Frankfurt School of Finance & Management
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Examination in the Bachelor of Science
Course title: Markets, Incentives and Ethical Management
Part: Markets and Incentives
Semester: 3
Lecturers: Prof. Dr. Heiko Karle, Prof. Dr. Markus Reisinger
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Aids: pocket calculator Casio FX-82 solar,
German-English Dictionary, English-English Dictionary

Please enter your student ID (matriculation number) and your group!

Student ID	Group

Please note:

The exam consists of 4 questions of which you will have to answer **all** questions. You have **90** minutes to complete the examination. The maximum of points to be reached is **90**. Please use the enclosed answer sheet to answer your questions and add your student ID on its cover.

We wish you all the best for your examination!

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Question	1	2	3	4	**Total**
Possible points:	27	26	10	27	**90**
Points achieved:					

Signature of corrector

Question 1 – General Equilibrium (27 points)

Suppose that there are two consumers A and B and two products x and y. The initial endowment W is such that consumer A is endowed with $(W_{XA}, W_{YA}) = (1,8)$ and consumer B is endowed with $(W_{XB}, W_{XB}) = (8,27)$. Both consumers have standard preferences, and their utility functions are the same and equal to

$$U_A(x_A, y_A) = x_A * y_A^2 \qquad U_B(x_B, y_B) = x_B * y_B^2.$$

(a) Determine consumer A and B's utility at their initial endowment. (2 points)

U_A=1*64=64 (1 point); U_B=8*729= 5832 (1 point).

(b) Explain how the set of allocations that are Pareto improvements compared to the initial endowment is characterized in an Edgeworth box? (2 points)

The area between the consumers' indifference curves passing through their initial endowment. (2 points)

(c) What is generally the condition for Pareto efficiency in an economy with two consumers A and B. (2 points)

The Pareto-efficiency condition is $MRS_A = MRS_B$ (i.e., the marginal rate of substitution of consumer A equals that of consumer B). (2 points)

(d) Determine (mathematically) the slopes of the indifference curves for the two consumers. (3 points)

$$MRS_A = -\frac{y_A}{2x_A} \text{ (1 ½ points)}$$

$$MRS_B = -\frac{y_B}{2x_B} \text{ (1 ½ points)}$$

(e) Determine (mathematically) the equation for the Pareto efficient allocations in the economy described above. (2 points)

$\frac{y_A}{x_A} = \frac{y_B}{x_B}$. (2 points)

(f) Describe verbally how you would use the equation for the Pareto efficient allocations and the initial endowment to determine the equation for the contract curve. (You do not need to calculate it mathematically!) (3 points)

Use that the sum of both consumers' endowment of x and y is equal to 9 and 35 (1 point).
The, solve for x_b and y_b, respectively. (1 point)

Then, plug in x_b and y_b into the equation for the Pareto efficient allocations and solve for y_a as a function of x_a. (1 point)

(g) The equation for the contract curve is $\mathbf{y_A = \frac{35}{9} * x_A}$.
Determine for each of the following three points whether it can be an equilibrium allocation in this economy:
(i) (xA = 1; yA = 8); (xB = 8; yB = 27);
(ii) (xA = 9; yA = 35); (xB = 0; yB = 0);
(iii) (xA = 1.7; yA = 6.61); (xB = 7.3; yB = 28.39).
Provide an explanation for your results. (6 points)

To be an equilibrium allocation, the allocation must fulfill two requirements:
it must be on the contract curve, and both consumer must be at least as well off as with the original endowment. (3 Points) Only (iii) satisfies both. (3 points; 1 point each).

