

**Examination in the Bachelor of Science**  
**Course title: Operations Management**  
**Semester: Winter 2016/2017**  
**Lecturer: Ullrich/Müller**  
**Group: BWL-BIS/WP, BWL-AIS, WI-AIS, MPE, BIM1, BIM2-BBF**

**Aids: Casio fx 82 solar, Casio fx 85 MS, Casio fx 85 GT plus, Dictionary, collection of formulae and statistical tables**

Please enter your student ID (matriculation number) and your group!

Student ID	Group
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Please note:

The exam consists of 5 questions of which you will have to answer **4** questions. If you answer all **5** questions only the first **4** will be evaluated. You have **80** minutes to complete the examination. The maximum of points to be reached is **80**. Please use the enclosed answer sheet to answer your questions and add your student ID on its cover.

Please always explain your solution in adequate depth with comments for each important step!

We wish you all the best for your examination!

Internal use only!

Question	1	2	3	4	5	Total
Possible points:	20	20	20	20	20	80
Points achieved:						

### Question 1

(20 points)

Mr. K's is a very popular hair salon. It offers high-quality hair-styling and physical relaxation services at a reasonable price, so it always has unlimited demand. The service process includes five activities that are conducted in the sequence described below. (The time required for each activity is shown in parenthesis):

Activity 1: Welcome a guest and offer homemade herb tea. (10 minutes)

Activity 2: Wash and condition hair. (10 minutes)

Activity 3: Neck, shoulder, and back stress release massage. (10 minutes)

Activity 4: Design the hair style and do the hair. (25 minutes)

Activity 5: Check out the guest. (5 minutes)

Three servers (S1, S2, and S3) offer the services in a worker-paced line. The assignment of tasks to servers is the following:

S1 does Activity 1.

S2 does Activities 2 and Activity 3.

S3 does Activities 4 and Activity 5.

Part a)

(4 points)

Which server is the bottleneck of the process?

*Solution:*

*S1 can process 1/10 customers per minute.*

*S2 can process 1/20 customers per minute.*

*S3 can process 1/30 customers per minute.*

*S3 has the lowest capacity and is hence the bottleneck.*

Part b)

(2 points)

What is the utilization of server 2?

*Solution:*

*Since we assume that there is unlimited demand, the flow rate is equal to the capacity of the process, i.e., 2 customers per hour.*

*The capacity of S2 is 3 customers per hour.*

*The utilization at S2 is  $2/3 = 66.7\%$ .*

Part c) (2 points)

What is the average labor utilization of the servers? Assume the process operates at its capacity.

*Solution:*

*labor content = 10+20+30 = 60 min.*

*total idle time = 20+10 = 30 min.*

*Average labor utilization = 60/(60+30) = 2/3 = 66.7%*

Part d) (4 points)

Assume a wage rate of \$18 per hour. What are the direct labor costs for one guest?

*Solution:*

*Direct labor costs = (Total wages) / (flow rate)*

*There are three employees with a wage of \$18/hr implying that the total wages per hour are given by 18x3 = \$54/hr.*

*We deduce that*

*Direct labor costs = 54/2 = \$27*

Part e) (8 points)

To increase the service rate, Mr. K's is considering two alternatives:

Alternative I: To hire a new employee to help any one (and only one) of the servers without changing the tasks performed by each server.

Alternative II: To redesign the assignment of tasks to servers. For this, Mr. K's is evaluating to reassign Activity 5 from S3 to S1.

What would be the costs of direct labor of serving one guest under each of the two alternatives? Assume that the system operates at its capacity.

*Solution:*

*Under Alternative I, the additional worker would help S3 and under this case the bottleneck would become S2 with a capacity of 3 customers/hr.*

*Direct labor costs = (18\*4)/3 = \$24*

*Under Alternative II, S3 would still be the bottleneck but the new capacity of S3 will be of 60/25=2.4 customers/hr.*

*Direct labor costs = (18\*3)/2.4 = \$22.5*

## Question 2

(20 points)

Dr. Y, a former Operations Management Professor, decided to change his life significantly. Rather than dealing every day with complicated formulas, he started his own micro-brew production. The key ingredient that he needs to purchase from his supplier is malt. His supplier charges \$35 per delivery (no matter how much is delivered) and \$1.20 per gallon. Dr. Y's annual holding cost per unit is 35% of the dollar value of the unit. Dr. Y uses 5000 gallons of malt per week (a year has 52 weeks). Because Dr. Y forgot basically everything what he once taught, he is now seeking advice from his former students.

