Examination in the Bachelor of Science
Course title: Markets, Incentives and Ethical Management
Part: Markets and Incentives
Semester: 2
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Group: 162 cohort
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Aids: pocket calculator Casio FX-82 solar

Please enter your student ID (matriculation number) and your group!

Student ID	Group

Please note:

The exam consists of 4 questions of which you will have to answer **all** questions. You have **90** minutes to complete the examination. The maximum of points to be reached is **90**. Please use the enclosed answer sheet to answer your questions and add your student ID on its cover.

We wish you all the best for your examination!

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Question	1	2	3	4	**Total**
Possible points:	34	14	30	12	**90**
Points achieved:					

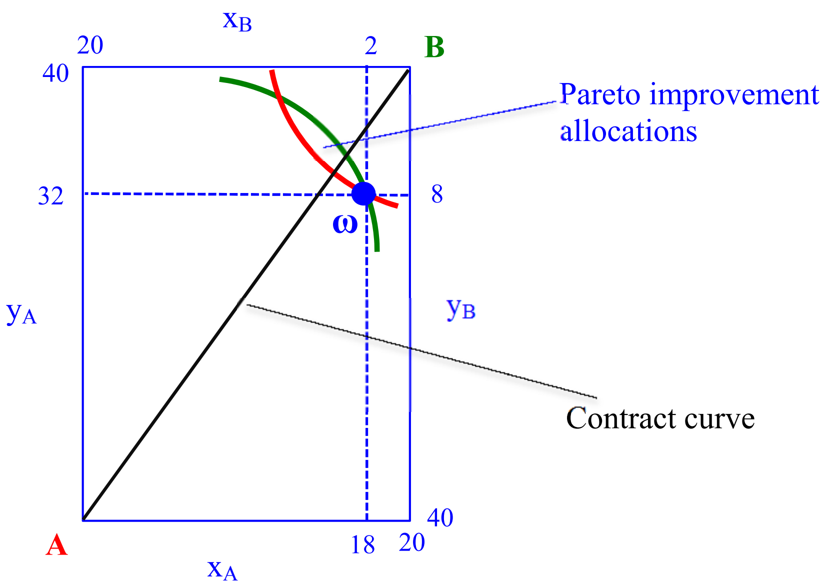
Signature of corrector

Question 1 – General Equilibrium (34 points)

Suppose that there are two consumers A and B and two products *x* and *y*. The initial endowment ω is such that consumer A is endowed with $(\omega_{X_A}, \omega_{Y_A}) = (18, 32)$ and consumer B is endowed with $(\omega_{X_B}, \omega_{Y_B}) = (2, 8)$. Both consumers have standard preferences and their utility functions are

$$U_A(x_A, y_A) = x_A^2 y_A^2 \quad \text{and} \quad U_B(x_B, y_B) = x_B^2 y_B^2.$$

(a) In the Edgeworth box, draw the consumers' indifference curves passing through the initial endowment and indicate the set of allocations that are Pareto improvements compared to the initial endowment. (6 points)



Allocation of points:
Edgeworth Box in general (1 point)
Correct notation of axes and numbers (1 point)
Initial endowment (1 point)
Indifference curves of the two consumers (2 points)
Pareto improvements (1 point)

(b) Determine the Pareto-efficiency condition. (3 points)

The Pareto-efficiency condition is $MRS_A = MRS_B$ (i.e., the marginal rate of substitution of consumer A equals the one of consumer B). (1 point)

$$MRS_A = -\frac{2x_A y_A^2}{2x_A^2 y_A} = -\frac{y_A}{x_A}$$ (1/2 points)

$$MRS_B = -\frac{2x_B y_B^2}{2x_B^2 y_B} = -\frac{y_B}{x_B}$$ (1/2 points)

Inserting these conditions in the Pareto-efficiency condition yields

$$\frac{y_A}{x_A} = \frac{y_B}{x_B}. \quad \text{(1 point)}$$

(c) Determine the equation of the contract curve and draw it in the Edgeworth box. (4 points)

Since $x_A + x_B = 20$ and $y_A + y_B = 40$, we have

$$x_B = 20 - x_A, \text{ (1 point)}$$

$$y_B = 40 - y_A \text{ (1 point)}.$$

Plugging these equalities into the Pareto-efficiency condition, yields the contract curve:

$$\frac{y_A}{x_A} = \frac{40 - y_A}{20 - x_A} \quad \text{(1 point)}$$

That is,

$$\mathbf{y_A = 2x_A} \quad \text{(1 point)}$$

(d) Calculate the demand functions of both consumers, assuming that the price of product y is the numeraire (i.e., $p_y = 1$). (8 points)

