Frankfurt School of Finance & Management
German Excellence. Global Relevance.

Examination in the Bachelor of Science
Course title: Markets, Incentives and Ethical Management
Part: Markets and Incentives
Semester: 2
Lecturers: Dr. Homayoon Moradi, Prof. Dr. Markus Reisinger, Dr. Roee Sarel
Group: 172
Examination date: 23th March 2019

Aids: pocket calculator Casio FX-82 solar,
German-English Dictionary, English-English Dictionary

Please enter your student ID (matriculation number) and your group!

Student ID	Group

Please note:

The exam consists of 4 questions of which you will have to answer **all** questions. You have **90** minutes to complete the examination. The maximum of points to be reached is **90**. Please use the enclosed answer sheet to answer your questions and add your student ID on its cover.

We wish you all the best for your examination!

Internal use only!

Question	1	2	3	4	**Total**
Possible points:	23	22	13	32	**90**
Points achieved:					

Signature of corrector

Question 1 – Game Theory (23 points)

Consider a situation with two symmetric firms. Each firm can invest in advertising to increase the demand of its product. The advertising levels are denoted by a_1 and a_2 for firm 1 and firm 2, respectively. Each firm has costs of investing in advertising, which are $(a_i)^2$, with $i=1,2$. The price in the market is equal to 1, while the demand of firm 1 is

$$1 + a_1 + a_1 a_2.$$

Similarly, the demand of firm 2 is

$$1 + a_2 + a_2 a_1.$$

Suppose first that the two firms choose their advertising levels simultaneously.

(a) Set up the profit function of firm 1 and determine the best-response function. (3 points)

$$\pi_1 = 1 + a_1 + a_1 a_2 - a_1^2 \quad \text{(1 point)}$$

$$\frac{\partial \pi_1}{\partial a_1} = 1 + a_2 - 2a_1 = 0 \quad \text{(1 point)}$$

$$\text{Best} - \text{response function:} \quad a_1 = \frac{1 + a_2}{2} \quad \text{(1 point)}$$

(b) Draw the best-response function in a diagram. Is the best-response function increasing or decreasing? Explain your result. (4 points)

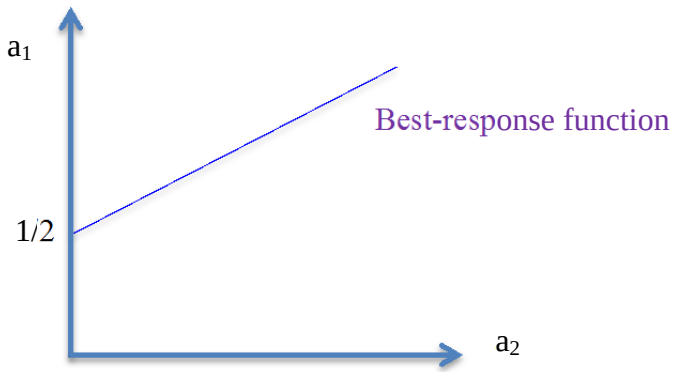

Diagram (1 1/2 points)
Best-response function is increasing (1 point)
The reason is that the demand of firm 1 increases is firm 2 sets a higher advertising level.
The two advertising levels are strategic complements (1 1/2 points)

(c) Solve for the advertising levels in the Nash equilibrium. (2 points).

Because the two firms are symmetric, in equilibrium: $a_1 = a_2$ (1 point)

From the best-response function, we can therefore write:

$$a_1 = \frac{1 + a_1}{2} \quad (1/2 \text{ point})$$

Solving this yields:

$$a_1 = a_2 = 1 \quad (1/2 \text{ point})$$

Suppose now that the two firms play the advertising game sequentially, that is, firm 1 sets its advertising level before firm 2 (i.e., firm 1 is a first mover and firm 2 is a second mover).

(d) Determine the subgame perfect equilibrium advertising levels of this sequential game. (4 points)

Game is solved by backward induction. Profit function of firm 2 in the second stage is:

$$\pi_2 = 1 + a_2 + a_2 a_1 - a_2^2 \quad (1/2 \text{ point})$$

$$\text{Best} - \text{response function: } a_2 = \frac{1 + a_1}{2} \quad (1/2 \text{ point})$$

We can insert this into the profit function of firm 1:

$$\pi_1 = 1 + a_1 + a_1 a_2 - a_1^2 = 1 + a_1 + a_1\left(\frac{1 + a_1}{2}\right) - a_1^2 = 1 + \frac{3}{2}a_1 - \frac{1}{2}a_1^2 \quad (1 \text{ point})$$

$$\frac{\partial \pi_1}{\partial a_1} = \frac{3}{2} - a_1 = 0 \quad (1/2 \text{ point})$$

$$a_1 = \frac{3}{2} \quad (1/2 \text{ point})$$

Inserting this back into the best-response function of firm 2 yields.

