Bachelor of Science
Operations Management
Collection of Formulae

Production processes

$$Average\ inventory = Average\ flow\ rate \cdot Average\ flow\ time$$

$$Capacity = \frac{m}{Processing\ time} = \frac{Units\ produced}{Time\ to\ produce\ the\ units}$$

$$Process\ capacity = min\{Capacity_1, Capacity_2, \ldots Capacity_n\}$$

$$Utilization = \frac{Flow\ Rate}{Capacity}$$

$$Implied\ utilization = \frac{Demand\ rate}{Capacity}$$

$$Flow\ rate = min\{Demand\ rate, Process\ capacity\}$$

$$Maximum\ flow\ rate = \frac{Demand\ rate\ for\ flow}{MAX(Implied\ utilization)}$$

$$Actual\ flow\ rate = min\{Demand\ rate, Maximum\ flow\ rate\}$$

$$Cycle\ time = \frac{1}{Flow\ rate} = \frac{Flow\ \ time}{Inventory}$$

$$Takt\ time = \frac{1}{Demand\ rate}$$

$$Cost\ of\ direct\ labor = \frac{Total\ wages}{Flow\ rate}$$

$$Total\ idle\ time\ = Cycle\ time \cdot No\ of\ workers - \ Labor\ content$$

$$Target\ manpower = \frac{Labor\ content}{Takt\ time}$$

$$Profit = Flow\ rate\ \cdot\ (Price - Variable\ Cost) - Fixed\ costs$$

$$Yield\ of\ resource = \frac{Flow\ rate\ of\ good\ output\ at\ the\ resource}{Flow\ rate\ of\ input} = 1 - \frac{Flow\ rate\ of\ defects\ at\ the\ resource}{Flow\ rate\ of\ input}$$

$$Process\ yield = \frac{Flow\ rate\ of\ good\ output\ of\ the\ process}{Flow\ rate\ of\ input\ to\ the\ process} = 1 - \frac{Flow\ rate\ of\ defects\ in\ the\ process}{Flow\ rate\ of\ input\ to\ the\ process}$$

$$Number\ of\ units\ started\ to\ get\ Q\ good\ units = \frac{Q}{Process\ yield}$$

$$Average\ labor\ utilization\ = \frac{Laborcontent}{Cycle\ time \cdot No\ of\ worker}$$

$$Time\ to\ produce\ a\ batch = Setup\ time + Batch\ size \cdot Processing\ tim$$

$$Capacity\ (given\ batch\ size) = \frac{Batch\ size}{Total\ Setup\ time\ + (Batch\ size\ \cdot\ Processing\ time)}$$

$$Recommended\ batch\ size = \frac{Target\ capacity\ \cdot\ Total\ setup\ time}{1 - (Target\ capacity\ \cdot\ Processing\ time)}$$

$$Maximum\ inventory = Batch\ size\ \cdot [1 - (Flow\ rate\ \cdot Processing\ time)]$$

$$Average\ inventory = \frac{Maximum\ inventory + Minimum\ inventory}{2}$$

Lean operations and statistical process control

$$Overall\ equipment\ effectiveness\ OEE = \frac{Value\text{-}adding\ time}{Total\ available\ time}$$

$$Value\text{-}adding\ percentage = \frac{Value\text{-}adding\ time\ of\ a\ flow\ unit}{Flow\ time}$$

$$Process\ capability\ index\ C_p = \frac{USL - LSL}{6\hat{\sigma}}$$

$$\bar{X} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$R = max\{x_1,\ x_2, \dots x_n\} - min\{x_1,\ x_2, \dots x_n\}$$

$$\bar{\bar{X}}\ =\ \frac{\bar{X}_1 + \bar{X}_2 + \cdots + \bar{X}_t}{t}$$

3-Sigma-Quality Control Chart

$$UCL =\ \bar{X} + (3 \cdot ESD\ \bar{X})$$

$$LCL =\ \bar{X} - (3 \cdot ESD\ \bar{X})$$

$$ESD\ \bar{X} = \frac{\hat{\sigma}}{\sqrt{n}}$$

$\hat{\sigma}$: $estimated\ standard\ deviation\ based\ on\ all\ n\ observations$

n: $sample\ size$

Queuing systems

m = number of servers
p = processing time
a = interarrival time
R = Average inflow rate = average arrival rate
CV_a = Coefficient of variation of the interarrival time a
CV_p = Coefficient of variation of the processing time p

$$Queue\ growth\ rate = Demand - Capacity$$

$$Length\ of\ queue\ at\ time\ T = T\ \cdot Queue\ growth\ rate$$

$$Time\ to\ serve\ the\ Qth\ person\ in\ the\ queue = \frac{Q}{Capacity}$$

$$Time\ to\ serve\ the\ person\ arriving\ at\ time\ T = T \cdot \left(\frac{Demand}{Capacity} - 1\right)$$

$$Average\ customer\ waiting\ time = \frac{1}{2} T \cdot \left(\frac{Demand}{Capacity} - 1\right)$$

$$T_q = p \cdot \left(\frac{Utilization}{1 - Utilization}\right) \cdot \left(\frac{CV_a^2 + CV_p^2}{2}\right)$$

$$T_q = \frac{p}{m} \cdot \left(\frac{Utilization^{\sqrt{2(m+1)}-1}}{1 - Utilization}\right) \cdot \left(\frac{CV_a^2 + CV_p^2}{2}\right)$$

$$Average\ interarrival\ time\ a = \frac{1}{R}$$

$$Utilization\ u = \frac{p}{a \cdot m}$$

$$Flow\ time\ = T_q + p$$

$$P_m(r) = \frac{\frac{r^m}{m!}}{1 + \frac{r^1}{1!} + \frac{r^2}{2!} + \cdots + \frac{r^m}{m!}}$$