Consumer A:

At the Pareto-optimal allocation, the MRS_A equals the price ratio:

$$\frac{y_A}{x_A} = p_X \text{ (1 ½ points)}$$

In addition, the budget constraint must be satisfied:

$$x_A p_X + y_A = 18p_X + 32 \text{ (1 ½ points)}$$

Solving the first condition for y_A and inserting in the second gives

$$2x_A p_X = 18p_X + 32 \text{ (1/2 points)}$$

Solving for x_A yields

$$x_A = 9 + \frac{16}{p_X} \text{ (1/2 points)}$$

Inserting this into y_A from above gives

$$y_A = 9p_X + 16 \text{ (1 point)}$$

Consumer B:

Here, the two conditions are

$$\frac{y_B}{x_B} = p_X \text{ (1 point)}$$

and

$$x_B p_X + y_B = 2p_X + 8 \text{ (1 point)}$$

Proceeding as in case of consumer A yields

$$x_B = 1 + \frac{4}{p_X} \text{ (1/2 points)}$$

and

$$y_B = p_X + 4 \text{ (1/2 points)}$$

(e) State the definition of competitive equilibrium (3 points).

A competitive equilibrium is a set of prices that satisfies the following two conditions:

1. markets clear (i.e., there is neither excess demand nor excess supply of any good), (1 ½ points)
2. given these prices, each subject maximizes his utility subject to the budget constraint. (1 ½ points)

(f) Calculate the competitive equilibrium set of prices if $p_y = 1$. (3 points)

$$x_A + x_B = 20 \rightarrow 9 + \frac{16}{p_X} + 1 + \frac{4}{p_X} = 20 \quad \text{(1 1/2 points)}$$

$$=> 10\, p_X + 20 = 20\, p_X \quad \text{(1 point)}$$

$$=> \quad p_X = 2 \text{ (1/2 points)}$$

(g) Explain why product *x* is more expensive than product *y*. (2 points)

Product x is more expensive because it is more scarce in the economy than product y. (2 points)

(h) Calculate the competitive equilibrium quantities. (2 points)

The competitive equilibrium quantities can be found by inserting $p_X^*=2$ into the consumers' demand functions. This gives

$\mathbf{x_A}^* = 9 + \frac{16}{2} = 17$ (1/2 points)

$\mathbf{y_A}^* = 9 * 2 + 16 = 34$ (1/2 points)

$\mathbf{x_B}^* = 1 + \frac{4}{2} = 3$ (1/2 points)

$\mathbf{y_B}^* = 2 + 4 = 6$ (1/2 points)

(i) List three key assumptions that are needed for the First Welfare Theorem to hold. (3 points)

The First Welfare Theorem works only under the following three key assumptions:
1. markets are competitive, i.e., **absence of market power**, (1 point)
2. **no externalities**, (1 point)
3. symmetric information (1 point).

Question 2 – Game Theory (14 points)

Consider a second-price auction with two bidders. The valuations of the two bidders are v_1 and v_2, and are unknown to each other. Each bidder simultaneously makes a bid for the object, where the bids are denoted by b_1 and b_2, respectively. The auctioneer will give the object to the bidder with the highest bid. The winning bidder has to pay the bid of the losing bidder, the losing bidder gets nothing and pays nothing.

(a) Demonstrate why it cannot be optimal for bidder i=1,2, to submit a bid b_i, which is below the valuation v_i. (6 points)

If b_i is below v_i, three things can occur:
(i) b_{-i} is above v_i. Then a bid b_i below v_i leads to a profit of 0, but so does a bid equal to v_i. It follows that increasing b_i to v_i gives the same profit (2 points)
(ii) b_{-i} is below b_i. Then a bid b_i below v_i leads to a profit of $v_i – b_{-i}$, but so does a bid equal to v_i. It follows that increasing b_i to v_i gives the same profit (2 points)
(i) b_{-i} is between b_i and v_i. Then the bid b_i below v_i leads to a profit of 0, but increasing b_i to v_i leads to a strictly positive profit of $v_i – b_{-i}$. (2 points)

(b) Provide a brief intuition why no bidder has the incentive to lower her bid below the true valuation. (2 points)