$$a_2 = \frac{5}{4} \, (1 \text{ point})$$

(e) Is the equilibrium advertising level of **firm 1** (i.e., the first mover) higher or lower than in the simultaneous game? Explain your result. (3 points)

It is higher in the sequential game than in the simultaneous game because 3/2 is larger than 1 (1 point)
The reason is that firm 1 benefits if firm 2 sets a higher advertising level. Because advertising levels are strategic complements, firm 1 can induce firm 2 to set a higher advertising level by advertising more itself (2 points)

(f) Is the equilibrium advertising level of **firm 2** (i.e., the second mover) higher or lower than in the simultaneous game. Explain your result (3 points).

It is higher in the sequential game than in the simultaneous game because 5/4 is larger than 1 (1 point)
The reason is that firm 1 sets a higher advertising level. This leads to a higher advertising level of firm 2 because firm 2 benefits at the margin from firm 1 advertising more. (2 points)

Consider now situations in general, that is, no longer related to the advertising game.

(g) State and explain a situation in which a player can benefit from committing to a particular action. What is the reason for the benefit through commitment? (4 points)

Such a situation could be bargaining (1 point)
If one is known to be a tough bargainer (i.e, a seller not lowering her price), one usually obtains a better deal in negotiations. In this case, a higher price form the buyer (1 ½ points)
The reason for this is that one limits her own flexibility to influence the action of someone else in a way that is preferable for yourself (1 ½ points)

Other situations could be a quantity competition game, in which firm setting its quantity first commits to set a higher quantity to induce the follower to produce a lower quantity
or a war situation, in which a general takes an action to prevent his army from running away and thereby commits that his army will be tough in a fight.

Question 2 – Oligopoly (22 points)

(a) Explain verbally why price competition between firms selling homogeneous products and having constant marginal costs (and no fixed costs) lead to an equilibrium in which prices are equal to marginal costs and firms make no profits. (4 points)

If a firm sets its price above marginal costs, the rival firm has the incentive to set its price slightly below. (1 point)
The reason is that such an undercutting of the rival's price allows a firm gain the entire demand because product are homogenous (i.e., perfect substitutes) and is therefore always profitable. (1 point)
Following this logic, each firm always undercuts until the price has reached marginal costs. Then undercutting is no longer profitable. (1 point)
If both firms set the price marginal cost, both make zero profits. However, no firm can do better because further undercutting leads to losses and increasing the price leads to zero demand and therefore also to zero profits. (1 point)

(b) What is the effect of an increase in the number of firms in the situation described in (a) that is, there are additional firms in the market with the same cost function as the firms already in the market. (2 points)

There is no effect (i.e., the outcome is still that all firms set the price equal to marginal cost). (1 point)
The logic described above holds for all firms, and therefore the outcome is unchanged. (1 point)

(c) State and briefly explain two strategic variables that can be important for competing firms **apart from prices and quantities**. Provide an industry example where the strategic variable is important. (6 points)

- Product differentiation (1 point)
  A firm produces a different variant of the product than the other firm (1 point)
  This is important in the automobile industry (BMW versus Mercedes) (1 point)
- Location of a firm (1 point)
  It is important for consumes how long they need to travel to the firm to buy a good (1 point)
  This is important for gasoline station (station in a city versus one on a motorway) (1 point)
- Information on the product (1 point)
  Firms may willingly reveal information of the product's ingredients (1 point)
  This is important in market for organic products (1 point)

Consider a market with two symmetric firms, firm 1 and firm 2. The two firms compete in prices. The demand functions of the two firms are

$$D_1 = 2 - 4p_1 + 2p_2 \quad \text{and} \quad D_2 = 2 - 4p_2 + 2p_1.$$

The costs of both firms are equal to zero.

(d) Do the two firms sell substitutes or complements? Briefly explain your result. (2 points)

The two firms sell substitutes. (1 point)
The demand of one firm is increasing if the other firm raises its price. (1 point)

(e) Are the products of the two firms differentiated or not? Briefly explain your result. (2 points)

The products are differentiated. (1 point)
The effect of a change in a firm's price on the demand of the firm is larger in absolute value than the effect of a change in the price of the rival. (1 point)

(f) Solve the game for the Nash equilibrium prices? (4 points)

$$\pi_1 = D_1(p_1, p_2) * p_1 = (2 - 4p_1 + 2p_2)p_1 \text{ (1 point)}$$

$$\frac{\partial \pi_1}{\partial p_1} = -4p_1 + (2 - 4p_1 + 2p_2) = 0 \Rightarrow p_1 = \frac{1}{4} + \frac{1}{4}p_2 \text{ (1 point)}$$

Firms are symmetric: $p_2 = p_1$(1 point)

$$p_2 = \frac{1}{4} + \frac{1}{4}p_2 \Rightarrow p_2 = p_1 = \frac{1}{3} \text{ (1 point)}$$

(g) Are the equilibrium prices above marginal costs? Briefly explain your result. (2 points)

Equilibrium prices are above marginal costs. (1 point)
The reason is that the products of the two firms are differentiated. This implies that undercutting the rival is not always profitable because the undercutting form dies not get the entire demand. (1 point)

Question 3 – Tacit Collusion (13 points)

Consider the situation in which two firms compete in an infinitely repeated game. They want to sustain the collusive outcome (i.e., each firm obtains half of the monopoly profit) via grim-trigger strategies.

