Digital Exam in Operations Management
Bachelor of Science

Winter term 2021
Lecturer: Brauch/Schäfer/Strohhecker/Thun
Date: 18.12.2021, 10:00 to 12:10

This exam consists of 6 question sections. You have to answer all questions included in these 6 sections. You have 130 minutes (120 + 10 minutes) to complete the examination. The maximum of points that can be reached is 120.

You are allowed to use the collection of formulae and statistical tables, and a non-programmable pocket calculator. Regardless of whether the exam is taken on campus or online, all participants must print out the formula and table sheets themselves and have it released by the proctor before the exam. Communication with fellow students or other persons is not allowed. Specifically exchanging the way of solving the exam problems with others is seen as a violation of the examination guidelines as well as the Honour Code and will result in evaluating the exam as failed.

Please note that trailing zeros (that is, any zero that appears to the right of both the decimal point and every digit other than zero) are automatically removed in Canvas. For example, if you are requested to enter the number 111.80 with two decimals, Canvas will show 111.8. When grading your solution, we will automatically add trailing zeros as requested. Please note also that the way how decimal and thousand separators are shown in edit fields and in some of the question texts depends on your Canvas language setting (to be found in >account >settings). We recommend to change this setting to "English" to reduce potential confusion.

It is recommended to include units in the descriptions of your solution path, as it is easier to self-detect errors. However, if the question does not explicitly ask for including units, no points are deducted in case units are missing.

To start, click the "Take the Quiz" button. When finished, click the "Submit Quiz" button. The exam will save and submit automatically when the time expires.

We wish you success in the exam.

Section I

Question 1 (1 P)
The company "Happy Birds" produces birdhouses each of which consists of one roof, one base plate, and four walls. The different parts are produced on one single CNC milling machine so that setups are necessary when changing from one part to another. The setups of the CNC-milling machine are not dependent on the part that will be produced next and require always half an hour. Milling a wall takes 30 seconds, milling a roof 60 seconds, and milling the base plate 120 seconds. The milled parts are stocked in a buffer before assembly of the birdhouses.

Assembly is conducted by employees in three steps. It takes 180 seconds to mount a single wall. The assembly of the base plate takes 120 seconds. The completion of a roof takes 300 seconds. Each of the three production steps in assembly is carried out by one employee. The five-day working week at "Happy Birds" includes 35 hours per employee with one shift each day. The hourly wage per employee amounts to 20 €.

The birdhouses are sold via their own online shop. The demand per week is on average 210 units.

Determine the implied utilization of the bottleneck process step in assembly. Enter your solution as percentage with two decimals.

Solution:
120,00

Question 2 (6 P)
Explain here in sufficient detail how you determined the solution to the question above.

As a flow unit, we consider a birdhouse or the components representing it.
Wall mounting: Processing time 3 minutes => 20 walls per hour (4 walls are needed per birdhouse) => 20/5 = 5 birdhouses per hour. *(2 P)*
Base plate: Processing time 2 minutes => 30 birdhouses per hour *(1 P)*
Roof mounting: Processing time 6 minutes => 10 birdhouses per hour *(1 P)*
The bottleneck is the wall mounting, as only 5 birdhouses per hour can be produced here. *(1 P)*
The implicit utilization at the bottleneck point in assembly thus amounts to
Demand rate = 210 birdhouses/week / 35 h/week = 6 birdhouses/h
Demand rate/flow rate = 6/5 = 1.2 *(1 P)*
Formula: 3600s/h / 180 s/wall

Question 3 (6 P)
Calculate the total idle time of all three operators in the assembly in minutes for one working day! Explain your solution in sufficient detail.

Solution:
The demand is greater than the capacity, so the idle time for wall mounting is 0, as this is where the bottleneck exists. *(1 P)*
The employee of the roof assembly could produce 10 roofs, but due to the bottleneck only has to produce 5 roofs. For a roof it needs 6 minutes = > 30 minutes of working time per hour = > 60 - 30 minutes = 30 minutes of idle time. Per 35/5 = 7 h working day, this results in 210 min. *(2 P)*
*The employee who installs the base plate needs 2 minutes per base plate. So he could produce 30 floor plates per hour, but only has to make 5. It takes 2 min./base plate * 5 base plates = 10 minutes. 60 minutes - 10 minutes = 50 minutes idle. Per working day, therefore, there are 7 · 50 = 350 min idle* *(2 P)*
Overall, this results in an idle time of 210 + 350 = 560 min over all three stages. *(1 P)*

Question 4 (7 P)
Determine the batch size (in component sets) for the CNC milling machine so that the milling process will not become the bottleneck! Explain your solution in sufficient detail.