The intuition is that if a bidder wins, her own bid does not influence the price she has to pay for the object. (1 point)
Therefore, by increasing the bid to the own valuation, a bidder does not need to pay more when winning. (1 point)

(c) Suppose the auction would not be played simultaneously but sequentially, that is, bidder 1 first submits her bid b_1 and afterwards bidder 2 submits her bid b_2. (Bidder 2 can observe the bid of bidder 1.) Would the outcome of the auction change compared to the simultaneous case? Explain your answer. (3 points)

The outcome would be unchanged. (1 point)
The reason is that whatever b_1 is, bidder 2 in the second stage has the incentive to bid her true valuation, by the same arguments as above (1 point).
Bidder 1 then has the same optimal strategy in the first stage as in the simultaneous game (1 point).

(d) In a first-price auction, the winning bidder needs to pay the bid she submitted, whereas all losing bidders still get nothing and pay nothing. Can it still be optimal in this case for a bidder to bid her true valuation? Explain your answer. (3 points)

It is not optimal to bid the true valuation (1 point).
The reason is that by doing so a bidder can never make a profit. If she wins, she has to pay her true valuation leading a profit of zero (1 point).
Lowering the bid below the true valuation always gives the bidder a positive probability to make a strictly positive profit (1 point).

Question 3 – Oligopoly and Tacit Collusion (30 points)

Consider a market with two firms, Firm 1 and Firm 2, which produce a homogeneous product. Suppose that the firms interact only once by setting their quantities simultaneously (**Cournot competition**). The inverse demand function is $p(Q) = 150 - Q$, where $Q = q_1+q_2$.
Each firm's marginal cost is 90.

(a) Determine the reactions functions of both firms. (4 points)

Firm 1's reaction function

$$\max_{q_1} \quad \pi_1 = (150 - q_1 - q_2)q_1 - 90q_1 \quad \text{(1 point)}$$

$$\frac{\partial \pi_1}{\partial q_1} = 150 - 2q_1 - q_2 - 90 = 0 \quad \text{(1 point)}$$

$$\boldsymbol{q_1 = 30 - \frac{1}{2}q_2} \quad \text{(1 point)}$$

By symmetry, Firm 2's reaction function is

$$\boldsymbol{q_2 = 30 - \frac{1}{2}q_1} \quad \text{(1 point)}$$

(b) Determine the Nash equilibrium quantities of both firms and the equilibrium profits. (4 points)

By symmetry we know that, in equilibrium, $q_1^*=q_2^*$. (1 point)

Substituting this condition into one of the reaction functions, we get

$$\mathbf{q_1^* = 30 - \frac{1}{2} q_1^*} \text{ (1 point)}$$

Solving for q_1^* yields

$$\mathbf{q_1^* = 20 = q_2^*} \quad \text{(1 point)}$$

$$p^* = 150 - 20 - 20 = 110$$

$$\boldsymbol{\pi_1}^* = \boldsymbol{\pi_2}^* = (110 - 90) * 20 = \mathbf{400} \quad \text{(1 point)}$$

Now, suppose that firms repeatedly interact for 4 periods and they simultaneously set their quantities in each period.

(c) Assuming that firms know that they will interact for exactly 4 periods, explain how to solve this finitely repeated game and why collusion cannot be sustained as a subgame perfect equilibrium. (5 points)

To determine the SPNE of this finitely repeated game we need to apply backward induction. (1 point)

We start from the 4th period. Since firms know that this is the last period in which they interact, they set their quantities to the one-shot Nash equilibrium level (i.e., q1*=q2*=20). (2 points)

Moving backward to the third period, firms anticipate their choices in the last period and so they will set again their quantities to the one-shot Nash equilibrium level. (1 point)

This argument carries on until the first period. Hence, because of the so-called "**last period effect**", there is a unique subgame perfect equilibrium in which the NE quantities are chosen in each period. (1 point)

Now, suppose that firms collude and jointly behave as a monopolist.