(a) Explain what a grim-trigger strategy is. Explicitly distinguish between the different payoffs that a firm can get when colluding and when not colluding. (6 points)

A grim-trigger strategy is as follows:
A firms starts be setting the price or quantity such that the firms in the industry obtain the monopoly profit. (For example, both firms set the monopoly price or with quantity competition, each firm sets half of the monopoly quantity). (1 point)
The firm continues with this strategy as long as both firms have done so in all previous periods. (1 point)
This is the cooperation phase. In this phase, each firm obtains as a payoff half of the monopoly profit (1 ½ points)
If, instead, one of the two firms has deviated from the cooperation strategy, each firm sets its action as in the Nash equilibrium of the stage game (e.g., price equal to marginal costs with Bertrand competition). (1 point)
This is the punishment phase and each firm obtains as payoff only the Nash equilibrium profit. (1 ½ points)

(b) Explain briefly why the firms can sustain collusion with the grim-trigger strategy only in case the future is sufficiently valuable for each firm (4 points)

When all firms follows the grim-trigger strategy, each one obtains in every period half of the monopoly profit. (1 point)
Instead, if a firm deviates, it obtains more than this profit today. (1 point)
However, from the next period onwards, it will be punished and obtains a smaller profit than half of the monopoly profit. (1 point)
Therefore, deviation brings an immediate gain but a loss in the future. The consequence is that collusion can only be sustained if the future is sufficiently valuable relative to the present. (1 point)

Suppose now that 2 additional firms compete in the market, that is, there are now 4 firms instead of only 2.

(c) Is collusion now easier or harder to sustain? Explain your result. (3 points)

Collusion becomes harder to sustain. (1 point)
The reason is that, when colluding, each firm now no longer obtains half of the monopoly profit but only one quarter of it. This makes collusion relative attractive compared to deviation. (2 points)

Question 4 – Asymmetric Information (32 points)

(a) State an economic situation with hidden information. Explain why hidden information can lead to adverse selection? What is the consequence for market efficiency? (4 points)

Used car market (1 point)
Hidden information means that the buyer does not observe the type of car that the selling (i.e., the quality). The buyer will determine the average value of the car. But then the high types are not willing to sell their car because the (opportunity) costs for these types are higher than the buyer's average valuation. Because high types are no longer present, the buyer adjusts her valuation downwards. Then even medium types can no longer break even and drop out of the market. In the end, only the low-quality cars remain, which is adverse selection. (2 points)
The outcome is inefficient because without hidden information, even high-quality cars and medium-quality cars could be traded between buyers and sellers. (1 point)

(b) Explain for the situation you described in (a) how signaling can improve the market outcome and why it allows the "good type" to distinguish from the "bad type". (4 points)

Signaling in the used car market could work in the following way:
The high-quality car seller gives an warranty to the buyer, promising to pay an amount of money in case the car turn out to be of low or medium quality (2 points)
The high-quality seller can afford to do so because he does not need to pay out the money as his the car is of high quality. Instead, issuing this warranty is not profitable (i.e., more costly) for the low- and the medium-quality seller because they need to pay the money. As a consequence, with the warranty, the buyer knows that only high-quality sellers will give this warranty and signaling therefore allows them to distinguish he different types. (2 points)

Consider the following situation of hidden action: A principal needs to hire an agent to perform a particular task. If the agent is successful with the task, the revenue for the principal is 500. Instead, if the agent is not successful with the task, the revenue to the principal is only 100.
The agent has two effort levels, e_1 with costs of 5 for the agent and e_2 with costs of 10 for the agent. The effect of the effort choice on the likelihood of the revenue is given by the following table:

Effort	prob(Revenue=500)	Prob(Revenue=100)
e_1	1/4	3/4
e_2	3/4	1/4

The principal is risk neutral and cares only about the expected revenue minus the expected wage payment to the agent. By contrast, the agent is risk averse, and her utility function is

$$\sqrt{w} - e,$$

where *w* is the wage she obtains from the principal and *e* is the effort level, which is either e_1 or e_2. The reservation utility of the agent equals 0.