Solution:
The three milled parts – walls, floor, roof – are considered as one set of components in the ratio required for a birdhouse – 4:1:1. *(1 P)*
The batch size L is to be selected so that the capacity of the CNC milling machine corresponds to the capacity of the assembly, i.e. target flow rate of the milling machine = capacity of the assembly. *(1 P)*
The capacity of the installation at the bottleneck is 5 birdhouses per hour = > 0.08333 birdhouse per min. Alternatively, of course, birdhouses per second or birdhouses per hour can also be calculated. The time unit must match the processing and set-up times used below. *(0.5 P)*
The production cycle consists of setting up for the production of the roof (30 minutes), the production of roofs (L · 1 minute per roof), the setting up for the production of walls (30 minutes) and the production of walls (L · 4 · 0.5 minutes = >L · 2 minutes for four walls), the setup for the production of floor slabs (30 minutes) and the production of floor slabs (L · 2 minutes per floor plate).
The total setup time in this production cycle is thus 30 + 30 + 30 = 90 minutes. *(1 P)*

The processing time for a set of components (roof, 4 walls, floor plate) is 5 minutes. *(1 P)*

The following formula is required:
Recommended batch size = (target flow rate · set-up time)/(1- target flow rate · processing time) *(1 P)*
= (0.08333 · 90) / (1 - 0.08333 · 5)
= 7.5 / 0.5833 = 12.85 => lot size 13 *(1 P)*
=> lot size = 13 subsets *(2 P)*
Specification of the solution in batch sizes for the individual milled parts
13 floor slabs, 13 roofs, 52 walls.

Section II

Question 5 (3 P)

The northern German garden center "Wiehnachtsboom" plans the selling of Christmas trees (Nordmann firs). As a sustainable operating company, the Christmas trees to be sold during the Christmas season are not felled. Instead, the Nordmann firs are purchased from a wholesaler who sells these firs in a pot. As a consequence, the firs have to be dugged out and potted ahead of the selling period (1st of December to 20st of December). Therefore, ad-hock replenishment of firs during the Christmas period is not possible.

The potted Christmas trees can be purchased from the wholesaler before the sales period for 50 Euro per piece. During the sales period, Nordmann firs are sold to customers for 100 Euro per piece. After Christmas, the sale begins. "Wiehnachtsboom" reduces the price to 40 euros per piece and offers free Germany-wide delivery for customers, which costs the center an average of 10 euros per fir. As a result, all unsold firs can be salvaged. The center forecast an average demand of 350 firs with a standard deviation of 50 firs.

What is the probability that an order of 300 fir trees is not sufficient to meet customer demand for Christmas trees? Enter your solution as a percentage with two decimal places.

Solution: 84.13%
z = (300 firs – 350 firs)/50 firs = - 1 *(1 P)*
The table of the distribution function of the standard normal distribution results in F(- 1) = 0.1587 = 15.87 %. The probability that the demand is less than or equal to 300 fir trees is thus 15.87 %. *(1 P)*
In all other cases, the purchased number of Nordmann firs is not sufficient. The probability of this is 100% - 15.87 % = 84.13%. *(1 P)*

Question 6 (8 P)

How many firs should "Wiehnachtsboom" order at the wholesaler to maximize the expected profit from selling Christmas trees. Explain your solution in sufficient detail.

Solution:
Given are EK = 50 €, VK = 100 €, RW = 30€, μ = 350, Ϭ = 50.
It is a newspaper seller problem with normally distributed demand, so the CR must be determined. *(1 P)*
For this purpose, the understock costs and the overstock costs must be determined.
The understock costs are 100 €/fir - 50 €/fir = 50 €/fir, as each fir could be sold for 100 € and costs only 50 €/fir in purchase. *(1 P)*
The overstock costs are 50 €/ fir - 40 €/ fir – 10 €/fir= 20 €/ fir *(2 P)*
The critical ratio is calculated using the following formula *(0.5P)*

$$CR = \frac{C_u}{C_o + C_u}$$

CR = $_{CU}$ / CU + CO = 50 €/fir / 50 €/fir + 20 €/fir = 50/70 = 0,7142 *.(1 P)*
In the table of the distribution function of the standard normal distribution, we look for 0.7142 and find:
z(0.7123) = 0.56
z(0.7157) = 0.57
The application of the rounding rule leads to the choice of z = 0.57. *(2 P)*
This results in Q = μ + z · Ϭ (0.5 P)*
Q= 350 firs + 0.57 · 50 firs = 378.5 firs = > 379* *(1 P)*

