Frankfurt School of Finance & Management
German Excellence. Global Relevance.

Examination in the Bachelor of Science
Course title: Markets, Incentives and Ethical Management
Part: Markets and Incentives
Semester: 3
Lecturers: Prof. Dr. Heiko Karle, Prof. Dr. Markus Reisinger
Examination date: 22th October 2020

Aids: pocket calculator Casio FX-82 solar,
German-English Dictionary, English-English Dictionary

Please enter your student ID (matriculation number) and your group!

Student ID	Group

Please note:

The exam consists of 4 questions of which you will have to answer **all** questions. You have **90** minutes to complete the examination. The maximum of points to be reached is **90**. Please use the enclosed answer sheet to answer your questions and add your student ID on its cover.

We wish you all the best for your examination!

Internal use only!

Question	1	2	3	4	**Total**
Possible points:	27	26	10	27	**90**
Points achieved:					

Signature of corrector

Question 1 – General Equilibrium (27 points)

Suppose that there are two consumers A and B and two products x and y. The initial endowment W is such that consumer A is endowed with $(W_{XA}, W_{YA}) = (1,8)$ and consumer B is endowed with $(W_{XB}, W_{XB}) = (8,27)$. Both consumers have standard preferences, and their utility functions are the same and equal to

$$U_A(x_A, y_A) = x_A * y_A^2 \qquad U_B(x_B, y_B) = x_B * y_B^2.$$

(a) Determine consumer A and B's utility at their initial endowment. (2 points)

(b) Explain how the set of allocations that are Pareto improvements compared to the initial endowment is characterized in an Edgeworth box? (2 points)

(c) What is generally the condition for Pareto efficiency in an economy with two consumers A and B. (2 points)

(d) Determine (mathematically) the slopes of the indifference curves for the two consumers. (3 points)

(e) Determine (mathematically) the equation for the Pareto efficient allocations in the economy described above. (2 points)

(f) Describe verbally how you would use the equation for the Pareto efficient allocations and the initial endowment to determine the equation for the contract curve. (You do not need to calculate it mathematically!) (3 points)

(g) The equation for the contract curve is $\mathbf{y_A = \frac{35}{9} * x_A}$.
Determine for each of the following three points whether it can be an equilibrium allocation in this economy:
(i) (xA = 1; yA = 8); (xB = 8; yB = 27);
(ii) (xA = 9; yA = 35); (xB = 0; yB = 0);
(iii) (xA = 1.7; yA = 6.61); (xB = 7.3; yB = 28.39).
Provide an explanation for your results. (6 points)

(h) What is the content of the First Welfare Theorem? (2 points)

(i) Now consider the German market for timber (Holz). After several very hot summers, the stock of spruce (Fichte) is at risk and must be lumbered, while the stock of other species such as beech (Buche) is not affected. How does the positive supply shock of spruce timber affect the price of beech timber? Explain your answer. (5 points)

Question 2 – Oligopoly (26 points)

Consider a market with two firms, firm 1 and firm 2, which produce a homogeneous product. Suppose that the firms simultaneously set their quantities (**Cournot competition**). The inverse demand function is $p(Q) = 40 - Q$, where $Q=q_1+q_2$. The cost function of each firm $i=1,2$, is $C_i = 10q_i$.

(a) Set up the profit function of firm 1 and determine the first-order condition. (4 points)

(b) Determine the quantities of both firms in the Nash equilibrium. (3 points)

(c) Suppose for this sub-question only that there are 5 firms in the market instead of 2. Each of the 5 firms has the same cost function given by $C_i = 10q_i$, with $i=1,2,3,4,5$, and the inverse demand function is still $p(Q) = 40 - Q$, where $Q=q_1+q_2+q_3+q_4+q_5$. What is the quantity of each firm in the Nash equilibrium? (3 points)

