Frankfurt School of Finance & Management
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**Examination in the Bachelor of Science**
**Course title: Markets, Incentives and Ethical Management**
**Part: Markets and Incentives**
**Semester: 2**
**Lecturers: Dr. Chiara Nardi, Dr. Ahmed Rashad, Prof. Dr. Markus Reisinger**
**Group: 162 cohort**
**Examination date: 22nd April 2017**

**Aids: pocket calculator Casio FX-82 solar**

Please enter your student ID (matriculation number) and your group!

| Student ID | Group |
| --- | --- |
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Please note:

The exam consists of 4 questions of which you will have to answer **all** questions. You have **90** minutes to complete the examination. The maximum of points to be reached is **90**. Please use the enclosed answer sheet to answer your questions and add your student ID on its cover.

We wish you all the best for your examination!

Internal use only!

| Question | 1 | 2 | 3 | 4 | **Total** |
| --- | --- | --- | --- | --- | --- |
| Possible points: | 34 | 14 | 30 | 12 | **90** |
| Points achieved: | | | | | |

Signature of corrector

## Question 1 – General Equilibrium (34 points)

Suppose that there are two consumers A and B and two products *x* and *y*. The initial endowment $\omega$ is such that consumer A is endowed with $(\omega_{X_A}, \omega_{Y_A}) = (18, 32)$ and consumer B is endowed with $(\omega_{X_B}, \omega_{Y_B}) = (2, 8)$. Both consumers have standard preferences and their utility functions are

$$U_A(x_A, y_A) = x_A^2 y_A^2 \quad \text{and} \quad U_B(x_B, y_B) = x_B^2 y_B^2.$$

**(a)** In the Edgeworth box, draw the consumers' indifference curves passing through the initial endowment and indicate the set of allocations that are Pareto improvements compared to the initial endowment. (6 points)

**(b)** Determine the Pareto-efficiency condition. (3 points)

**(c)** Determine the equation of the contract curve and draw it in the Edgeworth box. (4 points)

**(d)** Calculate the demand functions of both consumers, assuming that the price of product y is the numeraire (i.e., $p_y = 1$). (8 points)

**(e)** State the definition of competitive equilibrium (3 points).

**(f)** Calculate the competitive equilibrium set of prices if $p_y = 1$. (3 points)

**(g)** Explain why product *x* is more expensive than product *y*. (2 points)

**(h)** Calculate the competitive equilibrium quantities. (2 points)

**(i)** List three key assumptions that are needed for the First Welfare Theorem to hold. (3 points)

## Question 2 – Game Theory (14 points)

Consider a second-price auction with two bidders. The valuations of the two bidders are $v_1$ and $v_2$, and are unknown to each other. Each bidder simultaneously makes a bid for the object, where the bids are denoted by $b_1$ and $b_2$, respectively. The auctioneer will give the object to the bidder with the highest bid. The winning bidder has to pay the bid of the losing bidder, the losing bidder gets nothing and pays nothing.

**(a)** Demonstrate why it cannot be optimal for bidder i=1,2, to submit a bid $b_i$, which is below the valuation $v_i$. (6 points)

**(b)** Provide a brief intuition why no bidder has the incentive to lower her bid below the true valuation. (2 points)

**(c)** Suppose the auction would not be played simultaneously but sequentially, that is, bidder 1 first submits her bid $b_1$ and afterwards bidder 2 submits her bid $b_2$. (Bidder 2 can observe the bid of bidder 1.) Would the outcome of the auction change compared to the simultaneous case? Explain your answer. (3 points)

**(d)** In a first-price auction, the winning bidder needs to pay the bid she submitted, whereas all losing bidders still get nothing and pay nothing. Can it still be optimal in this case for a bidder to bid her true valuation? Explain your answer. (3 points)

## Question 3 – Oligopoly and Tacit Collusion (30 points)

Consider a market with two firms, Firm 1 and Firm 2, which produce a homogeneous product. Suppose that the firms interact only once by setting their quantities simultaneously (**Cournot competition**). The inverse demand function is $p(Q) = 150 - Q$, where $Q = q_1+q_2$.
Each firm's marginal cost is 90.

**(a)** Determine the reactions functions of both firms. (4 points)

**(b)** Determine the Nash equilibrium quantities of both firms and the equilibrium profits. (4 points)

Now, suppose that firms repeatedly interact for 4 periods and they simultaneously set their quantities in each period.

**(c)** Assuming that firms know that they will interact for exactly 4 periods, explain how to solve this finitely repeated game and why collusion cannot be sustained as a subgame perfect equilibrium. (5 points)

Now, suppose that firms collude and jointly behave as a monopolist.

**(d)** Determine the monopoly quantity and the monopoly profit. Moreover, assuming that firms equally split the market, find the quantity produced by each firm and the corresponding profit. (3 points)

Finally, assume that firms interact repeatedly for an infinite number of periods.

**(e)** Write down the grim trigger strategy that allows collusion to be sustained as a subgame perfect equilibrium by completing the following sentence. (2 points)

Produce ______________________ quantity in the first period. Keep producing ______________________ quantity as long as both firms have done so in all previous periods. If one firm has deviated from producing ______________________ quantity, produce the __________________ quantity forever.

**(f)** Determine the optimal deviation quantity and the corresponding profit. (4 points)

**(g)** Compute the discounted value of profits from collusion and the discounted value of profits from deviation. Determine the critical discount factor. (8 points)

## Question 4 – Asymmetric Information (12 points)

**(a)** Explain the difference between screening and moral hazard with respect to the point in time at which asymmetric information occurs. (3 points)

Consider the screening problem of a seller who faces a high-type buyer and a low-type buyer and can offer different contracts to them with respect to quality and price of the product. The high-type values quality more than the low type.

**(b)** Suppose the seller offers the first-best contracts (i.e., the contracts with symmetric information) to both buyer types. Explain why the high-type buyer has an incentive to accept the contract intended for the low type. (4 points)

**(c)** In a moral hazard problem, state the two constraints that an optimal contract needs to fulfill. Explain why each constraint must hold with equality at the optimal contract. (5 points)