Frankfurt School of Finance & Management
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Examination in the Bachelor of Science
Course title: Data Collection & Games and Incentives
Lecturers: Prof. Dr. Frederik Schwerter
Examination date: 14th December 2021

Aids: pocket calculator Casio FX-82 solar,
German-English Dictionary, English-English Dictionary

Please enter your student ID (matriculation number)!

Student ID

Please note:

The exam consists of 5 questions of which you will have to answer **all** questions. You have **120** minutes to complete the examination. The maximum of points to be reached is **120**. Please use the enclosed answer sheet to answer your questions and add your student ID on its cover.

We wish you all the best for your examination!

Internal use only!

Question	1	2	3	4	5	**Total**
Possible points:	24	30	34	12	20	**120**
Points achieved:						

Signature of corrector

Question 1 – General Equilibrium (24 points)

(a) What are the definitions of Pareto improvement and Pareto efficiency? (4 points)

A Pareto improvement is available if a change in the consumption bundle of all consumers implies a utility gain of at least one consumer and no utility loss for anyone. (2p)

Pareto efficient means that no pareto improvement is available. (2p)

Assume there are two consumers, A and B, and two goods, X und Y. The initial endowment is such that consumer A is endowed with $(X_A,Y_A)=(2,4)$ and consumer B is endowed with $(X_B,Y_B)=(4,2)$. Both consumers have standard preferences, and their utility functions are different and equal to

$$U_A(X_A,Y_A) = X_A^2\,Y_A \quad \text{and} \quad U_B(X_B,Y_B) = X_B Y_B^2$$

(b) Determine consumer A and B's utility at their initial endowment. (2 points)

In both cases, the utility is equal to "16" (one point for each one)

(c) Determine (mathematically) the slopes of the indifference curves for the two consumers. (4 points)

MRS_A = $-\frac{2Y_A}{X_A}$ und MRS_B = $-\frac{Y_B}{2X_B}$ (two points for each one)

(d) What is the definition of a competitive equilibrium? (3 points)

- Agents are price takers, 1 point
- Markets clear, 1 point
- Consumers max utility, 1 point

(e) Assume for this subquestion that the following prices were given:

$$p_X = 2 \quad and \quad p_Y = 1.$$

Given these prices and the consumers preferences, consumer A would maximize utility by consuming the bundle $(X_A = \frac{9}{2}, Y_A = \frac{9}{8})$ and consumer B would maximize utility by consuming the bundle $(X_B = \frac{10}{3}, Y_B = \frac{10}{3})$. Determine the excess supply and excess demand for the two goods. (3 Points)

Excess demand of X = (9/2 – 2) – (4 - 10/3) = 1.833 (1.5 points)
Excess supply of Y = (4-9/8) – (10/3 - 2) = 1.542 (1.5 points)

(f) Assume for this subquestion that the following prices were given:

$$p_X = 2 \quad and \quad p_Y = 1.$$

Given these prices and the consumers preferences, consumer A would maximize utility by consuming the bundle $(X_A = 4, Y_A = 2)$ and consumer B would maximize utility by consuming the bundle $(X_B = 2, Y_B = 4)$. Show that the given prices implement a competitive equilibrium. (3 Points)

For $(X_A = 4, Y_A = 2)$ and $(X_B = 2, Y_B = 4)$, we have that the MRS of A and B are the same and equal to the slope of the budget line (1.5 points):
MRS_A = $-\frac{2Y_A}{X_A} = -1$ (0.5 points)
MRS_B = $-\frac{Y_B}{2X_B} = -1$ (0.5 points)
$-\frac{p_B}{p_A} = -1$ (0.5 points)

(g) State the first welfare theorem as well as the three key assumptions behind the first welfare theorem. (5 points)

A competitive equilibrium is Pareto efficient (2 points)
Absence of market power (1 point)
No externalities (1 point)
Symmetric information (1 point)

Question 2 – Oligopoly (30 points)

Consider a market with two firms, firm 1 and firm 2, which produce a homogeneous product. Suppose that the firms simultaneously set their quantities (**Cournot competition**). The inverse demand function is $p(Q) = 20 - 2Q$, where $Q = q_1 + q_2$. The cost function of firm 1 is $C_1 = 4q_1$ and the cost function of firm 2 is $C_2 = 0$.

