Problem Set 4: Cartels and Tacit Collusion - Solution

Markets, Incentives and Ethical Management

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1. Consider a market with two firms which produce a homogeneous product. Firms are in Cournot-competition, that is, they compete in quantities. For simplicity marginal and fixed costs of both firms are zero. The inverse demand function is given by $P(Q) = 1 - Q$, with $Q = q_1 + q_2$. The firms play an infinitely repeated game and set their quantities simultaneously in each period. The common discount factor of both firms is $\beta < 1$.

   a.) Determine the critical discount factor above which firms can sustain the monopoly quantity as a subgame perfect equilibrium with the following grim trigger strategies: Produce half of the monopoly quantity in the first period. Keep producing half of the monopoly quantity as long as both firms have done so in all previous periods. If one firm has deviated from producing half of the monopoly quantity, produce the Nash equilibrium quantity forever. (Hint: Start by determining the monopoly quantity and respective monopoly profit. Then determine the optimal deviation of a firm, given that the other firm produces half the monopoly quantity, and the corresponding deviation profit. Then determine the static Nash equilibrium of the Cournot game and the resulting profits. Finally, compare the profit for a firm if it follows the collusive strategy of setting half of the monopoly quantity as long as no firm deviated from it with the profit of optimally deviating and obtaining the punishment profit from next period onwards.)

The monopoly profit is given by $\pi^M = P(Q^M)Q$ where $P(Q^M) = (1 - Q^M)$. Therefore we can rewrite the monopoly profit as

$$\pi^M = (1 - Q^M)Q^M.$$

FOC with respect to Q^M let us derive

$$\frac{\partial \pi^M}{\partial Q^M} = 1 - 2Q^M = 0.$$

The monopoly quantity is thus given by $Q^M = \frac{1}{2}$ which allows for computing the monopoly price to be $p^M = \frac{1}{2}$. The profit of either firm is thus $\pi^M_{i=1,2} = \frac{1}{2}\left(1 - Q^M\right)Q^M$ which is

$$\pi^M_{i=1,2} = \frac{1}{2}\left(1 - \frac{1}{2}\right)\frac{1}{2} = \frac{1}{8}.$$

Let us now determine the optimal deviation of firm 1, given that firm 2 produces half of the monopoly quantity which is $q_2^M = \frac{1}{2}Q^M = \frac{1}{2}\frac{1}{2} = \frac{1}{4}$.

Deviation profit of firm 1 is therefore

$$\pi_1^D = \left(1 - q_1 - \frac{1}{4}\right)q_1.$$

FOC with respect to q_1 can be written as

$$\frac{\partial \pi_1^D}{\partial q_1} = \frac{3}{4} - 2q_1 = 0$$

which lets us compute an optimal deviation quantity of $q_1^D = \frac{3}{8}$. The deviation profit of firm 1 is

$$\pi_1^D = \left(1 - \frac{3}{8} - \frac{1}{4}\right)\frac{3}{8} = \frac{9}{64}.$$

Finally we determine the static Nash outcome of the Cournot game. The profit of firm 1 under competition is

$$\pi_1^N = (1 - q_1 - q_2)\, q_1.$$

FOC with respect to q_1 are

$$\frac{\partial \pi_1^N}{\partial q_1} = 1 - 2q_1 - q_2 = 0.$$

The reaction function of firm 1 in a Cournot-Nash game is $q_1 = \frac{1-q_2}{2}$. Due to symmetry of the firms, the reaction function of firm 2 is $q_2 = \frac{1-q_1}{2}$. Inserting q_2 in q_1 and vice versa, lets us compute the equilibrium quantities $q_1^* = q_2^* = \frac{1}{3}$.
The Cournot-Nash profit of either firm is thus

$$\pi_{i=1,2}^N = \left(1 - \frac{2}{3}\right)\frac{1}{3} = \frac{1}{9}.$$

It is obvious that $\pi_{i=1,2}^D > \pi_{i=1,2}^M > \pi_{i=1,2}^N$.
We can now determine for which discount factor β collusion can be sustained. Following the collusive strategy in an infinitely repeated game allows either firm to gain a profit of

$$\pi_{i=1,2}^C = \pi_{i=1,2}^M \left(1 + \beta + \beta^2 + \beta^3 + ...\right) = \pi_{i=1,2}^M \frac{1}{1-\beta}$$

which is

$$\pi_{i=1,2}^C = \frac{1}{8}\left(1 + \beta + \beta^2 + \beta^3 + ...\right) = \frac{1}{8}\frac{1}{1-\beta}.$$

In contrast, deviating from the collusive strategy in an infinitely repeated Cournot game lets either firm generate a profit of

$$\pi_{i=1,2}^{NC} = \pi_{i=1,2}^D + \pi_{i=1,2}^N \left(\beta + \beta^2 + \beta^3 + ...\right) = \pi_{i=1,2}^D + \pi_{i=1,2}^N \frac{\beta}{1-\beta}$$

which is

$$\pi_{i=1,2}^{NC} = \frac{9}{64} + \frac{1}{9}\left(\beta + \beta^2 + \beta^3 + ...\right) = \frac{9}{64} + \frac{1}{9}\frac{\beta}{1-\beta}.$$

Collusion can therefore be sustained if

$$\begin{aligned}
\frac{1}{8}\frac{1}{1-\beta} &\geq \frac{9}{64} + \frac{1}{9}\frac{\beta}{1-\beta} \\
\frac{1}{8} &\geq \frac{9}{64}(1-\beta) + \frac{\beta}{9} \\
\frac{72}{64} &\geq \frac{81}{64}(1-\beta) + \beta \\
\frac{17}{64}\beta &\geq \frac{9}{64} \\
\beta &\geq \frac{9}{17}.
\end{aligned}$$

b.) What are the differences with respect to the deviation profit and the punishment profit of the Cournot model as compared to the Bertrand model considered in the lecture.

In contrast to the Bertrand model considered in the lecture, a Cournot setup allows a deviating firm to gain a positive profit in the static Nash equilibrium in the infinitely repeated game. Therefore, a firm obtains a positive profit also during the punishment phase. A Bertrand setup implies that firms set their prices equal to marginal costs in the punishment phase which leads to zero profits. This makes deviation more attractive in Cournot.

On the other hand, the deviation profit is larger with Bertrand competition. When deviating in Bertrand, a firm obtains the entire monopoly profit because its slightly undercuts the rival's monopoly price. Instead, with Cournot competition, the non-deviating firms still produces half of the monopoly quantity, and is therefore still active in the market. As a consequence, the deviating firm cannot get the entire monopoly profit.

Combining the results of the lecture and the problem set, we obtain that the critical discount factor is higher with Cournot competition ($\beta = 9/17$) than with Bertrand competition ($\beta = 1/2$). This implies that tacit collusion is more difficult to sustain with Cournot competition. Therefore, the first effect described above is stronger than the second.