(d) Explain (verbally) how the equilibrium price, the equilibrium profit, and the equilibrium quantity of each firm change in a Cournot model if the number of firms gets very large? (4 points)

(e) Consider now again the Cournot model described at the beginning of the question (i.e., the one with two firms) but suppose that the marginal cost of the two firms were different, that is, firm 1 has lower marginal cost than 10 and firm 2 has higher marginal cost.
Explain (verbally) how the quantities of the two firms change compared to the symmetric situation. (3 points)

(f) Explain why there is only a single price in a Cournot market. (3 points)

(g) Give one example of a market in which firms compete by setting quantities and there is a single market-clearing price. (1 point)

(h) Suppose that the quantity competition described at the beginning of the question is not played simultaneously but sequentially. In particular, firm 1 sets its quantity first, firm 2 observes this quantity, and then chooses its quantity.
Explain (verbally) how the quantities chosen in this sequential game differ from the ones of the simultaneous game. Provide the intuition for your result. (5 points)

Question 3 – Tacit Collusion (10 points)

Consider a situation with two firms (i.e., firm 1 and firm 2), which produce a homogenous product. The inverse demand function is linear and firms' production costs are zero. Competition occurs in prices (**Bertrand competition**). Suppose the two firms compete for an infinite number of periods. Each firm i has a discount factor δ_i between 0 and 1.

(a) Determine a reasonable strategy that allows the firms to sustain collusion if their discount factors are large enough. How is such a strategy called? (5 points)

(b) Order the following three situations from "collusion is most likely" to "collusion is least likely". Provide an explanation for your assessment. (5 points)

(i) ($\delta_1 = 0.7$; $\delta_2 = 0.7$);
(ii) ($\delta_1 = 0.4$; $\delta_2 = 0.4$);
(iii) ($\delta_1 = 0.8$; $\delta_2 = 0.3$).

Question 4 – Asymmetric Information (27 points)

Consider a used car market in which there are three types of car dealers: High (H), Medium (M), and Low (L) types. The car of high type is worth 20 to a buyer, the car of medium type is worth 15, and the car of low type is worth X. A high-type seller incurs a cost of 15 when selling the car, a medium-type seller incurs a cost of 13, and a low-type seller incurs a cost of 5.

Each seller knows the quality of the car he sells, but quality is not observable to buyers prior to purchase.

The probabilities for the different types are as follows: the probability that the car is of high type is 1/5, the probability that the car is of medium type is 3/5, and the probability that the car is of low type is 1/5.

The following table summarizes the aforementioned situation:

Type	Probability	Value	Cost
H	1/5	20	15
M	3/5	15	13
L	1/5	X	5

(a) Determine the maximum price that a buyer is willing to pay in case all the three qualities are offered. (3 points)

(b) Set up the condition that needs to be fulfilled so that the all three qualities (i.e., {H, M, L}) are offered in equilibrium. Determine the values of X so that this situation is indeed an equilibrium. (3 points)

(c) Suppose now that only the medium and the low types are active. Determine the probabilities for the medium and the low type in this situation. (2 points)

(d) Set up the condition that needs to be fulfilled so that only the medium and low quality (i.e., {M, L}) is offered in equilibrium. Determine the values of X so that this situation is indeed an equilibrium. (4 points)

(e) Explain how signaling in the used car market could work to overcome the problem of asymmetric information. (4 points)

(f) Focus now only on the sellers of type H and M. Suppose the seller of type H wants to signal the type of his car by offering the buyer a certificate with the following content: if the car turns out to be of type M, the seller pays the buyer an amount Y. Determine the values of Y, which allow type H to separate himself from type M. Briefly explain your solution. (3 points)

(g) Describe the fundamental trade-off of the principal-agent relationship with a risk-neutral principal and a risk-averse agent in case of asymmetric information. (5 points)

(h) Explain why in case of a risk-neutral agent no inefficiency occurs, despite the effect that there is asymmetric information. (3 points)