Examination in the Bachelor of Science
Course title: Markets, Incentives and Ethical Management
Part: Markets and Incentives
Semester: 3
Lecturers: Prof. Dr. Heiko Karle, Prof. Dr. Markus Reisinger, Prof. Dr. Frederik Schwerter
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Aids: pocket calculator Casio FX-82 solar,
German-English Dictionary, English-English Dictionary

Please enter your student ID (matriculation number) and your group!

Student ID	Group

Please note:

The exam consists of 4 questions of which you will have to answer **all** questions. You have **90** minutes to complete the examination. The maximum of points to be reached is **90**. Please use the enclosed answer sheet to answer your questions and add your student ID on its cover.

We wish you all the best for your examination!

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Question	1	2	3	4	**Total**
Possible points:	16	27	27	20	**90**
Points achieved:					

Signature of corrector

Question 1 – General Equilibrium (16 points)

(a) State the definition of competitive equilibrium in a market economy with two goods. (4 points)

In a competitive equilibrium, there are two prices with the following characteristics: (1 point)

- markets clear for both goods (2 points)
- all subjects maximize their respective objective functions (1 point)

(b) State the definition of the First Welfare Theorem. (2 points)

A competitive equilibrium is Pareto efficient. (2 points)

Suppose that there are two consumers A and B and two products *X* and *Y*. The initial endowment is such that consumer A is endowed with $(X_A, Y_A) = (5, 3)$ and consumer B is endowed with $(X_B, Y_B) = (7, 2)$. Both consumers have standard preferences and their utility functions are

$$U_A(X_A, Y_A) = X_A Y_A \quad \text{and} \quad U_B(X_B, Y_B) = X_B Y_B.$$

(c) Determine the utilities of consumers A and B at their respective initial endowment. (2 points)

$U_A=5*3=15$ (1 point), $U_B=7*2=14$ (1 point)

(d) Determine consumer A's and B's utility when consumer A gives 1 unit of good Y to consumer B. (2 point)

$U_A=5*2=10$ (1 point), $U_B=7*3=21$ (1 point)

(e) State whether the allocation in d) or in e) is preferred by a Rawlsian social welfare function. Explain your answer. (3 point)

The allocation with $U_A=15$ and $U_B=14$ is preferred from a Rawlsian perspective (1 point)
The Rawlsian social welfare function considers only the individual with the lower utility. This utility is 14 allocation with $U_A=15$ and $U_B=14$, but only 10 in the other case. (2 points)

(f) State whether the allocation in d) or e) is preferred by a Utilitarian social welfare function. Explain your answer. (3 point)

The allocation with U_A=10 and U_B=21 is preferred from a Utilitarian perspective (1 point)
The Utilitarian social welfare function considers the sum of the utilities. This sum is 31 in the allocation with U_A=10 and U_B=21, but only 29 in the other case. (2 points)

Question 2 – Oligopoly (27 points)

Consider price competition between two firms (firm 1 and 2). The firms sell a homogeneous product at constant marginal costs of c=5 for each firm. (There are no fixed costs). Prices are set simultaneously (**Bertrand competition**). The demand function is D(p)=10-p for a firm setting the lowest price alone, D(p)/2 for a firm setting the lowest price together with the other firm, and 0 for a firm not setting the lowest price. Assume that price differences can be infinitesimally small. Use the notation "$p - \varepsilon$" for slightly undercutting a price p of a rival firm.

(a) Determine the equilibrium prices and explain briefly why they constitute a Nash equilibrium. (4 points).

$$p_1 = p_2 = 5 \ \text{(1 point)}$$

Increasing the price above 5 leads to zero demand and therefore zero profit (2 points)
Decreasing the price leads to a loss per sold unit. Deviating is therefore not profitable. (1 point)

(b) Determine the corresponding profit of firm 1 and 2. (1 point)

Firms obtain zero profit. (1 point)

(c) Explain briefly why the Nash equilibrium in Bertrand competition cannot be determined by deriving the first-order conditions of the profit-maximizing firms. (3 points)

In Bertrand competition, the first-order conditions cannot be used because the demand and therefore the profit function is not continuous (and therefore not differentiable). (2 points)

Suppose now that only firm 1 is active in this market.

(d) Determine the profit function of firm 1. (1 point)

$$\pi_1 = (p_1 - 5)(10 - p_1) \ \text{(1 point)}$$

(e) What is the monopoly price that firm 1 will set? (2 points)

$$\frac{\partial \pi_1}{\partial p_1} = -(p_1 - 5) + (10 - p_1) = 0 \ \text{(1 point)}$$
$$p_1 = 7.5 \ \text{(1 point)}$$

Suppose next that both firms are active in the market but firm 2 has higher marginal costs of 6, i.e., c_1=5<c_2=6.