### Part a)

(3 points)

How many gallons should Dr. Y order from his supplier with each order? What are minimal annual inventory holding cost and minimal annual delivery cost?

*Solution:*

$$\text{Economic order quantity} = \sqrt{\frac{2 \times \text{Setup cost} \times \text{Flow rate}}{\text{Holding cost}}}$$

*We apply the EOQ formula to obtain Dr. Y's economic order quantity:*

$$\text{Economic order quantity} = \sqrt{\frac{2 \times 35 \times 5000 \times 52}{1.2 \times 0.35}} = \sqrt{\frac{18200000}{0.42}} = 6582.8 \quad (1P)$$

*Alternative way:*

*The fixed portion of the delivering cost is  $\$35 \times 5000 \times 52 / x$  per year =  $\$9,100,000 / x$ .*

*The average annual inventory holding cost is  $x/2 \times 1.2 \times 35\% = x/0.84$ .*

*Then the total weekly cost is  $\$35 \times 5000 \times 52 / x + x/2 \times 1.2 \times 35\%$ .*

*The cost is minimized at  $x = \sqrt{35 \times 5000 \times 52 / (0.6 \times 35\%)} = 6583$  gallons.*

*Minimal annual inventory holding cost:  $6583/2 \times 1.2 \times 35\% = 1382.43$  (1 P)*

*Minimal annual delivery cost =  $\$35 \times 5000 \times 52 / 6583 = 1382.35$  (1 P)*

*It is correct to also include purchasing cost in the annual delivery cost:  $1382.35 + 5000 \times 52 \times 1.2 = 313382.35$*

### Part b)

(2 points)

Suppose Dr. Y were to order 3800 gallons each time he orders. How many orders per year would he place on average? What time would lay in-between two orders?

*Number of orders per year =  $5000 \times 52 / 3800 = 68.42$  (1 P)*

*Time between two orders =  $3800/(5000*52) = 0.014615$  years = 0.76 weeks* (1P)

*Alternative solution =  $52 / 68.42 = 0.76$  weeks*

Part c) (7 points)

If Dr. Y places an order for 15000 gallons, then he will receive a 4% discount off the regular price of \$1.20. If Dr. Y were to do this with each order, what would be his average weekly total cost (in \$s)? When compared to your solution in a), what would you suggest? Order 15000 gallons and take the 4% discount or use the optimal order quantity calculated in a)?

*With a 4% discount, the malt is  $1.2*(1-0.04)=\$1.152$  per gallon.* (1P)

*Weekly purchase cost=  $\$1.152*5000=\$5760$ .* (1 P)

*Weekly delivery charges =  $5000/15000* \$35=\$11.67$ .* (1 P)

*Weekly inventory holding cost =  $15000/2 * \$1.152 * (35\%/52)=\$58.15$ .* (1 P)

*Total cost =  $\$5760 + \$11.67 + \$58.15=\$5830$*  (1 P)

*Total weekly cost based on the EOQ of 6583 is*

*Annual total cost from a)*

*Annual inventory holding cost:  $6583/2 * 1.2 * 35\% = 1382.43$*

*Annual delivery cost including purchasing cost =  $\$35 * 5000*52 / 6583 + 5000 * 52 * 1.2 = 313382.35$*

*Annual total cost =  $\$314764.78$  converted to weekly cost =  $\$6053.17$*  (1 P)

*Dr. Y is better off placing orders of 15000 gallons and taking the 4% discount than using EOQ as*

*$\$5830 < \$6053.17$*  (1 P)

Part d) (8 points)

*While driving home for Christmas, you cannot seem to get Little's Law out of your mind. You note that your average speed of travel is about 60 miles per hour. Moreover, the traffic report from radio FFM states that there is an average of 24 cars going in your direction on a one-quarter mile part of the highway. What is the flow rate of the highway (going in your direction) in cars per hour?*

