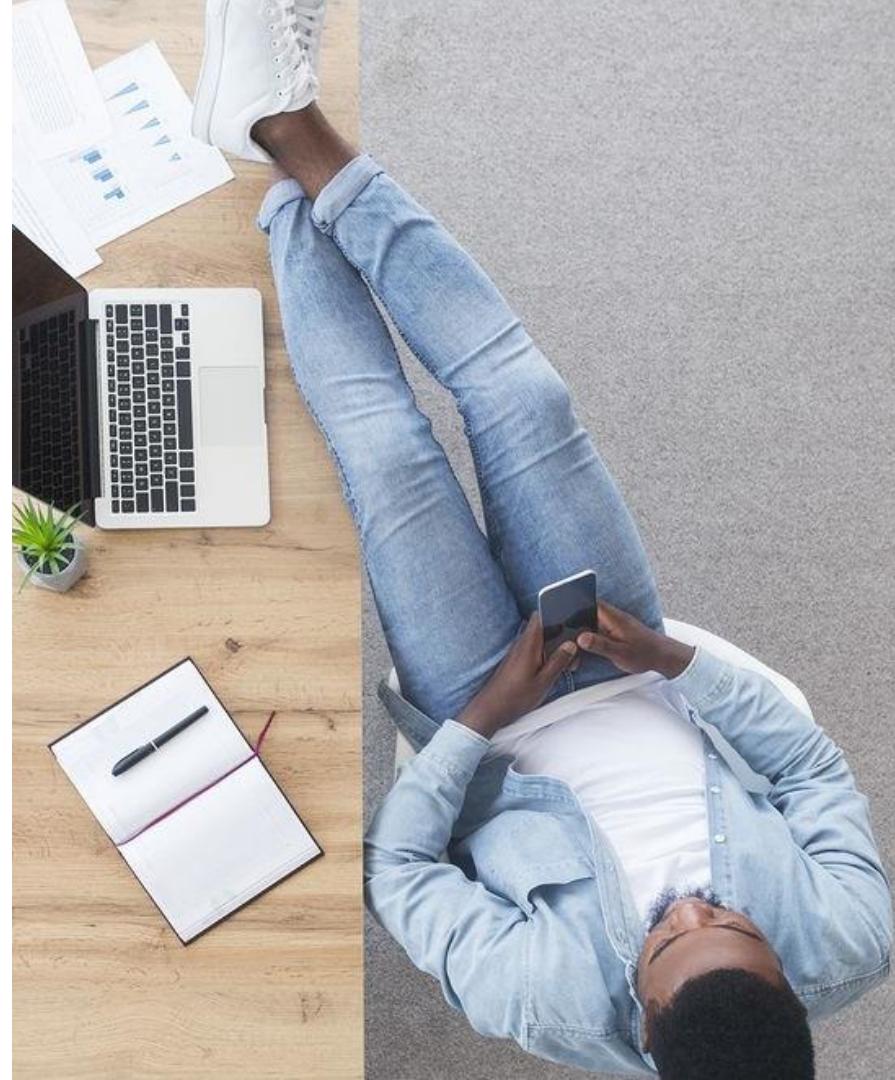
$$6.0 \frac{\text{FTE} \cdot \text{min}}{\text{unit}}$$

$$9.3 \frac{\text{FTE} \cdot \text{min}}{\text{unit}}$$

Two Reasons For Idle Time

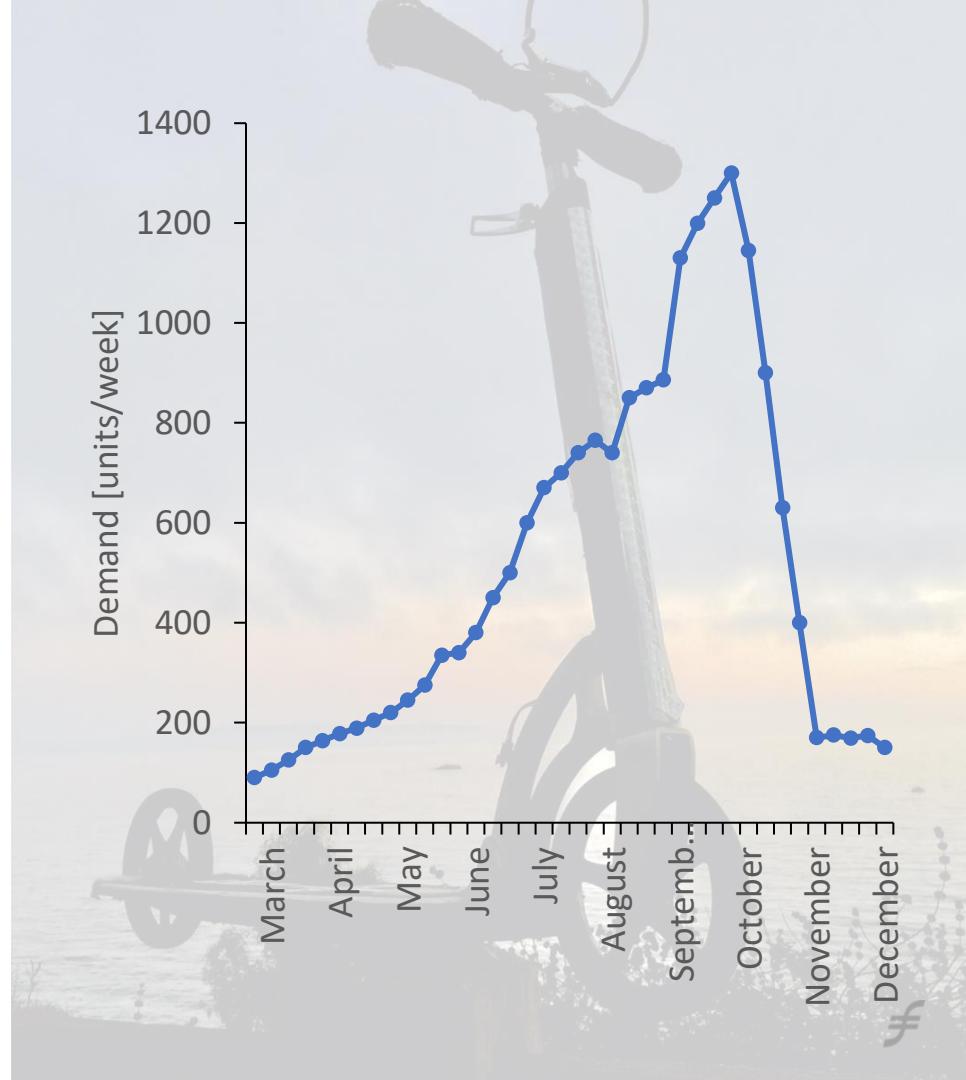
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-



Xootr Challenges

- How to measure process efficiency?
- **How to improve process efficiency?**
- How to scale up production?



How to Improve Process Efficiency?



Increase demand!



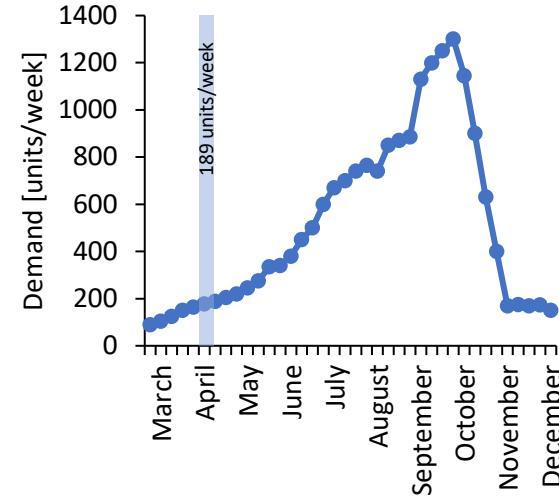
Improve the process

Performance of a Capacity Constrained System

$$\text{Cost of direct labor} = \frac{1,260 \frac{\text{€}}{\text{week}}}{159.0909 \frac{\text{units}}{\text{week}}} = 7.92 \frac{\text{€}}{\text{Unit}}$$

$$\begin{aligned} \text{Total idle time} &= 13.2 \frac{\text{min}}{\text{unit}} \cdot 3\text{FTE} - 31.5 \frac{\text{FTE} \cdot \text{min}}{\text{unit}} \\ &= 8.1 \frac{\text{FTE} \cdot \text{min}}{\text{unit}} \end{aligned}$$

$$\begin{aligned} \text{Average labor utilization} &= \frac{31.5 \frac{\text{FTE} \cdot \text{min}}{\text{unit}}}{13.2 \frac{\text{min}}{\text{unit}} \cdot 3\text{FTE}} = 0.7955 \end{aligned}$$

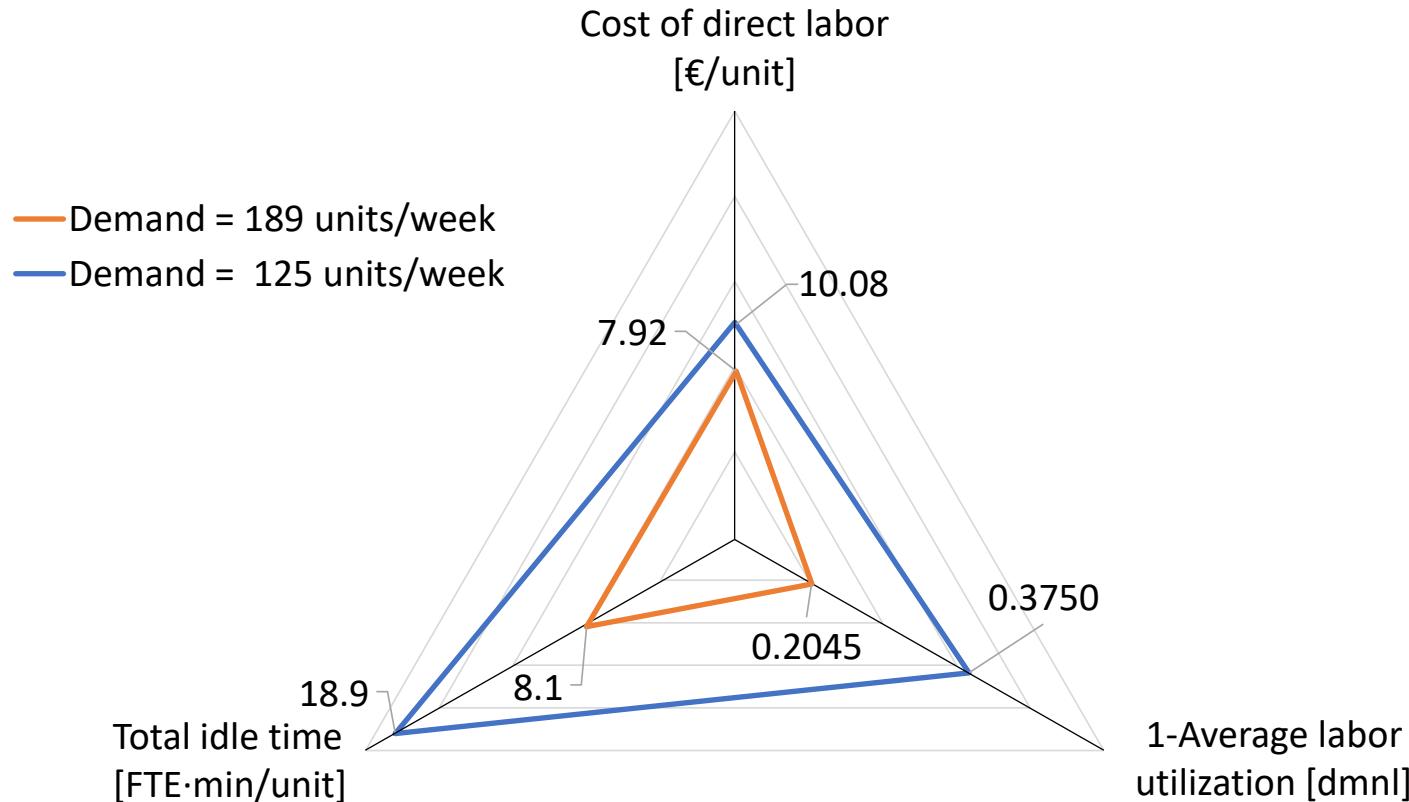


Demand rate: 189 units/week

Flow rate = Process capacity:
159.0909 units/week

Cycle time: 13.2 min/unit

Relation of Labor Efficiency Measures



How to Improve Process Efficiency?



Increase demand!



Improve the process

How to Improve a Process?

- Process improvement starts by looking at the bottleneck.
- Recap: bottleneck is a resource with the lowest capacity.
- Understanding the location of the bottleneck is critical for improving a process.



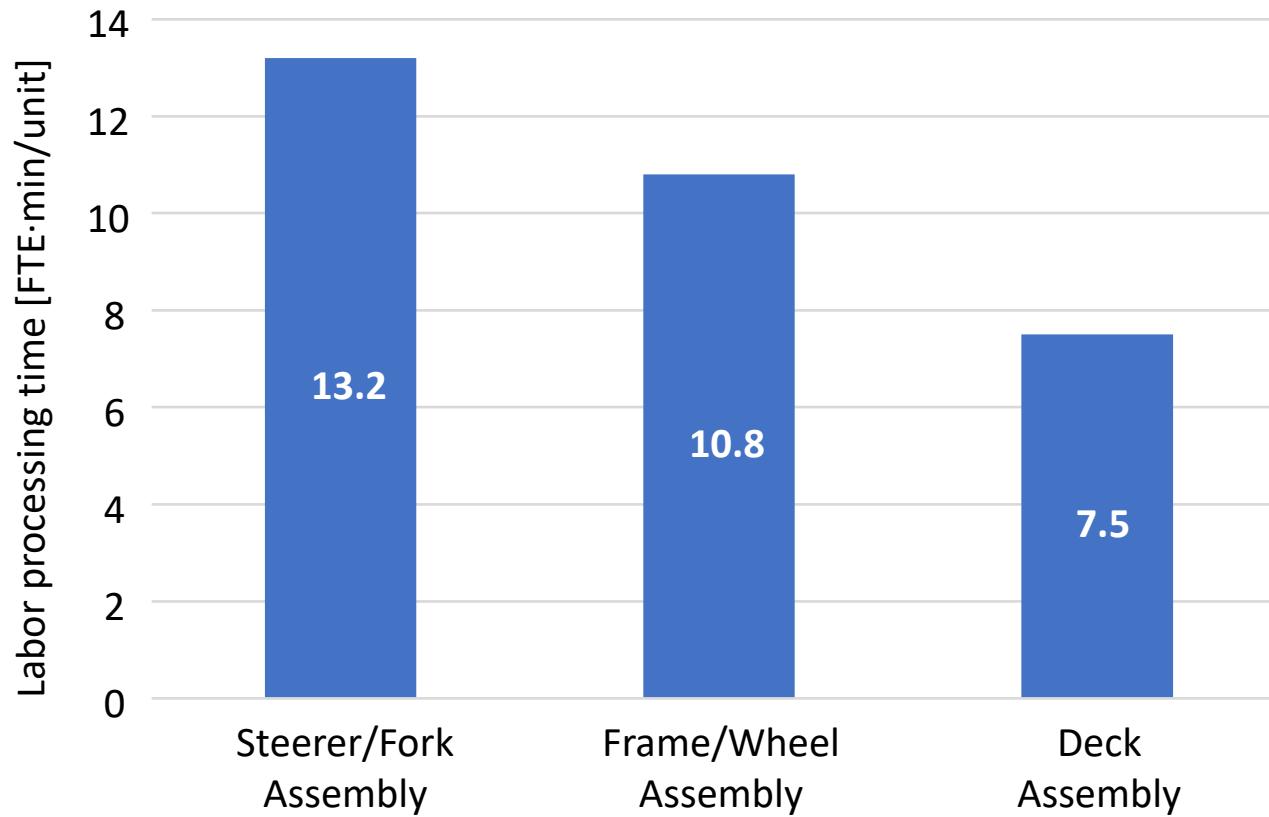
Off-Loading the Bottleneck...

...by:

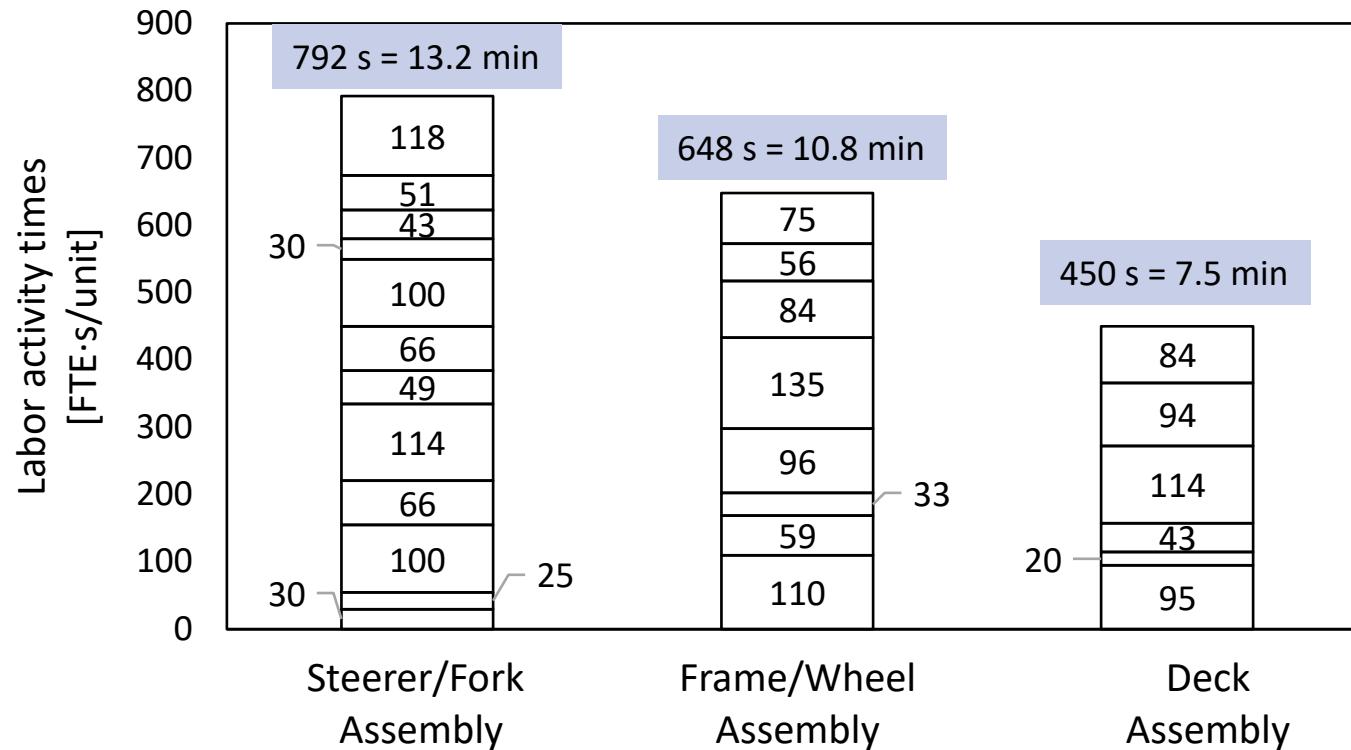
- line-balancing.
- automating some of the activities consuming time at the bottleneck by using technology.
- outsourcing some of the activities consuming time at the bottleneck.



Imbalance in Labor Processing Times



Line Balancing Requires a Micro-level Perspective



How to Balance a Process

Line Balancing

for a fixed sequence
of activities

for activities with
no fixed sequence

Heuristic approach

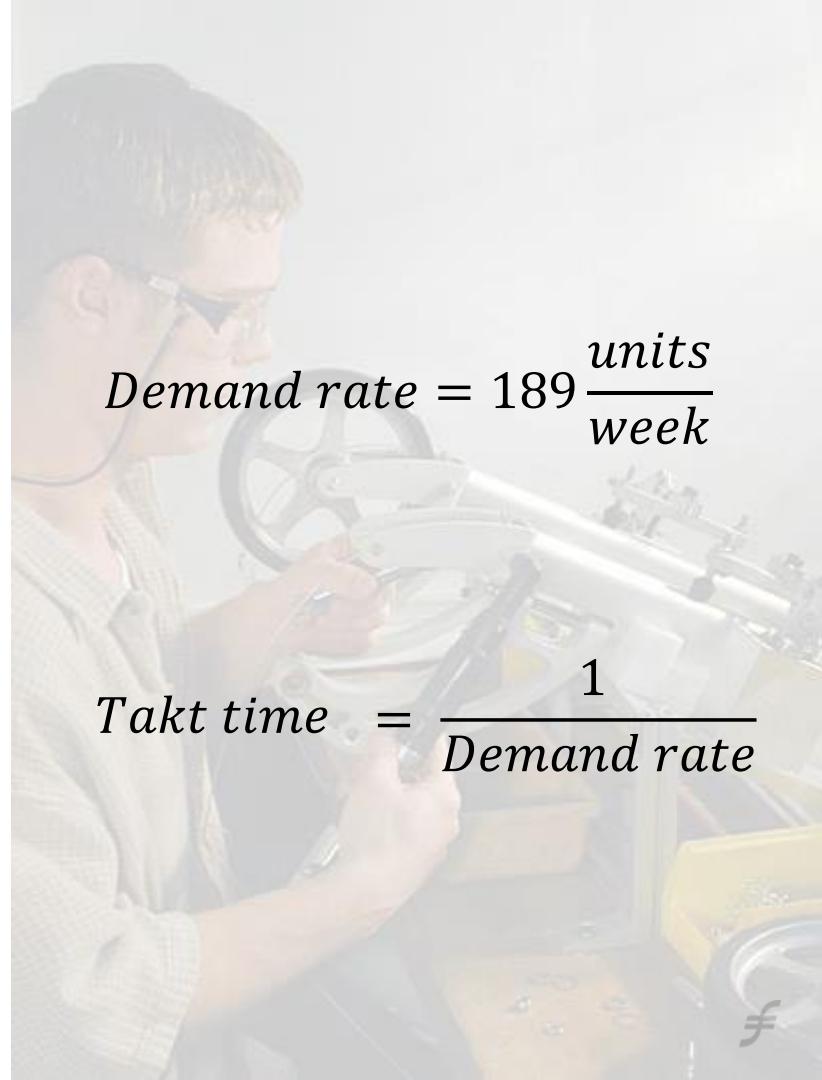
- Compute takt time.
- For each resource: keep on adding activities as long as processing time \leq takt time.

- Extra flexibility allows better results.
- Optimization difficult (beyond this module).

Takt Time

$$Demand\ rate = 189 \frac{\text{units}}{\text{week}}$$

$$Takt\ time = \frac{1}{Demand\ rate}$$



Re-assignment of Activities

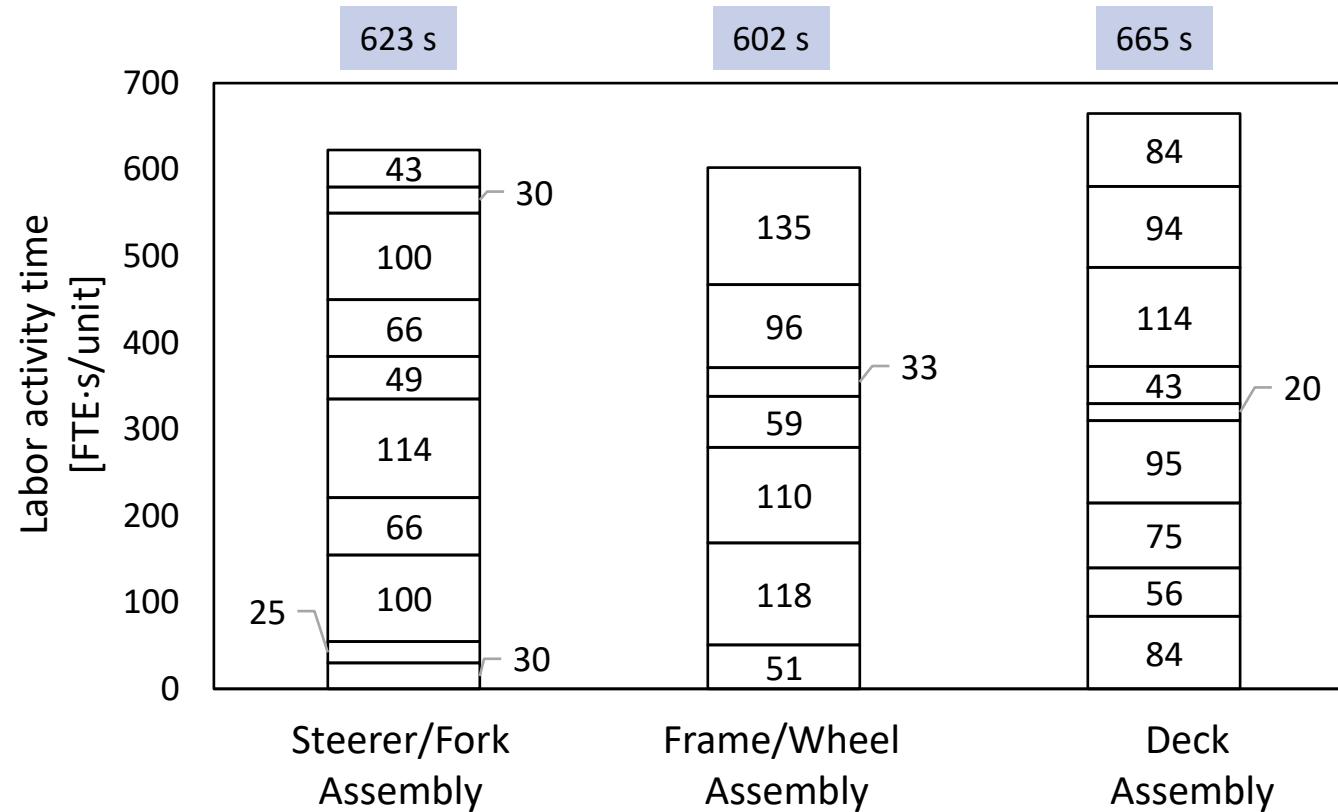
Tasks	Time per activity*	Worker	Tasks	Time per activity*	Worker
#1 Prepare cable	30		#14 Trim and cap cable	59	
#2 Move cable	25		#15 Place first rib	33	
#3 Assemble washer	100		#16 Insert axles and cleats	96	
#4 Apply fork, threading cable end	66		#17 Insert rear wheel	135	
#5 Assemble socket head screws	114		#18 Place second rib and deck	84	
#6 Steer pin nut	49		#19 Apply grip tape	56	
#7 Brake shoe, spring, pivot bolt	66		#20 Insert deck fasteners	75	
#8 Insert front wheel	100		#21 Inspect and wipe off	95	
#9 Insert axle bolt	30		#22 Apply decal and sticker	20	
#10 Tighten axle bolt	43		#23 Insert in bag	43	
#11 Tighten brake pivot bolt	51		#24 Assemble carton	114	
#12 Assemble handle cap	118		#25 Insert Xootr and manual	94	
#13 Assemble brake lever	110		#26 Seal carton	84	

*in seconds

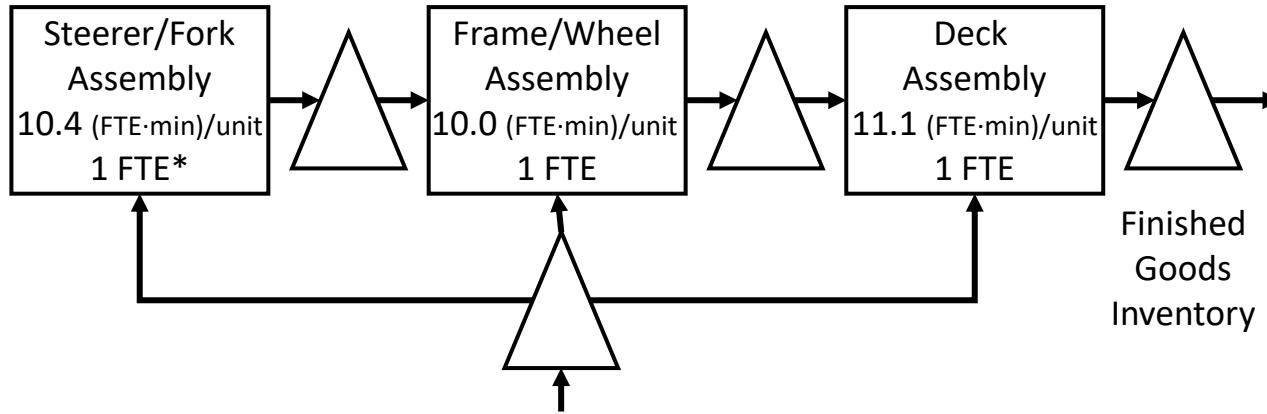
#14: Task number 14



After Line Balancing



Performance of the Balanced System



Worker work time: 35 hours/week

Workers get paid if they are idle

Demand rate: 189 units/week

Performance of the Balanced System

$$\text{Cost of direct labor} = \frac{1,260 \frac{\text{€}}{\text{week}}}{189 \frac{\text{units}}{\text{week}}} = 6.67 \frac{\text{€}}{\text{Unit}}$$

$$\begin{aligned} \text{Total idle time} &= 11.1 \frac{\text{min}}{\text{unit}} \cdot 3\text{FTE} \\ &\quad - 31.5 \frac{\text{FTE} \cdot \text{min}}{\text{unit}} = 1.8 \frac{\text{FTE} \cdot \text{min}}{\text{unit}} \end{aligned}$$

$$\begin{aligned} \text{Average labor utilization} &= \frac{31.5 \frac{\text{FTE} \cdot \text{min}}{\text{unit}}}{11.1 \frac{\text{min}}{\text{unit}} \cdot 3\text{FTE}} = 0.9450 \end{aligned}$$

Demand rate:

189 units/week

Flow rate:

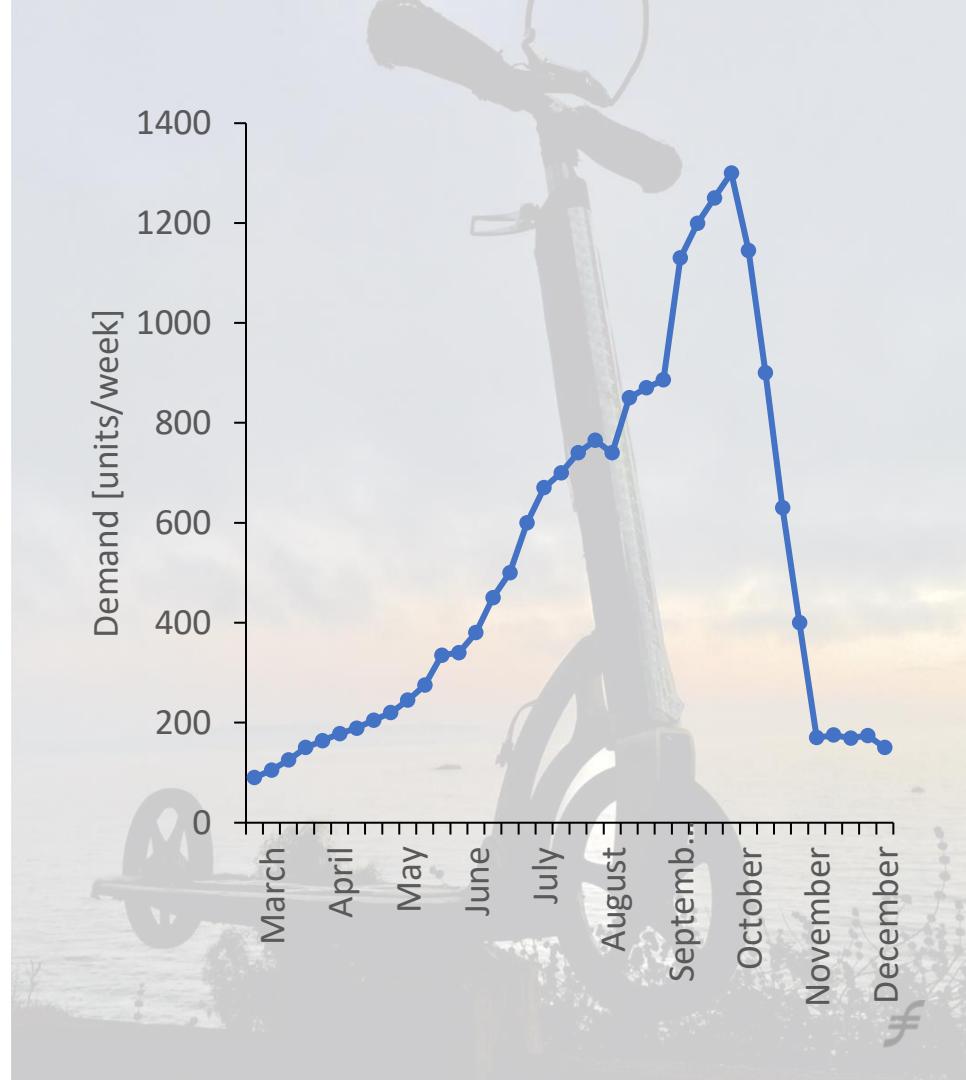
189 units/week

Cycle time:

11.1 min/unit

Xootr Challenges

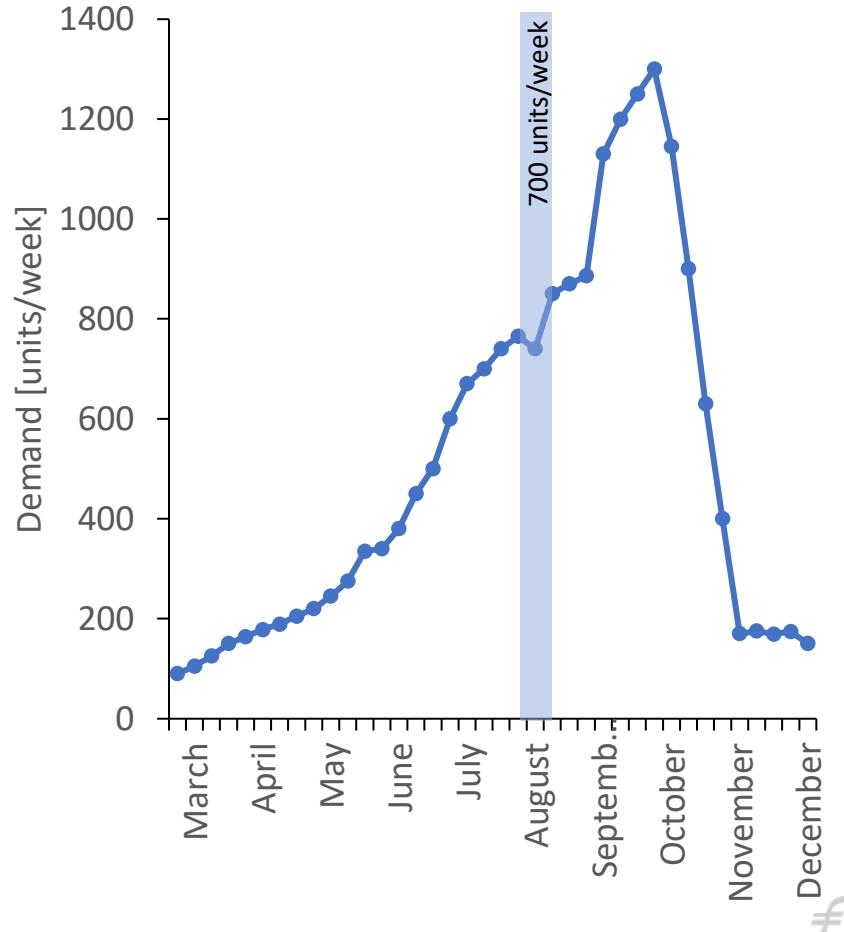
- How to measure process efficiency?
- How to improve process efficiency?
- **How to scale up production?**



How to scale up production to 700 units/week?

- How much time do we have to produce one unit?

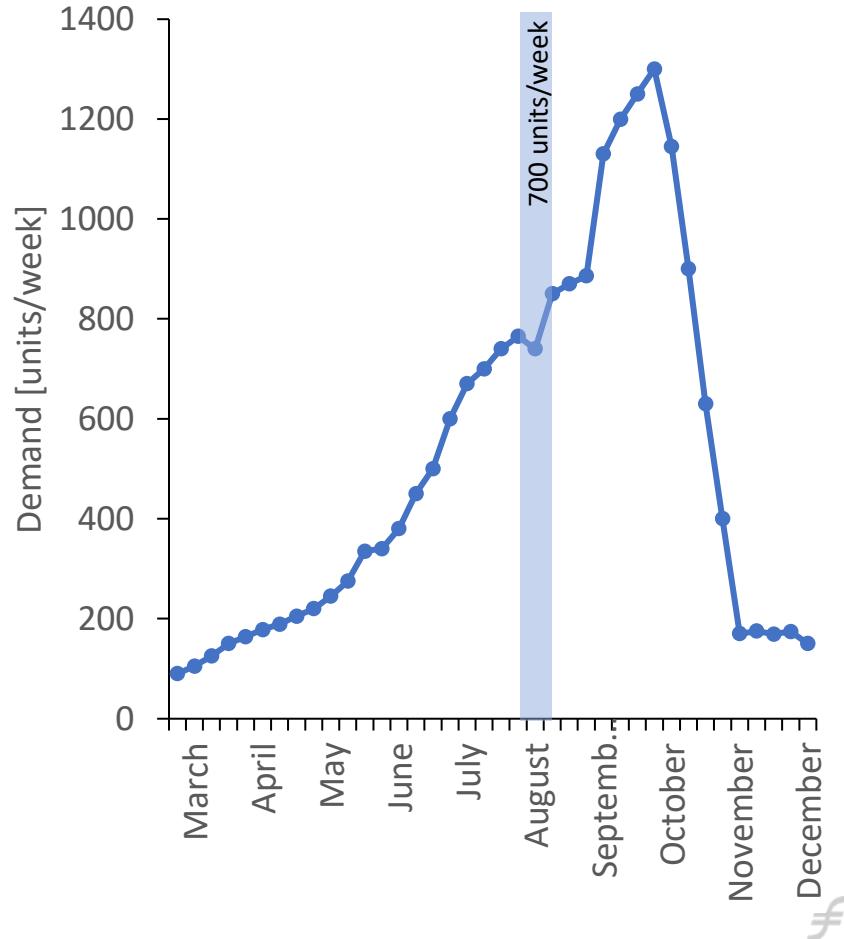
$$Takt\ time = \frac{1}{Demand\ rate}$$



How to scale up production to 700 units/week?

- How many FTE do we need?

$$\text{Target manpower} = \frac{\text{Labor content}}{\text{Takt time}}$$



Three Alternate Process Designs

- Specialization:
one assembly line with many stations.
- Replication:
a couple of parallel three-station lines.
- Generalization:
many parallel work cells.



Specialization – One Line With Many Stations

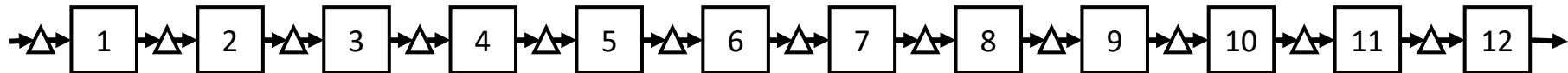
The line-balancing activity assignment rule is reused: For each resource, activities are added as long as processing time \leq takt time

Tasks	Time per activity [FTE·s/unit]	Time per worker			
		Worker 1	Worker 2	Worker 3	...
#1 Prepare cable	30	1	0	0	
#2 Move cable	25	1	0	0	
#3 Assemble washer	100	1	0	0	
#4 Apply fork, threading cable end	66	0	1	0	
#5 Assemble socket head screws	114	0	1	0	
#6 Steer pin nut	49	0	0	1	

...

How does the process look like?

Specialization – One Line With Many Stations



Why do we need 12 stations (=12 FTE) instead of 10.5?

Advantages of specialization

-
-
-

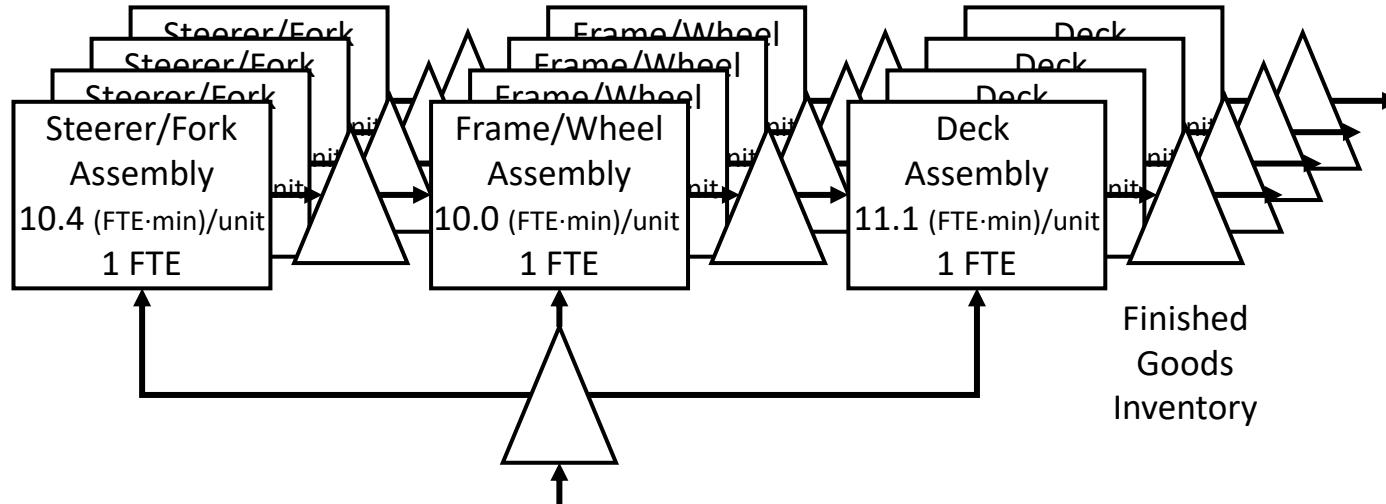
Disadvantages of specialization

-
-
-

Replication – Several Parallel Three-station Lines

Process capacity of one balanced line: 189.5 units/week

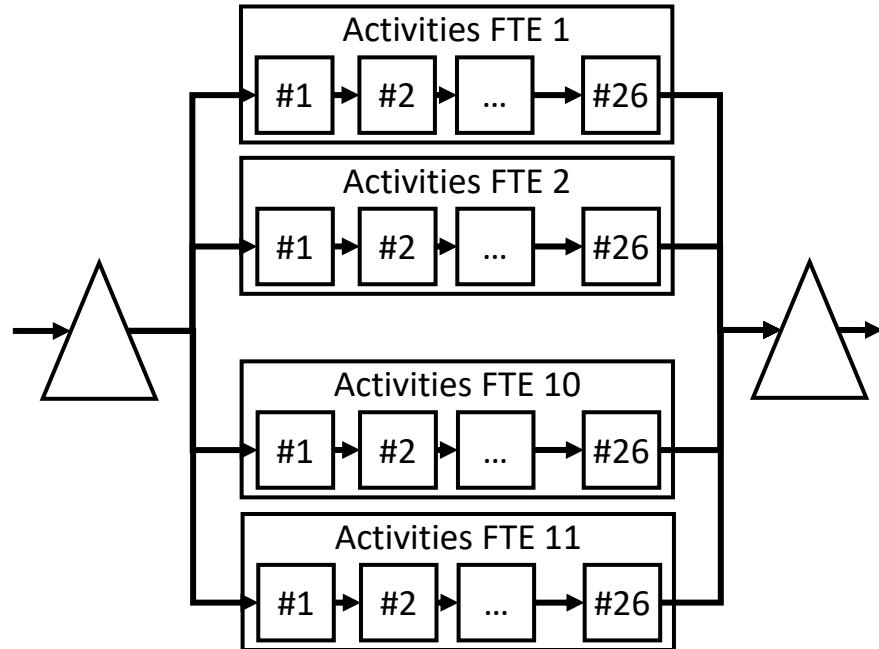
$$\text{Number of lines} = \frac{700 \frac{\text{units}}{\text{week}}}{189.5 \frac{\text{units}}{\text{week}}} = 3.7$$



Generalization - Many Parallel Work Cells

Assigning all activities to one work cell with one worker and use the [target manpower formula](#) and calculation:

$$\begin{aligned} \text{Target manpower} &= \frac{\text{Labor content}}{\text{Takt time}} \\ &= 10.5 \text{ FTE} \end{aligned}$$



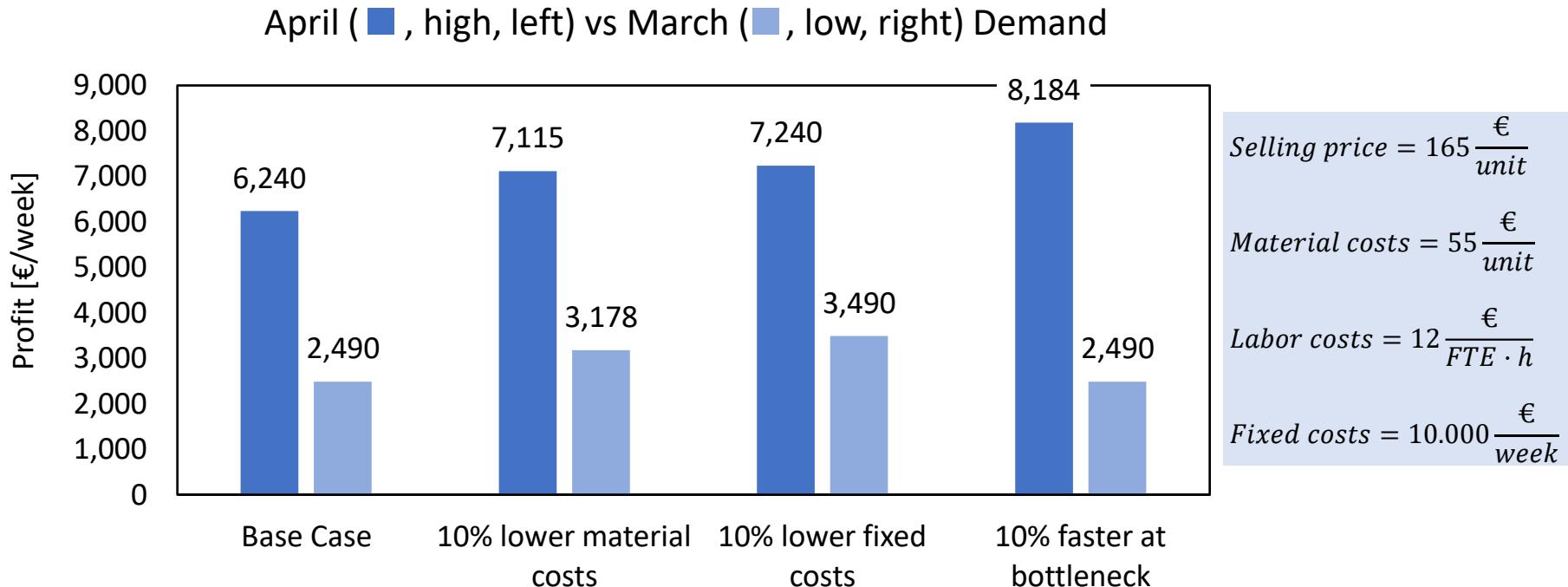
Financial Impact of Process Improvement

- The goal of operations in for-profit organizations is to help generate profits.
- Operations in non-for-profit organizations should help to avoid losses.

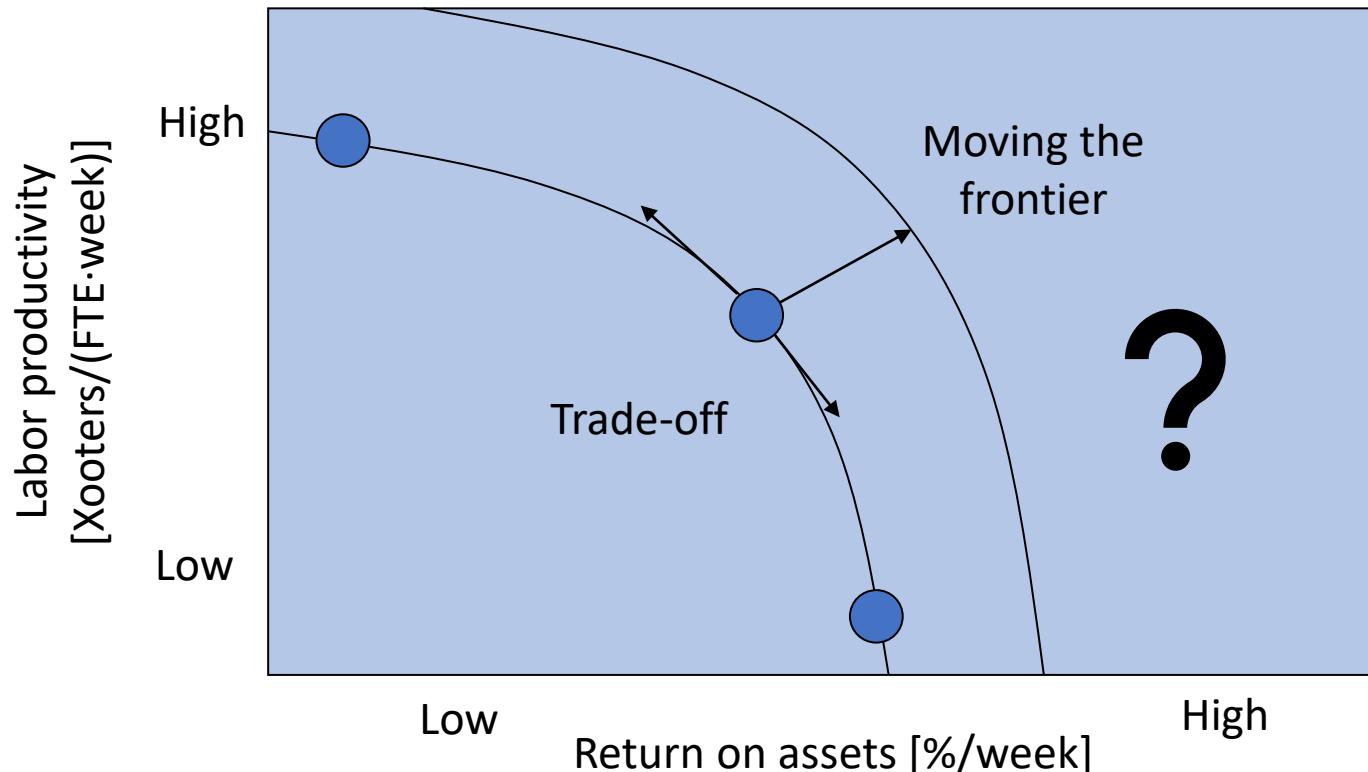
$$\begin{aligned} \textit{Profit} &= \textit{Flow rate} \\ &\cdot (\textit{Price} - \textit{Variable costs}) \\ &- \textit{Fixed costs} \end{aligned}$$



Profit Implications of Different Improvement Scenarios



Assessing the Impact of Line Balancing





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2023 Operations Management

PROCESS ANALYSIS WITH MULTIPLE FLOWS

**MULTIPLE FLOW UNITS,
SAME PROCESSING TIMES**



Capital One

- Capital One mission: help our customers succeed by bringing ingenuity, simplicity, and humanity to banking.
- Founded on the belief that the banking industry would be revolutionized by information and technology, beginning with credit cards.
- Today fifth-largest consumer bank and eighth-largest bank overall in the US.



Capital One Small Loans Department Challenges

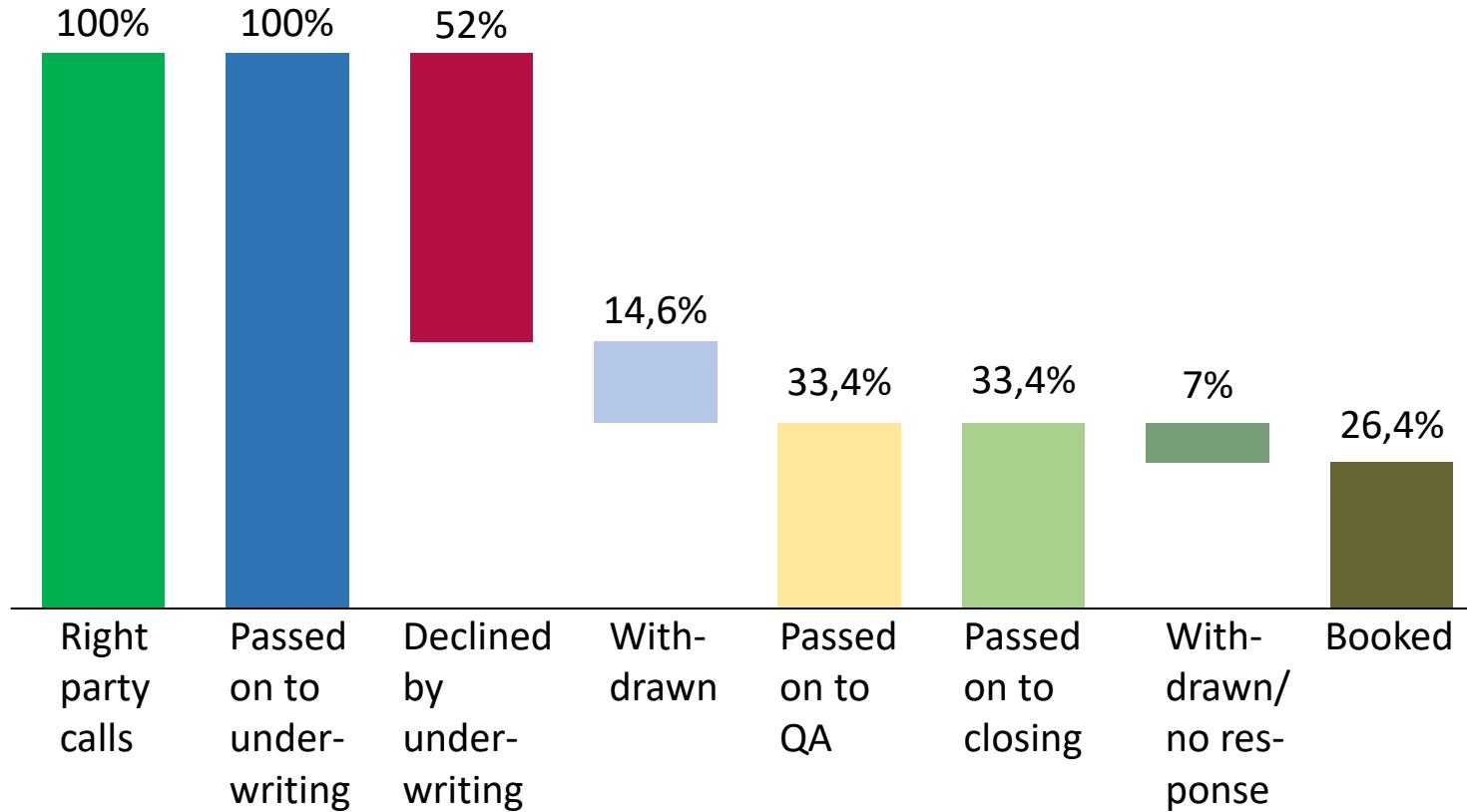
- How to determine demand for resources in a process with multiple flow units?
- How to find the bottleneck in a general process flow (with multiple flow units)?

Capital One Loan Approval Process

The marketing department solicits potential customers through direct mail and/or e-mail campaigns. Customers can answer by returning information concerning their name, the type of loan they are interested in, etc. This starts the loan approval process. It consists of the following five steps:

- An employee interviews the customer about his or her financial needs and collects all required information;
- Underwriting is the step where the decision of awarding the loan is made;
- Quality assurance is reviewing all documents prepared by the underwriter;
- Closing is in charge to print all relevant documents, prepare them for submission to the customer, and to call the customer to inform him or her that the loan is approved;
- The manager signs the final paperwork and releases the payment.

Not Every Customer Receives a Loan

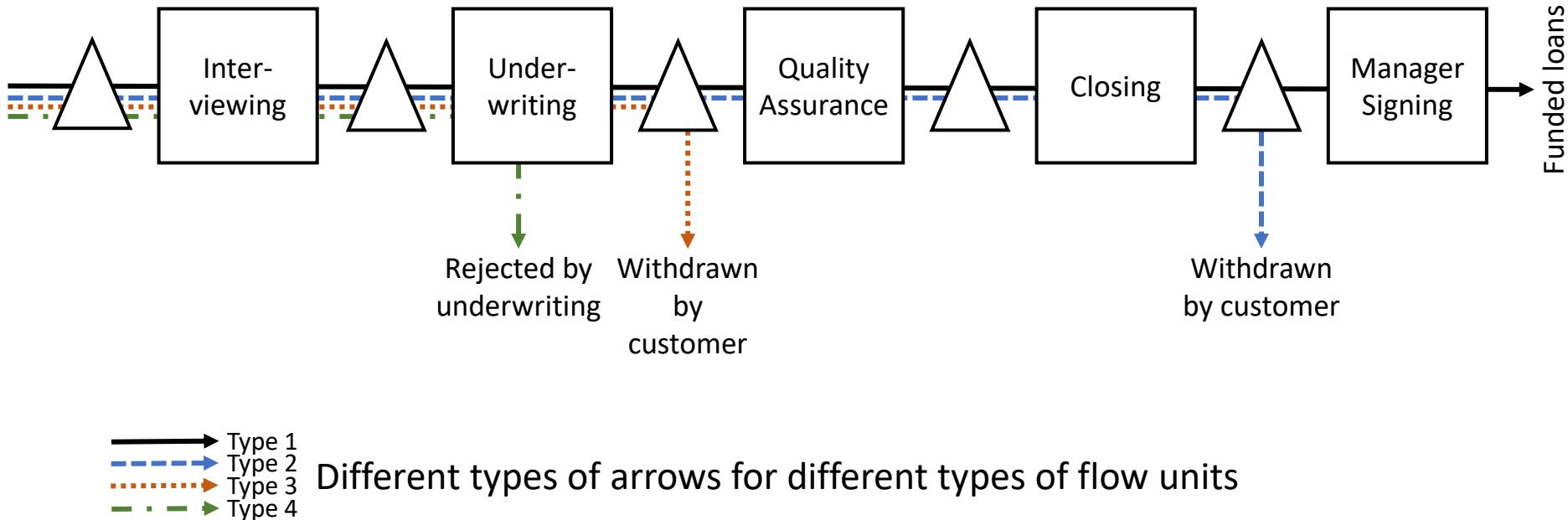


Demand Rates of Different Types of Flow Units

- On average, the Capital One loans department receives 110 loan applications per day by customers in total.
- This results in the following flow unit demand rates:

	Demand rate
Type 1 (funded loans)	29.04 apps/day
Type 2 (withdrawn after closing)	7.70 apps/day
Type 3 (withdrawn after underwriting)	16.06 apps/day
Type 4 (rejected by underwriting)	57.20 apps/day
Total	110.00 apps/day

Process Flow Diagram Showing Multiple Flow Units



Different types of arrows for different types of flow units

Demand Table

- A table depicting the resources (rows) and demand flows (columns) through a process.
- Integrates information from the process flow diagram with flow unit demand rates.

Resource	Flow unit				Total
	Type 1 [apps/day]	Type 2 [apps/day]	Type 3 [apps/day]	Type 4 [apps/day]	Demand [apps/day]
Interviewer	29.04	7.70	16.06	57.20	110.00
Underwriter	29.04	7.70	16.06	57.20	110.00
Quality assurance specialist	29.04	7.70	0	0	36.74
Closing employee	29.04	7.70	0	0	36.74
Manager	29.04	0	0	0	29.04



Capital One Small Loans Department Challenges

- How to determine demand for resources in a process with multiple flow units?
- How to find the bottleneck in a general process flow (with multiple flow units)?

Capital One Loan Approval Process

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- The manager signs the final paperwork and releases the payment.

Calculation of Capacity at Capital One

$$\text{Capacity} = \frac{\#FTE}{\text{Processing time}}$$

Capacity(Interviewers) =

Supply Data

No	Activity	Time per activity [FTE·min/app]	Number of Resources [FTE]
1	Interviewing	23	7
2	Underwriting	40	8
3	Quality assurance	25	2
4	Closing	70	6
5	Signing & Releasing	5	1

Capacity Levels at Capital One

Resource	Capacity [apps/min]	Capacity [apps/day]
Interviewers	0.3043	121.74
Underwriters	0.2000	80.00
Quality assurance specialists	0.0800	32.00
Closing employees	0.0857	34.29
Manager	0.2000	80.00

- Net work time at Capital One: 400 min/day
- Where is the bottleneck?

A General Definition of a Bottleneck

- So far:
The bottleneck is the resource with the lowest capacity.

- General:
The bottleneck is the resource with the highest implied utilization.

Formula used so far:

$$\frac{\text{Implied utilization}}{\text{Capacity}} = \frac{\text{Demand rate}}{\text{Capacity}}$$

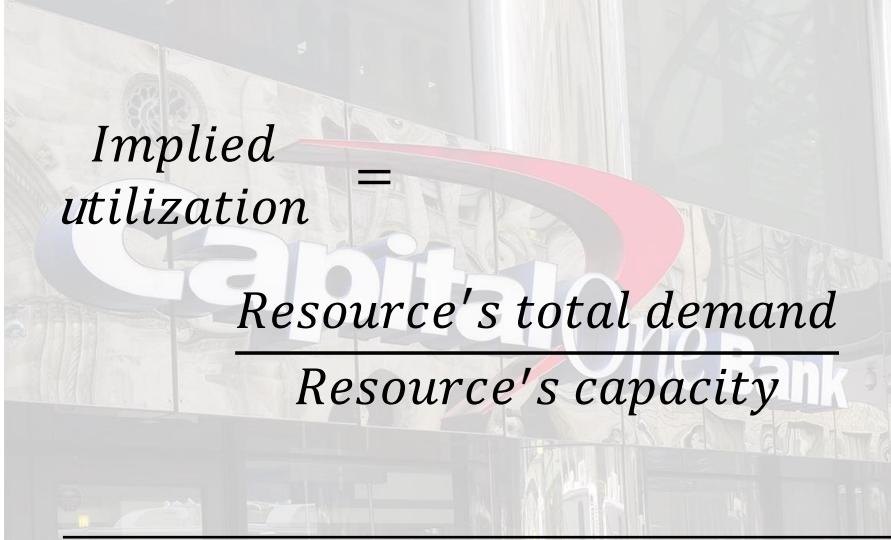
More generally:

$$= \frac{\text{Resource's total demand}}{\text{Resource's capacity}}$$

Calculation of Implied Utilization

Implied utilization =
(Interviewers)

$$\frac{110 \left[\frac{\text{apps}}{\text{day}} \right]}{121.74 \left[\frac{\text{apps}}{\text{day}} \right]} = 0.9036$$



Implied utilization =

$$\frac{\text{Resource's total demand}}{\text{Resource's capacity}}$$

Resource	Capacity [apps/day]	Total Demand [apps/day]	Implied Utilization
Interviewers	121.74	110.00	0.9036
Underwriters	80.00	110.00	1.3750
Quality assurance specialists	32.00	36.74	1.1481
Closing employees	34.29	36.74	1.0716
Manager	80.00	29.04	0.3630

Maximum and Actual Flow Rates

Max flow rate(Type 1) =

$$\text{Maximum flow rate} = \frac{\text{Demand rate for flow}}{\text{MAX(Implied utilization)}}$$

Assuming that all types of flow units are equally prioritized

Max flow rate(all types) =

$$\text{Actual flow rate} = \text{MIN} \left(\begin{array}{l} \text{Demand rate for flow,} \\ \text{Max flow rate} \end{array} \right)$$

How to find the bottleneck and the flow rates in a process with multiple flows?

- Step 1
Evaluate the demand table, with rows representing resources and columns representing different flows.
- Step 2
Evaluate the capacity for each resource. Be sure that the capacity is given in the same time unit as the demand rate.
- Step 3
Evaluate the implied utilization for each resource in the process:

$$\text{Implied utilization} = \frac{\text{Resource's total demand}}{\text{Resource's capacity}}$$

The resource with the highest implied utilization is the bottleneck.

- Step 4
If capacity is divided proportionally to demand for different type of flow units

$$\text{Maximum flow rate} = \frac{\text{Demand rate for flow}}{\text{MAX(Implied utilization)}}$$

Demand-constrained process: actual flow rates equal the demand rates.
Supply-constrained process: actual flow rates equal maximum flow rates.

TYPE OF FLOW UNIT DEPENDENT PROCESSING TIMES



Urgent Care Clinic

- Provides health care services to patients who feel sick but cannot get a timely appointment.
- Patients with severe conditions go to the emergency unit of the local hospital.
- Deals with mild cases, but can provide imaging diagnostic (x-ray) and some lab analysis.



Urgent Care Clinic Challenge

- How to find the bottleneck in a general process flow (with flow-dependent processing times)?

Five type of patients

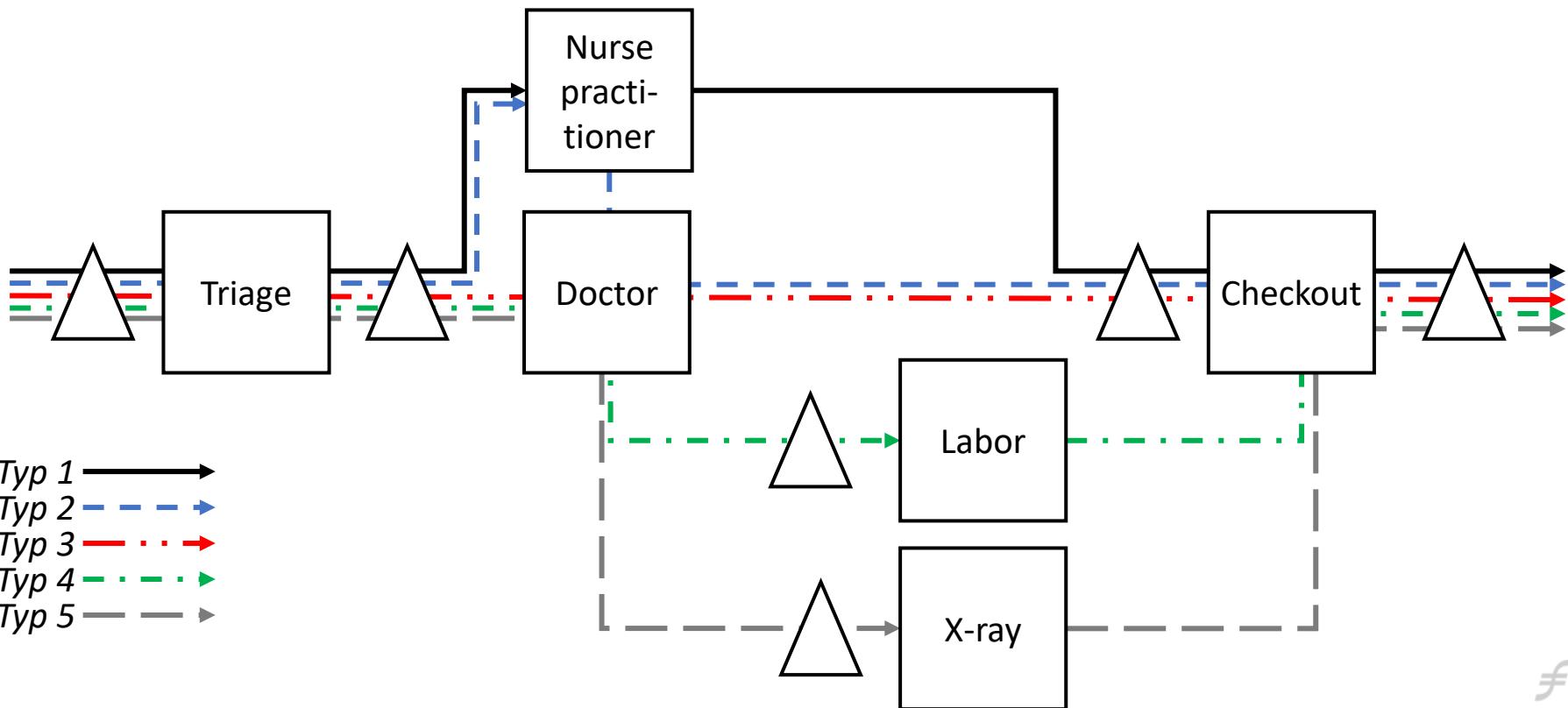
All patients go through an initial greeting and diagnosing stage (triage) and a final checkout step.

- Type 1 patients have only minor ailments and are seen by the nurse practitioner only;
- Type 2 patients are seen by the nurse practitioner, but the nurse practitioner also wants the doctor to look at the patient;
- Type 3 patients are seen by the doctor only;
- Type 4 patients are seen by the doctor, who the requests labs;
- Type 5 patients are seen by the doctor, who then requests X-rays.

Patient types	Demand for care [cases/day]	Demand for care [cases/h]
Type 1: Nurse only	20	2
Type 2: Nurse and doctor	5	0.5
Type 3: Doctor only	15	1.5
Type 4: Doctor with labs	15	1.5
Type 5: Doctor with X-rays	8	0.8

A day consists of 10 hours.