$$r = \frac{p}{a}\ or\ r = u \cdot m$$

$$Probability\{all\ m\ servers\ are\ busy\} = P_m(r)$$

$$Flow\ rate\ = \frac{1}{a} \cdot (1 - P_m)$$

$$Rate\ of\ lost\ demand = \frac{1}{a} \cdot P_m$$

Inventory management

Q = Order quantity	c = Purchase cost/purchase price per unit
h = Holding cost per unit and time	hp = Holding cost percentage per unit and time
K = Order, transportation, and setup costs	C = Total cost per time period (e.g. per year) relevant to decision making
C_u = Underage cost	C_o = Overage cost
μ = Expected demand	σ = Standard deviation of demand
F(Q) = Distribution function	I(Q) = Inventory function
S = Order-up-to level	b = backorder cost per unit and time
l = lead time	

$$Days\text{-}of\text{-}supply\ DOS = \frac{Inventory}{Average\ daily\ flow\ rate}$$

$$Inventory\ turns\ (per\ year)\ =\ \frac{Average\ (annual)\ flow\ rate}{Inventory}$$

Economic Order Quantity (EOQ)

$$h = h_p \cdot c$$

$$C(Q) = \frac{1}{2} \cdot h \cdot Q + K \cdot \frac{R}{Q}$$

$$Q^* = \sqrt{\frac{2 \cdot R \cdot K}{h}}$$

Newsvendor model

$$Critical\ ratio\ CR\ =\ \frac{c_u}{c_o\ +\ \mathrm{c}}$$

$$c \cdot \left(1 - F(Q)\right) = c_o \cdot F(Q)$$

$$F(Q)\ =\ \frac{c_u}{c_o\ +\ \mathrm{c}}$$

$$z\text{-}statistic\ or\ normalized\ order\ quantity\text{:}\ z\ =\ \frac{Q - \mu}{\sigma}$$

$$Q\ =\ \mu\ +\ z\ \cdot\ \sigma$$

$$\frac{A}{F} ratio = \frac{Actual\ demand}{Forecast}$$

$$\mu_{Demand} = \mu_{AF} \cdot Judgemental\ forecast$$

$$\sigma_{Demand} = \sigma_{AF} \cdot Judgemental\ forecast$$

$$Expected\ Inventory\ =\ \sigma \cdot I(z)$$

$$Expected\ sales = Q - Expected\ inventory$$

$$Expected\ lost\ sales = \mu_{Demand} - Expected\ sales$$

$$\begin{aligned} Expected\ profit \\ &= (Selling\ price\ \cdot\ Expected\ sales) \\ &+ (Discount\ price \cdot\ Expected\ inventory) - (Purchase\ price\ \cdot\ Q) \end{aligned}$$

$$In\text{-}stock\ probability = F(Q) = 1 - Stockout\ probability$$

$$Stockout\ probability = 1 - F(Q) = 1 - In\text{-}stock\ probability$$

$$Mismatch\ cost\ =\ (Co\ \cdot\ Expected\ inventory)\ +\ (Cu\ \cdot\ Expected\ lost\ sales)$$

$$= Maximum\ profit - Expected\ profit$$

$$Maximum\ profit\ =\ (Selling\ price\ -\ Purchase\ price)\ \cdot\ \mu$$

$$Expected\ pooled\ demand\ =\ 2 \cdot \mu$$

$$Standard\ deviation\ pooled\ demand\ =\ \sqrt{2(1 + Correlation)} \cdot \sigma$$

Order-up-to model

Inventory level = On-hand inventory – Back order

Inventory position = On-order inventory + Inventory level

A period's order quantity Q= Order-up-to level S – Inventory position

Inventory level at the end of a period = S – demand over l+1 periods

$$In\text{-}stock\ probability = prob\ \{Demand\ over\ l + 1\ periods\ \leq\ S\} = Critical\ ratio$$

$$Critical\ ratio = \frac{b}{h + b}$$

$$Order\text{-}up\text{-}to\ level\ S = \mu_{l+1} + z \cdot \sigma_{l+1}$$

$$Expected\ inventory = I(z) \cdot \sigma_{l+1}$$

$$In\text{-}stock\ probability = prob\{demand\ over\ l + 1\ periods \leq S\}$$

$$Stockout\ probability = prob\{demand\ over\ l + 1\ periods > S\}$$

$$Expected\ backorders = Expected\ inventory +\ demand\ over\ l + 1\ periods\ - S$$

$\mu_{\substack{Demand \\ short\ Period}} = \dfrac{\mu_{\substack{Demand \\ long\ Period}}}{n}$	$\mu_{\substack{Demand \\ long\ Period}} = n \cdot \mu_{\substack{Demand \\ short\ Period}}$
$\sigma_{\substack{Demand \\ short\ Period}} = \dfrac{\sigma_{\substack{Demand \\ long\ Period}}}{\sqrt{n}}$	$\sigma_{\substack{Demand \\ long\ Period}} = \sqrt{n} \cdot \sigma_{\substack{Demand \\ short\ Period}}$

Statistics

Mean

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Sample mean

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Standard deviation

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

Sample standard deviation

$$s = \sqrt{\frac{1}{N - 1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

Uniform distribution

$$\mu = \frac{uper\ bound - lower\ bound}{2}$$

$$\sigma = \sqrt{\frac{(uper\ bound - lower\ bound)^2}{12}}$$

$$Coefficient\ of\ variation = \frac{\sigma}{\mu} = \frac{s}{\bar{x}}$$