Question 7 (9 P)

What are the expected profit and the mismatch costs if "Wiehnachtsboom" orders the optimal quantity of firs from the wholesaler? (If you have not solved the previous question, please assume that "Wiehnachtsboom" orders 374 fir trees. Attention. This is not necessarily the correct solution to the previous task.) Explain your solution in sufficient detail.

Solution:
Given are EK = 50 €, VK = 100 €, RW = 30€, μ = 350, σ = 50.
Step 1: We need the following formula for the mismatch costs
Mismatch cost = Maximum profit - Expected profit (0.5 P)
Alternative formula

$$Mismatch\ cost\ =\ (Co\ \cdot\ Expected\ inventory)\ +\ (Cu\ \cdot\ Expected\ lost\ sales)$$

Step 2 (omitted for alternative formula):

$$Maximaler\ Gewinn = (Verkaufspreis - Einkaufspreis) \cdot \mu \qquad (0.5\ P)$$

Maximum profit = (€100 – €50/fir) · 350 firs = 17.500 € (1 P)
Step 3:
The expected profit is calculated as follows: (0.5 P)

$$\begin{aligned} Expected\ profit \\ &= (Selling\ price\ \cdot\ Expected\ sales) + (Discount\ price \cdot\ Expected\ inventory) \\ &- (Purchase\ price\ \cdot\ Q) \end{aligned}$$

Step 4
Q= 379 from previous Question*
Step 5: Determination of the expected inventory = σ · I(z) (0.5 P)
I(z), for the standard normal distribution from the corresponding table: I(0.57) = 0.7471 (1 P)
Expected inventory: σ · I(z)= 50 · 0.7471 = 37.355 firs (1 P)
Step 6
Calculate the expected sales quantity
Expected sales volume = Q − Expected stock = 379 – 37.355 = 341.645 firs* (1.5 P)
Step 7: Thus, all the required values for calculating the expected profit are available:

$$\begin{aligned} Expected\ profit \\ &= (Selling\ price\ \cdot\ Expected\ sales) + (Discount\ price \cdot\ Expected\ inventory) \\ &- (Purchase\ price\ \cdot\ Q) \end{aligned}$$

$$\begin{aligned} Expected\ profit\ &= \left(100\frac{€}{fir}\ \cdot\ 341{,}645\frac{firs}{season}\right) + \left(30\frac{€}{fir} \cdot 37{,}355\frac{fir}{season}\right) - \left(50\frac{€}{fir}\ \cdot\ 379\right) \\ &= 34.164{,}50\frac{€}{season} + 1.120{,}65\frac{€}{season} - 18.950\frac{€}{season} = 16.335{,}15\frac{€}{season} \end{aligned}$$

(2 P)

Step 8:
This results in the mismatch costs (mismatch costs = maximum profit - expected profit)
= 17.500 €/season – 16.335,15 €/season = 1.164,85 €/season (0.5 P)
Alternative, faster solution for step 8:

$$Mismatch\ cost\ =\ (Co\ \cdot\ Expected\ inventory)\ +\ (Cu\ \cdot\ Expected\ lost\ sales)$$

Mismatch cost = 20€/fir · 37,355 firs/season + 50€/fir · 350 – 341,645 firs/season = 1164,85€/season
{Grading rule: Rounding of expected values, specifically rounding of expected sales quantity and expected inventory, subtraction of 0.5 P each)

Section III

Question 8 (7 P)
In the small ski resort in the Berkshire Mountains in the east of Massachusetts the ski rental "Butternut" offers the newest model of the Völkl "RACETIGER SL" for rental. "Butternut" has 8 pairs of ski of this particular model for rental. Generally, skiers are interested in renting this model, but would be willing to rent an alternative model if the "Racetiger SL" is not available. On average skiers rent the model for six hours. On a daily basis, 12 skiers ask for the "Racetiger SL" whereby a normal skiing day has a length of 8 hours.