*Solution:*

*We look at 1 mile of the highway as our process. Since the speed is 60 miles per hour, it takes a car 1 minute to travel through the process (flow time). Thus,  $T = 1\text{min}$ .* (2 P)

*There are 24 cars on  $\frac{1}{4}$  of a mile, i.e. there are 96 cars on the 1 mile stretch (inventory).*

*Hence,  $I = 96$  cars.*

*Inventory=Flow Rate \* Flow Time: 96 cars=Flow Rate \* 1 minute. Thus, the Flow Rate is 96 cars per minute. Hence  $R = 96$  cars/min.* (2 P)

*This corresponds to  $96*60=5760$  cars per hour.* (2 P)

### Question 3

(20 points)

Stiff, Inc., an electronics systems integrator, is planning to design a key component for their next-generation product with Buyit. Stiff will integrate the component with some software and then sell it to consumers. Given the short life cycles of such products and the long lead times quoted by Buyit, Stiff only has one opportunity to place an order with Buyit prior to the beginning of its selling season. Stiff's demand during the season is normally distributed with a mean of 2,000 and a standard deviation of 800.

Buyit plans to sell the component for \$72 per unit to Stiff. Stiff incurs essentially no cost associated with the software integration and handling of each unit. Stiff sells these units to consumers for \$126 each. Stiff can sell unsold inventory at the end of the season in a secondary electronics market for \$40 each. The existing contract specifies that once Stiff places the order, no changes are allowed to it. Also, Buyit does not accept any returns of unsold inventory, so Stiff must dispose of excess inventory in the secondary market.

#### Part a)

(4 points)

What is the probability that Stiff's demand will be within 20% of its forecast? Hint: Recall that the expected value of a normally distributed random variable is its mean.

*Solution:*

*It is within 20% of the forecast if it is greater than 1600 and less than 2400. (1 P)*

*The z-statistic for 1600 is  $z=(1600-2000)/800=-0.50$  and the z-statistic for 2400 is  $z=(2400-2000)/800=0.50$ . (1 P)*

*From the standard normal distribution function table, we see that  $\Phi(-0.50)=0.3085$  and  $\Phi(0.50)=0.6915$ . So there is a 30.85% chance demand is less than 1600 and a 69.15% chance it is less than 2400. (1 P)*

*The chance it is between 1600 and 2400 is the difference in those probabilities:  $0.6915-0.3085=0.3830$ , i.e. 38.30%. (1 P)*

Part b) (3 points)

What is the probability that Stiff's demand will be more than 50% greater than its forecast?

*Solution:*

*The forecast is 2000 units. Demand is greater than 50% of the forecast if demand exceeds 3000 units. (1 P)*

*Find the z-statistic that corresponds to 3000 units:*

$$z = (Q - \mu) / \sigma = (3000 - 2000) / 800 = 1.25$$

*From the standard normal distribution function table, we find that  $\Phi(1.25) = 0.8944$ . Therefore, there is almost a 89% probability that demand is less than 3000 units. (1 P)*

*The probability that demand is greater than 3000 units is  $1 - \Phi(1.25) = 0.1056$ , or about 11%. (1 P)*

Part c) (5 points)

Under this contract, how many units should Stiff order to maximize its expected profit?

*Solution:*

*To find the expected profit-maximizing order quantity, first identify the underage and overage costs.*

*The underage cost is  $C_u = 126 - 72 = 54$  because each lost sale costs Stiff its gross margin. (1 P)*

*The overage cost is  $C_o = 72 - 40 = 32$  because each unit of leftover inventory can be sold for \$40. (1 P)*

*Now evaluate the critical ratio:*

$$C_u / (C_o + C_u) = 54 / (32 + 54) = 0.6279 \quad (1 P)$$

*Look up the critical ratio in the standard normal distribution function table:  $\Phi(0.32) = 0.6255$  and  $\Phi(0.33) = 0.6293$ , so choose  $z = 0.33$ . (1 P)*

*Convert the z-statistic into an order quantity:  $Q = \mu + z * \sigma = 2000 + 0.33 * 800 = 2267$ . (1 P)*

**For part d and e, assume Stiff orders 2200 units.**

Part d) (5 points)

What are Stiff's expected sales?