(f) Determine the equilibrium prices and explain briefly why they constitute a Nash equilibrium. (4 points)

$$p_1 = 6 - \varepsilon \, ; p_2 = 6 \quad (1 \text{ point})$$

Firm 1 serves the entire demand:
Increasing the price above $6 - \varepsilon$ leads to an equal split of demand or zero demand for firm 1 when firm 2 sets a price equal to 6 and therefore reduces firm 1's profit. (1 point)
Decreasing the price leads to a lower profit per sold unit and therefore reduces firm 1's profit. (1 point)

Firm 2 has zero demand:
Increasing the price above 6 leads to zero demand when firm 1 sets a price equal to $6-\varepsilon$ and therefore does not increase firm 2's profit. Decreasing the price leads to a loss per sold unit and therefore reduces profit. (1 point)

(g) Determine the corresponding profit of firm 1 and 2 when ε is approximately zero. (2 point)

$$\pi_1 = (6-5)(10 - 6) = 4; \;\; \pi_2 = 0 \;\; (1 \text{ point})$$

(h) Explain how the equilibrium prices in (f) are changing if firm 2's marginal costs increase further to 8, i.e., c_1=5<c_2=8. (3 points)

$$p_1 = p^M = 7.5 \, ; p_2 = 8 \quad (1 \text{ point})$$

It is optimal for firm 1 to set its price equal to the monopoly price of 7.5 and not to increase its price further than that. (2 points)

Assume that the demand functions of firm 1 and 2 have changed such that

$$D_1(p_1, p_2) = 10 - p_1 - \tfrac{1}{2}p_2 \qquad \text{and} \qquad D_2(p_1, p_2) = 10 - p_2 - \tfrac{1}{2}p_1.$$

(i) Do the two firms sell substitutes or complements now? Briefly explain your result. (3 points)

The two firms sell complements. (1 point)
The demand of one firm is decreasing if the other firm raises its price. (2 points)

(j) Solve the game for the Nash equilibrium prices when the marginal costs of both firms are equal to 5, i.e., $c_1=c_2=5$. (4 points)

$$\pi_1 = (p_1 - 5)\left(10 - p_1 - \frac{1}{2} * p_2\right) \text{ (1 point)}$$

$$\frac{\partial \pi_1}{\partial p_1} = -(p_1 - 5) + (10 - p_1 - \frac{1}{2} * p_2) = 0 \text{ (1 point)}$$

Firms are symmetric: $p_1 = p_2$ (1 point)

$$\frac{5}{2} p_1 = 15 \Rightarrow p_1 = p_2 = 6 \text{ (1 point)}$$

Question 3 – Tacit Collusion (27 points)

Consider a situation with two firms, firm 1 and firm 2, which produce a homogenous product. The inverse demand function is D(p) = 10 – 2p. Production cost functions are $2q_i$, i=1,2, for each firm, that is, $C_1(q_1) = 2q_1$ and $C_2(q_2) = 2q_2$.

(a) Suppose that firm 1 is a monopolist (that is, firm 2 does not participate in the market). Determine the monopoly price and firm 1's profit. (3 points)

$$\max_{q_M} \pi_M = (10 - 2q_M)q_M - 2q_M \quad \text{(1 point)}$$

$$\frac{\partial \pi_M}{\partial q_M} = 10 - 4q_M - 2 = 0$$

Therefore, $q_M = 2$ (1 point).

This implies a monopoly price of 6 and $\pi_M = 6 * 2 - 4 = 8$ (1 point)

(b) Instead of firm 1 being a monopolist, firm 1 now competes with firm 2. Competition occurs in prices (**Bertrand competition**). What are the equilibrium prices and equilibrium profits of this game? (3 points)

Equilibrium prices is equal to 2, as firms compete each other down to marginal costs (2 points)

Equilibrium profit is 0 for each firm (1 point)

(c) Suppose that instead of competing only for one period, the two firms compete in prices repeatedly for 50 periods. Explain why the Subgame Perfect Nash equilibrium in this repeated interaction is the same as the one in one-shot (single period) competition. (4 points)

As there is a finite number of periods, the game can be solved by backward induction (1 point).

In the last period, there is no future which implies that the last period works in the same way as the interaction in the one-shot game. (1 point)
The next-to-last period can then be treated as if it were the last period since the outcome in the last period does not depend on the outcome in the next-to-last period. (1 point)
This holds for all periods until the first one. (1 point)

Suppose now that the two firms compete for an infinite number of periods. Both firms have a common discount factor δ, which is between 0 and 1. Each firm follows a grim-trigger strategy.

