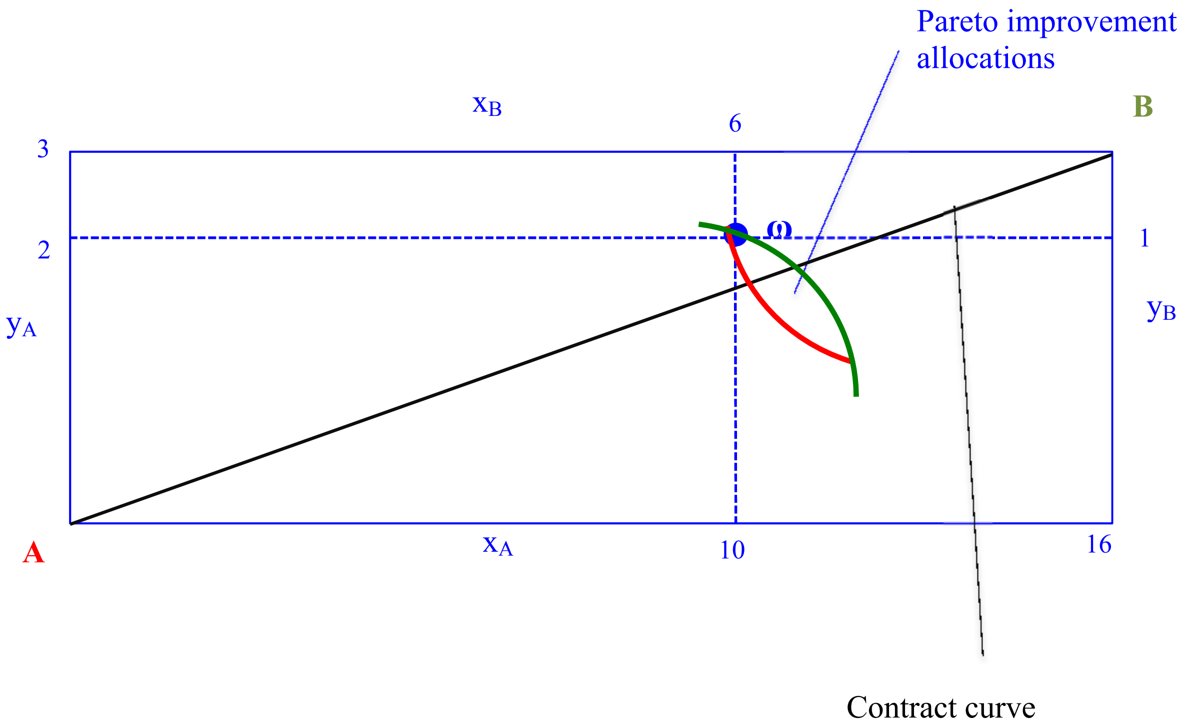
Question 1 – General Equilibrium (31 points)

Suppose that there are two consumers A and B and two products *x* and *y*. The initial endowment ω is such that consumer A is endowed with $(\omega_{X_A}, \omega_{Y_A}) = (10, 2)$ and consumer B is endowed with $(\omega_{X_B}, \omega_{Y_B}) = (6, 1)$. Both consumers have standard preferences and their utility functions are

$$U_A(X_A, Y_A) = X_A Y_A \quad \text{and} \quad U_B(x_B, y_B) = X_B Y_B.$$

(a) In the Edgeworth box, draw the consumers' indifference curves passing through the initial endowment. Indicate the set of allocations that are Pareto improvements compared to the initial endowment and briefly explain it. (6 points)



Allocation of points:

- Edgeworth Box in general (1 point)
- Correct notation of axes and numbers (1 point)
- Initial endowment (1 point)
- Indifference curves of the two consumers (2 points)
- Pareto improvements (1 point)

(b) Determine the Pareto-efficiency condition and briefly explain it. (4 points)

The Pareto-efficiency condition is $MRS_A = MRS_B$ (i.e., the marginal rate of substitution of consumer A equals that of consumer B). (1 point)

$$\mathrm{MRS_A} = -\frac{y_A}{x_A}$$ (1 point)

$$\mathrm{MRS_B} = -\frac{y_B}{x_B}$$ (1 point)