(h) What is the content of the First Welfare Theorem? (2 points)

(The allocation in) a competitive equilibrium is Pareto efficient. (2 points)

(i) Now consider the German market for timber (Holz). After several very hot summers, the stock of spruce (Fichte) is at risk and must be lumbered, while the stock of other species such as beech (Buche) is not affected. How does the positive supply shock of spruce timber affect the price of beech timber? Explain your answer. (5 points)

The higher supply of spruce timber leads to a lower price and a higher quantity of spruce timber being traded (2 points). Due to a substitution effect, consumers will demand more spruce timber for heating than beech timber (1 point). Therefore, the demand of beech timber is decreased which leads to a lower quantity and a lower price of beech timber. (2 points).
(If someone interpreted the supply shock of spruce timber as negative but argued correctly after, then (s)he gets 4 out of 5 points.)

Question 2 – Oligopoly (26 points)

Consider a market with two firms, firm 1 and firm 2, which produce a homogeneous product. Suppose that the firms simultaneously set their quantities (**Cournot competition**). The inverse demand function is $p(Q) = 40 - Q$, where $Q=q_1+q_2$. The cost function of each firm $i=1,2$, is $C_i = 10q_i$.

(a) Set up the profit function of firm 1 and determine the first-order condition. (4 points)

$$\pi_1 = (40 - q_1 - q_2)q_1 - 10q_1 \text{ (2 points)}$$

$$\frac{\partial \pi_1}{\partial q_1} = 30 - 2q_1 - q_2 = 0 \text{ (2 points)}$$

(b) Determine the quantities of both firms in the Nash equilibrium. (3 points)

Since the game is symmetric, we can use symmetry, which gives $q_1=q_2$. (1 point)

Using this in the first-order condition yields: $30 - 3q_1 = 0$ (1 point)

Solving this: $q_1 = q_2 = 10$ (1 point)

(c) Suppose for this sub-question only that there are 5 firms in the market instead of 2. Each of the 5 firms has the same cost function given by $C_i = 10q_i$, with $i=1,2,3,4,5$, and the inverse demand function is still $p(Q) = 40 - Q$, where $Q=q_1+q_2+q_3+q_4+q_5$. What is the quantity of each firm in the Nash equilibrium? (3 points)

The profit function of a firm is now:
$(40 - q_1 - q_2 - q_3 - q_4 - q_5)q_1 - 10q_1$ (1 point)

The first-order condition is: $(30 - 2q_1 - q_2 - q_3 - q_4 - q_5) = 0$ (1 point)

Symmetry implies that $q_1 = q_2 = q_3 = q_4 = q_5$, which can be used in the first-order condition to get: $30 - 6q_1 = 0$. As a consequence, the quantity of each firm in equilibrium is 5. (1 point)

(d) Explain (verbally) how the equilibrium price, the equilibrium profit, and the equilibrium quantity of each firm change in a Cournot model if the number of firms gets very large? (4 points)

If the number of firms gets very large, each firm produces a small but positive quantity, which implies that also the aggregate quantity gets very large. (1 point)

The equilibrium price then gets closer and closer to marginal costs. (1 point)

The equilibrium profit becomes zero (i.e., no profit margin). (1 point)

The equilibrium quantity of each firm becomes negligibly small. (1 point)

(e) Consider now again the Cournot model described at the beginning of the question (i.e., the one with two firms) but suppose that the marginal cost of the two firms were different, that is, firm 1 has lower marginal cost than 10 and firm 2 has higher marginal cost.
Explain (verbally) how the quantities of the two firms change compared to the symmetric situation. (3 points)

Firm 1's quantity increases (1 point)
Firm 2's quantity decreases (1 point)
The reason is that firm 1 has a larger margin per unit due to its lower marginal costs, whereas the opposite holds for firm 2. (1 point)

(f) Explain why there is only a single price in a Cournot market. (3 points)

In Cournot, both firms bring their quantity to the market. The sum of these quantities constitutes the supply. (1 point)
The price then adjusts so that this quantity is also demanded, that is, demand equals supply. (1 point)
Since the firms' goods are homogeneous, there is only one price. (1 point)

(g) Give one example of a market in which firms compete by setting quantities and there is a single market-clearing price. (1 point)

Electricity market (1 point)
Other examples would be gas market, oil market, etc.