Process Flow Diagram



Flow-dependent Processing Times

Patient types	Processing times [FTE · min/case]					
	Triage nurse	Nurse			Check- out clerk	
		practiti oner	Doctor	Lab	X-ray	
Type 1: Nurse only	6	20	0	0	0	4
Type 2: Nurse and doctor	6	30	5	0	0	4
Type 3: Doctor only	6	0	15	0	0	4
Type 4: Doctor with labs	6	0	20	5	0	4
Type 5: Doctor with X-rays	6	0	20	0	20	4

Abstract Flow Unit “One Minute of Work”

- With patient type dependent processing times, using “one patient” or “one patient” as flow unit is no longer adequate.
- Consistent measurement of capacity “Patient/Hour” would not be possible.
- Changing the flow unit to the abstract concept of “one minute of work” is necessary.



Demand Rate

- Consider again the nurse practitioner.
- Type 1 patients require 20 minutes of work, and type 2 patients 30 minutes.
- Type 3 to 5 patients require no work at all.

*Demand for work
at resource i*

$$= \sum_{Type=1}^n p_{Type,i} \cdot d_{Type}$$

p: processing time, d: demand

For example, demand for the nurse practitioner resource in minutes of work per hour:

$$\begin{aligned}
 &= 2.0 \left[\frac{\text{cases}}{h} \right] \cdot 20 \left[\frac{\text{FTE} \cdot \text{min}}{\text{case}} \right] \\
 &\quad + 0.5 \left[\frac{\text{cases}}{h} \right] \cdot 30 \left[\frac{\text{FTE} \cdot \text{min}}{\text{case}} \right] \\
 &= 55 \left[\frac{\text{FTE} \cdot \text{min}}{h} \right]
 \end{aligned}$$



Demand Rate in Minutes of Work

Resource	Demand for work created by patient					Total
	Type 1	Type 2	Type 3	Type 4	Type 5	
Triage nurse	12	3	9	9	4.8	37.8
Nurse practitioner	40	15	0	0	0	55
Doctor	0	2.5	22.5	30	16	71
Lab	0	0	0	7.5	0	7.5
X-ray	0	0	0	0	16	16
Checkout clerk	8	2	6	6	3.2	25.2

in [FTE·min/h]

Capacity

Resource	#FTE	Capacity [FTE· min/h]
	[FTE]	
Triage nurse	1	60
Nurse practitioner	2	120
Doctor	1	60
Lab	1	60
X-ray	1	60
Checkout clerk	1	60

*Capacity at
resource i*

$$= \#FTE(i) \cdot 60 \left[\frac{\text{min}}{\text{h}} \right]$$

For example

*Nurse
practitioner capacity*

$$= 2[FTE] \cdot 60 \left[\frac{\text{min}}{\text{h}} \right]$$

$$= 120 \left[\frac{FTE \cdot \text{min}}{\text{h}} \right]$$

Calculation of Implied Utilization

Implied utilization =
(Nurse practitioner) =

$$\frac{55 \left[\frac{\text{FTE} \cdot \text{min}}{h} \right]}{120 \left[\frac{\text{FTE} \cdot \text{min}}{h} \right]} = 0.4583$$

Implied utilization =

Demand rate in minutes of work

Capacity in minutes of work

Resource	Capacity [FTE·min/h]	Demand [FTE·min/h]	Implied utilization [dmnl]
Triage nurse	60	37.8	0.6300
Nurse practitioner	120	55.0	0.4583
Doctor	60	71.0	1.1833
Lab	60	7.5	0.1250
X-ray	60	16.0	0.2667
Checkout clerk	60	25.2	0.4200

How to find the bottleneck and the flow rates in a process with flow-dependent processing times?

- Step 1
Compute the workload for each resource and type of demand.
- Step 2
Sum the workloads for each resource across all demand types.
- Step 3
Compute each resource's capacity (available working time).
- Step 4
For each resource, compute:

$$\text{Implied utilization} = \frac{\text{Demand rate in minutes of work}}{\text{Capacity in minutes of work}}$$

The resource with the highest implied utilization is the bottleneck.

- Step 5
If capacity is divided proportionally to demand for different type of flow units:

$$\text{Maximum flow rate} = \frac{\text{Demand rate for flow}}{\text{MAX(Implied utilization)}}$$

Demand-constrained process: actual flow rates = demand rates.

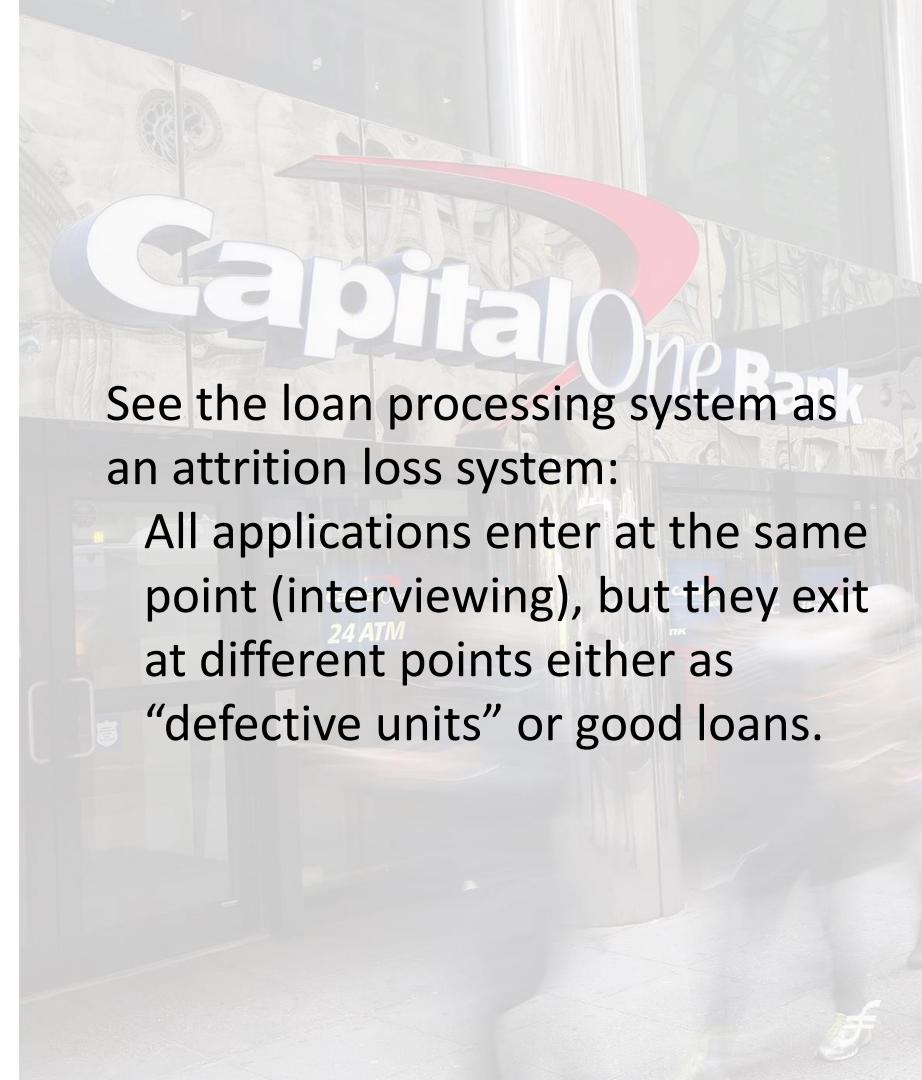
Supply-constrained process: actual flow rates = maximum flow rates.

ATTRITION LOSS SYSTEMS



Two Additional Capital One Challenges

- How many interviews to conduct to get a certain output Q of funded loans?
- What capacity of each resource is needed to produce a certain output Q of funded loans?



Calculation of the Yield of a Resource

Yield of resource =

$$\frac{\text{Flow rate of good output at the resource}}{\text{Flow rate of input}}$$

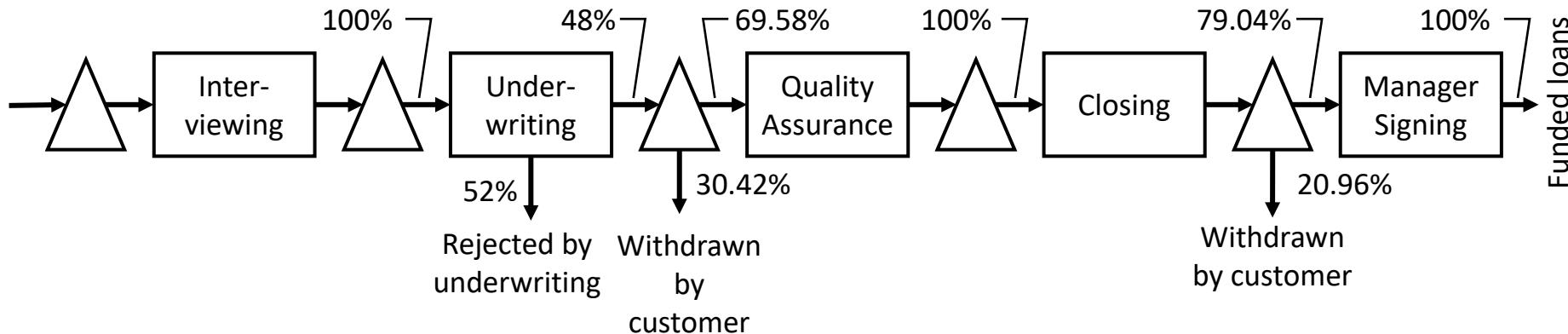
$$1 - \frac{\text{Flow rate of defects at the resource}}{\text{Flow rate of input}}$$

Process yield =

$$\frac{\text{Flow rate of good output of the process}}{\text{Flow rate of input to the process}}$$

$$1 - \frac{\text{Flow rate of defects in the process}}{\text{Flow rate of input to the process}}$$

Process Flow Diagram Showing Attrition Losses



$$\text{Process yield} = 1 \cdot 0.48 \cdot 0.6958 \cdot 1 \cdot 0.7904 \cdot 1 = 0.264 \left[\frac{\text{good apps}}{\text{app}} \right]$$

$$\text{Yield of underwriters} = \frac{33.4 \text{ [good apps]}}{100 \text{ [apps]}} = 0.334 \left[\frac{\text{good apps}}{\text{app}} \right]$$

Number of Interviews to Conduct

- Assume that Capital One sets the goal to have an output of 100 funded loans (good apps):

Number of interviews to conduct =

$$\frac{\text{Number of units started to get } Q \text{ good units}}{\text{Process yield}}$$

Analogous calculation to determine the capacity of each resource in the process so that the desired output is achieved.

REWORK



Cardiac Intensive Care Unit (CICU)

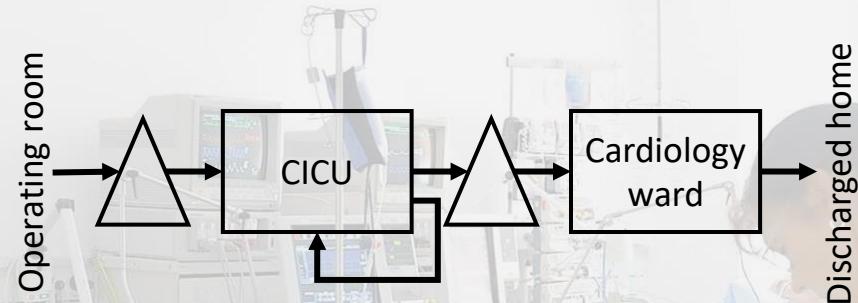
- Specialized hospital wards dedicated to caring for people with serious or acute heart problems.
- Staffed around the clock by nurses, technicians, and physicians specially trained.
- Specialized equipment for cardiac monitoring, testing and treatment.



CICU Challenges

- How to find the bottleneck and the capacity in a process with rework?

CICU Process Flow Diagram



CICU Data

- 12 beds in the CICU, 18 beds in the cardiology ward.
- Patients are admitted to the CICU at the rate of 1.2 patients per day.
- Patients without complications stay for 5 days.
- Patients developing a complication (20%) stay for 13 days.
- Patients discharged from CICU to the cardiology ward spend there four days.
- No rework of rework.

Defining the Average Patient as Flow Unit

Calculation of weighted average processing times:

$$WAPT_{CICU} =$$

Implied utilization =

$$\frac{\text{Demand rate}}{\text{Capacity}}$$

$$\text{Capacity} = \frac{m}{WAPT}$$

WAPT: weighted average processing time

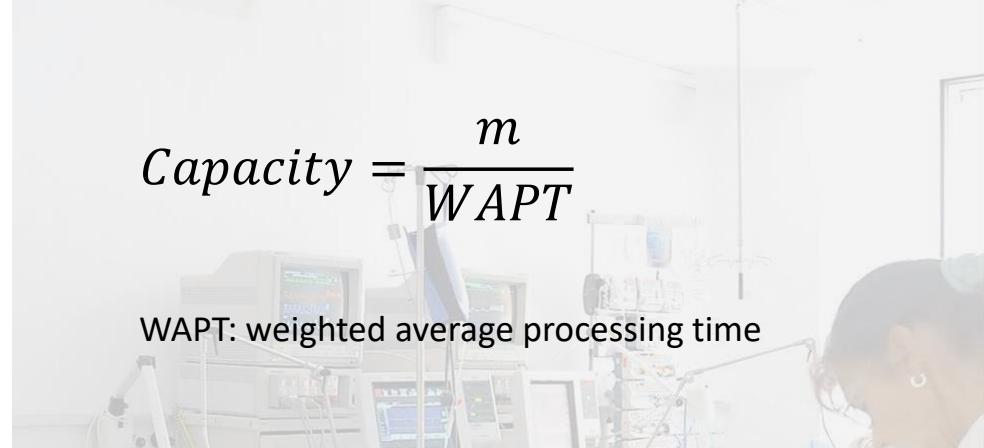
$$WAPT_{Ward} =$$

Capacities

Calculation of capacities:

$$\text{Capacity} = \frac{m}{WAPT}$$

WAPT: weighted average processing time



Implied utilization and bottleneck

Calculation of implied utilizations (iu):

Implied utilization =

$$\frac{\text{Demand rate}}{\text{Capacity}}$$

The bottleneck in the process
is CICU, not Ward.

How to find the bottleneck and the capacity in a process with rework?

- Step 1

At each resource with rework, evaluate the weighted average processing time.

- Step 2

Evaluate the capacity of each resource using the formula:

$$\text{Capacity} = \frac{m}{WAPT}$$

- Step 3

Evaluate for each resource the implied utilization:

$$\text{Implied utilization} = \frac{\text{Demand rate}}{\text{Capacity}}$$

- The resource with the highest implied utilization is the bottleneck.



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PROCESS INTERRUPTIONS



PRODUCTION CYCLES DURING PROCESS INTERRUPTIONS





Xootr Component Production

- Two components for a Xootr are made with a CNC milling machine:
 - Steer support
 - Metal rib
- Bill of material: each Xootr needs one steer support and two ribs (that is also called one component set).
- Xootr LLC has a single milling machine.

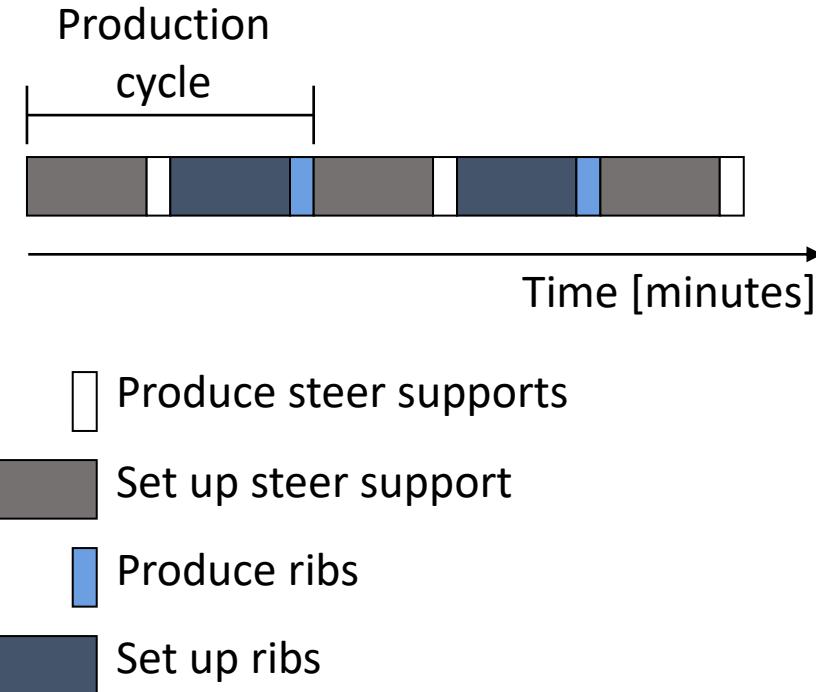


Milling Machine Setup and Production

- First step to produce a part: set up the CNC milling machine
- Set up consists of:
 - i. installing the correct tooling
 - ii. calibrating the equipment to the correct dimensions
- Set up time: 60 minutes
(not dependent on the part produced)
- Set up time is non-value adding
- Once the milling machine is set up, it can make the parts quickly
- Production rates
 - Steer supports: 1 unit/minute
 - Ribs: 2 units/minute



Production Cycle of the Milling Machine



- A production cycle is a repeating sequence of produced units.
- To evaluate capacity, we must define the flow unit:
 - one Xootr, or, equivalently, Or
 - one component set (consisting of 1 steer support and 2 ribs needed per Xootr)
- The batch size is the number of flow units produced in one cycle.
 - For example, if we produce 12 steer supports and 24 ribs in a production cycle, such that we can produce 12 Xootrs, the batch size is said to be 12 component sets.

SETUPS AND CAPACITY

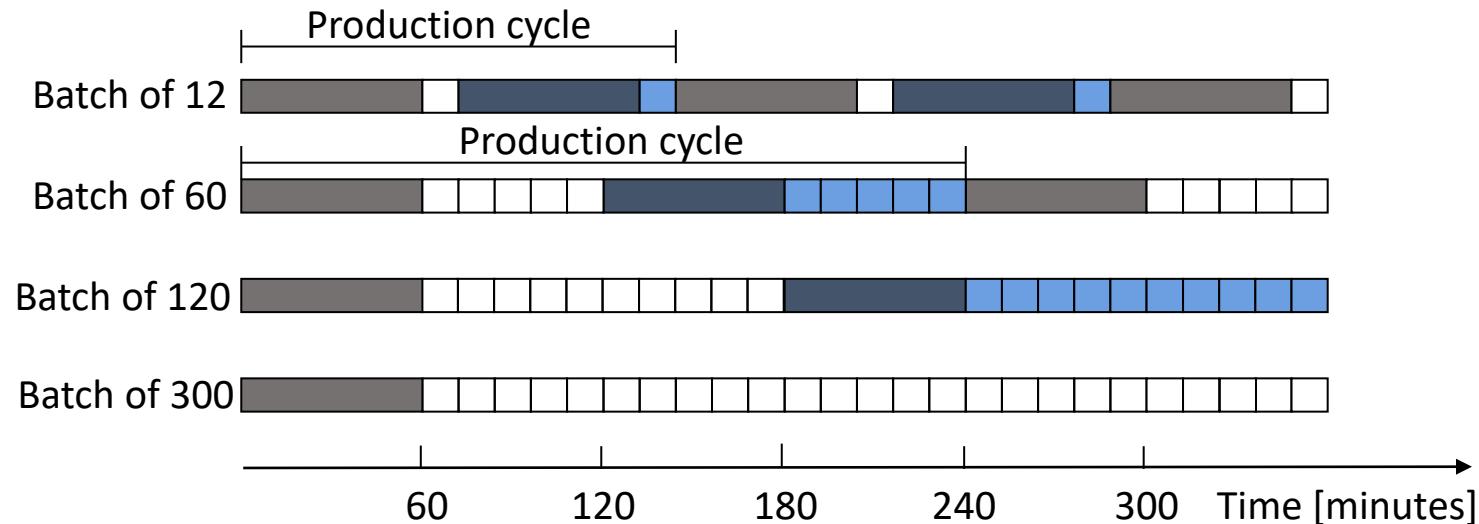


Xootr Component Production Challenges

- What is the capacity of the milling machine?
- What is the utilization of the milling machine?
- What is the average/maximum inventory of ribs and steer supports?
- What batch size minimizes inventory while not constraining the flow through the process?



The Impact of Batch Size on Capacity – What is Your Intuition?



Produce steer supports (1 box corresponds to 12 units = 12 scooters)

Set up steer support

Produce ribs (1 box corresponds to 24 units = 12 scooters)

Set up ribs

Capacity of a Resource with Setups

- So far, we defined capacity as (Topic 3):

$$\text{Capacity} = \frac{m}{\text{Processing time}} = \frac{\text{Units produced}}{\text{Time to produce the units}}$$

- For batch processes with setups, the time to produce a batch is given by:

$$\text{Time to produce a batch} = \text{Setup time} + \text{Batch size} \cdot \text{Processing time}$$

where the batch size is the number of flow units produced in one “cycle”.

- Thus, the capacity of a resource with setups is:

$$\text{Capacity} = \frac{\text{Batch size}}{\text{Total setup time} + (\text{Processing time} \cdot \text{Batch size})}$$

where total setup time is the sum of all setup times in one “cycle”.

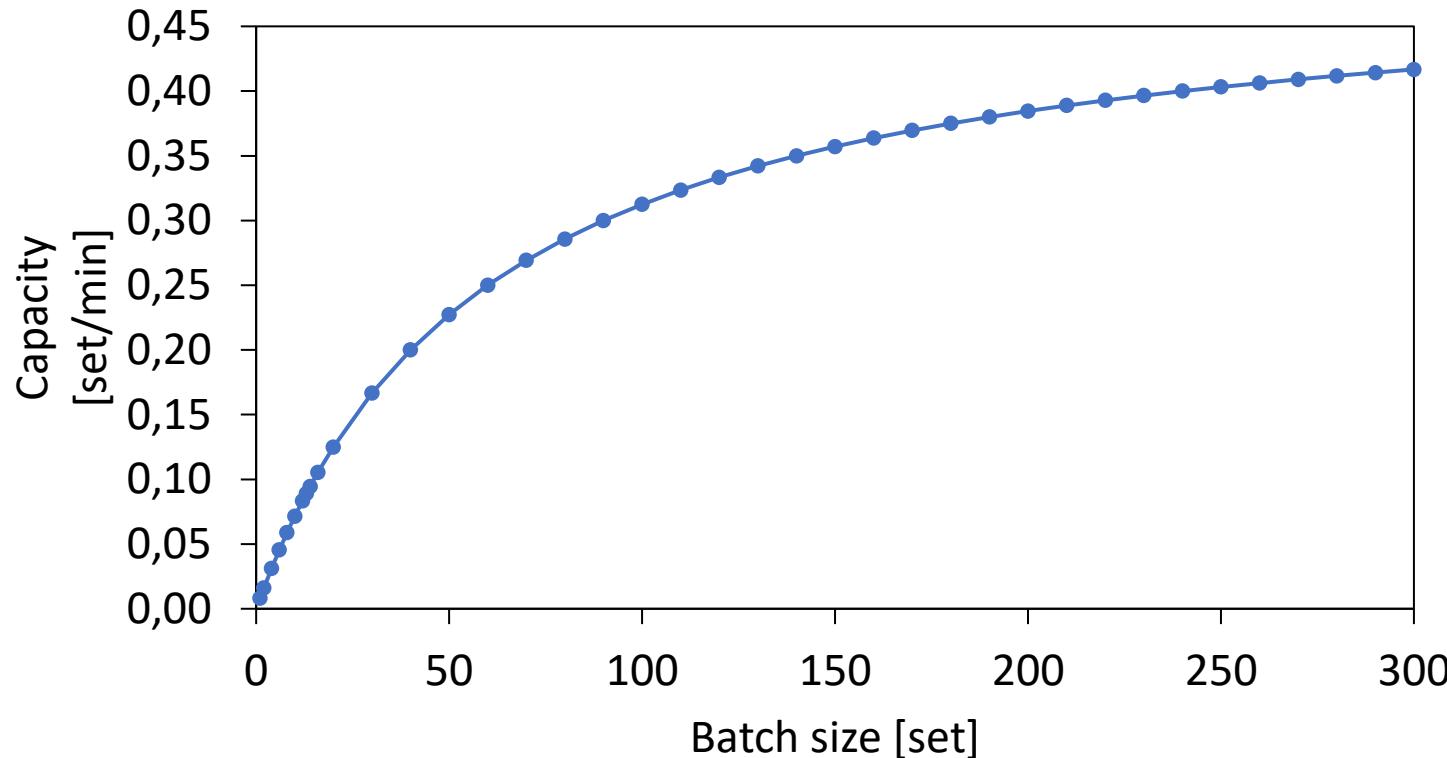
Capacity of the Milling Machine

- Using a batch size of 100 component sets, capacity is:

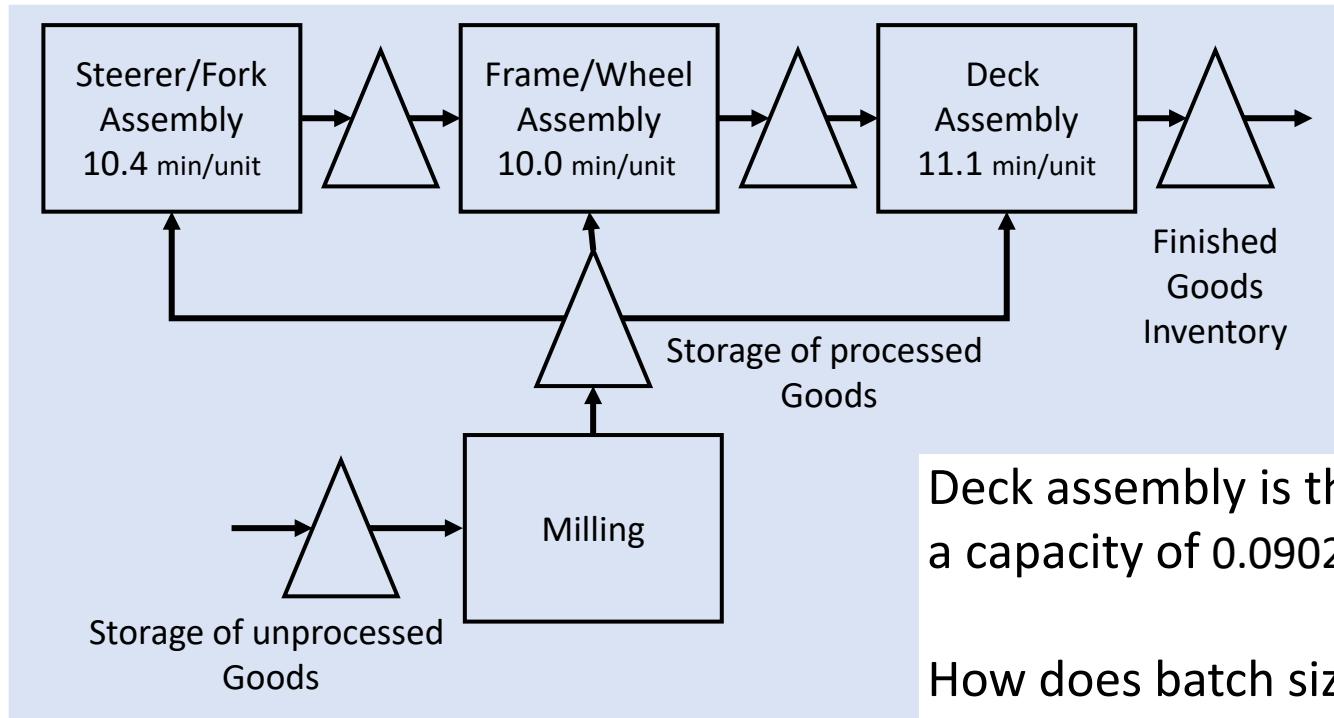
$$\text{Capacity} = \frac{\text{Batch size}}{\text{Total setup time} + (\text{Processing time} \cdot \text{Batch size})}$$

- Total setup time (sum of all setup times in the cycle):
- Processing time per component set:
- Capacity:

Relation of Milling Machine Capacity and Batch Size



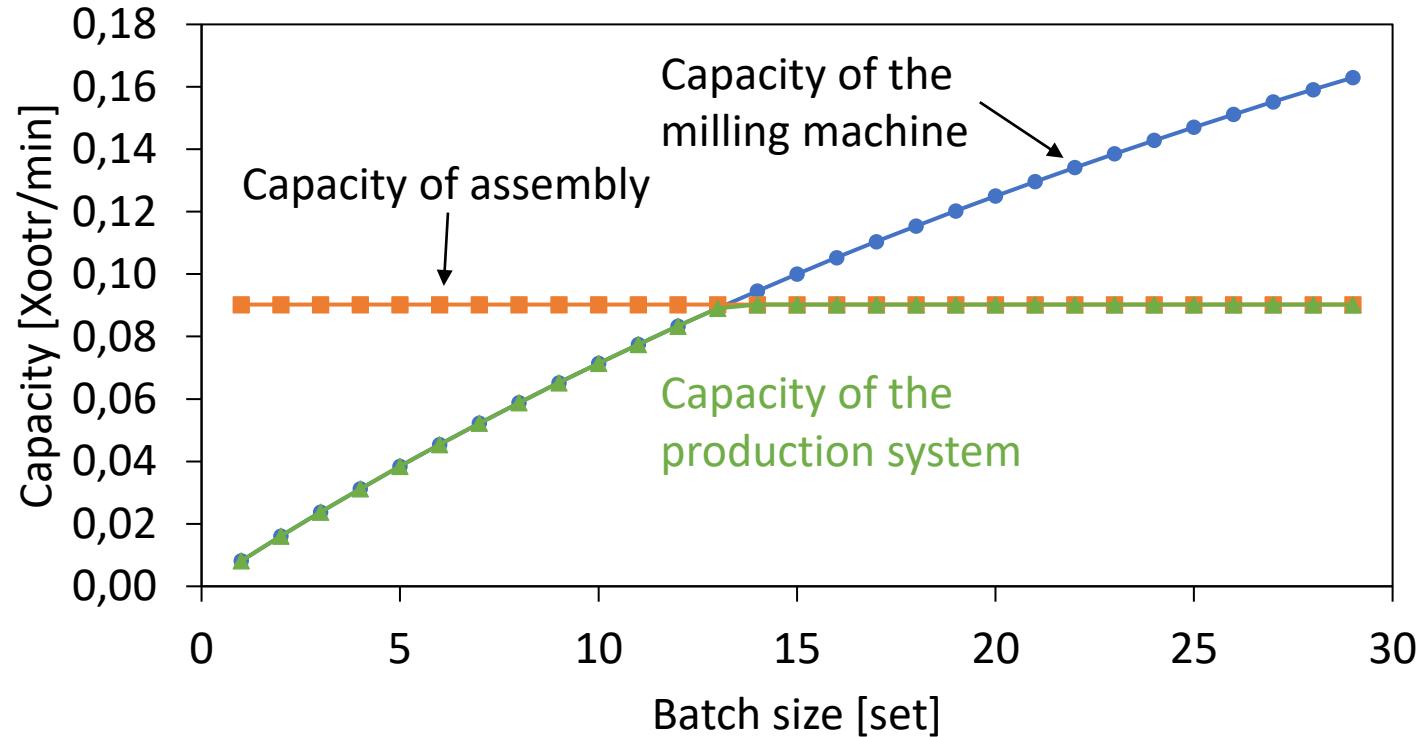
Xootr Milling and Assembly



Deck assembly is the bottleneck with a capacity of 0.0902 Xootr per min.

How does batch size variations at milling affect the capacity of the production system?

Capacity of the System



Xootr Component Production Challenges

- What is the capacity of the milling machine?
- **What is the utilization of the milling machine?**
- What is the average/maximum inventory of ribs and steer supports?
- What batch size minimizes inventory while not constraining the flow through the process?



Utilization of the Milling Machine

Assuming a capacity-constrained process, the flow rate at a batch size of 100 sets is

Flow rate =

*Output
rate when =
producing*

Utilization =

Formula from Topic 3

$$\text{Utilization} = \frac{\text{Flow rate}}{\text{Capacity}}$$

does not consider non-value adding set up times.

Therefore, generalized formula required:

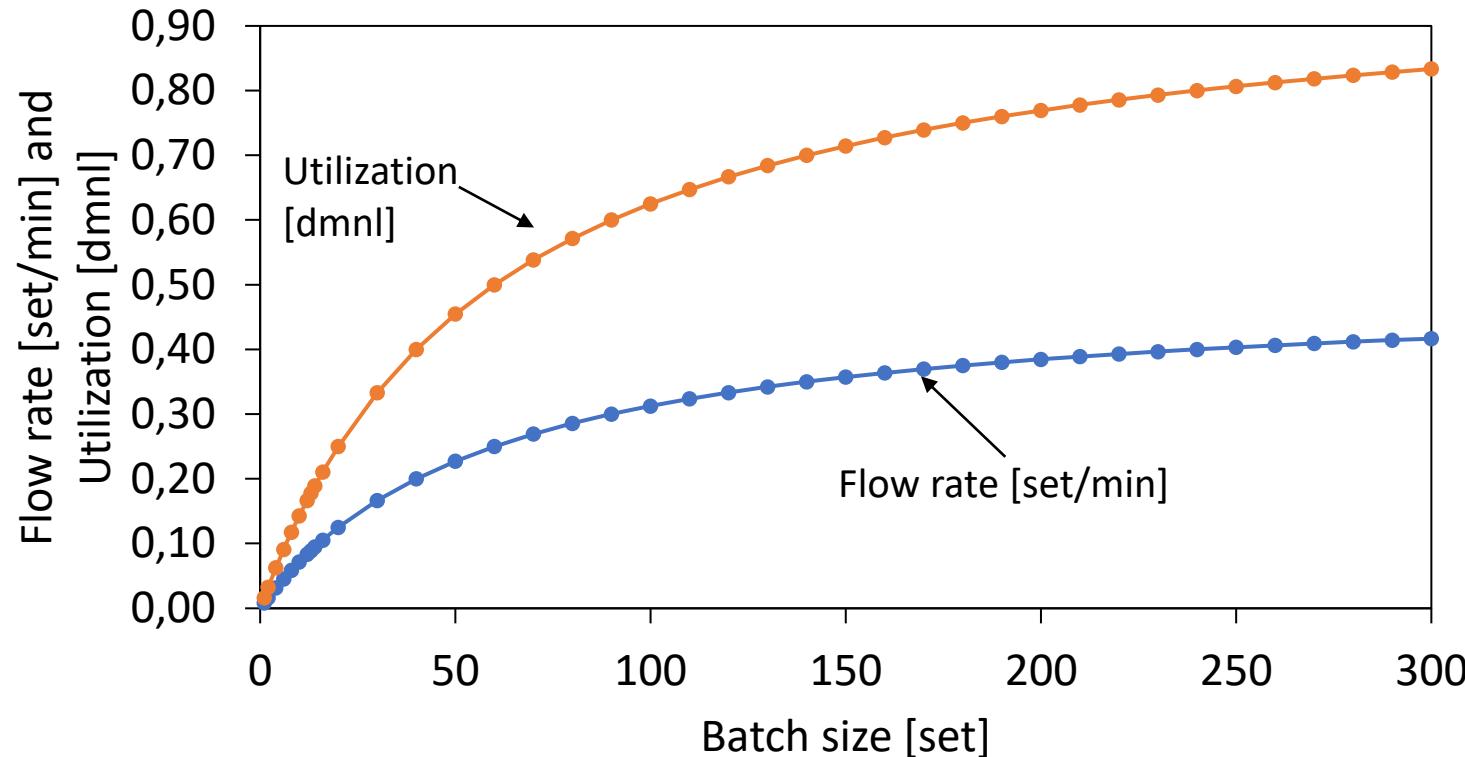
Utilization =

$$\frac{\text{Flow rate}}{\text{Output rate when producing}}$$

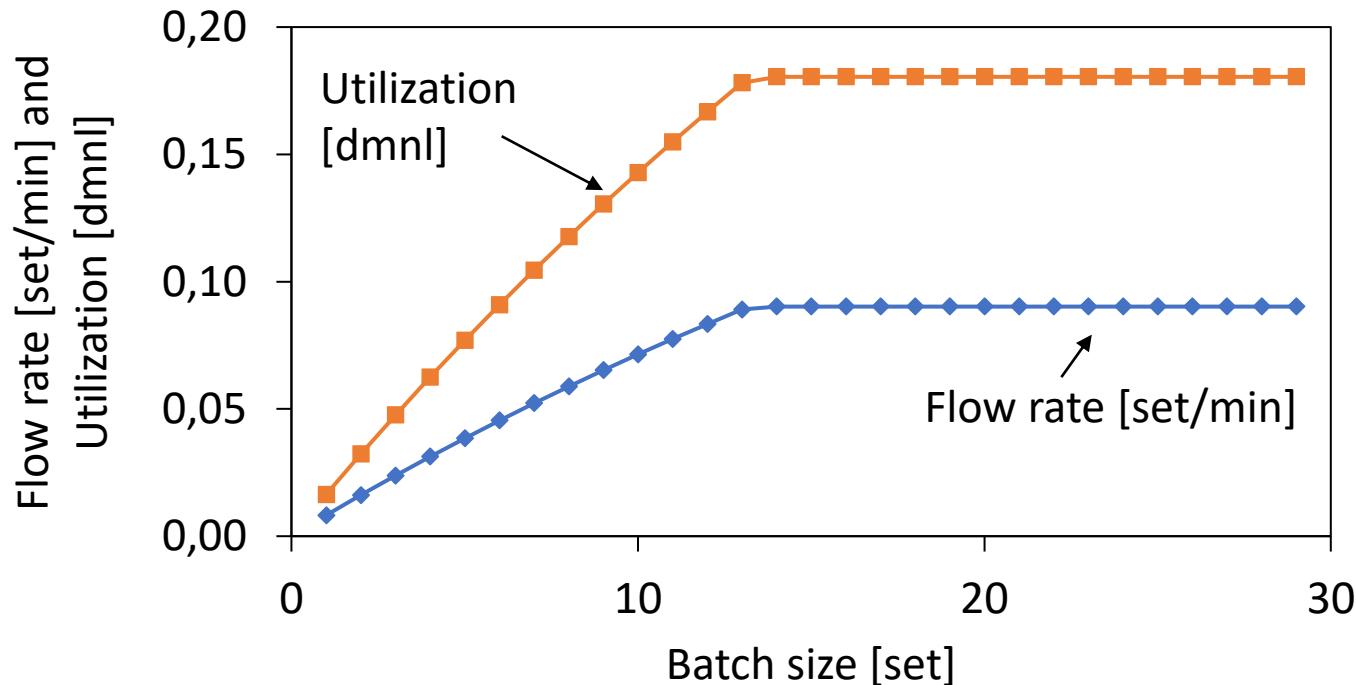
Output rate when producing =

$$\frac{m}{\text{Processing time}}$$

Flow Rate and Utilization as a Function of Batch Size



The Production System Perspective (Assembly Included)



SETUPS AND INVENTORY

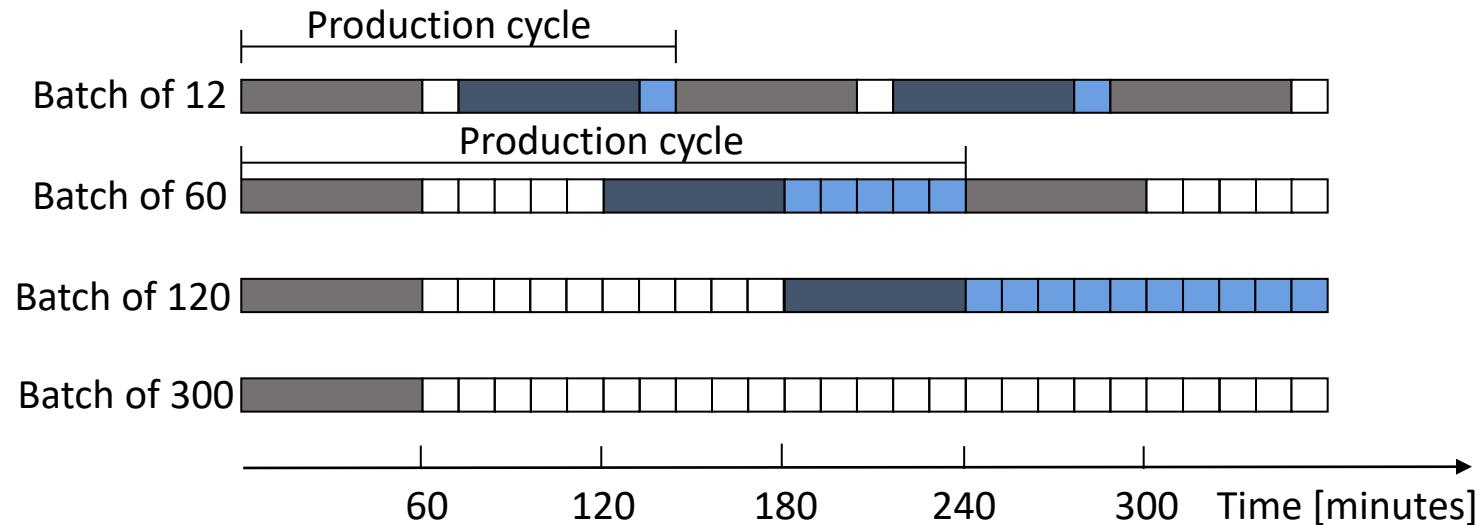


Xootr Component Production Challenges

- What is the capacity of the milling machine?
- What is the utilization of the milling machine?
- **What is the average/maximum inventory of ribs and steer supports?**
- What batch size minimizes inventory while not constraining the flow through the process?



The Impact of Batch Size on Inventory – What is Your Intuition?



Produce steer supports (1 box corresponds to 12 units = 12 scooters)

Set up steer support

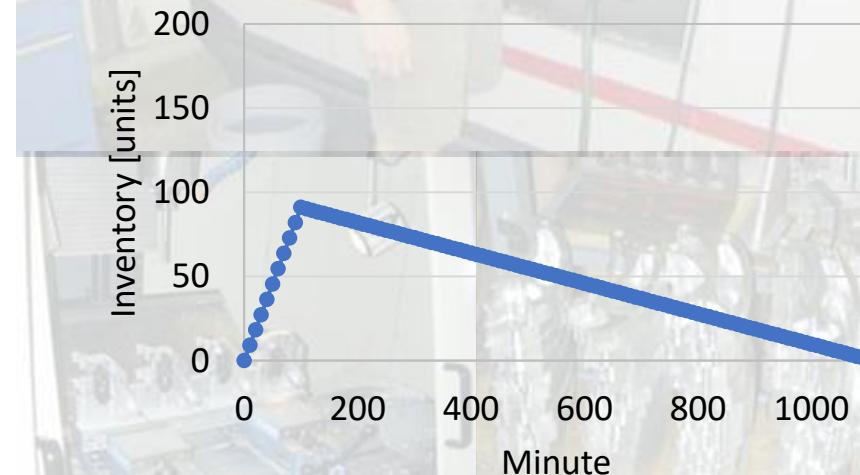
Produce ribs (1 box corresponds to 24 units = 12 scooters)

Set up ribs

Maximum Inventory Steer Support

- Steer support inventory net increase rate when producing:
- Time to produce one batch:

Set-up time: 60 minutes
Processing times
Steer support: 1 min/unit
Rib: 0.5 min/unit
Assembly capacity: 0.0902 Xootr per min
Batch size: 100 component sets



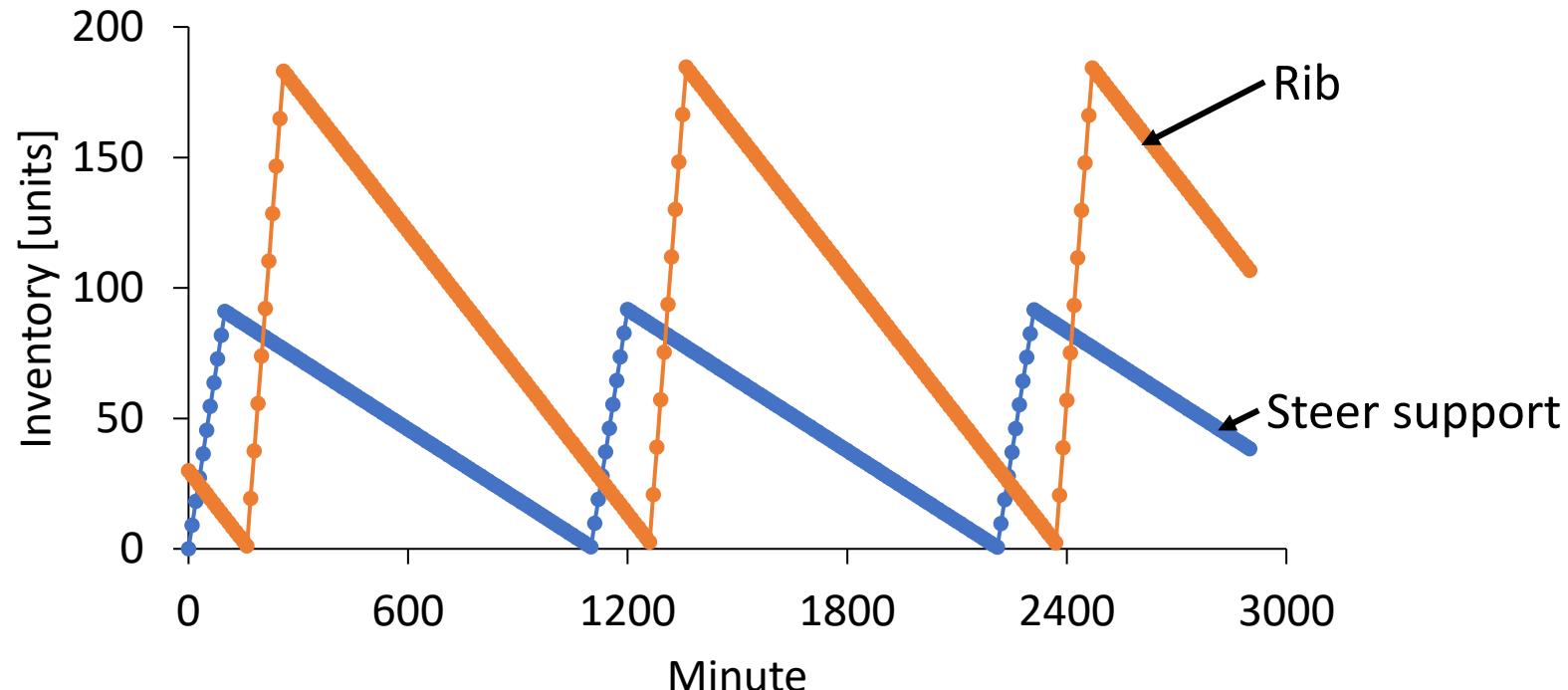
Maximum Inventory Steer Support

Maximum inventory

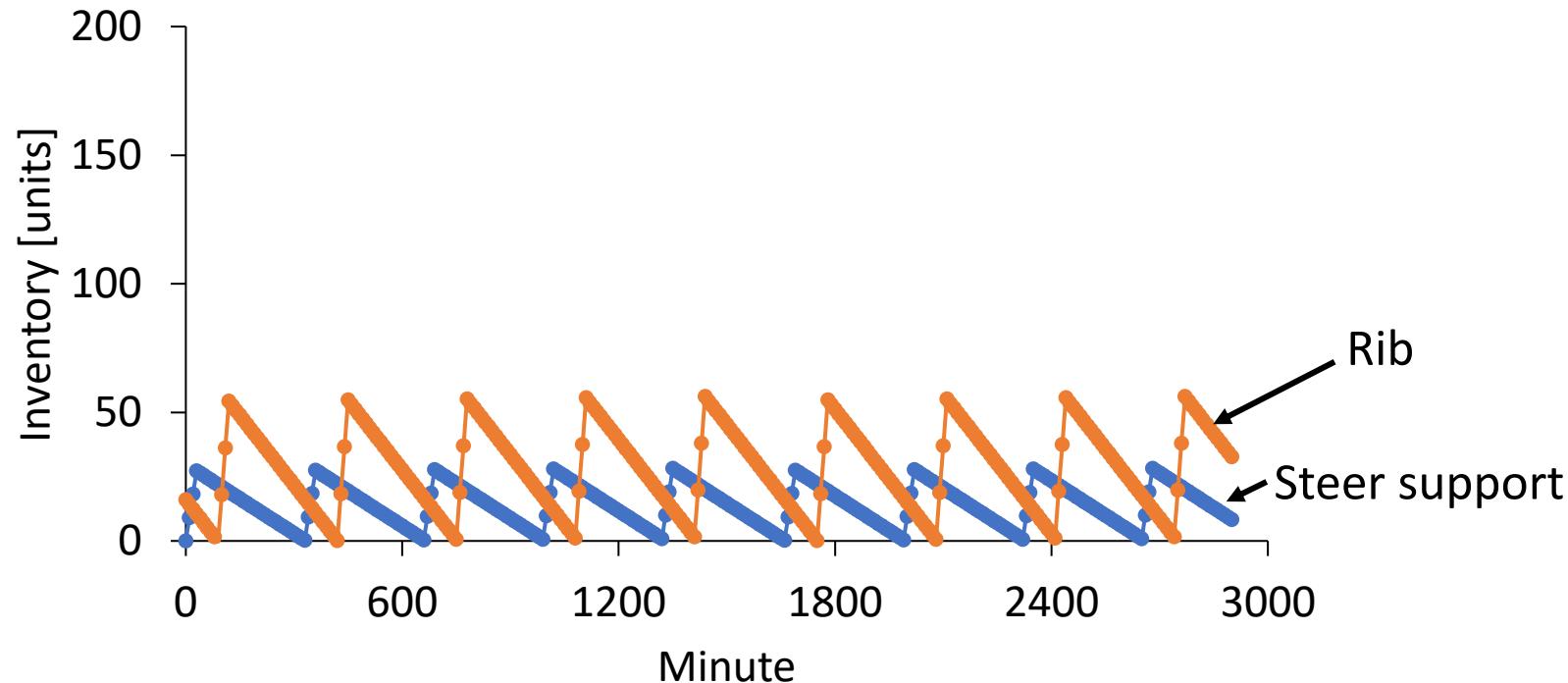
$$\begin{aligned} &= \text{Batch size} \cdot \text{Processing time} \left(\frac{1}{\text{Processing time}} - \text{Flow rate} \right) \\ &= \text{Batch size} \cdot [1 - (\text{Flow rate} \cdot \text{Processing time})] \end{aligned}$$

Maximum inventory steer support

Inventory Over Time (Batch Size = 100)



Inventory Over Time (Batch Size = 30)



Average Inventory

- Steer support (Batch size 100 or 30)

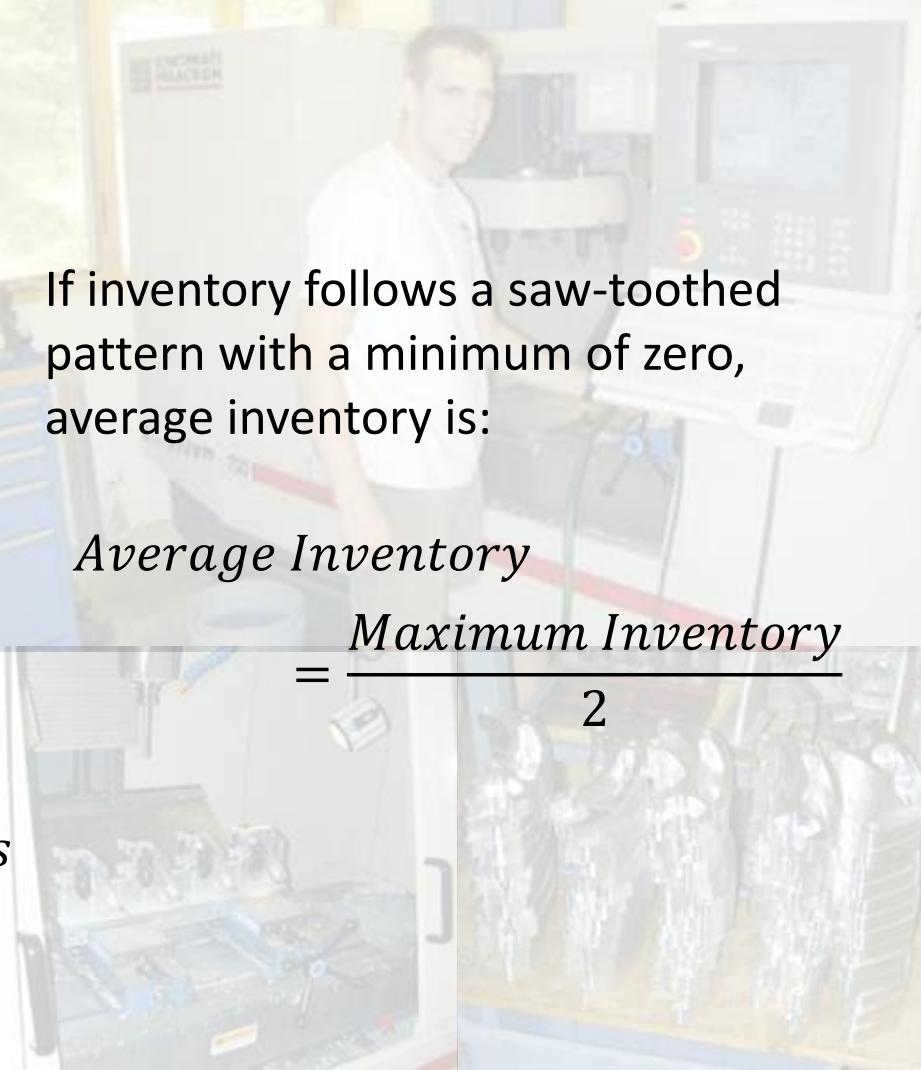
$$= \frac{90.9774 \text{ units}}{2} = 45.4887 \text{ units}$$

$$= \frac{27.2932 \text{ units}}{2} = 13.6466 \text{ units}$$

- Ribs (Batch size 100 or 30)

$$= \frac{181.9549 \text{ units}}{2} = 90.9774 \text{ units}$$

$$= \frac{54.5864 \text{ units}}{2} = 27.2932 \text{ units}$$

A photograph showing a man in a white shirt and tie standing in front of a computer monitor in what appears to be a factory or laboratory environment. In the foreground, there is a shelving unit with various electronic components and tools. A sign on the wall behind him reads "ENGINES REACH".

If inventory follows a saw-toothed pattern with a minimum of zero, average inventory is:

Average Inventory

$$= \frac{\text{Maximum Inventory}}{2}$$

RECOMMENDED BATCH SIZE



Xootr Component Production Challenges

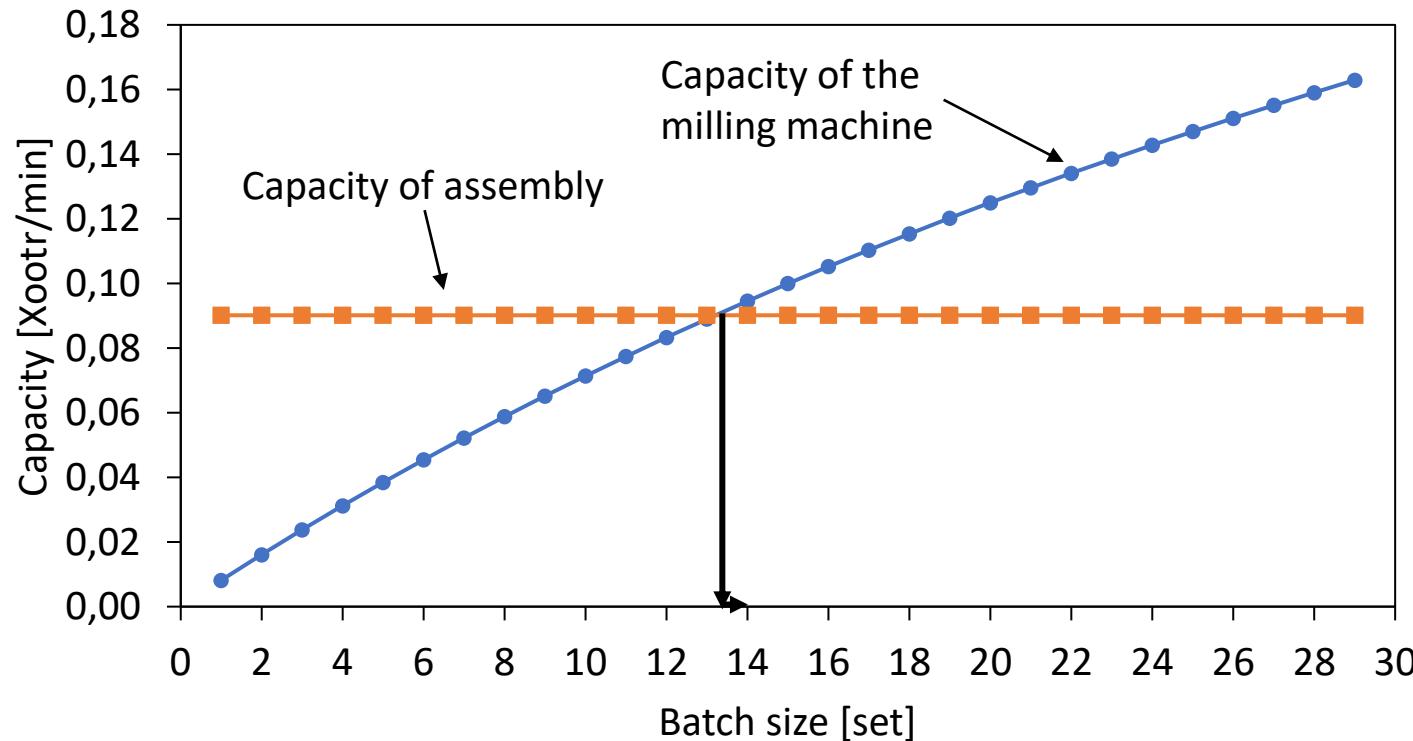
- What is the capacity of the milling machine?
- What is the utilization of the milling machine?
- What is the average/maximum inventory of ribs and steer supports?
- **What batch size minimizes inventory while not constraining the flow through the process?**



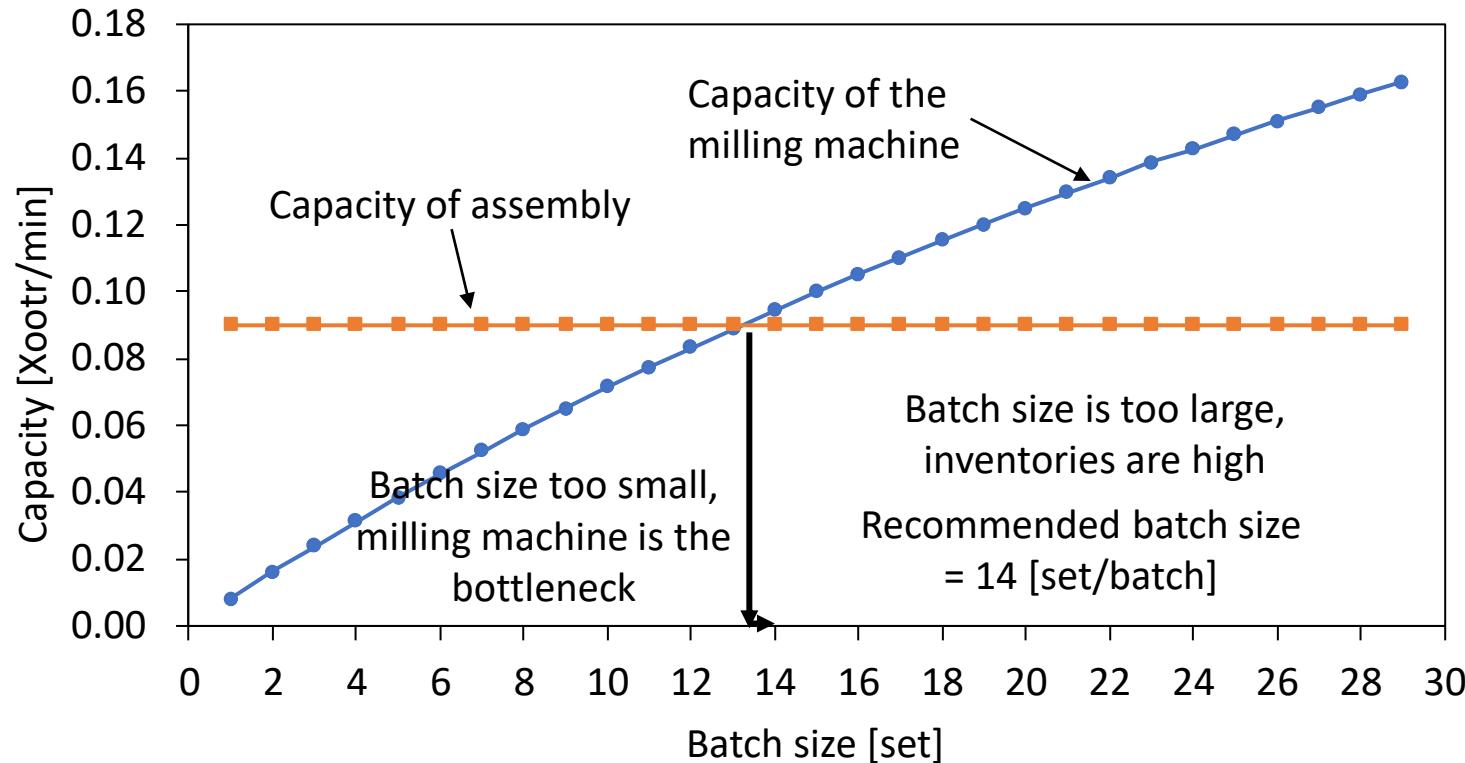
Choosing a Batch Size in a Process with Setups

- Conflicting objectives
 - Large batches \Rightarrow large inventory, but gains in capacity.
 - Small batches \Rightarrow small inventory, but losses in capacity.
- Important observation
 - Capacity at the bottleneck resource is extremely valuable (in a capacity-constrained process).
 - Capacity at a non-bottleneck resource is free (assuming that costs related to this resource are fixed).
- Implications
 - If setup occurs at the bottleneck \Rightarrow increase the batch size.
 - If setup occurs at **non**-bottleneck \Rightarrow decrease the batch size.

Recommended Batch Size for the Milling Machine



Recommended Batch Size for the Milling Machine



Choosing the Batch Size to Meet a Target Capacity

- Start with the capacity equation:

$$\text{Capacity} = \frac{\text{Batch size}}{\text{Total setup time} + (\text{Processing time} \cdot \text{Batch size})}$$

- Understand capacity as target capacity and set it equal to the flow rate .
- Rearrange the above equation to yield an equation for the recommended batch size:

$$\text{Recommended batch size} = \frac{\text{Target capacity} \cdot \text{Total setup time}}{1 - (\text{Target capacity} \cdot \text{Processing time})}$$

Recommended Batch Size

- Recommended batch size (in component sets):

Set-up time:

Steer support: 60 min

Rib: 60 min

Processing times:

Steer support: 1 min/unit

Rib: 0.5 min/unit

Bill of materials:

Steer support: 1 unit/xootr

Rib: 2 units/xootr

Assembly capacity: 0.0902 Xootr per min

Total setup time: 120 min

Total processing time: 2 min/set

PRODUCT VARIETY



Jürgen Langbein

- Founded 1965 in Hamburg
- Production of fonds, soups, sauces, and pastes
- 3000m² factory
- Typical size of kettle: 400 l



Unternehmen

| UNTERNEHMEN

Unternehmen

Eine der feinsten Adressen für Gourmets und Feinschmecker. Dieser Herstellungsbetrieb für nationale und internationale Feinkost liegt in Kaltenkirchen, Schleswig-Holstein. Die Produktion von maritimen Frischpasteten, die bis auf das Jahr 1880 zurückgeht, ist heute noch die Basis aller innovativen Weiterentwicklungen. Das Sortimentsportfolio besteht aus Gourmet-Fonds, -Suppen, -Saucen und -Pasten.

Die Erfolgsgeschichte begann 1965 mit der Gründung des Unternehmens durch Jürgen Langbein. In der Hindenburgstraße in Hamburg wurden die ersten Produkte wie Krebs- und Hummerpaste hergestellt. Die Idee, die dahinter steckte, war, dass auch außerhalb der Fangsaison der Krebse die Hamburger Feinschmecker-Restaurants ihre Gäste mit Spezialitäten verwöhnen könnten.

Jürgen Langbein Soup Production Challenge

- What impact has product variety in processes with set-ups on process performance?
- Which strategies are available?



Unternehmen

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Setup Time and Product Variety: Two Soups

- Consider a process that makes two kinds of soup.
- Define the flow unit to be one liter of soup.
- Assume we iterate between vegetable and tomato production.
- Define the batch to be the total quantity (in liter) of vegetable and tomato soup.
- What batch size minimizes inventory?



	Vegetable	Tomato
Demand [l/h]	100	75
Setup time [h]	0.4	0.4
Production rate [l/h]	300	300

Two Soup Analysis

- Our target capacity (or flow rate) is $100 + 75 = 175$ liter per hour.
- Each batch involves producing vegetable **and** tomato soup, so the total setup time is $2 \times 0.4 = 0.8$ hour per batch.
- Processing time = $1/300$ hour per liter.
- Recommended batch size:

- Production:

Three Soups and Expanding Demand

- Process output: three types of soup (vegetable, tomato, potato)
- Now suppose one kind of soup is added and demand for the others remains the same.
- A batch is still a set of vegetable, tomato and potato soup.
- What batch size minimizes inventory?



	Vegetable	Tomato
Demand [l/h]	100	75
Setup time [h]	0.4	0.4
Production rate [l/h]	300	300

	Potato
Demand [l/h]	50
Setup time [h]	0.4
Production rate [l/h]	300



Three Soups Expanding Demand Analysis

- Desired capacity (or flow rate) is $100 + 75 + 50 = 225 \text{ l/hour}$.
- Each batch involves producing vegetable, tomato and potato soup, so the total setup time is $0.4 \text{ h} + 0.4 \text{ h} + 0.4 \text{ h} = 1.2 \text{ hours per batch}$.
- Processing time = $1/300 \text{ hour per liter}$.
- Recommended batch size:
- Production:

Jürgen Langbein Soup Production Challenge

- What impact has product variety in processes with set-ups on process performance?
- **Which strategies are available?**



Unternehmen

| UNTERNEHMEN

Unternehmen

“Die bewährten und geschätzten Langbein-Spezialitäten sind ideale Begleiter für Genießer. Sie basieren auf herausragendem handwerklichen Können und überzeugen in Vielfalt und Qualität.”

Eine der feinsten Adressen für Gourmets und Feinschmecker. Dieser Herstellungsbetrieb für nationale und internationale Feinkost liegt in Kaltenkirchen, Schleswig-Holstein. Die Produktion von maritimen Frischpasten, die bis auf das Jahr **1880** zurückgeht, ist heute noch die Basis aller innovativen Weiterentwicklungen. Das Sortimentsportfolio besteht aus Gourmet-Fonds, -Suppen, -Saucen und -Pasten.

