Problem Set 3: Oligopoly - Solution

Markets, Incentives and Ethical Management

Prof. Dr. Markus Reisinger

1. Asymmetric Bertrand competition. Consider the Bertrand model of the lecture, but assume that the firms face different marginal costs: let $c_1 > c_2$. To exclude uninteresting cases, assume that c_1 is smaller than the monopoly price of the industry. To simplify the analysis, assume that prices can only be changed in small discrete increments of ε (e.g. $\varepsilon = 1$ Euro cent).

   (a) What is the equilibrium outcome?

   The equilibrium is $p_2 = c_1 - \varepsilon$ and $p_1 = c_1$. This is an equilibrium since firm 1 makes zero profits and no sales. Firm 1 could only obtain positive sales when offering the good at a price below c_1. At the same time, firm 2 cannot do better because lowering the price would reduce its profits, while increasing the price to c_1 would result in a discrete drop in profits (firm 2's price increases only marginally, but firm 2 loses half of the sales to firm 1).

   (b) What would happen if we increased the number of firms by introducing firms that have the same or higher marginal production cost as firm 1?

   Nothing would happen. All other firms would set $p = c_1$. Thus, firm 2's best response would still be to set $p_2 = c_1 - \varepsilon$ and get all the sales.

   (c) How would you interpret the effect that firm 1 has on the market outcome?

   Although firm 1 makes no sales in equilibrium, it has an impact on the equilibrium outcome - its marginal cost constitutes an upper bound on the price that firm 2 can charge. (This is the reason why we assumed that c_1 is below the monopoly price. Otherwise, firm 2 could charge the monopoly price and firm 1 would have no effect on the outcome).

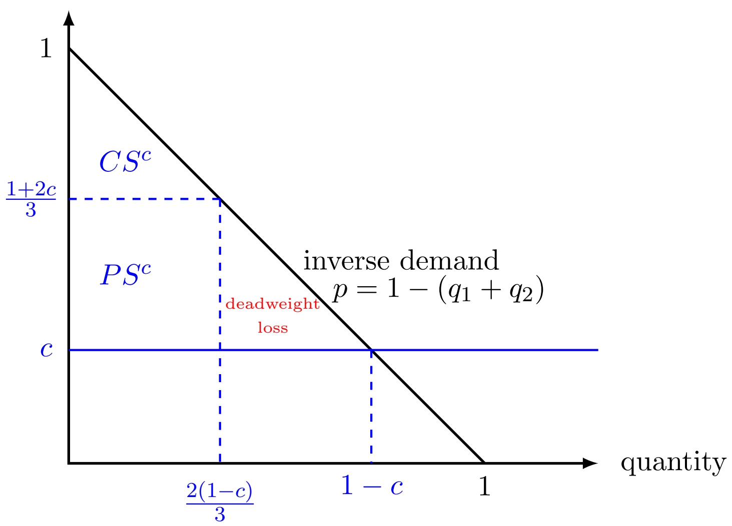
   In some sense, this scenario can be described as "potential competition": If a firm has a monopoly position, it can be limited in exploiting its dominance by the threat of entry of other firms once it charges too high prices.

2. Welfare in Cournot duopoly. Consider the symmetric Cournot duopoly discussed in the lecture. Calculate the welfare loss, i.e., the deadweight loss that arises from the firms' market power (their ability to set prices above marginal cost).

   Hint: To determine the consumer and producer surplus and the social welfare, it is helpful to plot the inverse demand function.

From the lecture, we know that the equilibrium price and quantities in Cournot duopoly are determined by $p^c = \frac{1+2c}{3}$ and $q_1^c = q_2^c = \frac{1-c}{3}$, implying that $\pi_1^c = \pi_2^c = \frac{(1-c)^2}{9}$. The deadweight loss is determined by the loss in total surplus (social welfare) if we consider duopolistic Cournot competition instead of perfect competition. Thus, we only need to compare the level of the total surplus of the duopolistic Cournot equilibrium with the one of the perfectly competitive equilibrium. Generally, the total surplus (or the social welfare) TS is the sum of producer surplus (PS = sum of producer profits)

Figure 1:



and consumer surplus (CS = aggregated benefit of those consumers whose valuation for the good exceeds the market price).

Figure 1 is a helpful tool to determine the producer and consumer surplus and the total surplus. The respective expressions for the consumer and producer surplus in the duopolistic Cournot equilibrium can be derived easily by employing a simple triangle calculation:

$$\begin{aligned} PS^c &= \pi_1 + \pi_2 = \frac{2}{9}(1-c)^2, \\ CS^c &= \frac{1}{2}\left(1 - \frac{1+2c}{3}\right)\left(\frac{2(1-c)}{3}\right) \\ &= \frac{1}{2}\left(\frac{2}{3}(1-c)\right)^2 \\ &= \frac{2}{9}(1-c)^2. \end{aligned}$$

