# Problem Set 2: Refresher on Game Theory - Hints

Markets, Incentives and Ethical Management

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1. Find all pure strategy Nash-Equilibria of the games *a.*) to *d.*) and the subgame perfect equilibria of *e.*) to *h.*).

**a.)**

| | | **Player 2** L | **Player 2** R |
|---|---|---|---|
| **Player 1** | U | $\underline{2},\underline{3}$ | 3,2 |
| | D | 1,2 | 2,1 |

If Player 2 plays L then Player 1´s best response is U.

If Player 2 plays R then Player 1´s best response is U.

If Player 1 plays U then Player 2´s best response is L.

If Player 1 plays D then Player 2´s best response is L.

⇒ The unique Nash-Equilibrium is U,L

(The strategies D for Player 1 and R for Player 2 are strictly dominated)

**b.)**

| | | **Player 2** L | **Player 2** R |
|---|---|---|---|
| **Player 1** | U | $\underline{3},\underline{3}$ | 8,2 |
| | D | 2,8 | 6,6 |

If Player 2 plays L then Player 1´s best response is U.

If Player 2 plays R then Player 1´s best response is U.

If Player 1 plays U then Player 2´s best response is L.

If Player 1 plays D then Player 2´s best response is L.

⇒ The unique Nash-Equilibrium is U,L.

(The strategies D for Player 1 and R for Player 2 are strictly dominated. This game represents a Prisoner´s dilemma.)

**c.)**

| | | **Player 2** L | **Player 2** R |
|---|---|---|---|
| **Player 1** | U | -1,3 | 3,2 |
| | D | 2,1 | 2,3 |

If Player 2 plays L then Player 1´s best response is D.

If Player 2 plays R then Player 1´s best response is U.

If Player 1 plays U then Player 2´s best response is L.

If Player 1 plays D then Player 2´s best response is R.

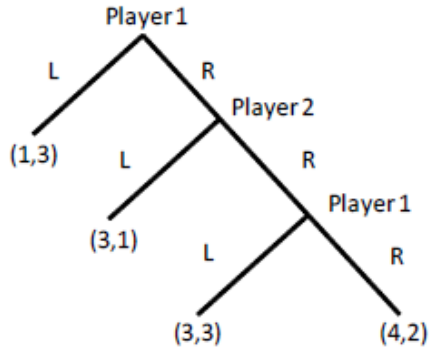
⟹ There is no pure strategy Nash-Equilibrium.

**d.)**

| | | Player 2 | | |
|---|---|---|---|---|
| | | L | M | R |
| **Player 1** | U | 2,3 | 1,2 | 1,2 |
| | M | 3,2 | $\underline{2}$,$\underline{3}$ | 4,1 |
| | D | $\underline{4}$,$\underline{3}$ | 1,2 | 3,1 |

There are two Nash-Equilibria, with L,D and M,M.

**e.)**




Solving by backward induction:

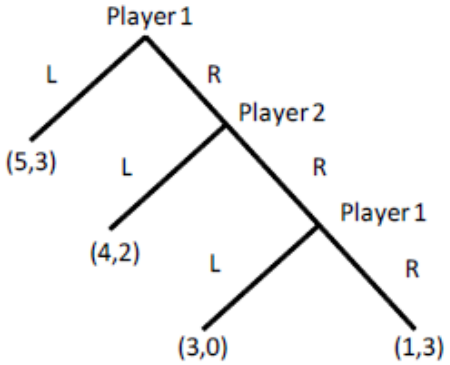
In node three Player 1 would choose R (4 instead of 3)

Knowing this Player 2 would choose R in node 2 (2 instead of 1)

In node one Player 1 chooses R (4 instead of 1)

$\Longrightarrow$ The outcome of the subgame perfect NE is $(4, 2)$, with R,R,R.

**f.)**



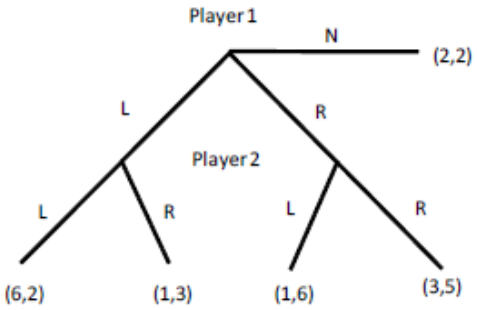

In node three Player 1 would choose L (3 instead of 1)

Knowing this Player 2 would choose L in node 2 (2 instead of 0)

In node one Player 1 chooses L (5 instead of 4)

$\Longrightarrow$ Outcome of the subgame perfect NE is $(5, 3)$, where player 1 just chooses L in node one.

**g.)**




Player 2 knows in which node she is (i.e. she knows if Player 1 has chosen L or R)

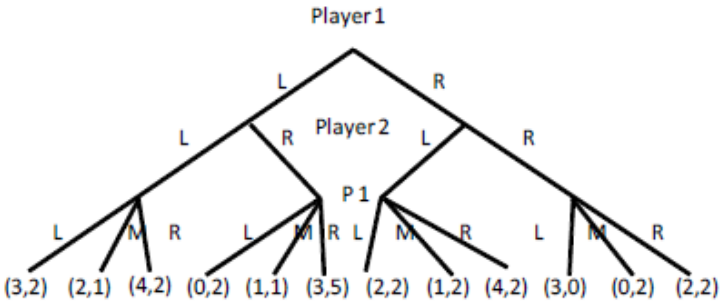
If Player 1 has chosen L Player 2 will choose R (3 instead of 2)

If Player 1 has chosen R Player 2 will choose L (6 instead of 5)

Player 1 chooses N (2 instead of 1)

$\Longrightarrow$ The outcome of the subgame perfect NE is $(2, 2)$, where player 1 just chooses N in node one.

**h.)**




$\Longrightarrow$ The outcome of the subgame perfect NE is $(4, 2)$, with actions along the equilibrium path of R,L,R

2. Translate the following situation of strategic interaction into a game by employing a payoff matrix (normal form representation) and determine the the outcome (i.e. the Nash-Equilibrium):

   - There are two competing firms (e.g. Audi, BMW) with the same Original equipment manufacturer (OEM) (e.g. Bosch).
   - Each firm can undertake a joint quality improvement project with the OEM at a cost of 6 each.
   - Both firms will benefit from improved quality of the OEM and get an additional profit of 7.
   - Only one of the firms has to make the investment (the other is able to free-ride).
   - There will be no additional improvement if both firms invest (still benefit of 7).
   - Without the investment the lower quality leads to a loss of 3 for the two firms.
   - The firms decide simultaneously about the investment.

   There are two firms which have two strategies: Invest or Not Invest.
   They choose simultaneously and know the situation and the other firm's preferences (maximum profit).

   Outcomes (payoff matrix):

| | | **Audi** | |
|---|---|---|---|
| | | Not Invest | Invest |
| **BMW** | Not Invest | -3,-3 | 7,1 |
| | Invest | 1,7 | 1,1 |

   There are two Nash-Equilibria (Invest, Not Invest and Not-Invest, Invest). Both firms want to have the quality improvement but would prefer the other firm to do the project with the OEM. If one firm is able to make the credible threat not to invest the other firm will invest (because 1 is better than -3).