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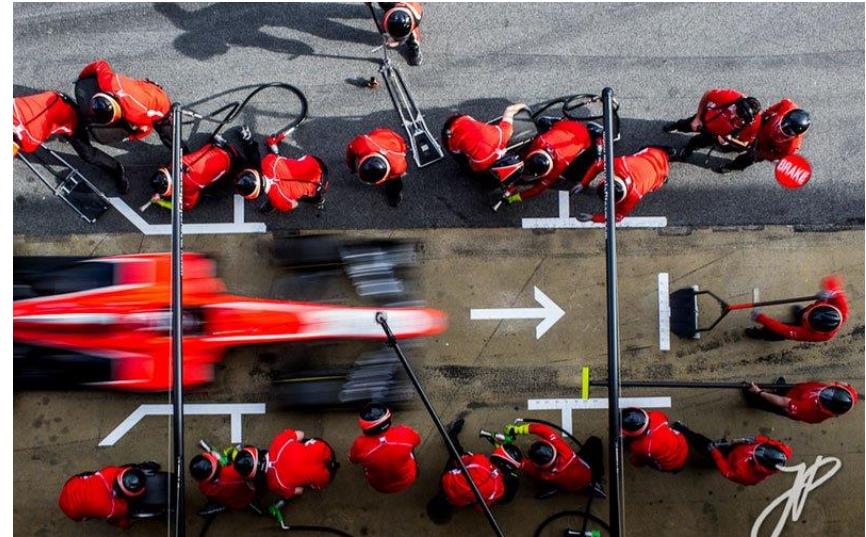
Henry Ford's Famous Proclamation



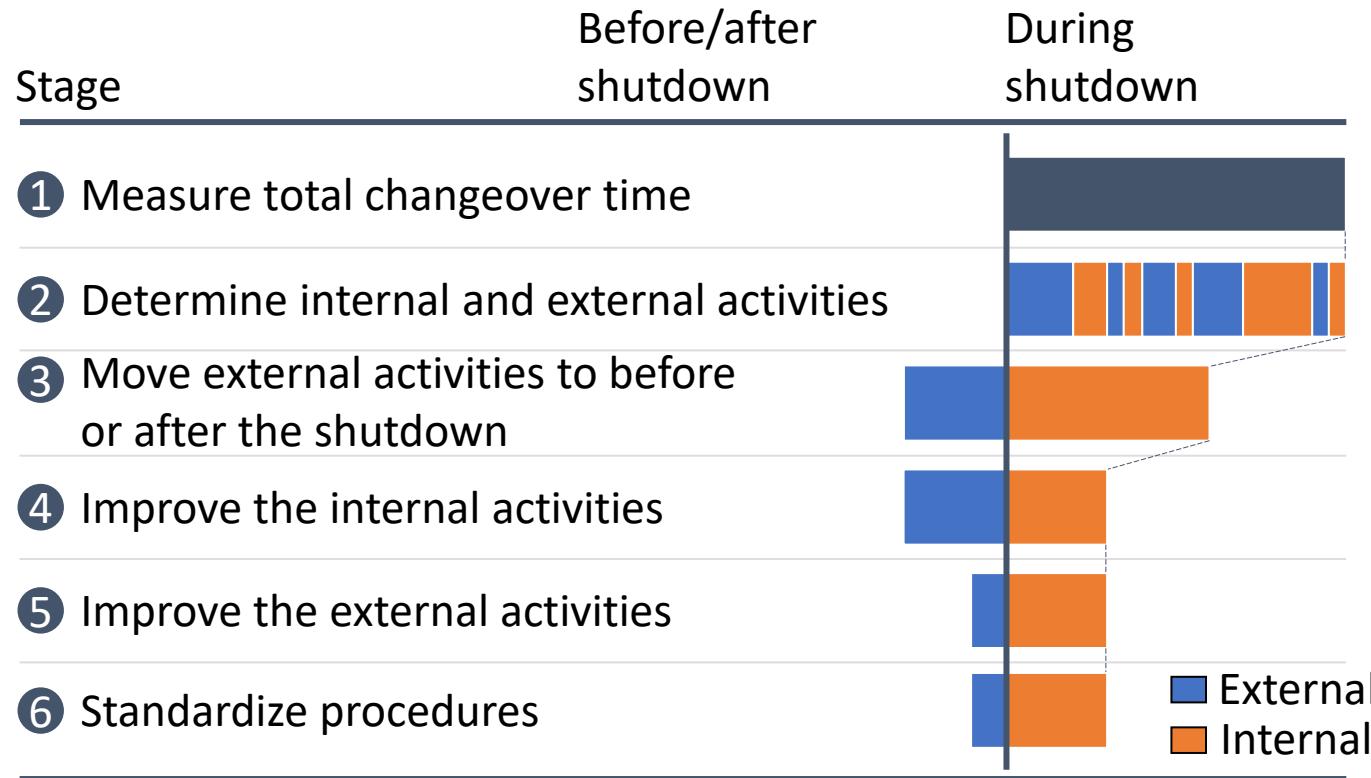
“Customers can have any color they want, as long as it is black.”

Reduction of Set-up Time with SMED

- Single minute exchange of die (SMED) method.
- Paradigm: Setup times should take less than 10 minutes.
- Idea: Divide setup-related tasks into internal and external setup tasks.
 - Internal tasks: can only be executed while the machine is stopped.
 - External tasks: can be done while the machine is still operating, i.e., before or after the actual changeover occurs.

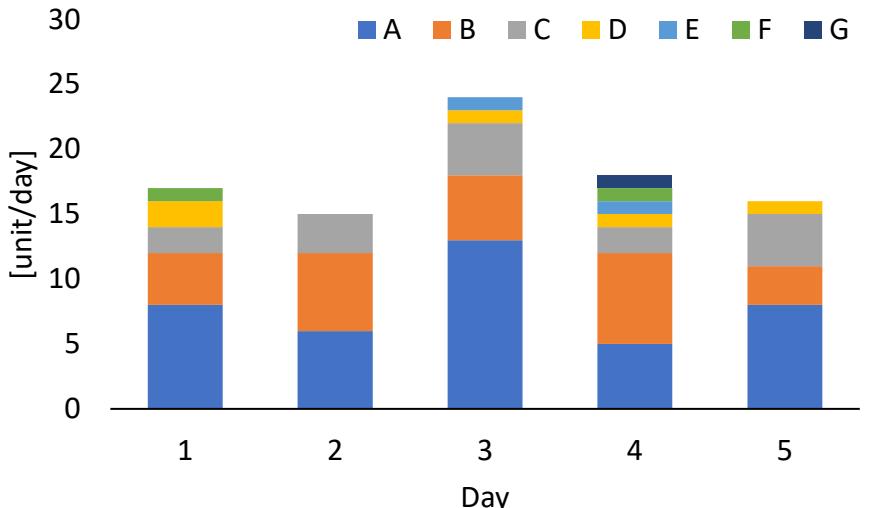


Setup Time Reduction with the 6-stage SMED approach

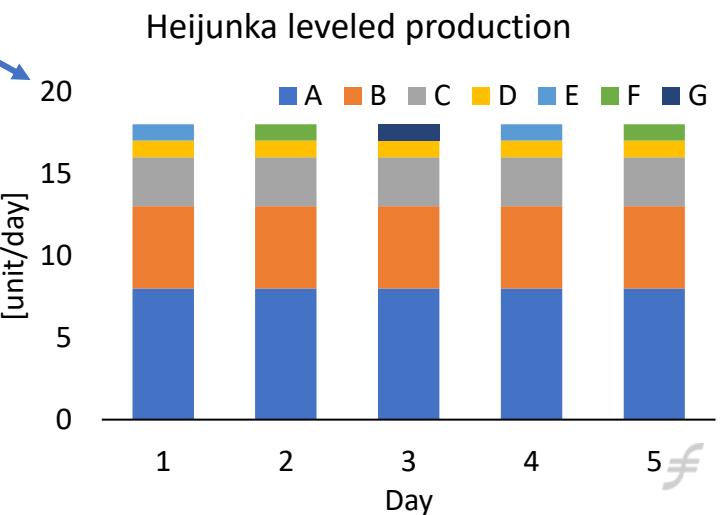
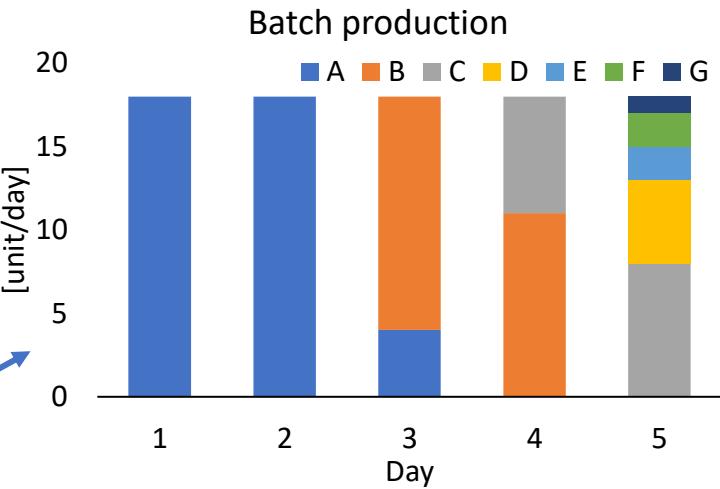


Smooth the flow - Heijunka

Example: Demand rate of 7 products A - G



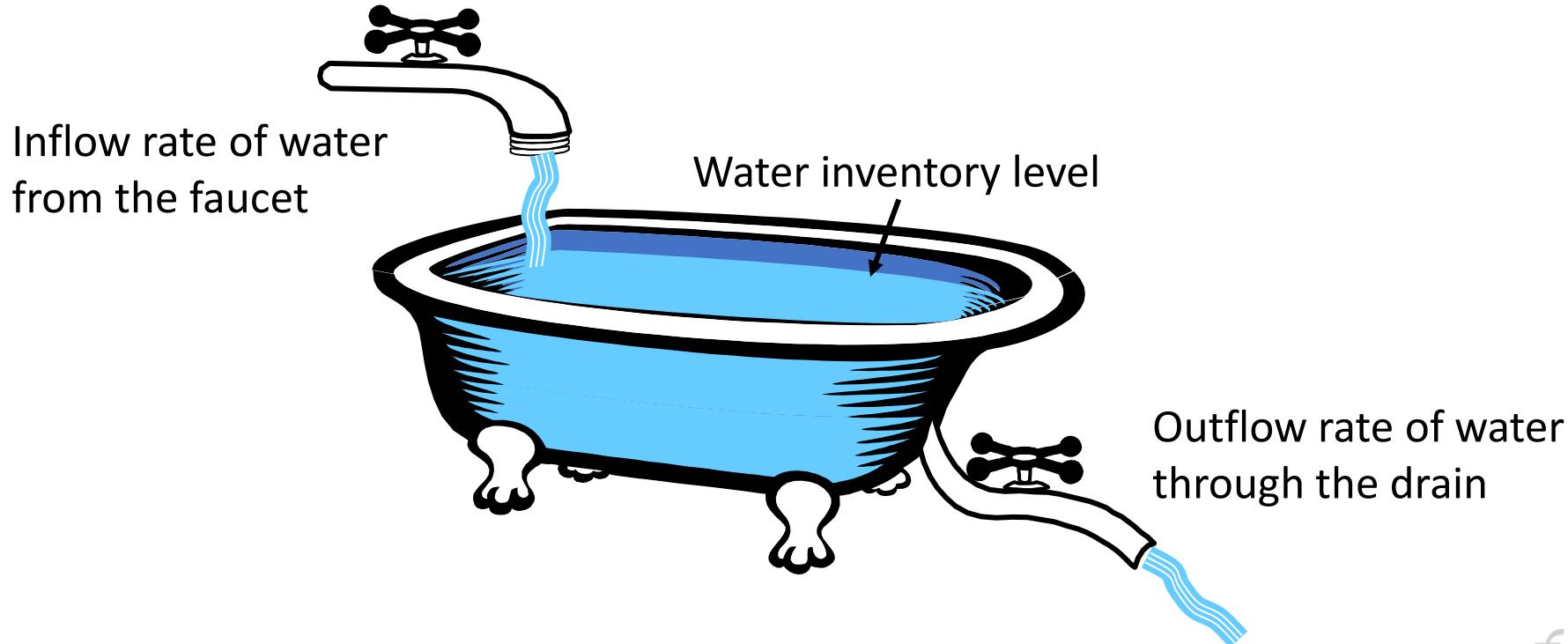
	Product						
Weekly demand [units/week]	A	B	C	D	E	F	G
	40	25	15	5	2	2	1



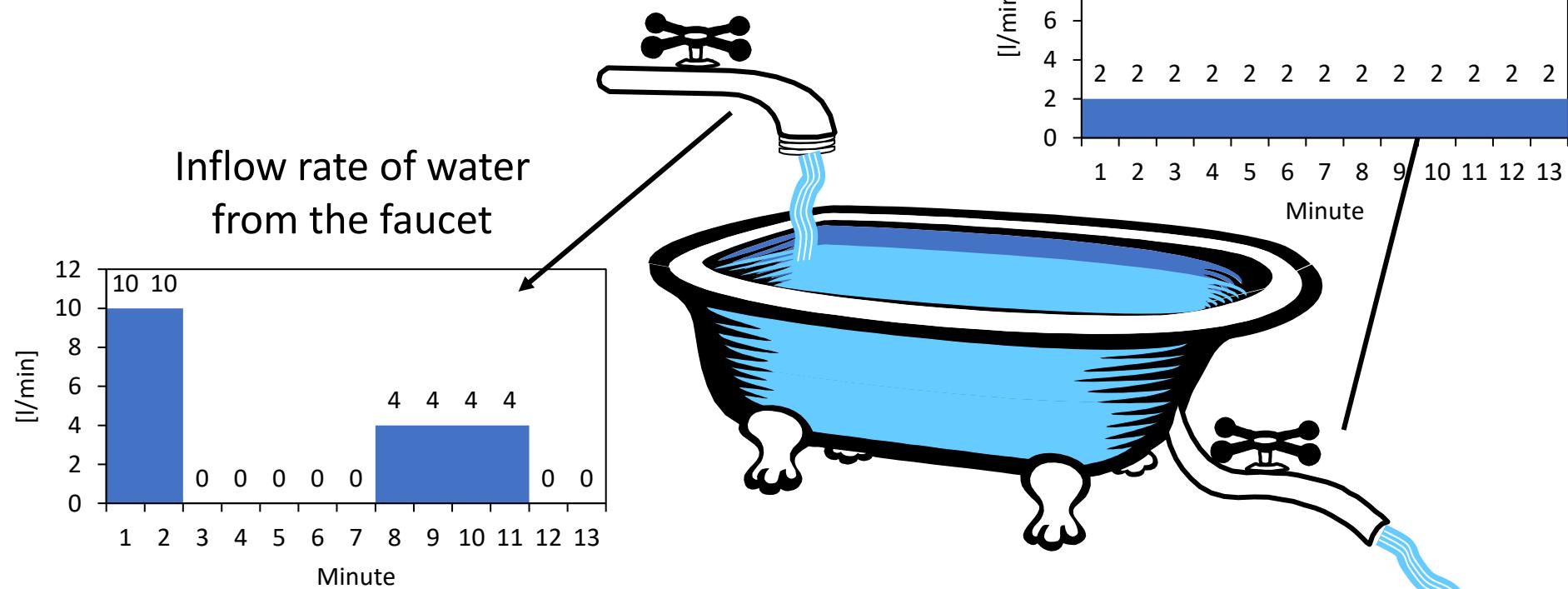
EXTRA



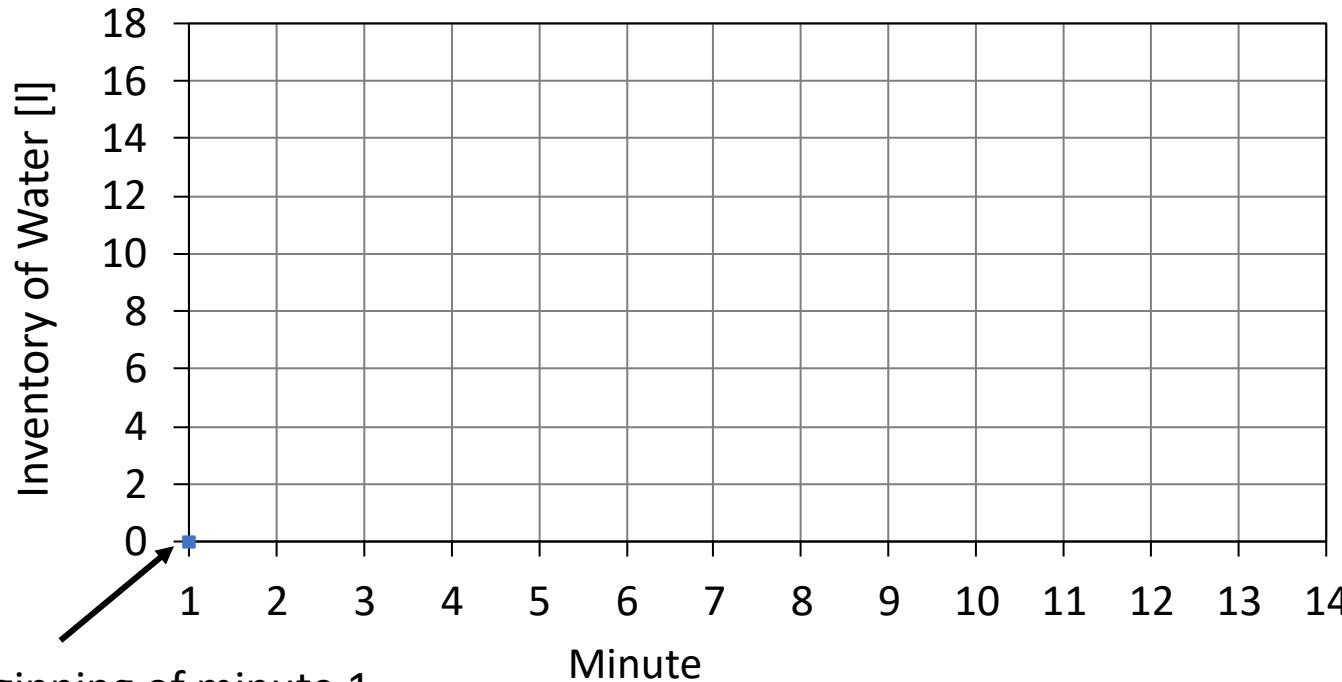
Inventory of Water in a Bathtub



What is the inventory of water?



Inventory of Water



At the beginning of minute 1
the bathtub is empty!

Three Soups and Dividing Demand

- Suppose one kind of soup is added, but total demand remains the same: 175 l/hour.
- A batch is still a set of vegetable, tomato and potato soup.
- What batch size minimizes inventory?



	Vegetable	Tomato
Demand [l/h]	80	65
Setup time [h]	0.4	0.4
Production rate [l/h]	300	300

	Potato
Demand [l/h]	30
Setup time [h]	0.4
Production rate [l/h]	300



Three Soups Dividing Demand Analysis

- Desired capacity (or flow rate) is $80 + 65 + 30 = 175 \text{ l/hour}$.
- Each batch involves producing vegetable, tomato and potato soup, so the total setup time is $0.4 \text{ h} + 0.4 \text{ h} + 0.4 \text{ h} = 1.2 \text{ hours per batch}$.
- Processing time = $1/300 \text{ hour per liter}$.
- Recommended batch size:

- Production:



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Frankfurt School

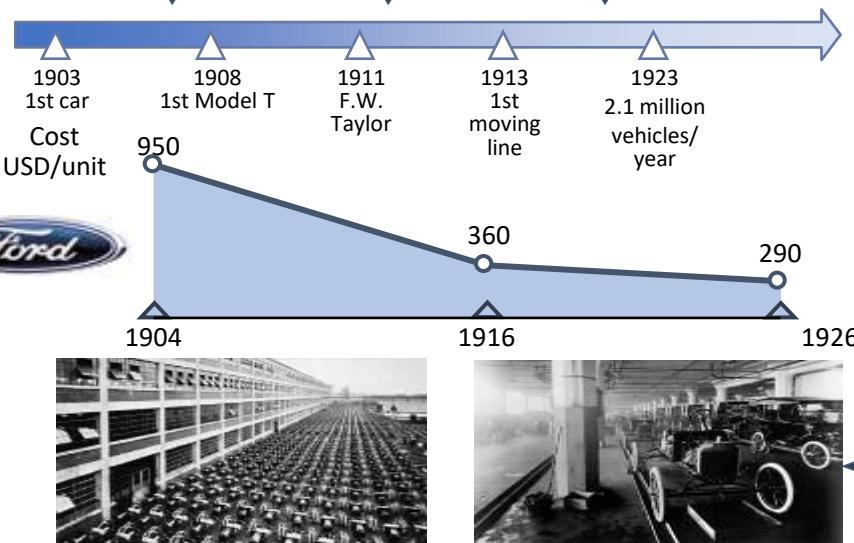
2023 Operations Management



LEAN OPERATIONS AND THE TOYOTA PRODUCTION SYSTEM

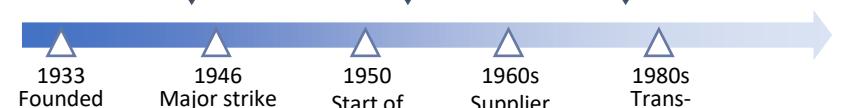
Ford vs Toyota Production System

- Taylorism: “The one best way”
- Moving line ensuring working at same pace
- Process driven by huge, rapid machinery with inflexible batch production



Mass production driven by economies of scale impossible

- Low production volume (1950):
Ford 3,656,000 – Toyota 11,000
- Low productivity (Japan 1/9 of US)
- Lack of resources



Key idea of TPS: systematic elimination of non-value-adding activities

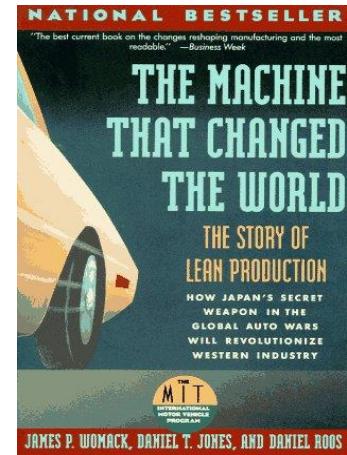
Key idea of Ford: cost reduction through cheap labor, economies of scale, and high utilization
→ cheap car for everyone

Some Facts and Figures

Assembly Plants, 1986

	Ford Framingham	Toyota Takaoka
Gross Assembly Hours per Car ^a	40.7	18
Assembly Defects per 100 Cars ^b	130	45
Assembly Space per Car ^c	8.1	4.8
Inventories of Parts (average)	2 weeks	2 hours

- a) Gross assembly hours per car are calculated by dividing total hours of effort in the plant by the total number of cars produced
- b) Defects per car were estimated from the J.D. Power Initial Quality Survey for 1987 (www.jdpower.com)
- c) Assembly Space per Car is square feet (0,093 m²) per vehicle per year, corrected for vehicle size



Lean Operations

Lean = Elimination of waste

Two categories of waste

Waste of time
of a resource

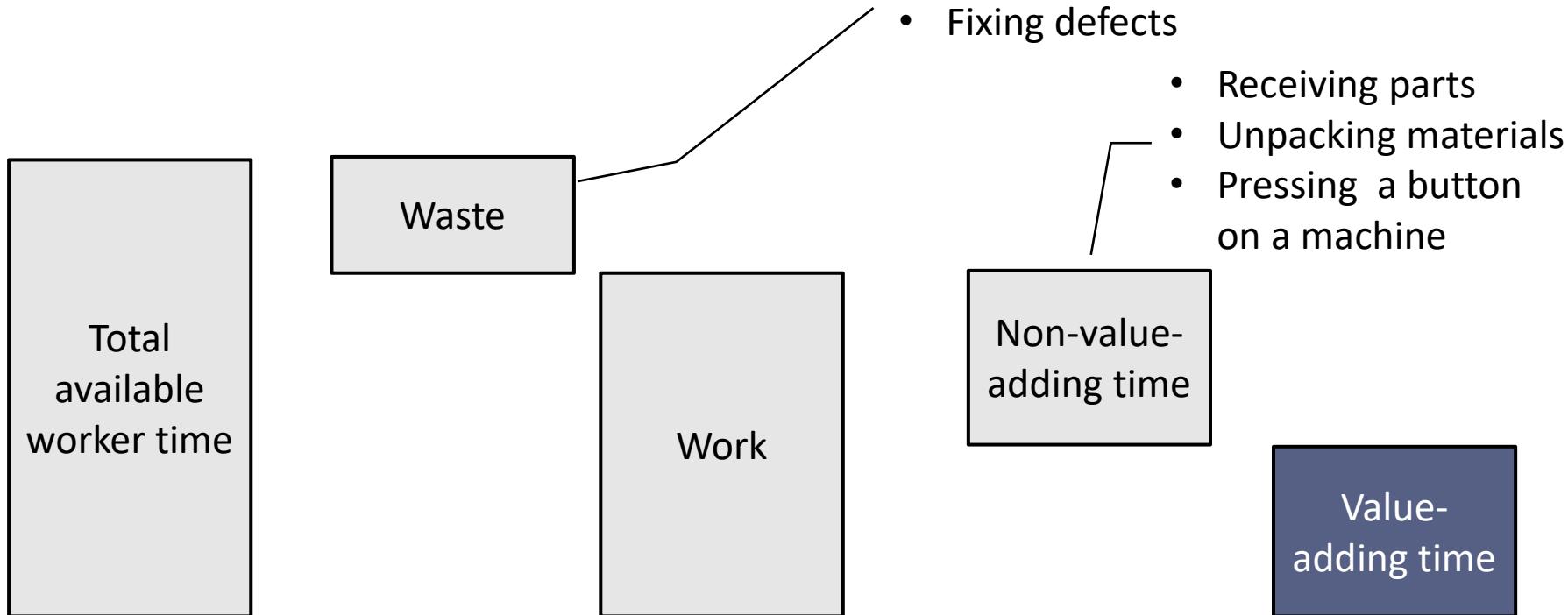
Waste of time
of a flow unit

Seven Sources of Production Waste to be Eliminated

1. Waiting time (of resources and flow units)
2. Overproduction (too much too soon)
3. Excess inventory (result of overproduction, inadequate inventory models)
4. Unnecessary transporting (inadequate physical layout)
5. Overprocessing (spending more time on a flow unit than necessary, processing waste)
6. Rework (takes away time that could be used in regular production)
7. Unnecessary motions and movement (Taylor: There are many ways to perform a task, but there is only one right way.)

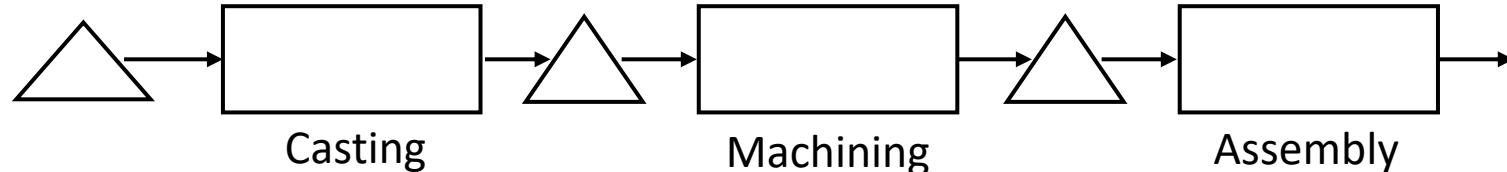


Waste of Time at a Resource

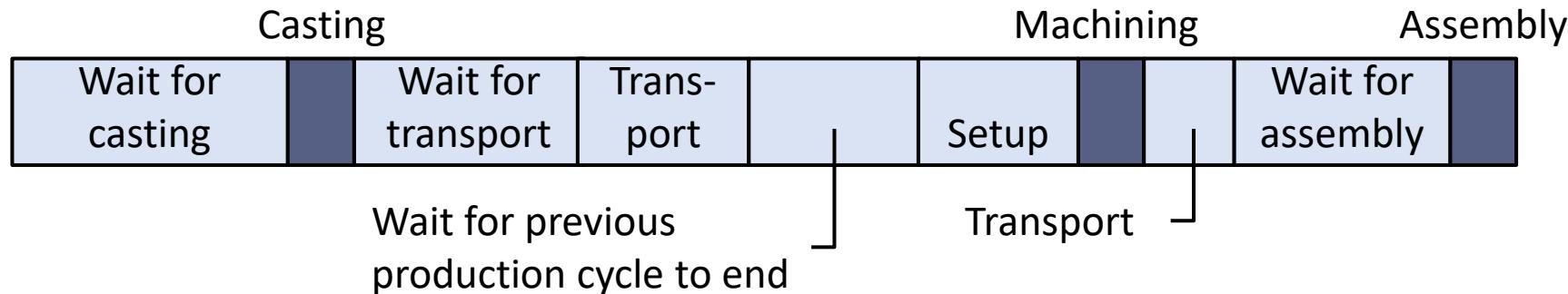


$$\text{Overall Equipment Effectiveness OEE} = \frac{\text{Value-adding time}}{\text{Total available time}}$$

Waste of Time of a Flow Unit

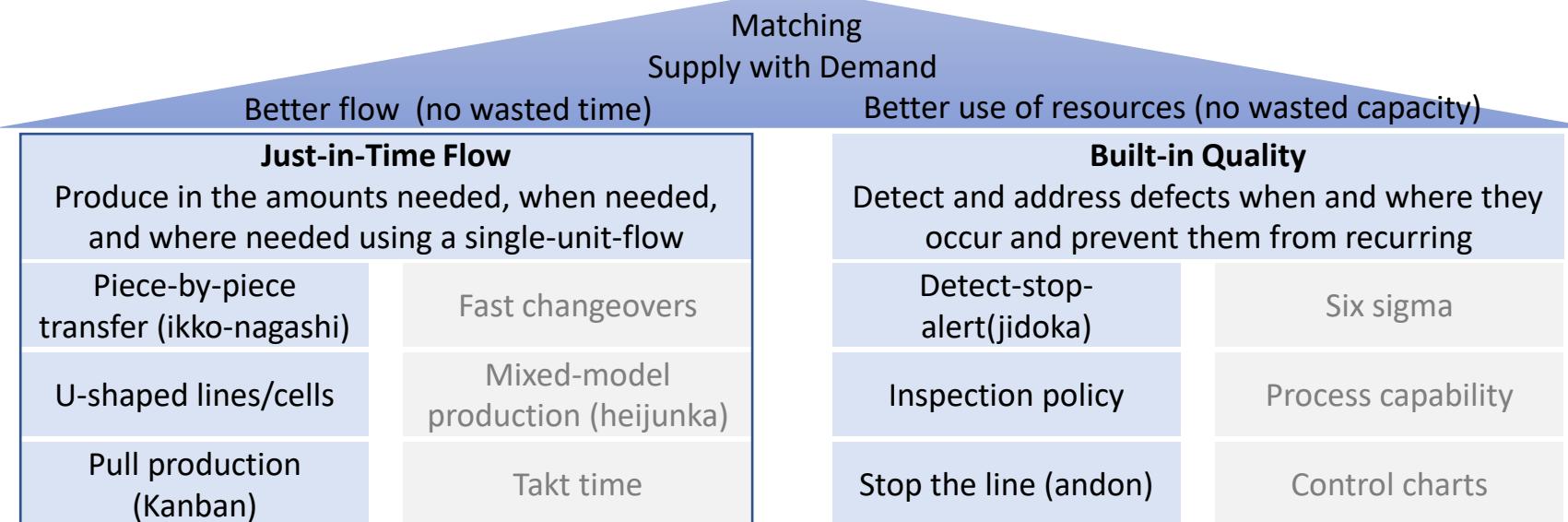


Perspective of one flow unit:



$$\text{Value-adding percentage} = \frac{\text{Value-adding time of a flow unit}}{\text{Flow time}}$$

The Toyota Production System House



Process Improvement: Problem solving and process improvement are led by the workers (kaizen)

Go and see yourself (genchi genbutso)

Root cause

Control charts

Pareto analysis

Foolproofing

Stable Processes: Create a stable and smooth flow with little variability and built-in flexibility

Level production

Built-in flexibility

Variability and flow



Just-in-time Flow

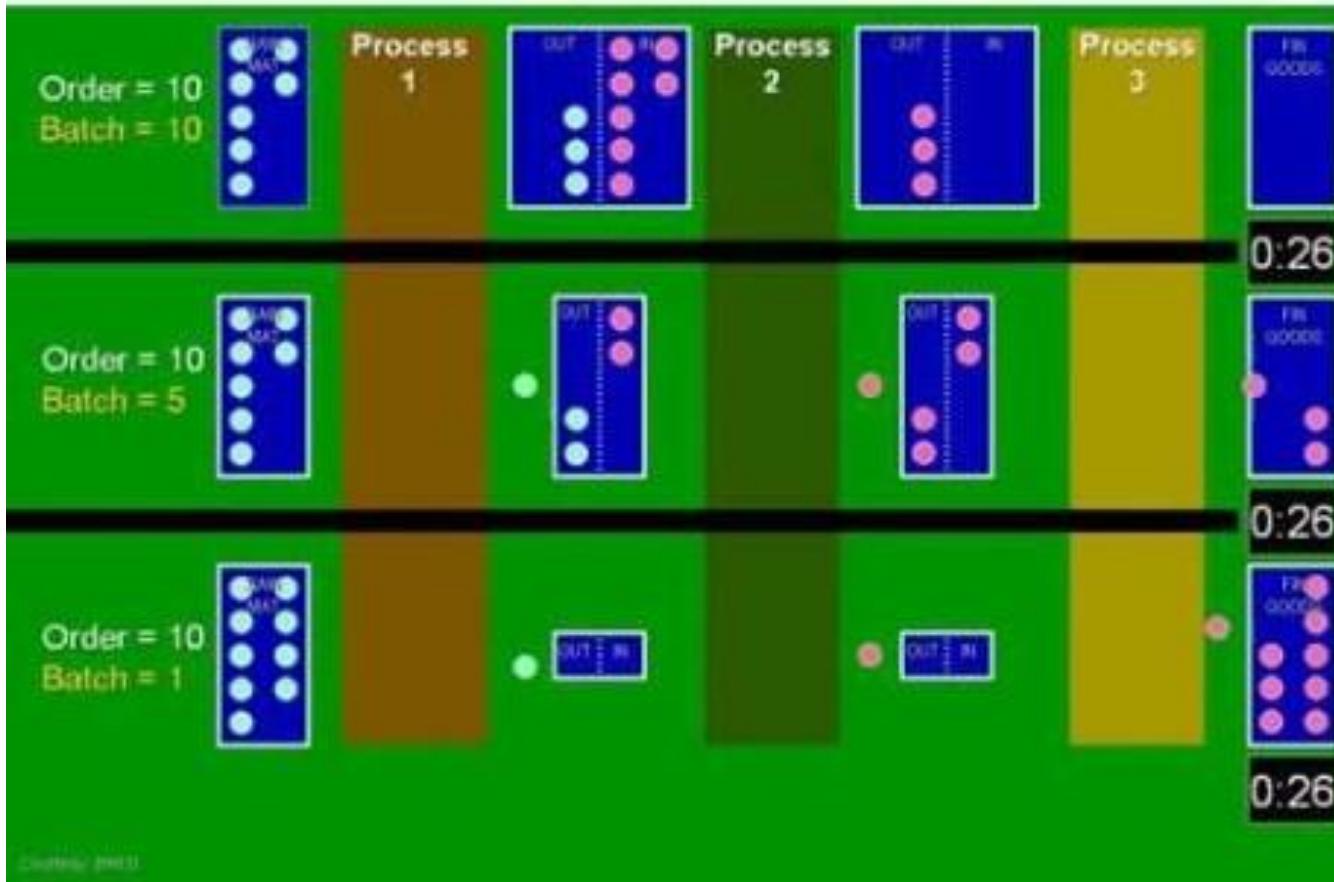
Just-in-time goal

- Create a supply process that forms a smooth flow with its demand, thereby giving customers exactly what they need, when they need it
- Run like the tortoise, not the hare

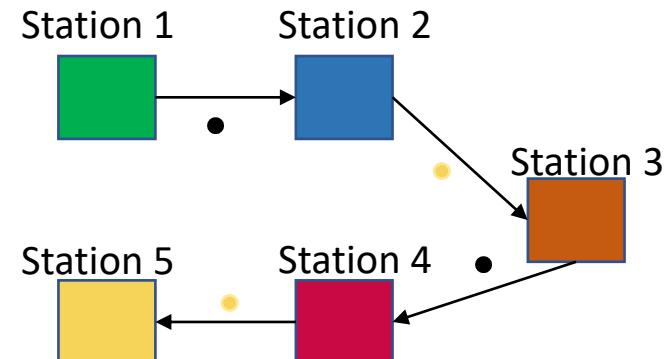
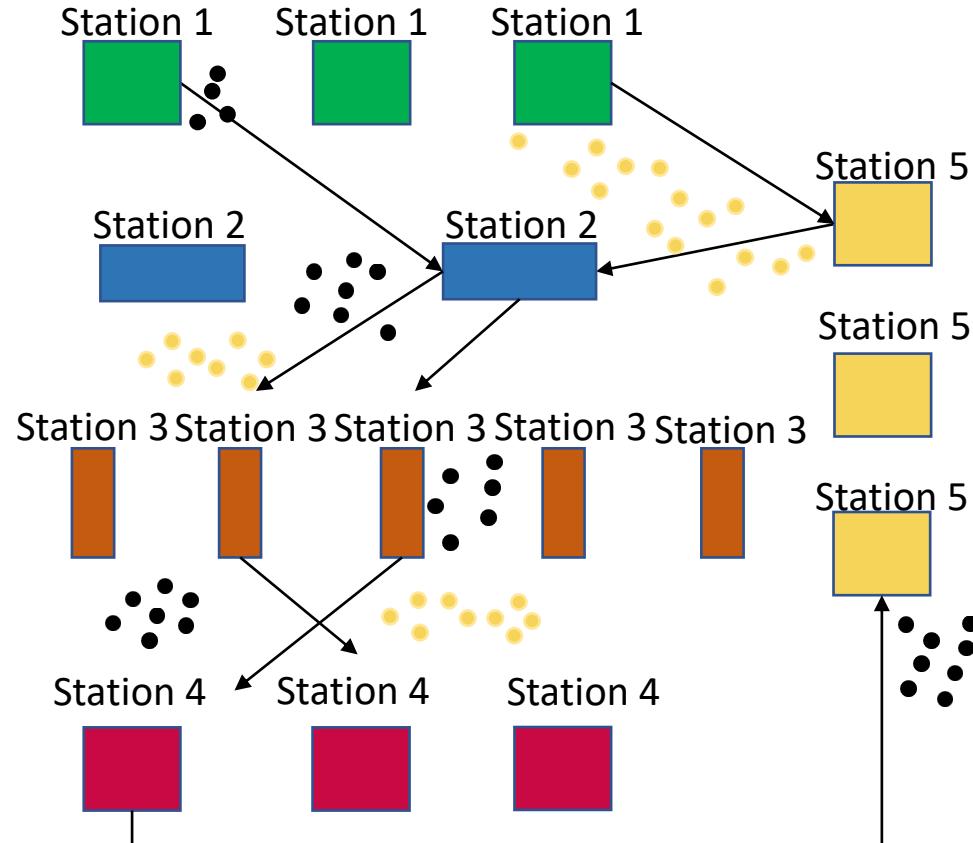


THE TORTOISE AND THE HARE

Flow – One piece flow versus Batch Production



U-shaped Work Cell with Piece-by-piece Transfer



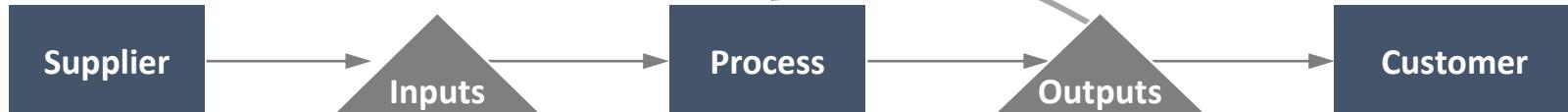
Push versus Pull

- Push: Flow units are allowed to enter the system independent of current amount of inventory in the system and independent of customer demand

PUSH: Input availability triggers execution



PULL: Outputs trigger execution

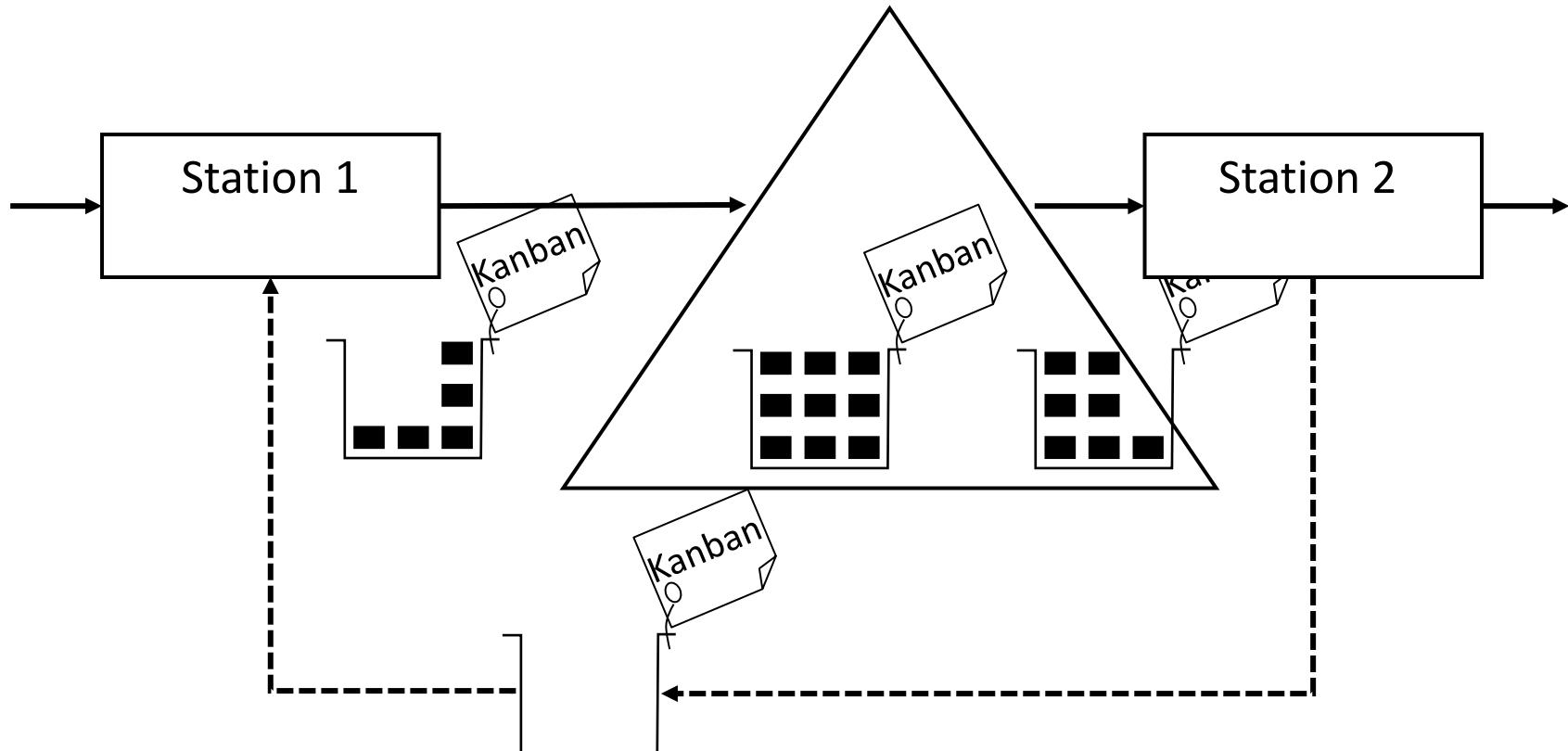


- Pull: the resource furthest downstream (i.e., closest to the market) triggers production

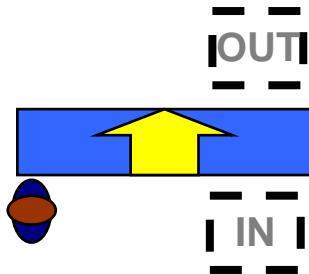
Implementation of a Pull System: Kanban

- Kanban-based pull (also known as supermarket)
- Returnable parts containers circulate between upstream and downstream resources
- The upstream resource is allowed to produce only, when it received an empty container → the arrival of an empty container triggers production
- Kanban refers to the card attached to the container which includes “production orders”
- Main advantage: there can never be more inventory in the system than has been authorized by Kanban cards; production stops if all containers are full

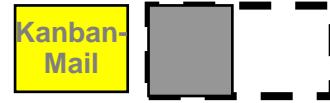
Kanban control overview



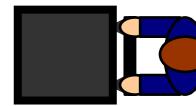
Explanation of Symbols Used in Kanban Animation



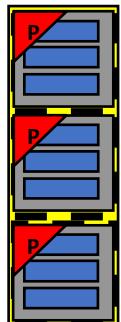
Machine with operator,
input and output area



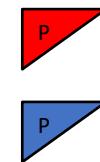
Kanban-Mail and
KLT-bin



Logistics-staff



Kanban-buffer with Kanbans ()
and small load KLT* box ()
filled with components ().

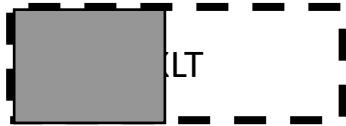


Kanban- Lot
size
= 3
Components

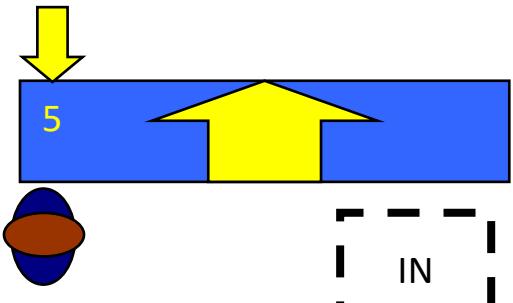
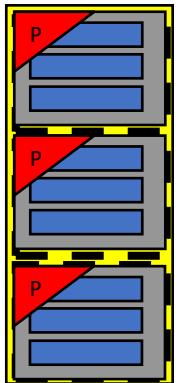
* KLT = Kleinladungsträger, Eurocontainer

Station 2 (Sink)

Kanban-Mail

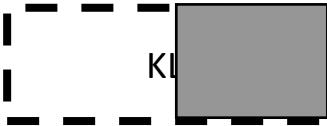


IN

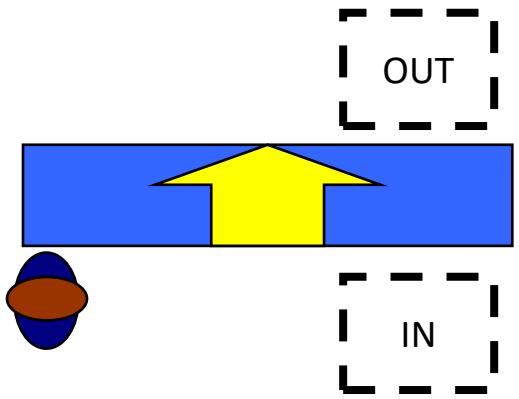
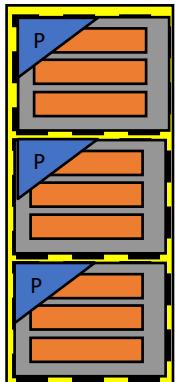


Station 1 (Source)

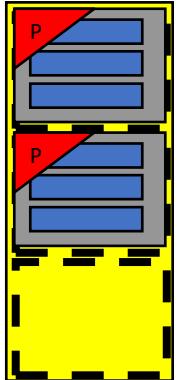
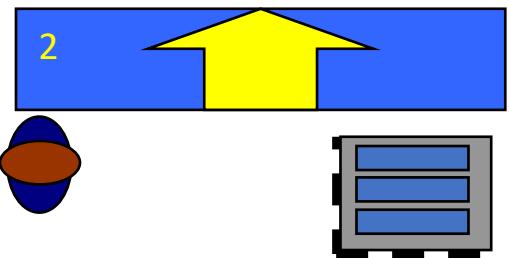
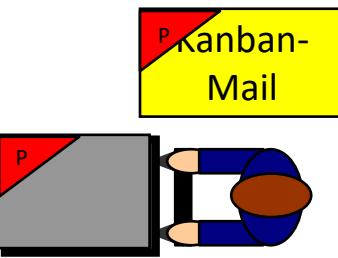
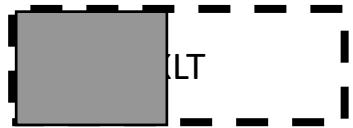
Kanban-Mail



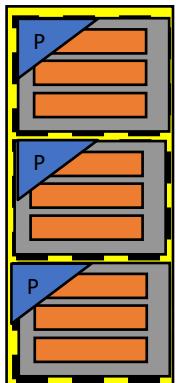
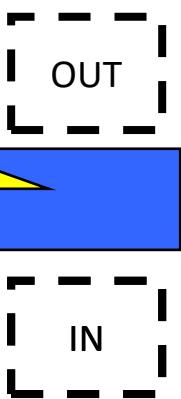
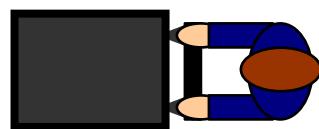
OUT



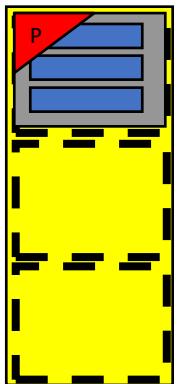
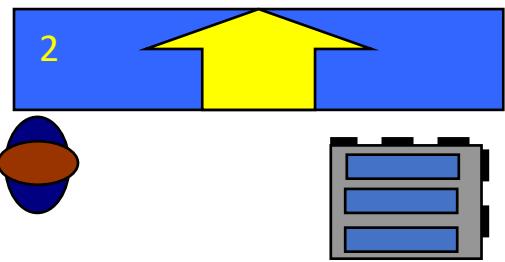
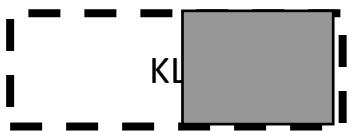
Station 2 (Sink)



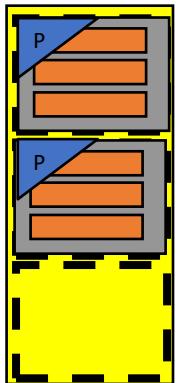
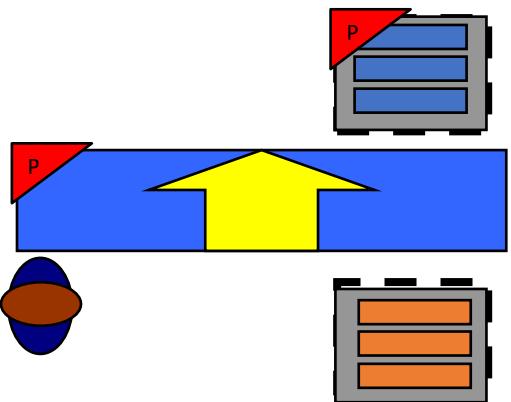
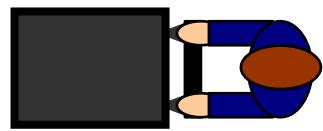
Station 1 (Source)



Station 2 (Sink)

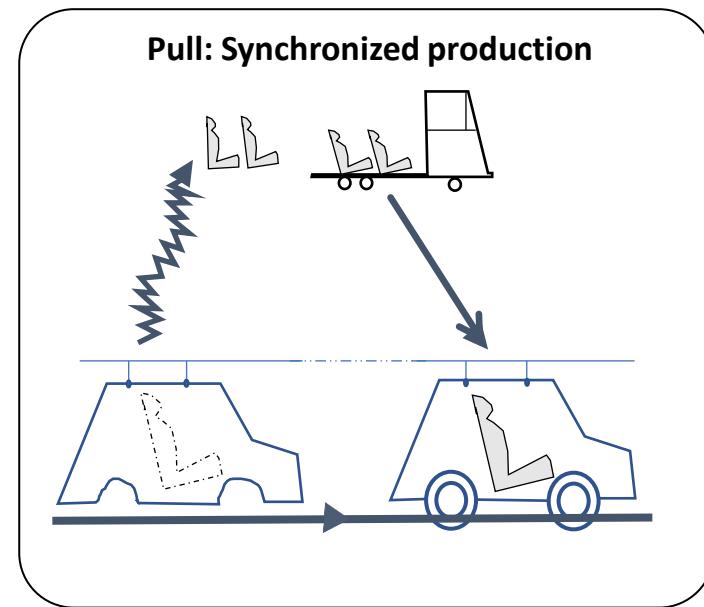


Station 1 (Source)



Implementation of a Pull System: Make-to-Order Process

- A Pull system can also be implemented by Make-To-Order processes
- Part produced for specific order (at supplier)
- Shipped right to assembly → Real time synchronization
- Make-to-Order: In a pure make-to-order setting production is purely triggered by the end-customer
- Common for low-volume products with high customization; customers must be willing to wait; no finished products inv.



When to use Kanban, When to use Make-to-Order?

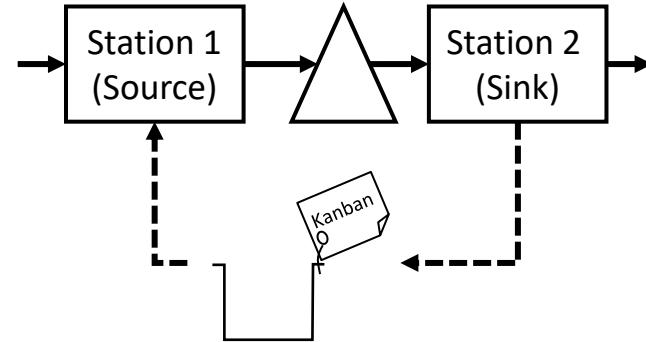
Pull: Make-to-order production



Make-to-Order should be used when:

- products are processed in low volume and high variety.
- customers are willing to wait for orders.
- it is expensive or difficult to store products.

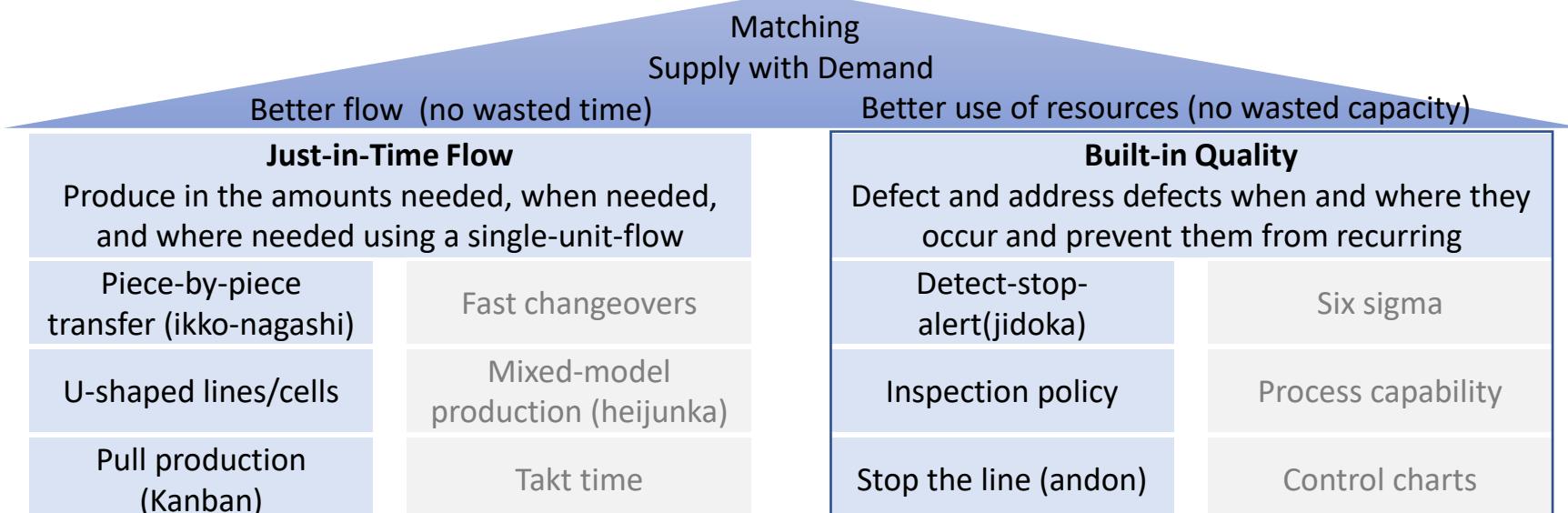
Pull: Kanban



Kanban should be used for products that:

- are processed in high volumes and limited varieties.
- are required with a short lead time → it is reasonable to preproduce (#Kanban cards).
- have low storing costs and efforts.

The Toyota Production System House



Process Improvement: Problem solving and process improvement are led by the workers (kaizen)

Go and see yourself (genchi genbutso)

Root cause

Control charts

Pareto analysis

Foolproofing

Stable Processes: Create a stable and smooth flow with little variability and built-in flexibility

Level production

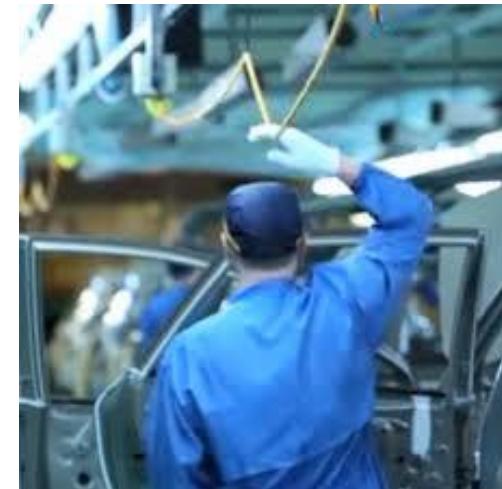
Built-in flexibility

Variability and flow



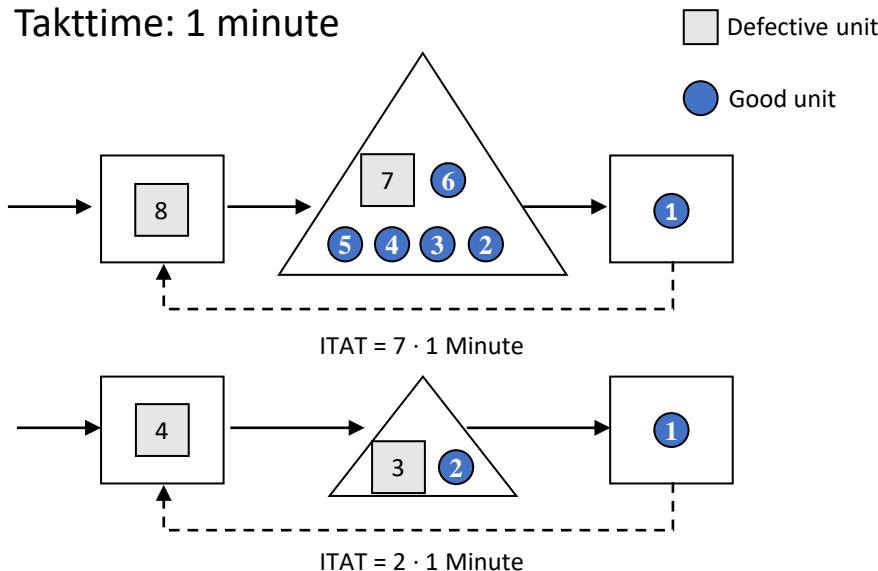
Detect-Stop-Alert (Jidoka)

- To approach “zero defects”, TPS relies on
 - Defect prevention
 - Rapid defect detection
 - Strong worker responsibility
- Jidoka
 - If, despite defect prevention, a defect occurs, TPS attempts to detect this defect ASAP
 - Stop the machine/assembly immediately if a defect is detected: Andon cord
 - Inspection policy: In contrast to GM/Ford where quality inspection comes at the end of the process every worker proves quality such that quality inspection is “built in” the process (tsukurikomi).
- Large WIP inventories delay the detection of quality issues (opposite of Jidoka)

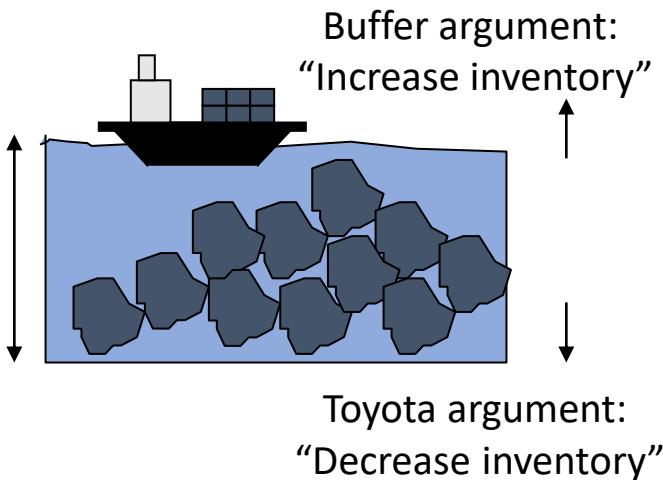


Inventory Delays the Detection of Errors

Takttime: 1 minute

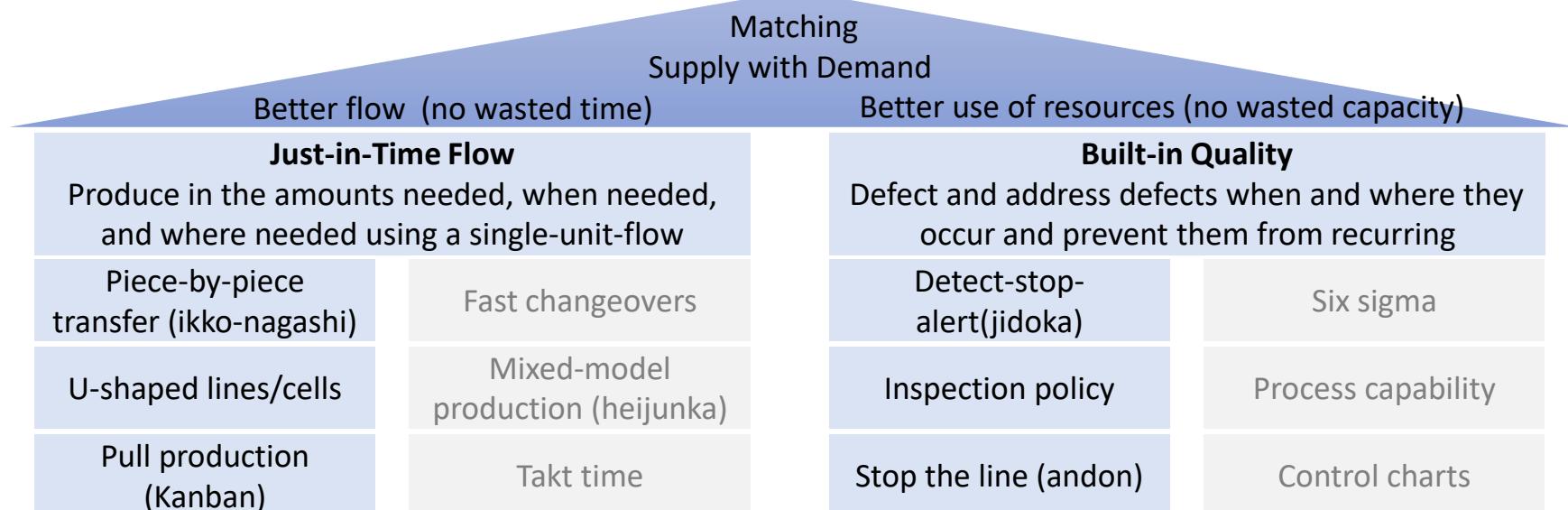


Inventory in process



- Inventory leads to a longer ITAT (Information turnaround time) \Rightarrow slow feed-back and no learning
- TPS philosophy: expose problems instead of hiding them
- However, zero inventory may become a problem! ... (right figure)

The Toyota Production System House



Process Improvement: Problem solving and process improvement are led by the workers (kaizen)

Go and see yourself (genchi genbutso)

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Level production

Built-in flexibility

Variability and flow



Kaizen

- Continuous improvement
 - Build an organizational culture and value system that stresses improvement of all processes
 - Part of everyone's job
- Respect for people
 - People are treated as knowledge workers
 - Engage physical and mental capabilities
 - Empower employees



Genchi Genbutsu

- The Ohno Circle: Stand in the circle and observe
- Gary Convis: President moves to shop floor
- Sienna Chief Engineer: "I must drive through all 50 states and all Canadian provinces and territories and Mexico"
- Engineering: Go and see where the actual parts are made and tested





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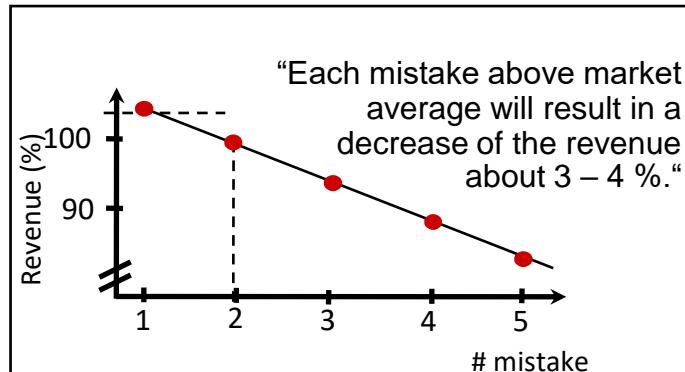
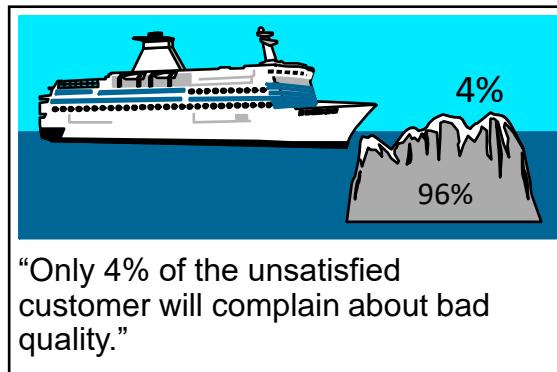
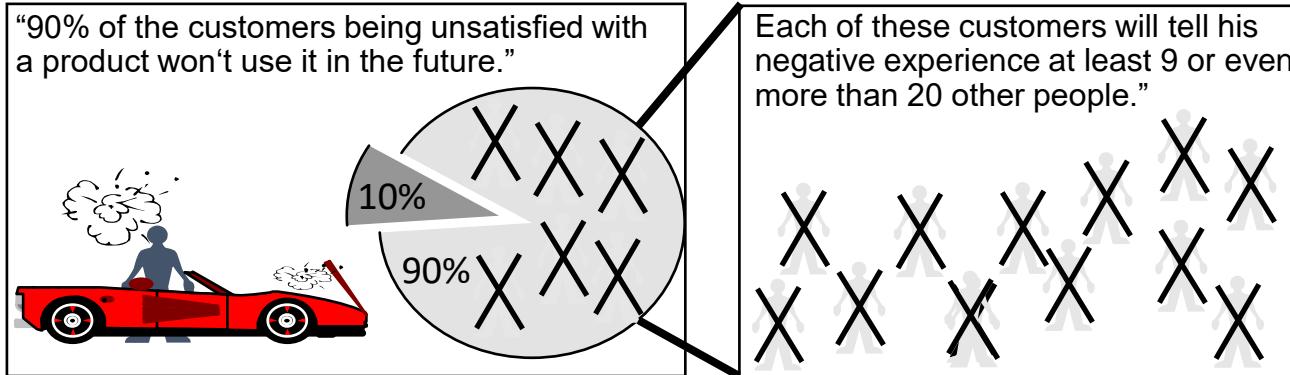
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2023 Operations Management



QUALITY AND STATISTICAL PROCESS CONTROL

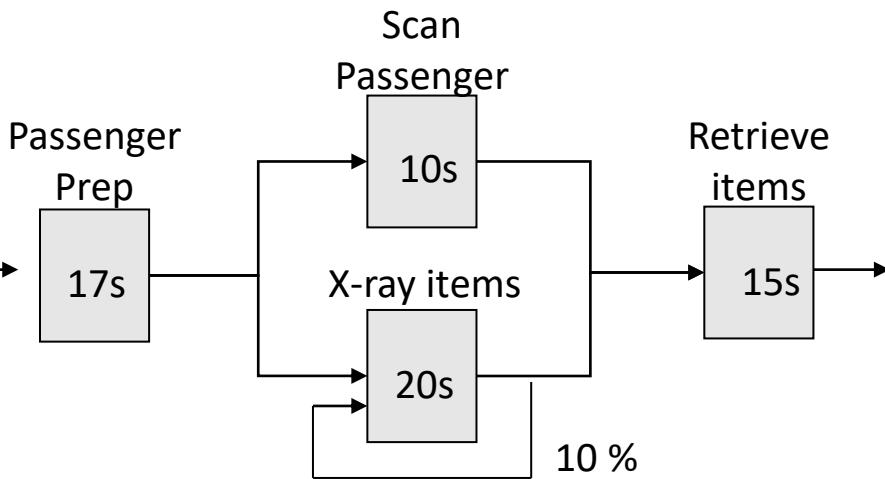
Effect of Quality Issues- Customers



Quelle: Robert L. Desatnik: Long Live the King, in: Quality Progress, April 1989.