(h) Suppose that the quantity competition described at the beginning of the question is not played simultaneously but sequentially. In particular, firm 1 sets its quantity first, firm 2 observes this quantity, and then chooses its quantity.
Explain (verbally) how the quantities chosen in this sequential game differ from the ones of the simultaneous game. Provide the intuition for your result. (5 points)

Firm 1's quantity is larger than in the simultaneous game (1 point)
Firm 2's quantity is lower (1 point)

Firm 1 chooses in the first stage a quantity so that firm 2 changes its action compared to the simultaneous game in a way that is beneficial for firm 1. (1 point)

Since firm 1 benefits from a lower quantity of firm 2, and quantities are strategic substitutes (i.e., firm 2 reduces its quantity when firm 1 produces more), firm 1 sets a higher quantity than in the simultaneous game. (2 points)

Question 3 – Tacit Collusion (10 points)

Consider a situation with two firms (i.e., firm 1 and firm 2), which produce a homogenous product. The inverse demand function is linear and firms' production costs are zero. Competition occurs in prices (**Bertrand competition**). Suppose the two firms compete for an infinite number of periods. Each firm i has a discount factor δ_i between 0 and 1.

(a) Determine a reasonable strategy that allows the firms to sustain collusion if their discount factors are large enough. How is such a strategy called? (5 points)

Grim-trigger strategy: (1 point)
Start by setting the monopoly price in the first period. (1 point)
Keep setting the monopoly price as long as both have done so in all previous periods. (1 point)
If one firm has deviated from setting the monopoly price in a previous period, set the Nash equilibrium price of 0 forever. (2 points; 1 point for p=0)

(b) Order the following three situations from "collusion is most likely" to "collusion is least likely". Provide an explanation for your assessment. (5 points)

(i) $(\delta_1 = 0.7;\ \ \delta_2 = 0.7)$;
(ii) $(\delta_1 = 0.4;\ \ \delta_2 = 0.4)$;
(iii) $(\delta_1 = 0.8;\ \ \delta_2 = 0.3)$.

What is most critical for deviations is the lowest discount factor of both firms. (2 points) Therefore, the ranking is (i), (ii), (iii). (3 points; 1 point for each correct binary ranking)

Question 4 – Asymmetric Information (27 points)

Consider a used car market in which there are three types of car dealers: High (H), Medium (M), and Low (L) types. The car of high type is worth 20 to a buyer, the car of medium type is worth 15, and the car of low type is worth X. A high-type seller incurs a cost of 15 when selling the car, a medium-type seller incurs a cost of 13, and a low-type seller incurs a cost of 5.

Each seller knows the quality of the car he sells, but quality is not observable to buyers prior to purchase.

The probabilities for the different types are as follows: the probability that the car is of high type is 1/5, the probability that the car is of medium type is 3/5, and the probability that the car is of low type is 1/5.

The following table summarizes the aforementioned situation:

Type	Probability	Value	Cost
H	1/5	20	15
M	3/5	15	13
L	1/5	X	5

(a) Determine the maximum price that a buyer is willing to pay in case all the three qualities are offered. (3 points)

The maximum price that the buyer is willing to pay is the expected value: (1 point)

$$E[v\,|S = \{H, M, L\}] = \frac{1}{5}\,20 + \frac{3}{5}\,15 + \frac{1}{5}\,X \text{ (2 points)}$$

(b) Set up the condition that needs to be fulfilled so that the all three qualities (i.e., {H, M, L}) are offered in equilibrium. Determine the values of X so that this situation is indeed an equilibrium. (3 points)

{H, M, L} constitutes an equilibrium if the maximum price that the buyer will accept is greater than or equal to the cost of the H-type seller. (1 point)

That is,

$$E[v\,|S = \{H, M, L\}] \geq 15$$
$$\frac{1}{5}\,20 + \frac{3}{5}\,15 + \frac{1}{5}\,X \geq 15 \text{ (1 point)}$$
$$4 + 9 + \frac{1}{5}\,X \geq 15$$
$$\frac{1}{5}\,X \geq 2$$
$$X \geq 10 \text{ (1 point)}$$