Determine the probability that the "RACETIGER SL" is available for a customer entering the rental shop. Explain your solution in sufficient detail.

Solution:
Since the skiers would switch to an alternative model, these are impatient customers, so that the Erlang loss model is used here.
m = 8 pairs
12 skiers in eight hours => 1.5 skiers / hour => 40 min. / skier => a = 40 min; *(2 P)*
Rental period 6 hours => p = 6 hours = 360 min. *(1 P)*
Offered Load r = p / a = 360 / 40 = 9 *(1 P)*
p(m=8; r=9) = 0.2852 (no pair available) *(2 P)*
We are looking for the counter-probability (ski is available): 1 - p = 1 – 0.2852 = 0.7108 => 71.08 % *(1 P)*

Question 9 (3 P)
Butternut's price for renting the "RACETIGER SL" is 30 $ per skiing day and pair of skis. How much revenue can "Butternut" expect from renting the "RACETIGER SL"? Enter your solution with two decimals.

*Flow rate = 12 customers/day * 0.7108 = 8.5296 customers/day* *(2 P)*
*Revenue = 8.5296 customers/day * $30/customer = 255.89/day* *(1P)*

Question 10 (3 P)
"Butternut" would like to achieve that more skiers can make the wonderful experience of testing the "RACETIGER SL". Hence, the manager of "Butternut" considers limiting the lending period to 4 hours. Determine for this scenario the probability that a pair of "RACETIGER SL" is not available for rent when a customer enters the shop. Provide your solution as percentage with two decimals.

From above, the following can be adopted:
m = 8 pairs;
12 skiers in eight hours => 1.5 skiers / hour => 40 min. / skier => a = 40 min;
New rental period 4 hours => p = 4 hours = 240 min.
Offered Load r = p / a = 240 / 40 = 6
p(m=8; r=6) = 0.1219 (no pair available)

Question 11 (7 P)
Despite the limitation of the lending period, "Butternut" is not happy with the service level achieved. "Butternut" has the objective that 95 out of 100 interested skiers can rent a pair. Hence, "Butternut" considers buying additional pairs of the "RACETIGER SL" for 500 $. After how many days would this expense pay off? Explain your solution in sufficient detail.

Solution:
The targeted probability that skis are available is 0.95, so the probability that all skis are rented must be less than 0.05. (1 P)
Offered load r = p / a = 240 / 40 = 6 (from previous task) *(1 P)*

The table of loss probabilities of the Erlang loss system gives the probability of 0.0431 for r = 6 for m = 10:

Erlang Loss Table

	m									
r = p/a	1	2	3	4	5	6	7	8	9	10
1.00	0.5000	0.2000	0.0625	0.0154	0.0031	0.0005	0.0001	0.0000	0.0000	0.0000
1.50	0.6000	0.3103	0.1343	0.0480	0.0142	0.0035	0.0008	0.0001	0.0000	0.0000
2.00	0.6667	0.4000	0.2105	0.0952	0.0367	0.0121	0.0034	0.0009	0.0002	0.0000
2.50	0.7143	0.4717	0.2822	0.1499	0.0697	0.0282	0.0100	0.0031	0.0009	0.0002
3.00	0.7500	0.5294	0.3462	0.2061	0.1101	0.0522	0.0219	0.0081	0.0027	0.0008
3.50	0.7778	0.5765	0.4021	0.2603	0.1541	0.0825	0.0396	0.0170	0.0066	0.0023
4.00	0.8000	0.6154	0.4507	0.3107	0.1991	0.1172	0.0627	0.0304	0.0133	0.0053
4.50	0.8182	0.6480	0.4929	0.3567	0.2430	0.1542	0.0902	0.0483	0.0236	0.0105
5.00	0.8333	0.6757	0.5297	0.3983	0.2849	0.1918	0.1205	0.0700	0.0375	0.0184
5.50	0.8462	0.6994	0.5618	0.4358	0.3241	0.2290	0.1525	0.0949	0.0548	0.0293
6.00	0.8571	0.7200	0.5902	0.4696	0.3604	0.2649	0.1851	0.1219	0.0751	0.0431
6.50	0.8667	0.7380	0.6152	0.4999	0.3939	0.2991	0.2174	0.1501	0.0978	0.0598
7.00	0.8750	0.7538	0.6375	0.5273	0.4247	0.3313	0.2489	0.1788	0.1221	0.0787