Effect of Quality Issues

– Rework



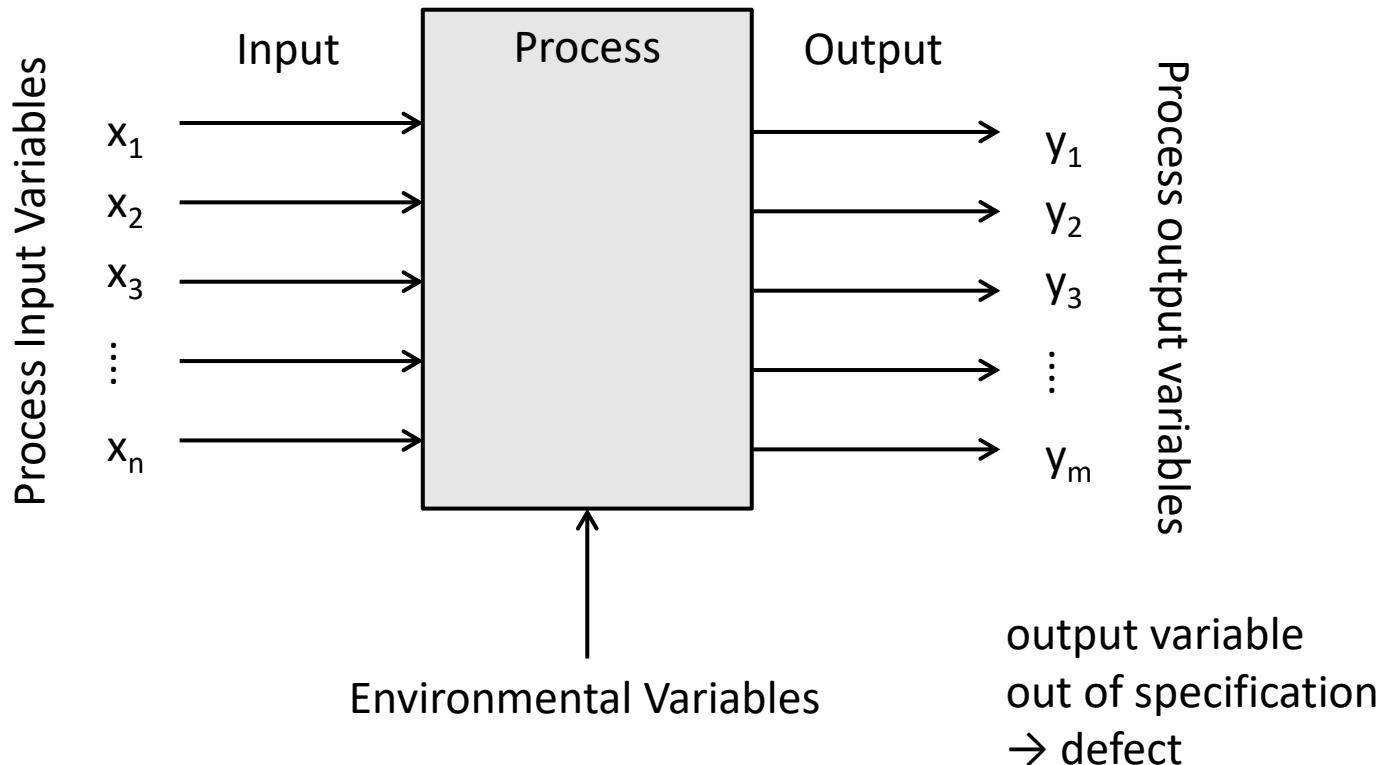
- Process flow diagram:
Airport security check
- Bottleneck: X-ray items
- Capacity without quality issues/rework: 3 [passengers/min]
- Suppose: 10% of passenger baggage need to be rescanned
 - Avg. processing time of X-ray items = $0.9 \cdot 20 \text{ [s]} + 0.1 \cdot 40 \text{ [s]} = 22 \text{ [s]}$
 - Capacity decreases to $1/22 \text{ [passengers/s]} \cdot 60 \text{ [s]/[min]} = 2.73 \text{ [passengers/min]}$
- The system's capacity is reduced by 9 % due to re-work!

Variation

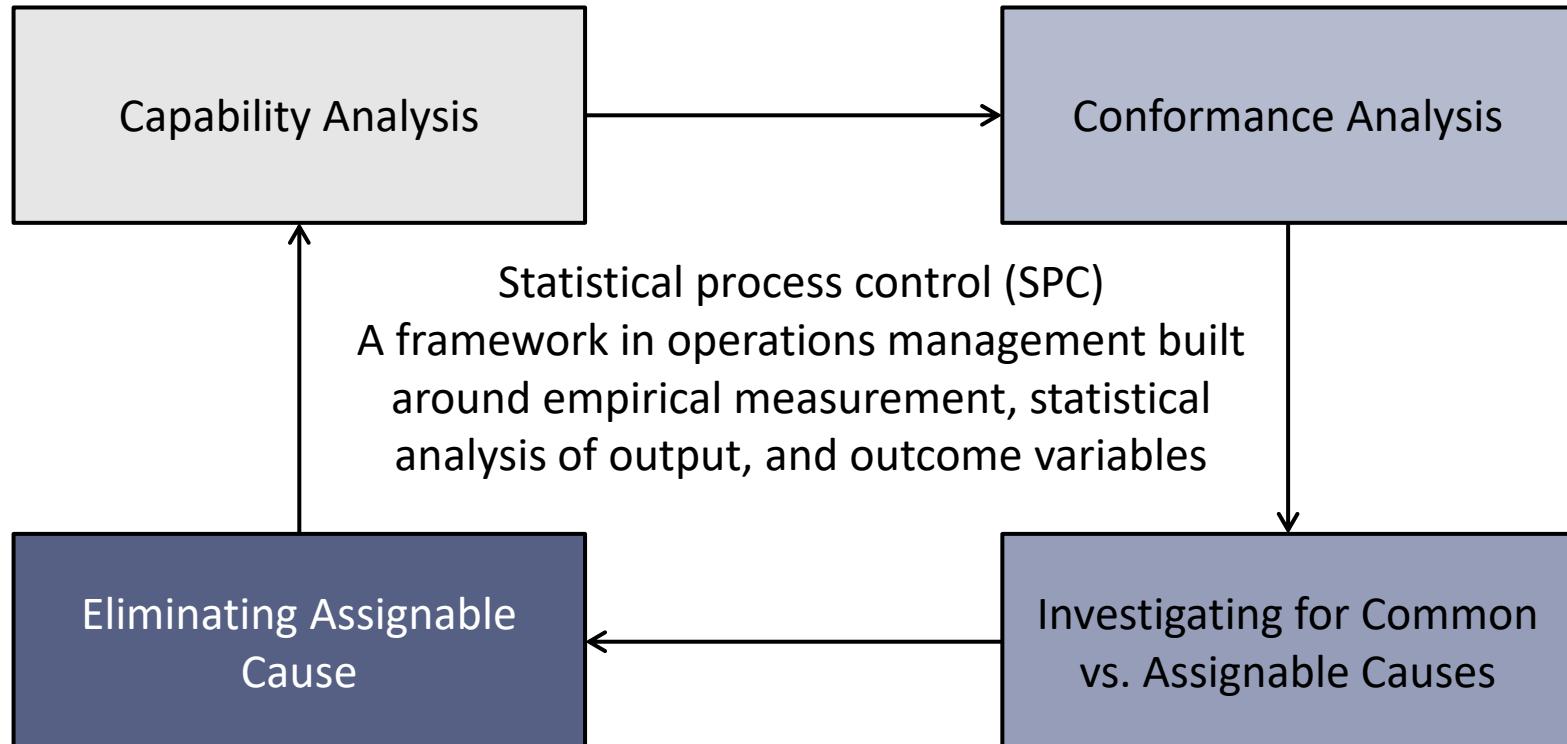
- Variation exists everywhere.
- Defects are driven by variability.



Variation in a Process

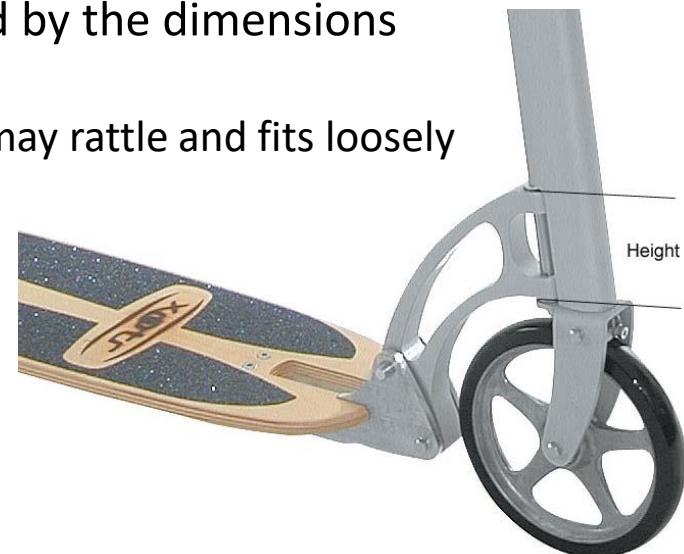


The Statistical Process Control Framework



Capability Analysis: The Xootr Example

- Steer support part - a critical component of the Xootr scooter
- The “height” of the steer support must closely match the opening in the lower handle
- The “height” of the steer support is specified by the dimensions as falling between 79.900 and 80.000 mm
 - If the height is less than 79.900 mm the part may rattle and fits loosely
→ lower specification limit (LSL)
 - If the height is more than 80.000 mm it does not fit in the gap
→ upper specification limit (USL)
- Specification limits determine which units are acceptable/defective



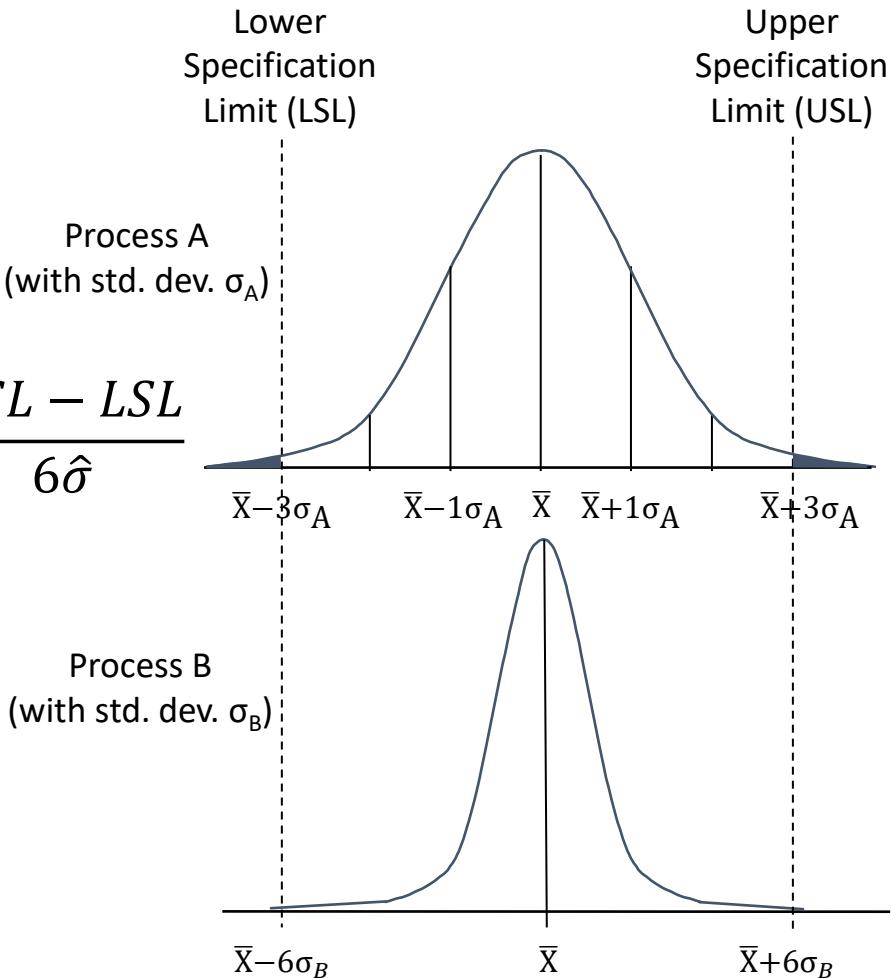
Sources of Variation in the CNC Milling Process

- The steer support is refined by a high-precision CNC machine
- Nevertheless, there is variation in the output, resulting from
 - CNC machine characteristics, e.g. positioning error,
 - Changing raw material quality,
 - Changes in temperature,
 - CNC operator mistakes, etc.



Determining the Process Capability Index

- The process capability measure, C_p , relates the process tolerance to the actual variation captured by the estimated standard deviation.
- Process tolerance in relation to actual variation
- $C_p=1$: probability of 99.7% that specifications are met \Rightarrow “3-Sigma”
- $C_p=0.6$: probability of 92.8%, that specifications are met
- $C_p=2.0$: probability of 99.999998%, that specifications are met
 \Rightarrow „6-Sigma“



Process Capability & Defect Probability

- Probability {part too small}
 $= \text{NORM.DIST}(79.900, 79.950, 0.017291, \text{TRUE}) = 0.001915954$
- Probability {part too big}
 $= 1 - \text{NORM.DIST}(80.000, 79.950, 0.017291, \text{TRUE}) = 0.001915954$
- Probability {part defective}
 $= 0.001915954 + 0.001915954 = 0.003831908$
- Defective parts per million (PPM)
 $= 0.003831908 \cdot 1,000,000 = 3831.9$

LSL: 79.900 mm

USL: 80.000 mm

Xootr engineers estimate a standard deviation of 0.017291 mm.

$$C_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{80.000 - 79.900}{6 \cdot 0.017291} = 0.96389$$

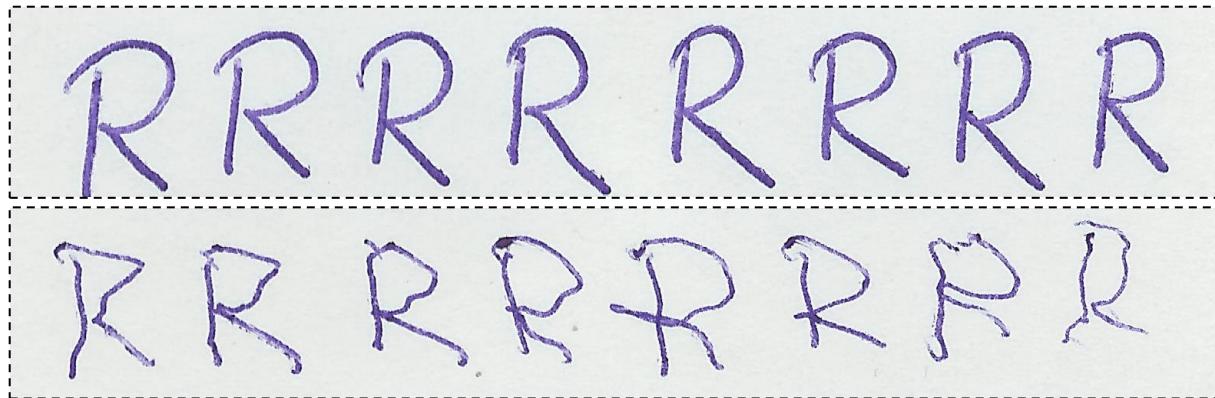


Conformance Analysis - Two Types of Variation

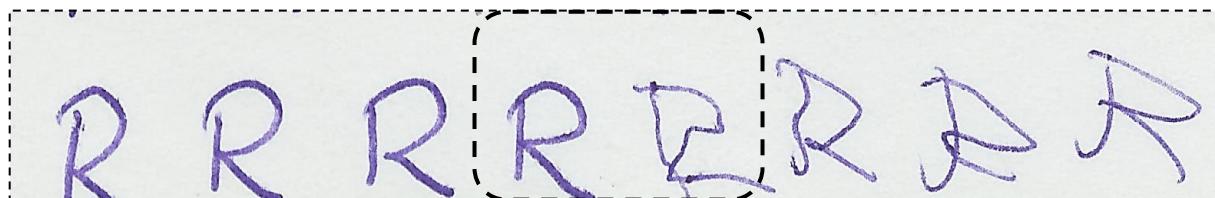
- Common-cause variations („natural“ variation)
 - No two pieces are exactly identical, but the variation is “within the tolerance”
 - Natural, purely stochastic variation; noise within the system
 - Plotting common cause variation looks like a normal distribution
- Special-cause variation, also assignable-cause variation
 - New, unanticipated, emergent or previously neglected phenomena, e.g., error in programming a machine, operator error, material defect, etc.
 - Assignable causes of variation change the underlying statistical distribution of a process
 - Process improvement projects typically address these assignable causes

Two Types of Variation

Common cause (Low level of Variation)



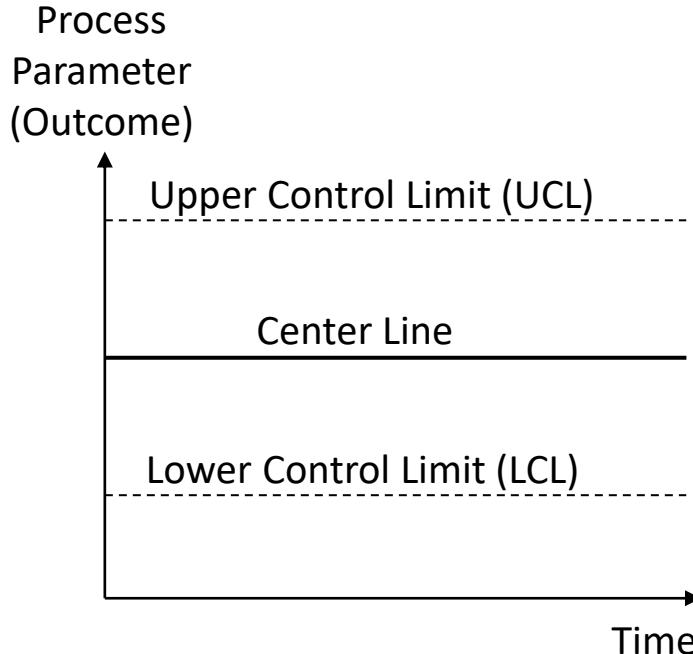
Common cause (High level of Variation)



Assignable Cause Variation

Conformance analysis is about identifying assignable cause variation as soon as possible

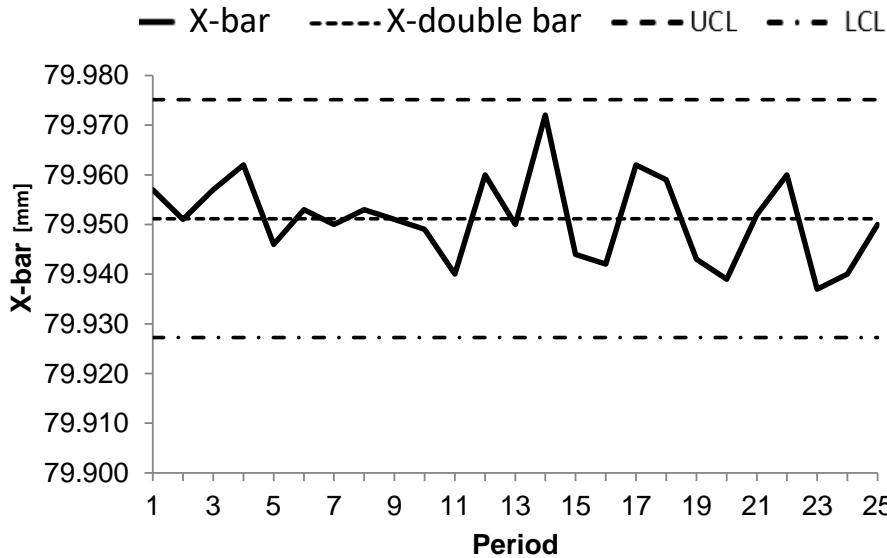
Detect Abnormal Variation



Control charts

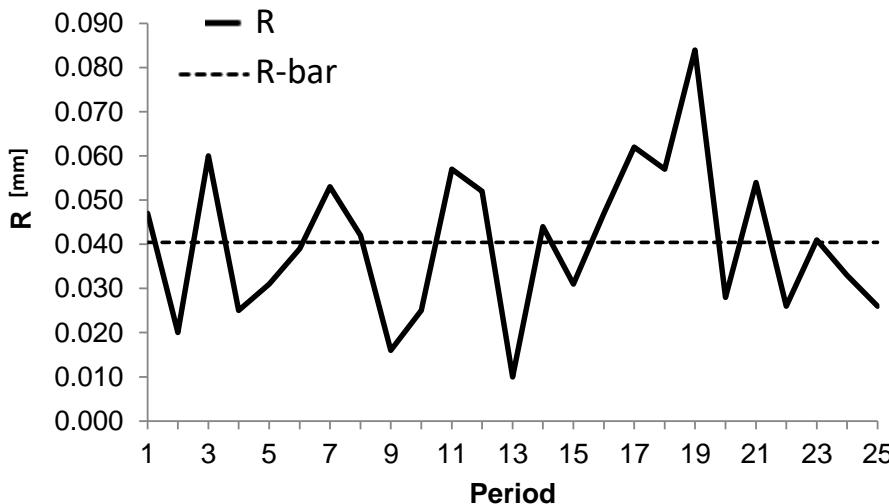
- Graphical tools to statistically distinguish assignable and common cause variation
- Track process outcome over time
 - Mean
 - Standard deviation
- Distinguish between
 - common cause variation (within control limits)
 - assignable cause variation (outside control limits)
- Two types of control charts:
 - X-bar chart
 - R (range)-chart
 - The type of chart (X-bar or R) determines the Y-Axis

Two Types of Control Charts: X-bar Chart



- Y-axis corresponds to the mean of a sample (e.g., a day)
- Dotted line (X-double bar) is computed from the mean over the sample means
- Used to document trends over time and to identify unexpected jumps (e.g., due to new operator)

Two Types of Control Charts: R-chart



- Y-axis corresponds to the range of a sample (e.g., a day)
- Range: difference between the highest and the lowest value in a sample OR the standard deviation
- Dotted line is computed from the mean over the sample ranges

Formulas

Period	Height of steer support parts in mm					Mean (X-bar)	R
	x1	x2	x3	x4	x5		
1	79.941	79.961	79.987	79.940	79.956	79.957	0.047
2	79.953	79.942	79.962	79.956	79.944	79.951	0.020
3	79.926	79.986	79.958	79.964	79.950	79.957	0.060
4	79.960	79.970	79.945	79.967	79.967	79.962	0.025
5	79.947	79.933	79.932	79.963	79.954	79.946	0.031
6	79.950	79.955	79.967	79.928	79.963	79.955	0.039
7	79.971	79.960	79.941	79.962	79.918	79.950	0.053
8	79.970	79.952	79.946	79.928	79.970	79.953	0.042
9	79.960	79.957	79.944	79.945	79.948	79.951	0.016
10	79.936	79.945	79.961	79.958	79.947	79.949	0.025
11	79.911	79.954	79.968	79.947	79.918	79.940	0.057
12	79.950	79.955	79.992	79.964	79.940	79.960	0.052
13	79.952	79.945	79.955	79.945	79.952	79.950	0.010
14	79.973	79.986	79.942	79.978	79.979	79.972	0.044
15	79.931	79.962	79.935	79.953	79.937	79.944	0.031
16	79.966	79.943	79.919	79.958	79.923	79.942	0.047
17	79.960	79.941	80.003	79.951	79.956	79.962	0.062
18	79.954	79.958	79.992	79.935	79.953	79.959	0.057
19	79.910	79.950	79.947	79.915	79.994	79.943	0.084
20	79.948	79.946	79.943	79.935	79.920	79.939	0.028
21	79.917	79.949	79.957	79.971	79.968	79.952	0.054
22	79.973	79.959	79.971	79.947	79.949	79.960	0.026
23	79.920	79.961	79.937	79.935	79.934	79.937	0.041
24	79.937	79.934	79.931	79.934	79.964	79.940	0.033
25	79.945	79.954	79.957	79.935	79.961	79.950	0.026
Average (X double bar)					79.951	0.040	

$$\bar{X} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

$$R = \max\{x_1, x_2, \dots, x_n\} - \min\{x_1, x_2, \dots, x_n\}$$

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_t}{t}$$

Defining the UCL and LCL

- Objective: Set UCL and LCL such that we can claim with 99.7% confidence that the process has “gone out of control,” if we observe \bar{X} or R outside the control limits

$$UCL = \bar{X} + (3 \times ESD \bar{X})$$

$$LCL = \bar{X} - (3 \times ESD \bar{X})$$

$$ESD \bar{X} = \frac{\hat{\sigma}}{\sqrt{n}}$$

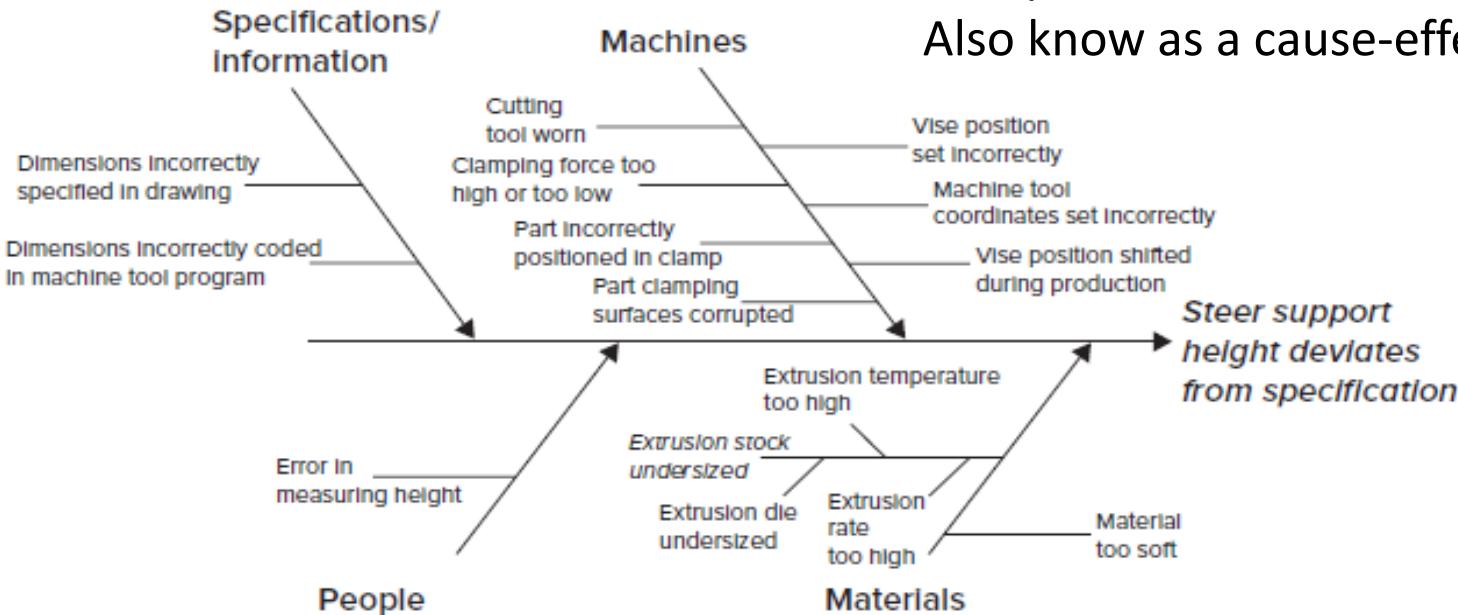
- $\hat{\sigma}$ = Estimated standard deviation based on all sample observation from all days
- If a process “goes out of control”, we say an assignable cause has occurred with a 99.7% probability

Investigating Assignable Causes

- Defect occurrence reflects some abnormal variation in the outcome variable.
 - Result of abnormal variation in input or environmental variable
- Control Chart will alert an assignable cause variation has occurred
 - Our job to look for what input or environmental variables led to this result – to investigate the root cause for the abnormal variation
- First step- to create a more careful diagram illustrating the relationship between the outcome variable and the various input and environmental variables.

Fishbone/Ishikawa Diagram

A structured way to brainstorm about the potential root causes that have led to a change in an outcome variable. This is done by mapping out all input and environmental variables. Also known as a cause-effect diagram.

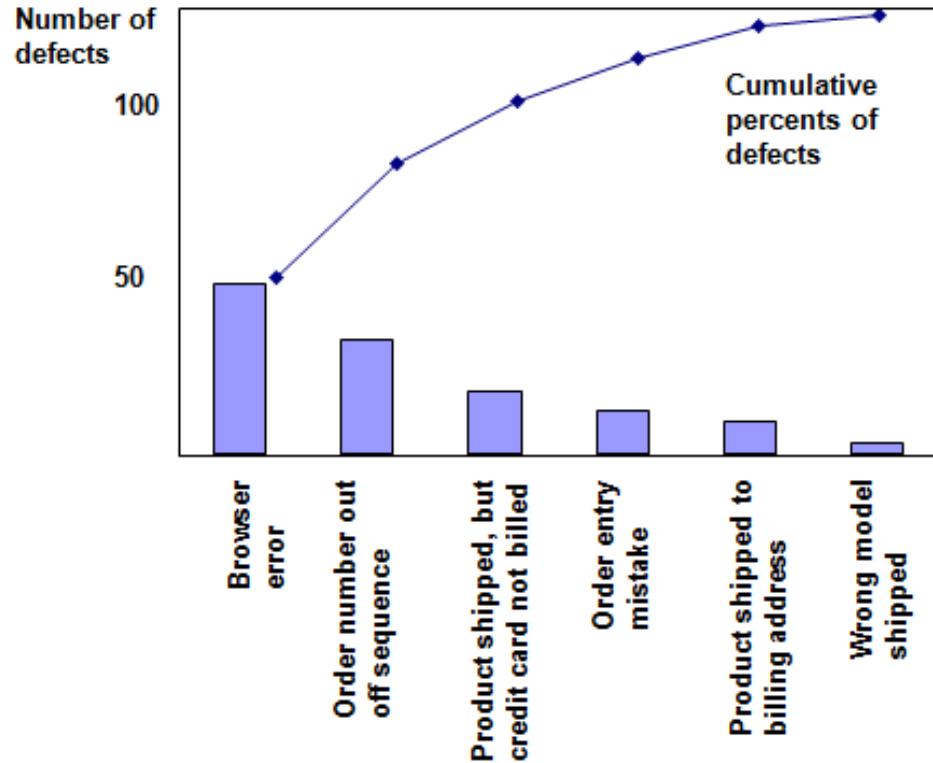


Investigating Assignable Causes with the Five Whys Technique

- Brainstorming technique that helps employees to find the root cause of a problem by asking the question “Why” at least five times:
- Avoids stopping too early and not having found the real root cause.
- An example of a problem is: The vehicle will not start.
 - **Why?** – The battery is dead. (First why)
 - **Why?** – The alternator is not functioning. (Second why)
 - **Why?** – The alternator belt has broken. (Third why)
 - **Why?** – The alternator belt was well beyond its useful service life and not replaced. (Fourth why)
 - **Why?** – The vehicle was not maintained according to the recommended service schedule. (Fifth why, a root cause)

Investigating Assignable Causes: Pareto Analysis

- Graphical way to identify the most important causes of process defects
- Order root causes in decreasing order of frequency of occurrence
- 80-20 logic



Given the multiple potential root causes of a defect (uncovered by the Fishbone/Five Whys technique) it is desirable to find the root cause for most of the problems.

How to Eliminate Assignable Causes

- Design robust processes

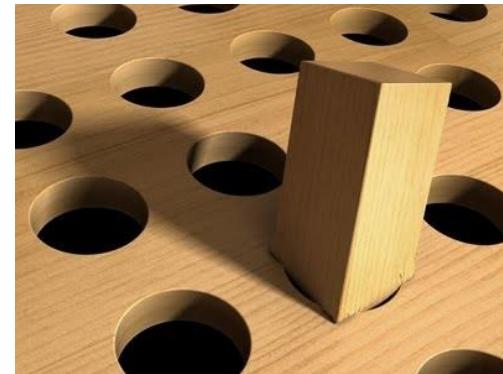
Robust processes can tolerate variation in input variables and environmental variables without leading to a defect. They are resistant against variation in input and environmental strategies.

- How can we make a process robust?

- Over engineering – make the process so that can do well, even under exceptional conditions
- Early warning signs on input and environmental variables There typically exists some lag time between the occurrence of a variation at this point an alert would be placed providing for earlier intervention to the root cause of the variation
- Fool-proofing – Change the work in such a way that the operator cannot complete the task if a mistake has been made. The side benefit provides the operator feedback thus reducing the likelihood of future reoccurrence of defective outputs.



Foolproofing (Poka-yoke)



EXTRA



Probability of Error Table Solution

- LSL: 79.900 mm
- USL: 80.000 mm
- Height of the steer support:
 - Standard deviation $\sigma = 0.017291$ mm
 - Mean $\mu = 79.950$ mm.
- $\Pr(\text{Height} < 79.900 \text{ mm})?$

$$z = \frac{79.900 - 79.950}{0.017291} = -2.8918$$

z-Transformation : $z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution Function Table, $\phi(z) | F_{(0,1)}(z)$

<i>z</i>	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	0.00
-3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.6	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
-3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
-3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
-3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
-3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
-2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
-2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
-2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
-2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
-2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
-2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
-2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
-2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
-2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
-2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228

$$\Pr\{\text{Height} \leq 79.900 \text{ mm}\} = 0.0019$$

Probability of Error Table Solution

- LSL: 79.900 mm
- USL: 80.000 mm
- Height of the steer support:
 - Standard deviation $\sigma = 0.017291$ mm
 - Mean $\mu = 79.950$ mm.
- $\Pr(\text{Height} > 80.000 \text{ mm})?$

$$z = \frac{80.000 - 79.950}{0.017291} = 2.8918$$

z-Transformation : $z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution Function Table, $\phi(z) | F(0,1)(z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$\Pr\{\text{Height} \leq 80.000 \text{ mm}\} = 0.9981$
 $\Pr\{\text{Height} > 80.000 \text{ mm}\} = 1 - 0.9981$
 $= 0.0019$



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Frankfurt School

2023 Operations Management

SERVICE SYSTEMS WITH PATIENT CUSTOMERS

QUEUES WHEN DEMAND EXCEEDS SUPPLY



Ski Butternut Rental Shop

- Small ski resort in Berkshire Mountains of eastern Massachusetts
- 10 lifts, 22 trails
- 1 rental shop

The screenshot shows the homepage of skibutternut.com. At the top, there's a navigation bar with links for Tubing, Free Mobile App, Buy Now, and Directions. The main logo 'Ski Butternut' is on the left, with 'THE MOUNTAIN', 'TICKETS/PASSES', 'LESSONS/PACKAGES', and 'RENTALS' menu items below it. A large image of a ski lodge with a clock tower is centered. On the left side of the page, there's a sidebar with 'SKI & SNOWBOARD RENTALS' and links for 'Daily Rentals', 'Demo Skis', and 'Season Rentals'. On the right, a large blue button says 'BUY RENTALS'. The overall theme is a professional website for a ski resort.

Daily Rentals

We offer rental packages for the same

BUY RENTALS

Ski Butternut Rental Shop Challenges

- What is the maximum length of the queue at the shop?
- How long does a customer have to wait?
- What is the average customer wait time?
- How can peak demand be managed?

The screenshot shows the official website for Ski Butternut. At the top, there's a navigation bar with links for Tubing, Free Mobile App, Buy Now, and Directions. The main logo for 'Ski Butternut' is on the left, with a red swoosh graphic. Below the logo, there are tabs for THE MOUNTAIN, TICKETS/PASSES, LESSONS/PACKAGES, and a highlighted blue tab for RENTALS. The central part of the page features a large photograph of a modern wooden ski rental building with a prominent gabled roof and a clock tower, surrounded by snow and pine trees. In front of the building, several skiers are standing with their equipment. To the left of the photo, a dark sidebar contains the text 'SKI & SNOWBOARD RENTALS' and a list of services: Daily Rentals, Demo Skis, and Season Rentals. To the right of the photo, the word 'Daily Rentals' is displayed in large text, followed by a subtext 'We offer rental packages for the same' and a large blue button labeled 'BUY RENTALS'.

Ski Butternut Rental Shop Challenges

- **What is the maximum length of the queue at the shop?**
- How long does a customer have to wait?
- What is the average customer wait time?
- How can peak demand be managed?

The screenshot shows the homepage of skibutternut.com/rentals/. At the top, there's a navigation bar with links for 'Tubing', 'Free Mobile App', 'Buy Now', and 'Directions'. The main logo 'Ski Butternut' is on the left, with 'THE MOUNTAIN', 'TICKETS/PASSES', 'LESSONS/PACKAGES', and 'RENTALS' below it. A large banner image of a snowy mountain slope with ski lifts is in the background. Overlaid on the banner is the text 'Some data:' followed by a bulleted list. In the bottom right corner of the banner, there's a 'BUY RENTALS' button.

Some data:

- Rental customers are unusually disciplined.
- They arrive at a constant rate of 1.5 customers per minute between 8:00 and 10:00.
- Employees in the shop are experienced and work quickly. But they still can only manage to process 1.25 customers per minute.

Maximum Length of the Queue

- Rental shop queue growth rate

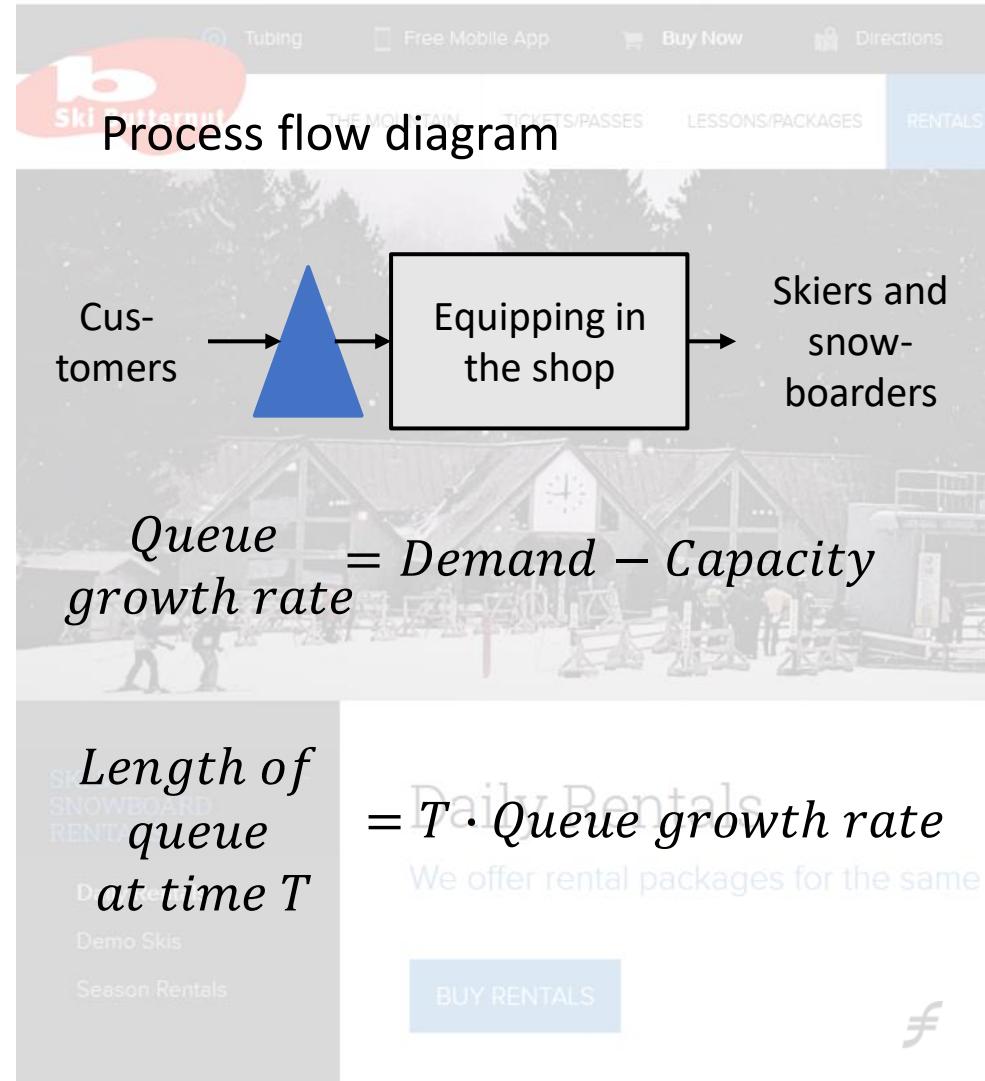
$$= 1.5 \frac{\text{cust.}}{\text{min}} - 1.25 \frac{\text{cust.}}{\text{min}}$$

$$= 0.25 \frac{\text{cust.}}{\text{min}}$$

- Length of queue when last customer arrives

$$= 120\text{min} \cdot 0.25 \frac{\text{cust.}}{\text{min}}$$

$$= 30 \text{ cust.}$$



Ski Butternut Rental Shop Challenges

- What is the maximum length of the queue at the shop?
- **How long does a customer have to wait?**
- What is the average customer wait time?
- How can peak demand be managed?

The screenshot shows the homepage of skibutternut.com/rentals/. The top navigation bar includes links for Tubing, Free Mobile App, Buy Now, and Directions. The main menu has options for THE MOUNTAIN, TICKETS/PASSES, LESSONS/PACKAGES, and RENTALS. A large banner image shows a snowy mountain scene with a ski lift. Overlaid on the banner is the text "Some data:" followed by a bulleted list of challenges. In the bottom left corner of the banner, there is a small watermark or overlay text that reads "Daily Rentals" and "for the same".

Some data:

- Rental customers are unusually disciplined.
- They arrive at a constant rate of 1.5 customers per minute between 8:00 and 10:00.
- Employees in the shop are experienced and work quickly. But they still can only manage to process 1.25 customers per minute.

How long does a customer wait?

- Waiting time of last customer arriving (30th in the queue):

$$= \frac{30 \text{ cust.}}{1.25 \frac{\text{cust.}}{\text{min}}} = 24 \text{ min}$$

- Alternatively:

$$= 120 \text{ min} \cdot \left(\frac{1.5 \frac{\text{cust.}}{\text{min}}}{1.25 \frac{\text{cust.}}{\text{min}}} - 1 \right)$$

The screenshot shows the Ski Butternut website with several formulas overlaid on a background image of a ski resort building.

Waiting time of Qth person in the queue = $\frac{Q}{\text{Capacity}}$

Waiting time of person arriving at time T = $T \cdot \left(\frac{\text{Demand}}{\text{Capacity}} - 1 \right)$

SKI & SNOWBOARD RENTALS

Daily Rentals
Demo Skis
Season Rentals

BUY RENTALS

Tubing Free Mobile App Buy Now Directions

Ski Butternut Rental Shop Challenges

- What is the maximum length of the queue at the shop?
- How long does a customer have to wait?
- **What is the average customer wait time?**
- How can peak demand be managed?

The screenshot shows the homepage of skibutternut.com/rentals/. At the top, there's a navigation bar with links for 'Tubing', 'Free Mobile App', 'Buy Now', and 'Directions'. The main logo 'Ski Butternut' is on the left, with 'THE MOUNTAIN', 'TICKETS/PASSES', 'LESSONS/PACKAGES', and 'RENTALS' below it. A large background image of a snowy mountain slope with ski lifts is visible. Overlaid on the bottom right is a semi-transparent box containing the text 'Some data:' followed by a bulleted list. In the bottom left corner of the page, there are links for 'SKI & SNOWBOARD RENTALS', 'Daily Rentals', 'Demo Skis', and 'Season Rentals'. A 'BUY RENTALS' button is also present. A small Facebook icon is in the bottom right corner.

Some data:

- Rental customers are unusually disciplined.
- They arrive at a constant rate of 1.5 customers per minute between 8:00 and 10:00.
- Employees in the shop are experienced and work quickly. But they still can only manage to process 1.25 customers per minute.

How long does the average customer wait?

- Average customer waiting time

$$= \frac{1}{2} 120 \text{ min} \cdot \left(\frac{1.5 \frac{\text{cust.}}{\text{min}}}{1.25 \frac{\text{cust.}}{\text{min}}} - 1 \right)$$

$$= 12 \text{ min}$$

The screenshot shows the Ski Butternut website with a banner at the top for 'THE MOUNTAIN' and 'TICKETS/PASSES'. Below the banner, there's a large image of a snowy mountain scene. Overlaid on the image are two blue arrows: one pointing down from the text 'Duration of excess demand' to the term 'Demand' in the formula, and another pointing up from the term 'Capacity' in the formula to the text 'Implied utilization Daily Rentals'.

Duration of excess demand

Average customer waiting time

$$= \frac{1}{2} T \cdot \left(\frac{\text{Demand}}{\text{Capacity}} - 1 \right)$$

Implied utilization

Daily Rentals

We offer rental packages for the same

SKI & SNOWBOARD RENTALS

Daily Rentals

Demo Skis

Season Rentals

BUY RENTALS

Tubing

Free Mobile App

Buy Now

Directions

Ski Butternut

Ski Butternut Rental Shop Challenges

- What is the maximum length of the queue at the shop?
- How long does a customer have to wait?
- What is the average customer wait time?
- **How can peak demand be managed?**

The screenshot shows the homepage of [Ski Butternut](https://skibutternut.com/). At the top, there's a navigation bar with links for 'Tubing', 'Free Mobile App', 'Buy Now', and 'Directions'. Below the navigation is the 'Ski Butternut' logo. The main content area features a large image of a snowy mountain scene with a ski lift. Overlaid on this image is the text 'Some data:' followed by a bulleted list of challenges. In the bottom left corner of the main image, there's a semi-transparent box containing text about daily rentals, demo skis, and season rentals, along with 'BUY RENTALS' and social media icons.

Some data:

- Rental customers are unusually disciplined.
- They arrive at a constant rate of 1.5 customers per minute between 8:00 and 10:00.
- Employees in the shop are experienced and work quickly. But they still can only manage to process 1.25 customers per minute.

SKI & SNOWBOARD RENTALS

Daily Rentals

Demo Skis

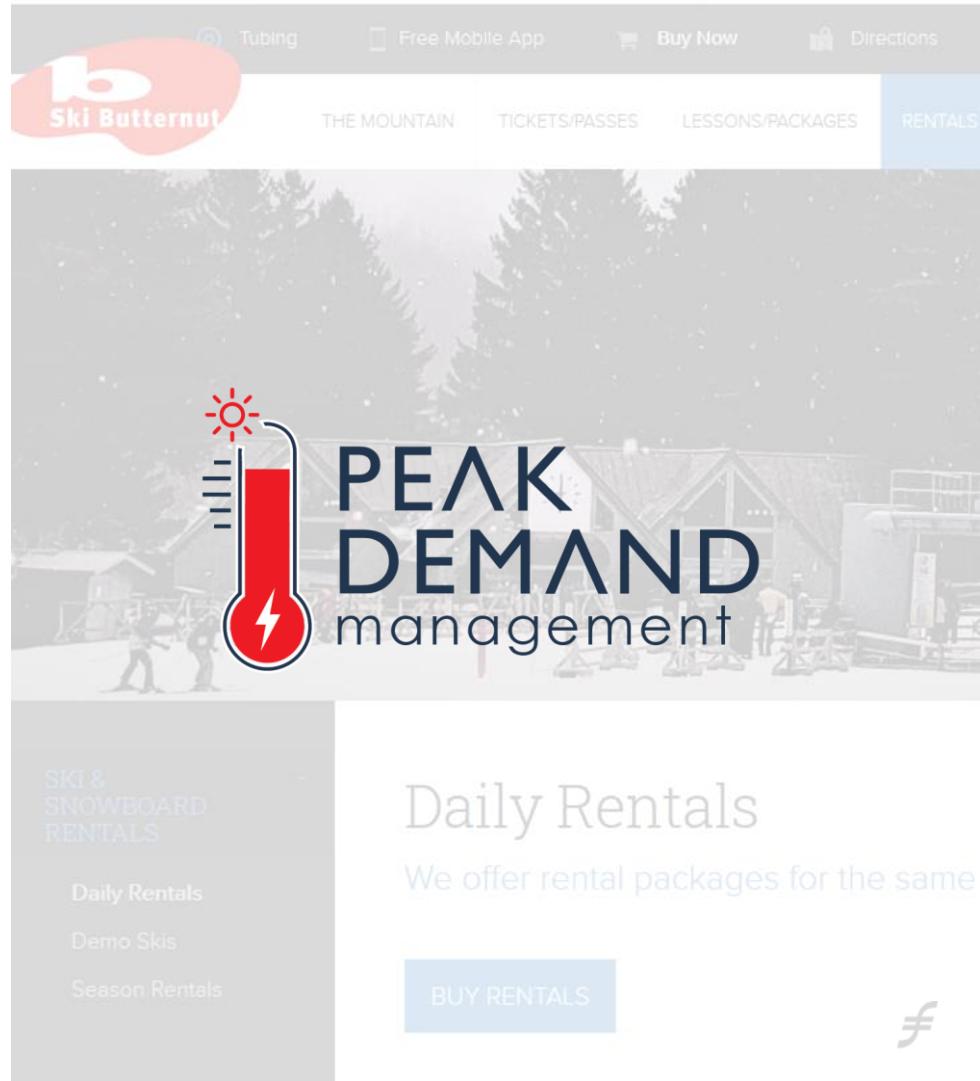
Season Rentals

BUY RENTALS

Facebook icon

Managing Peak Demand

- Reduce peak demand by
 - Peak-load pricing
 - Off-peak discount
- Increase capacity
 - Increase resources
 - Pre-processing



QUEUES WHEN DEMAND AND SUPPLY ARE VARIABLE - ONE SERVER



Frankfurt Key Services

- Primary address for copying keys
- Opening hours: 9:00 to 19:00
- Many years of experience, highest precision, large selection of blanks
- Fast: full copy in 4 minutes on average

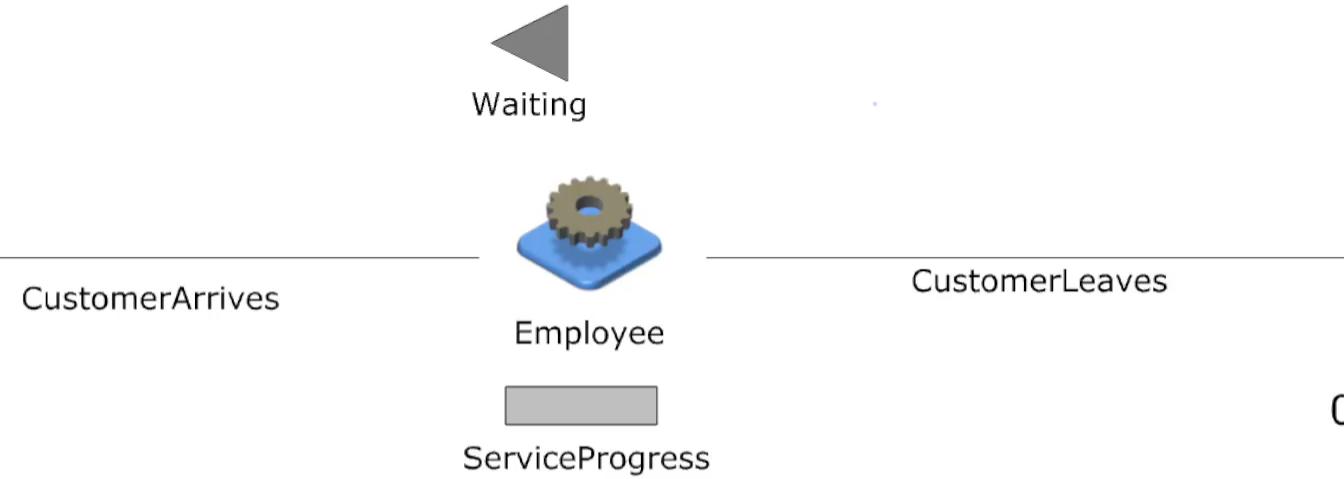
Frankfurter
Schlüsseldienst



Frankfurt Key Service

10:00 to 11:00 on a typical day

09:55:00.000



Key Services Challenges

- What is the capacity?
- How long do customers have to wait on average?
- How many customers wait on average in or in front of the shop?

Frankfurter
Schlüsseldienst



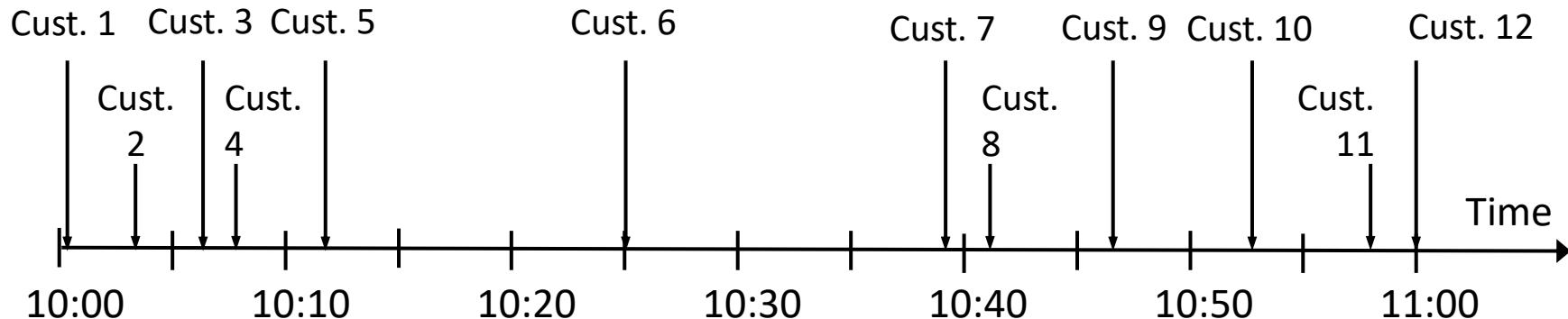
Key Services Challenges

- **What is the capacity?**
- How long do customers have to wait on average?
- How many customers wait on average in or in front of the shop?

Frankfurter
Schlüsseldienst

- one employee serves
- customers wait patiently

Key Service: Arrival and Operating Data



Customer	1	2	3	4	5	6	7	8	9	10	11	12
Customer												
Arrival Time	10:01	10:04	10:06	10:07	10:11	10:28	10:40	10:41	10:46	10:53	10:59	11:00
Service Time [min]	4	10	1	5	1	13	2	1	1	7	1	2

Capacity of Frankfurter Schlüsseldienst

Customer	Service time [min]
1	4
2	10
3	1
4	5
5	1
6	13
7	2
8	1
9	1
10	7
11	1
12	2

Key Services Challenges

- What is the capacity?
- **How long do customers have to wait on average?**
- How many customers wait on average in or in front of the shop?

Frankfurter
Schlüsseldienst

- one employee serves
- customers wait patiently



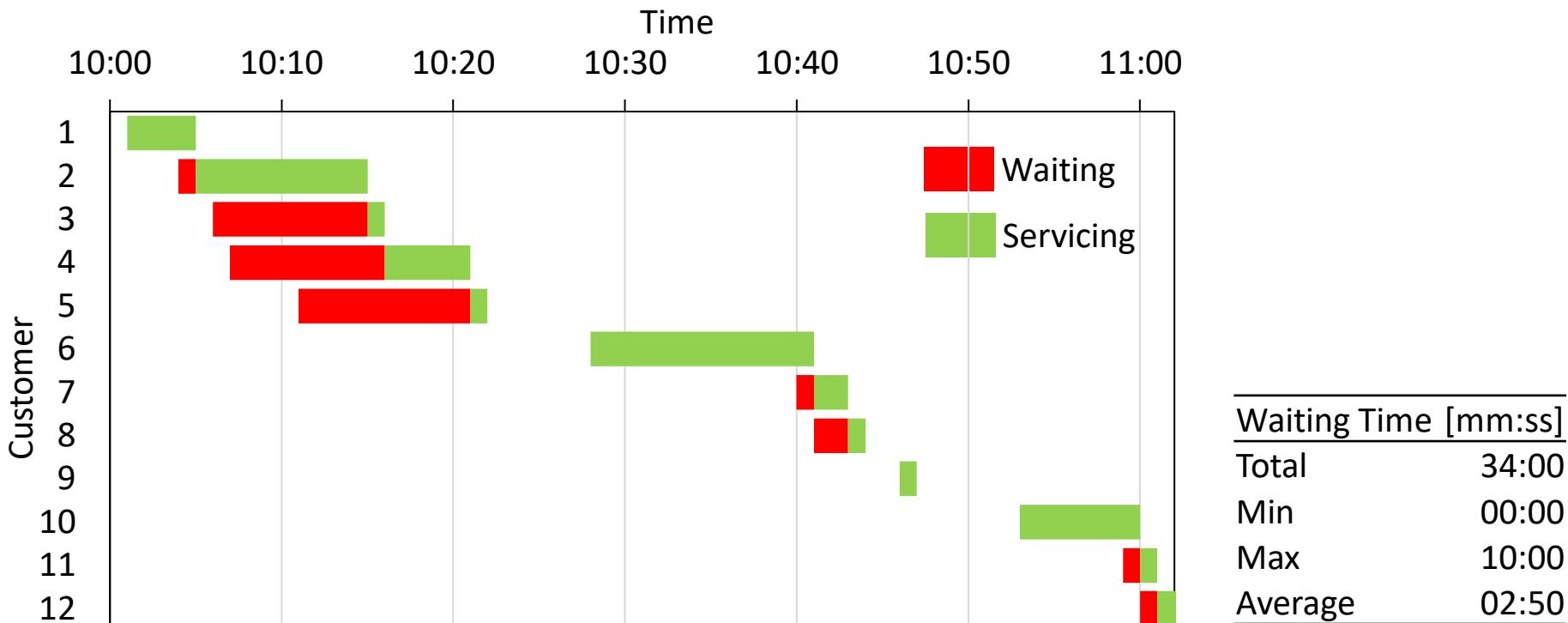
Calculation of Waiting Times

Cust.	Customer arrival time	Service time	Start time of service	End time of service	Waiting time
1	10:01	04:00			
2	10:04	10:00			
3	10:06	01:00			
4	10:07	05:00	10:16	10:21	00:09
5	10:11	01:00	10:21	10:22	00:10
6	10:28	13:00	10:28	10:41	00:00
7	10:40	02:00	10:41	10:43	00:01
8	10:41	01:00	10:43	10:44	00:02
:	:	:	:	:	:

in[hh:mm]



Waiting and Service Times as Gantt-Diagram



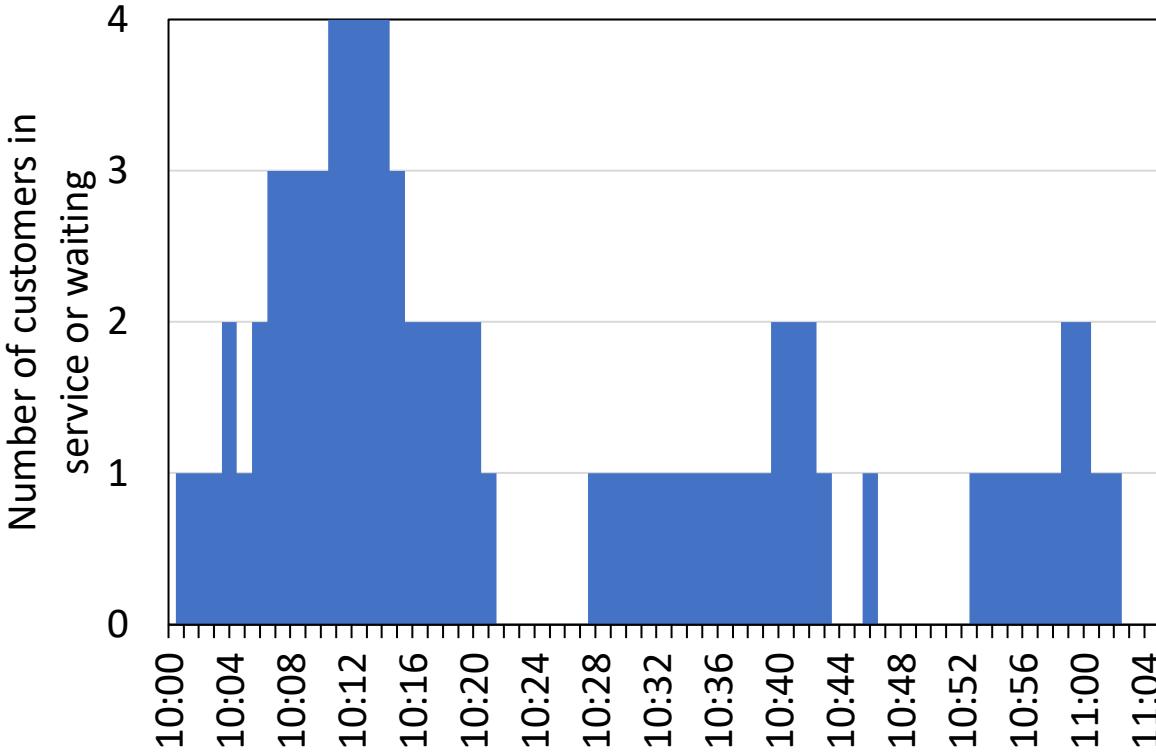
Key Services Challenges

- What is the capacity?
- How long do customers have to wait on average?
- **How many customers wait on average in or in front of the shop?**

Frankfurter
Schlüsseldienst

- one employee serves
- customers wait patiently

Number of Customers C (either Waiting or in Service)



On average:

10:00 to 11:02

→ 63 min

$$\bar{C} = \frac{\sum_{t=1}^{63} C_t}{63}$$
$$= 1.3016 \text{ [Customers]}$$

tel-inform Inbound Call Center

- Modern, owner-managed service company with around 300 employees
- Three defining pillars: tradition, social commitment and humanity
- Excellent customer service, e.g. for tax consultants

0800 1004334 - 001

Rückruf anfordern

tel-inform | Kundenservice mit Charakter

[Home](#) » [Telefonservice](#) » Steuerberater

Büro- und
Telefonservice für
Steuerberater: Mit
uns sind Sie auch in
Terminen erreichbar



Die meisten denken, dass der Beruf eines Steuerberaters lediglich aus dem Erstellen von Steuererklärungen und Bilanzen besteht. Dass der Beruf eines Steuerberaters viel komplexer und abwechslungsreicher ist, weiß kaum einer! Sie als **Steuerberater** müssen Anrufe entgegennehmen, Ihre Mandanten beraten und Termine wahrnehmen. Durch die Beantwortung von Kundenanfragen verlieren Sie kostbare Zeit, die Sie mit einem **Telefonservice** an Ihrer Seite effizienter nutzen können. Während Sie Ihre Termine

tel-inform Inbound Call Center Challenges

In a system with variability in customer arrivals and processing and with limited availability of past data,

- what is the (average) flow rate?
- what is the (average) flow time?
(waiting plus processing time)
- what is the (average) inventory?
(number of customers waiting and in service)

tel-inform call center data

1 employee Kundenservice mit Charakter

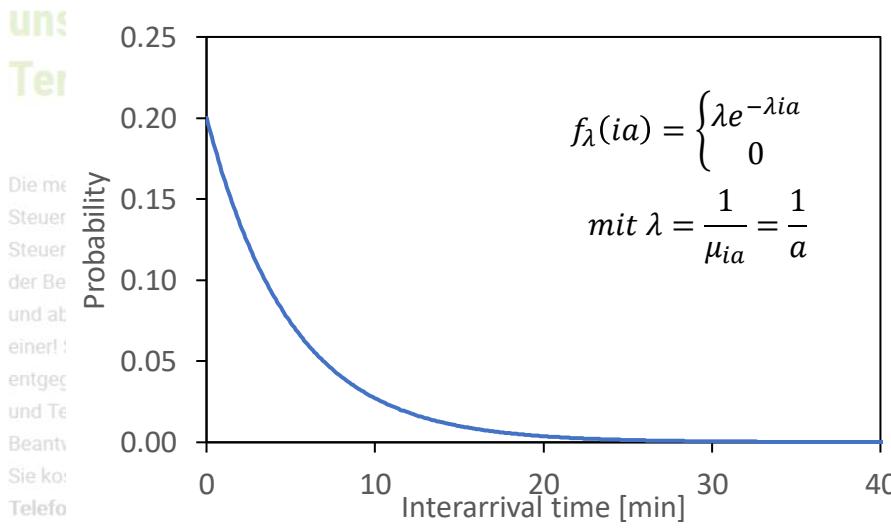
avg service time $p = 4 \text{ min}$

(uniformly distributed between 2 and 6 min)

$$\text{avg arrival rate} = 12 \frac{\text{calls}}{\text{h}}$$

avg interarrival time $a =$

exponentially distributed interarrival times ia:



tel-inform Average Flow Rate

*Demand rate = average inflow rate
= average arrival rate*

Flow rate =

$$\min \left\{ \begin{array}{l} \text{Demand rate,} \\ \text{Process capacity} \end{array} \right\}$$

Variability in customer arrivals and processing...

- ...does **not** affect the calculation of the average flow rate!
- ...does **complicate** the calculation of the average flow time!

tel-inform Inbound Call Center Challenges

In a system with variability in customer arrivals and processing and with limited availability of past data,

- what is the (average) flow rate?
- what is the (average) flow time?
(waiting plus processing time)
- what is the (average) inventory?
(number of customers waiting and in service)

tel-inform call center data

1 employee Kundenservice mit Charakter

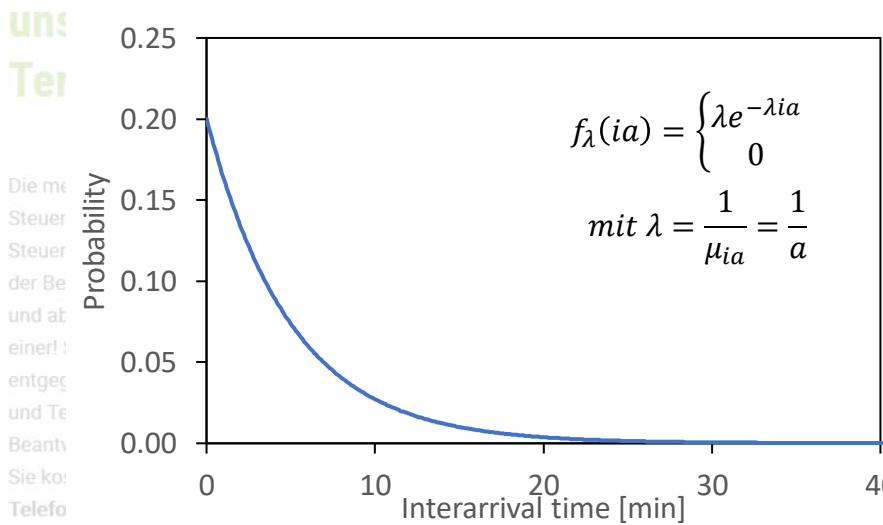
avg service time $p = 4 \text{ min}$

(uniformly distributed between 2 and 6 min)

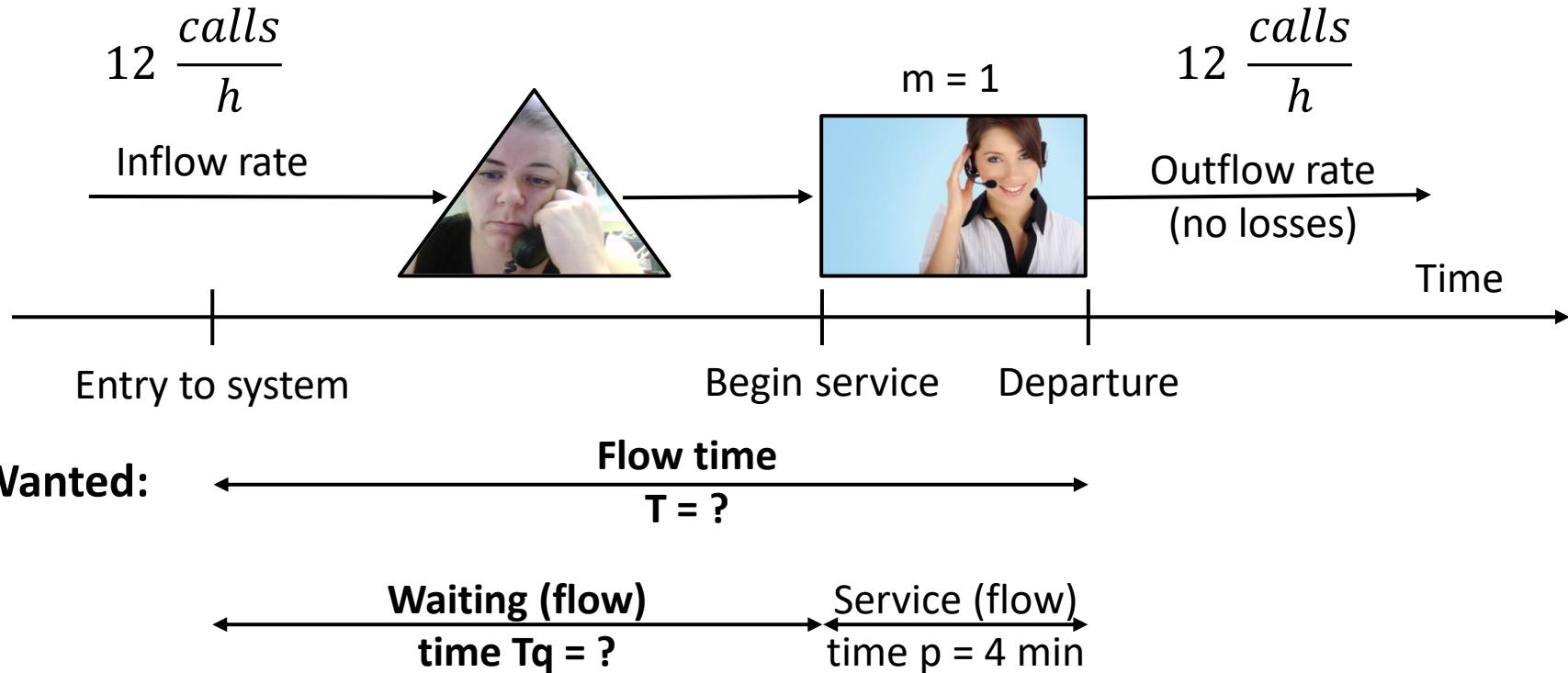
$$\text{avg arrival rate} = 12 \frac{\text{calls}}{\text{h}}$$

avg interarrival time $a =$

exponentially distributed interarrival times ia:



tel-inform Process Flow Diagram and Data (6:00 to 7:00)



One Server Waiting Time Formula

$$\text{Flow Time } T = T_q + p$$

We know p (service flow time), but we need the following waiting formula to calculate T_q :

$$T_q = p \cdot \left(\frac{\text{Utilization}}{1 - \text{Utilization}} \right) \cdot \left(\frac{CV_a^2 + CV_p^2}{2} \right)$$

CV_a^2 : Coefficient of variation of the interarrival time a

CV_p^2 : Coefficient of variation of the processing time p

$$\text{Coefficient of variation } CV = \frac{\text{Standard deviation}}{\text{Mean}}$$



Requirements of the Waiting Time Formula

- Constant average arrival rates
- Server utilization $< 100\%$
- Exponentially distributed interarrival times
- Arbitrarily distributed service times

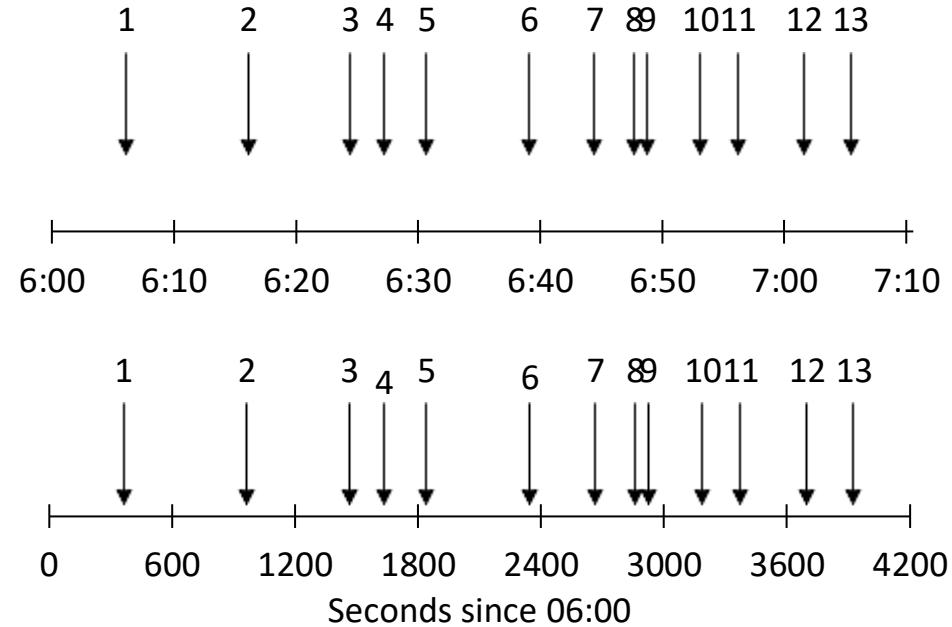


Interarrival Time vs Arrival Time

Call	Call arrival time [hh:mm:ss]	Interarrival time [s]
1	06:06:05	
2	06:16:06	00:10:01
3	06:24:27	00:08:21
4	06:27:15	00:02:48
5	06:30:42	00:03:27
6	06:39:05	00:08:23
7	06:44:27	00:05:22
8	06:47:42	00:03:15
9	06:48:46	00:01:04
10	06:53:08	00:04:23
11	06:56:13	00:03:05
12	07:01:38	00:05:25
13	07:05:27	00:03:48

Call

Call



a is the average interarrival time!