Consider first the case in which the principal can observe the effort level of the agent (symmetric information).

(c) What is the optimal wage level that the principal pays in case he wants to implement the **high** effort level e_2? What is the expected profit of the principal? (3 points)

As the principal can observe the effort level, he only needs to make sure that the agent participates:

$$\sqrt{w_H} - e_2 \geq 0 \Rightarrow \sqrt{w_H} \geq 10. \text{ (1 point)}$$

The lowest possible wage that the principal needs to pay to induce the agent to participate is therefore:

$$w_H = 100. \text{ (1 point)}$$

Expected profit:

$$\frac{3}{4} * 500 + \frac{1}{4} * 100 - 100 = 375 + 25 - 100 = 300. \text{ (1 point)}$$

(d) Determine the optimal wage level and the expected profit of the principal in case he wants to implement the **low** effort level e_1? Does the principal optimally implement effort level e_1 or e_2 with symmetric information? (3 points)

Participation constraint:

$$\sqrt{w_L} - e_1 \geq 0 \Rightarrow w_L = 25. \text{ (1 point)}$$

Expected profit with the low effort level:

$$\frac{1}{4} * 500 + \frac{3}{4} * 100 - 25 = 125 + 75 - 25 = 175 \text{ (1 point)}$$

Because 175 is lower than 300, the principal wants to implement the high effort level e_2. (1 point)

(e) Explain briefly why the principal optimally pays a flat wage to the agent (i.e., a wage that is not conditional on the revenue) in case the effort level is observable. (2 points)

The reason is that the agent is risk averse whereas the principal is risk neutral. (1 point)
Setting a wage, which is not flat, exposes the agent to risk and implies that the principal must pay a higher expected wage. (1 point)

Suppose now that the principal cannot observe the effort level of the agent (asymmetric information).

(f) Describe the fundamental trade-off of the principal-agent relationship in case of asymmetric information. (4 points)

The fundamental trade-off is between incentives and insurance. (1 point)
The first goal of the principal is two motivate the agent to spend high effort. This can only work with a steep contract, in which the agent gets rewarded when the outcome is good (i.e., revenue for the principal is high). (1 point)
The second goal is to provide insurance to the agent because the agent is risk averse and the principal risk neutral. This can be achieved in the optimal way with a flat contract, in which the agent gets the same wage, whatever the revenue of the principal is. (1 point)
Both goals are in contradiction to each other and can never be optimally fulfilled at the same time. (1 point)

(g) Formally state the two constraints that the optimal contract must fulfill with asymmetric information if the principal wants to implement the high effort level e_2. (4 points)

Participation constraint:

$$\frac{1}{4}\sqrt{w_L}+\frac{3}{4}\sqrt{w_H}-10\geq 0 \text{ (2 points)}$$

Incentive compatibility constraint:

$$\frac{1}{4}\sqrt{w_L}+\frac{3}{4}\sqrt{w_H}-10\geq \frac{3}{4}\sqrt{w_L}+\frac{1}{4}\sqrt{w_H}-5 \text{ (2 points)}$$

(h) Determine the optimal wage levels that the principal pays the agent. (4 points)

At the optimal contract, the two constraints must hold with equality. This implies

$$\frac{1}{4}\sqrt{w_L}+\frac{3}{4}\sqrt{w_H}-10=0$$

and

$$\frac{1}{4}\sqrt{w_L}+\frac{3}{4}\sqrt{w_H}-10=\frac{3}{4}\sqrt{w_L}+\frac{1}{4}\sqrt{w_H}-5$$

Multiplying the participation constraint with 2 and rewriting the incentive compatibility constraint yields:

$$\begin{aligned}\frac{1}{2}\sqrt{w_L}+\frac{3}{2}\sqrt{w_H}&=20\\ -\frac{1}{2}\sqrt{w_L}+\frac{1}{2}\sqrt{w_H}&=5 \text{ (2 points)}\end{aligned}$$

Adding the two equations yields:

$$2\sqrt{w_H} \geq 25$$
$$\sqrt{w_H} = 12.5$$
$$w_H = 156.25 \text{ (1 point)}$$

Inserting this into the second equation:

$$-\frac{1}{2}\sqrt{w_L} + 6.25 = 5 \Rightarrow \sqrt{w_L} = 2.5$$
$$w_L = 6.25 \text{ (1 point)}$$

(i) What is the associated profit of the principal at the optimal contract? (2 points)

$$\frac{1}{4} * (100 - 6.25) + \frac{3}{4}(500 - 156.25) = 281.25 \text{ (2 points)}$$

(j) Is it optimal for principal to implement the high effort level e_2 instead of the low effort level e_1? (2 points)

Because 281.25 is larger than 175, which is the profit the principal can obtain with implementing the low effort level, the principal prefers to implement he high effort level. (2 points)