*Solution:*



If  $Q=2200$ , then the corresponding z-statistic is  $z=(Q-\mu)/\sigma=((2200-2000))/800=0.25$ .

(2 P)

From the standard normal distribution loss table, we see that  $L(0.25)=0.2863$

(1 P)

Expected lost sales are then  $\sigma*L(z)=800*0.2863=229$ .

(1 P)

Finally, recall that expected sales equal expected demand minus expected lost sales:

Expected sales =  $2000 - 229=1771$ .

(1 P)

Part e)

(3 point)

What is the probability that Stiff has lost sales of 600 units or more?

*Solution:*

Stiff incurs 600 or more units of lost sale if demand exceeds the order quantity by 600 or more units; that is, if demand is 2800 units or greater.

(1 P)

The z-statistic that corresponds to 2800 is  $z=(Q-\mu)/\sigma = (2800-2000)/800=1$ . In the standard normal distribution function table,  $\Phi(1)=0.8413$ .

(1 P)

Demand exceeds 2800 with a probability of  $1 - \Phi(1)=15.9\%$ .

(1 P)

## Question 4

(20 points)

Part a)

(5 points)

Suppose a firm uses an A/F ratios approach to come up with a demand forecast. The average A/F ratio from historical data turns out to be 0.8. What is the best conclusion that can be drawn from this observation? Please justify your answer in 3-4 sentences.

- a) The optimal order quantity should be 0.8 times the forecast.
- b) The critical ratio of the product in the past was 0.8.
- c) The coefficient of variation of demand will be 0.8.
- d) There is a bias in the forecasting process: forecasts are on average higher than demand.
- e) There is a bias in the forecasting process: forecasts are on average lower than demand.
- f) None of the above

*Solution:*

*(d) is correct, since an A/F ratio lower than 1 means that the actual demand is lower than the forecasted demand. (a) is incorrect because the optimal order quantity depends also on the critical ratio. (b) and (c) are incorrect since the A/F ratio determines neither the critical ratio nor the coefficient of variation. (e) is incorrect, since the bias is in the opposite direction as described in (d).*

Part b)

(5 points)

Which of the following Normal demand distributions would result in the highest expected profit for the newsvendor model (keeping cost/revenue parameters the same)? Please justify your answer in 3-4 sentences.

- a) Mean 20, standard deviation 0.
- b) Mean 20, standard deviation 10.
- c) Mean 20, standard deviation 20.
- d) Mean 10, standard deviation 0.
- e) Mean 10, standard deviation 10.
- f) Mean 10, standard deviation 20.
- g) It is not possible to determine which of the above has the highest expected profit because it would depend on the particular cost and revenue parameters.

*Solution:*

*Higher average demand and lower standard deviation would lead to higher profit in the newsvendor model. Therefore a) yields to the highest profit.*

Part c)

(5 points)

Which kind of product is likely to have the highest mismatch cost as a % of its maximum profit? Please justify your answer in 3-4 sentences.

- a) High demand variability, large gross margin, high salvage value
- b) High demand variability, large gross margin, low salvage value
- c) High demand variability, low gross margin, high salvage value
- d) High demand variability, low gross margin, low salvage value
- e) Low demand variability, large gross margin, high salvage value
- f) Low demand variability, large gross margin, low salvage value
- g) Low demand variability, low gross margin, high salvage value
- h) Low demand variability, low gross margin, low salvage value

*Solution:*

*Products with low critical ratios and high coefficient of variation will have the highest mismatch cost as a % of their maximum profit. Recall that  $CR = (\text{cost of underage}) / (\text{cost of underage} + \text{cost of overage})$ . Products with low gross margin will have low costs of underage, while products with low salvage value will have high costs of overage, which together yield to low critical ratios. Answer (D) yields to low CR and high coefficient of variation due to the above mentioned factors and is thus the correct answer.*

Part d)

(5 points)

Suppose the newsvendor model describes a firm's operations decision. Is it possible to have positive expected lost sales and positive expected left over inventory? Choose the best answer and justify your answer in 3-4 sentences.