How to Determine CV_a at tel-inform?

- Way 1: Based on the empirical observation that interarrival times are typically exponentially distributed
 - For an exponential distribution, it can be shown that mean = standard deviation
 - Therefore, $CV_a = 1$
- Way 2: Based on data
 - Using sample mean \bar{x} as estimator of the population mean
 - Using sample standard deviation s as estimator of the population standard deviation

$$s = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2} \quad \text{or Excel function } STDEV.S$$

How to Determine CV_p at tel-inform?

- Way 1: Use prior knowledge about the probability distribution type and parameters
 - tel-inform service time p is uniformly distributed between 2 and 6 min

$$\mu = \frac{\text{upper bound} + \text{lower bound}}{2} =$$

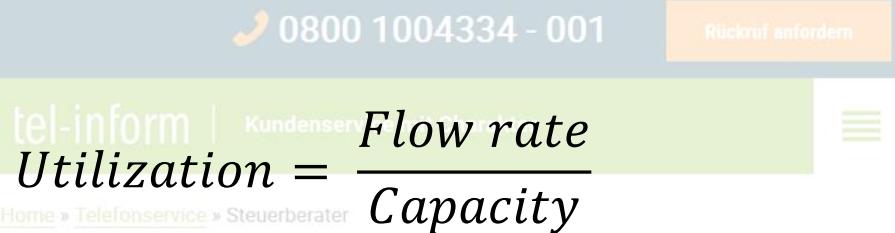
$$\sigma = \sqrt{\frac{(\text{upper bound} - \text{lower bound})^2}{12}} =$$

$$CV_p = \frac{\sigma}{\mu} =$$

- Way 2: Based on data (see way 2 for CV_a)

Utilization at tel-inform?

Utilization =



tel-inform | Kundenservice für Steuerberater

Utilization = $\frac{\text{Flow rate}}{\text{Capacity}}$

Home » Telefonservice » Steuerberater

Flow rate = $\min \left\{ \frac{\text{Demand rate},}{\text{Capacity}} \right\}$

Demand rate = $\frac{1}{a} \left[\frac{\text{call}}{\text{min}} \right] = \frac{[1\text{call}]}{[\text{min}]}$

Capacity = $\frac{1}{p} \left[\frac{\text{call}}{\text{min}} \right] = \frac{[1\text{call}]}{[\text{min}]}$

Die Minuten, die es dauert, dass der Beruf eines Steuerberaters lediglich aus dem Erstellen von Steuererklärungen und Bilanzen besteht. Dass der Beruf eines Steuerberaters viel komplexer und abwechslungsreicher ist, weiß kaum einer! Sie als Steuerberater müssen Anrufer entgegennehmen, Ihre Mandanten beraten und auf Kundenanfragen antworten. Beantwortung von Kundenanfragen verliert Sie kostbare Zeit, die Sie mit einem Telefonservice an Ihrer Seite effizienter nutzen können. Während Sie Ihre Termine

$$\text{Utilization} = \frac{\frac{1}{a}}{\frac{1}{p}} = \frac{p}{a}$$



Waiting time at tel-inform?

$$T_q = p \cdot \left(\frac{\text{Utilization}}{1 - \text{Utilization}} \right) \cdot \left(\frac{CV_a^2 + CV_p^2}{2} \right)$$

$$\text{Utilization} = 0.8000$$

$$CV_a = 1.000$$

$$CV_p = 0.2887$$

tel-inform Inbound Call Center Challenges

In a system with variability in customer arrivals and processing and with limited availability of past data,

- what is the (average) flow rate?
- what is the (average) flow time?
(waiting plus processing time)
- what is the (average) inventory?
(number of customers waiting and in service)

tel-inform call center data

1 employee Kundenservice mit Charakter

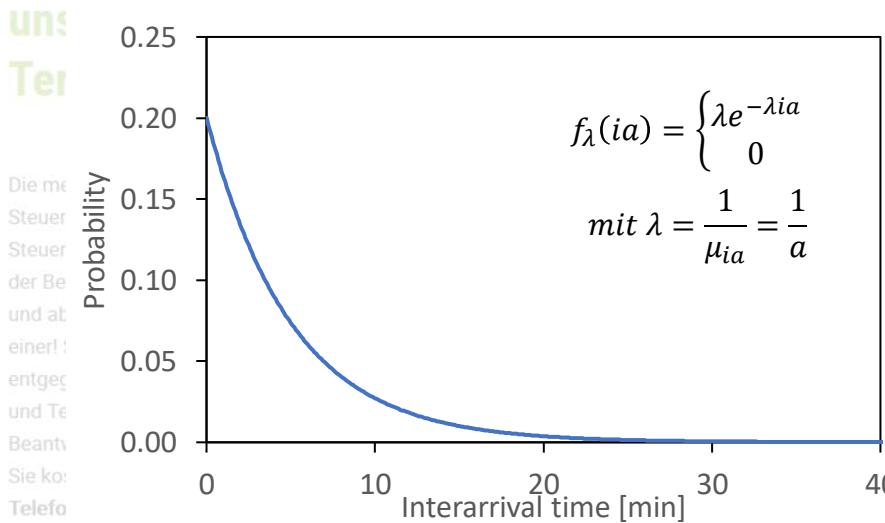
avg service time $p = 4 \text{ min}$

(uniformly distributed between 2 and 6 min)

$$\text{avg arrival rate} = 12 \frac{\text{calls}}{\text{h}}$$

avg interarrival time $a =$

exponentially distributed interarrival times ia:



Average Inventory

0800 1004334 - 001

Rückruf anfordern

Little's Law:
Inventory = $I_p = R \cdot p$

[Home](#) > [Telefonservice](#) > [Steuerberater](#)

$$\text{Inventory} = \text{Flow rate} \cdot \text{Flow time}$$

Inventory waiting in queue:

Telefonservice für
Steuerberater. Mit
uns sind Sie auch in
Terminen erreichbar



Inventory in service:

Die Steuerberater lediglich aus dem Erstellen von Steuererklärungen und Bilanzen besteht. Dass der Beruf eine Steuerberater viel komplexer und abwechslungsreicher ist, weiß kaum einer! Sie als Steuerberater müssen Anrufe entgegennehmen, Ihre Mandanten beraten und Termine wahrnehmen. Durch die Beantwortung von Kundenanfragen kostbare Zeit, die Sie mit einem

$$I_p = R \cdot p$$

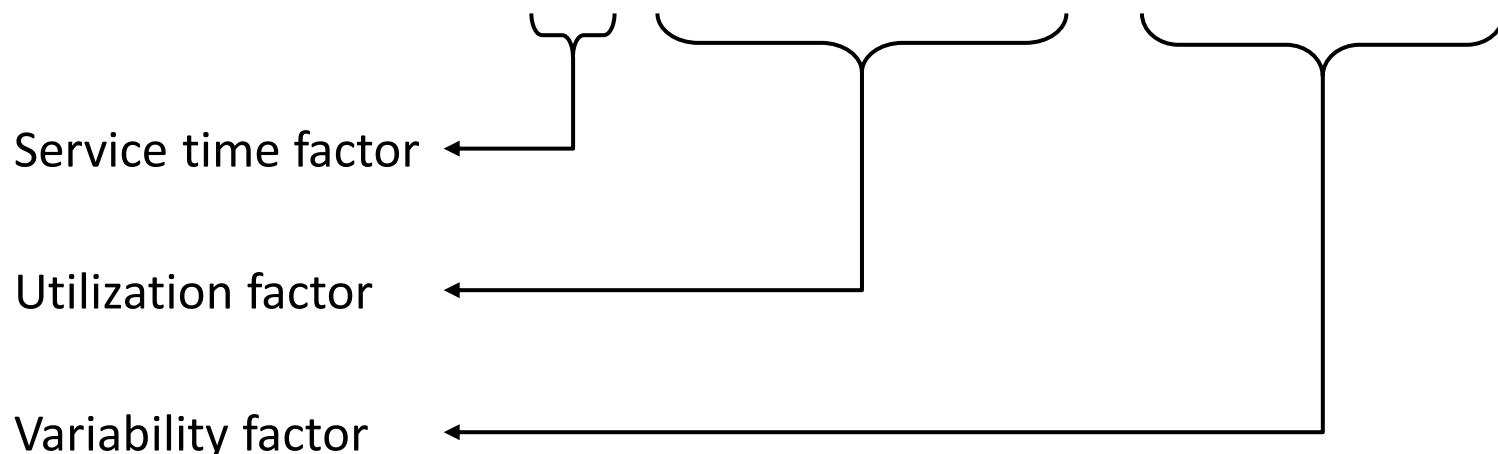
$$I_p = u \cdot \text{maximum calls in service}$$

nutzen können. Während Sie Ihre Termine

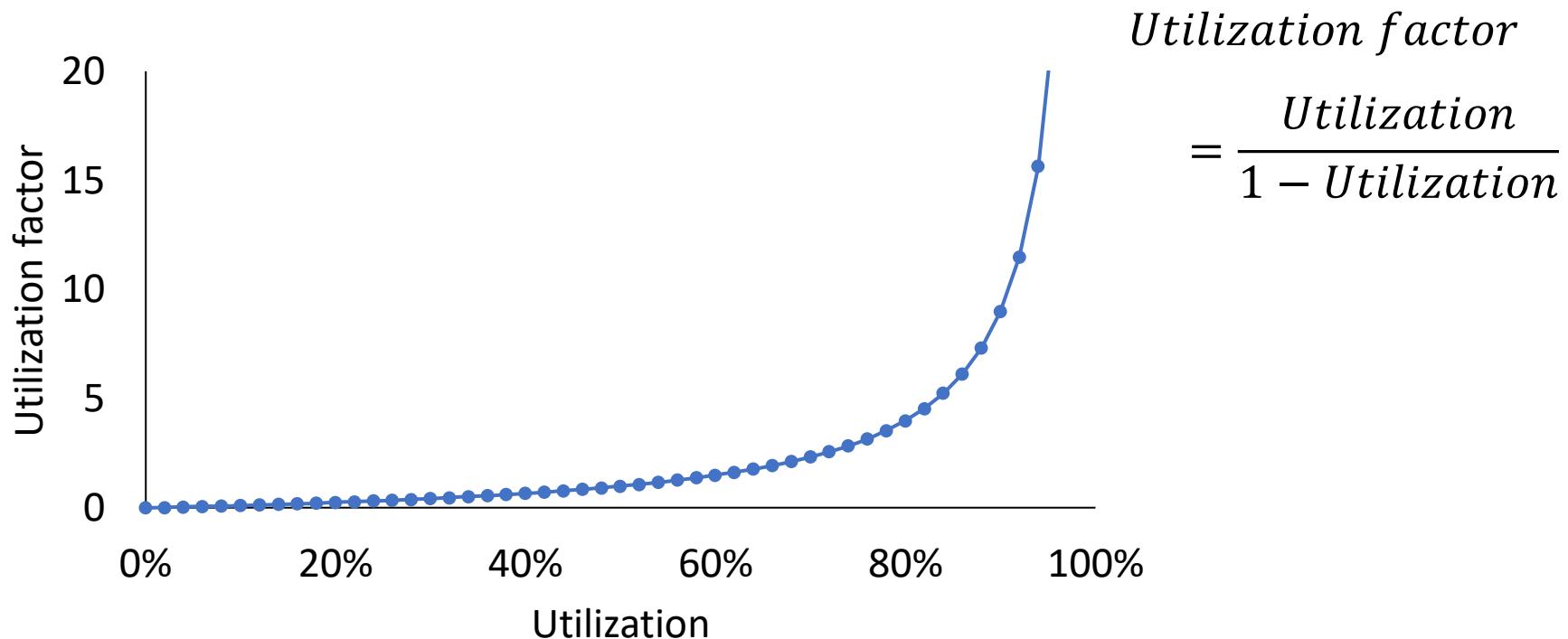


The Key Drivers of Waiting Time

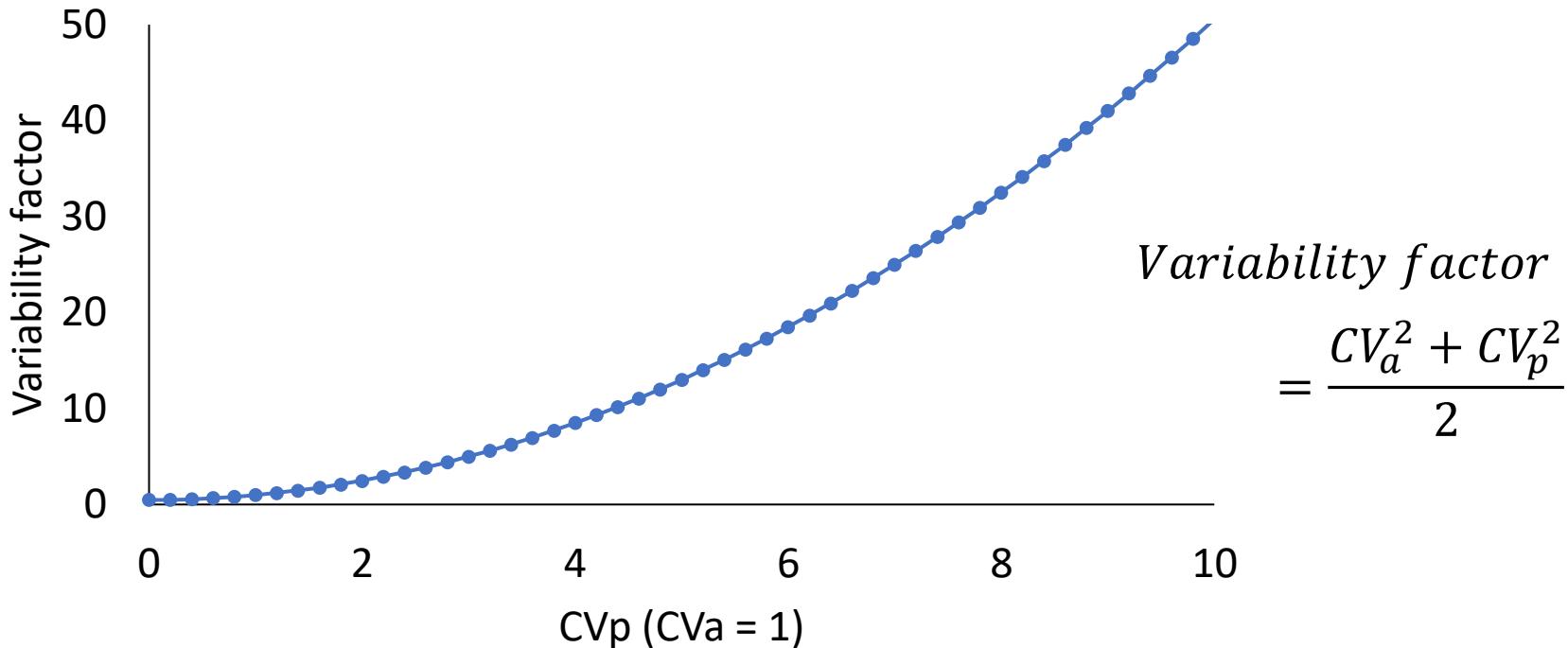
$$T_q = p \cdot \left(\frac{\text{Utilization}}{1 - \text{Utilization}} \right) \cdot \left(\frac{CV_a^2 + CV_p^2}{2} \right)$$



Utilization Factor as Function of Utilization



Variability Factor as Function of CVp



QUEUES WHEN DEMAND AND SUPPLY ARE VARIABLE - MULTIPLE SERVER



Fraport Terminal 2 Challenges

How to organize the self-service check-in machines?

- Multiple machines with multiple queues (one per machine)?
- Multiple machines with one common queue?

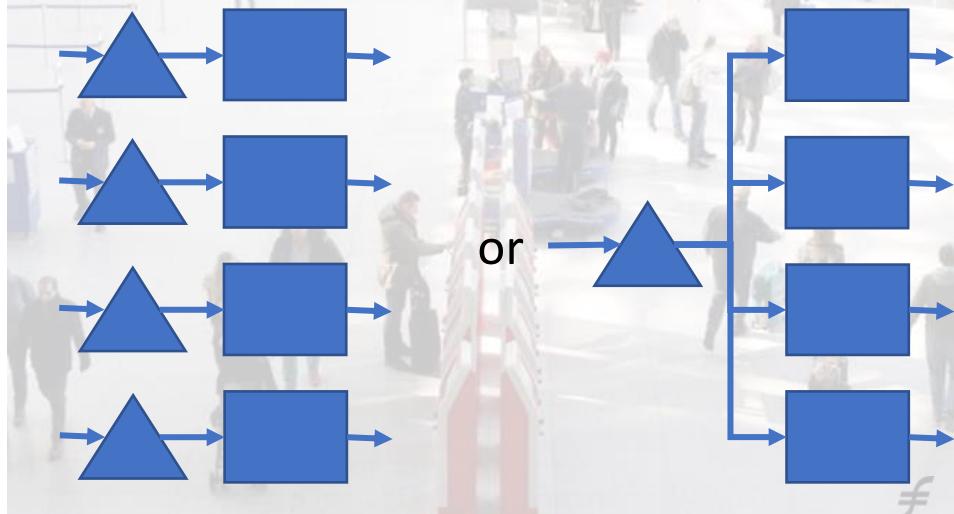


Fraport Terminal 2 Challenges

How to organize the self-service check-in machines?

- Multiple machines with multiple queues (one per machine)?
- Multiple machines with one common queue?

- 4 self-service check-in machines
- p is exponentially distributed with a mean of 1 minute per passenger
- a is exponentially distributed with a mean of 1/3 minutes



Self-service Check-in Machine Utilization

$$\text{Utilization} = \frac{p}{a \cdot m}$$

- 4 self-service check-in machines
- p is exponentially distributed with a mean of 1 minute per passenger
- a is exponentially distributed with a mean of 1/3 minutes
- Arriving passengers are split equally into four lines



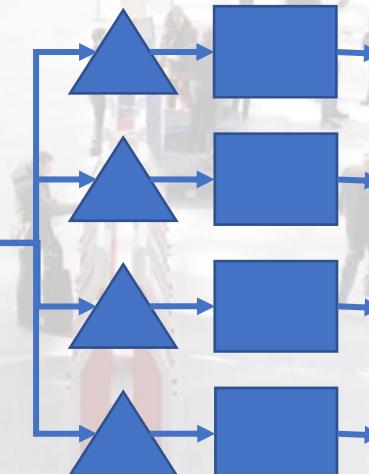
Waiting Time in One of the Four Check-in Lines

$$T_q = p \cdot \left(\frac{\text{Utilization}}{1 - \text{Utilization}} \right) \cdot \left(\frac{CV_a^2 + CV_p^2}{2} \right)$$

Number of Passengers Waiting in Line

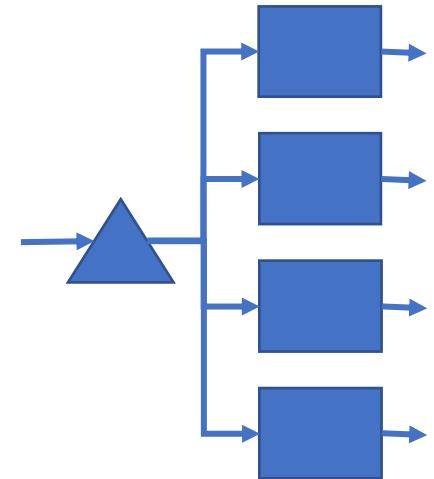
$$I_q = R \cdot T_q$$

- 4 self-service check-in machines
- p is exponentially distributed with a mean of 1 minute per passenger
- a is exponentially distributed with a mean of 1/3 minutes
- Arriving passengers are split equally into four lines



Waiting Time in the Shared Check-in Line

$$T_q = \frac{p}{m} \cdot \left(\frac{\text{Utilization}^{\sqrt{2(m+1)}-1}}{1 - \text{Utilization}} \right) \cdot \left(\frac{CV_a^2 + CV_p^2}{2} \right)$$



Number of Passengers Waiting in the One Line

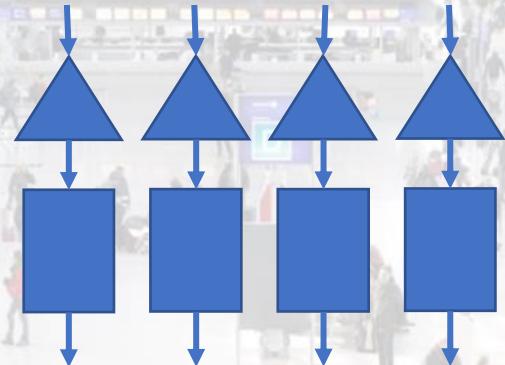
$$I_q = R \cdot T_q$$

- 4 self-service check-in machines
- p is exponentially distributed with a mean of 1 minute
- a is exponentially distributed with a mean of 1/3 minutes
- Arriving passengers are split equally into four lines



Comparison

Dedicated queue system



Utilization

75 %

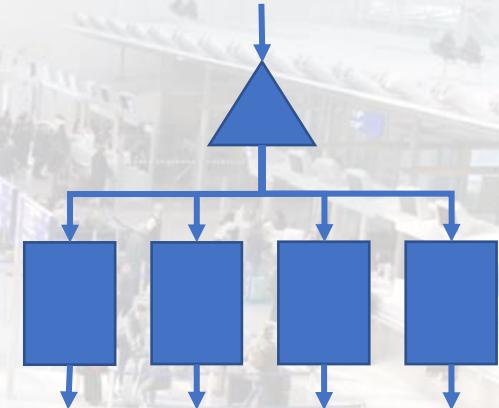
Waiting time
per passenger

3 min

Number of passengers
waiting in line(s)

$$4 \cdot 2.25 = 9 \text{ passengers}$$

Pooled queue system



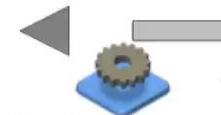
75 %

0.5368 min

1.6105 passengers

Fraport Check-In 4 check-in machines with separate queues

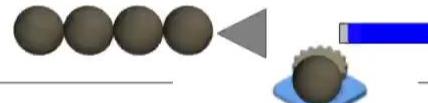
2022-Jan-01 02:36:26.112



107

Utilisation: 64.75 %

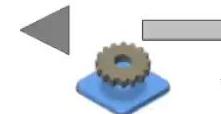
Average Wait Time: 1.5868 min



112

Utilisation: 69.98 %

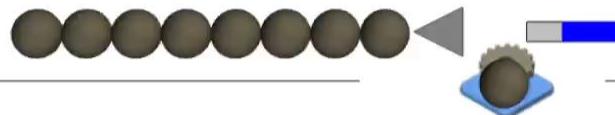
Average Wait Time: 2.0211 min



100

Utilisation: 61.20 %

Average Wait Time: 0.9198 min



138

Utilisation: 91.69 %

Average Wait Time: 6.2618 min

Average Wait Time: 2.6974 min

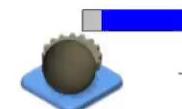
Average Utilisation: 71.90 %

Current Total Customers Waiting: 12

Average Total Customers Waiting: 9.07

Fraport Check-In 4 check-in machines with one shared queues

2022-Jan-01 02:29:38.752



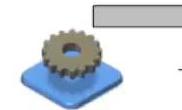
133

Utilisation: 81.63 %



117

Utilisation: 79.62 %



103

Utilisation: 71.75 %



97

Utilisation: 61.62 %

Average Wait Time: 0.3460 min

Average Utilisation: 73.65 %

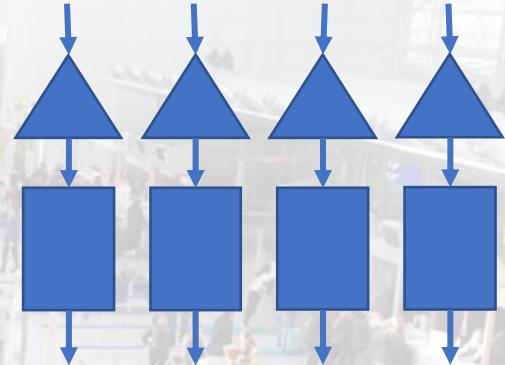
Current Total Customers Waiting: 0

Average Total Customers Waiting: 1.05

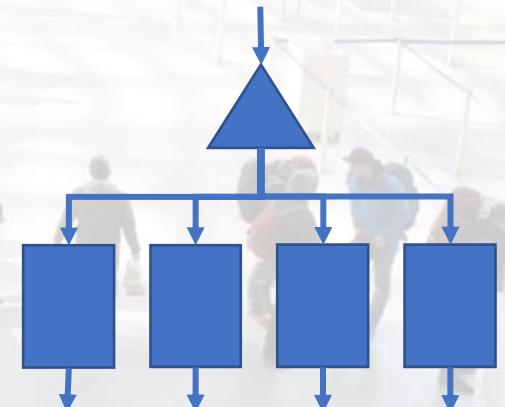
The Power of Pooling

- Servers do neither work more nor work faster. Utilization is the same.
- Pooled process uses available capacity more effectively.
- Pooling prevents the case that one resource is idle (e.g., no passengers at check-in machine 1) while the other faces a backlog of work (e.g., multiple passengers in queue at machine 2).
- Pooling identical resources balances the load for the servers, leading to shorter waiting times.

Dedicated queue system



Pooled queue system



Limitations to Pooling

- Pooling may require workers to have a broader set of skills, which may require more training and higher wages.
- Pooling may disrupt the customer – server relationship.
- Pooling may increase the time-in-queue for one customer class at the expense of another.
- Pooling may require investment and lead to higher fixed costs.

SYSTEM SIZE AND ECONOMIES OF SCALE



Fraport Terminal 2 Challenges

- What is the minimum number of machines that must be installed?
- How does the size of machine pools affect the waiting time?



What is the Minimum Number of Machines?

$$\text{Minimum number of servers} > \frac{p}{a}$$

- p is exponentially distributed with a mean of 1 minute
- a is exponentially distributed with a mean of 1/3 minutes
- If less than the minimum number of machines are available, number of passengers in the system could explode:

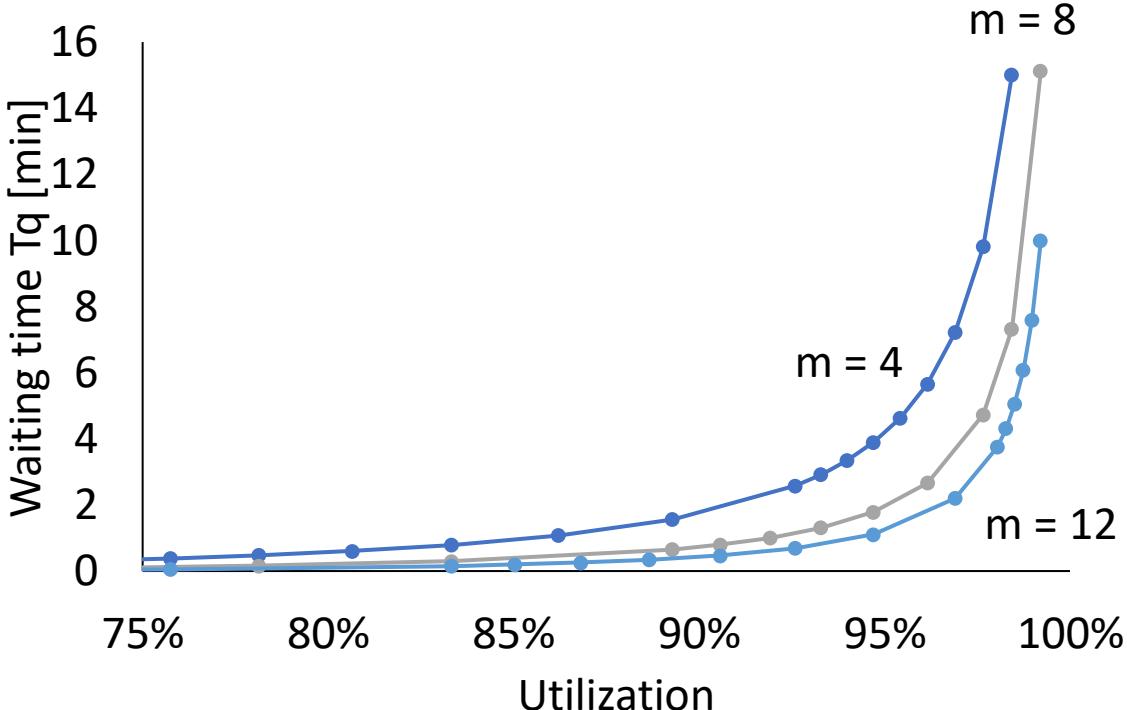


Fraport Terminal 2 Challenges

- What is the minimum number of machines that must be installed?
- How does the size of machine pools affect the waiting time?



Economies of Scale



The same applies:

The waiting time for customers increases progressively with the capacity utilisation of the check-in machines.

But:

The larger the pool of machines, the later the increase occurs and the less sensitive the waiting time reacts to an increase in occupancy.

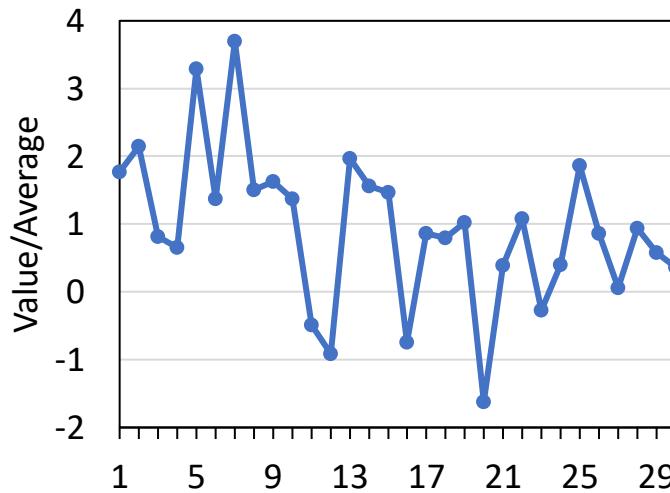
EXTRA



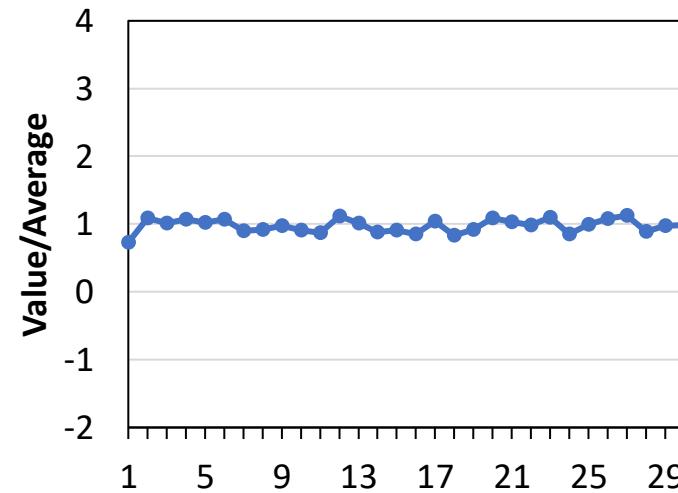
Coefficient of Variation – A Measure of Variability

$$\text{Coefficient of Variation } CV = \frac{\text{Standard Deviation}}{\text{Mean}}$$

$$CV = \frac{10}{10} = 1$$

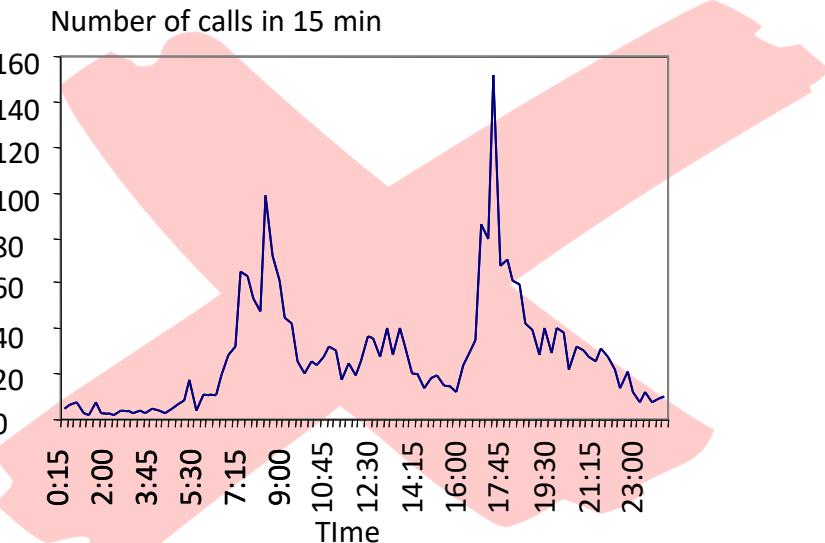
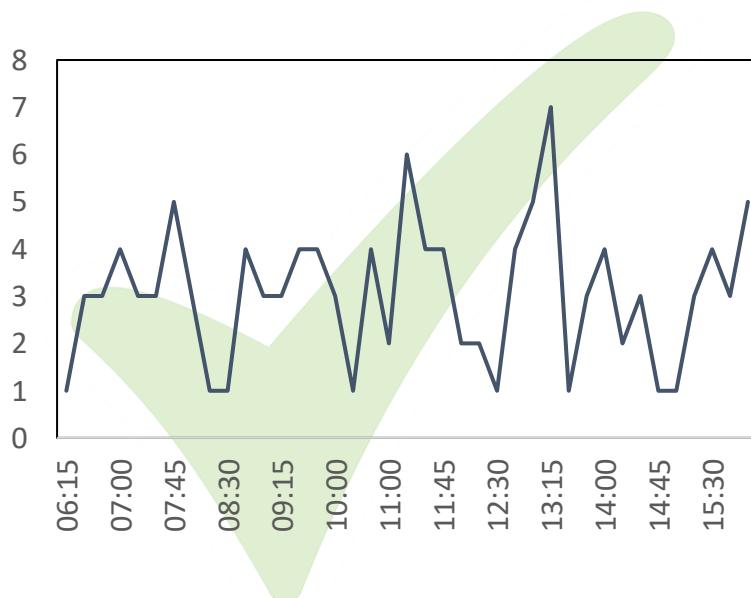


$$CV = \frac{10}{100} = 0.1$$



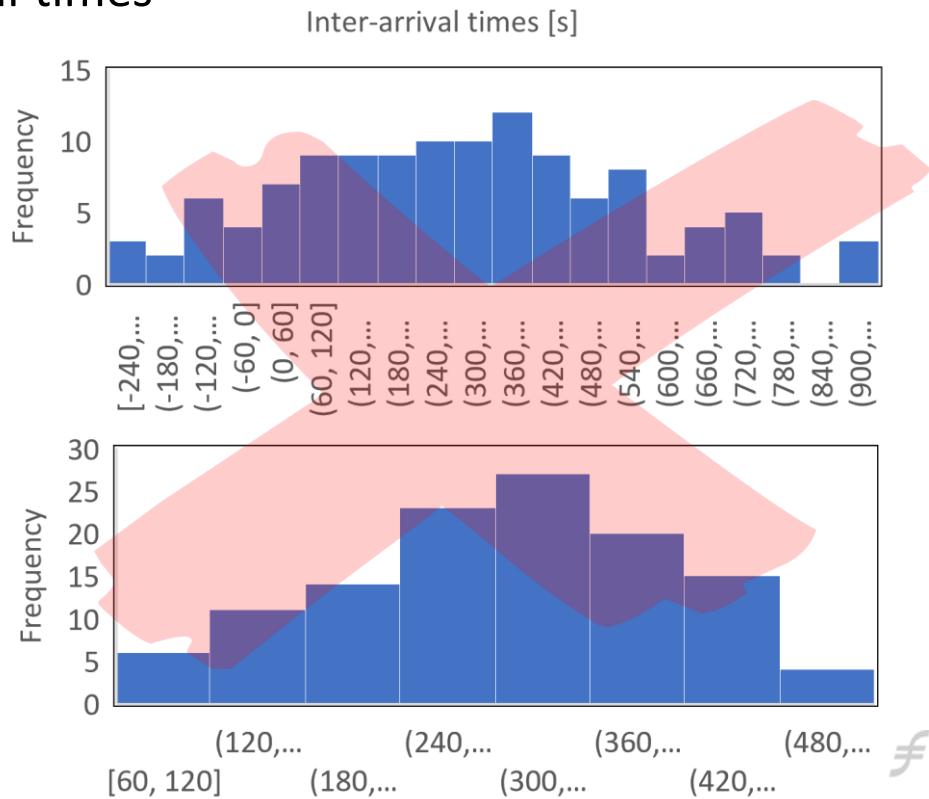
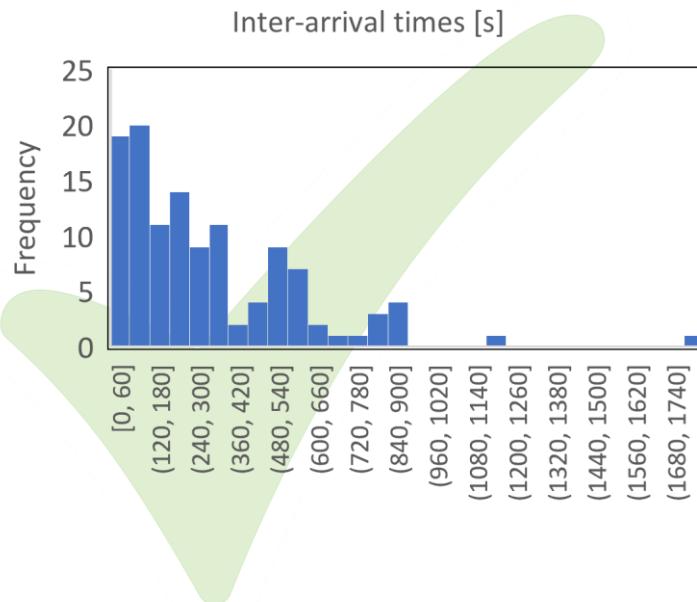
Prerequisites for the waiting time formula

1. Constant average arrival rates



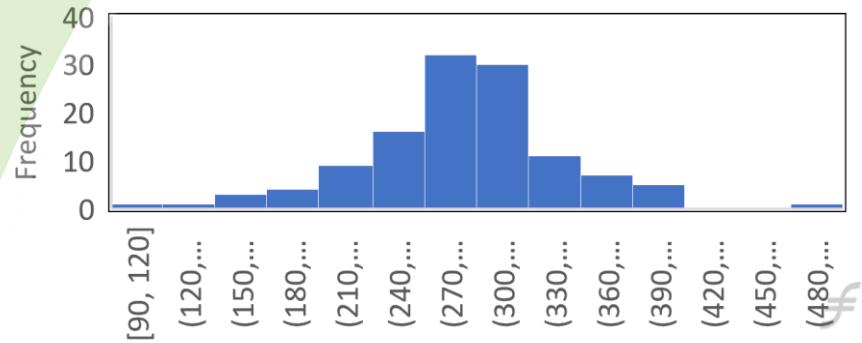
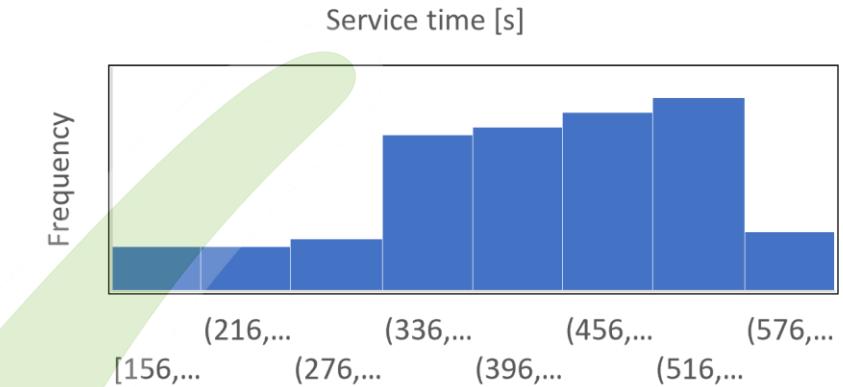
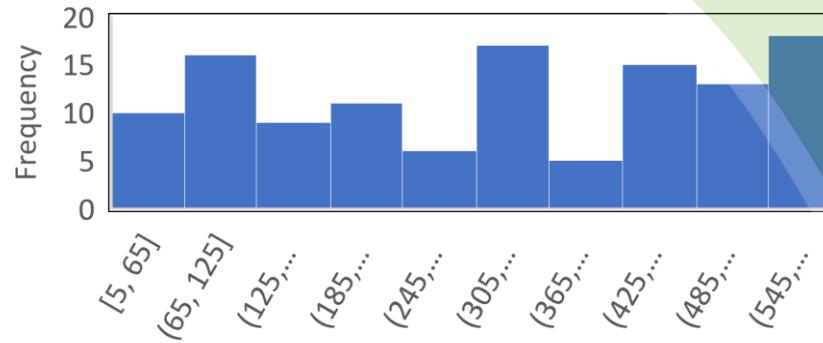
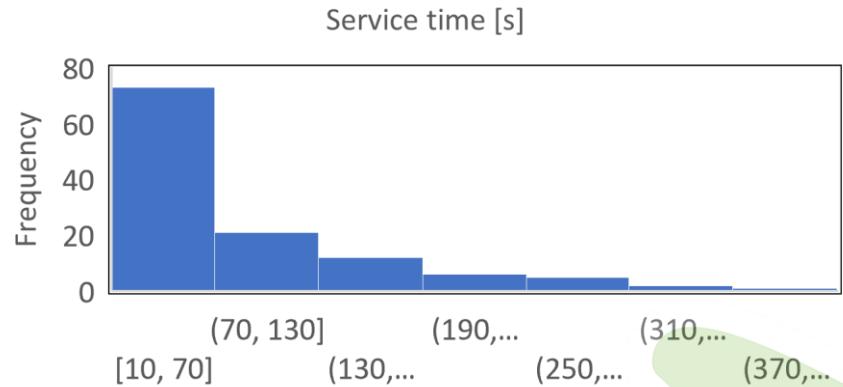
Prerequisites for the waiting time formula

2. Exponentially distributed inter-arrival times



Prerequisites for the waiting time formula

3. Arbitrarily distributed operating/service times





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Frankfurt School

2023 Operations Management

SERVICE SYSTEMS WITH IMPATIENT CUSTOMERS



LOST DEMAND IN QUEUES WITH NO BUFFERS



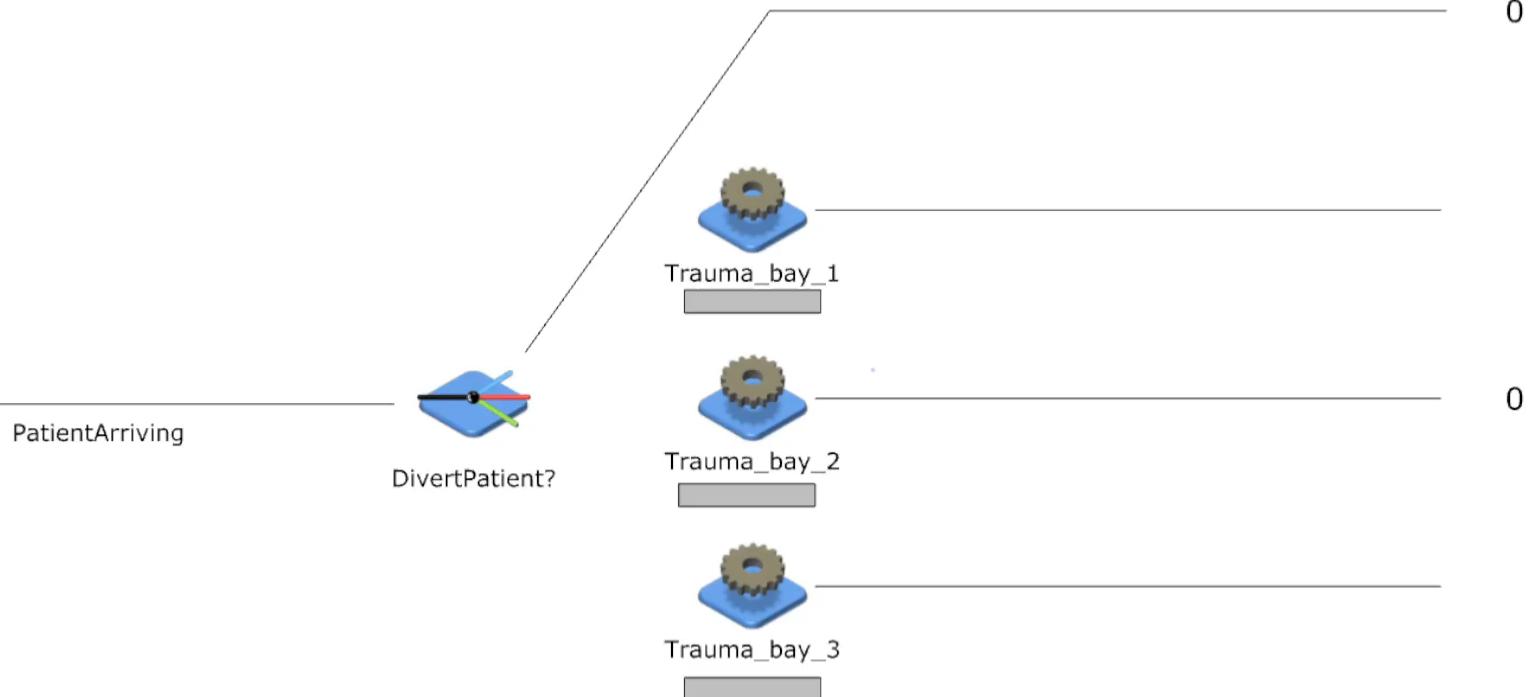
Trauma Center

- Hospital equipped and staffed to provide care for patients suffering from major traumatic injuries such as falls, motor vehicle collisions, or gunshot wounds
- Full range of specialists and equipment available 24 hours a day^[1]
- Operation of a trauma center is extremely expensive



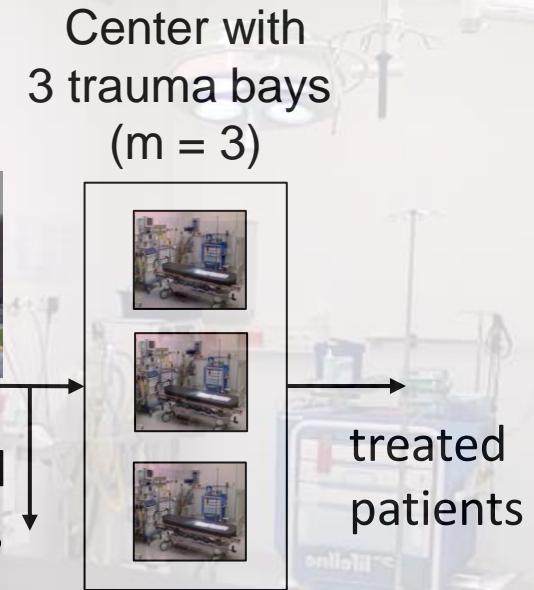
Trauma Center

01-Jan-2022 08:00:00



Trauma Center Challenge

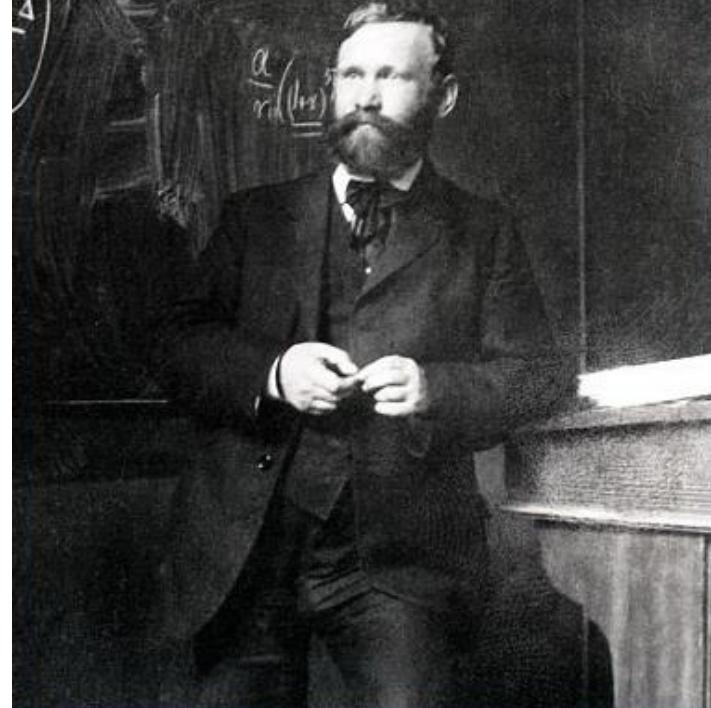
- What is the probability that all trauma bays are utilized?
- How many trauma patients need to be diverted?
- What is the utilization of the trauma bays?
- How many trauma bays are needed to guarantee a diversion probability of less than 1%?



- Demand:
One trauma case comes in every 3 hours.
- Processing:
Patient stays in trauma bay for an average of 2 hours.

The Erlang Loss Model

- There are m servers.
- Customers are processed by a single server.
- The average interarrival time is a . The coefficient of variation CV_a is 1. (Interarrival times are exponentially distributed.)
- The average processing time is p for any distribution..
- Customers do not wait. At most m customers are in the system.



Agner Krarup Erlang (1878 –1929) was a Danish mathematician, statistician and engineer, who invented the fields of traffic engineering and queueing theory.

Capacity and Implied Utilization

Capacity

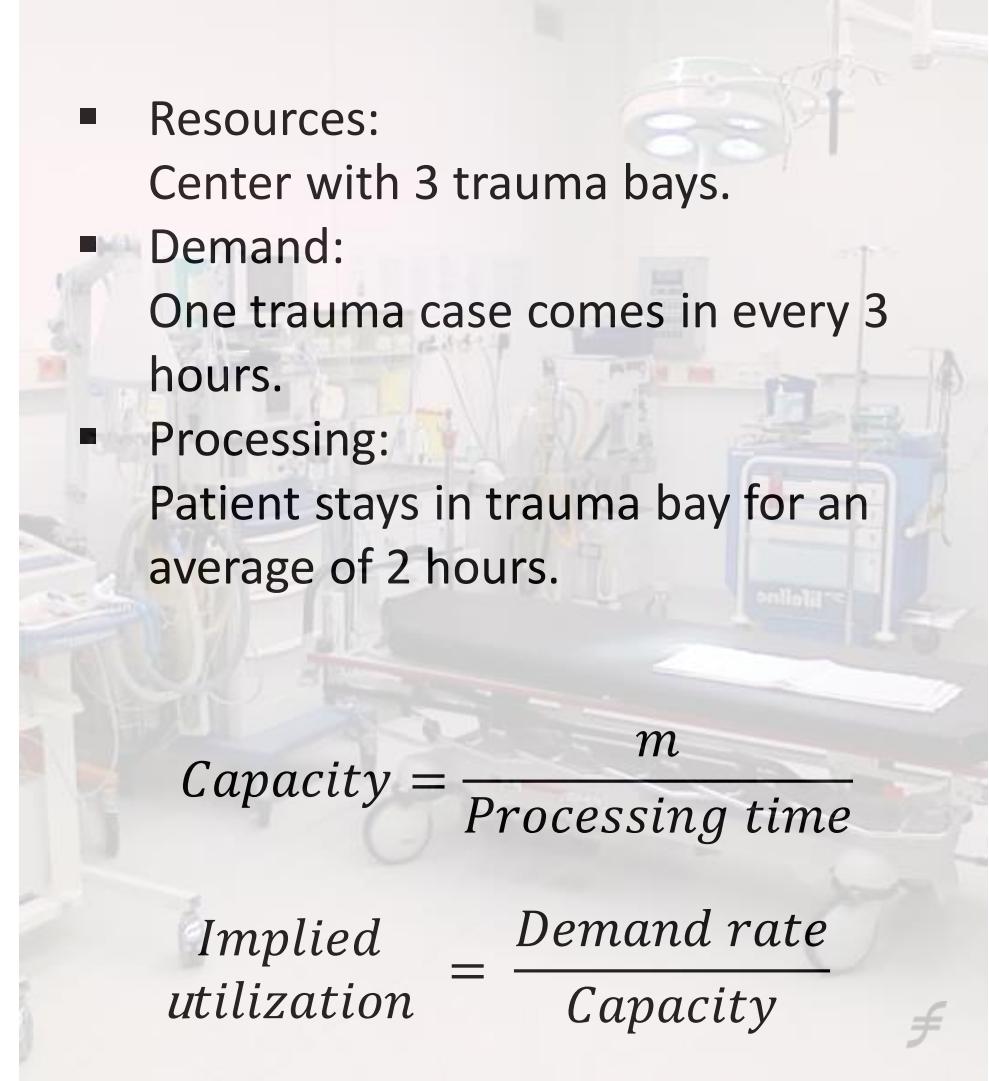
Demand rate

Implied utilization

- Resources:
Center with 3 trauma bays.
- Demand:
One trauma case comes in every 3 hours.
- Processing:
Patient stays in trauma bay for an average of 2 hours.

$$\text{Capacity} = \frac{m}{\text{Processing time}}$$

$$\text{Implied utilization} = \frac{\text{Demand rate}}{\text{Capacity}}$$



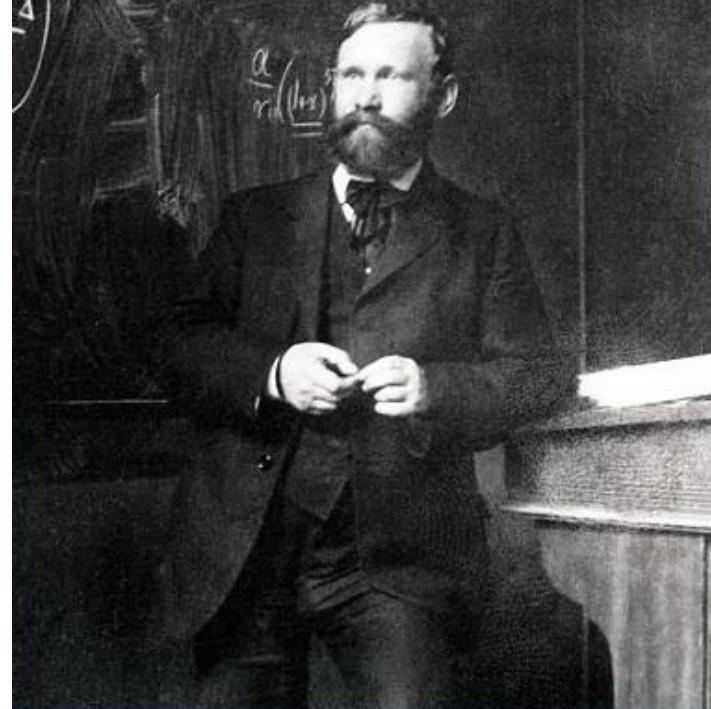
The Erlang Loss Formula

The probability that all servers are busy is given by the following „nasty“ formula:

$$P_m(r) = \frac{\frac{r^m}{m!}}{1 + \frac{r^1}{1!} + \frac{r^2}{2!} + \dots + \frac{r^m}{m!}}$$

r is the so-called **offered load** – a measure of how much demand is loaded on the system relative to the capacity of one server:

$$r = \frac{p}{a}$$



Agner Krarup Erlang (1878 –1929) was a Danish mathematician, statistician and engineer, who invented the fields of traffic engineering and queueing theory.

Using the Erlang Loss Table

Offered load:

$$r = \frac{p}{a} =$$

$$P_m(r) = P(\) =$$

Erlang Loss Table

$r = p/a$	m						
	1	2	3	4	5	6	7
0.10	0.0909	0.0045	0.0002	0.0000	0.0000	0.0000	0.0000
0.20	0.1667	0.0164	0.0011	0.0001	0.0000	0.0000	0.0000
0.25	0.2000	0.0244	0.0020	0.0001	0.0000	0.0000	0.0000
0.30	0.2308	0.0335	0.0033	0.0003	0.0000	0.0000	0.0000
0.33	0.2481	0.0393	0.0043	0.0004	0.0000	0.0000	0.0000
0.40	0.2857	0.0541	0.0072	0.0007	0.0001	0.0000	0.0000
0.50	0.3333	0.0769	0.0127	0.0016	0.0002	0.0000	0.0000
0.60	0.3750	0.1011	0.0198	0.0030	0.0004	0.0000	0.0000
0.67	0.4012	0.1185	0.0258	0.0043	0.0006	0.0001	0.0000
0.70	0.4118	0.1260	0.0286	0.0050	0.0007	0.0001	0.0000
0.75	0.4286	0.1385	0.0335	0.0062	0.0009	0.0001	0.0000

Offered Load vs Implied Utilization

- In common:
both measures give a sense of the amount of demand relative to the capacity of the system
- In contrast:
 - Offered load **does not** account for the number of servers in a system
 - Implied utilization does consider the actual number of servers

$$r = u \cdot m = \frac{2}{9} \cdot 3 = \frac{2}{3}$$

$$r = \frac{p}{a} = \frac{2h}{3h} = \frac{2}{3}$$

Implied utilization

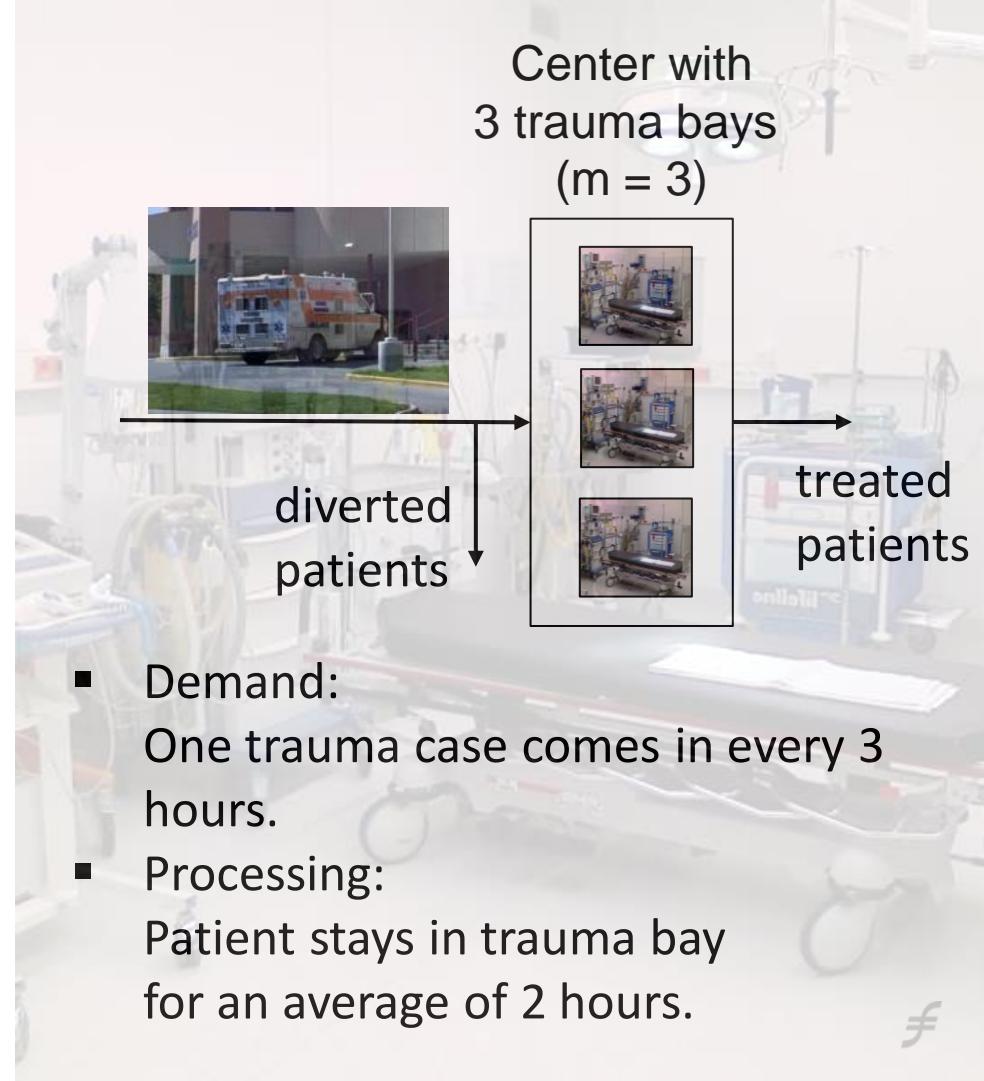
$$= \frac{\text{Demand rate}}{\text{Capacity}}$$

$$= \frac{\frac{1}{a}}{\frac{m}{p}} = \frac{p}{m \cdot a} = \frac{2}{3 \cdot 3}$$



Trauma Center Challenge

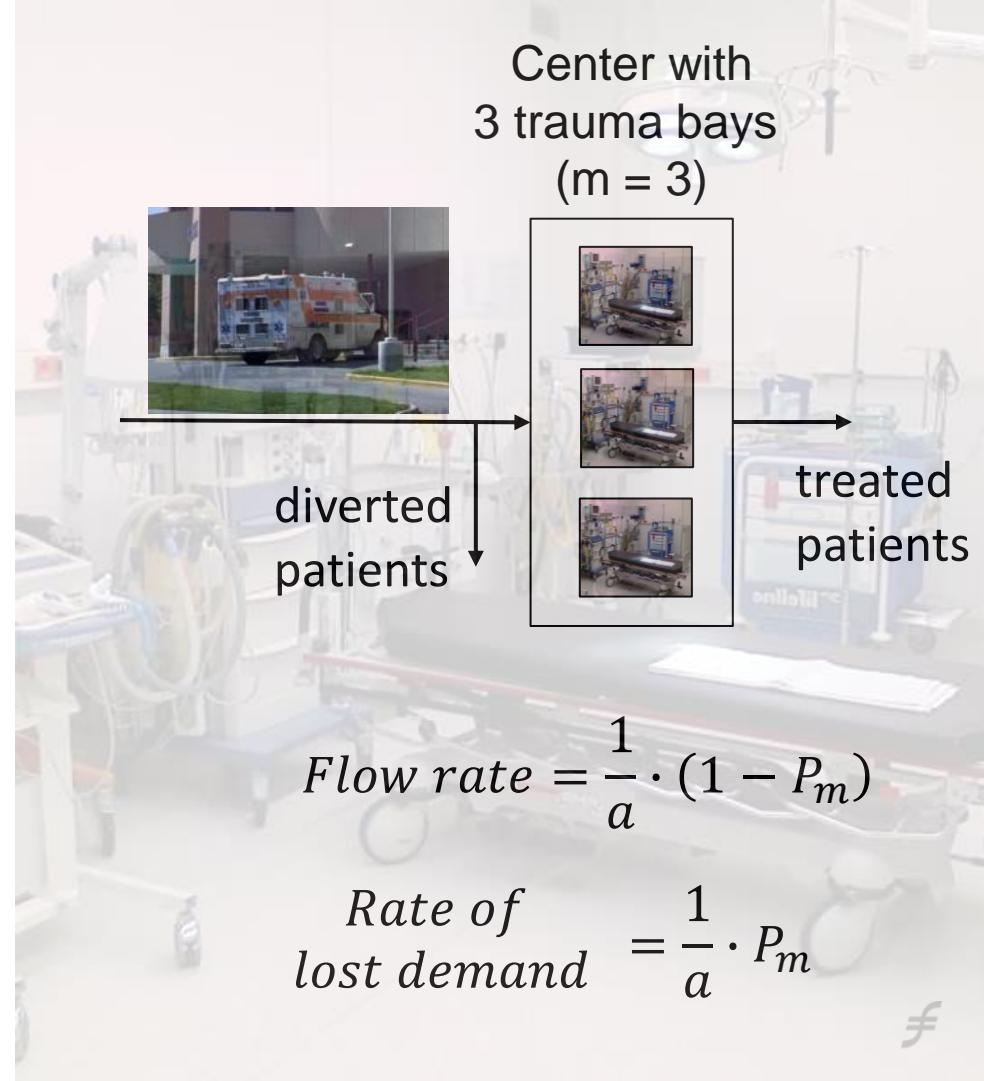
- What is the probability that all trauma bays are utilized?
- **How many trauma patients need to be diverted?**
- What is the utilization of the trauma bays?
- How many trauma bays are needed to guarantee a diversion probability of less than 1%?



Rate of Diverted Patients

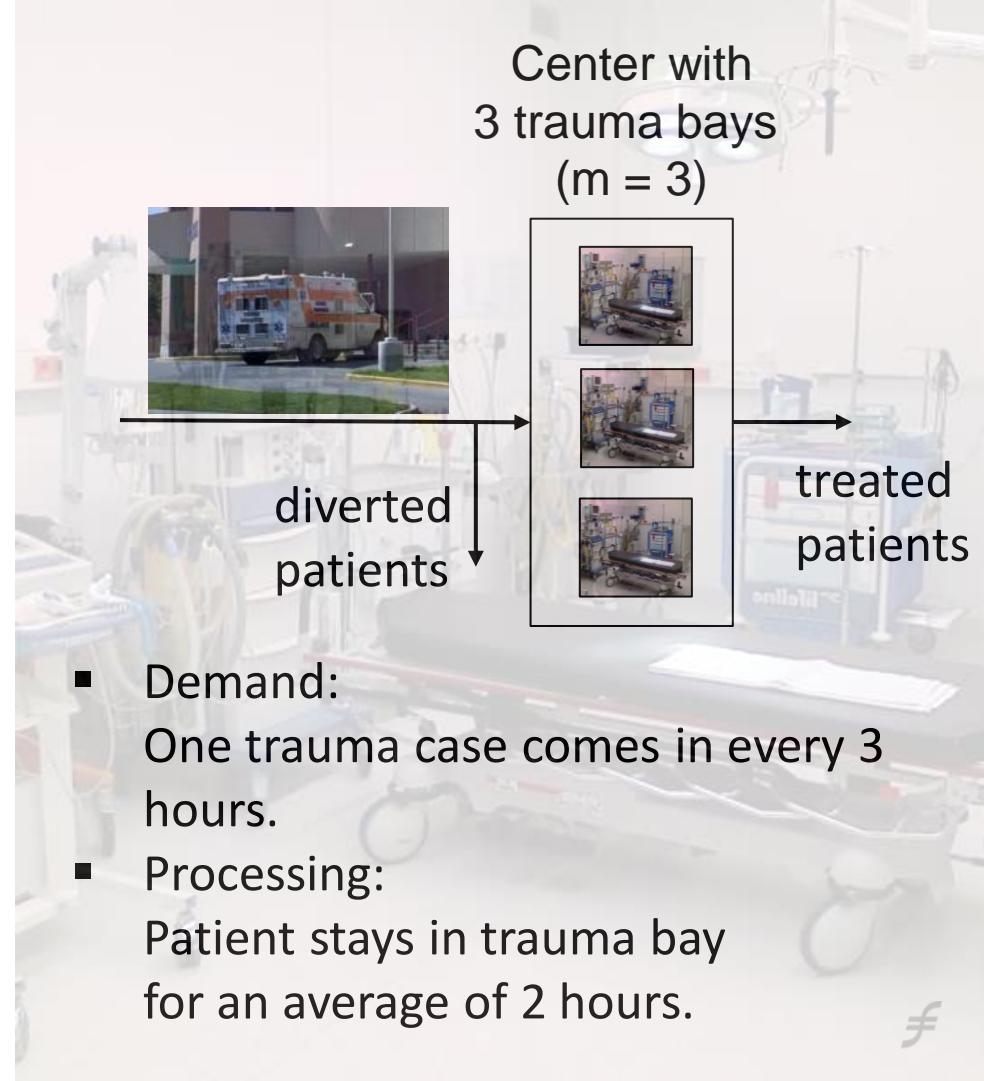
Rate of lost demand

Flow rate



Trauma Center Challenge

- What is the probability that all trauma bays are utilized?
- How many trauma patients need to be diverted?
- **What is the utilization of the trauma bays?**
- How many trauma bays are needed to guarantee a diversion probability of less than 1%?