Inserting these conditions in the Pareto-efficiency condition yields

$$\frac{\mathbf{y_A}}{\mathbf{x_A}} = \frac{\mathbf{y_B}}{\mathbf{x_B}}.$$ (1 point)

(c) Determine the equation of the contract curve and draw it in the Edgeworth box. (5 points)

Since $x_A + x_B = 16$ and $y_A + y_B = 3$, we have

$x_B = 16 - x_A$, (1 point)

$y_B = 3 - y_A$ (1 point).

Plugging these equalities into the Pareto-efficiency condition, yields the contract curve:

$$\frac{y_A}{x_A} = \frac{3-y_A}{16-x_A}$$ (1 point)

That is,

$$\mathbf{y_A} = \frac{\mathbf{3}}{\mathbf{16}} * \mathbf{x_A}$$ (1 point) and drawing it (1 point).

(d) State the definition of competitive equilibrium. (4 points)

A competitive equilibrium is a set of prices that satisfies the following two conditions:
1. markets clear (i.e., there is neither excess demand nor excess supply of any good), (2 points)
2. given these prices, each subject maximizes his utility subject to the budget constraint. (2 points)

(e) State the definition of the First Welfare Theorem. (2 points)

A competitive equilibrium is (Pareto) efficient. (2 points)

(f) Indicate where in the Edgeworth box the competitive equilibrium must lie according to the first welfare theorem and your answer in (a). (2 points)

The competitive equilibrium must lie on the **contract curve** and on **the set of Pareto improvements over the initial endowment**. (2 points)

The next questions are about distributional issues regarding the two individuals and the welfare with different social welfare functions.

(g) Determine the utilities of consumers A and B at their respective initial endowment. (1 point)

$U_A(10,2) = 20$ $U_B(6,1) = 6$ (1/2 point each)

(h) Determine consumer A and B's utility when consumer A gives 1 unit of good Y to consumer B. (1 point)

$U_A(10,1) = 10$ $U_B(6,2) = 12$ (1/2 point each)

(i) State whether the allocation in g) or in h) is preferred by a Rawlsian welfare function? Explain your answer. (3 point)

The Rawlsian welfare function is $Wr = \min\{UA,UB\}$. Only the minimum utility in the society matters. (1 point)

In h) $Wr=10>6=Wr$ in g). (2 points)

(j) State whether the allocation in g) or h) is preferred by a Utilitarian welfare function? Explain your answer. (3 point)

The Utilitarian welfare function is $Wu = UA+UB$. Only the sum of the utilities in the society matters but not their distribution. (1 point)

In g) $Wu=26>22=Wu$ in h). (2 points)

Question 2 – Oligopoly (20 points)

Consider two firms, firm 1 and firm 2, which produce a homogenous product. The inverse demand function is $P = 10 - Q$. Production cost functions are $C_1(q_1) = 2q_1$ for firm 1 and $C_2(q_2) = 4q_2$ for firm 2.

(a) Suppose that both firms compete in prices (Bertrand Competition). Determine the equilibrium prices and the equilibrium profit of each firm. (4 points)

Firm 2 sets a price of 4 (1 point)
Firm 1 sets a price slightly below 4 (1 point)
Firm 2's profit is 0, as it obtains no demand (1 point)
Firms 1's profit is almost (4-2) * (10-4) = 2*6=12 (1 point)

(b) Suppose now that firms compete in quantities (Cournot competition). Determine the equilibrium quantities and the equilibrium profit of each firm. (7 points)