Thus, total surplus from Cournot duopoly is:

$$TS^c = PS^c + CS^c = \frac{4}{9}(1-c)^2.$$

Under perfect competition, the total surplus is maximized in equilibrium and we refer to this as first best outcome (FB). In this case, the equilibrium price equals the firms' marginal cost: $p^{FB} = c$, resulting in a quantity that is given by $q^{FB} = 1 - c$. Thus, firms make zero profits and the consumer surplus is equal to the total surplus. The consumer surplus under perfect competition is given by

$$CS^{FB} = \frac{1}{2}(1-c)(1-c) = \frac{(1-c)^2}{2}.$$

Since the producer surplus is zero, this implies that $TS^{FB} = (1-c)^2/2$. Therefore, the deadweight loss (welfare loss) due to imperfect competition (here: duopolistic Cournot competition) is:

$$WL = TS^{FB} - TS^{Cournot} = \left(\frac{1}{2} - \frac{4}{9}\right)(1-c)^2 = \frac{1}{18}(1-c)^2.$$

The deadweight loss can also be derived directly from Figure 1. The triangular area that indicates the deadweight loss is determined by

$$WL = \frac{1}{2}\left(\left[(1-c) - \frac{2(1-c)}{3}\right]\left[\frac{1+2c}{3} - c\right]\right) = \frac{1}{18}(1-c)^2.$$

3. How are prices determined in the Cournot model? Can equilibrium prices paid for products differ among firms?

In the Cournot model, the equilibrium price is the one that clears the market, and it is equal to the inverse demand $p(q^{Cournot})$. Finding the equilibrium price may be done by an "auctioneer", for instance, a computer in the electricity market. Under Cournot competition, one may suppose that such an "auctioneer" considers the offered quantities and knows the market demand function, and then selects the market clearing price, that is, the "auctioneer" finds the highest price at which all offered units are sold. The resulting equilibrium allocation is then equivalent to the outcome in the quantity-setting game. Because of this, and because the model involves homogeneous products, there can never be different prices for products from different firms.

4. Consider a Cournot model in which n identical firms are active, with a cost function

$$C_i(q_i) = \frac{1}{2}q_i^2, \quad \forall i.$$

Assume that the market demand is given by $D(p) = 1-p$. Calculate the the price and the quantity of the symmetric Cournot equilibrium.

First, we need to determine the inverse demand function—i.e., we need to solve the demand function that determines the demanded quantity for the price (the price that clears the market is so that all offered units are sold: $D(p) = \sum_{j=1}^{n} q_j$):

$$p = 1 - \sum_{j=1}^{n} q_j$$

The inverse demand function is then used to set up the maximization problem of firm i (i.e., the profit function that has to be maximized $\pi_i = p_i q_i - C_i(q_i)$):

$$\max_{q_i} \left(1 - \sum_{j=1}^{n} q_j\right) q_i - \frac{1}{2}q_i^2.$$

Taking the derivative with respect to q_i gives the first-order condition:

$$1-\sum_{j=1}^{n} q_j - q_i - q_i = 0.$$

By symmetry, we know that in a symmetric equilibrium $q_1 = q_2 = ... = q_n$. Thus, $\sum_{j=1}^{n} q_j = nq_i$ and the first-order condition can be simplified to:

$$\begin{aligned} 1-(n+2)\,q_i &= 0, \\ q_i &= \frac{1}{n+2}, \qquad \forall i. \end{aligned}$$

Implying:

$$\begin{aligned} q^{Cournot} &= \frac{n}{n+2} \\ p^{Cournot} &= \frac{2}{n+2}. \end{aligned}$$

5. Consider a market with two firms $i = 1, 2$. The firms face the following demand functions:

$$\begin{aligned} q_1 &= \tfrac{1}{2} - p_1 + \tfrac{1}{4}p_2; \\ q_2 &= \tfrac{1}{2} - p_2 + \tfrac{1}{4}p_1. \end{aligned}$$

To simplify the description, suppose that costs of each firm are 0.

(a) The demand functions are interdependent, that is, the demand for product 1 depends not only on the own price but also on the p_2, and vice versa.
Do the firms produce substitutes or complements?
Are the products differentiated or not?

The products are substitutes. This can be seen from the demand functions, as the demand for product i increases in the price of product $-i$. For example, if product 2 becomes more expensive, consumers buy more products from firm 1, which implies that the products are substitutes.

The products are differentiated. The demand for product i (i.e., q_i) reacts stronger to a change in p_i than to a change in p_{-i}. Specifically, q_i falls by 1 unit if p_i raises, but only increases by 1/4 units of p_{-i} raises. Therefore, in absolute terms, the change is larger in own price than in the rival's price which can only happen if products are differentiated.

(b) Assume that the two firms compete in prices. Calculate the Nash equilibrium for this game. (At the right point of your calculations, use the symmetry between the two firms.)

Firm 1 maximizes:

$$\begin{aligned}\pi_1 &= q_1(p_1,p_2)\times p_1 \to \max_{p_1} \\ &= (\frac{1}{2}-p_1+\frac{1}{4}p_2)\times p_1\end{aligned}$$

First-order-condition:

$$\begin{aligned}-p_1+(\frac{1}{2}-p_1+\frac{1}{4}p_2) &= 0 \\ \frac{1}{2}-2p_1+\frac{1}{4}p_2 &= 0\end{aligned}$$

Since firms are symmetric, we can directly solve for the equilibrium price $\Longrightarrow p_1 = p_2$

$$\begin{aligned}\frac{1}{2}-2p_1+\frac{1}{4}p_1 &= 0 \\ \frac{7}{4}p_1 &= \frac{1}{2} \\ p_1 &= \frac{2}{7} = p_2\end{aligned}$$

Since the products are differentiated, equilibrium prices are above marginal costs! Therefore, firms make positive profits.