- Demand:
One trauma case comes in every 3 hours.
- Processing:
Patient stays in trauma bay for an average of 2 hours.

Utilization of the Trauma Bays

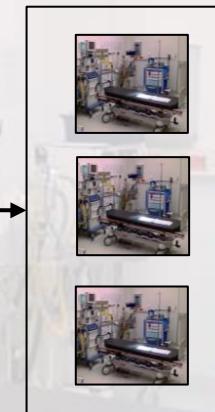
Utilization

Average number of occupied servers

Center with
3 trauma bays
($m = 3$)



diverted patients



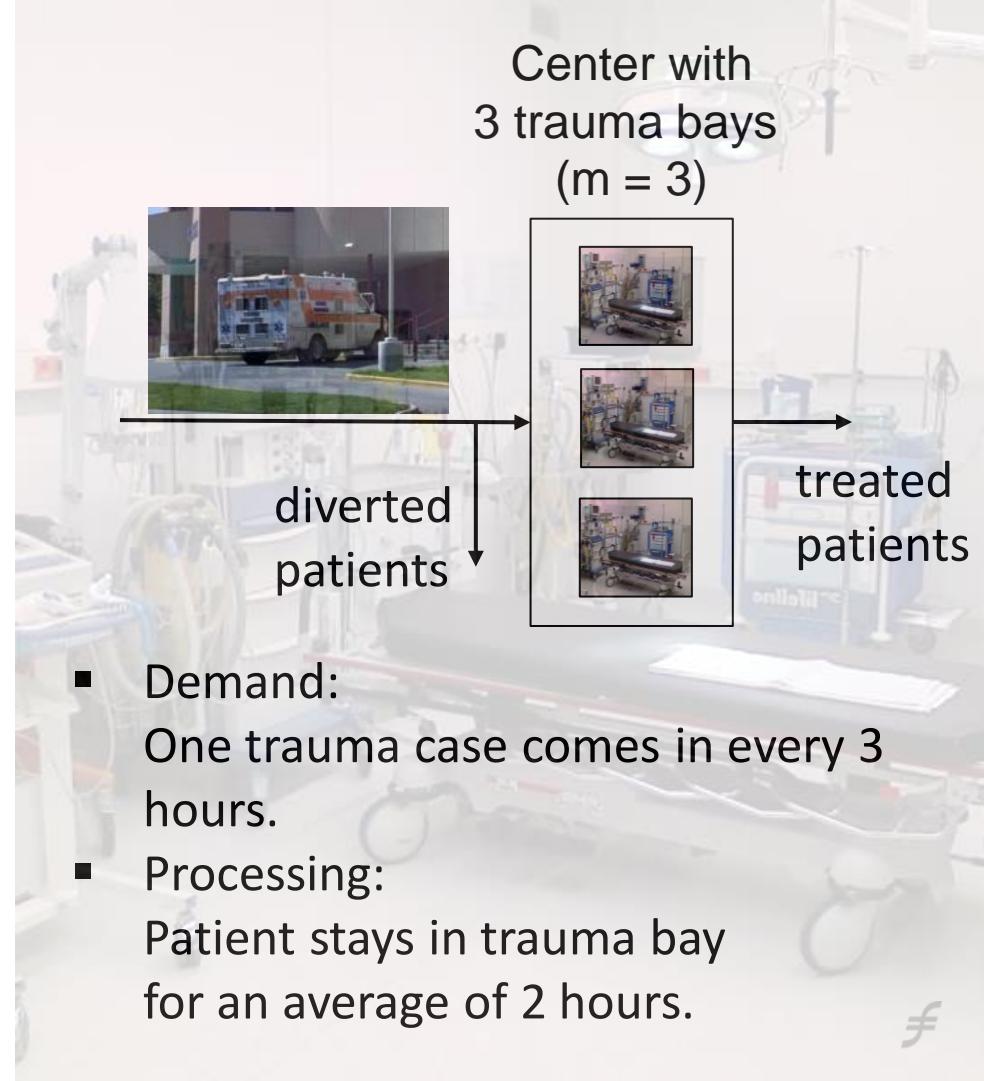
treated patients

$$\text{Utilization} = \frac{\text{Flow rate}}{\text{Capacity}}$$

$$= \frac{\text{Processing time}}{m} \cdot \text{Flow rate}$$

Trauma Center Challenge

- What is the probability that all trauma bays are utilized?
- How many trauma patients need to be diverted?
- What is the utilization of the trauma bays?
- **How many trauma bays are needed to guarantee a diversion probability of less than 1%?**



Trauma Bays Needed

We begin with the offered load:

$$r = \frac{p}{a} = \frac{2h}{3h} = \frac{2}{3}$$

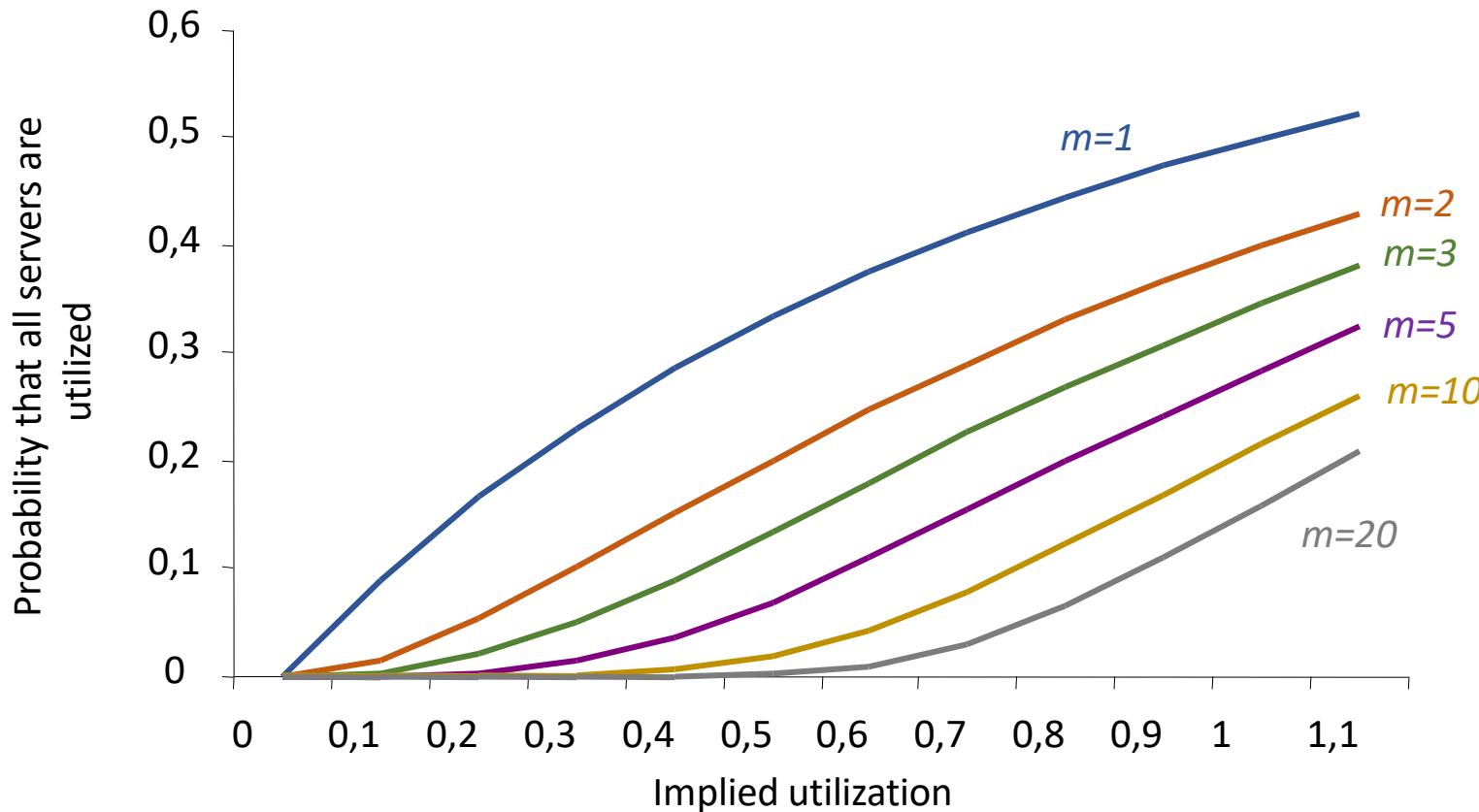
Erlang Loss Table

$r = p/a$	m						
	1	2	3	4	5	6	7
0.10	0.0909	0.0045	0.0002	0.0000	0.0000	0.0000	0.0000
0.20	0.1667	0.0164	0.0011	0.0001	0.0000	0.0000	0.0000
0.25	0.2000	0.0244	0.0020	0.0001	0.0000	0.0000	0.0000
0.30	0.2308	0.0335	0.0033	0.0003	0.0000	0.0000	0.0000
0.33	0.2481	0.0393	0.0043	0.0004	0.0000	0.0000	0.0000
0.40	0.2857	0.0541	0.0072	0.0007	0.0001	0.0000	0.0000
0.50	0.3333	0.0769	0.0127	0.0016	0.0002	0.0000	0.0000
0.60	0.3750	0.1011	0.0198	0.0030	0.0004	0.0000	0.0000
0.67	0.4012	0.1185	0.0258	0.0043	0.0006	0.0001	0.0000
0.70	0.4118	0.1260	0.0286	0.0050	0.0007	0.0001	0.0000
0.75	0.4286	0.1385	0.0335	0.0062	0.0009	0.0001	0.0000

MANAGING A QUEUE WITH IMPATIENT CUSTOMERS



Economies of Scale



A Second Perspective on Economies of Scale

Offered Load, r	Number of Servers, m	P_m	Implied Utilization	Utilization
1	4	0.015	25%	25%
5	10	0.018	50%	49%
10	17	0.013	59%	58%
25	34	0.016	74%	72%
50	61	0.017	82%	81%
100	113	0.018	88%	87%

Number of servers required for different offered loads to achieve a probability that all servers are busy that is no more than 0.02

Pooling of Erlang Loss Systems

	A	B	A+B = C
a	0.20	0.20	0.10
p	0.15	0.15	0.15
m	2	2	4
Demand rate = 1/a	5	5	10
Capacity = m/p	13.33	13.33	26.67
Implied utilization = p/(a x m)	37.5%	37.5%	37.5%
Offered load, r = p/a	0.75	0.75	1.5
P_m	0.1385	0.1385	0.048
Flow rate = $(1- P_m)/a$	4.31	4.31	9.52
Utilization = Flow rate/Capacity	32%	32%	36%

The performance of two separate Erlang loss systems, A and B, compared to the performance of the pooled system that combines the two together



The Performance of Different Levels of Pooling

	Number of Independent Systems				
	16	8	4	2	1
a	0.2000	0.1000	0.0500	0.0250	0.0125
p	0.15	0.15	0.15	0.15	0.15
m	2	4	8	16	32
Demand rate = 1/a	5	10	20	40	80
Capacity = m/p	13.33	26.67	53.33	106.67	213.33
Implied utilization = p/(a x m)	37.5%	37.5%	37.5%	37.5%	37.5%
Offered load, r = p/a	0.75	1.5	3	6	12
P _m	0.1385	0.048	0.0081	0.0003	0.0000
Flow rate = (1- P _m)/a	4.31	9.52	19.84	39.99	80.0
Utilization = Flow rate/Capacity	32.3%	35.7%	37.2%	37.5%	37.5%

Note: The column header is the number of independent systems. For example, the first column has 16 independent systems, while the second column describes pooling pairs of systems to create 8 independent systems. In the last column there is only one system with then 32 resources.



Trauma Center with 2 Buffers

01-Jan-2022 10:05:52



0



DivertPatient?



Trauma_bay_1



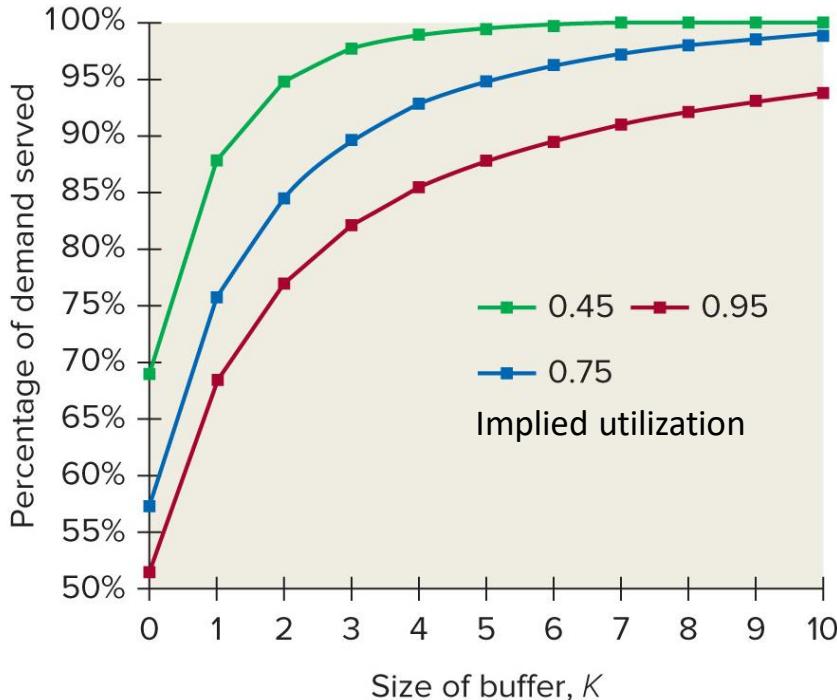
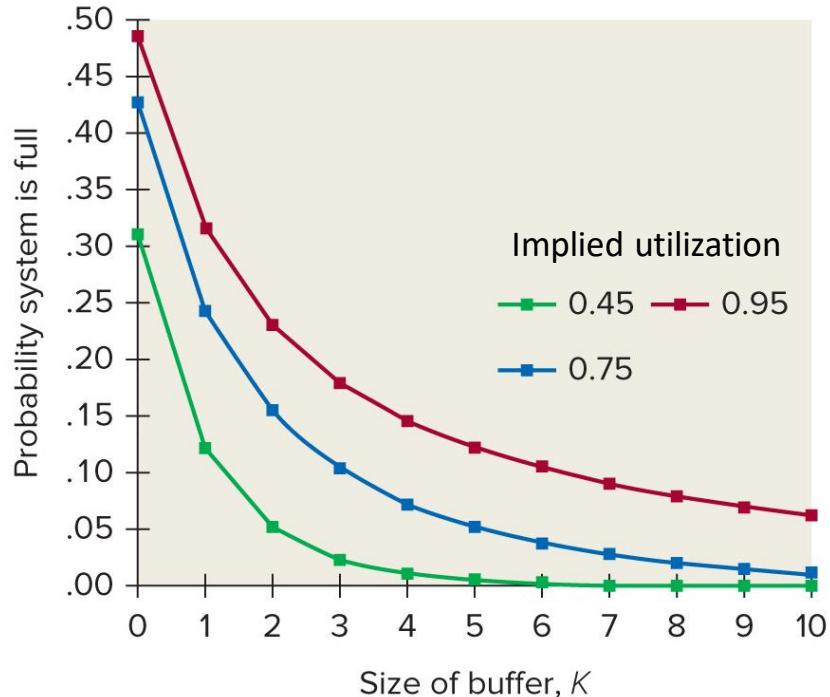
Trauma_bay_2



Trauma_bay_3

21

Impact of Increasing the Buffer Size

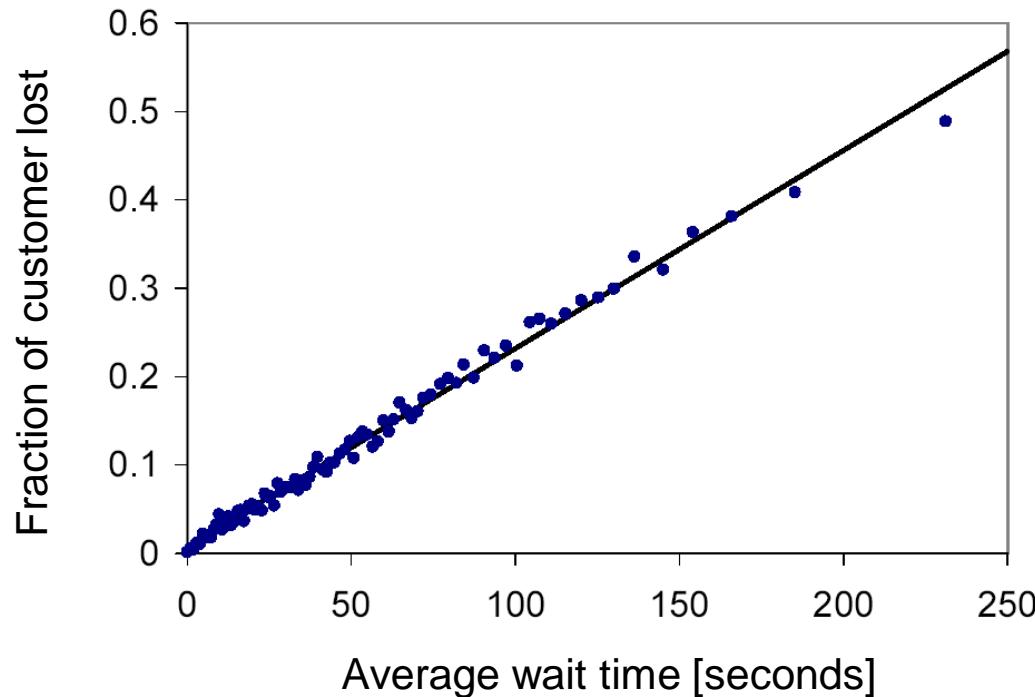


Issues with Buffers

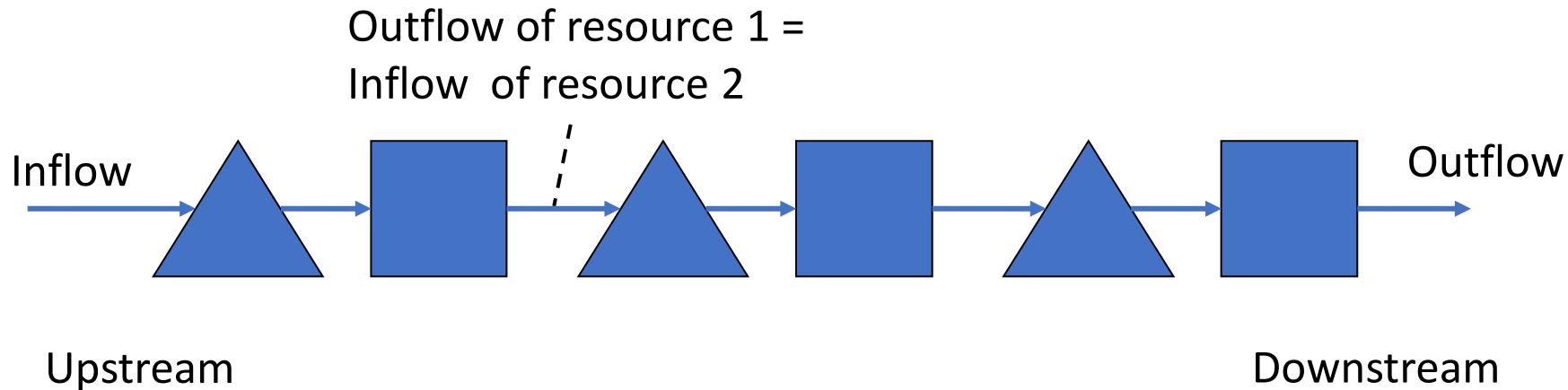
- Some customers might not be able to wait
- Balking: Customers choose not to join a queue
- Reneging/abandoning: Customers choose to leave the queue after joining it



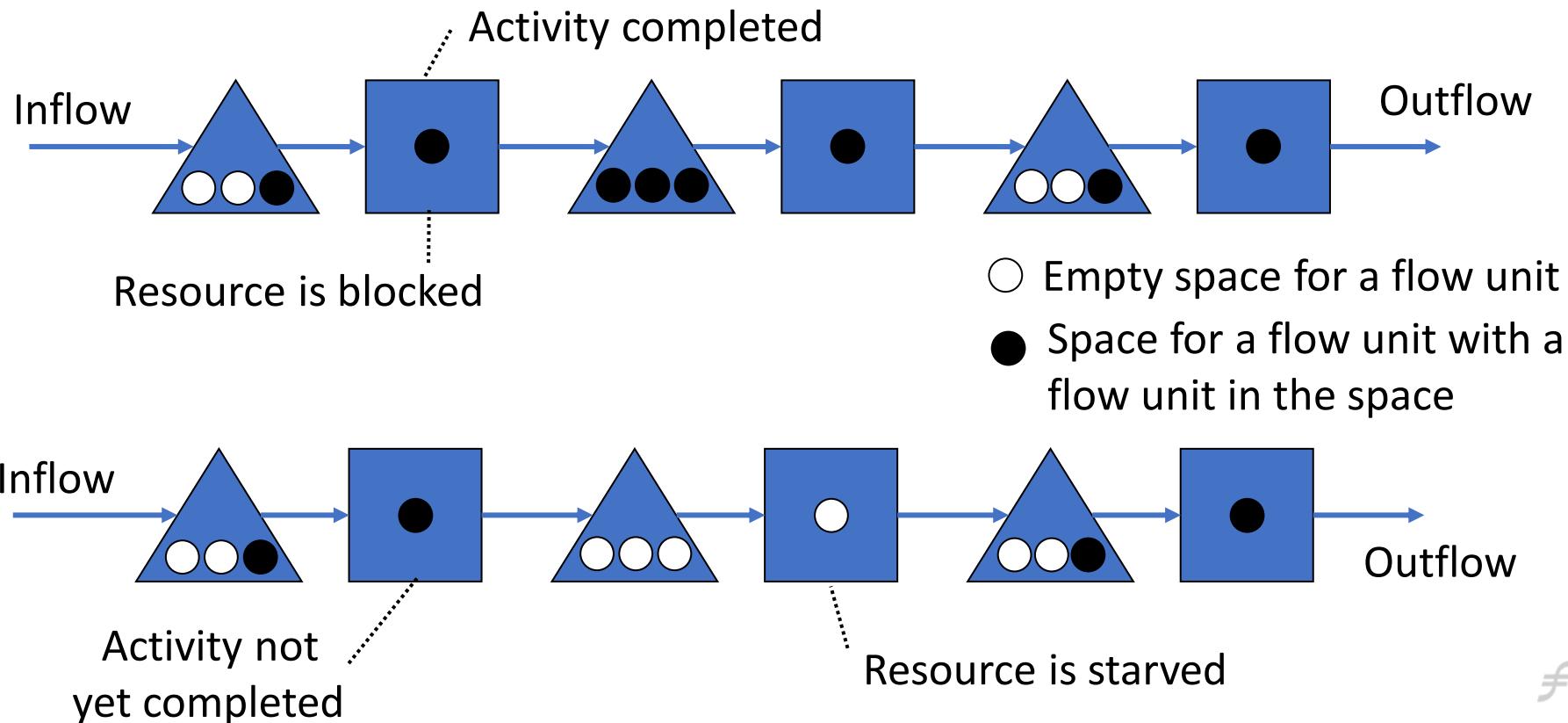
Impact of Waiting Time on Customer Loss



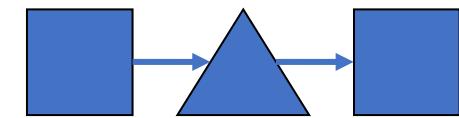
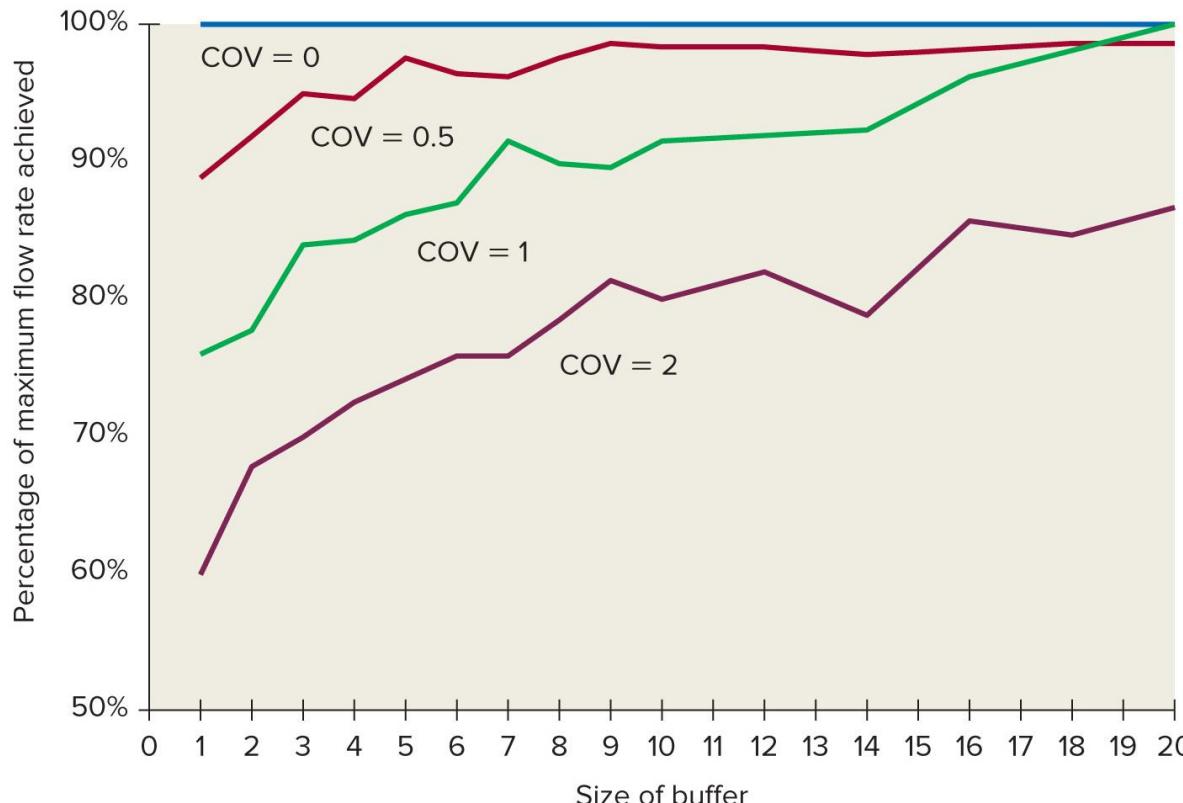
A Process with Sequential Resources



The Concepts of Blocking and Starving



Impact of Variability on a Tandem Process



The flow rate of a two-resource tandem queue as a percentage of the flow rate without any variability. Different levels of variability in processing times (COV) are displayed on different lines. The coefficient of variation of the arrival process is always 1.

Buffer or Suffer

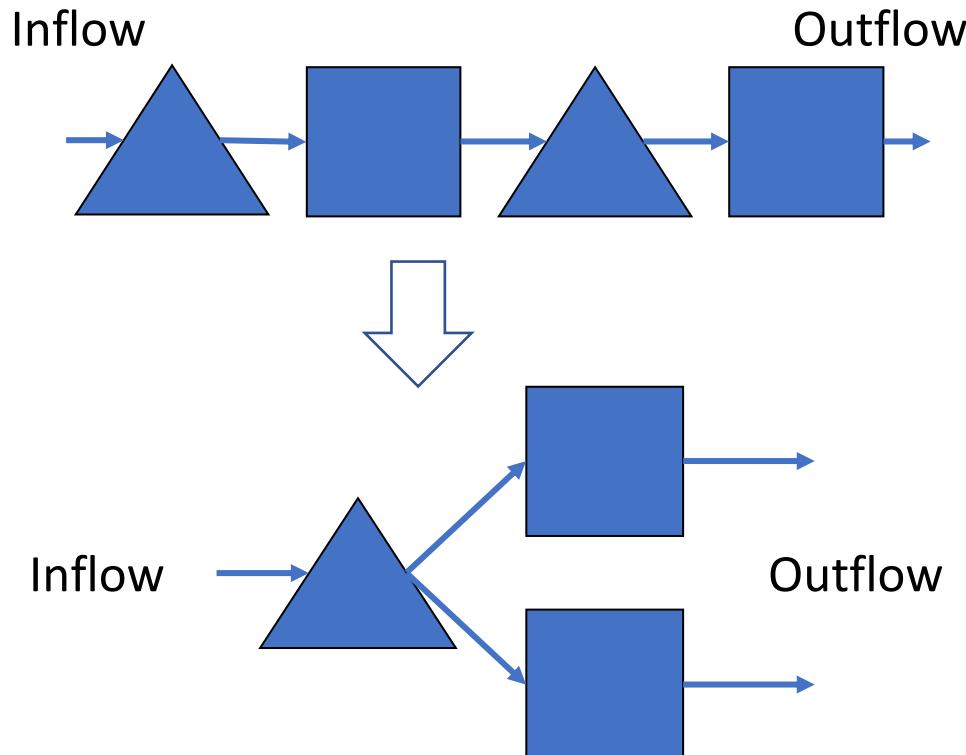
For a special process with applied resources:

- The greater the variability at constant buffer stock size, the lower the capacity.
- The larger the size of the buffer stock at constant variability, the higher the capacity.

But be aware of the disadvantages of buffers!



Parallel instead of Sequential Production Designs



But – parallelisation
also has disadvantages 

Reduce Variability in Processing Times

Standard working procedures

- Task/movement analysis
- Development of best practice
- Documentation of guidelines and instructions how a task is to be completed
- Continuous monitoring and updating





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2023 Operations Management

INTRODUCTION TO INVENTORY MANAGEMENT

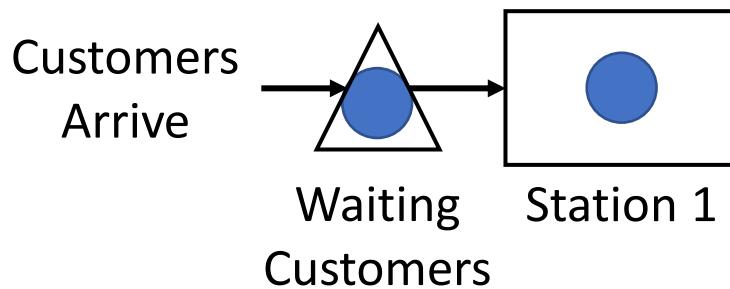


INVENTORY MANAGEMENT

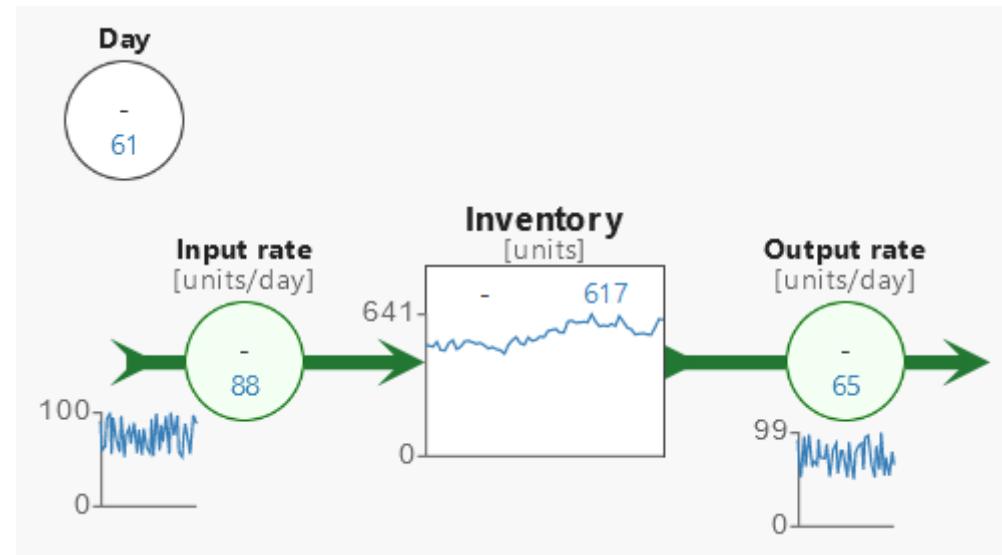


Graphical Symbols for Inventory

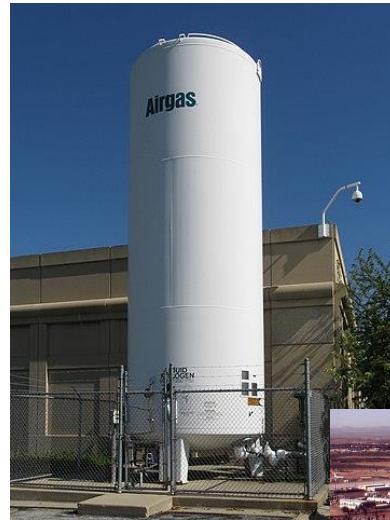
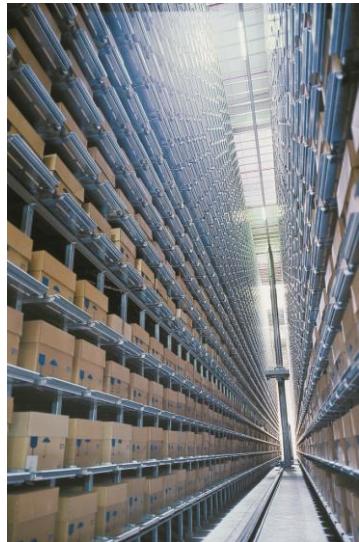
Process Flow Diagram



Stock Flow Diagram



Inventory, Store, and Warehouse



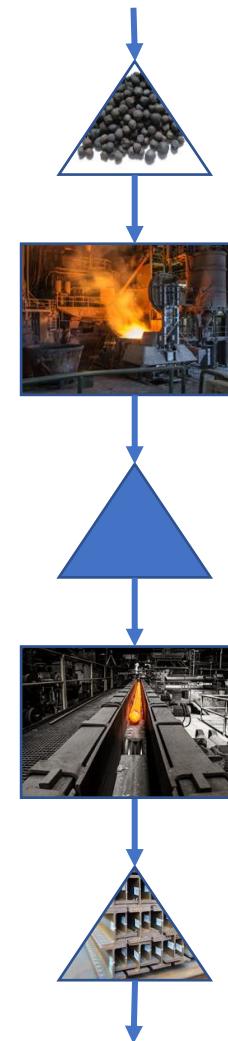
Management of Inventories



Right quantity of the right product in the right place at the right time

Types of Inventory

- Raw material inventory
- Work-in process inventory (WIP)
- Finished product inventory



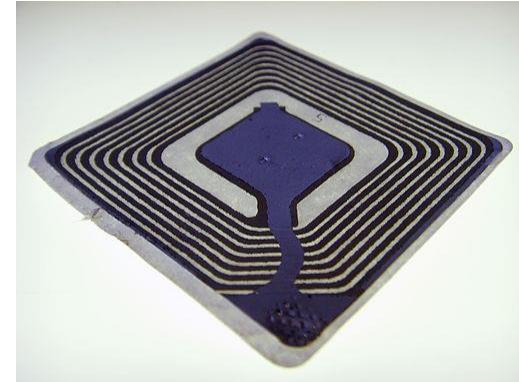
Inventory before
a process

Inventory inside
a process

Inventory after
a process

Inventory Management Capabilities

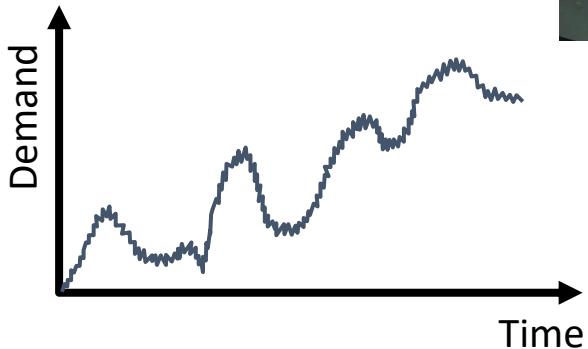
- Forecasting
- Product and demand tracking
- Analytical skills
- Product transportation and handling assets



Reasons for Holding Inventories

- Flow time
- Seasonality (in combination with rigid capacity)
- Batching
- Buffers
- Uncertain demand
- Pricing

$$I = R \cdot T$$



HOW TO MEASURE INVENTORY



Bakery „Backhaus Heislitz“

- Family business since 1886
- New production building in Kriftel (opening 2012)
- Approx. 100 employees
- 10 locations
- 12 types of rolls
- Aroma cupboard for „maturing“ bread roll dough



Backhaus Heislitz Challenges

- What is the hours-of-supply for the inventory in the „aroma cupboard“?
- What is the inventory turnover for the aroma cupboard?



Backhaus Heislitz Challenges

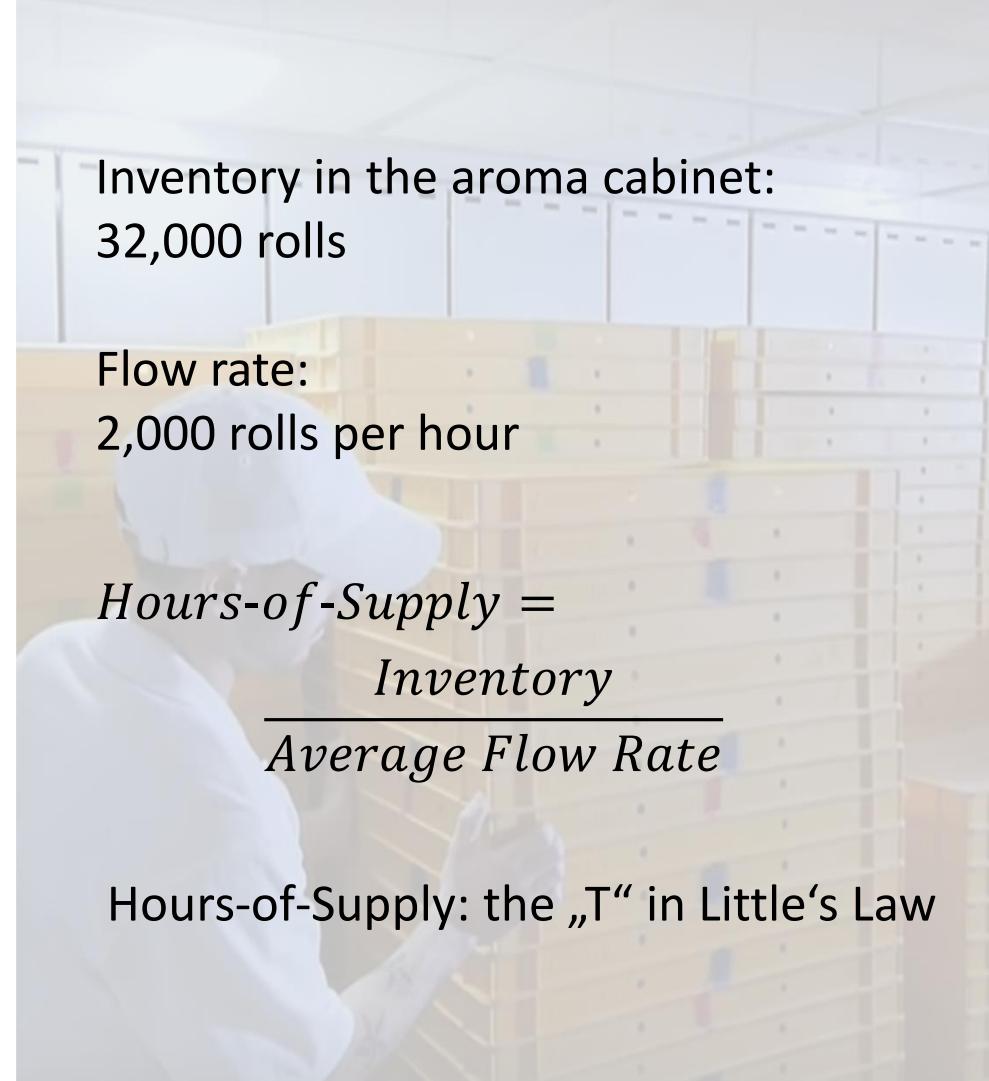
Hours-of-Supply =

Inventory in the aroma cabinet:
32,000 rolls

Flow rate:
2,000 rolls per hour

$$Hours\text{-}of\text{-}Supply = \frac{Inventory}{Average\ Flow\ Rate}$$

Hours-of-Supply: the „T“ in Little's Law



Backhaus Heislitz Challenges

Inventory =

Average Flow Rate =

Hours-of-Supply =

Inventory in the aroma cabinet:

- 25,000 Urweck
- 7,000 Krustis

Flow rates:

- 1,500 Urweck per hour
- 500 Krustis per hour

Production costs:

- 0.20 € per Urweck
- 0.40 € per Krusti

Hours-of-Supply =

$$\frac{\text{Stock}}{\text{Average Flow Rate}}$$

Backhaus Heislitz Challenges

- What is the hours-of-supply for the inventory in the „aroma cupboard“?
- What is the inventory turnover for the aroma cupboard?



Backhaus Heislitz Challenges

Inventory Turns =

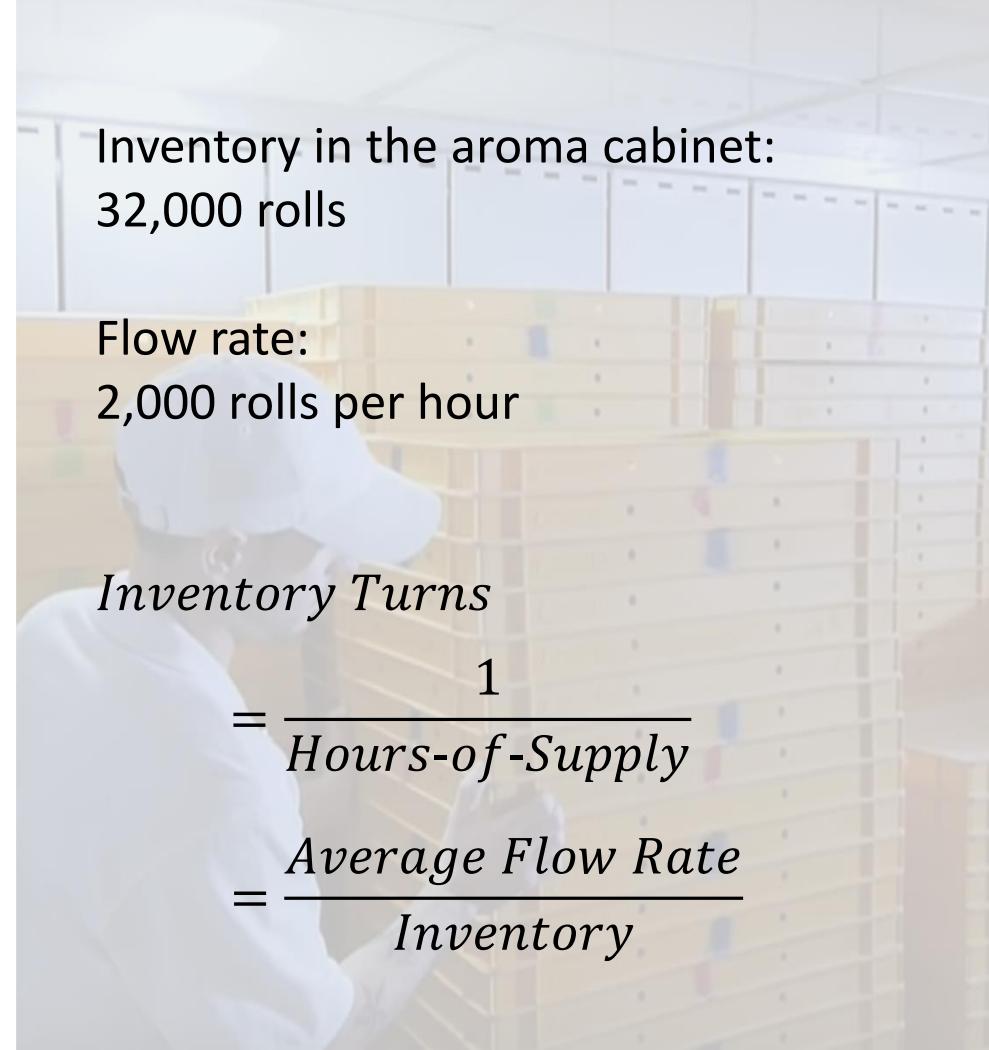
Inventory in the aroma cabinet:
32,000 rolls

Flow rate:
2,000 rolls per hour

Inventory Turns

$$= \frac{1}{\text{Hours-of-Supply}}$$

$$= \frac{\text{Average Flow Rate}}{\text{Inventory}}$$



Benchmark – Inventory Turns for Retail Segments

Retail Segment	Examples	Annual Inventory Turns [1/Years]	Days-of-Supply [Days]
Jewelry	Tiffany	1.68	217
Hobby, toy/game stores	Toys R Us	2.99	122
Department store	Sears, JCPenney	3.87	94
Radio, TV, consumer electronics	Best Buy	4.10	89
Variety stores	Johl's, Target	4.45	82
Apparel and accessory	Ann Taylor, Gap	4.57	80
Drug and proprietary stores	Rite Aid, CVS	5.26	69
Home furniture/equipment	Bed Bath & Beyond	5.44	67
Catalog, mail-order	Spiegel, Lands' End	8.60	42
Food stores	Safeway, Walmart	10.78	34

Cachon/Terwiesch, 2020, S. 304



Benchmark – Inventory Turns for Selected Retailers

Company	Categories	Annual Inventory Turns	Days-of-Supply
Macy's	Apparel, cosmetics, home furnishings	3.1	117
Kohl's	Apparel, footwear, soft home products	3.3	111
Lowe's	Hardware, home improvement	3.9	95
Sears	Hardware, apparel, home furnishings, appliances	3.9	94
Home Depot	Hardware, home improvement	4.6	80
Ace Hardware	Hardware	6	60
Target	Apparel, grocery, home furnishings, electronics	6.4	57
Walmart	Apparel, grocery, home furnishings, electronics	8.2	44
Costco	Apparel, grocery, home furnishings, electronics	12.2	30
Kroger	Grocery	12.3	30
Safeway	Grocery	12.7	29
SuperValu	Grocery	17.3	21
Whole Foods	Grocery	20.2	18



Covestro

- Leading global supplier of innovative, sustainable and versatile high-tech polymer materials
- Before Sept. 1, 2015 Bayer MaterialScience
- > 1.000 products, 16.500 employees
- Headquarters: Leverkusen



Covestro Challenges

- What is Covestro's Years-of-Supply measure for 2019 and 2020?
- What is Covestro's Inventory Turnover measure for 2019 and 2020?

Covestro – Annual Report 2019

Covestro Group Consolidated Statement of Financial Position

	Dec. 31, 2019	Dec. 31, 2020
	€ million	€ million
Current assets		
Inventory	1,916	1,663
Trade accounts receivable	1,561	1,593
Other financial assets	27	1144
Other receivables	359	295
Claims for income tax refunds	104	55
Cash and cash equivalents	748	1404
Assets held for sale	12	36
	4,727	6,190

Covestro Group Consolidated Income Statement

	2019	2020
	€ million	€ million
Sales		
Sales	12,412	10,706
Cost of goods sold	-9,658	-8,207
Gross profit	2,754	2,499
Selling expenses	-1,380	-1,195
Research and development expenses	-266	-262
General administration expenses	-372	-310
Other operating income	181	63
Other operating expenses	-65	-99
EBIT	852	696



Covestro Inventory metrics

Years-of-supply =

	Dec. 31, 2019	Dec. 31, 2020
	€ million	€ million
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STOCKOUT AND HOLDING COSTS



Consequences of a Stockout

- Customer waits for the item to become available
- Lose the sale of one item, but the customer purchases another item
- Lose the sale
- Lose the sale and lose the customer

Stockout Cost

- Stockout costs are opportunity costs
- Opportunity costs are difficult to determine
- A company needs to know where it is on the „stockout cost spectrum“



Inventory Holding Cost

- Opportunity cost of capital
- Inventory storage cost
- Obsolescence cost
- Spoilage and shrinking cost

Inventory Holding Cost Percentage and per Unit

- Inventory holding cost percentage h_p : ratio of the cost to hold an item in inventory during a designated time period relative to the cost c to purchase the item.
- Inventory holding cost per unit h : cost to hold an item in inventory during a designated time period.

$$h = h_p \cdot c$$

Example:

If an item costs € 50 and the firm assigns an annual holding cost percentage of 30%, then it costs the firm $0.3 \cdot € 50 = € 15$ to hold that item in inventory for an entire year.

Inventory Holding Cost for One Single Unit

Question:

How much does it cost the retailer to hold a dress that it purchased for 50€?

Data:

- The holding cost of inventory at a retailer is 35 % per year.
- The retailer's annual turns are 3.

Antwort:

Holding Cost for one year:

The dress is not in stock for whole year:

Holding Cost for one dress:



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2023 Operations Management

INVENTORY MANAGEMENT WITH STEADY DEMAND



THE ECONOMIC ORDER QUANTITY



Circored Iron Ore Reduction Plant

- Iron ore fines is purchased from ore mines in South America
- Circored process uses hydrogen to reduce the ore
- Direct Reduced Iron (DRI) is briquetted to produce Hot Briquetted Iron (HBI)
- Plant output (HBI): sold to steel mills known as “mini-mills”



Iron Ore Reduction Plant Challenges

- What is the yearly cost of ordering and transporting iron ore fines?
- What is the yearly cost of holding iron ore fines inventory?
- What is the cost minimal order quantity?
- How often is an order placed?

Plant production rate R:
657,000 tons per year

Ordering and transportation (O&T) cost
K: \$100,000 per shipment using
a Capesize vessel (100,000 t)

Inventory holding cost percentage h_p :
20 % per year

Purchasing price per ton of iron ore
fines: \$100

Annual Ordering and Transportation Cost

Annual O&T Cost C_{OT}

$$= K \cdot \frac{R}{Q}$$

Plant production rate R:
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Ordering and transportation (O&T) cost
K: \$100,000 per shipment using
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Inventory holding cost percentage h_p :
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Purchasing price per ton of iron ore fines:
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Iron Ore Reduction Plant Challenges

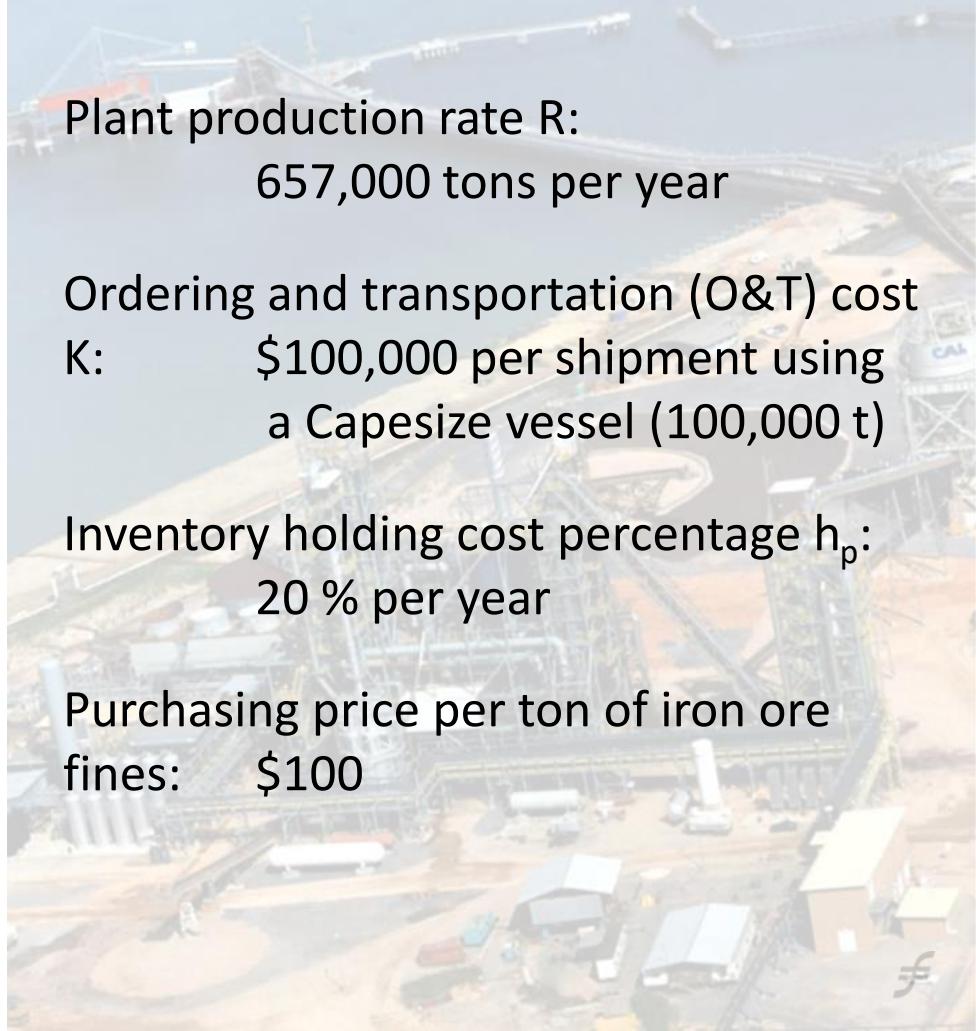
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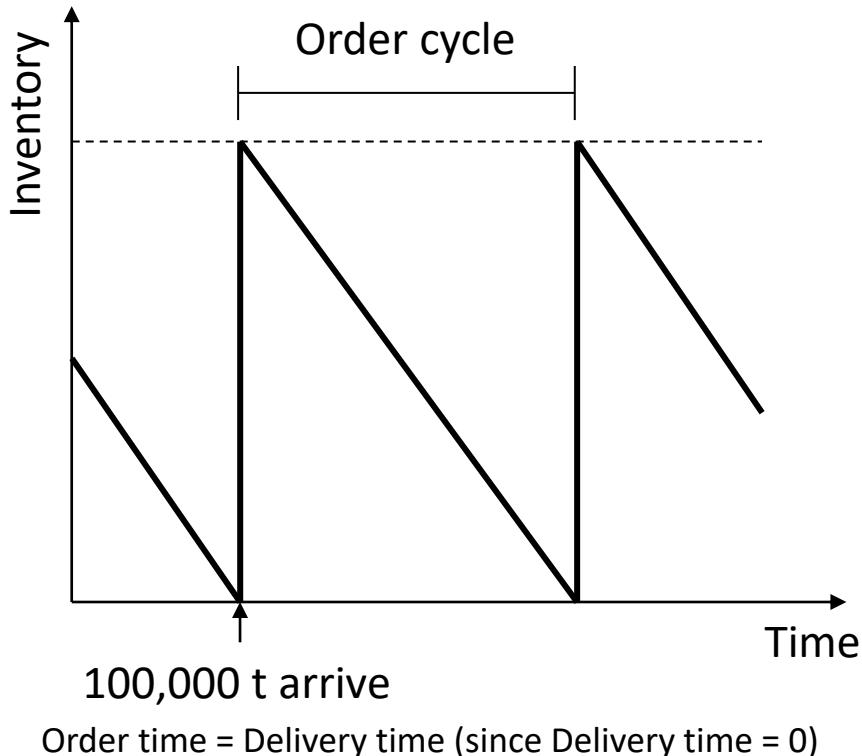
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Purchasing price per ton of iron ore
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Annual Inventory Holding Costs



Plant production rate R:

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K: \$100,000 per shipment using
a Capesize vessel (100,000 t)

Inventory holding cost percentage h_p :

20 % per year

Purchasing price per ton of iron ore fines:

\$100

Annual Inventory Holding Costs

Annual inventory holding cost C_H

$$= \frac{1}{2} \cdot h \cdot Q$$

Plant production rate R:

657,000 tons per year

Ordering and transportation (O&T) cost

K: \$100,000 per shipment using
a Capesize vessel (100,000 t)

Inventory holding cost percentage h_p :

20 % per year

Purchasing price per ton of iron ore fines:

\$100

$$h = 0.2 \frac{1}{\text{year}} \cdot 100 \frac{\$}{t} = 20 \frac{\$}{\text{year} \cdot t}$$

Iron Ore Reduction Plant Challenges

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- **What is the cost minimal order quantity?**
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Inventory holding cost percentage h_p :
20 % per year

Purchasing price per ton of iron ore
fines: \$100

Optimal Order Quantity

Minimize total annual inventory holding and O & T costs $C(Q)$

$$C(Q) = \frac{1}{2} \cdot h \cdot Q + K \cdot \frac{R}{Q} \rightarrow \min$$

$$(1) \quad \frac{dC(Q)}{dQ} = 0$$

$$(3) \quad K \cdot \frac{R}{Q^2} = \frac{1}{2} \cdot h$$

$$(5) \quad Q^2 = \frac{2 \cdot R \cdot K}{h}$$

$$(2) \quad \frac{1}{2} \cdot h - K \cdot \frac{R}{Q^2} = 0$$

$$(4) \quad \frac{1}{Q^2} = \frac{1}{2} \cdot h \frac{1}{R \cdot K}$$

$$(6) \quad Q^* = \sqrt{\frac{2 \cdot R \cdot K}{h}}$$

Optimal Order Quantity

$$Q^* = \sqrt{\frac{2 \cdot R \cdot K}{h}}$$

Plant production rate R:

657,000 tons per year

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Inventory holding cost percentage h_p :
20 % per year

Purchasing price per ton of iron ore
fines: \$100

Order frequency

Order frequency

$$= \frac{\text{Flow rate } R}{\text{Order quantity } Q}$$

Plant production rate R:

657,000 tons per year

Ordering and transportation (O&T) cost

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a Capesize vessel (100,000 t)

Inventory holding cost percentage h_p :

20 % per year

Purchasing price per ton of iron ore fines:

\$100

Optimal Annual Total Cost C(Q*)

$$C(Q^*) = \frac{1}{2} \cdot h \cdot Q^* + K \cdot \frac{R}{Q^*}$$

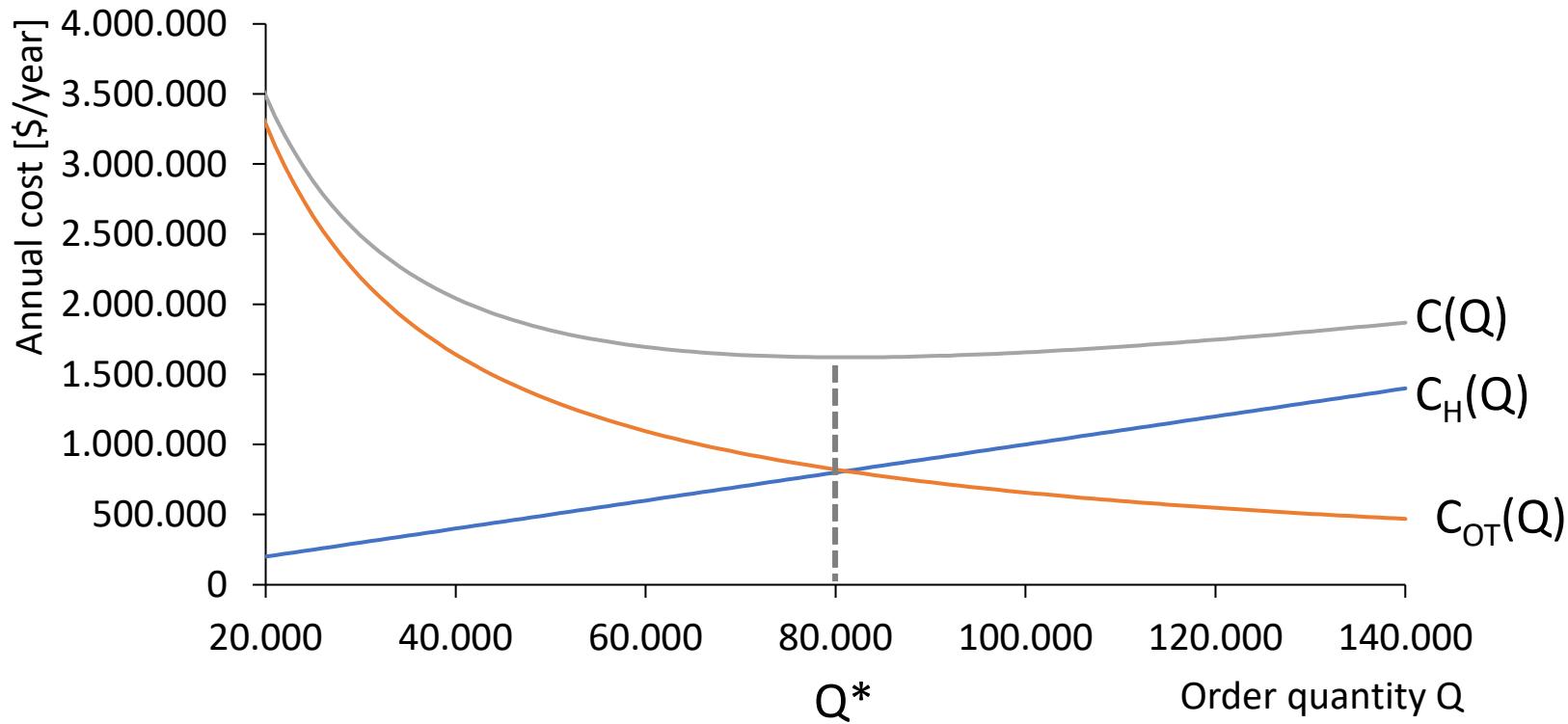
$$Q^* = 36,249.14t$$

$$\begin{aligned} h &= 0.2 \frac{1}{year} \cdot 500 \frac{\$}{t} \\ &= 100 \frac{\$}{year \cdot t} \end{aligned}$$

$$K = 100,000\$$$

$$R = 657,000 \frac{t}{year}$$

EOQ Costs as a Function of Q



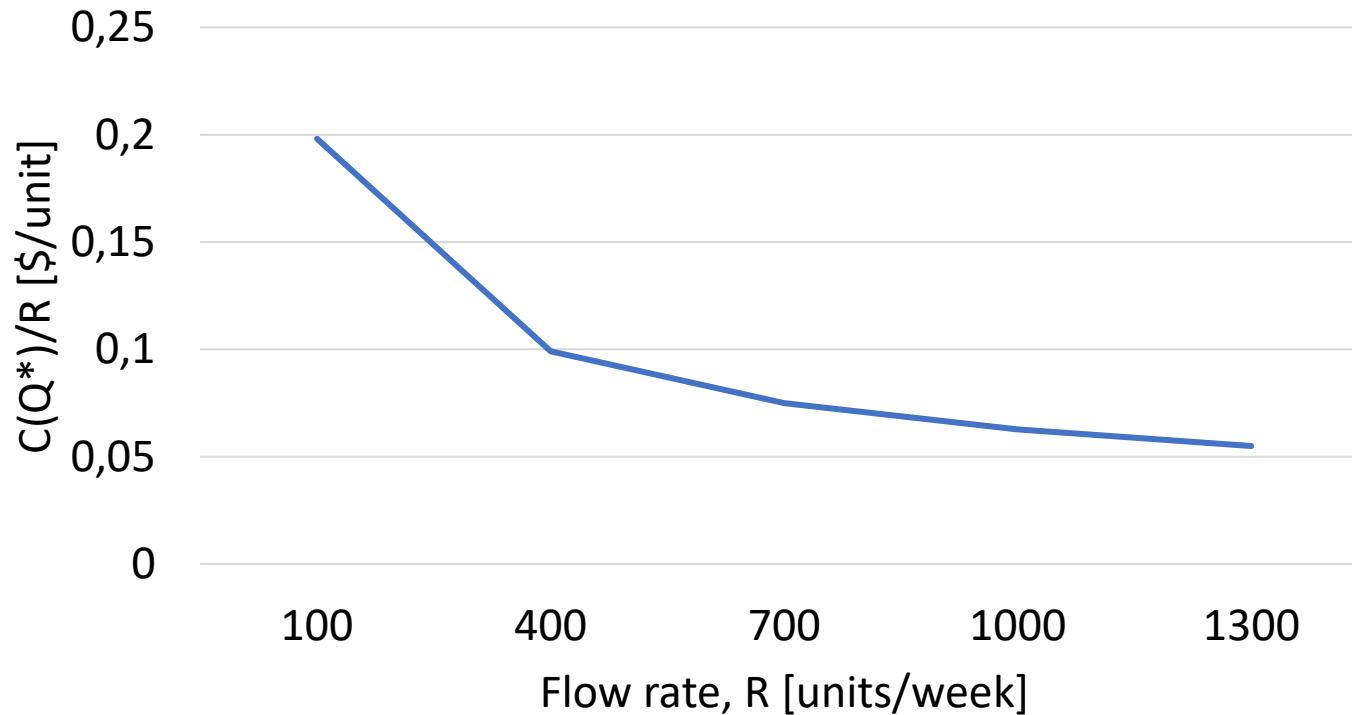
Xootr LLC Handle Caps Purchasing Challenge

- Xootr LLC purchases pairs of handle caps from a supplier in Taiwan for \$0.85 per pair
- Ordering and transportation from Taiwan to the US is \$300 per shipment
- Production rate is 700 Xootr/week
- Annual inventory holding cost percentage is 40%



What is the optimal caps EOQ? What is $C(Q^*)$?

EOQ and Economies of Scale



QUANTITY CONSTRAINTS AND DISCOUNTS



Xootr LLC Handle Caps Purchasing Challenge

- What is the impact of order quantity constraints on Q?
- Should the Xootr purchasing manager accept a price discount for a higher Q than the EOQ or not?