(a) Set up the profit functions of firm 1 and firm 2. Determine the first-order conditions of firm 1 and firm 2. (6 points)

Pi_1 = (20-2(q_1 + q_2))q_1 – 4q_1 (1.5p)

FOC: 20-4q_1-2q_2-4=0 (1.5p)

Pi_2 = (20-2(q_1 + q_2))q_2 (1.5p)

FOC: 20-4q_2-2q_1=0 (1.5p) ⇔

(b) Determine the quantities of both firms in the Nash equilibrium. (6 points)

(8-q_2)/2=q_1
(10-q_1)/2=q_2

(8-(10-q_1)/2)/2=q_1 ⇔ (8-5)/2 + q_1/4 = q_1 ⇔ 3/2 = (3/4)q_1

2=q_1 (3p)
q_2=4 (3p)

(c) Determine the profits of both firms in the Nash equilibrium. Explain why they differ. (4 points)

p=20-2*2-2*4=8

pi_1 = 8*2 – 4*2 = 8 (1.5p)
pi_2 = 8*4 = 32 (1.5p)

Firm 1 produces at higher costs and produces less. (1p)

(d) Assume for this subquestion that the production technology of firm 2 is available also to firm 1 as well as to two additional firms that entered the market. Consequently, all four firms produce at no cost, such that $C_1 = C_2 = C_3 = C_4 = 0$. The inverse demand function maintains to be $p(Q) = 20 - 2Q$, where we now have $Q = q_1 + q_2 + q_3 + q_4$. Determine the quantities of the four firms in the Nash equilibrium. (6 points)

Take any company, say 1: pi_1 = (20-2(q1+q2+q3+q4))q1; FOC: 20-4q1-2q2-2q3-2q4=0

Symmetry: q1=q2=q3=q4, so 20-4q1-2q2-2q3-2q4=0 ⇔ 20-10q1=0 ⇔ q1 = 2 = q2=q3=q4 (6p)

(e) Discuss verbally the differences between standard Cournot competition and standard Bertrand competition. (8 points)

Price vs quantity competition (2p for each)

In B: even with only two firms, outcomes are like in perfect competition (2p)

In C: firm profits decrease in the number of competitors, perfect competition only for infinitely many competitors (2p)

Question 3 – Tacit Collusion (34 points)

Consider a situation with two firms (i.e., firm 1 and firm 2), which produce a homogenous product at the same marginal costs c. Competition occurs in prices (**Bertrand competition**). The two firms share the demand of their product equally in case they choose the same price. The firm with the lowest price supplies the entire demand in case their prices differ.

Assume that the monopoly profit in the market is 1, $\pi^M = 2$. If both firms produce at marginal costs, $p_1 = p_2 = c$, the profit of each firm is 0, $\pi_1 = 0,\ \pi_2 = 0$. If one firm chooses to charge the monopoly price, while the other firm undercuts the monopoly price by as little as possible, the profit of firm with monopoly price is zero and the profit of the firm who

undercuts the monopoly price is equal to $x \leq 2$. If firm 2 charges monopoly price and firm 1 undercuts the monopoly price, then $\pi_1 = x \; and \; \pi_2 = 0$. If firm 1 charges monopoly price and firm 2 undercuts the monopoly price, then $\pi_1 = 0 \; and \; \pi_2 = x$.

Assume that both firms have the same discount factor δ between 0 and 1, $0 < \delta < 1$.

NOTE: there was a typo in the question text, the monopoly profit is 2, $\pi^M = 2$. Subquestions (e) and (f) depend on the precise size of the monopoly profit and hence answers that implicitly assume a profit equal 1 or equal to 2 will be evaluated to be correct.

(a) Suppose both firms compete in one period only. Determine the prices of both firms in Nash equilibrium. (4 points)

p=c for each firm (2p for each firm)

(b) Please discuss the meaning of the discount factor δ. Do smaller values of δ imply smaller or greater discounting of future periods? (2 points)

Smaller values of delta (ie smaller discount factors) imply more discounting (2p)

(c) Suppose the two firms compete over **a finite** number of periods that is greater than 100. In principle, could firms sustain collusion if their discount factors are large enough? If yes, (i) determine the strategy and (ii) state how it is called. If no, explain why not by (i) describing how firms behave instead and (ii) how they do so. (6 points)

No (2p)
Backwards induction (2p)
Because of backwards induction, firms never face a future of cooperation (collusion) and hence face no incentive to cooperate (collude) already now (1p), so they play Nash equ in each period (1p).

(d) Suppose the two firms compete over **an infinite** number of periods. Determine a reasonable strategy that allows the firms to sustain collusion, that is all firms set their prices equal to the monopoly price and share the monopoly profit, if their discount factors are large enough. How is such a strategy called? (6 points)

Yes (1p)
Grim trigger strategy (1p): cooperate/collude via monopoly price setting in the first period (1) and in every period afterwards if the competitor did so in the previous period (1); do not cooperate/collude once competitor did not cooperate/collude in the previous period (1) by setting price according to Nash eq (1).