(2 P)

*Expenses of 2 * $500/pair = $1000 are required.*
From Question before: p(m=8; r=6) = 0.1219 (no pair available), i.e. pair available 1 – 0.1219 = 0.8781. *(1 P)*
Turnover
12 customers/day (0.9569 - 0.8781) · $30/customer = $28.36/day *(1 P)*
Payback period = $1000/ $28.36/day = 35.26 days *(1 P)*
Or more detailed with more intermediate steps:
Flow rate = 12 customers/day · 0.8781 = 10.5372 customers/day
Revenue = 10.5372 customers/day · $30/customer = 316.12 /day
New flow rate
Flow rate = 12 customers/day S 0.9569 = 11.4828 customers/day
Revenue = 11.4828 customers/day · $30/customer = $344.48/day
Revenue growth = $344.48/day - $316.12/day = $28.36/day

Section IV

Question 12 (2 P)
A producer of tomato juice orders tomatoes from a wholesaler. Per Year, 100 tons of tomatoes are required. For each delivery the wholesaler charges 50 € independent of the order quantity. Keeping one ton of tomatoes for one year in inventory costs the producer 100 € (whereby one year equals 52 weeks or 365 days). The purchasing cost of one ton of tomatoes is € 120 €.

What are the costs for the juice producer to store 10 tons of tomatoes for one week? Please enter your solution with four decimals.

Solution: 100 €/(t · year) / 52 weeks/year · 10 t =19,2308 € *(2 P)*

Question 13 (4 P)
Determine the order quantity that minimizes the total cost for delivery and inventory holding (if any order quantity is possible).

Solution:
R = 100 tons / year, K = 50 €, h = 100 € per ton(3 P)
*$_{Qopt}$ = √(2 * 100 * 50 / 100) = 10 tons*

$$Q^* = \sqrt{\frac{(2 \cdot 100 \cdot 50)}{100}} = 10\ t \text{(3 P)}$$
Q = 10 tons*

Please use your solution for the question above to answer the following two questions. If you were unable to determine this solution, please make a plausible assumption.

Question 14 (2 P)
How often in a year does the juice producer order from the wholesaler?

n = R / $_{Qopt}$ = 100 tons / year / 10 tons = 10 times / year

Question 15 (6 P)
What is the sum out of the weekly delivery cost, inventory holding cost, and the purchasing cost?

Solution:
*K = $_{KB}$ + KL + KE = 120 € / t * 100 t + 500 €/year + 500 € / year = 12,000 € + 1,000 € = 13,000 €*
Division by 52 weeks = 250 €/week
Alternative solution with formula from the formula collection:
$$C(Q) = \frac{1}{2} \cdot h \cdot Q + K \cdot \frac{R}{Q}$$
= 0.5 · 100 €/ton · 10 t + 50 € · (100 tons/year / 10 t) = 1000 €(4 P)
Purchase costs = 120 € · 100 t = 12000 €(1 P)
Sum of all costs per week = 13,000 € / 52 weeks = 250 €/week (1 P)

Question 16 (3 P)
The customers of the juice producer cancel their purchasing contracts that were granting constant order quantities for tomato juice. Hence, the juice producer is now confronted with fluctuating demand. Since the wholesaler announced a delivery time of two months in the future at the same time, the juice producer decides to place his orders for tomatoes on a monthly basis using the order-up-to approach (OUT model) with a base stock level S of 15 tons.

Beginning of next month, the manager identifies an on-hand inventory of 3 tons of tomatoes. 7 tons of placed orders are still open. Two of these 7 tons are already waiting in front of the factory gates. How many tons of tomatoes does the juice producer order?

Solution:
Order quantity = target stock – physical stock – open order quantity + backlog
The 2t tomatoes waiting for admission are still part of the open order.
There is no backlog.
Order quantity = 15 – 3 – 7 = 5 t

Question 17 (3 P)
An unexpected high demand for juice in this current month results in an inventory level of -4 tons of tomatoes at the beginning of the next month. What quantity will then be ordered if the quantities ordered in preceding months arrive as expected? (See preceding question.)