Xootr LLC Handle Caps Purchasing Challenge

- What if handle caps need to be ordered in boxes containing 5,000 units?

$$Q^* = 8,015 \text{ units}$$

$$h = 0.006538 \frac{\$}{\text{unit} \cdot \text{week}}$$

$$K = 300\text{\$}$$

$$R = 700 \frac{\text{units}}{\text{week}}$$



Xootr LLC Handle Caps Purchasing Challenge

- Should the Xootr purchasing manager order 10,000 units if this gets them a 5% discount from the supplier?

$$Q^* = 8,015 \text{ units}$$

$$h = 0.006538 \frac{\$}{\text{unit} \cdot \text{week}}$$

$$K = 300\text{\$}$$

$$R = 700 \frac{\text{units}}{\text{week}}$$



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INVENTORY MANAGEMENT WITH PERISHABLE DEMAND



THE NEWSVENDOR MODEL



Fruchtknall

- Family business proposing
Ripe fruits
 - + highest quality
 - + fast transport
 - = unique taste
- Direct import from Thailand with express airfreight
- Unique shopping experience
 - Order terminals
 - <15 minutes delivery time



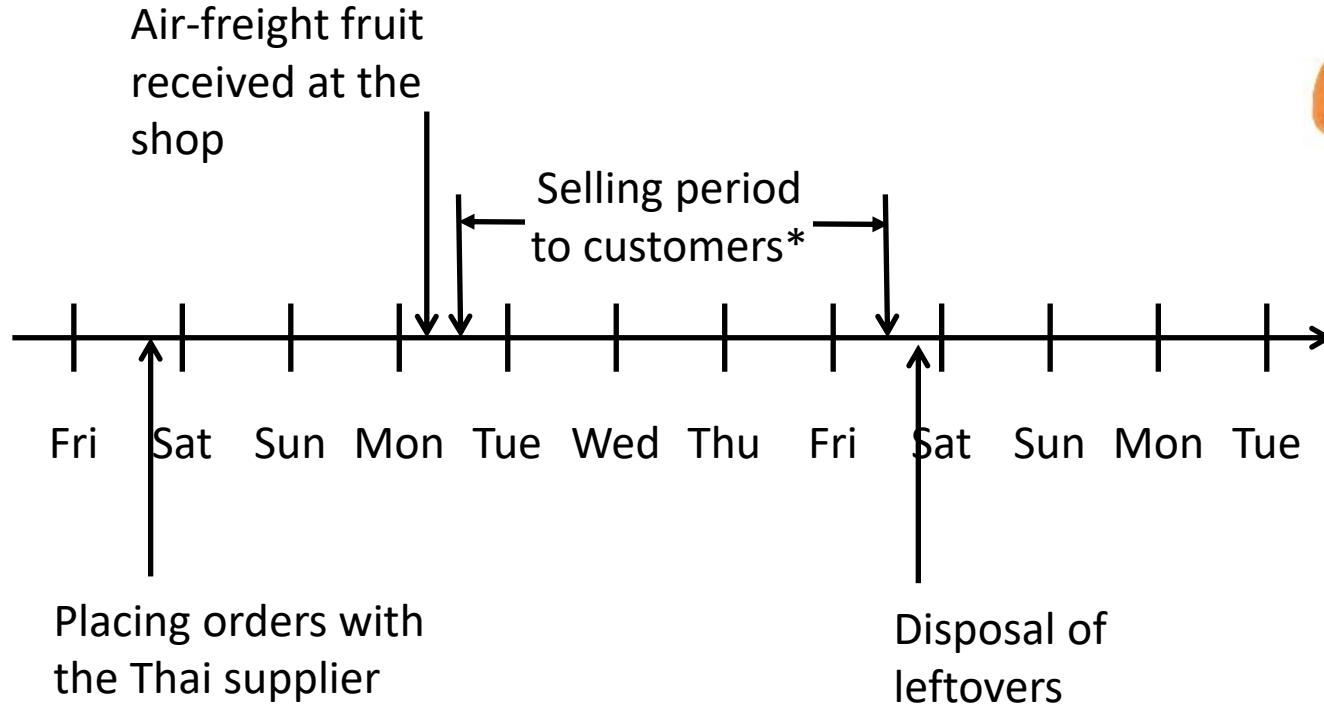
Fruchtknall Data



	Mango	Papaya
Selling price	6.00	13.00€/unit
Purchase price	3.50	2.50€/unit
Salvage value	0.00	0.00€/unit
Salvage cost	0.00	0.00€/unit



Fruchtknall Air-freight Fruit Timeline



Fruchtknall Challenge

- How is the profit contribution of a specific week calculated?
- Ordering how many mangos and papayas maximizes weekly profit contributions?



A Week's Profit Contribution

$Profit_{Mango} =$

$Profit_{Papaya} =$

Demand and order quantity of last week:

	Mango	Papaya
Demand	130	8 Unit/Week
Order quantity	120	10 Unit/Week
Selling price	6.00	13.00 Euro/Unit
Purchase price	3.50	2.50 Euro/Unit

Mangos:

Demand > Order quantity

Sales volume = 120 unit

Papayas:

Demand < Order quantity

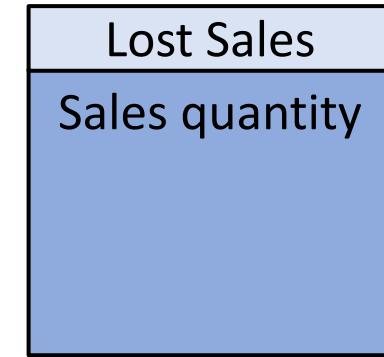
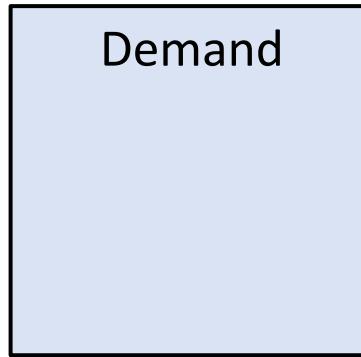
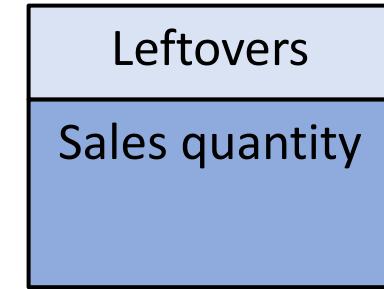
Sales volume = 8 unit

Fruchtknall Challenge

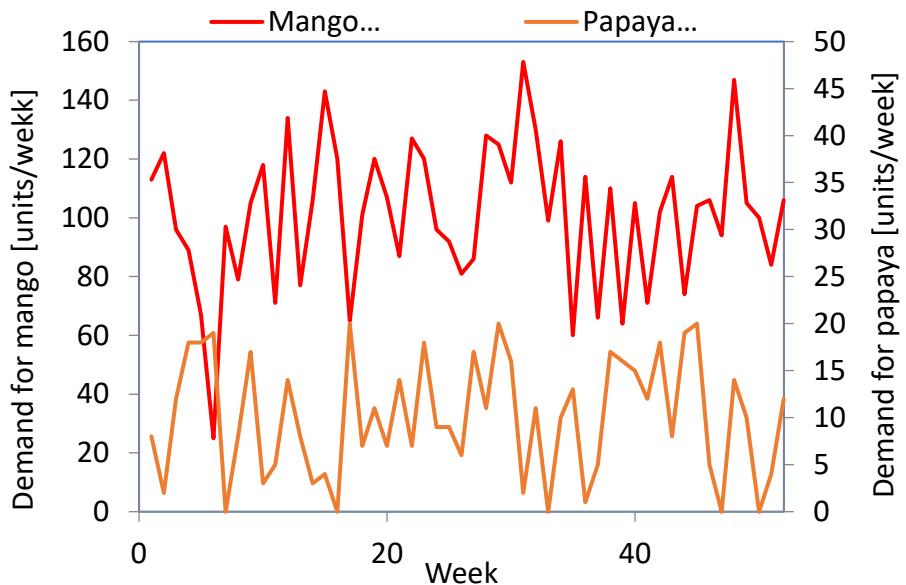
- How is the profit contribution of a specific week calculated?
- **Ordering how many mangos and papayas maximizes weekly profit contributions?**



Impact of Orders and Demand on Sales

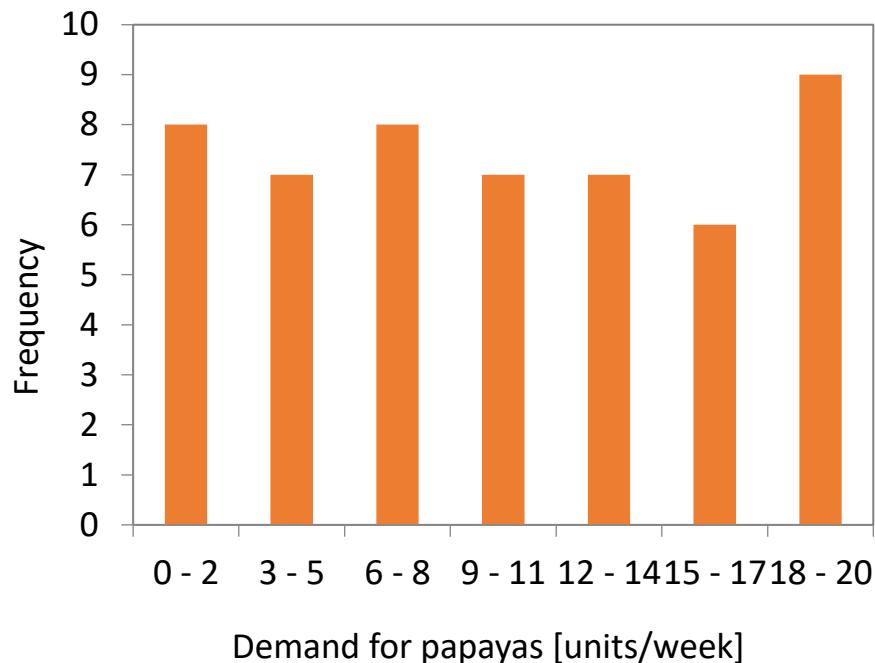
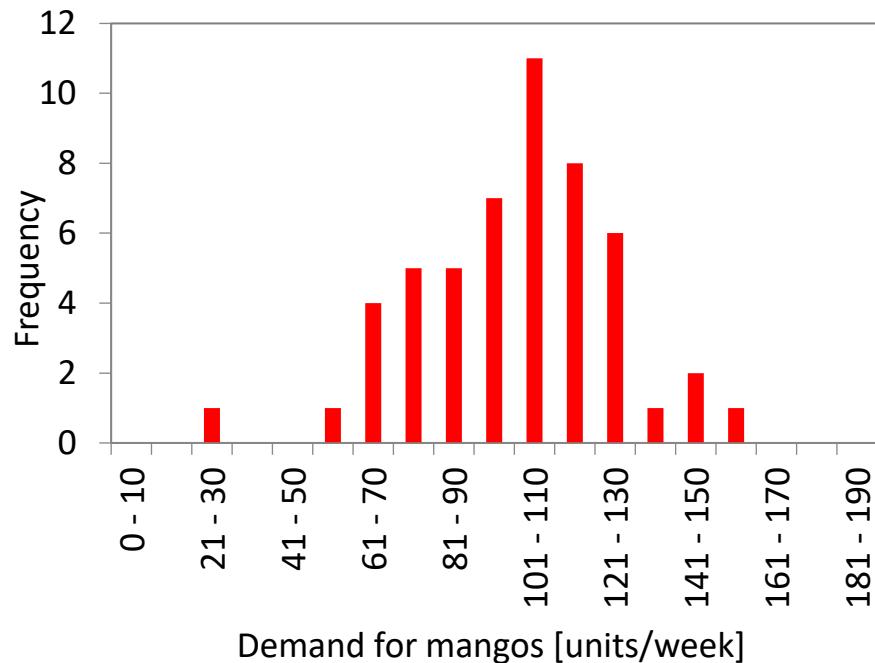


Forecasting Demand

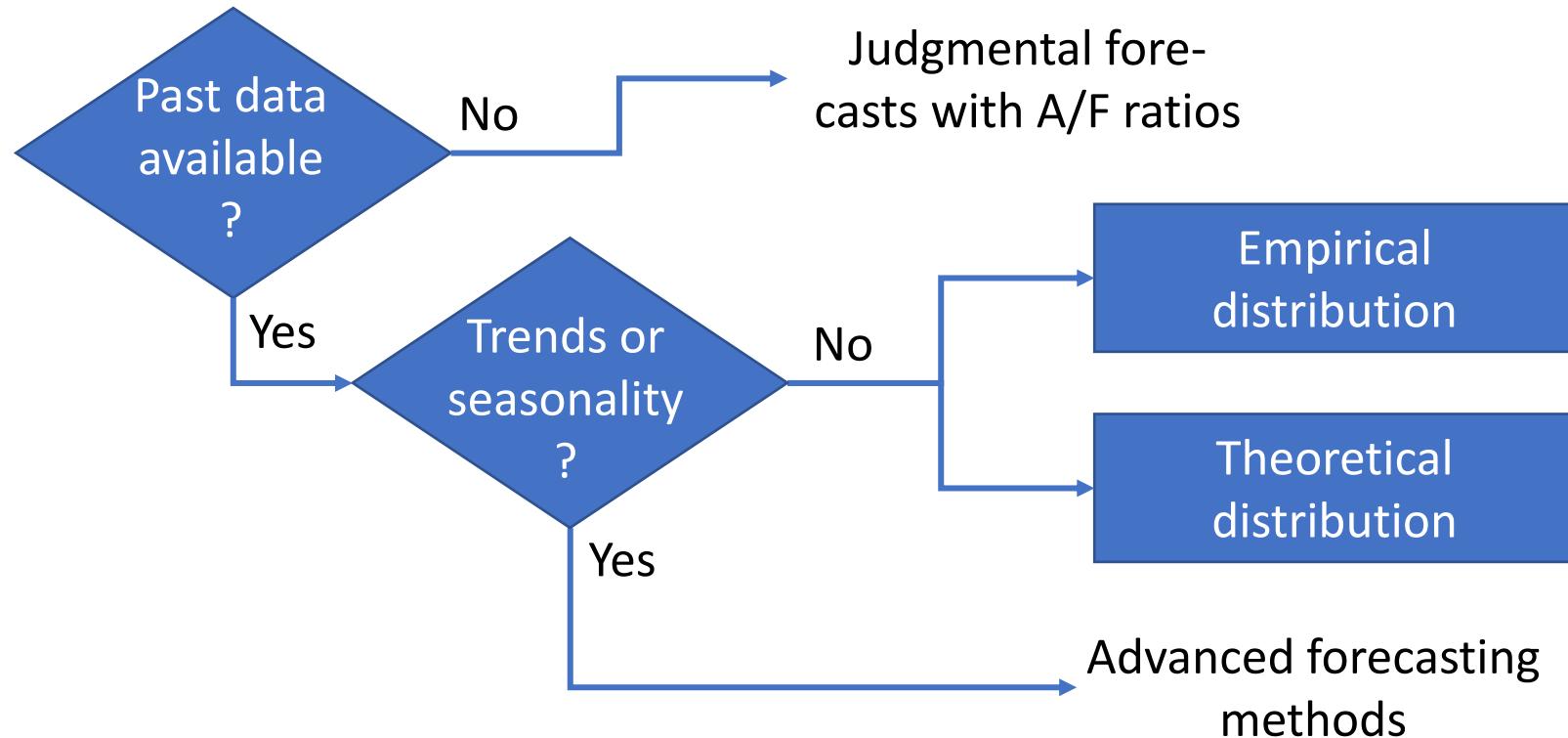


Week	Mango [units/week]	Papaya [units/week]
1	113	8
2	122	2
3	96	12
4	89	18
5	67	18
6	25	19
7	97	0
8	79	8
9	105	17
10	118	3
11	71	5
12	134	14
⋮	⋮	⋮

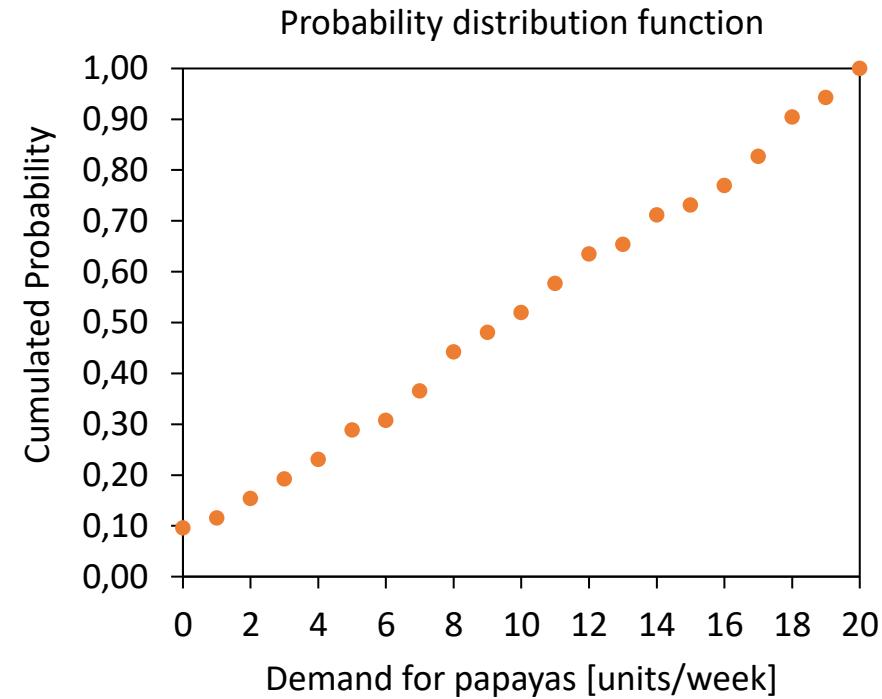
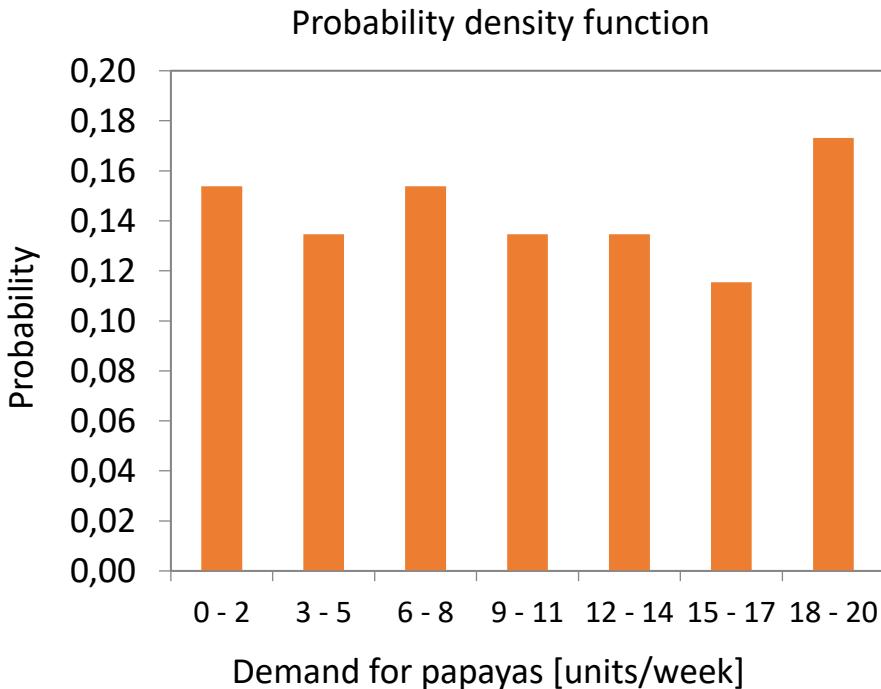
Histogram of Past Demand



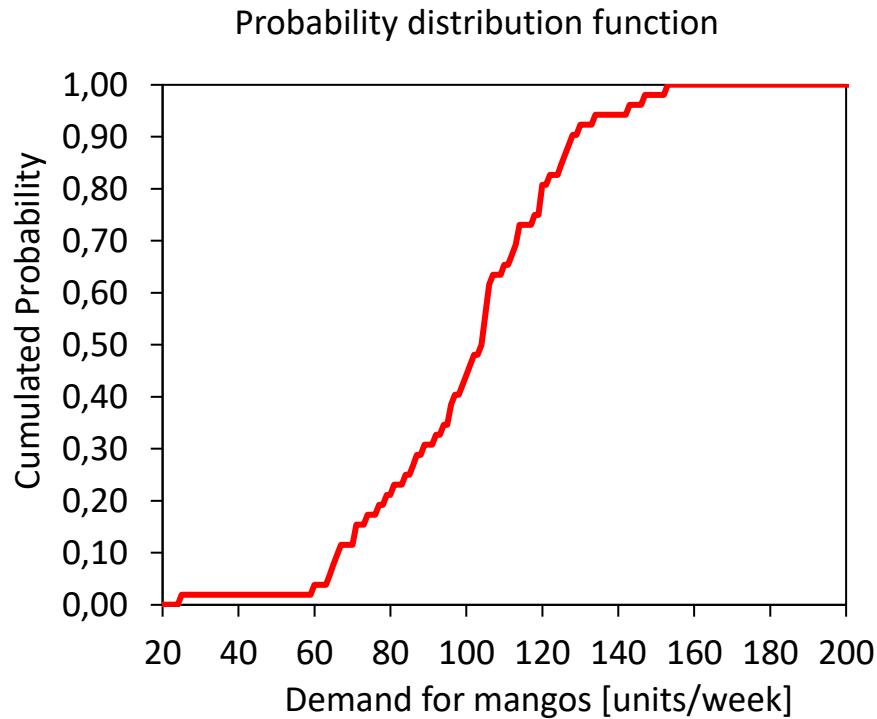
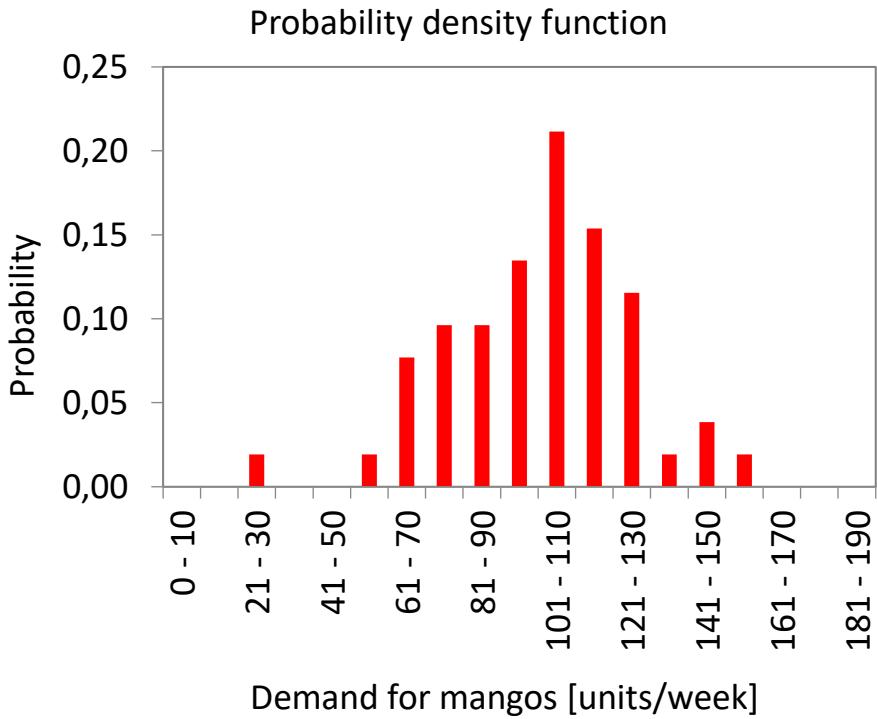
How to Construct a Forecast for Variable Demand?



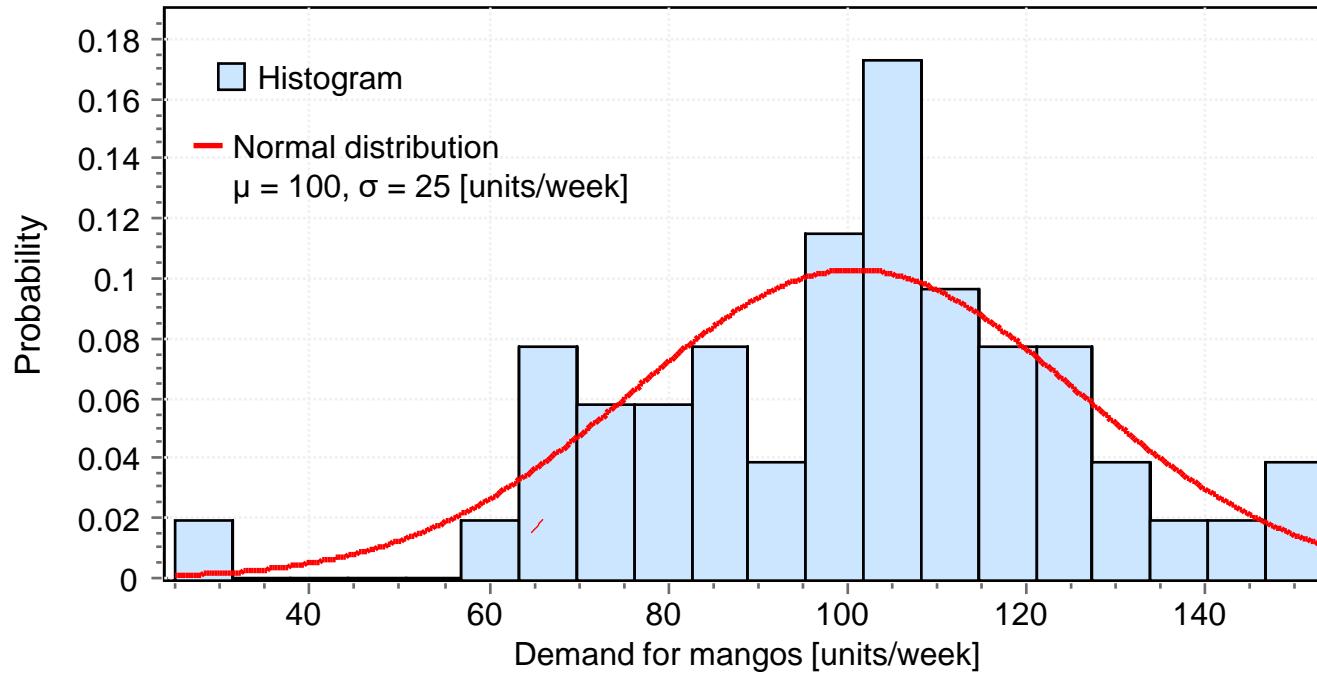
Empirical Distribution for Papaya Demand



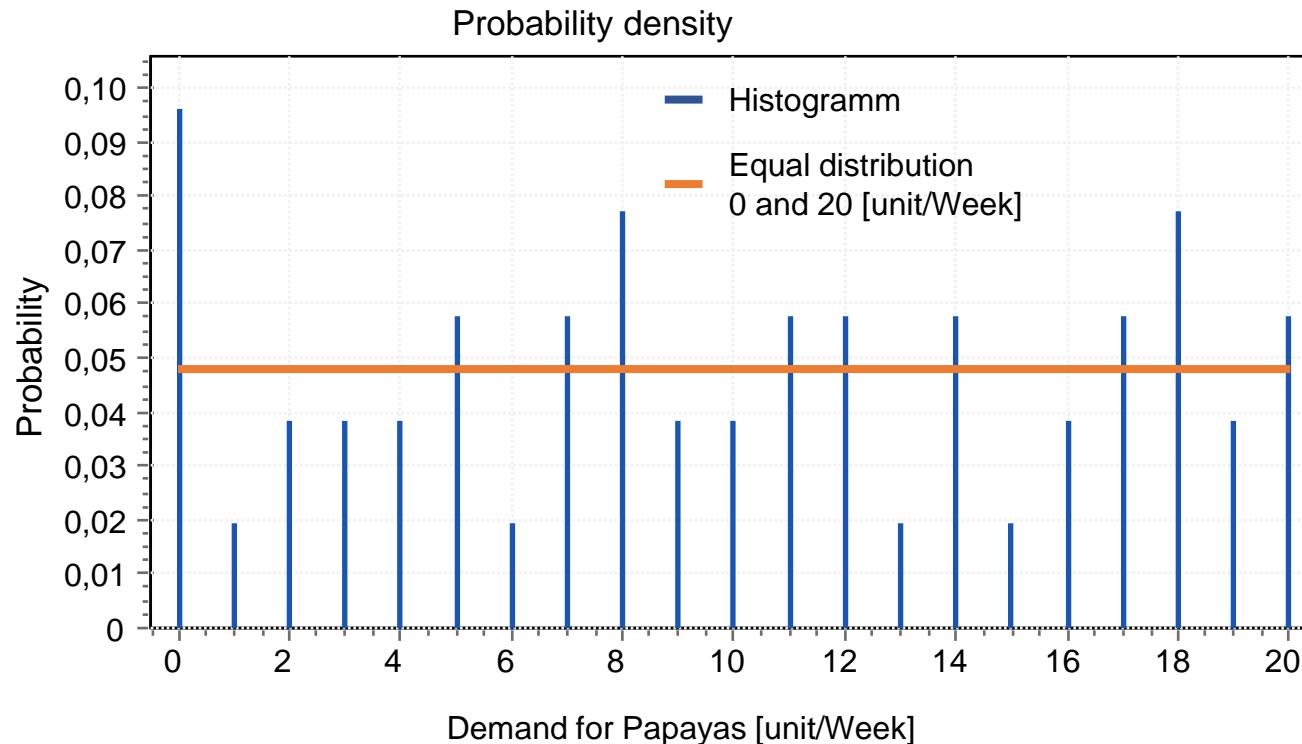
Empirical Distribution for Mango Demand



Fitting a Theoretical Distribution



Theoretical distribution of Papaya - Demand



What would you decide?



Two Ways of Determining Optimal Q*

... taking into account that demand is uncertain.

- Simulation
 - For each possible Q , draw a large number of demand values from the demand distribution and determine profit distribution (average, min, max, stddev)
 - Find the Q that results in the highest – average – profit
 - Type of „brute force approach“
- Analytic
 - Determine an expected profit function (or expected costs functions) that can be solved for the optimal Q

Simulation with Q = 10 Papayas/week

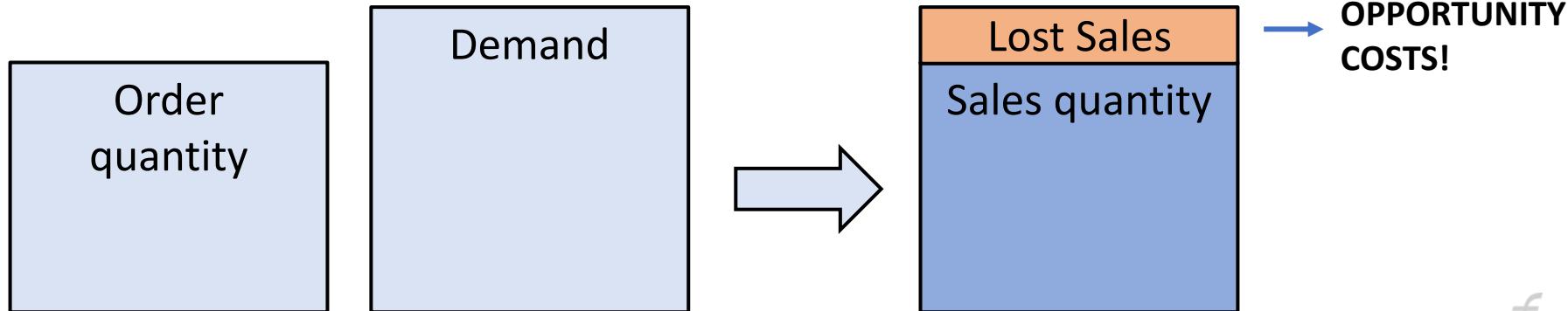
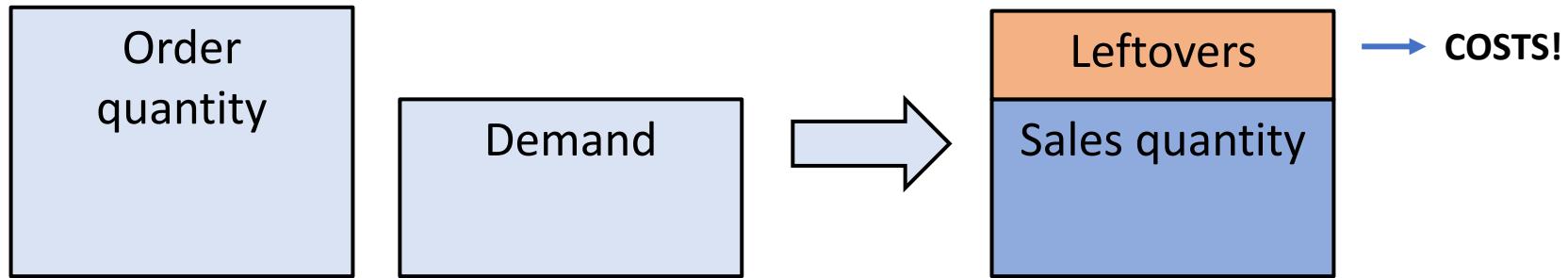
Papaya	
Selling price	13.00€/unit
Purchase price	2.50€/unit
Salvage value	0.00€/unit
Salvage cost	0.00€/unit

Replica- tion	Demand [unit/week]	Sales [unit/week]	Lost sales [unit/week]	Leftovers [units/week]	Sales revenue [€/week]	Profit [€/week]
1	18	10	8	0	130	105
2	16	10	6	0	130	105
3	19	10	9	0	130	105
4	2	2	0	8	26	1
5	15	10	5	0	130	105
6	19	10	9	0	130	105
7	14	10	4	0	130	105
8	5	5	0	5	65	40
9	4	4	0	6	52	27
10	11	10	1	0	130	105

Average profit: 80.30 €/week

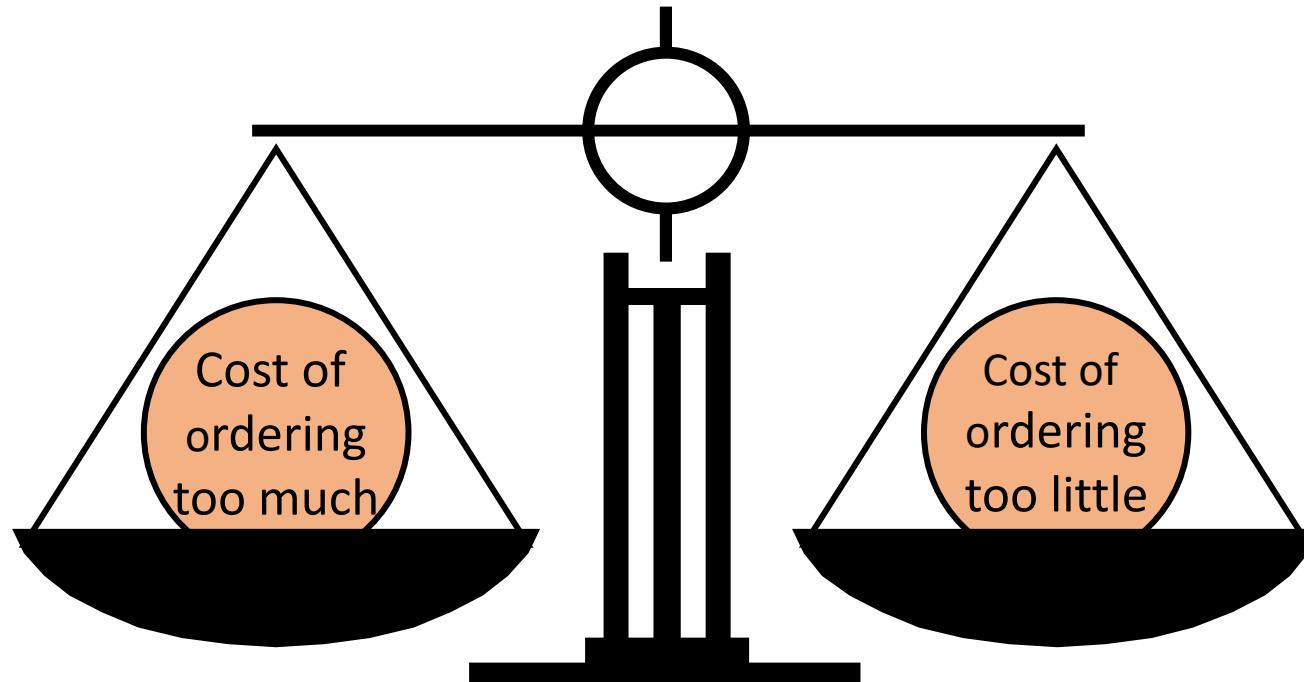


Towards an Analytic Solution for Q^*



Optimal Q^* by Balancing...

... the cost of ordering too much vs. cost of ordering too little



Overage and Underage Costs

Overage costs = cost of ordering too much

Question: How much does it cost to order a fruit too much?

- Answer: Purchase price + Salvage cost – Salvage value
- Mango: $3.50 + 0 - 0 = 3.50 \text{ €/unit}$
- Papaya: $2.50 + 0 - 0 = 2.50 \text{ €/unit}$

Underage costs = cost of ordering too little

Question: How much does it cost to order a fruit too little?

- Answer: Selling price - Purchase price = (lost) Profit per unit
- Mango: $6.00 - 3.50 = 2.50 \text{ €/unit}$
- Papaya: $13.00 - 2.50 = 10.50 \text{ €/unit}$

	Mango	Papaya
Selling price	6.00	13.00 €/unit
Purchase price	3.50	2.50 €/unit
Salvage value	0.00	0.00 €/unit
Salvage cost	0.00	0.00 €/unit

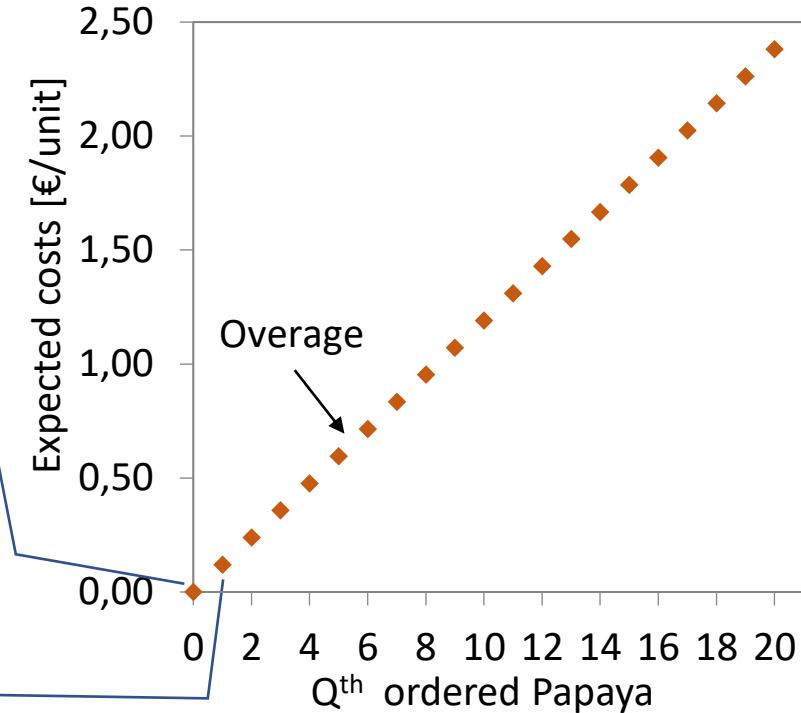




Expected Overage Cost of Qth Papaya

If Mr. Müller does not order any papaya at all, then none can be leftover. The expected overage costs are therefore 0 €/unit.

When Mr. Müller orders a papaya, he expects...
.....that this is leftover with a probability of 1/21.
...that the leftover inventory cost is 2.50 €/unit · 1/21 = 0.12 €/unit.



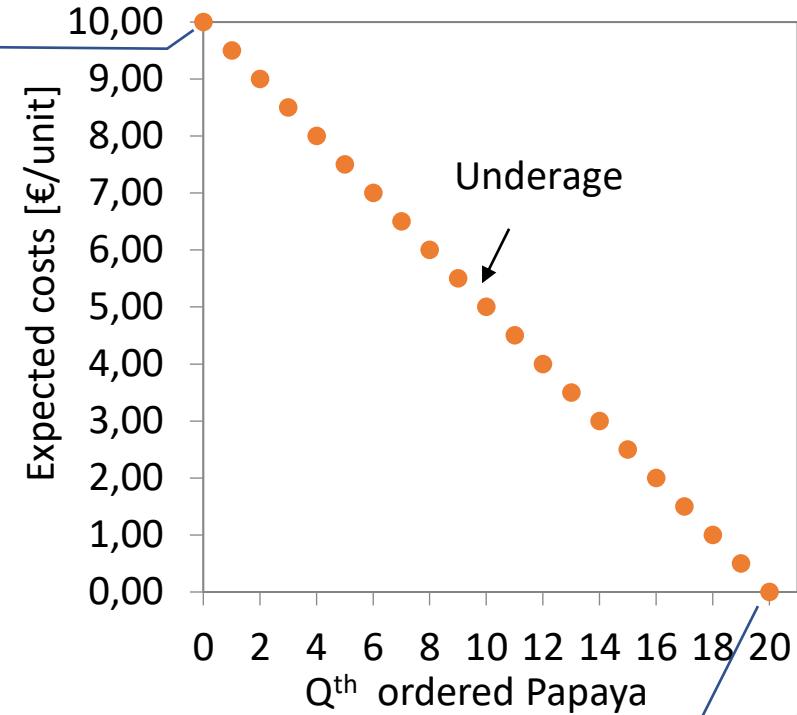
Expected Underage costs for Qth Papaya

When Mr. Müller does not order any papaya, he expects ...

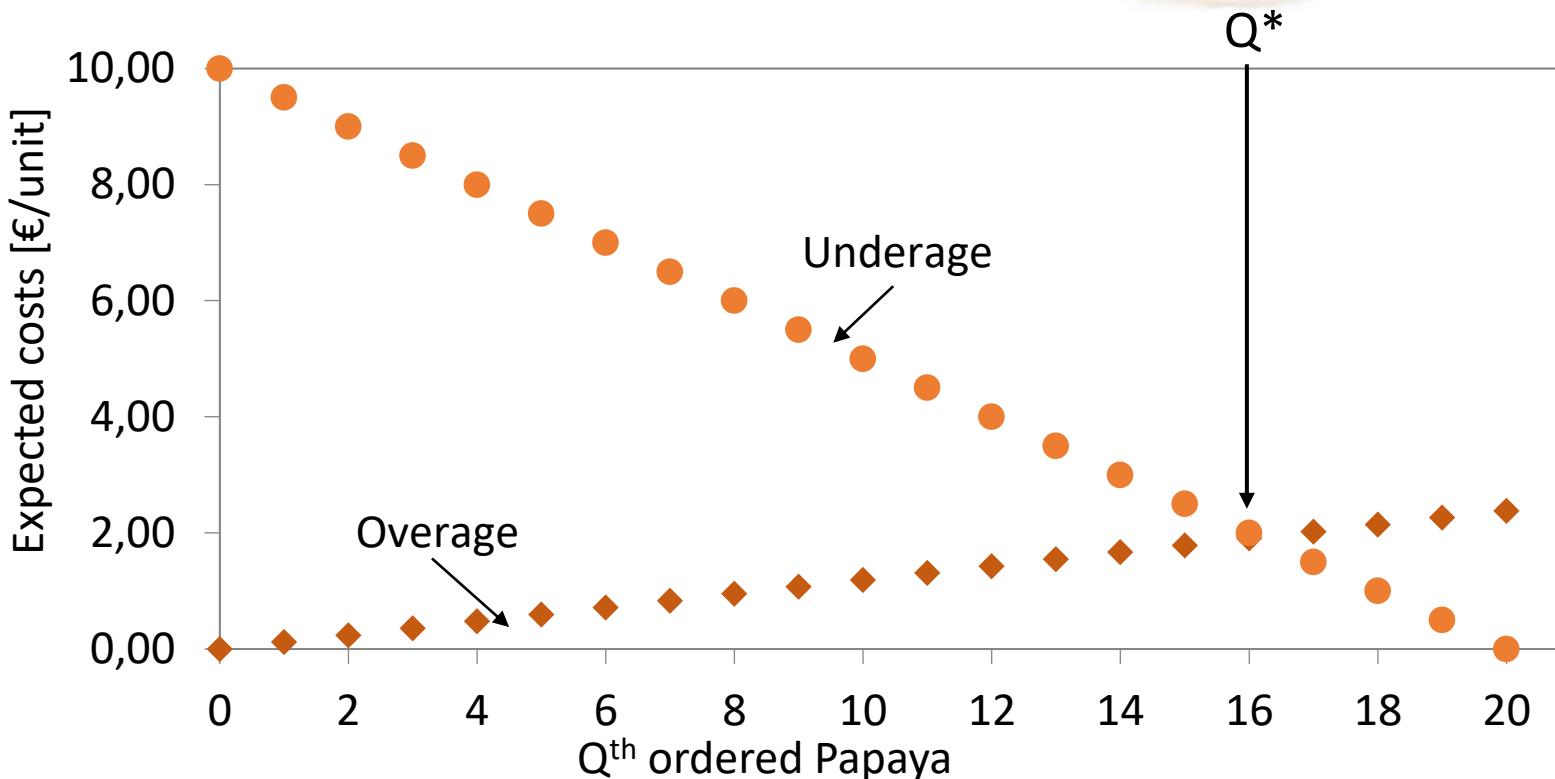
...that he could have sold one with a probability of 20/21.

... the underage costs to be
 $10.50 \text{ €/unit} \cdot 20/21 = 10.00 \text{ €/unit}$.

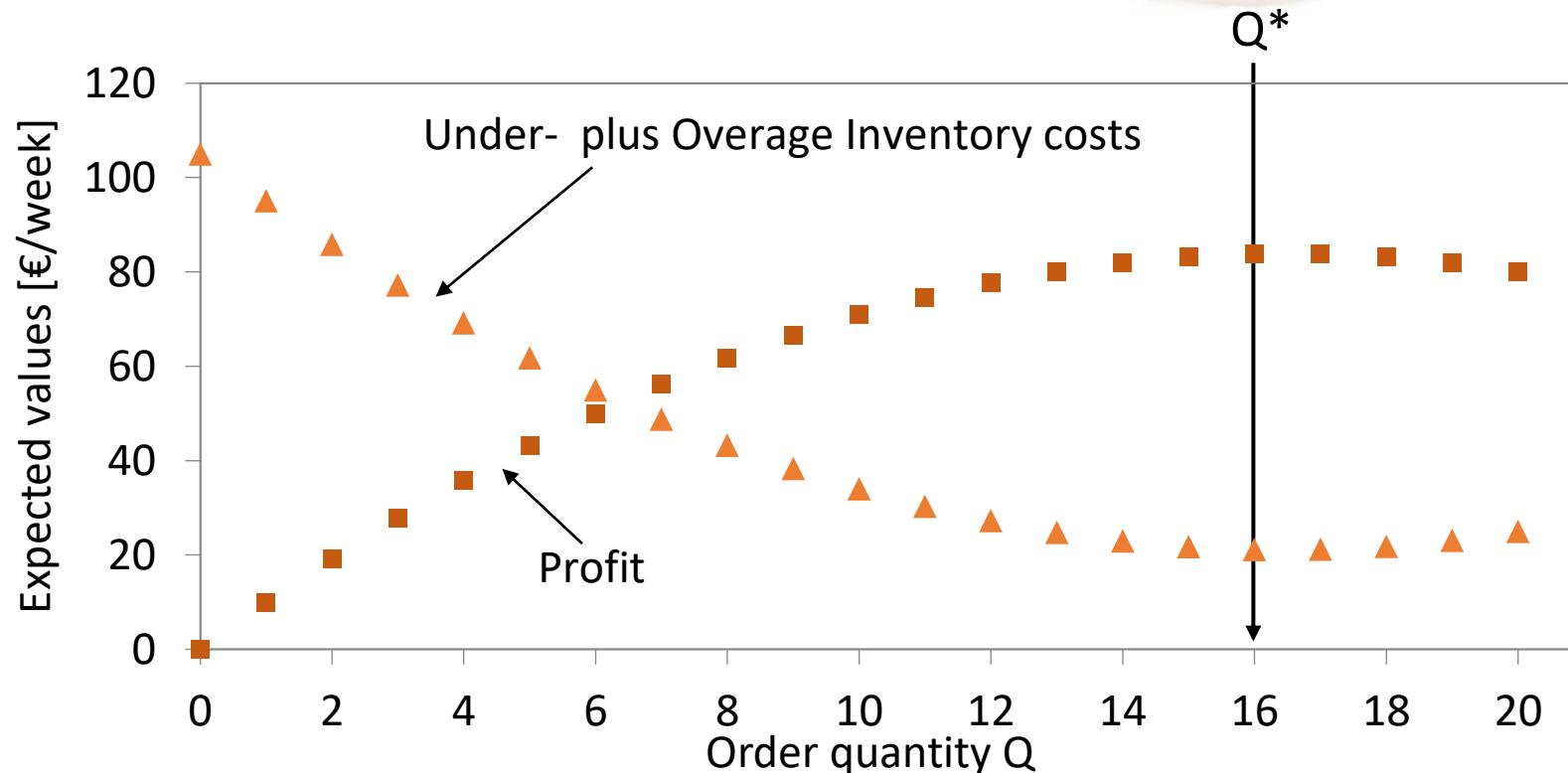
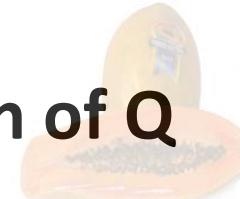
When Mr. Miller orders 20 papayas, he expects never to have too few papayas and therefore no understocking costs.



Q^* - Balance Between Expected Unit Costs



Expected Weekly Profits as a Function of Q



Towards a Formula for Q^* : Critical Ratio

- Expected underage cost based on the Q^{th} unit = $C_u \cdot (1 - F(Q))$
- Expected overage cost based on the Q^{th} unit = $C_o \cdot F(Q)$
- Both expected costs are in balance at:

$$C_u \cdot (1 - F(Q)) = C_o \cdot F(Q)$$

- Dissolving to $F(Q)$ gives the famous critical ratio:

$$F(Q) = \frac{C_u}{C_u + C_o}$$



Fruchtknall Critical Ratio

Optimal solution:

$$p(D \leq Q^*) = CR$$

	Mango	Papaya
Selling price	6.00	13.00€/unit
Purchase price	3.50	2.50€/unit
Salvage value	0.00	0.00€/unit
Salvage cost	0.00	0.00€/unit
Overage cost Co	3.50	2.50€/unit
Underage cost Cu	2.50	10.50€/unit

$$\text{Critical Ratio } CR = \frac{C_u}{C_u + C_o}$$

Q* Using the Theoretical Distribution Function

- Here, the **theoretical distribution function table** for Papaya demand is used: Uniform(0, 20)
- Search the cumulated probability that is nearest but larger than the CR
- Set Q to the demand in the same line:

$$Q^* = 16 \text{ units}$$

Demand D [units/week]	Cumulated Probability F(D)
0	0.04761905
1	0.0952381
2	0.14285714
3	0.19047619
4	0.23809524
5	0.28571429
6	0.33333333
7	0.38095238
8	0.42857143
9	0.47619048
10	0.52380952
11	0.57142857
12	0.61904762
13	0.66666667
14	0.71428571
15	0.76190476
16	0.80952381
17	0.85714286
18	0.9047619
19	0.95238095
20	1

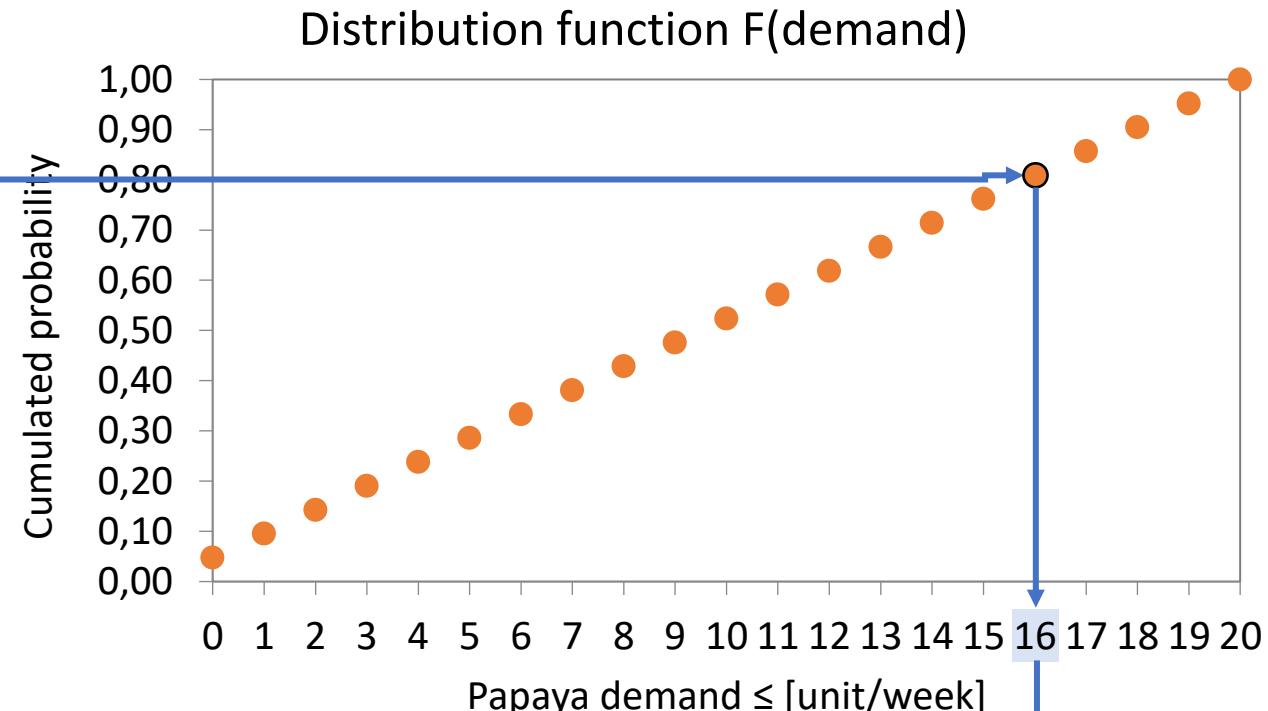


$$CR_{Papaya} = 0.8077$$

Obtaining Q^* From a Graphical Distribution Function

$$CR_{Papaya} = 0.8077$$

$$Q^* = 16 \text{ units}$$



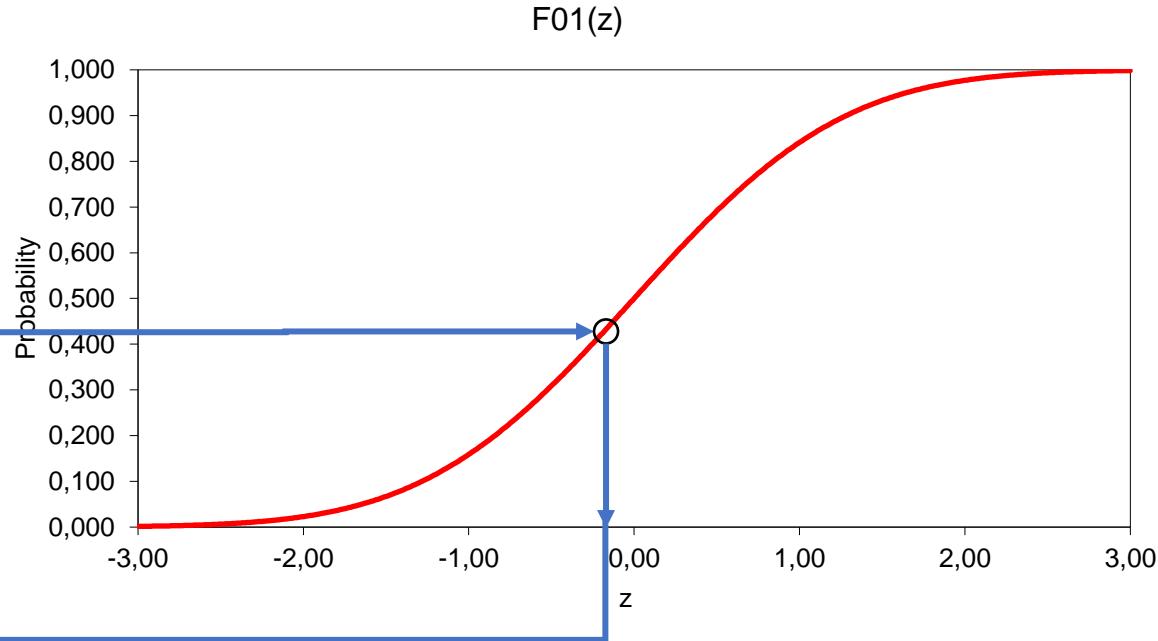
Mango Q*: Using the Standard Normal

...distribution function to determine Q^* for mangos with normally distributed demand (mean = 100 units per week, stdev = 25 units per week)



$$CR_{Mango} = 0.4167$$

$$z = -0.21$$



Converting z to Q^* for Mangos

$$z = \frac{Q - \mu}{\sigma}$$



$$Q = \mu + z \cdot \sigma$$



Using the Standard Normal F(z)-Table

- Find the critical ratio inside the Standard Normal Distribution Function Table
- If the critical ratio falls between two values in the table, choose the greater z-statistic... this is called the round-up rule

$$z = -0.21$$

z	Density $f_{01}(z)$	Distribution $F_{01}(z)$
-0.30	0.381388	0.382089
-0.29	0.382515	0.385908
-0.28	0.383606	0.389739
-0.27	0.384663	0.393580
-0.26	0.385683	0.397432
-0.25	0.386668	0.401294
-0.24	0.387617	0.405165
-0.23	0.388529	0.409046
-0.22	0.389404	0.412936
-0.21	0.390242	0.416834
-0.20	0.391043	0.420740
-0.19	0.391806	0.424655
-0.18	0.392531	0.428576
-0.17	0.393219	0.432505
-0.16	0.393868	0.436441
-0.15	0.394479	0.440382
-0.14	0.395052	0.444330
-0.13	0.395585	0.448283

$CR_{Mango} = 0.4167$



Conclusion

- Optimal order quantities
 - For Mangos: 95 unit/Week
 - For Papayas: 16 unit/Week
- Optimal order quantity almost always \neq Average
- Optimal order quantity does not change from week to week



USING A/F RATIOS TO MODEL DEMAND



O'Neill's Hammer 3/2 Wetsuit

- O'Neill Inc. is a designer and manufacturer of apparel, wetsuits, and accessories
- One of their products is a wetsuit for professional surfers
- The „Hammer“ Wetsuit is produced by O'Neill's contract manufacturer TEC Group in Asia



O'Neill's Hammer 3/2 Wetsuit

- TEC Group provides many benefits to O'Neill (low cost, sourcing expertise, flexible capacity, etc.), but they have a three-month lead time on all orders
 - O'Neill needs to order the Hammer 3/2 on Nov. 1st, to have an expected arrival of the products on Jan. 31st
 - Selling season for the Hammer 3/2: February to July
 - Leftover inventory is sold at a discounted price at the end of the season



O'Neill's Hammer 3/2 Wetsuit Economics

Hammer 2/3

Selling price 190\$/unit

Purchase price from TEC 110\$/unit

Discount price* 90\$/unit

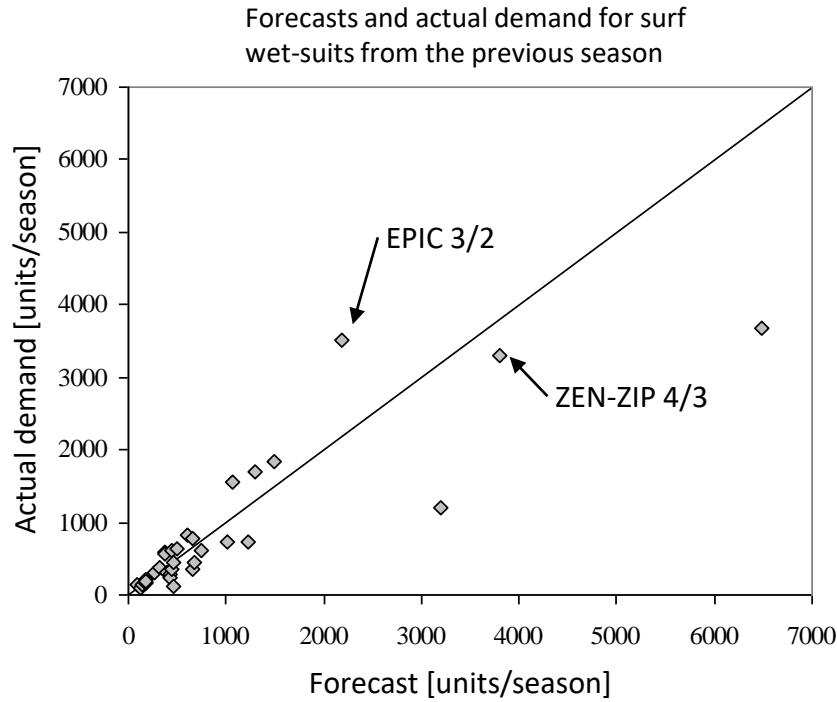
Salvage costs 0\$/unit

*at end of season



O'Neill's Hammer 3/2 Wetsuit Forecast

- Based on past sales data for similar products marketing and sales team forecasts 3,200 units of the Hammer 3/2 to be sold in the spring season.
- Unfortunately, the judgmental forecast is rather poor: actual demand deviates from the forecasts by more than 25% in 50% of the cases.
- The forecast becomes better over time (once the first units have been sold), but the long lead time prevents O'Neill from later orders.



O'Neill's Hammer 3/2 Wetsuit Challenges I

- How can the limited accuracy of the judgmental forecasts be considered?
- What is the optimal order quantity to be placed at TEC on 1st November?



A/F Ratio to Account for Forecast Errors

- Relative measure of forecast error
- Compute the A/F ratio based on past observations as:

$$A/F \text{ ratio} = \frac{\text{Actual demand}}{\text{Forecast}}$$

A/F ratio = 1 → perfect forecast

A/F ratio > 1 → too low forecast

A/F ratio < 1 → too high forecast

Product description	Forecast	Actual demand [units/season]	Error*	A/F Ratio **
JR ZEN FL 3/2	90	140	-50	1.5556
EPIC 5/3 W/HD	120	83	37	0.6917
JR ZEN 3/2	140	143	-3	1.0214
WMS ZEN-ZIP 4/3	170	163	7	0.9588
HEATWAVE 3/2	170	212	-42	1.2471
JR EPIC 3/2	180	175	5	0.9722
WMS ZEN 3/2	180	195	-15	1.0833
ZEN-ZIP 5/4/3 W/HOOD	270	317	-47	1.1741
WMS EPIC 5/3 W/HD	320	369	-49	1.1531
EVO 3/2	380	587	-207	1.5447
JR EPIC 4/3	380	571	-191	1.5026
WMS EPIC 2MM FULL	390	311	79	0.7974
HEATWAVE 4/3	430	274	156	0.6372
ZEN 4/3	430	239	191	0.5558
EVO 4/3	440	623	-183	1.4159
ZENFL 3/2	450	365	85	0.8111
HEAT 4/3	460	450	10	0.9783
ZEN-ZIP 2MM FULL	470	116	354	0.2468
HEAT 3/2	500	635	-135	1.2700
WMS EPIC 3/2	610	830	-220	1.3607
WMS ELITE 3/2	650	364	286	0.5600
ZEN-ZIP 3/2	660	788	-128	1.1939
ZEN 2MM S/S FULL	680	453	227	0.6662
EPIC 2MM S/S FULL	740	607	133	0.8203
EPIC 4/3	1020	732	288	0.7176
WMS EPI 4/3	1060	1552	-492	1.4642
JR HAMMER 3/2	1220	721	499	0.5910
HAMMER 3/2	1300	1696	-396	1.3046
HAMMER S/S FULL	1490	1832	-342	1.2295
EPIC 3/2	2190	3504	-1314	1.6000
ZEN 3/2	3190	1195	1995	0.3746
ZEN-ZIP 4/3	3810	3289	521	0.8633
WMS HAMMER 3/2 FULL	6490	3673	2817	0.5659

*Error = Forecast - Actual Demand

** A/F Ratio = Actual Demand divided by Forecast

A/F-based Forecasting

Schritt 1: Estimate the distribution of the A/F ratios determined
 From the data using statistical software (e.g., Kolmogorov- Smirnov
 Or Anderson-Darling test)
 → Normal distribution hypothesis is not rejected



Step 2: Determination of the
 normal distribution

parameters

$\mu = 0.9975$

$\sigma = 0.3690$

Product description	Forecast	Actual demand [units/season]	Error	A/F Ratio
JR ZEN FL 3/2	90	140	-50	1.5556
EPIC 5/3 W/HD	120	83	37	0.6917
JR ZEN 3/2	140	143	-3	1.0214
...
ZEN 3/2	3190	1195	1995	0.3746
ZEN-ZIP 4/3	3810	3289	521	0.8633
WMS HAMMER 3/2 FULL	6490	3673	2817	0.5659
Average	941.82	824.36	117.45	0.9978
Standard deviation	1283.56	949.73	671.12	0.3638

Using Historical A/F Ratios to Model Demand

- Judgmental forecast for the Hammer 3/2: 3,200 units per season
- Determine a normally distributed demand model:

$$\mu_{Demand} = \mu_{AF} \cdot \frac{Judgemental}{forecast}$$

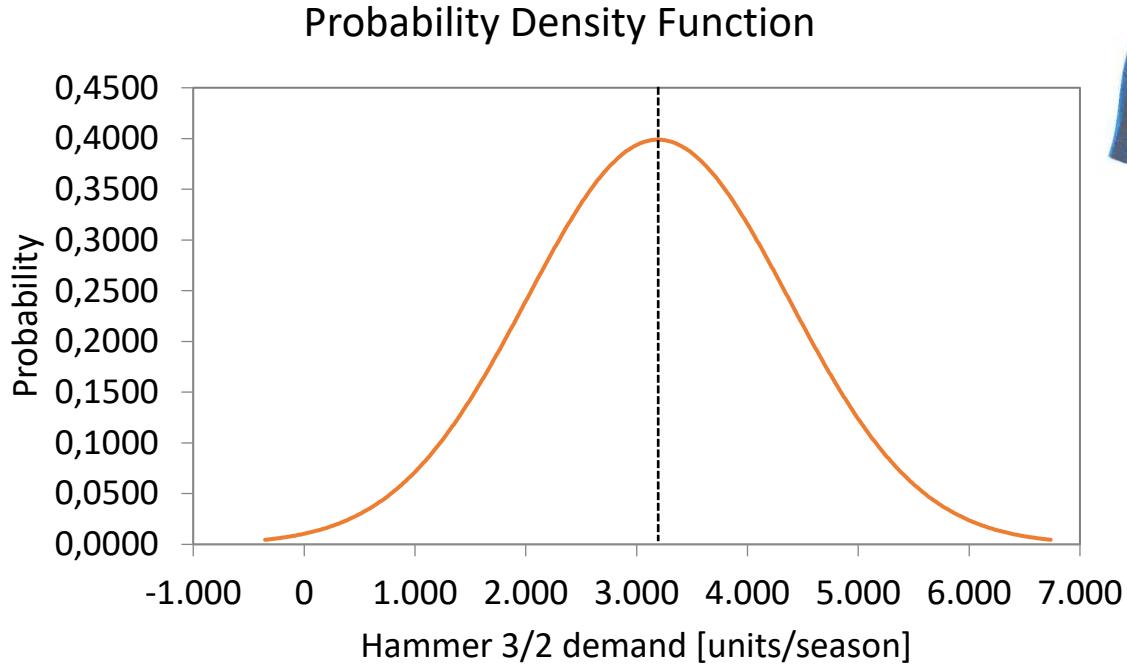
$$\sigma_{Demand} = \sigma_{AF} \cdot \frac{Judgemental}{forecast}$$

$$\mu_{\substack{Demand \\ Hammer 3/2}} = 0.9975 \cdot 3,200 \frac{\text{units}}{\text{season}} = 3,192 \frac{\text{units}}{\text{season}}$$

$$\sigma_{\substack{Demand \\ Hammer 3/2}} = 0.3690 \cdot 3,200 \frac{\text{units}}{\text{season}} = 1,181 \frac{\text{units}}{\text{season}}$$



O'Neill's Hammer 3/2 Wetsuit Demand Model



O'Neill's Hammer 3/2 Wetsuit Challenges I

- How can the limited accuracy of the judgmental forecasts be considered?
- **What is the optimal order quantity to be placed at TEC on 1st November?**

Steps to determine Q^*

Step 1: Calculate CR

Step 2: Determine z

Step 3: Convert z into Q^*



Hammer 3/2 Critical Ratio

Step 1: Determine CR

$$CR_{Hammer\ 3/2} =$$

	Hammer 2/3
Selling price	190\$/unit
Purchase price from TEC	110\$/unit
Discount price*	90\$/unit
Salvage costs	0\$/unit

*at end of season

$$Critical\ Ratio\ CR = \frac{C_u}{C_u + C_o}$$



Optimal Order Quantity Q* Hammer 3/2

Step 2: Determine Q such that there is a probability (Demand $\leq Q$) = CR

Step 2a: Determine z

Methode 1: Statistical Table method

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621

Step 2b: Covert z into Q*

Methode 2: Computer method

For example, in Excel use:

$$z = \text{NORM.S.INV}(CR)$$

NEWSVENDOR PERFORMANCE MEASURES



O'Neill's Hammer 3/2 Wetsuit Challenges II

If O'Neill orders 4,000 Hammer 3/2 wetsuits...

- ...what is expected inventory?
- ...what is expected sales (expected lost sales)?
- ...what is expected profit?
- ...what is the in-stock probability?
- ...what is the stockout probability?