(e) Suppose again the two firms compete over an infinite number of periods. For each of the following discount factors and values of x, state and determine whether collusion (via monopoly price setting) can be an equilibrium outcome or not (8 points):

Collusion can be SPE if $\frac{\pi^M}{2}\frac{1}{1-\delta} \geq x$. If $\pi^M = 2$, then $\frac{1}{1-\delta} \geq x$. If $\pi^M = 1$, then $\frac{1}{2(1-\delta)} \geq x$

i. $\delta = 0.00001, x = \frac{1}{5}$
yes, the inequalities hold (2p). x is too low to provoke deviation from collusion for any firm.

ii. $\delta < \frac{1}{3}, x = \frac{3}{2}$
no, the inequalities do not hold (2p). There is too much discounting in light of the high incentive to deviate.

iii. $\delta = \frac{2}{5}, x = \frac{7}{4}$
no (2p), there is too much discounting in light of the high incentive to deviate. Delta >=3/7 would have been required to sustain collusion

iv. $\delta = \frac{3}{5}, x = 2$
yes for $\pi^M = 2$ and no for $\pi^M = 1$ (2p), there is too little discounting to make deviation profitable.

(f) Now assume that 3 new firms enter the market. These new firms also produce at marginal costs c, but their discount factor is always equal to one, $\delta_3 = \delta_4 = \delta_5 = 1$. (i) all firms share demand equally if they all choose the same price; (ii) if four firms charge the monopoly price and one firm undercuts the monopoly price, the firms with monopoly prices make a profit of zero and the undercutting firm makes a profit of $x \leq 2$.
Return to the cases i., ii., iii., and iv. of subquestion 3d. Would the arrival of new firms in the market change your responses to whether collusion can be an equilibrium outcome? (8 points)

For $\pi^M = 2$:
still no collusion in (ii) and (iii), 2p each.
Still collusion in (i), 2p
No collusion in (iv), 2p

For $\pi^M = 1$:
still no collusion in (ii) and (iii), 2p each.
No collusion in (i) and (iv), 2p each.

Question 4 – Asymmetric Information (12 points)

Jane is responsible for the profits of all restaurants in region Z of the restaurant chain M. How high the profits of the restaurants in region Z are depends on how hard Jane works but also on many other factors she cannot influence and which cannot be observed by her or Kim, the owner of the restaurant chain. Kim wants to induce Jane to work hard but cannot (at reasonable cost) monitor Jane's effort level.

(a) Assume both Jane and Kim are **risk neutral**. Also assume that Kim wants to offer Jane a contract that induces Jane to provide high effort. Describe and explain verbally what the optimal contract would be. (4 points)

Franchise contract: Kim receives a fixed payment from Jane (1p), who gets all realized profits (2p). Jane is hence incentivized to put in high effort (1p)

(b) Assume Jane is **risk averse**, while Kim maintains to be **risk neutral**. Again assume that Kim wants to offer Jane a contract that induces Jane to provide high effort. (i) Describe and explain verbally what trade-off Kim faces in designing an optimal contract. (ii) Describe and explain verbally how the optimal contract will look like. (iii) Name the two restrictions/constraints that Kim would have to take care of when proposing a contract. (8 points)

Trade-off: Jane wants to be insured against small earnings -> flat wage (2p).
Kim wants high effort: Flat wage incentivizes Jane to shirk and put in too little effort (1p). A wage that increases in the profits of Jane's restaurants would incentive Jane to put in more effort, non-flat wage (1p). To overcome this: Kim will propose a bonus contract (1p) that entails some risk, but less than in the franchise contract (1p).
Participation constraint: Jane agrees to work for Kim (1)
Incentive compatibility constraint: Janes agrees to put in high effort (1)

Question 5 – Data Collection (20 points)

a) State the definition of a third factor. (6 points)

A third factor is correlated with the dependent variable (3p)
A third factor is correlated with the independent variable (3p)

Assume you collected data on 50k workers of a company. All of these workers are working in factories that are all located in region A. For each worker, you collected data on (i) whether or not the worker participated in on-the-job training a year ago and (ii) how productive the workers are in their job. The training program was intended to improve worker's productivity. In analyzing the effect of the training program on worker's productivity, the status of the training program is the independent variable, while worker's productivity is dependent variable.

b) Based on the above situation, explain the difference in questions pertaining to prediction and causality. (7 points)

A prediction question would be like: Assume we know the training status of workers in a different region. We could predict the productivity of workers from the other region who are trained and those who are not trained based on the group averages of region A. (3.5p)

A causality question would be like: assume we take a worker and increase her training, how much would the productivity of that worker increase? (3.5p)

c) Assume that you do not know for certain why or why not workers participated in the training program in region A. You observe, however, that workers in region A who

participated in the training program are significantly more productive than workers who did not participate in the training program. Assume your colleague Bob wants to convince your CEO to implement on-the-job training programs in all other regions in which your company has factories. You are not sure whether you agree with Bob. You argue that the training program would be costly. Bob argues that your collected data proves that the training program would make workers more productive and hence likely increases revenues as well. Would you agree with Bob's statement? Explain your response. (7 points)

Not necessarily (1p): the data may reveal only a correlation (2p) and not a causal relationship (2p): third factors may be at play (2p).