Firm 1's reaction function

$$\max_{q_1} \quad \pi_1 = (10 - q_1 - q_2)q_1 - 2q_1 \quad \text{(1 point)}$$

$$\frac{\partial \pi_1}{\partial q_1} = 10 - 2q_1 - q_2 - 2 = 0 \quad \text{(1 point)}$$

$$q_1 = 4 - \frac{1}{2}q_2 \quad \text{(1/2 point)}$$

Firm 2's reaction function

$$\max_{q_2} \quad \pi_2 = (10 - q_1 - q_2)q_2 - 4q_2 \quad \text{(1/2 point)}$$

$$\frac{\partial \pi_2}{\partial q_2} = 10 - 2q_2 - q_1 - 4 = 0 \quad \text{(1/2 point)}$$

$$q_2 = 3 - \frac{1}{2}q_1 \quad \text{(1/2 points)}$$

Inserting q_1 into q_2 yields $q_2 = 3 - \frac{1}{2}(4 - \frac{1}{2}q_2) = 1 + \frac{1}{4}q_2$ (1 point)
Solving this for q_2 yields $q_2 = \frac{4}{3}$. (1/2 points)
Inserting it back into q_1 yields $q_1 = 10/3$. (1/2 points)

The resulting profits are
$\pi_1 = \frac{\left(10 - \frac{10}{3} - \frac{4}{3}\right)10}{3} - \frac{20}{3} = \frac{100}{9}$ (1/2 points)
and $\pi_2 = \frac{\left(10 - \frac{10}{3} - \frac{4}{3}\right)4}{3} - \frac{16}{3} = \frac{16}{9}$. (1/2 points)

(c) Is **firm 2** (i.e., the less efficient firm) better with price competition or with quantity competition? Provide an explanation to your answer. (3 points)

Firm 2 is better off with quantity competition (1 point)
Explanation:
The firm is not driven out of the market as in price competition (1 point)
It therefore obtains positive profits (1 point)

(d) Is **firm 1** (i.e., the more efficient firm) better with price competition or with quantity competition? What is the explanation for your result. (3 points)

Firm 1 is better off with price competition (1 point)
The firm obtains a profit of 12 with price competition, which is larger than 100/9 (1 point)
Explanation:
The firm cannot price the rival out of the market in quantity competition (1 point)

(e) Would firm 2 (i.e., the less efficient firm) be better off if the competition was sequential and it was setting its price before firm 1 does (that is, firm 2 chooses its price first, firm 1 observes this price, and sets its own price). Provide an explanation to your answer. (3 points)

No firm 2 would not better off and still obtain zero profits (1 point)
The firm will still not be willing to set a price below its marginal costs of 4 (1 point)
Firm 1 would then set its price slightly below, which implies that firm 2 gets no sales (1 point)

Question 3 – Oligopoly and Tacit Collusion (25 points)

Consider a situation with two firms, firm 1 and firm 2, which produce a homogenous product. The inverse demand function is $P = 5 - Q$. Production cost functions are $2q_i$, i=1,2, for each firm, that is, $C_1(q_1) = 2q_1$ and $C_2(q_2) = 2q_2$.

(a) Suppose that firm 1 is a monopolist (that is, firm 2 does not participate in the market). Determine the monopoly price and firm 1's profit. (4 points)

$$\max_{q_M} \quad \pi_M = (5 - q_M)q_M - 2q_M \quad \text{(1 point)}$$

$$\frac{\partial \pi_M}{\partial q_M} = 5 - 2q_M - 2 = 0 \quad \text{(1 point)}$$

$q_M = \frac{3}{2}$, which implies a monopoly price of 5-3/2=3.5 (1 point)

$$\pi_M = \frac{\left(5-\frac{3}{2}\right)3}{2} - \frac{6}{2} = \frac{9}{4} \text{ (1 point)}$$

(b) Instead of firm 1 being a monopolist, firm 1 now competes with firm 2. Competition occurs in prices (Bertrand competition). What are the equilibrium prices and equilibrium profits of this game? (3 points)

Equilibrium prices is equal to 2, as firms compete each other down to marginal costs (2 points)

Equilibrium profit is 0 for each firm (1 point)

(c) Suppose that instead of competing only for one period, the two firms compete in prices repeatedly for 100 periods. Explain why the Subgame Perfect Nash equilibrium in this repeated interaction is the same as in the one-shot (single period) competition. (4 points)

As there is a finite number of periods, the game can be solved by backward induction (1 point).
In the last period, there is no future which implies that the last period works in the same way as the interaction in the one-shot game. (1 point)
The next-to-last period can then be treated as if it were the last period since the outcome in the last period does not depend on the outcome in the next-to-last period. (1 point)
This holds for all periods until the first one. (1 point)

Suppose now that the two firms compete for an infinite number of periods. Both firms have a common discount factor δ, which is between 0 and 1. Each firm follows a grim-trigger strategy.