(d) Formulate a reasonable grim-trigger strategy that allows the firms to sustain collusion if the discount factor δ is large enough. (5 points)

Grim-trigger strategy:
Start by setting the monopoly price of 6 in the first period. (1 point)
Keep setting the monopoly price of 6 as long as both have done so in all previous periods. (2 points)
If one firm has deviated from setting the monopoly price in a previous period, set the Nash equilibrium price of 2 forever. (2 points)

(e) To determine the best deviation from a grim-trigger strategy, state the optimal one-period best-response of a firm, given that the competitor sets its price at the monopoly level. What is the resulting profit? (3 points).

The best response is to set the price slightly below the monopoly price. In this case, at 6 – e. (2 points)
The resulting profit is then (almost) equal to (6-2)*(2)=8. (1 point)

(f) Determine now the critical discount factor above which firms can sustain tacit collusion when following a grim trigger strategy. (5 points)

The discounted value of profits from collusion is

$$\pi^{Collusion} = 4\,\frac{1}{1-\delta} \quad \text{(1 points)}$$

The discounted value of profits from deviation is

$$\pi^{Deviation} = 8 + 0\,\frac{\delta}{1-\delta} \quad \text{(1 points)}$$

The critical discount factor can be found by equating the aforementioned discounted values.

$$\pi^{Collusion} = \pi^{Deviation} \qquad \text{(1 point)}$$

$$4\,\frac{1}{1-\delta} = 8$$

$\delta = \mathbf{1/2}$ (2 points)

Suppose that firms would compete in quantities instead of prices. They still follow a grim-trigger strategy to sustain a collusive outcome in which each firm produces half of the monopoly quantity.

(g) Explain verbally how the deviation profit of a firm differs when firms collude by setting quantities as compared to the price setting case. (4 points)

When deviating in quantity competition, the rival still produces a positive quantity (1 point)
Therefore, the firms does not get the entire monopoly profit. (2 points)
Instead, with Bertrand competition, it gets the full monopoly profit. (1 point)

Question 4 – Asymmetric Information (20 points)

Consider the following signaling situation:

There are two types of cars: one has high quality and is worth 50 to a buyer, and one has low quality and is worth 20 to a buyer. Both types are equally likely (that is, they each have a probability of 0.5).

The owner of each type of car can go to a car repair shop to implement "cosmetic" changes to the car, which do not affect the quality. However, implementing a level K of cosmetic changes is less expensive for the owner of a high-quality car than for the owner of a low-quality car. Specifically, implementing one unit of cosmetic changes costs 2 for the owner of a high-quality car but 5 for the owner of a low-quality car.

(a) Suppose that implementing cosmetic changes was not possible, what is the expected value of a car to a buyer? (2 points)

½ (50+20)=35 (2 points)

Consider the following constellation: Low-quality owners choose K=0, high-quality owners choose K= K*>0, and buyers pay 50 when seeing a level K= K* but only 20 when seeing a car with K=0.

(b) Determine for which values of K* neither the owner of low-quality car nor the owner off a high-quality car have an incentive to deviate from the constellation describec above. (5 points)

High-quality:
50-2 K* > 20, which is equivalent to K* < 15 (2 points)

Low-quality:
20> 50-5 K*, which is equivalent to K* > 6 (2 points)

No deviation incentive if 6 < K* < 15. (1 point)

(c) Out of these levels, which level of K* is the most-efficient one for social surplus? Give a short explanation. (3 points)

K* = 6 is most efficient (1 point)
Higher levels of K* do not improve quality (1 point)
But higher levels are costly (1 point)

(d) Explain verbally why signaling allows a buyer to disentangle the quality types although cosmetic changes do not improve the quality of a car. (4 points)

High-quality sellers choose a level so that low-quality sellers do not want to mimic (2 points)
This works because quality is positive correlated with low cost of changes (2 points)

Hidden action occurs in several economic situations.

(e) State and briefly explain a situation of hidden action. (2 points)

Labor relationship: An employer cannot observe how hard an employee works (2 points)
Insurance: An insurance company cannot observe the effort level of an insure to avoid a damage (2 points)

(f) Consider a situation of hidden action in which both parties (i.e., the principal and the agent) are risk neutral. Describe a contract that can avoid inefficiencies in such a situation and briefly explain your result. (4 points)

The principal sells the project to the agent (1 point)
The agent can then keep all money from the project for herself (2 points)
She therefore has perfect incentives to choose the efficient effort level. (1 point)