Solution:
Order quantity = target stock – physical stock – open order quantity + backlog
Net Inventory = Physical Inventory – Backlog
Order Quantity = Target Stock – Net Inventory – Open Order Quantity
Open order quantity from previous month = 7 t – 2 t + 5 t = 10 t
Order quantity = 15 – -4 - 10 = 9 t

Section V

Question 18 (3 P)

The barber shop "The great quiff" is located in Seckenheim (a suburb of Mannheim) and is heavily frequented by potential customers. On average each hour five customers enter the barber shop. The standard deviation of the interarrival time is 5 minutes. Due to a lack of alternatives customers are willing to wait to be served. The average processing time for serving one customer equals 55 minutes with a standard deviation of 15 minutes. The barber shop employs five barbers who can fulfil a broad range of customer wishes. Chairs exist in a sufficient number so that these will not become a bottleneck. The shop includes a waiting area where the customers can wait until they will be served.

What is the average utilization of a barber? Enter your solution as percentage with two decimals.

Solution:
The following applies to the average utilization:

$$Auslastung\ u = \frac{p}{a \cdot m}$$

The processing time p, the number of servers m and the intermediate arrival time a are required.
m = 5 (hairdressers)
p (processing time) = 55 min.
a = 60 min/hour / 5 customers/hour = 12 min
This results in the average capacity utilization of a hairdresser:
*u = 55 / (5 * 12) = 0.9166 => 91.66%*

Question 19 (10 P)

How many customers wait on average in the waiting area of the barber shop? Explain your solution in sufficient detail.

Solution:
The waiting time formula is required here: (0.5 P)

$$T_{q=} \frac{p}{m} \cdot \left(\frac{Auslastung^{\sqrt{2(m+1)}-1}}{1 - Auslastung}\right) \cdot \left(\frac{CV_a^2 + CV_p^2}{2}\right)$$

In addition, the coefficients of variation CVa and CVp must be determined. The following applies to the coefficients of variation:
CV = Standard Deviation / Mean
CVa = 5 / (60 / 5) = 0.4167 (2 P)
CVp = 15 / 55 = 0.2727 (1P)
Furthermore, m = 5 (0.5 P)
Tp = 55 / 5 · (0.9166 $^{\surd(2(5+1))\ -\ 1}$ */ 1 - 0.9166) * (0.41672 + 0.27272) / 2*
= 11 · ($^{0.91662.464}$ */ 0.0834) * (0.1736 + 0.0743)/2 = 106.42 * 0.1239 = 13.18 min.* (3 P)
Use of Littles Set I = R · T (0.5 P)
We have just calculated T.
R is 5 customers per hour = 5/60 = 1/12 customer per minute (1 P)
I = 13.8 min · 1/12 customers/min = 1.0988 customers (1 P)

Question 20 (4 P)

How many full-time barbers does Managing Director Herta Holle have to hire in order to undercut a utilization of 75%, but to get as close as possible to it? Explain your solution in sufficient detail.

Solution:

$$Auslastung\ u = \frac{p}{a \cdot m}$$

Dissolve the formula according to m (2 P)

$m = \frac{p}{a \cdot u}$

The processing time p and a from question 17 are to be used.

p (processing time) = 55 min., a = 12 min

$m = \frac{55}{12 \cdot 0{,}75}$ *=6.1*

From this we conclude on an m of 7, so that two more hairdressers have to be set. *(2 P)*

Alternative solution:

According to the formula for the utilization u, this decreases linearly with the increase of m, i.e. by a gradual increase of m by integers, since only full-time employees are to be hired, the utilization decreases (which is desired). In this respect, m is to be increased from 5 to 6, i.e. calculation of the load with m = 6

*u = 55 / (6 * 12) = 0.9166 => 76.39%*

Capacity utilization has fallen but is still above the required maximum of 75%. Therefore, m should be further increased:

*u = 55 / (7 * 12) = 0.9166 => 65.48%*

This falls below the target value, so that m = 7 must be set.

Question 21 (3 P)

Frankfurt Student Consulting analyzes the service design of the barber shop. The consultants praise that pooling is implemented. What were the negative consequence of abandoning the pooling concept?

Solution:

Pooling means that there is a common queue for multiple resources and one flow unit always goes to the next free server. The renunciation of pooling would mean allowing a separate queue for each hairdresser. This would increase the average waiting time, because blockage and starvation can occur at the same time. It may be that a customer is waiting for "his" hairdresser to become free, while the neighboring hairdresser has nothing to do at the moment.