- a) No - if there is left over inventory then there should not be lost sales.
- b) No - if expected lost sales is positive, then expected left over inventory must be negative.
- c) No - actual demand can differ from sales.
- d) Yes - they are both expectations over numerous possible outcomes, among which there will be no outcome in which both are positive.
- e) Yes - as long as the underage cost is greater than the overage cost.

*Solution:*

*Answer d) is correct. Since expected lost sales and expected sales are averages over all possible outcomes, they are both positive. But in each outcome (realization), there will be no lost sales if items are left over, and a sale would never be lost if there are items left over. However, if averages are taken over all possible outcomes, they are both positive.*

### Question 5

(20 points)

Quick Print Inc. uses plain and three-hole-punched paper for copying needs. Demand for each paper type is highly variable. Weekly demand for the plain paper is estimated to be normally distributed with mean 100 and standard deviation 65 (measured in boxes). Each week, a replenishment order is placed to the paper factory and the order arrives five weeks later. All copying orders that cannot be satisfied immediately due to the lack of paper are back-ordered. The inventory holding cost is about \$1 per box per year.

Part a)

(4 points)

Suppose that Quick Print decides to establish an order-up-to level of 700 for plain paper. At the start of this week, there are 523 boxes in inventory and 180 boxes on order. How much will Quick Print order this week?

*Solution:*

*The inventory position of Quick Print is: Inventory position = On-order inventory + inventory level = 523 + 180 = 703. Because the inventory position exceeds the order-up-to level, 0 units should be ordered.*

Part b)

(4 points)

What is Quick Print's optimal order-up-to level for plain paper if Quick Print operates with a 99 percent in-stock probability?

*Solution:*

*From the Standard Normal Distribution Function Table, we find that  $\Phi(2.33)=0.9901$ .*

*Hence, we choose  $z = 2.33$ .* (2 P)

*The optimal order-up-to level for plain paper, given that Quick Print operates with a 99 percent in-stock probability, is:*

$$S = \mu + z * \sigma = 600 + 2.33 * 159.22 = 971. \quad (2 P)$$

Part c)

(4 points)

Suppose the order-up-to model is used to manage inventories. The firm is planning changes that will reduce the lead time to receive replenishments, but the firm is also planning on making its assortment more "fashionable," which essentially means that forecast uncertainty is likely to rise for all products. What can be said about the likely change in the firm's on-order inventory? Please justify your answer.

- a) It will surely decrease.
- b) It will remain the same.
- c) It will surely increase.
- d) More information is needed to determine the impact of these changes.

*Solution:*

*Answer a) is correct.*

*From Little's Law, on-order inventory = Lead time  $\times$  Expected demand in one period.*

*A change in forecast uncertainty influences the standard deviation of demand but does not influence either the length of the lead time or the expected demand in one period. Hence, these changes (reducing the lead time and making products more fashionable) will reduce the on-order inventory due to the reduction in the lead time (more fashionable products has no impact on the on-order inventory)*

Part d)

(8 points)

A firm manages its inventory with an order up-to model. Each period is one day, the lead time is 2 days, the base stock level is 10 and their inventory position at the start of a day (before they submit an order for that day) is -4. Which of the following is definitely true? Please justify your answer in depth.

- a) Demand was 4 units yesterday.
- b) Demand was 10 units yesterday.
- c) There are 4 units backordered.
- d) There are 14 units on-order before they order today.
- e) After today's order there will be 14 units on-order.
- f) They will receive more inventory today.
- g) None of the above.

*Answer = g: With an inventory position of -4, we know they have no on-hand inventory, and the sum of on-order minus backorders is -4. We don't know how many are on-order or backordered. Therefore, they have at least 4 units backordered, but they could have more, in which case the amount on-order would vary, ruling out c, d, and e. We don't know when those backorders occurred, so there are many possibilities for demand yesterday (ruling out a and b). We also don't know the how many were ordered 2 days ago, so we don't know how much will arrive today (ruling out f).*