(d) Formulate a reasonable grim-trigger strategy that allows the firms to sustain collusion if the discount factor δ is large enough. (5 points)

Grim-trigger strategy:
Start by setting the monopoly price of 3.5 in the first period. (1 point)
Keep setting the monopoly price of 3.5 as long as both have done so in all previous periods. (2 points)
If one firm has deviated from setting the monopoly price in a previous period, se the Nash equilibrium price of 2 forever. (2 points)

(e) To determine the best deviation from a grim-trigger strategy, state the optimal one-period best-response of a firm, given that the competitor sets its price at the monopoly level. What is the resulting profit? (3 points).

The best response is he set the price slightly below the monopoly price. In this case, at 3.5 – e (1 point).
The resulting profit is then (almost) equal to (5-3.5)*(3.5-2)=9/4. (2 points)

(f) Determine now the critical discount factor above which firms can sustain tacit collusion when following a grim trigger strategy. (6 points)

The discounted value of profits from collusion is

$$\pi^{Collusion} = 9/8 \frac{1}{1-\delta} \quad \text{(1 points)}$$

The discounted value of profits from deviation is

$$\pi^{Deviation} = 9/4 + 0 \frac{\delta}{1-\delta} \quad \text{(2 points)}$$

The critical discount factor can be found by equating the aforementioned discounted values.

$$\pi^{Collusion} = \pi^{Deviation} \qquad \text{(1 points)}$$

$$9/8 \frac{1}{1-\beta} = 9/4$$

$$\delta = \mathbf{1/2} \quad \text{(2 points)}$$

Question 4 – Asymmetric Information (14 points)

Consider the following signaling situation:

There are two types of cars: one has high quality and is worth 50 to a buyer, and one has low quality and is worth 20 to a buyer. Both types are equally likely (that is, they each have a probability of 0.5).

The owner of each type of car can go to a car repair shop to implement "cosmetic" changes to the car, which do not affect the quality. However, implementing a level λ of cosmetic changes is less expensive for the owner of a high-quality car than for the owner of a low-quality car. Specifically, implementing one unit of cosmetic changes costs 2 for the owner of a high-quality car but 5 for the owner of a low-quality car.

(a) Suppose that implementing cosmetic changes was not possible, what is the expected value of a car to a buyer? (2 points)

½ (50+20)=35 (2 points)

Consider the following constellation: Low-quality owners choose $\lambda=0$, high-quality owners choose $\lambda=\lambda^*>0$, and buyers pay 50 when seeing a level $\lambda=\lambda^*$ but only 20 when seeing a car with $\lambda=0$.

(b) Determine for which values of λ^* neither the owner of low-quality car nor the owner off a high-quality car have an incentive to deviate. (5 points)

High-quality:
$50-2\lambda^* > 20$, which is equivalent to $\lambda^* < 15$ (2 points)

Low-quality:
$20 > 50-5\lambda^*$, which is equivalent to $\lambda^* > 6$ (2 points)

No deviation incentive if $6 < \lambda^* < 15$. (1 point)

(c) Out of these levels, which level of λ^* is the most-efficient one for social surplus? Give a short explanation. (3 points)

$\lambda^* = 6$ is most efficient (1 point)
Higher levels of λ^* do not improve quality (1 point)
But higher levels are costly (1 point)

(d) Explain verbally why signaling allows a buyer to disentangle the quality types although cosmetic changes do not improve the quality of a car. (4 points)

High-quality owners choose a level so that low-quality owners do not want to mimic (2 points)
This works because quality is positive correlated with low cost of changes (2 points)