Section VI

Question 22 (4 P)
In the manufacturing area of an industrial company, a CNC-milling-machine is available for one 8 hour shift per day. An analysis has revealed that the CNC-milling-machine is the bottleneck of the capacity-constrained production system. The machine can produce 20 parts per hour (no matter whether a part has to be reworked or not). 10 % of the milled parts are defect and have to be re-processed on the CNC-milling-machine (re-processing takes the same time and is 100% free of errors). Calculate the flow rate of the production process in good parts per shift. Enter your answer with two decimals.

Solution:
Cap = 20 parts / hour
This results in a processing time of 60 min/hour / 20 parts/hour = 3 min/part *(1 P)*
Error rate = 10 %
*The average processing time is therefore 0.9 * 3 + 0.1 * (3 +3) = 3.3 min* *(2 P)*
The conversion into a flow rate/capacity results in
= 1 / 3.3 min = 10/33 parts/min
*Conversion to parts per hour: 60 min/hour * 10/33 = 200/11 = 18.18 parts/h*
*Conversion to parts per shift = 18.18 * 8 = 145.45 parts/shift* *(1 P)*

Question 23 (4 P)
By implementing the concept of Total Quality Managements (TQM) the percentage of defective parts for which rework is required can be reduced by 50 %. By how many percent can the capacity of the milling process (measured in good parts per shift) be increased? Explain your solution in sufficient detail.

Solution:
What is needed is the solution from the previous question:
Capacity = flow rate from preliminary question = 200/11 parts/hour = 18.18 parts/h
*The average processing time is now 0.95 * 3 + 0.05 * (3 +3) = 3.15 min* *(2 P)*
*This results in the capacity: 1/3.15 *60 = 400/21 = 19.05 parts/h* *(1 P)*
And an increase in capacity by:
(400/21 – 200/11)/ 200/11 = 200/231 / 200/11 = 1/21 = 4.76% *(1 P)*

Question 24 (3 P)
The CNC-milling-machine produces parts with a mean length of 46 mm and a standard deviation of 0.3 mm. The lower specification limit is 45.5 mm and the upper specification limit equals 46.3 mm. Calculate the process capability index of the CNC-milling-machine. Explain your solution in sufficient detail.

Solution
μ = 46; σ = 0.3; USL = 46.4; LSL = 45.6
Cp = (USL - LSL) / 6 0.3 = 0.8 / 1.8 = 0.444

Question 25 (5 p)
What is the probability that the length of a part is outside the specification limits? Explain your solution in sufficient detail.

Solution
p (too short): z = (45.5 – 46) / 0.3 = - 0.5/0.3 =-5/3 = -1.6667 *(1P)*
p (too short) = p(-1.67) = 0.0475 = 4.75% *(1 P)*
p (too long) = 1 - p(shorter than 46.3)
z = (46.3 – 46) / 0.3 = 0.3/0.3 = 1 *(1 P)*
p (1) = 0.8413 *(0.5 P)*
p (too long) = 1 - 0.8413 = 0.1587 = 15.87 % *(1 P*

p (outside) = 4.75% + 15.87% = 20.62% *(0.5 P)*
The mean is not exactly in the middle between USL and LSL, so the initially determined probability cannot simply be doubled.

Question 26 (4 P)
The production manager is extremely dissatisfied with the high number of errors occurring during milling. He tries to find reasons for the low process capability index. List and explain two potential reasons for the CNC machine's low process capability.

Solution:
The process capability is low because the standard deviation of the length of the machined parts is high, i.e. the length varies greatly. This can have the following causes:

- *The machine is inadequately or not maintained at all, which means that, for example, the positioning of the tool is inaccurate*
- *The machine is insufficiently calibrated (adjusted) so that there are deviations between the target and the actual position of the workpiece.*
- *The programming of the CNC machine has errors, so that, for example, the specified length of the workpiece varies.*
- *The workpiece has a poor material quality and expands more than planned during milling, for example.*
- *The temperature of the environment fluctuated more than the specification of the machine prescribes.*

Answers other than those listed above may also be correct. It is important that an answer refers to specific problems in the milling process and does not name general, non-specific quality management problems.