(c) Suppose now that only the medium and the low types are active. Determine the probabilities for the medium and the low type in this situation. (2 points)

Probability for the medium type: $\frac{\frac{3}{5}}{\frac{3}{5}+\frac{1}{5}} = 3/4$ (1 point)

Probability for the low type: $\frac{\frac{1}{5}}{\frac{3}{5}+\frac{1}{5}} = 1/4$ (1 point)

(d) Set up the condition that needs to be fulfilled so that only the medium and low quality (i.e., {M, L}) is offered in equilibrium. Determine the values of X so that this situation is indeed an equilibrium. (4 points)

The maximum price that the buyer will accept is:

$\mathrm{E}[\mathrm{v}\,|\mathrm{S} = \{\mathrm{M},\mathrm{L}\}] = \frac{\frac{3}{5}}{\frac{3}{5}+\frac{1}{5}} 15 + \frac{\frac{1}{5}}{\frac{3}{5}+\frac{1}{5}} X$ (1 point)

{M, L} constitutes an equilibrium if and only if the maximum price that the buyer will accept is greater than or equal to the cost of the M-type seller. (1 point)

That is,

$$\mathrm{E}[\mathrm{v}\,|\mathrm{S} = \{\mathrm{M},\mathrm{L}\}] \geq 13$$

$$\frac{\frac{3}{5}}{\frac{3}{5}+\frac{1}{5}} 15 + \frac{\frac{1}{5}}{\frac{3}{5}+\frac{1}{5}} X \geq 13 \text{ (1 point)}$$

$$\frac{3}{4} 15 + \frac{1}{4} X \geq 13$$

$$45 + X \geq 52$$

$$X \geq 7 \text{ (1 point)}$$

(e) Explain how signaling in the used car market could work to overcome the problem of asymmetric information. (4 points)

The seller of type H offers the buyer a guarantee in form of a warranty (1 point)
This warranty states that the seller will pay the buyer a certain amount of the car does not turn out to be of type H. (1 point)
The seller of type H is willing to do do because he never has to pay this amount. (1 point)
However, a seller of type M or L has to pay this amount (with some probability) as their cars are not of type H. Therefore, they do not want to mimic the seller of type H. (1 point)

(f) Focus now only on the sellers of type H and M. Suppose the seller of type H wants to signal the type of his car by offering the buyer a certificate with the following content: if the car turns out to be of type M, the seller pays the buyer an amount Y. Determine the values of Y, which allow type H to separate himself from type M. Briefly explain your solution. (3 points)

If the buyer knows that the car is of type H, she would be willing to pay 20. (1 point)
However, she would be willing to pay only 15 for an M type. (1 point)
Therefore, to prevent the seller of type M to mimic type H, $Y \geq 5$. (1 point)

(g) Describe the fundamental trade-off of the principal-agent relationship with a risk-neutral principal and a risk-averse agent in case of asymmetric information. (5 points)

There are two targets in this relationship: efficient risk sharing and inducing the agent to work hard (i.e., giving the right incentives) (1 point)
Efficient risk sharing can be achieved when the risk neutral principal bears all the risk, and the agent has no risk (1 point)
The wage of the agent should therefore optimally be the same, regardless of the revenue (1 point)
However, with such a contract, the agent has no incentive to work hard, as she is not rewarded for working hard. (1 point)

Working hard can be achieved with a contract that pays the agent a lot if the outcome is good but only little if the outcome is bad. (1 point)

(h) Explain why in case of a risk-neutral agent no inefficiency occurs, despite the effect that there is asymmetric information. (3 points)

With a risk-neutral agent, only one goal exist, namely to set the right incentives (1 point)
The principal can then sell the project to the agent, i.e., the agent pays a fixed payment to the principal and can keep every Euro that the project pays off. (This is similar to a franchise contract.) (2 points)