(d) Determine the monopoly quantity and the monopoly profit. Moreover, assuming that firms equally split the market, find the quantity produced by each firm and the corresponding profit. (3 points)

If firms collude, they behave as in a monopoly and solve the following problem:

$$\max_{p_M} \quad \pi_M = \ (p_M - c)\, Q_M = (150 - Q_M - 90)\, Q_M \ \text{(1 point)}$$

$$\frac{\partial \pi_M}{\partial Q_M} = 150 - \ 2Q_M - 90 = 0 \quad \rightarrow \quad \mathbf{Q_M} = \mathbf{30} \text{ (1/2 points)}$$

Thus, the monopoly profit is

$$\boldsymbol{\pi_M} = (150 - 30 - 90)\ 30 = \ \mathbf{900} \ \text{(1/2 points)}$$

If firms equally share the market, each of them produces

$$q_i^M = \frac{1}{2}\, Q_M = \ \frac{1}{2}\, 30 = \mathbf{15} \text{ (1/2 points)}$$

and earns half of the monopoly profit.

$$\boldsymbol{\pi_1} = \boldsymbol{\pi_2} = \ \frac{1}{2}\, \pi_M = \ \frac{1}{2}\, 900 = \mathbf{450} \text{ (1/2 points)}$$

Finally, assume that firms interact repeatedly for an infinite number of periods.

(e) Write down the grim trigger strategy that allows collusion to be sustained as a subgame perfect equilibrium by completing the following sentence. (2 points)

Produce _______________________ quantity in the first period. Keep producing _______________________ quantity as long as both firms have done so in all previous periods. If one firm has deviated from producing _______________________ quantity, produce the ___________________ quantity forever.

half of the monopoly (1/2 points); half of the monopoly (1/2 points); half of the monopoly (1/2 points); (Cournot) Nash equilibrium (1/2 points)

(f) Determine the optimal deviation quantity and the corresponding profit. (4 points)

A firm deviates by choosing $\mathbf{q^D}$ that satisfies his reaction function, given that the other firm sticks to produce half of the monopoly quantity (i.e., 15). (1 point)

Hence, using the firm's reaction function derived in subquestion (a) we get

$$\mathbf{q^D} = \mathbf{30} - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{15} = \mathbf{22.5} \quad \text{(1 1/2 points)}$$

The corresponding profit is

$$\boldsymbol{\pi^D} = (150 - 22.5 - 15 - 90) * 22.5 = \ \mathbf{506.25} \ \text{ (1 1/2 points)}$$

(g) Compute the discounted value of profits from collusion and the discounted value of profits from deviation. Determine the critical discount factor. (8 points)

The discounted value of profits from collusion is

$$\pi^{Collusion} = 450 \frac{1}{1-\beta} \quad \text{(2 points)}$$

The discounted value of profits from deviation is

$$\pi^{Deviation} = 506.25 + 400 \frac{\beta}{1-\beta} \quad \text{(2 points)}$$

The critical discount factor can be found by equating the aforementioned discounted values.

$$\pi^{Collusion} = \pi^{Deviation} \qquad \text{(2 points)}$$

$$450 \frac{1}{1-\beta} = 506.25 + 400 \frac{\beta}{1-\beta}$$

$$\boldsymbol{\beta = 0.53} \quad \text{(2 points)}$$

Question 4 – Asymmetric Information (12 points)

(a) Explain the difference between screening and moral hazard with respect to the point in time at which asymmetric information occurs. (3 points)

With screening, asymmetric information is present at the point in time when the two parties (principal and agent) sign the contract because the buyer knows her reservation price but the seller does not know the buyer's reservation price. (2 points)
Moral hazard occurs only after the two parties have signed the contract (1 point)

Consider the screening problem of a seller who faces a high-type buyer and a low-type buyer and can offer different contracts to them with respect to quality and price of the product. The high-type values quality more than the low type.

(b) Suppose the seller offers the first-best contracts (i.e., the contracts with symmetric information) to both buyer types. Explain why the high-type buyer has an incentive to accept the contract intended for the low type. (4 points)

The seller offers contracts to each type so that they do not receive any surplus. (1 point)
Since the high-type values quality more than the low type, his utility from the contract intended for the low type is higher than the utility for the low type. (1 point)
However, the seller can only charge a payment that equals the utility for the low type. (1 point)

This implies that the high type obtains a positive utility from the low-type contract (1 point).

(c) In a moral hazard problem, state the two constraints that an optimal contract needs to fulfill. Explain why each constraint must hold with equality at the optimal contract. (5 points)

Participation constraint and incentive-compatibility constraint (1 point)
If the participation constraint would not bind, the principal could reduce the wage, the agent would still participate, and the principal would obtain a higher profit (2 points).
If the incentive-compatibility constraint would not bind, the principal could lower the wage for the case in which the agent works hard (i.e., choses the correct action), the agent would still work hard, and the principal obtains a higher profit. (2 points)