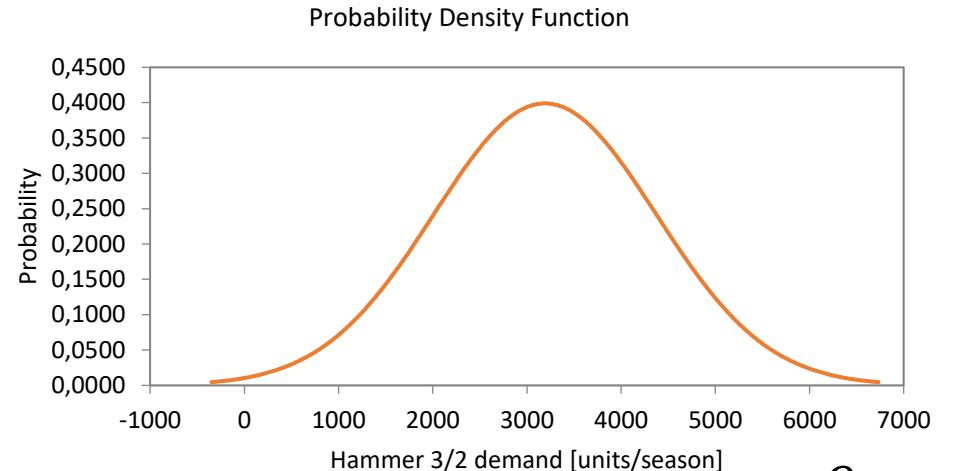


Converting Q to the Standard Normal z

$$Q = 4,000 \frac{\text{units}}{\text{season}}$$

$$z = \frac{4,000 \frac{\text{units}}{\text{season}} - 3,192 \frac{\text{units}}{\text{season}}}{1,181 \frac{\text{units}}{\text{season}}}$$

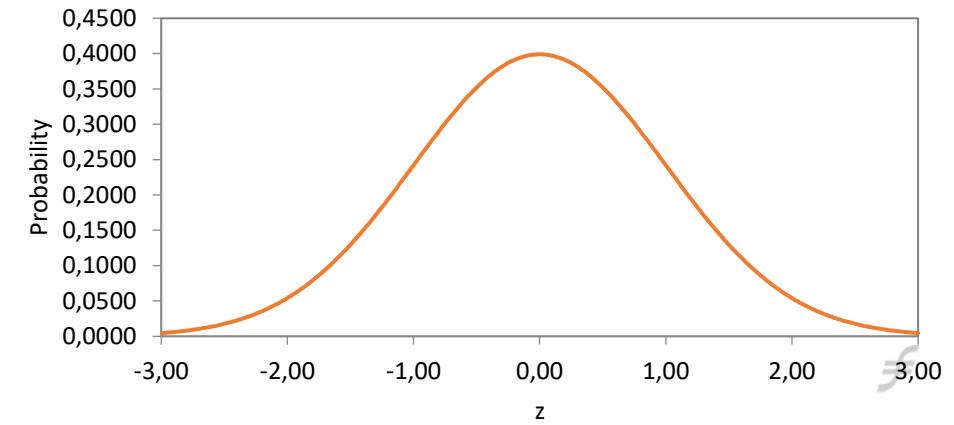
$$z = 0.6842$$



↓

$$Z = \frac{Q - \mu}{\sigma}$$

Standard Normal Probability Density Function



Expected Inventory

$$z = 0.6842$$

Standard Normal Inventory Function Table, $I(z)$

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.3989	0.4040	0.4090	0.4141	0.4193	0.4244	0.4297	0.4349	0.4402	0.4456
0.1	0.4509	0.4564	0.4618	0.4673	0.4728	0.4784	0.4840	0.4897	0.4954	0.5011
0.2	0.5069	0.5127	0.5186	0.5244	0.5304	0.5363	0.5424	0.5484	0.5545	0.5606
0.3	0.5668	0.5730	0.5792	0.5855	0.5918	0.5981	0.6045	0.6109	0.6174	0.6239
0.4	0.6304	0.6370	0.6436	0.6503	0.6569	0.6637	0.6704	0.6772	0.6840	0.6909
0.5	0.6978	0.7047	0.7117	0.7187	0.7257	0.7328	0.7399	0.7471	0.7542	0.7614
0.6	0.7687	0.7759	0.7833	0.7906	0.7980	0.8054	0.8128	0.8203	0.8278	0.8353
0.7	0.8429	0.8505	0.8581	0.8658	0.8734	0.8812	0.8889	0.8967	0.9045	0.9123
0.8	0.9202	0.9281	0.9360	0.9440	0.9520	0.9600	0.9680	0.9761	0.9842	0.9923
0.9	1.0004	1.0086	1.0168	1.0250	1.0333	1.0416	1.0499	1.0582	1.0665	1.0749
1.0	1.0833	1.0917	1.1002	1.1087	1.1172	1.1257	1.1342	1.1428	1.1514	1.1600

$$I(0.69) = 0.8353$$

- Expected inventory: expected number of units left over at the end of the selling period (before salvaging).
- Intuition: The more we order the higher is the chance that demand is lower than our order and the higher is inventory.

$$\text{Expected inventory} = \sigma \cdot I(z)$$

$$\begin{aligned} \text{Expected inventory} &= 1,181 \cdot 0.8353 \\ &= 986.50 \end{aligned}$$

Expected Sales

Expected Sales

$$= 4000 \frac{\text{units}}{\text{season}} - 986.50 \frac{\text{units}}{\text{season}}$$

$$= 3,013.50 \frac{\text{units}}{\text{season}}$$

- Expected sales: expected number of units sold in the selling period (before salvaging).
- Intuition: The more we order the lower is the chance that demand is higher than Q .

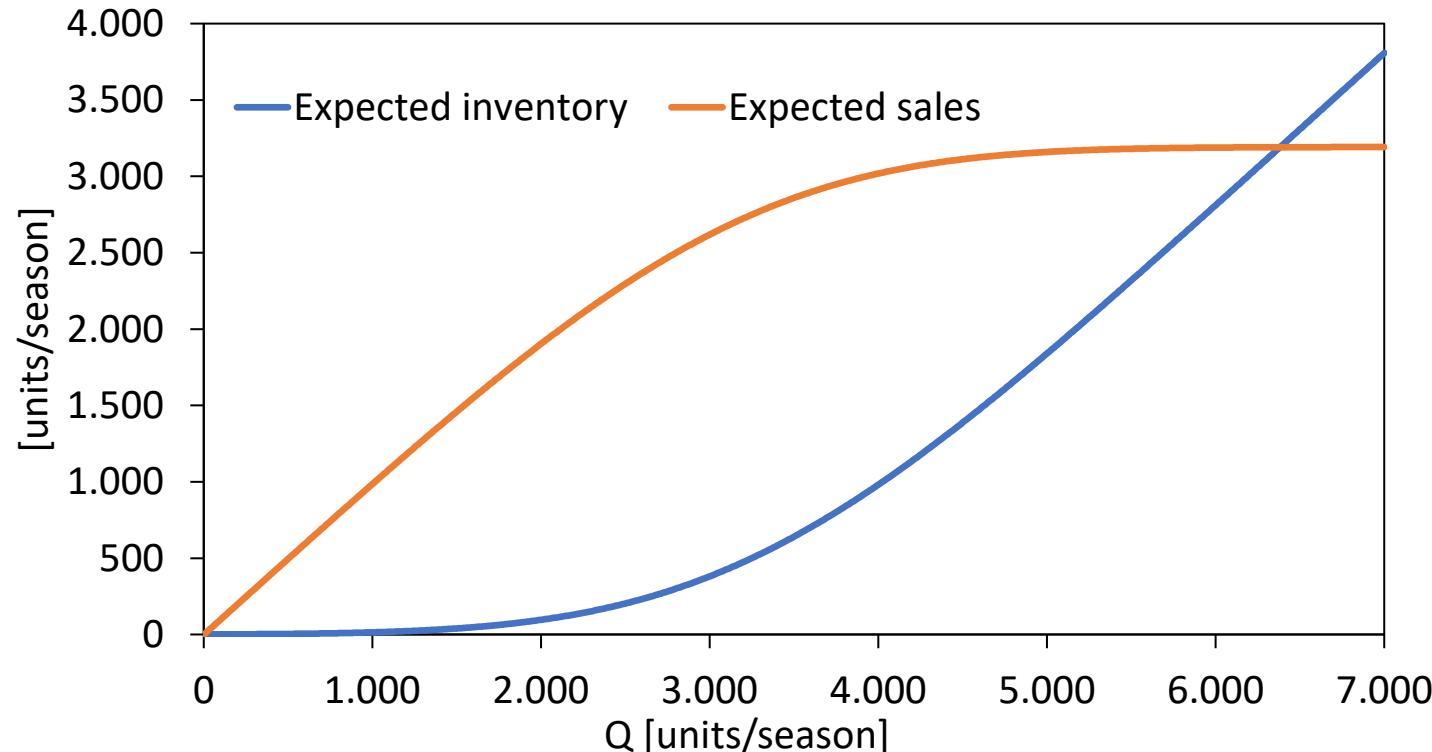
$$\text{Sales} + \text{Inventory} = Q$$

$$\frac{\text{Expected sales}}{\text{sales}} + \frac{\text{Expected inventory}}{\text{inventory}} = Q$$

$$\frac{\text{Expected sales}}{\text{sales}} = Q - \frac{\text{Expected inventory}}{\text{inventory}}$$



Expected Inventory and Sales as a Function of Q



Expected Profit

Expected profit

$$= 190 \frac{\$}{unit} \cdot 3,013.50 \frac{units}{season}$$

$$+ 90 \frac{\$}{unit} \cdot 986.50 \frac{units}{season}$$

$$- 110 \frac{\$}{unit} \cdot 4,000 \frac{units}{season}$$

$$= 221,350 \frac{\$}{season}$$

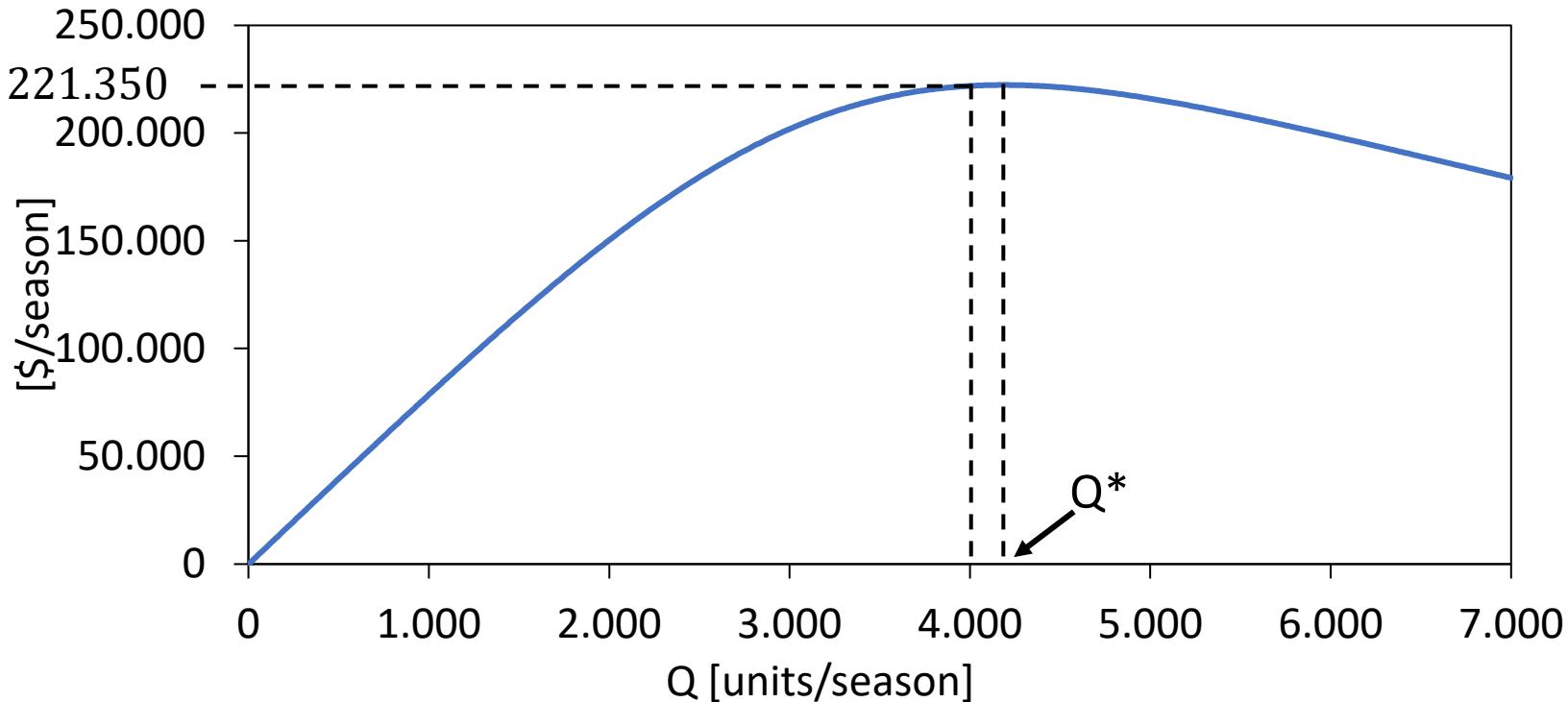
Expected profit

$$= \frac{\text{Selling price}}{\text{Purchase price}} \cdot \frac{\text{Expected sales}}{\text{Expected inventory}}$$

$$+ \frac{\text{Discount price}}{\text{Purchase price}} \cdot \frac{\text{Expected sales}}{\text{Expected inventory}}$$

$$- \frac{\text{Purchase price}}{\text{Purchase price}} \cdot Q$$

Expected Profit as a Function of Q



In-stock Probability

$$Q = 4,000 \frac{\text{units}}{\text{season}}$$

$$z = \frac{4,000 \frac{\text{units}}{\text{season}} - 3,192 \frac{\text{units}}{\text{season}}}{1,181 \frac{\text{units}}{\text{season}}}$$

$$z = 0.6842$$

Using the standard normal distribution function table:

$$\begin{aligned} \text{In-stock probability} &= \\ F(0.6842) &= 0.7549 \end{aligned}$$

- In-stock probability: probability all demand is satisfied
- All demand is satisfied if demand is the order quantity, Q, or smaller
- The distribution function tells us the probability that demand is Q or smaller!

$$\text{In-stock probability} = F(Q)$$

Stockout Probability

$$Q = 4,000 \frac{\text{units}}{\text{season}}$$

$$z = \frac{4,000 \frac{\text{units}}{\text{season}} - 3,192 \frac{\text{units}}{\text{season}}}{1,181 \frac{\text{units}}{\text{season}}}$$

$$z = 0.6842$$

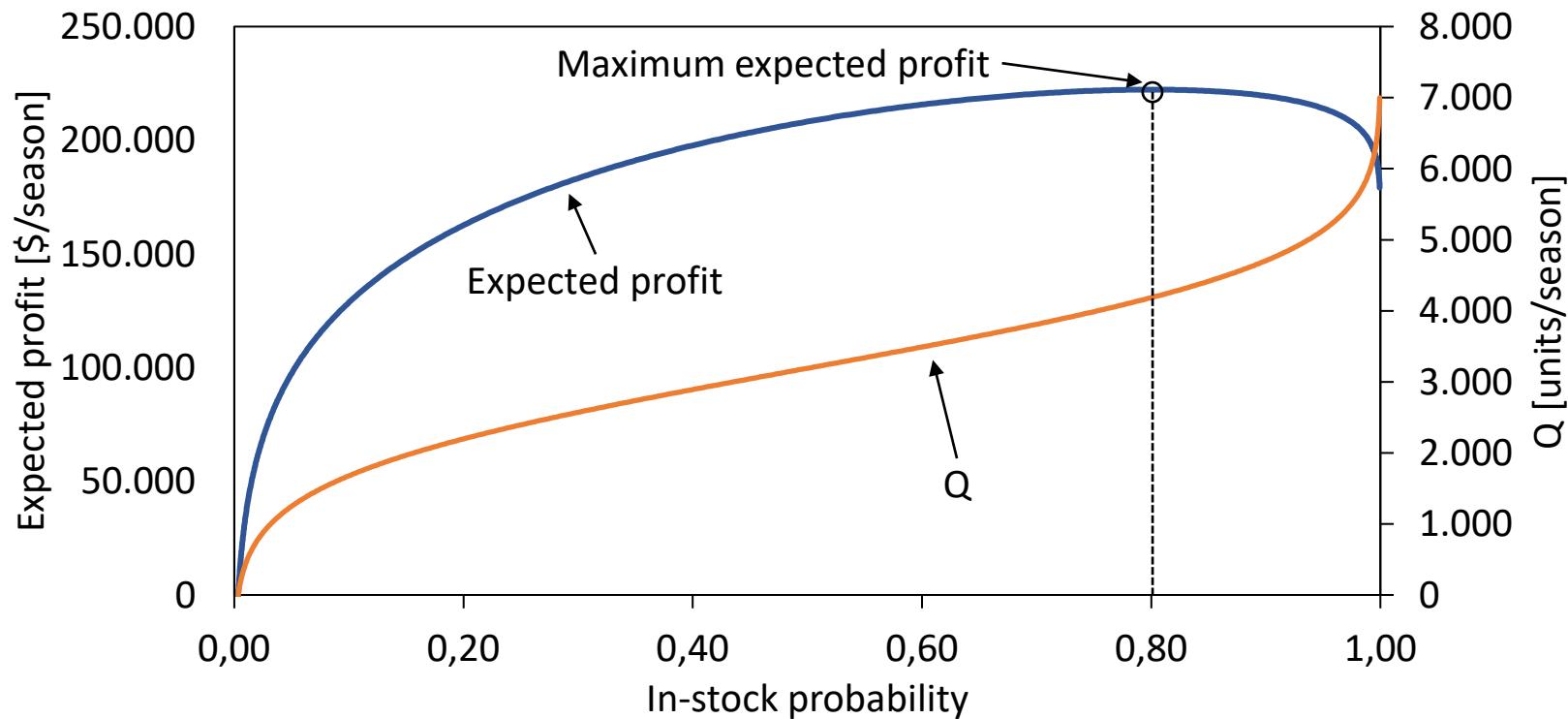
Using the standard normal distribution function table

$$\begin{aligned} \text{Stockout probability} &= 1 - F(0.6842) \\ &= 1 - 0.7549 = 0.2451 \end{aligned}$$

- Stockout probability: probability that some demand is **not** satisfied
- Some demand is not satisfied if demand exceeds the order quantity, Q
- Thus

$$\begin{aligned} \text{Stockout probability} &= 1 - F(Q) \\ &= 1 - \text{In-stock probability} \end{aligned}$$

Expected Profit and In-stock Probability Trade-off



O'Neill's Hammer 3/2 Wetsuit Challenge III

- How many Hammer 3/2 should O'Neill order to guarantee an in-stock probability of 99%?



Choose Q to Achieve 99% In-stock Probability

In the Standard Normal Distribution Function Table, we find:

$$F(2.32) = 0.9898$$

$$F(2.33) = 0.9901$$

Choose $z = 2.33$ to satisfy the 99% in-stock probability constraint

$$Q = 3,192 \frac{\text{units}}{\text{season}} + 2.33 \cdot 1,181 \frac{\text{units}}{\text{season}}$$

$$Q = 5,944 \frac{\text{units}}{\text{season}}$$

- Step 1:
Find the z-statistic that yields the target in-stock probability
- Step 2:
Convert the z-statistic into an order quantity for the actual demand distribution

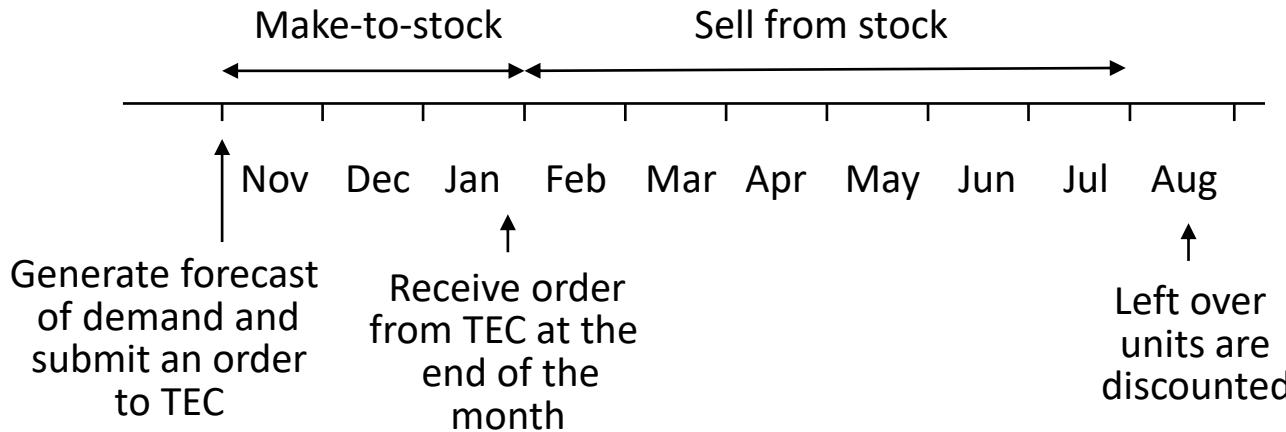
$$Q = \mu + z \cdot \sigma$$

MISMATCH COSTS IN THE NEWSVENDOR MODEL



The Newsvendor Dilemma

- “Newsvendors” can manage, but not avoid, the dilemma:
 - Order too much → inventory is left over at the end of the selling period
 - Order too little → opportunity cost of lost sales
- Dilemma arises because the newsvendor commits to its entire supply before demand occurs: Make-to-Stock system



Maximum Profit

- Maximum profit (=highest possible expected profit) is used as benchmark.
- The maximum profit is the profit without any mismatch costs, i.e., every unit is sold and there are no lost sales:

$$\begin{aligned} \text{Maximum profit} &= \text{Expected demand } \mu \cdot \\ &\quad (\text{Selling price} - \text{Purchase price}) \end{aligned}$$

$$\text{Maximum profit} = 3,192 \frac{\text{units}}{\text{season}} \cdot (190 - 110) \frac{\$}{\text{unit}} = 255,360 \frac{\$}{\text{season}}$$

- Maximum profit depends only on two factors: expected demand and profit per unit sold.

The Demand-Supply Mismatch Costs

- Maximum profit does not suffer from the two types of mismatch costs:
 - Cost of inventory (because of ordering too much)
 - Opportunity cost of stockouts (because of ordering too little)
- Expected profit suffers from mismatch costs, thus:
$$\text{Expected profit} = \text{Maximum profit} - \text{Mismatch costs}$$
- Rearranging for mismatch costs yields:
$$\text{Mismatch costs} = \text{Maximum profit} - \text{Expected profit}$$

Expected Profit for Hammer 3/2 Q*

$$\mu_{\text{Demand Hammer } 3/2} = 3,192 \frac{\text{units}}{\text{season}}$$

$$\sigma_{\text{Demand Hammer } 3/2} = 1,181 \frac{\text{units}}{\text{season}}$$

$$Q^* = 4,196 \frac{\text{units}}{\text{season}}$$

$$\text{Selling price} = 190 \frac{\$}{\text{unit}}$$

$$\text{Purchase price} = 110 \frac{\$}{\text{unit}}$$

$$\text{Discount price} = 90 \frac{\$}{\text{unit}}$$

Hammer 3/2 Mismatch Costs

- For the Hammer 3/2, mismatch cost are

$$\$255,360 - \$222,307 = \$33,053 \text{ per season.}$$

- Alternative way to determine mismatch costs:

$$\begin{aligned} \text{Mismatch costs} &= c_u(\mu - \text{expected sales}) + c_o \cdot \text{expected inventory} \\ &= c_u \cdot \text{expected lost sales} + c_o \cdot \text{expected inventory} \end{aligned}$$

$$\begin{aligned} \text{Mismatch costs} &= 80 \frac{\$}{\text{unit}} (3,192 - 3,062.27) \frac{\text{unit}}{\text{season}} \\ &+ 20 \frac{\$}{\text{unit}} \cdot 1,133.73 \frac{\text{unit}}{\text{season}} = 33,053 \frac{\$}{\text{season}} \end{aligned}$$

Significance of Mismatch Costs

- Hammer 3/2 mismatch costs \$33,053 per season account for:

$$\frac{33,053 \frac{\$}{\text{season}}}{222,307.34 \frac{\$}{\text{season}}} = 14.87\% \text{ of expected profits}$$

$$\frac{33,053 \frac{\$}{\text{season}}}{683,867.34 \frac{\$}{\text{season}}} = 4.83\% \text{ of expected revenues}$$

- Typical net profit in the apparel industry: 2% to 5% of revenue
→ Mismatch costs are significant

Factors Impacting Mismatch Costs

Mismatch costs depend on:

- Demand variability (measured as the coefficient of variation)

AND

- $\text{Critical ratio} = \frac{c_u}{c_o + c_u}$

What do you think?

- If demand variability increases, what happens to mismatch costs?
- If the critical ratio increases, what happens to mismatch costs?

Mismatch Cost...

... as percentage of the maximum profit

Coefficient of variation	Critical ratio					
	0.4	0.5	0.6	0.7	0.8	0.9
0.10	10%	8%	6%	5%	3%	2%
0.25	24%	20%	16%	12%	9%	5%
0.40	39%	32%	26%	20%	14%	8%
0.55	53%	44%	35%	27%	19%	11%
0.70	68%	56%	45%	35%	24%	14%
0.85	82%	68%	55%	42%	30%	17%
1.00	97%	80%	64%	50%	35%	19%

STRATEGIES TO MANAGE A NEWSVENDOR SYSTEM



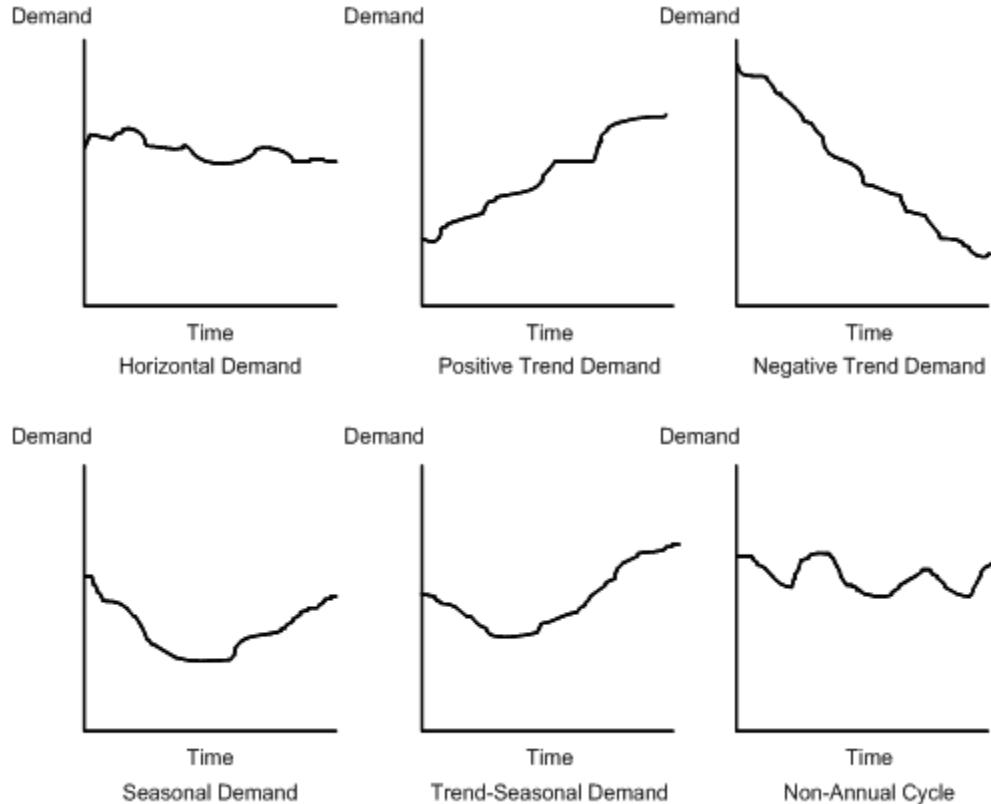
Strategies for the Newsvendor

- Do not sell the product
- Increase the profit margin, thus increase the critical ratio
- Reduce demand uncertainty (improve forecasting, product pooling)
- Quick response: order or produce late in the season
- Make-to-order



Improve Demand Forecasting

- Many improvement options, e.g.
 - Analyze demand patterns and use the most adequate forecasting technique
 - Track forecasting accuracy and learn from errors
 - ...
- Limit to this strategy: most forecasts are still wrong



Demand Pooling

- Reduce the variety offered to customers by combining, or pooling, similar products
- Using statistical economies of scale: aggregating demand into a large scale tends to reduce uncertainty (coefficient of variation ↓).

Impact of demand pooling of two products with identical demand distribution:

Expected pooled demand:

$$\mu_{pooled} = 2 \cdot \mu$$

Standard deviation pooled demand:

$$\sigma_{pooled} = \sqrt{2(1 + Correlation)} \cdot \sigma$$

Hammer 3/2 Demand Pooling

$$\mu_{pooled} = 2 \cdot 3,192 = 6,384 \frac{\text{units}}{\text{season}}$$

$$\begin{aligned}\sigma_{pooled} &= \sqrt{2(1 + 0)} \cdot 1,181 \frac{\text{units}}{\text{season}} \\ &= 1,670.19 \frac{\text{units}}{\text{season}}\end{aligned}$$

$$CV = \frac{1,670.19}{6,384} = 0.2616$$

Selling the Hammer 3/2 to two customer segments with identically distributed, uncorrelated demand:

Surfing



Scuba diving



$$\mu = 3,192 \frac{\text{units}}{\text{season}} \quad \mu = 3,192 \frac{\text{units}}{\text{season}}$$

$$\sigma = 1,181 \frac{\text{units}}{\text{season}} \quad \sigma = 1,181 \frac{\text{units}}{\text{season}}$$

$$CV = \frac{1,181}{3,192} = 0.3700$$

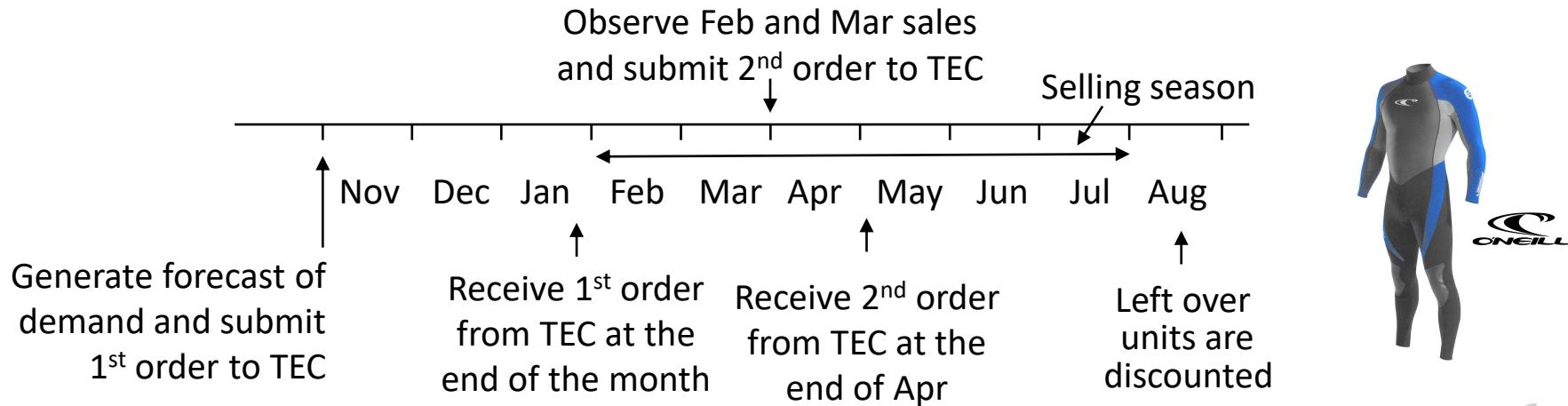
Quick Response

- Capability to respond to updated demand information, e.g. by using reactive capacity
- Reactive capacity allows the firm to react to changes in its demand forecast
- Examples of reactive capacity
 - Spare capacity
 - Flexible suppliers



Quick Response with Reactive Capacity at O'Neill

- TEC allows O'Neill to place a second order with fast 4-week delivery.
- TEC charges a premium of 20 % per unit (\$132 vs. \$110) in the second order.
- There are no restrictions imposed on the 2nd order quantity.
- O'Neill's updated forecast of total season sales is (nearly) perfect end of March.



Impact of O'Neill's 2nd Order Option on the CR

- One order only
- Economics
 - Selling price: 190 \$/unit
 - Purchase price: 110 \$/unit
 - Discount price: 90 \$/unit
- 1st order 1st November, 2nd order 31st March if updated demand forecast higher than Q of first order
- Purchase price 2nd order: 132 \$/unit
- Overage cost do not change
- Underage cost:
 $132 - 110 = 22 \text{ \$/unit}$

$$\begin{aligned} \text{Critical Ratio CR} &= \\ \frac{(190 - 110)}{(190 - 110) + (110 - 90)} &= 0.8 \end{aligned}$$

$$\begin{aligned} \text{Critical Ratio CR} &= \\ \frac{(132 - 110)}{(132 - 110) + (110 - 90)} &= 0.5238 \end{aligned}$$

Impact of O'Neill's 2nd Order Option on the Exp. Profit

	No quick response	Quick response
Q* first order	4,196	3,263 units/season
Expected Q second order	-	436.43 units/season
Expected inventory	1,133.73	507.43 units/season
Expected sales	3,062.27	2,755.57 units/season
Expected lost sales	129.73	0.00 units/season
Maximum profit	255,360.00	255,360 \$/season
Expected profit	222,307	235,610 \$/season
Mismatch costs	33,053	19,750 \$/season

Make-to-Order

- An item's production begins after the customer for the item is known.
- Units are generally delivered to a customer immediately after production is completed, thereby not spending time in inventory.
- Opposite of make-to-stock



Make-to-Order Pros and Cons

Make-to-Order	
Advantages	Disadvantages
<ul style="list-style-type: none">▪ No leftover (finished-goods) inventory▪ No lost sales▪ Mismatch costs can be entirely eliminated <p>→ Particularly firms with high CVs and low critical ratios benefit from switching to Make-to-Order.</p>	<ul style="list-style-type: none">▪ Customers must wait for order to be fulfilled (willingness depends on products) → queuing system with potentially high wait time▪ Component inventory is necessary → some inventory risk remains▪ If production is far away → high shipment costs (for each order)

Make-to-Order can work...

- ... if customers are patient.
- ... customers have a strong preference for variety.
- ... production is reasonably quick.
- ... leftover inventory is expensive.



Hybrid Production Systems

- **Mass customization** – each customer's order is unique, customized to her/his exact preference.
- **Assemble-to-order** – a make-to-order system in which a product is assembled from a set of standardized modular components after an order is received.



MASS CUSTOMIZATION

Good for blue jeans:
Good for education.



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2023 Operations Management

INVENTORY MANAGEMENT WITH FREQUENT ORDERS



THE ORDER-UP-TO MODEL



Medtronic

- Designer and manufacturer of medical devices
- Well known for cardiac rhythm products, in particular, pacemakers
- Founded in 1949
- [More](#) about Medtronic's history



Pacemaker InSync ICD Model 7272

- Inventory held at three levels:
manufacturing plants, distribution
centers, field locations
- Our focus:
The distribution center (DC) in
Mounds View, Minnesota
- Demand for a pacemaker occurs
when it is implanted in a patient
via surgery

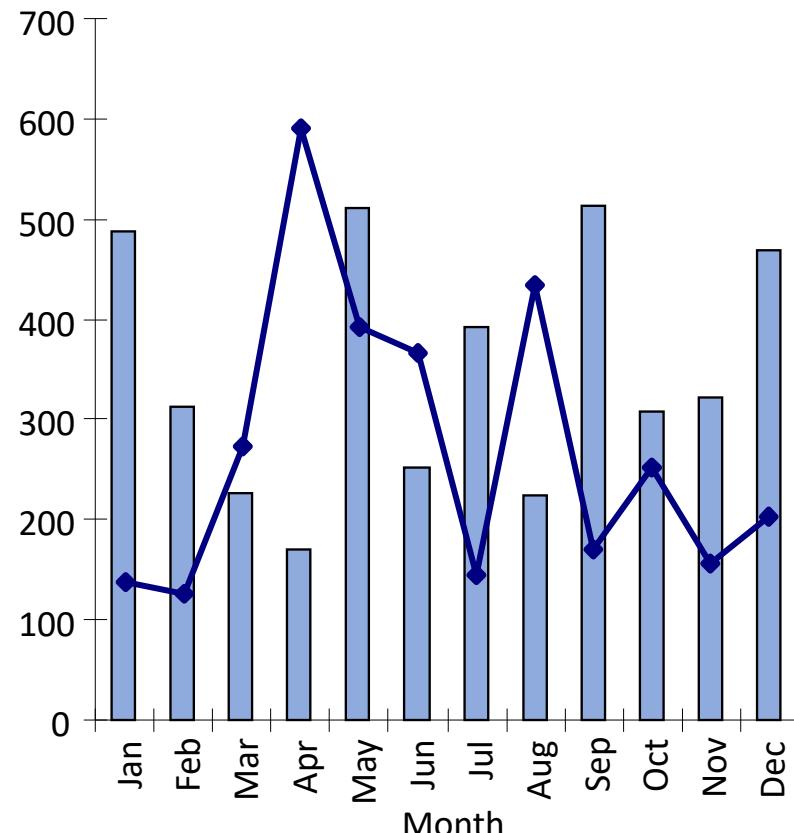


Sales and Replenishment

- Sales representatives attend each surgery and deliver precisely the model that the surgeon requests from their trunk or consignment inventory.
- Sales reps replenish their inventory by calling an order to Medtronic's distribution center. If the requested model is in stock at the distribution center, they receive it typically one day later (rarely more than two days).
- The Mounds View distribution center requests replenishments from the production facilities on a weekly basis. With the InSync pacemaker, there is a three-week lead time to receive an order.

InSync Demand and Inventory at DC

- Average monthly demand = 349 units
- Standard deviation of monthly demand = 122.4
- Considerable variation
 $CV = \frac{122.4}{349} = 0.35$
- No indication for seasonality



Columns: Demand in units/month
Line: end-of-month inventory in units



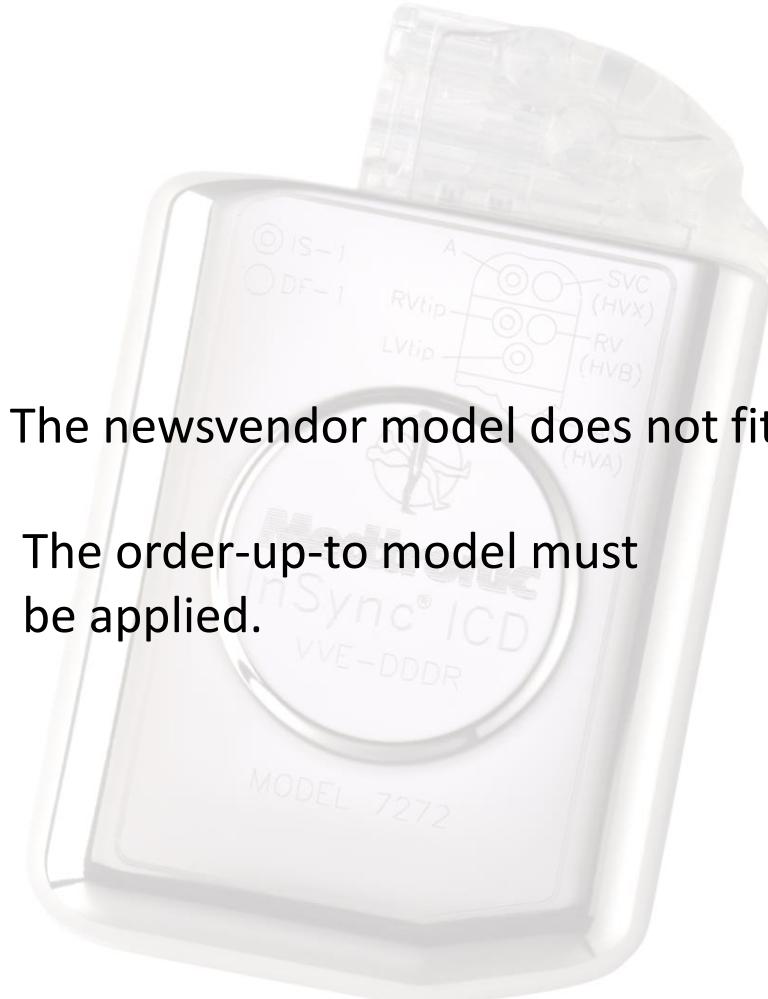
Pacemaker InSync ICD Model 7272 Challenge

- What is the optimal order quantity at the distribution center (DC) in Mounds View, Minnesota?



Pacemaker Characteristics

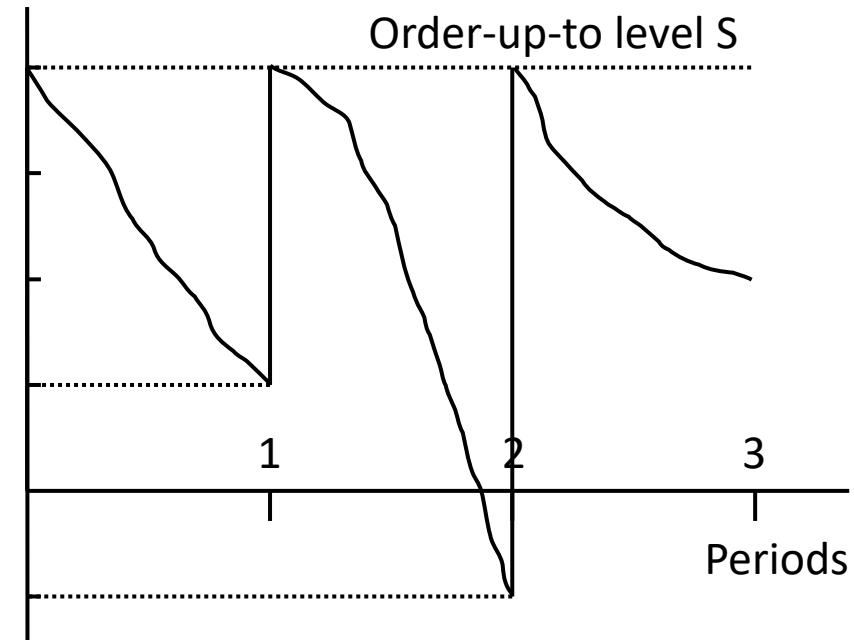
- InSync pacemakers
 - Have very high gross margins
 - Do not perish
 - Have a rather long life cycle and can be replenished multiple times
- Target:
 - Minimize inventory investment while maintaining a very high service target (99.9% in-stock probability)



- ⇒ The newsvendor model does not fit.
- ⇒ The order-up-to model must be applied.

The Order-up-to Model

- Considers a single product that is sold over a long time horizon
- Demand follows a random distribution
- Opportunities to replenish inventory occur in regular time intervals
- Time between two ordering opportunities is called period
- All periods are of the same duration (e.g., one hour, one week, one month)
- Each period's demand is the outcome of a single random distribution



Order-up-to Model Terminology

- On-order inventory/pipeline inventory: number of units that have been ordered but have not been received
 - On-hand inventory: number of units physically in inventory ready to serve demand
 - Backorder: total amount of demand that has not been satisfied
 - All backordered demand is eventually filled; there are no lost sales!
 - Back-ordered demand is fulfilled once inventory is available again
 - Inventory level = On-hand inventory – Backorder
 - Inventory position $= \text{On-order inventory} + \text{Inventory level}$
 $= \text{On-order inventory} + \text{On-hand inventory} - \text{Backorder}$
 - Order up-to level, S
 - maximum inventory position we are allowed, also called base stock level



Order-up-to Model Implementation

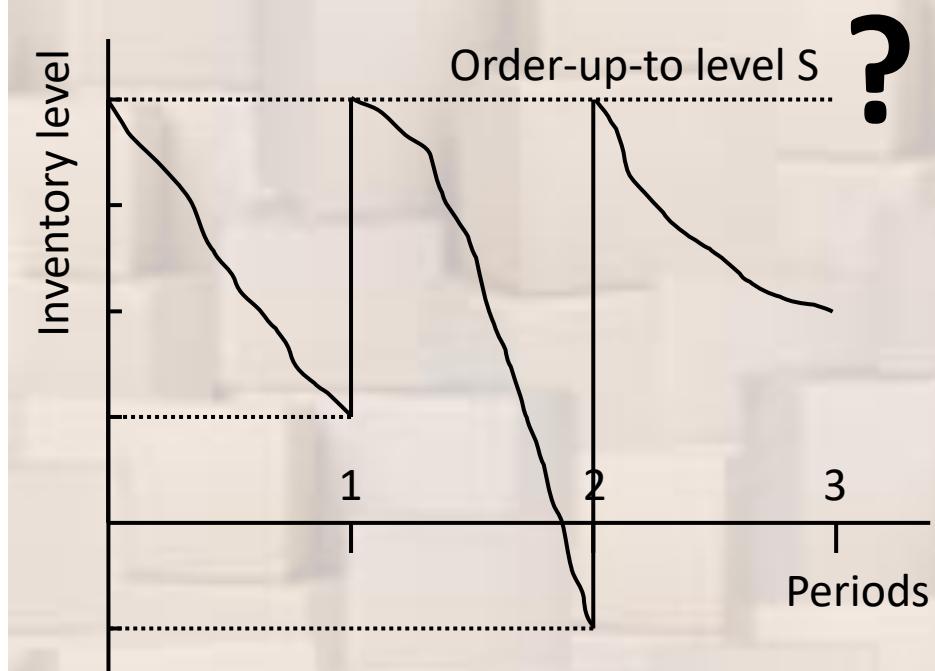
- The implementation of the order-up-to policy is intuitive:
 - Check inventory position at the beginning of the period
 - If inventory position < S (order-up-to level) \Rightarrow order enough to fill inventory position up to S
- Therefore, each period's order quantity = $S - \text{Inventory position}$
 - Consider a sales rep's trunk inventory of the InSync pacemaker :
 - At the beginning of a period an inventory level of -4 units is observed (4 units are back-ordered)
 - On-order inventory is 1 unit
 - Suppose that the chosen order-up-to level is $S = 3$
 - How many units should the sales rep order at the period's beginning?
- Note: A period's order quantity = the previous period's demand
 - \Rightarrow Pull system because inventory is ordered in response to demand
 - \Rightarrow 1-for-1 ordering policy

How to set the order-up-to level S?

Intuitive relations:

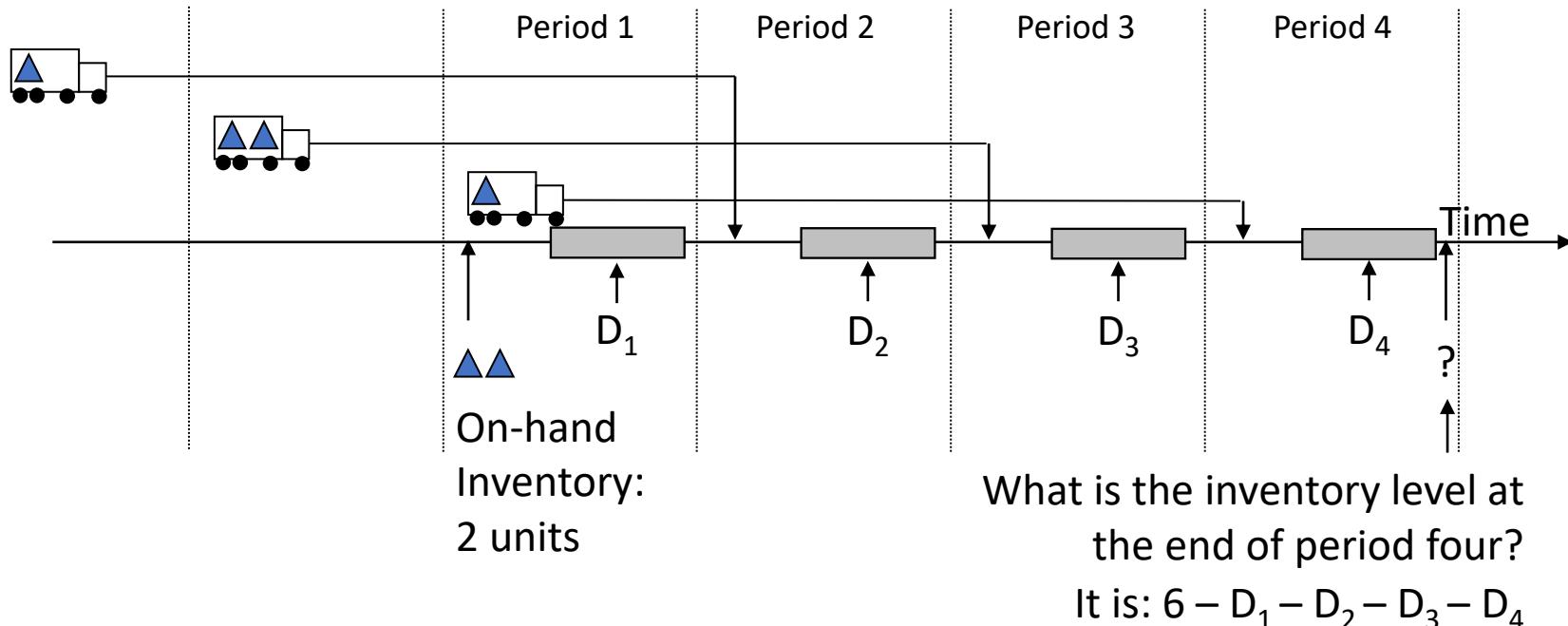
- High S
 - ⇒ high inventory level
 - ⇒ high service level
 - ⇒ high inventory holding costs
- Low S
 - ⇒ low inventory level
 - ⇒ low service level
 - ⇒ low inventory holding costs

⇒ Important to understand the inventory level (= on-hand inventory – backorder) in the order-up-to model



The End-of-Period Inventory Level

Suppose: $S = 6$, lead time $l = 3$



Pacemaker InSync ICD Model 7272 Challenge

Adapted to the order-up-to model:

- What is the optimal order-up-to level at the distribution center if Medtronics service target is 99.9 % in-stock probability?



Order-up-to Level at the Distribution Center

- The Mounds View distribution center requests replenishments from the production facilities on a weekly basis. With the InSync pacemaker, there is a three week lead time to receive an order.
 - one period = one week
 - (production) lead time = three weeks = three periods
- Demand distributions required for
 - one (focal) period and
 - $I+1$ periods (= 4 periods)



Distribution Center Demand Distribution

Average weekly demand =

Std. dev weekly demand =

We know:

- Average monthly demand:
349 units/month
- Std. dev. of monthly demand:
122.38 units/month
- One period = one week
- $I+1$ periods = 4 weeks

We need to convert demand distribution

- from one long period (months) to
- n short periods (weeks).

We know that there are $52/12 = 4.33$ weeks per month.

Converting a Demand Distribution

From a long period length (e.g., a month) into n short periods (e.g., 4.33 weeks):

$$\mu_{\text{Demand}_{\text{short Period}}} = \frac{\mu_{\text{Demand}_{\text{long Period}}}}{n}$$

$$\sigma_{\text{Demand}_{\text{short Period}}} = \frac{\sigma_{\text{Demand}_{\text{long Period}}}}{\sqrt{n}}$$

From n short period length (e.g., 4.33 weeks) into one long period (e.g., a month):

$$\mu_{\text{Demand}_{\text{long Period}}} = n \cdot \mu_{\text{Demand}_{\text{short Period}}}$$

$$\sigma_{\text{Demand}_{\text{long Period}}} = \sqrt{n} \cdot \sigma_{\text{Demand}_{\text{short Period}}}$$

The above formulas assume the same demand distribution represents demand in each period and demands across periods are independent of each other.

99.9% In-stock Probability at the Distribution Center

- The distribution center is in stock in a period if all demand is satisfied in that period.
- If on-hand inventory at the distribution center is 50 pacemakers at the beginning of a period and demand is 50 units in that period, the center is in stock even though the end of period inventory is 0.
- The center is not in stock if one or more units are backordered at the end of the period, which means that the inventory level at the end of the period is negative.



99.9% In-stock Probability at the Distribution Center

- Determine the z-value
- Determine order-up-to level S

*End-of-period inventory =
S – demand over $l + 1$ periods*

*In-stock probability =
prob(demand over $l + 1$ periods $\leq S$)*

This formula is a good approximation for high in-stock probabilities.

Order-up-to level

$$S = \mu_{l+1} + z \cdot \sigma_{l+1}$$

$$\mu_{4 \text{ weeks}} = 4 \cdot 80.54 = 322.15 \frac{\text{units}}{4 \text{ weeks}}$$

$$\sigma_{4 \text{ weeks}} = \sqrt{4} \cdot 58.79 = 117.58 \frac{\text{units}}{4 \text{ weeks}}$$

PERFORMANCE MEASURES



InSync ICD Model 7272 Challenges II

Suppose that $S = 625$ at the DC,
what is...

- ...expected on-hand inventory?
- ...in-stock probability?
- ...stockout probability?
- ...expected on-order inventory?
- ...expected back order?



Expected On-hand Inventory

- In general, the following is true:
On-hand inventory at the end of a period equals the order-up-to level S , minus demand over $l+1$ periods, or zero, whichever is greater.
- The order-up-to level, S , is like an order quantity in the newsvendor model: it determines the amount of inventory that is available in the system before demand occurs.
- The available inventory is reduced by demand, leaving the firm potentially with some inventory.
- In the newsvendor model, the inventory after demand is called leftover inventory. In the order-up-to model, it is called on-hand inventory.

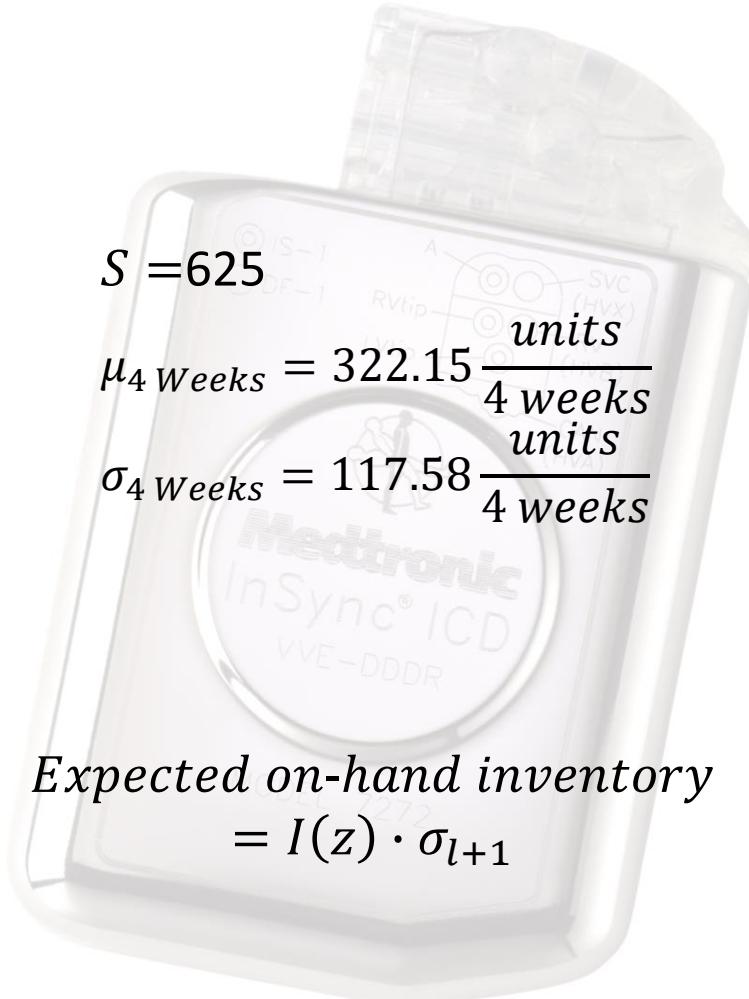
Expected On-hand Inventory I

$$S = 625$$

$$\mu_{4 \text{ weeks}} = 322.15 \frac{\text{units}}{4 \text{ weeks}}$$

$$\sigma_{4 \text{ weeks}} = 117.58 \frac{\text{units}}{4 \text{ weeks}}$$

Expected on-hand inventory
 $= I(z) \cdot \sigma_{l+1}$



Stockout and In-stock Probabilities

- The in-stock probability is the probability all demand is filled in a period:

$$\text{In-stock probability} = \text{prob}(\text{demand over } l + 1 \text{ periods} \leq S)$$

- The stockout probability is the probability at least one unit is backordered in a period

$$\text{Stockout probability} = \text{prob}(\text{demand over } l + 1 \text{ periods} > S)$$

$$= 1 - \text{prob}(\text{demand over } l + 1 \text{ periods} \leq S)$$

In-stock and Stockout Probabilities at the DC for S = 625

- Step 1: Determine demand distribution parameters for l+1 periods
- Step 2: Evaluate the z-statistic
- Step 3: Use the z-statistic to look up in the standard normal distribution function table the probability the standard normal demand is z or lower.
- Step 4: In-stock probability = F(z) and stockout probability = 1 - F(z)
 - With S = 625, the in-stock probability is 0.9951 → 99.51%
 - With S = 625, the stockout probability is 1-0.9951 = 0.0049 → 0.49%

$$\mu_{4 \text{ weeks}} = 322.15 \frac{\text{units}}{4 \text{ weeks}} \quad \sigma_{4 \text{ weeks}} = 117.58 \frac{\text{units}}{4 \text{ weeks}}$$

$$z = \frac{S - \mu}{\sigma} = \frac{625 - 322.15}{117.58} = 2.58$$

z	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.3	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974

Expected On-order Inventory

$$\mu_{\text{weekly demand}} = 80.54 \frac{\text{units}}{\text{week}}$$

- Expected on-order inventory (also pipeline inventory), is the average amount of inventory on order at any given time

Flow time = lead time = 3 weeks

- We use Little's law

$$\text{Inventory} = \text{Flow rate} \cdot \text{Flow time}$$

- to evaluate expected on-order inventory:

On-order inventory =

$$\text{On-order inventory} = \mu_{\text{period}} \cdot l$$

Expected Back Order

- Expected back order is the expected number of backorders at the end of any period.
- If demand over $l+1$ periods is larger than S , inventory level at the end of the period is negative (=backorders).

*Expected backorders =
Expected inventory
+ demand over $l + 1$ periods
- S)*

Expected Back Order

$$S = 625$$

$$\mu_{4 \text{ weeks}} = 322.15 \frac{\text{units}}{4 \text{ weeks}}$$

$$\sigma_{4 \text{ weeks}} = 117.58 \frac{\text{units}}{4 \text{ weeks}}$$

*Expected on-hand inventory
= 303.54*

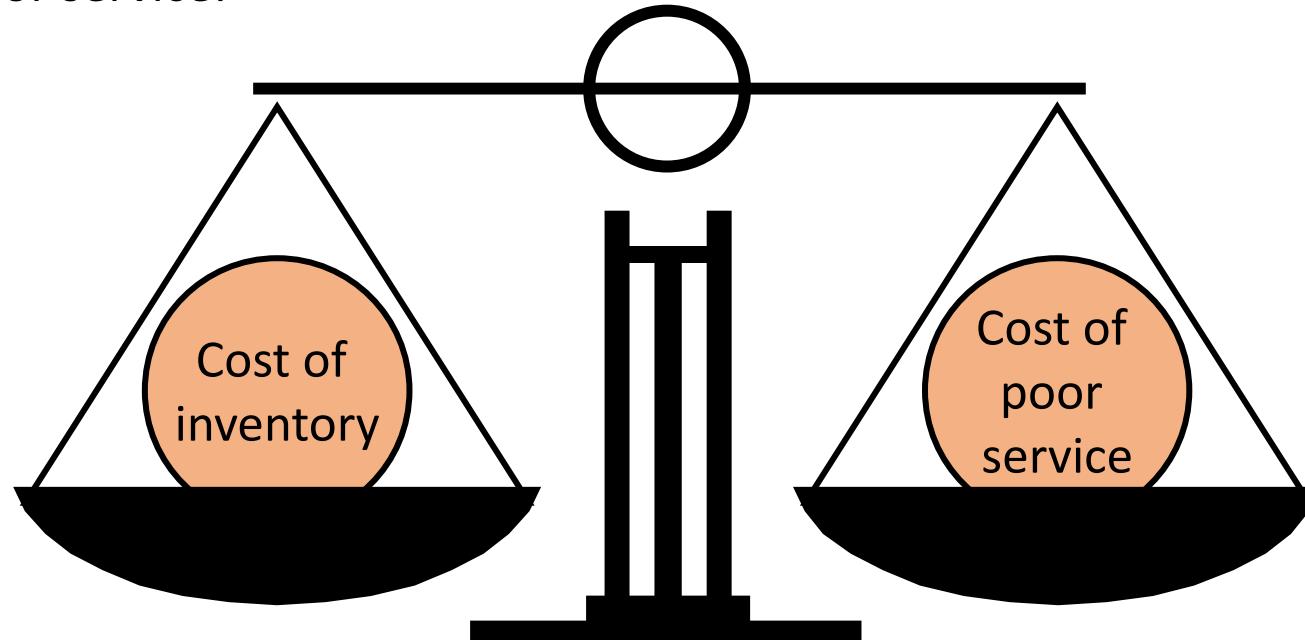
*Expected backorders
= Expected inventory
+ demand over $l + 1$ periods – S*

CHOOSING AN APPROPRIATE SERVICE LEVEL



Choosing an Appropriate Service Level

An appropriate service level minimizes the cost of holding inventory plus the cost of poor service:



Cost of Holding Inventory

- Holding cost: cost of holding one unit of inventory over one year (as % of the item's cost)
- Suppose annual holding cost rate (h) = 35% and gross margin = 75%
- Then, the (purchase) cost of one unit is
$$(1 - 0.75) \cdot \text{selling price} = 0.25 \cdot \text{selling price}$$
- Annual holding cost are then
$$0.35 \cdot 0.25 \cdot \text{selling price} = 0.0875 \cdot \text{selling price}$$
- Given 260 days per year, the daily holding cost of an item is:
$$(0.0875 \cdot \text{selling price}) / 260 = 0.000337 \cdot \text{selling price}$$

Cost of Poor Service

- Cost of poor service
 - A natural measure may be the cost of a backorder (b)
 - Often assumed to be the cost of lost sale (i.e., the gross margin)
- Suppose gross margin = 75%
then, the “backorder penalty cost” is $b = 0.75 \cdot \text{selling price}$

Choosing an Appropriate Service Level

- “Too-much versus Too-little” challenge
 - S too high \rightarrow excessive holding cost
 - S too low \rightarrow excessive backorder costs
- Using the newsvendor logic with $C_O = h$ and $C_U = b$
 - Newsvendor:
choose Q such that the critical ratio equals the probability demand $\leq Q$
 - Analogously in the order-up-to model:
choose S such that $\text{Prob}(\text{Demand over } l+1 \text{ periods} \leq S)$
 - Condition for minimizing holding and backorder costs:

$$\text{Prob}(\text{Demand over } l + 1 \text{ periods} \leq S) = \frac{b}{b + h}$$

Optimal Service Level

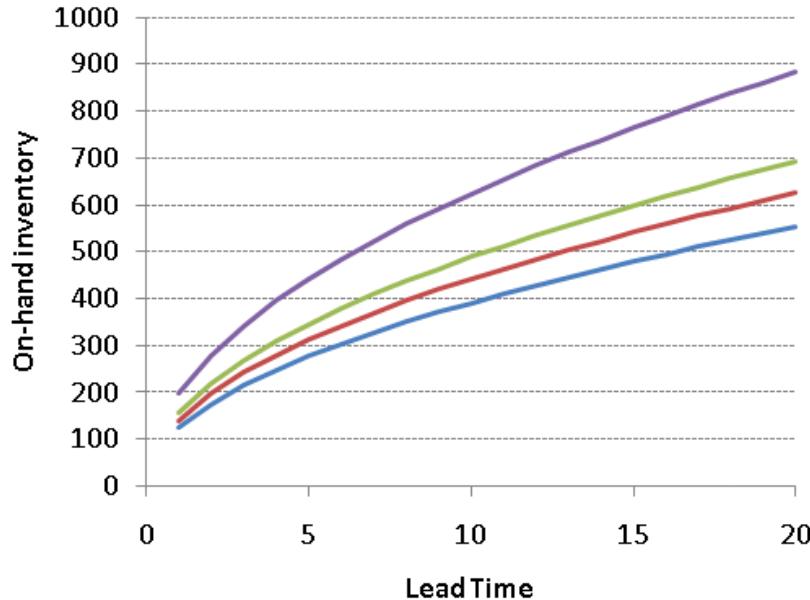
- Recall:
 - The holding cost in a period of an item: $h = 0.000337 \cdot \text{selling price}$
 - Backorder penalty cost in a period: $b = 0.75 \cdot \text{selling price}$

$$\text{Critical ratio} = \frac{b}{b + h} =$$

- Note:

Although holding inventory is not cheap (35%/year), we should maintain an extremely high in-stock probability of 99.96%.

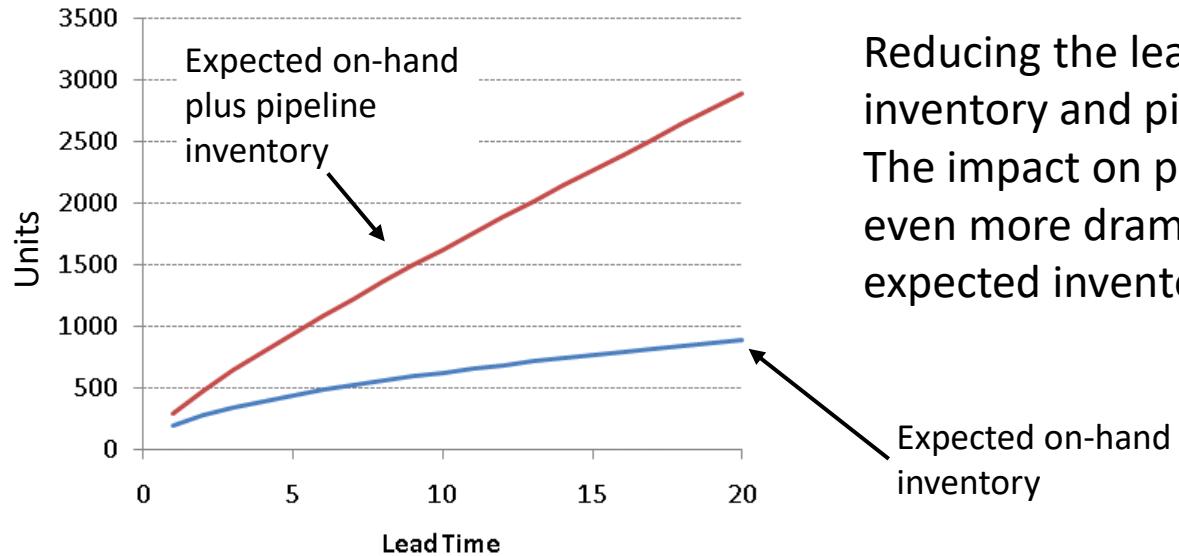
Shorten lead times and you will reduce inventory



Reducing the lead time reduces expected inventory, especially as the target in-stock increases.

The impact of lead time on expected inventory for four in-stock targets, 99.95%, 99.5%, 99.0% and 98%, top curve to bottom curve respectively. Demand in one period is normally distributed with mean 100 and standard deviation 60.

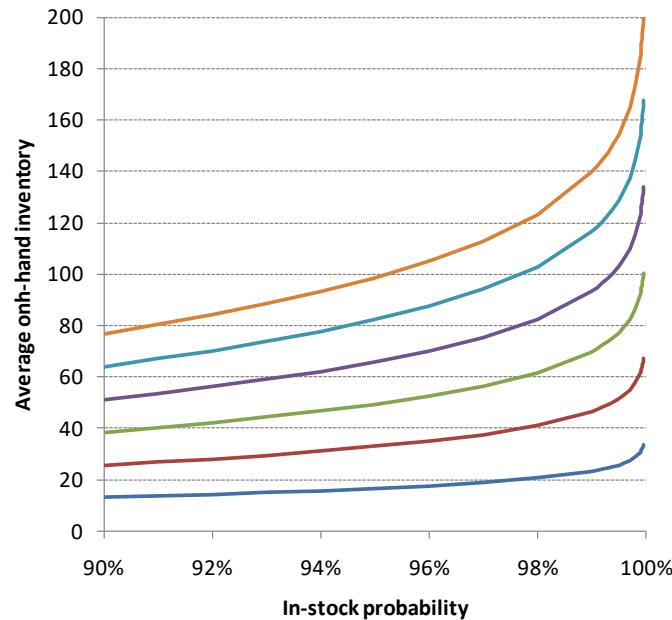
Do not forget about pipeline inventory



Reducing the lead time reduces expected inventory and pipeline inventory. The impact on pipeline inventory can be even more dramatic than the impact on expected inventory.

99.95% in-stock requirement and normally distributed demand in one period with a mean of 100 and standard deviation of 60.

Increase service and you need to add inventory



- The higher the targeted service, the more inventory needed.
- Inventory increases at an increasing rate as the target service approaches 100%.
- The higher the uncertainty of demand the more inventory increases with increasing target in-stock probability.

The tradeoff between inventory and in-stock probability with Normally distributed demand and a mean of 100 over $(l+1)$ periods. The curves differ in the standard deviation of demand over $(l+1)$ periods: 60, 50, 40, 30, 20, 10 from top to bottom.

Order-up-to Model vs. Newsvendor Model

- Both models have uncertain future demand, but there are differences

	Newsvendor	Order up-to
Inventory obsolescence	After one period	Never
Number of replenishments	One (maybe two or three with some reactive capacity)	Unlimited
Demand occurs during replenishment	No	Yes

- Newsvendor applies to short life cycle products with uncertain demand and the order up-to applies to long life cycle products with uncertain, but stable, demand.



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